APPLgrid news

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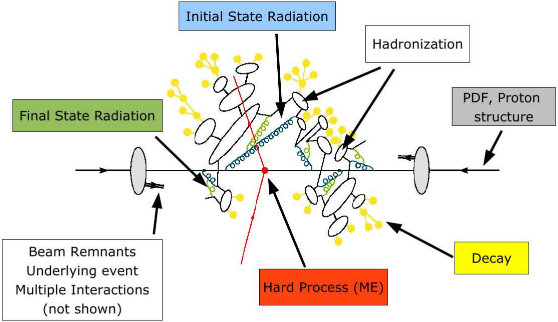
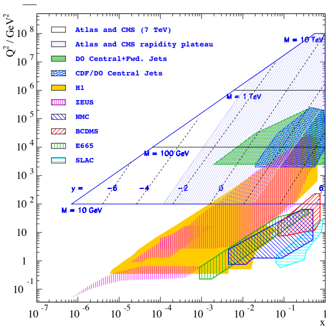
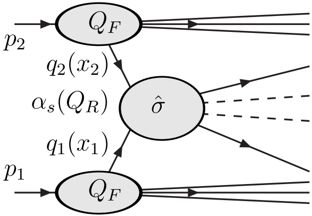
Proton-proton collision





- hard scattering can be calculated to NLO(NNLO) precision
- description of showers and non-perturbative effects comes from MC
- PDFs and strong coupling are determined from precision data (LEP, HERA, TEVATRON, . . .).

N^{x}LO QCD cross section



$$\frac{d\sigma}{dX} \sim \sum_{(i,j,p)} \int d\Gamma \alpha_s^p(Q_R^2)\, q_i(x_1, Q_F^2) q_j(x_2, Q_F^2)\, \frac{d\hat{\sigma}^{ij}_{(p)}}{dX}(x_1, x_2, Q_F^2, Q_R^2;\ S)$$

- Coupling and parton density functions are non-perturbative inputs to calculation (extracted from data)
- Perturbative coefficients are essentially independent from PDF functions due to factorization theorem

Calculating NLO cross-sections
takes a long time ($\sim$ *days*/*weeks*/*months*)

$\Longrightarrow$ we can split calculation into two parts

APPLGRID method

Eur.Phys.J.C66:503-524,2010.

- Step 1 (long run): Collect perturbative weights to grids .
  - binning
  - interpolation
  - initial flavours decomposition : $13 \times 13 \rightarrow \mathcal{L}$ ($\mathcal{L} \sim 10$)

$$\frac{d\hat{\sigma}^{ij}_{(p)}}{dX}(x_1, x_2, Q^2_F, Q^2_R;\ S) \xrightarrow{3D-grid} w^{(p)(l)}(x_1^m, x_2^n, Q^{2^k})\ (Q^2_R \equiv Q^2_F)$$

- Step 2 ($\sim$ 10–100 ms): Convolute grid with PDF's .
  - integral $\rightarrow$ sum
  - any coupling, PDF

Details of the method (I)

Interpolation

- user defined interpolation orders $n_y, \quad n_\tau$

$$f(x, Q^2) = \sum_{i=0}^{n_y} \sum_{\iota=0}^{n_\tau} f_{k+i,\kappa+\iota} \; I_i^{(n)} \left(\frac{y(x)}{\delta y} - k \right) \; I_\iota^{(n')} \left(\frac{\tau(Q^2)}{\delta \tau} - \kappa \right)$$

Subprocess decomposition

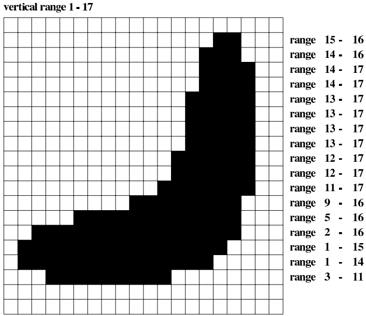
$13 \times 13 \to \mathcal{L}$ due to the symmetries of the ME weights

$$\sum_{m,n} \nu_{mn}^{(l)} \, f_{m/H_1} \left(x_1, Q^2 \right) f_{n/H_2} \left(x_2, Q^2 \right) \equiv F^{(l)} \left(x_1, x_2, Q^2 \right) ,$$

“generalised“ PDFs depend on the process and the perturbative order

Details of the method (II)

Phasespace optimisation



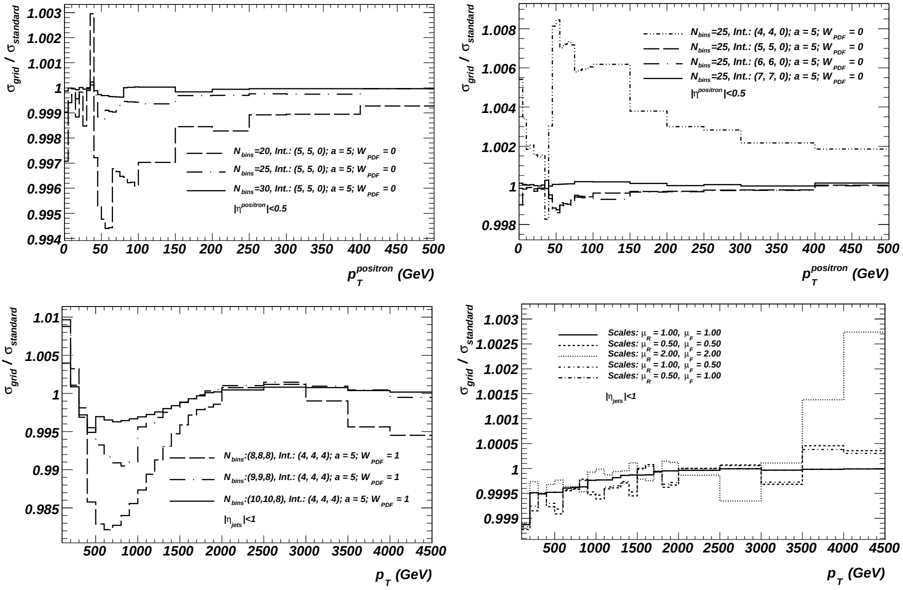
User just defines max/min possible values of x, Q^2. The optimisation procedure finds appropriate limits for each subprocess/order/observable bin.

$\longleftarrow$ $x_1 x_2$−phasespace

Final result

$$\frac{d\sigma}{dX} = \sum_p \sum_{l=0}^{L} \sum_{m,n,k} w_{m,n,k}^{(p)(l)} \left(\frac{\alpha_s(Q_k^2)}{2\pi} \right)^{p_l} F^{(l)}\left(x_{1\,m}, x_{2\,n}, Q_k^2\right)$$

APPLGRID accuracy.



Scale dependence

Having the weights $w_{m,n,k}^{(p)(l)}$ determined separately order by order in α_s, it is straightforward to vary the renormalisation μ_R and factorisation μ_F scales a posteriori.

We assume scales to be equal

$$\mu_R = \mu_F = Q$$

in the original calculation.

Introducing ξ_R and ξ_F corresponding to the factors by which one varies μ_R and μ_F respectively,

$$\mu_R = \xi_R \times Q$$
$$\mu_F = \xi_F \times Q$$

Master formula and Scale dependence at NLO

Then for arbitrary ξ_R and ξ_F we may write:

$$
\begin{aligned}
\frac{d\sigma}{dX}(\xi_R,\xi_F) = \sum_{l=0}^{L}\sum_{m}\sum_{n}\sum_{k}\Bigg\{ & \left(\frac{\alpha_s\left(\xi_R^2 Q^2{}_k\right)}{2\pi}\right)^{p_{\mathrm{LO}}} \\
& \times W_{m,n,k}^{(\mathrm{LO})(l)} F^{(l)}\left(x_{1\,m}, x_{1\,n}, \xi_F^2 Q^2{}_k\right) + \left(\frac{\alpha_s\left(\xi_R^2 Q^2{}_k\right)}{2\pi}\right)^{p_{\mathrm{NLO}}} \\
& \times \Bigg[\left(W_{m,n,k}^{(\mathrm{NLO})(l)} + 2\pi\beta_0 p_{\mathrm{LO}} \ln \xi_R^2\, W_{m,n,k}^{(\mathrm{LO})(l)}\right) F^{(l)}\left(x_{1\,m}, x_{2\,n}, \xi_F^2 Q^2{}_k\right) \\
& - \ln \xi_F^2\, W_{m,n,k}^{(\mathrm{LO})(l)} \\
& \qquad \times \left(F^{(l)}_{q_1\to P_0\otimes q_1}\left(x_{1\,m}, x_{2\,n}, \xi_F^2 Q^2{}_k\right) + F^{(l)}_{q_2\to P_0\otimes q_2}\left(x_{1\,m}, x_{2\,n}, \xi_F^2 Q^2{}_k\right)\right)\Bigg]\Bigg\},
\end{aligned}
$$

where $F^{(l)}_{q_1\to P_0\otimes q_1}$ is calculated as $F^{(l)}$, but with q_1 replaced with $P_0 \otimes q_1$ (LO splitting function convoluted with PDF), and analogously for $F^{(l)}_{q_2\to P_0\otimes q_2}$.

Master formula and Scale dependence at NNLO

$$\frac{d\sigma}{dX}(\xi_R, \xi_F) = \sum_{l=0}^{L}\sum_{m}\sum_{n}\sum_{k} F^{(l)}\left(x_{1\,m}, x_{2\,n}, \xi_F^2 {Q^2}_k\right) \times$$

$$\left\{ \left(\frac{\alpha_s\left(\xi_R^2 {Q^2}_k\right)}{2\pi}\right)^{p_{\mathrm{LO}}} W_{m,n,k}^{(\mathrm{LO})(l)} \right.$$

$$\left. + \left(\frac{\alpha_s\left(\xi_R^2 {Q^2}_k\right)}{2\pi}\right)^{p_{\mathrm{NLO}}} W_{m,n,k}^{(\mathrm{NLO})(l)} + \left(\frac{\alpha_s\left(\xi_R^2 {Q^2}_k\right)}{2\pi}\right)^{p_{\mathrm{NNLO}}} W_{m,n,k}^{(\mathrm{NNLO})(l)} \right\}$$

$$+ \sum_{l=0}^{L}\sum_{m}\sum_{n}\sum_{k} \left(\frac{\alpha_s\left(\xi_R^2 {Q^2}_k\right)}{2\pi}\right)^{p_{\mathrm{NNLO}}} \times \left\{ \text{Scale variations terms} \right\}$$

Scale dependence at NNLO

$L_R = \log \xi_R^2, \; L_F = \log \xi_F^2$

$$
\begin{aligned}
= \quad & L_R \left(p_{\mathrm{NLO}} \beta_0 W_{m,n,k}^{(\mathrm{NLO})(l)} + p_{\mathrm{LO}} \beta_1 W_{m,n,k}^{(\mathrm{LO})(l)} \right) F^{(l)} \\
+ \quad & L_R^2 p_{\mathrm{NLO}} \beta_0^2 W_{m,n,k}^{(\mathrm{LO})(l)} F^{(l)} \\
- \quad & L_F \left(W_{m,n,k}^{(\mathrm{NLO})(l)} \left[F^{(l)}_{q_1 \to P_0 \otimes q_1} + F^{(l)}_{q_2 \to P_0 \otimes q_2} \right] \right. \\
& \qquad \left. + W_{m,n,k}^{(\mathrm{LO})(l)} \left[F^{(l)}_{q_1 \to P_1 \otimes q_1} + F^{(l)}_{q_2 \to P_1 \otimes q_2} \right] \right) \\
+ \quad & L_F^2 W_{m,n,k}^{(\mathrm{LO})(l)} \left(F^{(l)}_{q_1 \to P_0 \otimes q_1 ; q_2 \to P_0 \otimes q_2} + \frac{1}{2} F^{(l)}_{q_1 \to P_0 \otimes P_0 \otimes q_1} + \frac{1}{2} F^{(l)}_{q_2 \to P_0 \otimes P_0 \otimes q_2} \right. \\
& \left. + \frac{1}{2} \beta_0 F^{(l)}_{q_1 \to P_0 \otimes q_1} + \frac{1}{2} \beta_0 F^{(l)}_{q_2 \to P_0 \otimes q_2} \right) \\
- \quad & L_R L_F p_{\mathrm{NLO}} \beta_0 W_{m,n,k}^{(\mathrm{LO})(l)} \left(F^{(l)}_{q_1 \to P_0 \otimes q_1} + F^{(l)}_{q_2 \to P_0 \otimes q_2} \right)
\end{aligned}
$$

Interface to cross section calculators

- NLOJET++ : Jet production in $pp(\bar{p})-$ and $ep-$ collisions.
  - $2 \to 2$ and $2 \to 3$ at NLO; $2 \to 4$ at LO www.desy.de/∼znagy/Site/NLOJet++.htm.
- MCFM : parton-level NLO(NNLO) QCD cross sections calculator for various femtobarn-level processes at hadron-hadron colliders.
  - V, $V + nJet$, $V + b\bar{b}$, VV, $Q\bar{Q}$,... $(\sim \mathcal{O}(300))$ mcfm.fnal.gov/
- SHERPA : Simulation of High-Energy Reactions of PArticles in lepton-lepton, lepton-photon, photon-photon, lepton-hadron and hadron-hadron collisions.
  - A huge amount of scattering processes sherpa.hepforge.org.
- aMC@NLO : A framework for the computation of hard events at the NLO or LO, to be subsequently showered (infrared-safe observables at the NLO or LO).
  - Matrix elements calculations from Madgraph 5 amcatnlo.web.cern.ch/amcatnlo/; madgraph.phys.ucl.ac.be/.
- DYNNLO : NNLO calculation of Drell-Yan processes at hadron colliders theory.fi.infn.it/grazzini/dy.html
- NNLOJET : NNLO calculation of vector boson, V+jet, Higgs, inclusive jet/dijet at hadron colliders

Current developments : MCFM v9.0

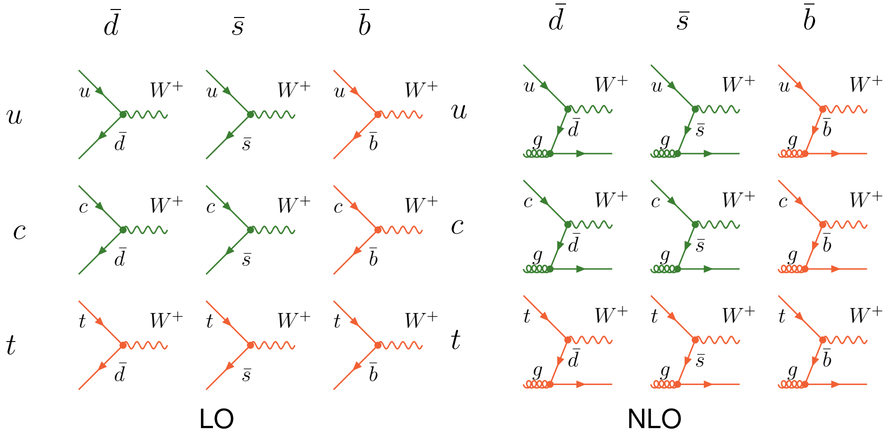
- MCFM group has released new version on 25.09.2019 [arXiv:1909.09117]
  - Newest version 10.1 (Jan 10th, 2022) [arXiv:2107.04472]
- Features: improved integration algorithm, better control of numerical accuracy, new code structure, new processes for off-shell SM and SMEFT single-top-quark production, high-pt treatment of mass effects in H+jet process,
- several NNLO color-singlet processes (V, H, VH)
- APPLgrid interface is off. The old one wouldn't work anyway, due to many changes in the MCFM code
- In contact with authors to re-enable the APPLgrid interface for the new versions
  - LO, REAL, VIRT contributions already implemented and validated
  - NNLO contribution is based on the SCET approach (interface is finalised)

Bridge to the MCFM code

- The mcmf-bridge user interface is re-written
  - dynamic library
  - use of cmake
  - based on the yaml steering file
  - allows to fully configure the observable w/o recompiling the package

```
---
##
## Grid 1
##
 - name: Wpt3
   directory: grid
   ## alphas power at LO
   power: 0
   ## number of loops
   nloops: 2
   genpdf: "mcfm-wp:mcfm-wp:mcfm-wp-nnlo"
   transform: f2
   ## nbins, interpolation order, minvalue, maxvalue
   xbinning:   [10, 5, 1e-9, 1]
   q2binning:  [5, 3, 100, 100000]
   ## observable bin boundaries
   observable: [ 10, 20, 30, 50, 100, 150, 200 ]
---
##
## Grid 1
##
 - name: Wpt1
   directory: grid
   ## alphas power at LO
   power: 0
   ## number of loops
   nloops: 2
   genpdf: mcfm-wp
   transform: f2
   ## nbins, interpolation order, minvalue, maxvalue
   xbinning:   [10, 5, 1e-9, 1]
   q2binning:  [5, 3, 100, 100000]
   ## observable bin boundaries
   observable: [ 10, 20, 30, 50, 100, 150, 200 ]
---
```

APPLGRID subprocesses for $W^{\pm}$ production (I)



APPLGRID subprocesses for $W^{\pm}$ production (II)

The weights for W^{+}-production can be organized in six possible initial state combinations (calculated using MCFM)

$$\bar{D}U: \quad F^{(0)}\left(x_1, x_2, Q^2\right) = \sum_{j=1,3,5} f_{-j/H_1}\left(x_1\right) \sum_{i=2,4,6} f_{i/H_2}\left(x_2\right) V_{ij}^2$$

$$U\bar{D}: \quad F^{(1)}\left(x_1, x_2, Q^2\right) = \sum_{i=2,4,6} f_{i/H_1}\left(x_1\right) \sum_{j=1,3,5} f_{-j/H_2}\left(x_2\right) V_{ij}^2$$

$$\bar{D}g: \quad F^{(2)}\left(x_1, x_2, Q^2\right) = \sum_{i=1,3,5} f_{-i/H_1}\left(x_1\right) \left(V_{iu}^2 + V_{ic}^2 + V_{it}^2\right) f_{0/H_2}\left(x_2\right)$$

$$Ug: \quad F^{(3)}\left(x_1, x_2, Q^2\right) = \sum_{i=2,4,6} f_{i/H_1}\left(x_1\right) \left(V_{id}^2 + V_{is}^2 + V_{ib}^2\right) f_{0/H_2}\left(x_2\right)$$

$$g\bar{D}: \quad F^{(4)}\left(x_1, x_2, Q^2\right) = \sum_{i=1,3,5} f_{0/H_1}\left(x_1\right) \left(V_{iu}^2 + V_{ic}^2 + V_{it}^2\right) f_{-i/H_2}\left(x_2\right)$$

$$gU: \quad F^{(5)}\left(x_1, x_2, Q^2\right) = \sum_{i=2,4,6} f_{0/H_1}\left(x_1\right) \left(V_{id}^2 + V_{is}^2 + V_{ib}^2\right) f_{i/H_2}\left(x_2\right)$$

We separate $u\bar{d}$ from $\bar{d}u$ in order to get the right rapidity distribution for the electron, because of the chiral nature of the $W^{\pm}$ couplings

NNLO contribution

the rest is specifically for the W-production, but it is the *"same"* for other processes

$$d\sigma^{\mathrm{NNLO}} = \alpha_s^2 \sum_{i,j} V_{\mathrm{CKM}}^{i,j} \times \mathrm{weight}^{i,j} \times \text{kinematic factor}$$

NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NaN	NaN	NaN	NaN	NaN	NaN	NaN	0.4853E+00	NaN	0.4853E+00	NaN
NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NaN	NaN	NaN	NaN	NaN	NaN	NaN	0.4853E+00	NaN	0.4853E+00	NaN
NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NaN	NaN	0.4483E+01	NaN	0.4483E+01	NaN	NaN	NaN	NaN	NaN	NaN
NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
NaN	NaN	0.4483E+01	NaN	0.4483E+01	NaN	NaN	NaN	NaN	NaN	NaN
NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN

$\mathrm{weight}^{i,j}$ is constructed from two beam functions, for each incoming proton, and soft and hard matrix elements

Beam functions

Parton momentum fraction :

born event x_0, after emission $x = x_0/z$

- Zero order beam function
  - $q, g : f_{q,g}(x_0)$
- First order beam function
  - $g : f_g(x_0),\ f_g(x),\ \sum_{q,\bar{q}} f(x)$
  - $q : f_q(x_0),\ f_q(x),\ f_g(x)$
- Second order beam function
  - $g : f_g(x_0),\ f_g(x),\ \sum_{q,\bar{q}} f(x)$
  - $q : f_q(x_0),\ f_q(x),\ f_g(x),\ \sum_{q} f(x)\ \sum_{\bar{q}} f(x),\ f_{\bar{q}}(x)$

Sub-processes at NNLO

process	$x_{1,0}\,;\,x_{2,0}$	$x_1\,;\,x_{2,0}$	$x_{1,0}\,;\,x_2$	$x_1\,;\,x_2$
$u\bar{d}$	+	+	+	+
ug			+	+
$g\bar{d}$		+		+
gg				+
$\sum_q f_q \bar{d}$		+		
$\sum_{\bar{q}} f_{\bar{q}} \bar{d}$		+		
$\bar{u}\bar{d}$		+		
$u \sum_q f_q$			+	
$u \sum_{\bar{q}} f_{\bar{q}}$			+	
ud			+	

SCET interface status

- Beam functions order-0,1,2 are implemeted
- Sub-process decomposition (depends on the partonic sub-processes at the LO)
  - $W^+ \Rightarrow$ done
  - $W^- \Rightarrow$ in progress
  - $Z^0 \Rightarrow$ next step
- Next steps
  - Multi-thread calculations (default for MCFM, turned off for the mcfm-bridge now)
  - Use non-ROOT version of the APPLgrid

Summary

Precision measurements of QCD improve knowledge of PDFs and strong coupling constant and might facilitate discoveries at the LHC.

- APPLgrid is an open project, complete source code is available as HEPforge package
- A posteriori evaluation of uncertainties from renormalisation and factorisation scale variations, strong coupling measurement and PDFs error sets in a very short time
- Allows rigorous inclusion of collider data in a PDF/strong coupling fit.
- Other functionality, such as $\sqrt{S}$ rescaling, change of initial hadrons
- A list of QCD and electroweak processes can be studied
  - Jet production studied using NLOJET++ , NNLOJET
  - Drell-Yan production via NNLOJET, MCFM, DYNNLO
  - Many other processes via MCFM , SHERPA, aMC@NLO, ContactInteractions
    - W/Z/γ; W/Zγ+jet; $t\bar{t}$, $b\bar{b}$, $c\bar{c}$; W+charm, any process with a basic decomposition