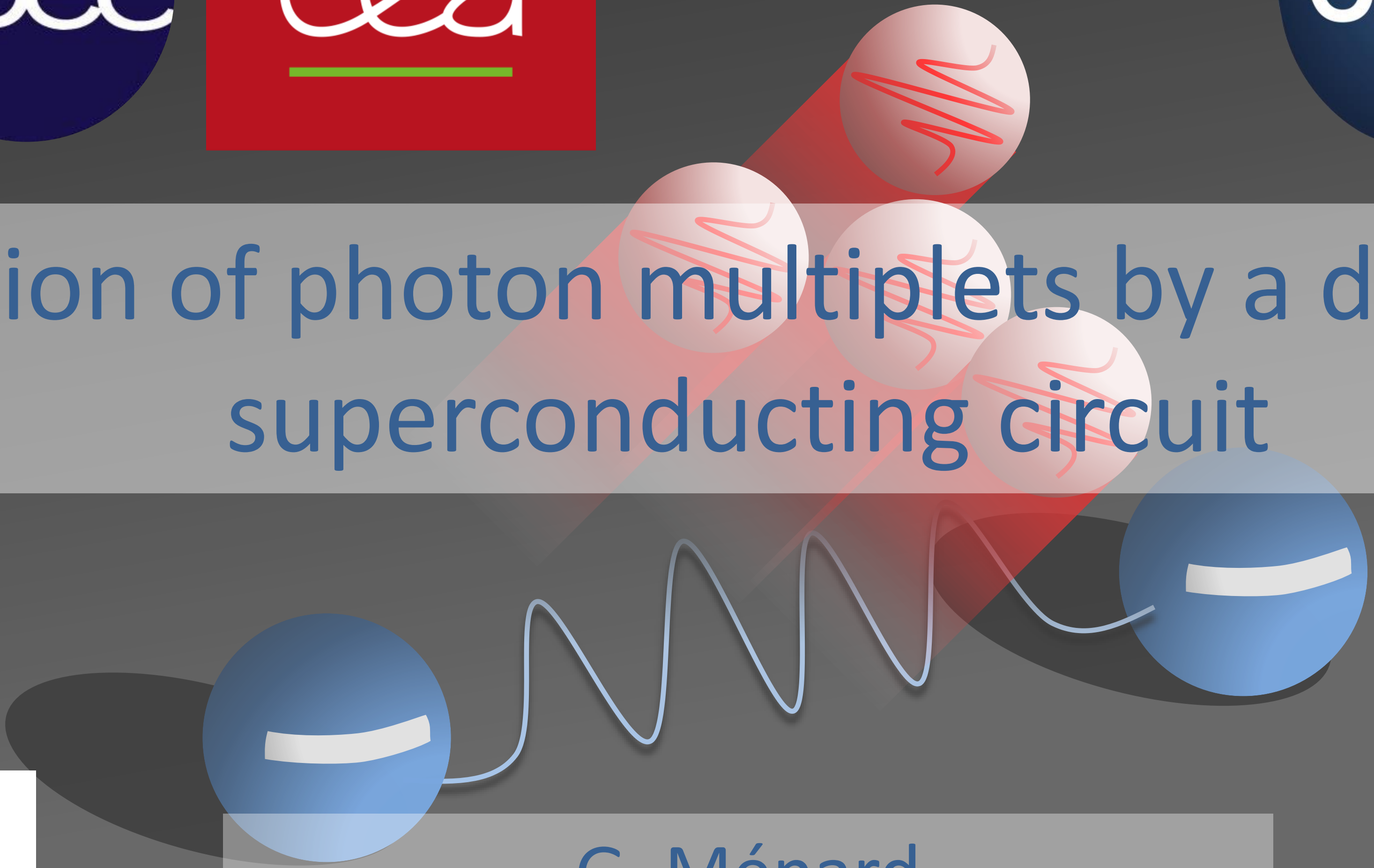
Emission of photon multiplets by a dc-biased superconducting circuit



G. Ménard

150 ans SFP 06/07/2023

Société Française de Physique | 150 ANS D'ENGAGEMENT POUR LA PHYSIQUE

GNE

A. Peugeot

F. Portier

I. Mukharsky

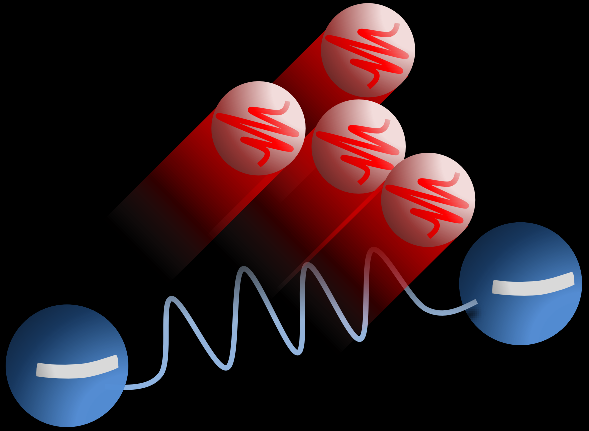
C. Altimiras

Quantronique

D. Vion

D. Estève

C. Rolland
Z. Iftikhar
P. Roche
H. Le Sueur
P. Joyez

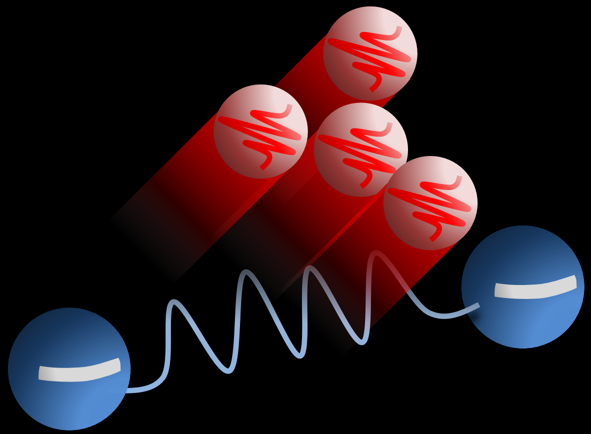


IQST Ulm university (theory)

C. Padurariu

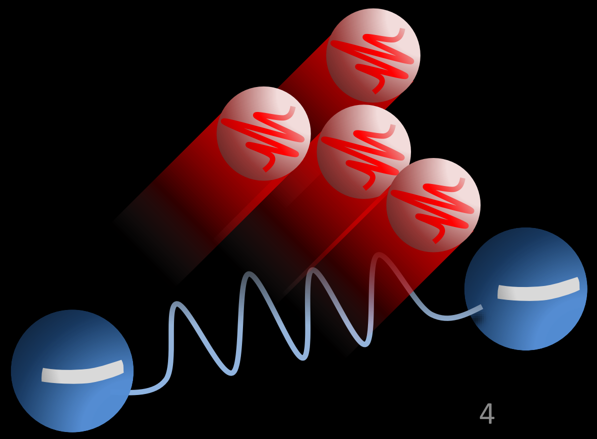
B. Kubala

J. Ankerhold

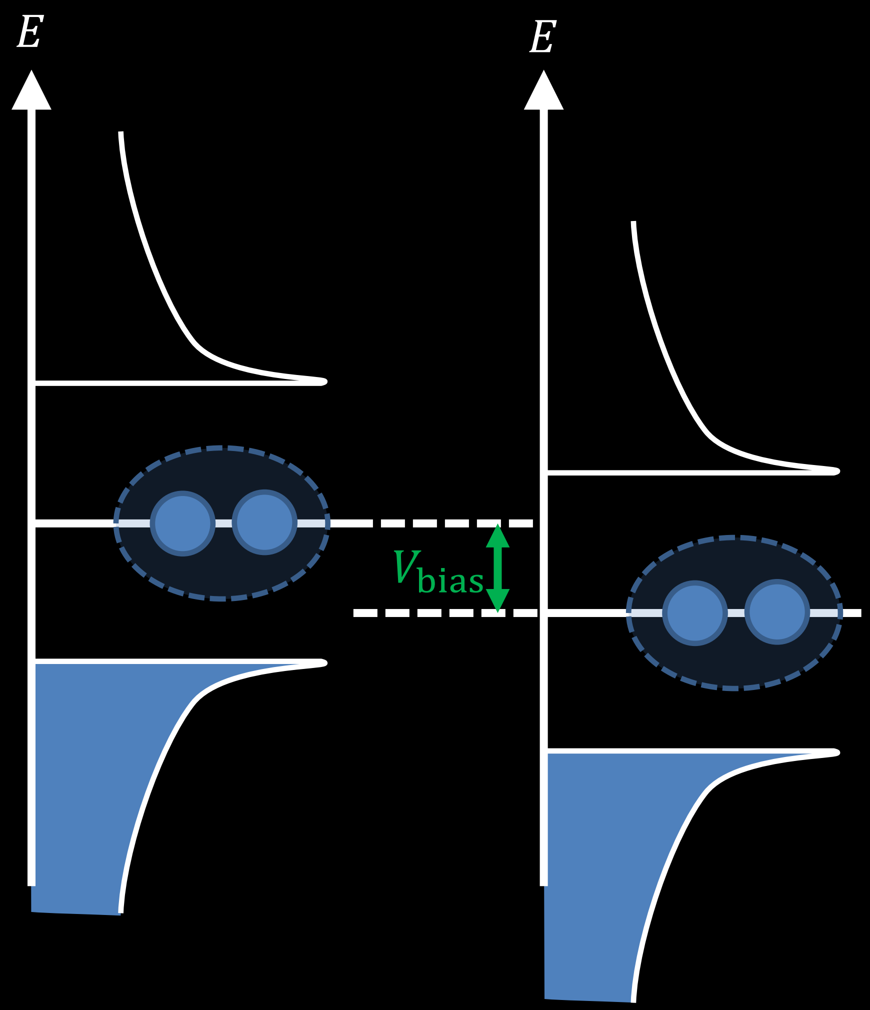


Outline

- Introduction, photoemission
- Experimental setup
- Photoemission in cQED
- Fano factor
- Conclusion

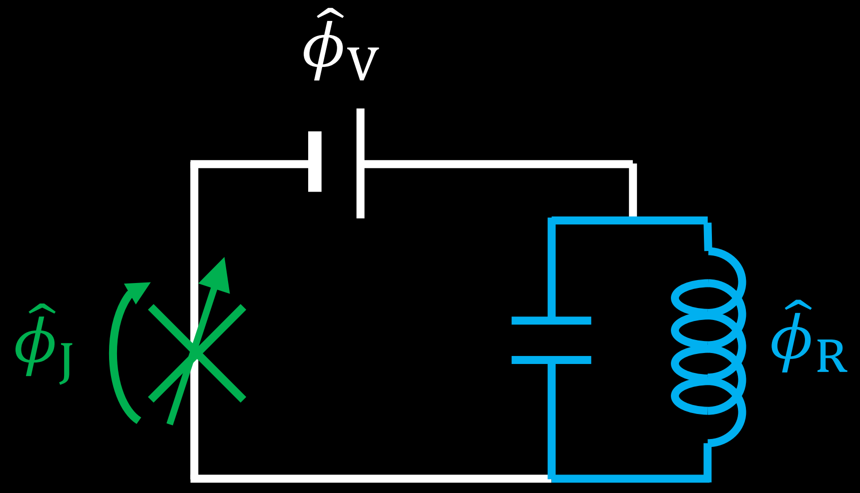


Normal metal comparison (T=0)



Josephson junction with bias voltage V_{bias}

Josephson effect



Josephson junction with bias voltage V_{bias}

Resonator in circuit

Josephson Hamiltonian

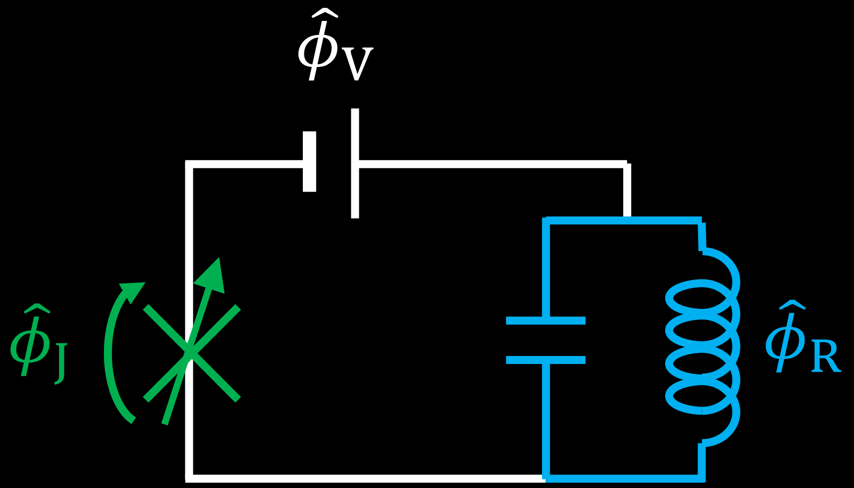
$$\hat{H}_J = -E_J \cos \hat{\phi}_{\mathrm{J}}$$

Phase

$$\hat{\phi}_{\mathrm{V}} = \omega_J t = \hat{\phi}_{\mathrm{J}} + \hat{\phi}_R$$

$$\hat{\phi}_{\mathrm{R}} = \sqrt{\alpha}\left(\hat{a}^\dagger + \hat{a}\right)$$

Inelastic tunneling of Cooper pair



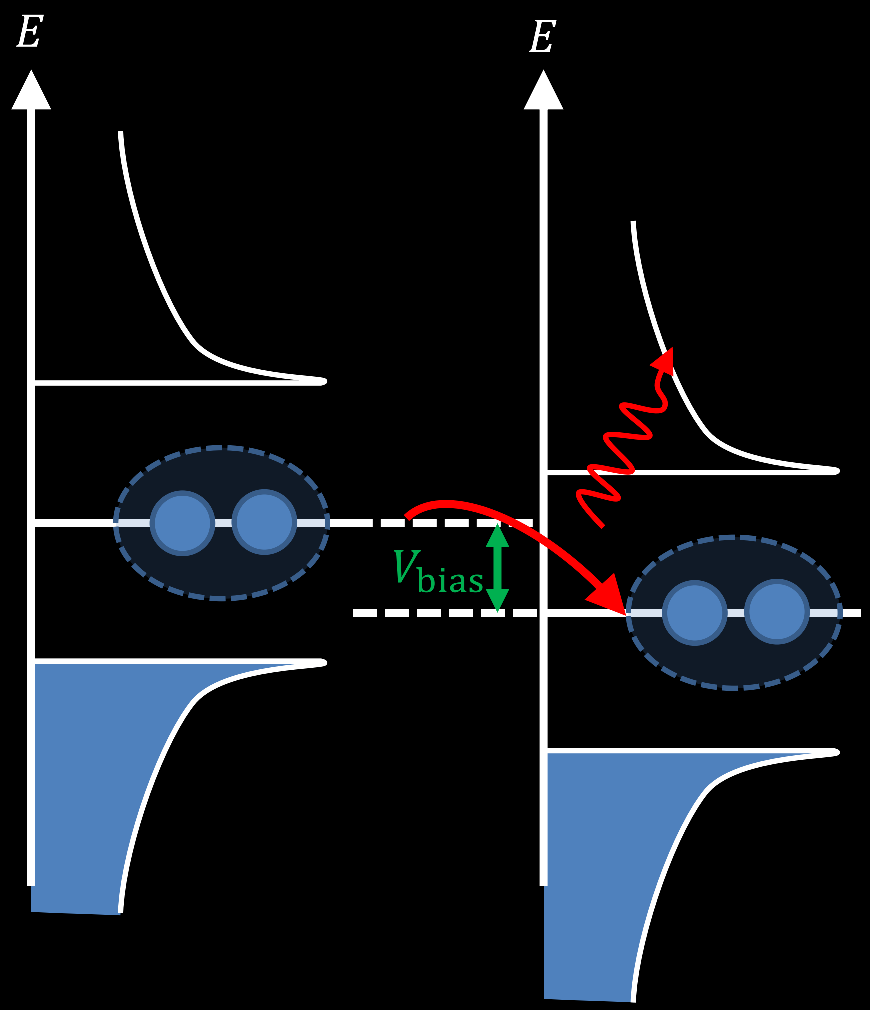
Coupling Josephson junction & microwave resonator

$$\hat{H} = h\nu_R \hat{a}^\dagger \hat{a} - E_J \cos\left(2\pi\nu_J t - \sqrt{\alpha}\left(\hat{a}^\dagger + \hat{a}\right)\right)$$

Resonator

Josephson junction

Inelastic tunneling of Cooper pair



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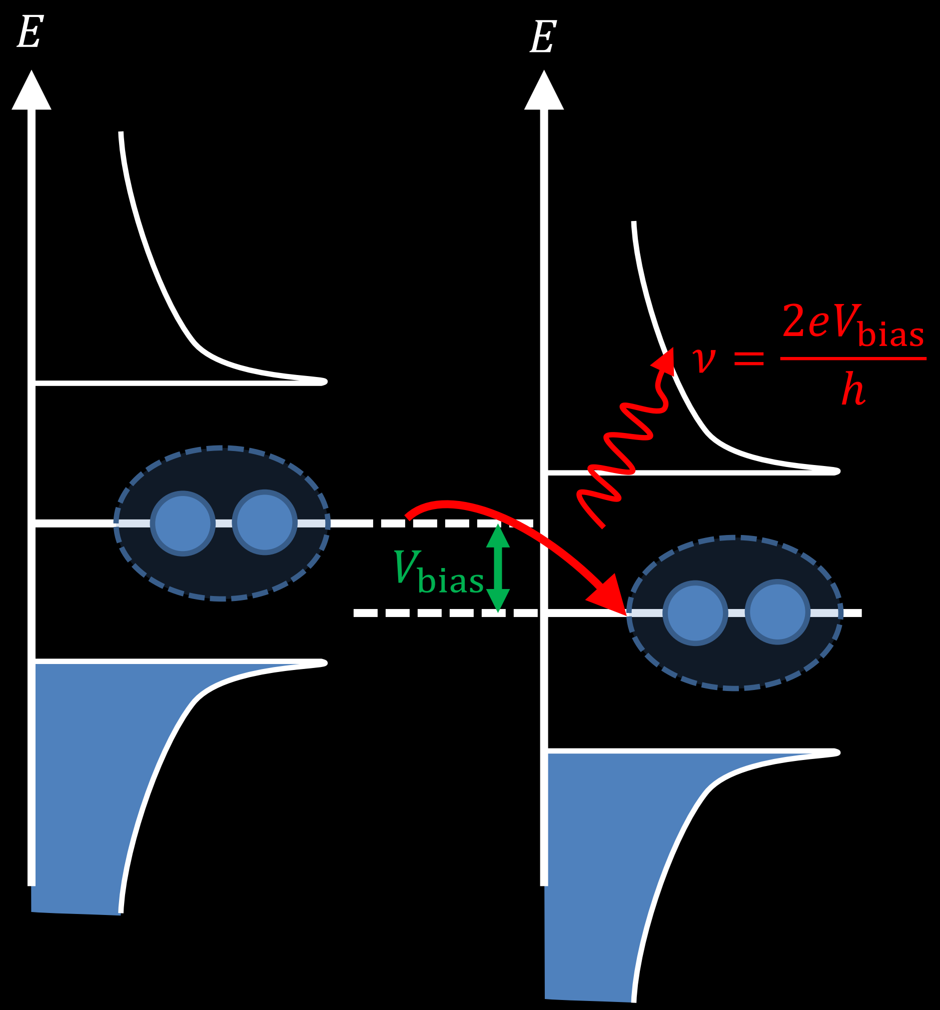


Coupling Josephson junction & microwave resonator

$$\hat{H} = h\nu_R \hat{a}^\dagger \hat{a} - E_J \cos\left(2\pi\nu_J t - \sqrt{\alpha}(\hat{a}^\dagger + \hat{a})\right)$$

Josephson frequency:

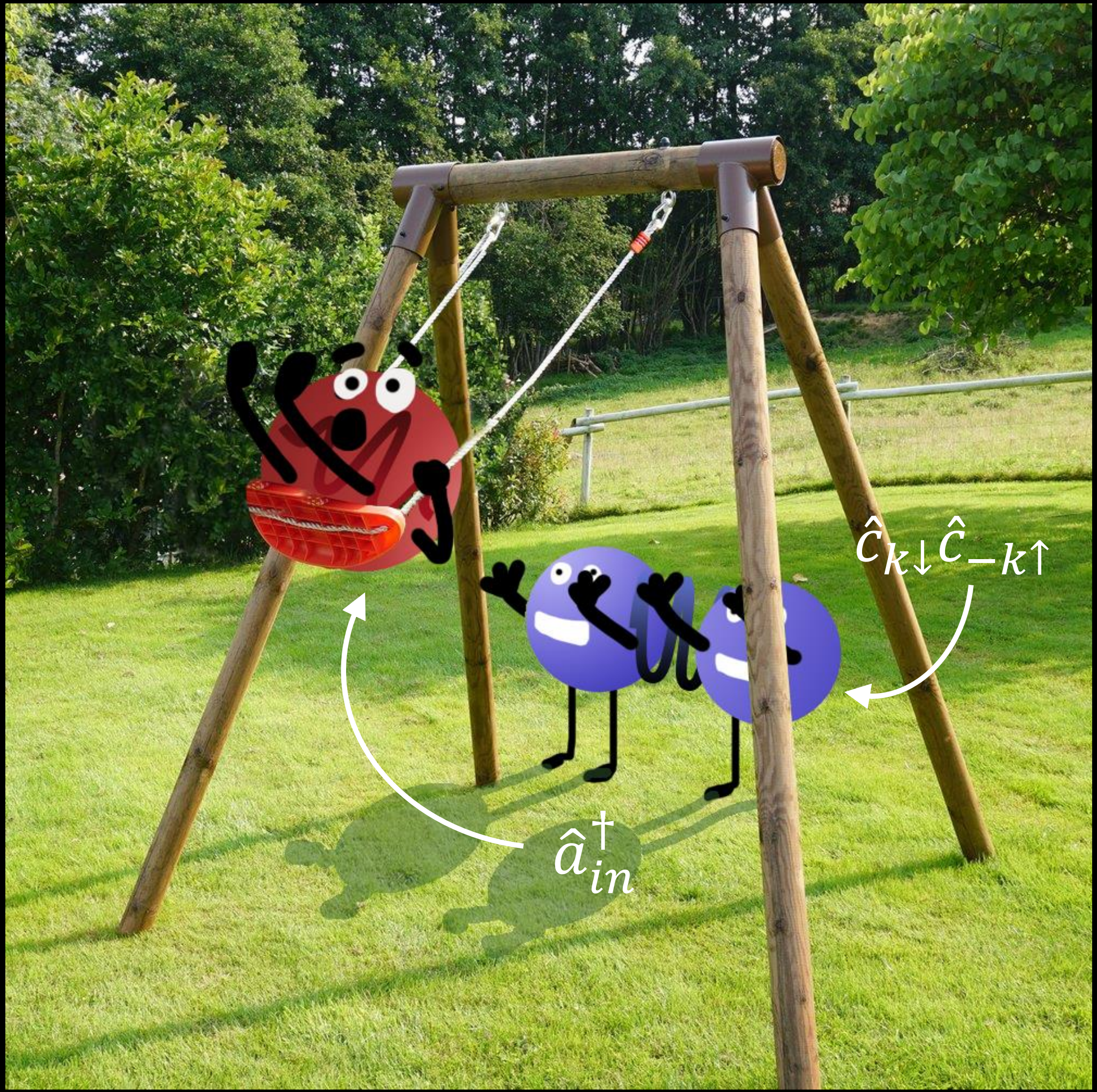
$$\nu_J = \frac{2eV}{h}$$

$$\Gamma_{rad} \propto \alpha = \frac{4\pi Z_R}{R_k}$$

$$\left(R_k = \frac{h}{e^2}\right)$$

α can be tuned by properly designing the circuit

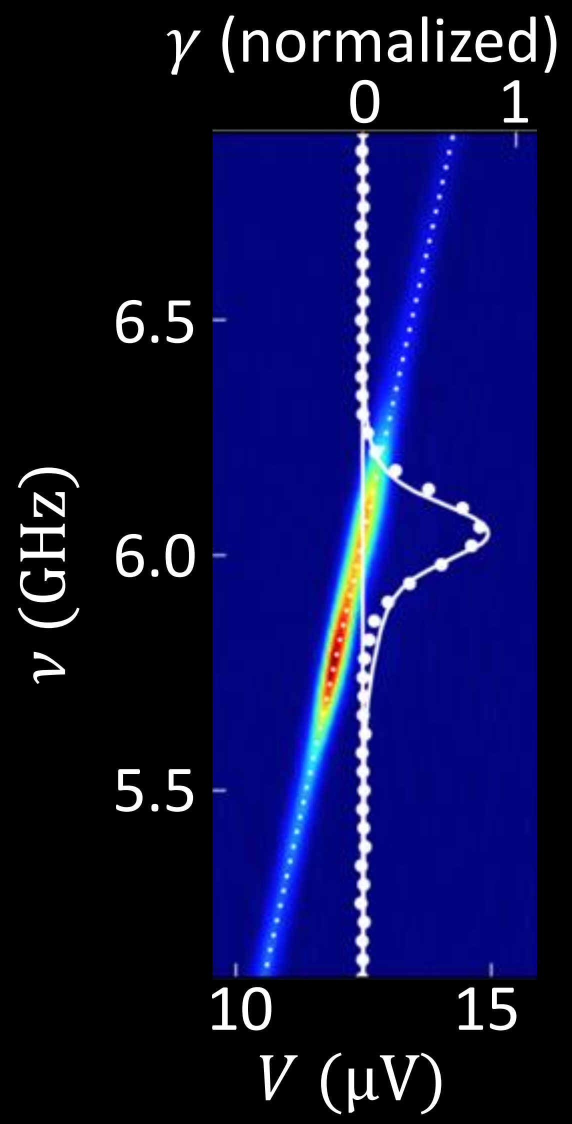
Inelastic tunneling of Cooper pair



Visual representation

(not to scale)

Inelastic tunneling of Cooper pair

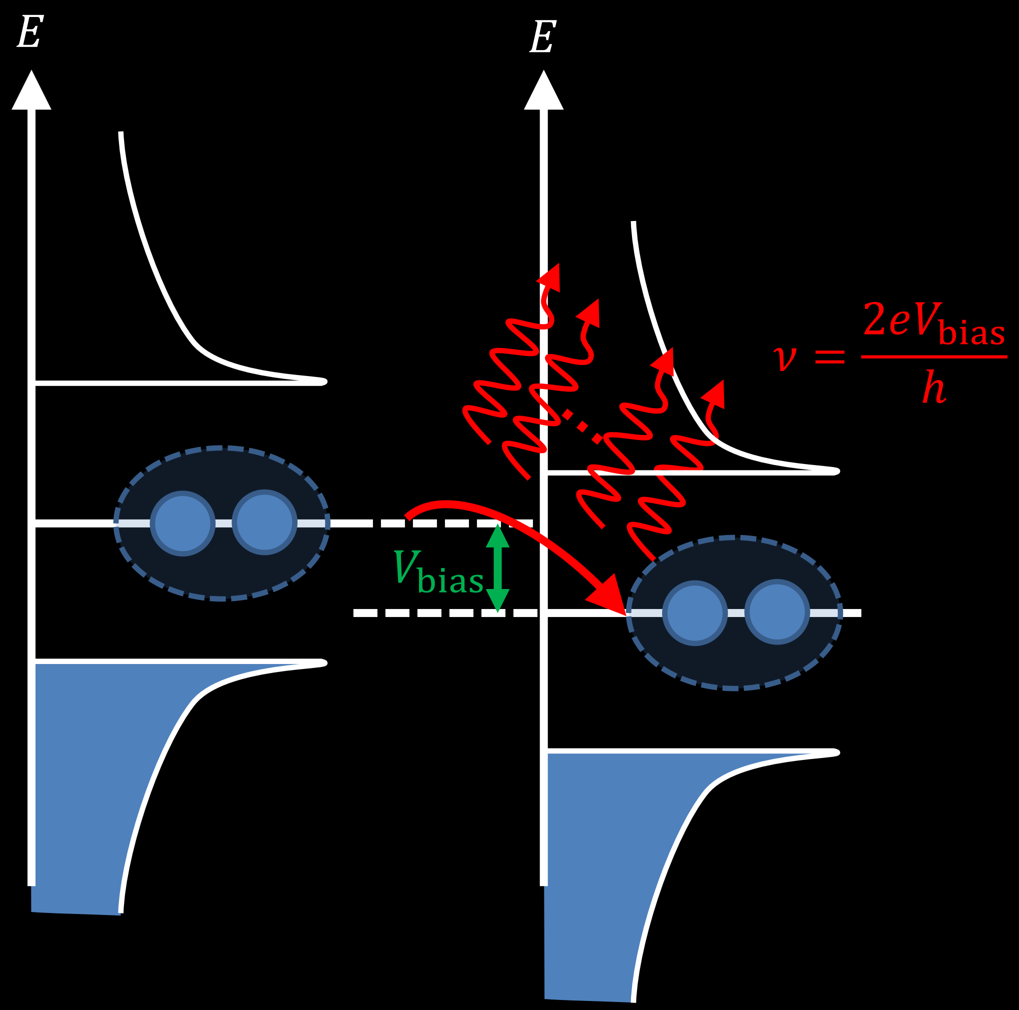


- DC polarization at voltage V
- Recording Power spectrum density (PSD) in finite frequency band (5-7 GHz)
- Resonator centered at 6,1 GHz

Main emission line follows:
$$2eV = h\nu$$

M. Hofheinz et al. Phys. Rev. Lett. **106**, 217005 (2011)

Inelastic tunneling of Cooper pair



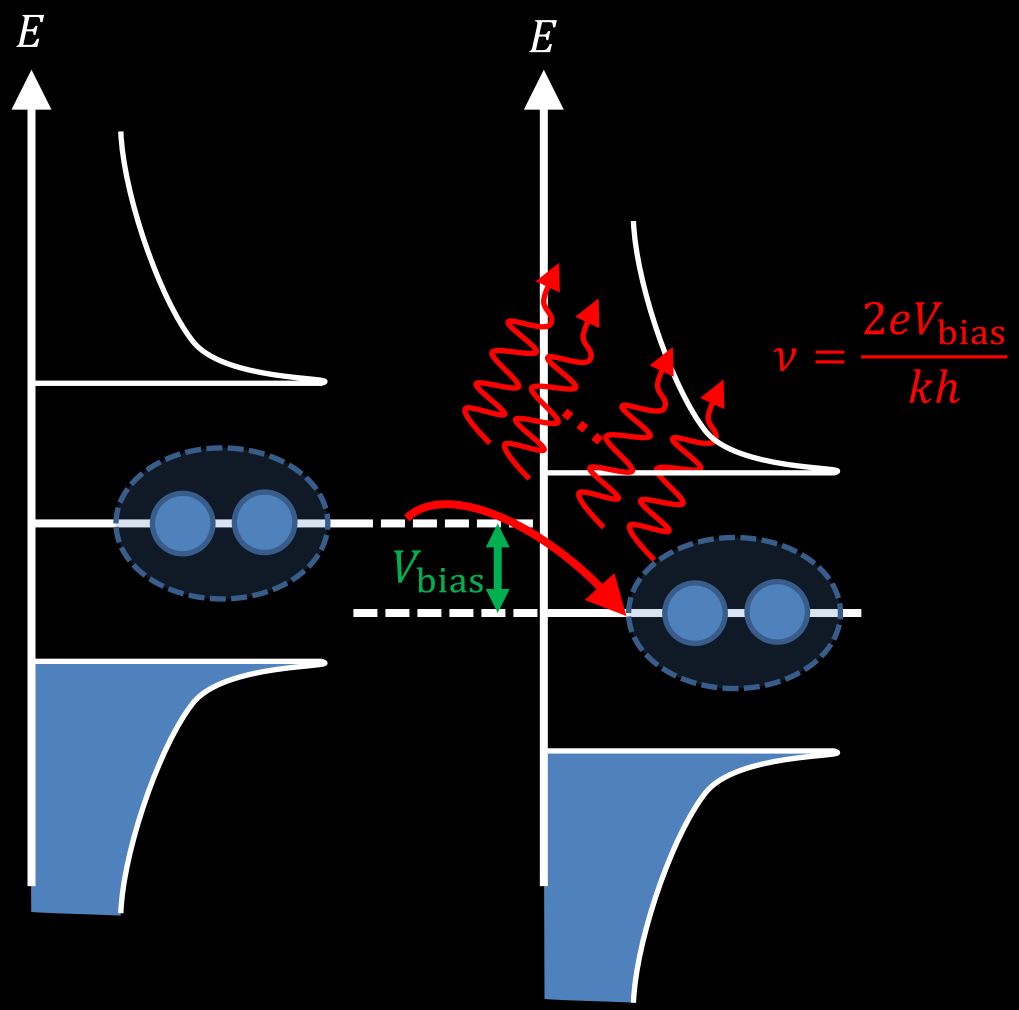
Emission occurs at

$$\nu_J = \frac{2eV}{kh}$$

With a rate

$$\Gamma_{rad} \simeq \alpha^k$$

Inelastic tunneling of Cooper pair



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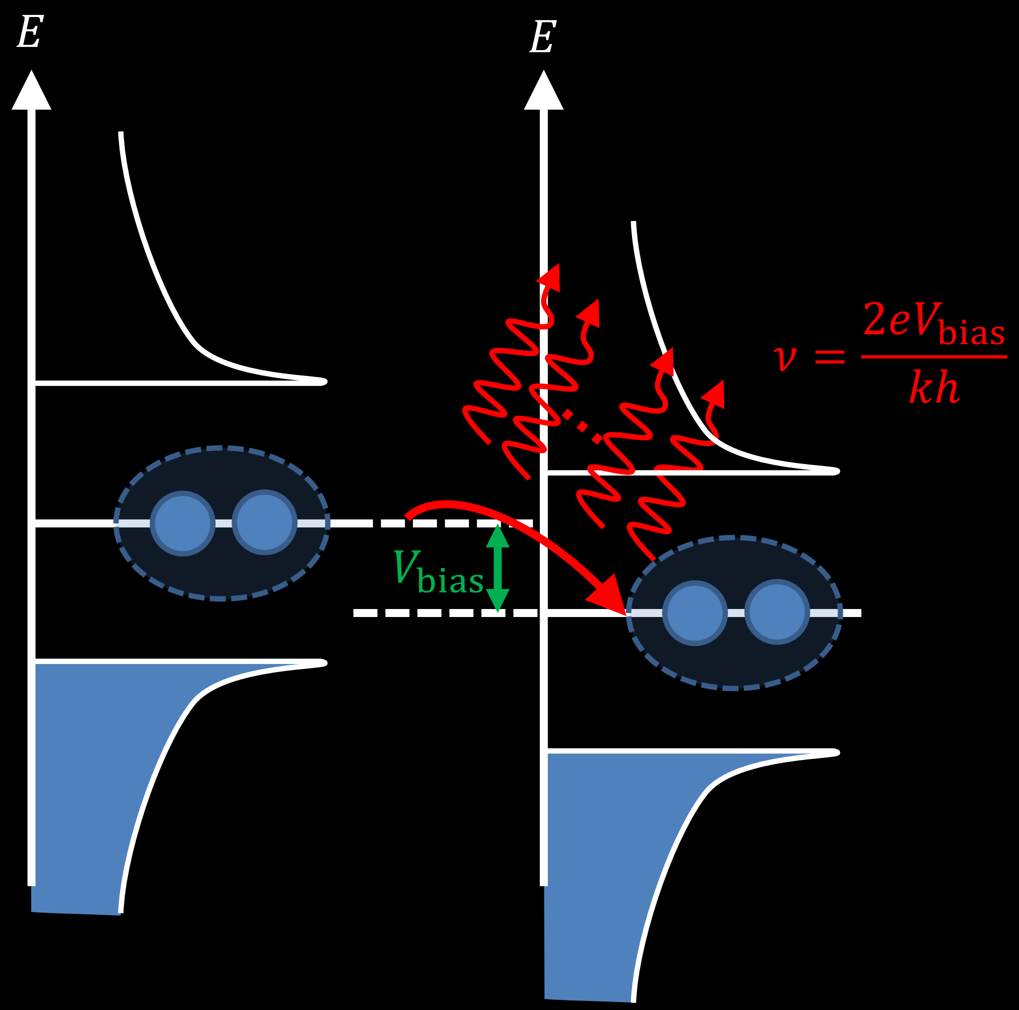
$$\hat{H} = h\nu_R \hat{a}^\dagger \hat{a} - E_J \cos(\hat{\phi}_J)$$

$$\hat{\phi}_J = \hat{\phi}_V - \hat{\phi}_R$$

Voltage source: $\hat{\phi}_V = \nu_J t$

Resonator: $\hat{\phi}_R = \sqrt{\alpha}(\hat{a}^\dagger + \hat{a})$

Inelastic tunneling of Cooper pair



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Voltage source: $\hat{\phi}_V = \nu_J t$

Resonator: $\hat{\phi}_R = \sqrt{\alpha}(\hat{a}^\dagger + \hat{a})$

Large α: no perturbative treatment

Inelastic tunneling of Cooper pair

$$\hat{H}_J \approx e^{i\hat{\phi}} + e^{-i\hat{\phi}}, \qquad \hat{\phi}_R = \sqrt{\alpha}\left(\hat{a}^\dagger + \hat{a}\right)$$

Rotating wave approximation: $e^{\pm i\hat{\phi}} \rightarrow \hat{a}^{(\dagger)^k}$

Inelastic tunneling of Cooper pair

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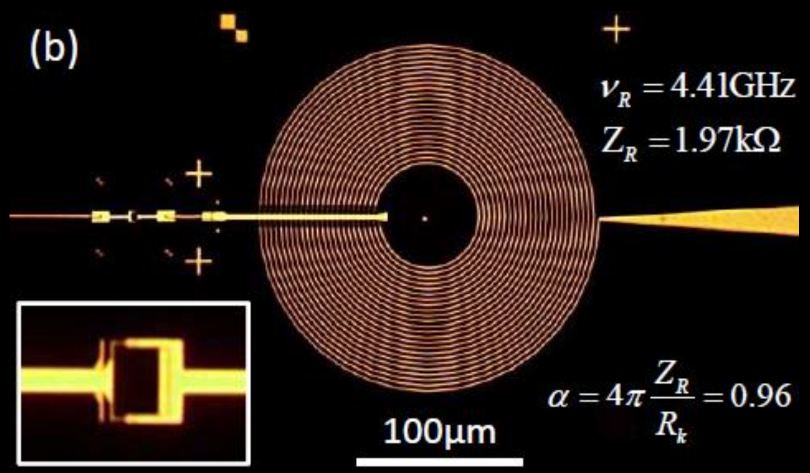
Rotating wave approximation: $e^{\pm i\hat{\phi}} \rightarrow \hat{a}^{(\dagger)^k}$

$$\hat{H}_k = -\frac{E_J e^{-\frac{\alpha}{2}}}{2}\,\alpha^{\frac{k}{2}}\left[e^{-i\delta t}\hat{B}_k\left(i\hat{a}^\dagger\right)^k + \mathrm{h.c.}\right]$$

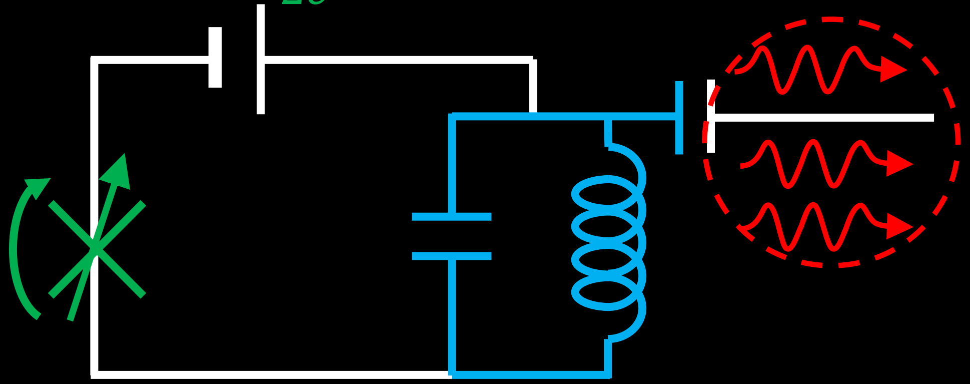
($\hat{B}_k$: diagonal in Fock basis involving generalized Laguerre polynomials)

Squeezing Hamiltonian

Experimental setup



$$V_k = k\frac{h\nu_R}{2e}$$



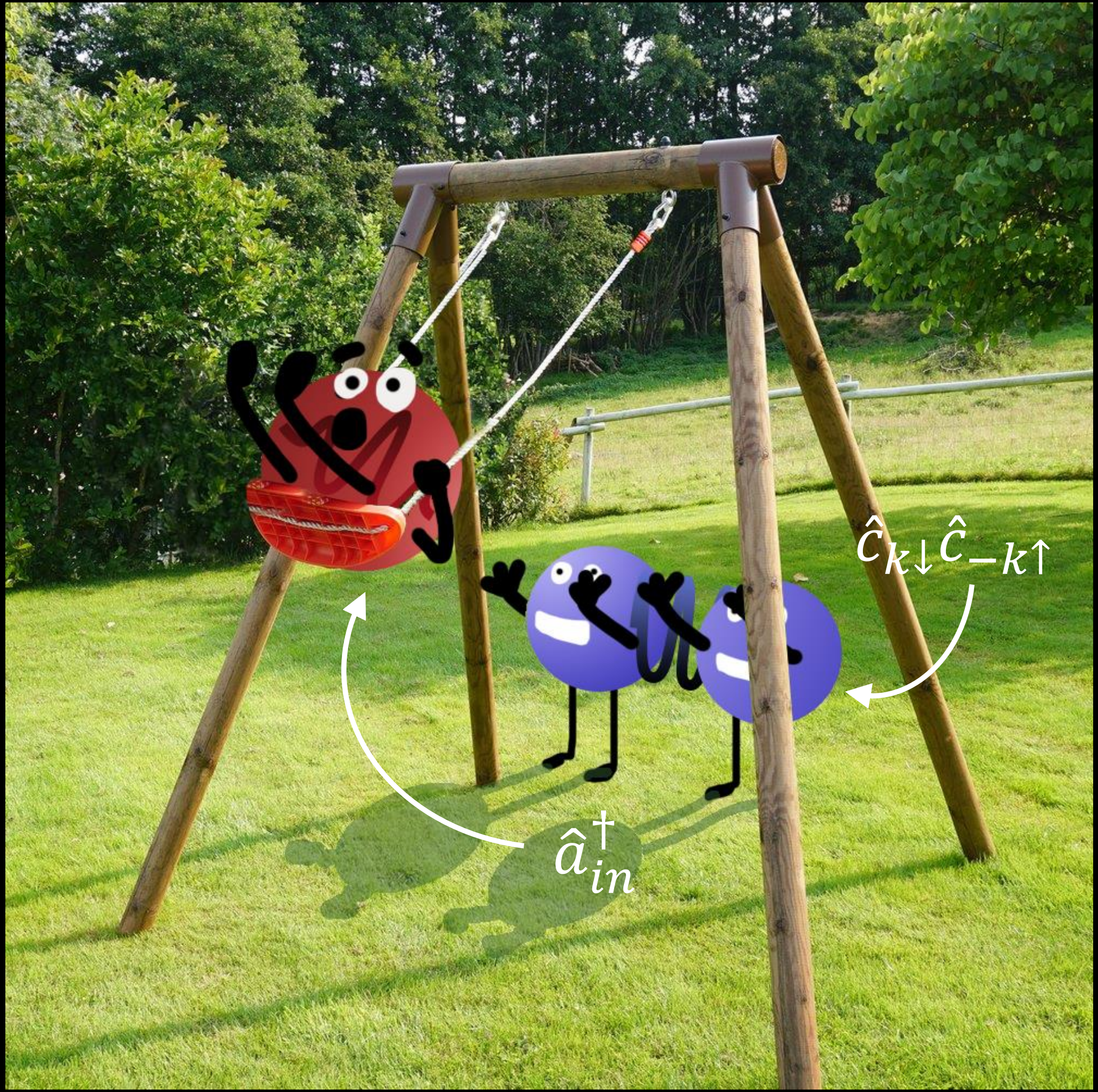
Nb Spiral inductor connected to dc-biased Al/AlOx/Al SQUID

$\alpha \simeq 1$

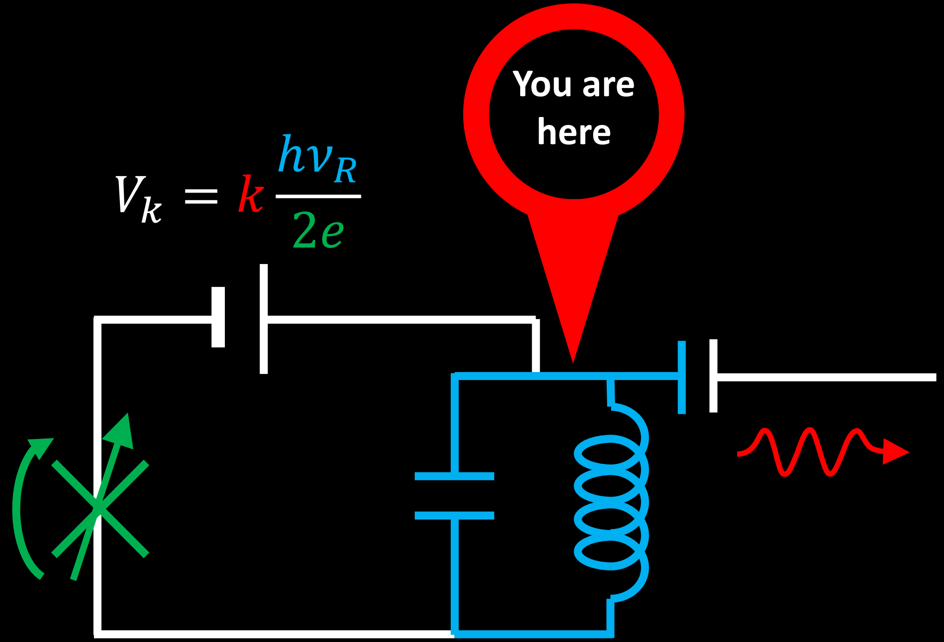
(NB: same sample but 3 different experimental runs in 2 different fridges)

C. Rolland et al. Phys. Rev. Lett. **122**, 186804 (2019)
G. Ménard et al. Phys. Rev. X **12**, 021006 (2022)

Experimental setup

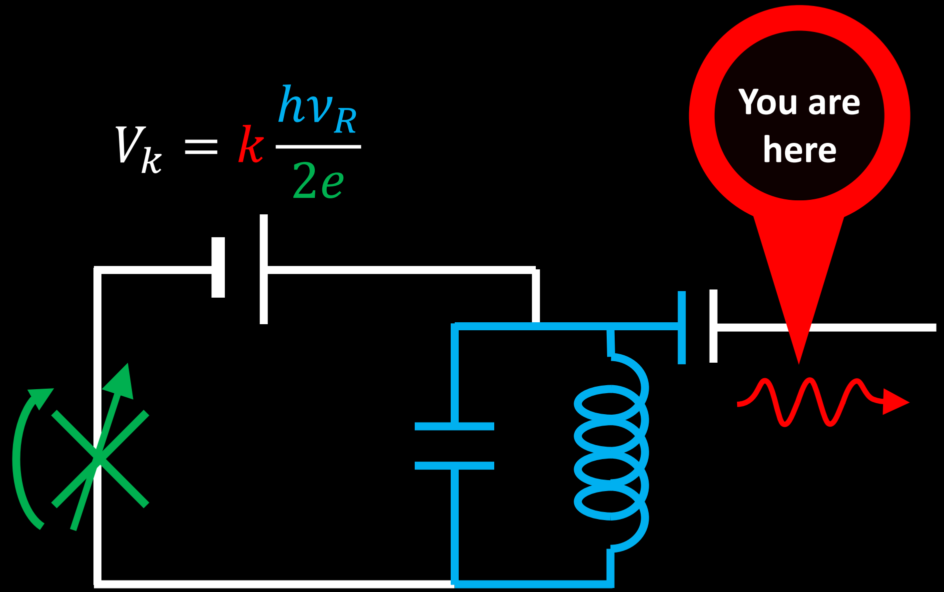


$$V_k = k\frac{h\nu_R}{2e}$$



The photons live in the resonator but cannot be directly measured.

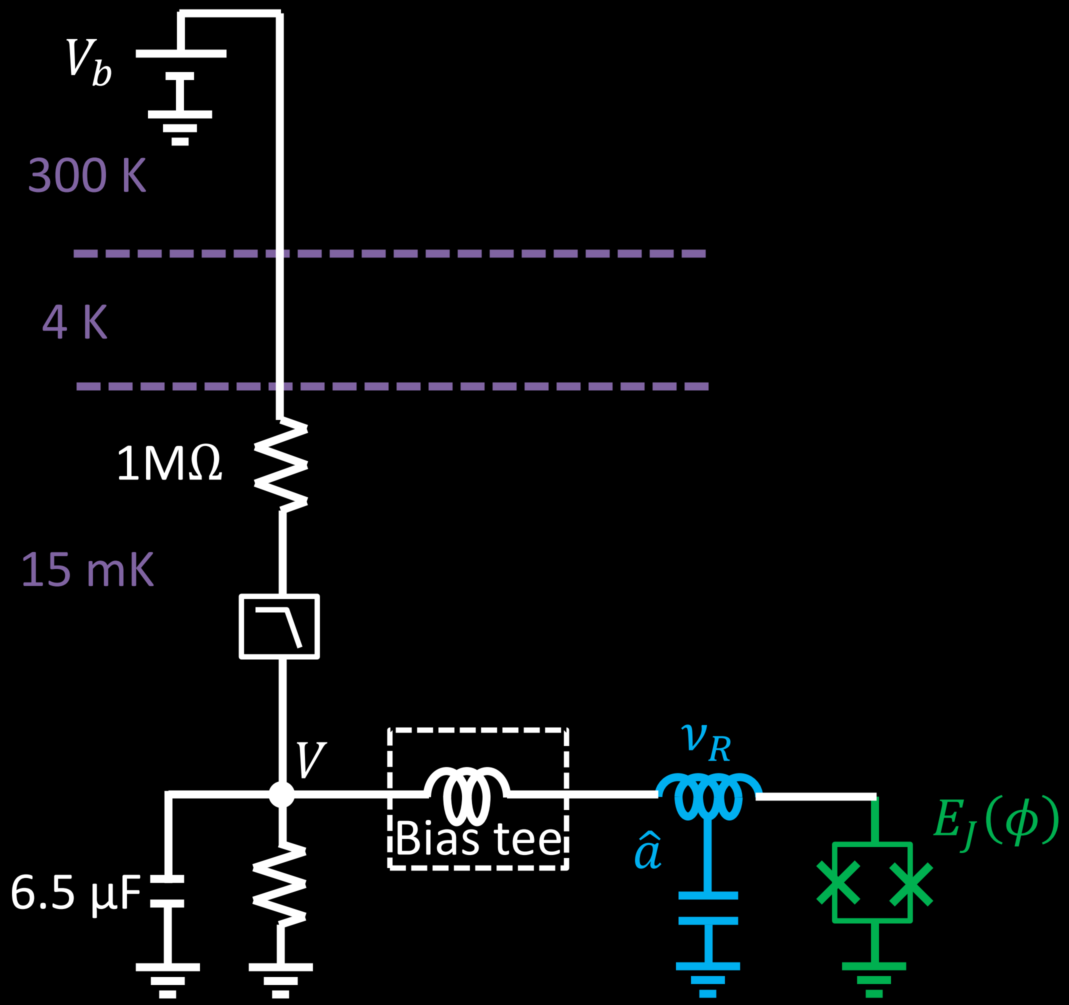
Experimental setup



We only measure the leakage from the resonator

$$\hat{a}_{out} = \kappa \hat{a}_{in}$$

Experimental setup

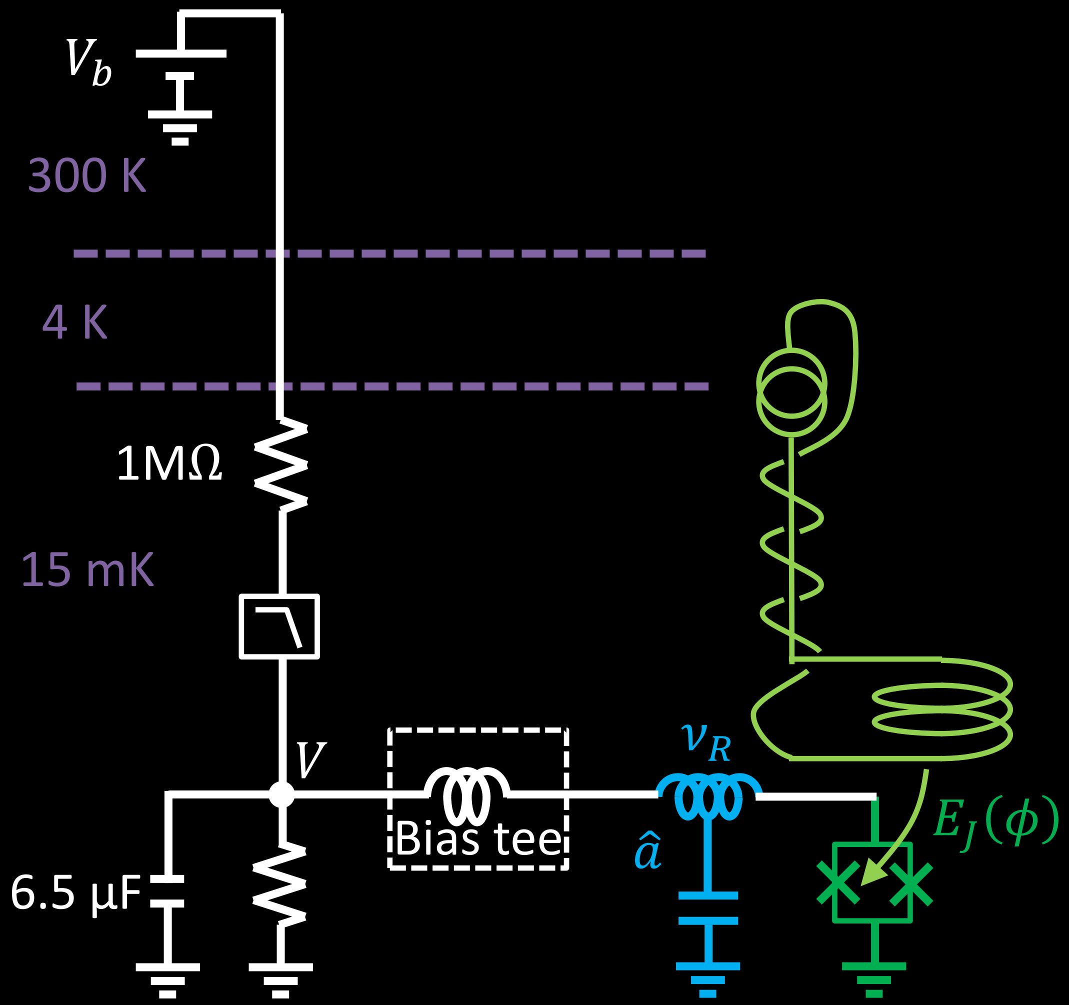


Measurement circuit:

- Polarization (V_b)
- Resonator (ν_R)
- SQUID $(\hat{\phi}_J)$

G. Ménard et al. Phys. Rev. X **12**, 021006 (2022)

Experimental setup

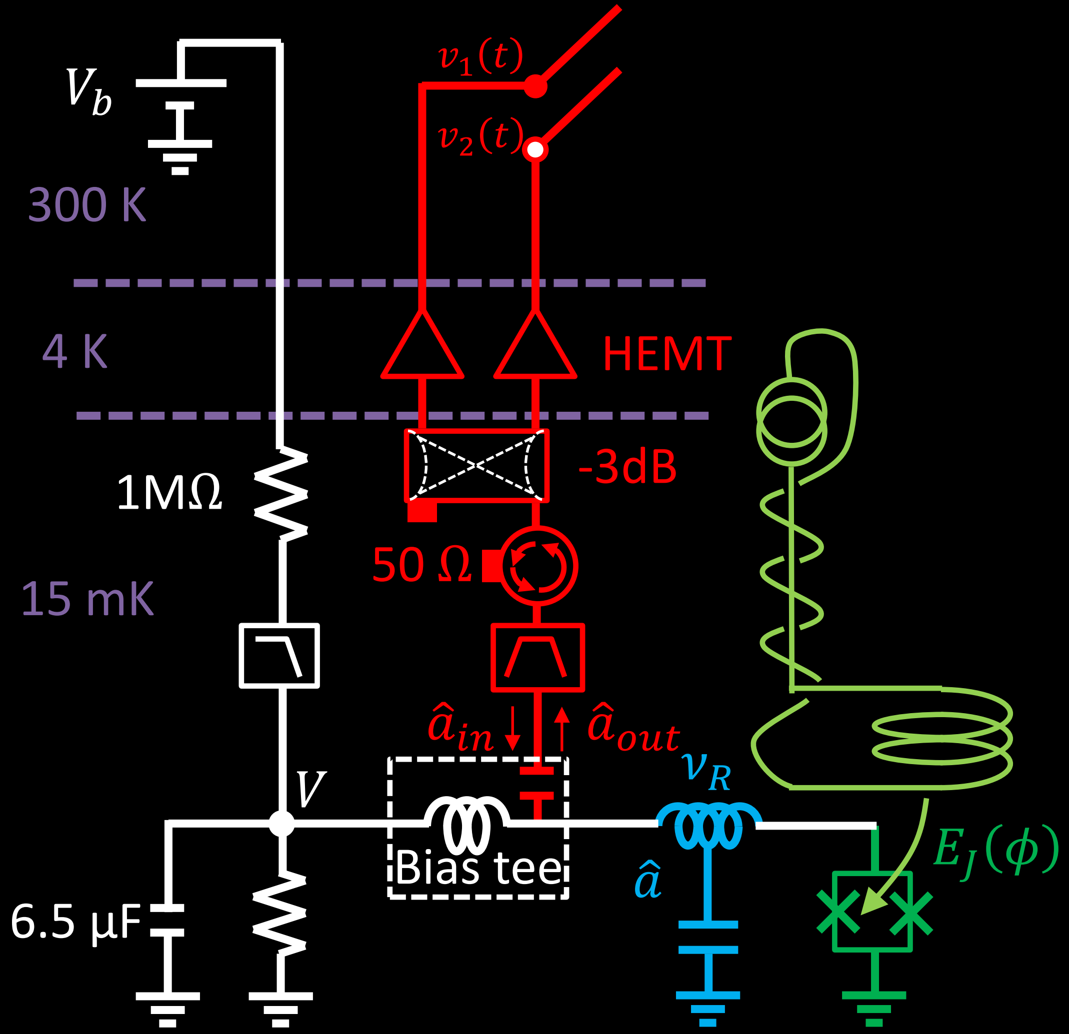


Measurement circuit:

- Polarization (V_b)
- Resonator (ν_R)
- SQUID $(\hat{\phi}_J)$
- Magnetic tuning (E_J)

G. Ménard et al. Phys. Rev. X **12**, 021006 (2022)

Experimental setup



Measurement circuit:

- Polarization (V_b)
- Resonator (ν_R)
- SQUID $(\hat{\phi}_J)$
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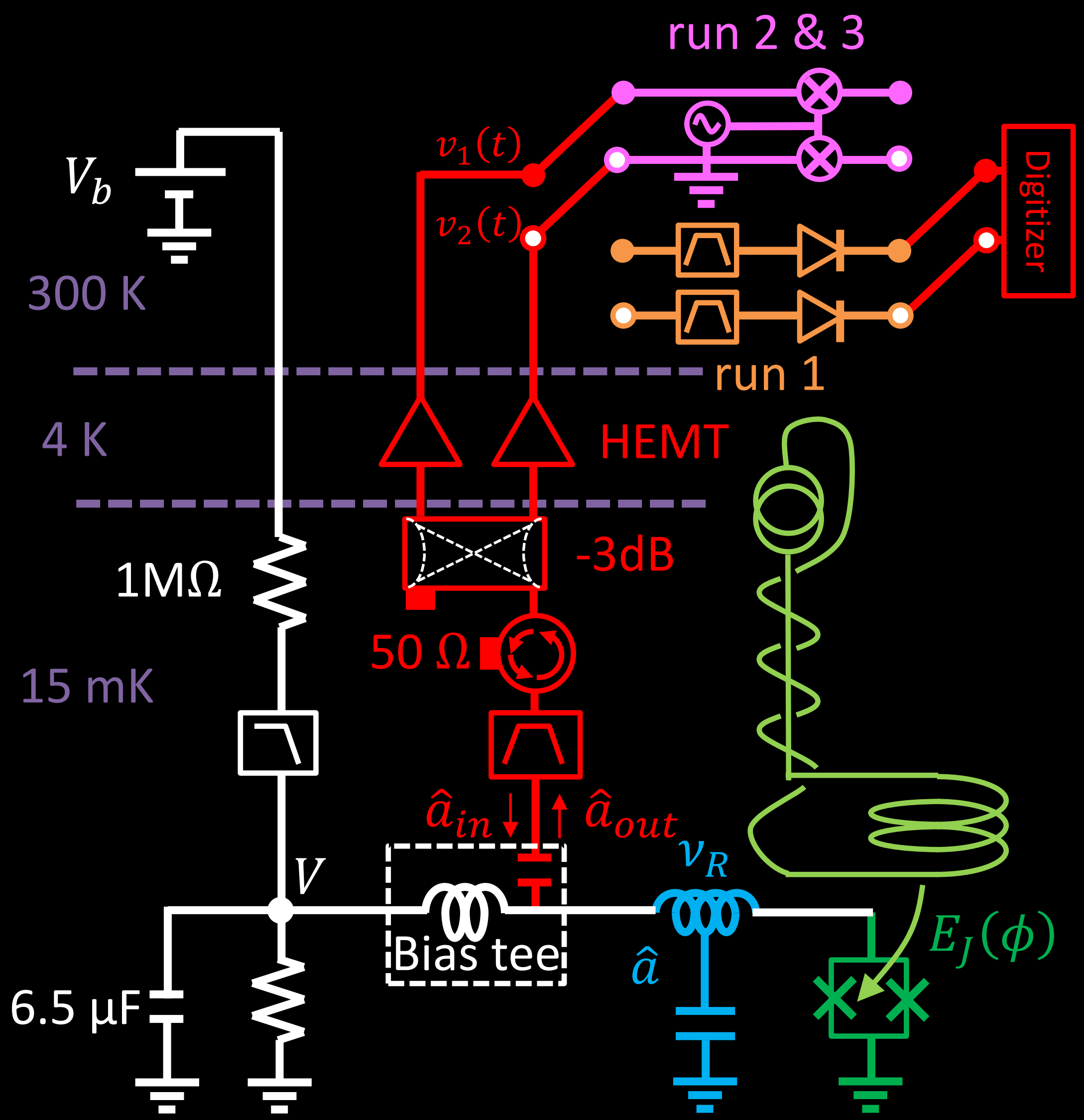
- Measurement $(\hat{a}_{\text{out}})$

Experimental setup



Measurement circuit:

- Polarization (V_b)
- Resonator (ν_R)
- SQUID $(\hat{\phi}_J)$
- Magnetic tuning (E_J)
- Measurement $(\hat{a}_{\text{out}})$

- Heterodyning
- Signal integration (diode)

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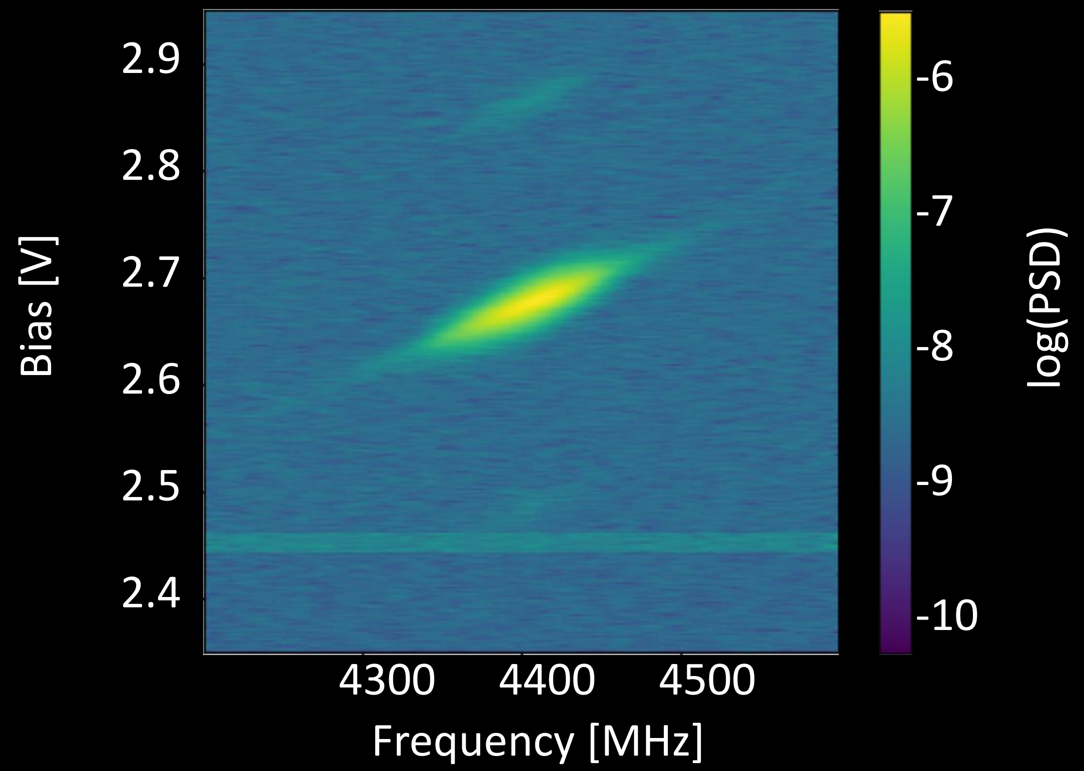
Experimental setup

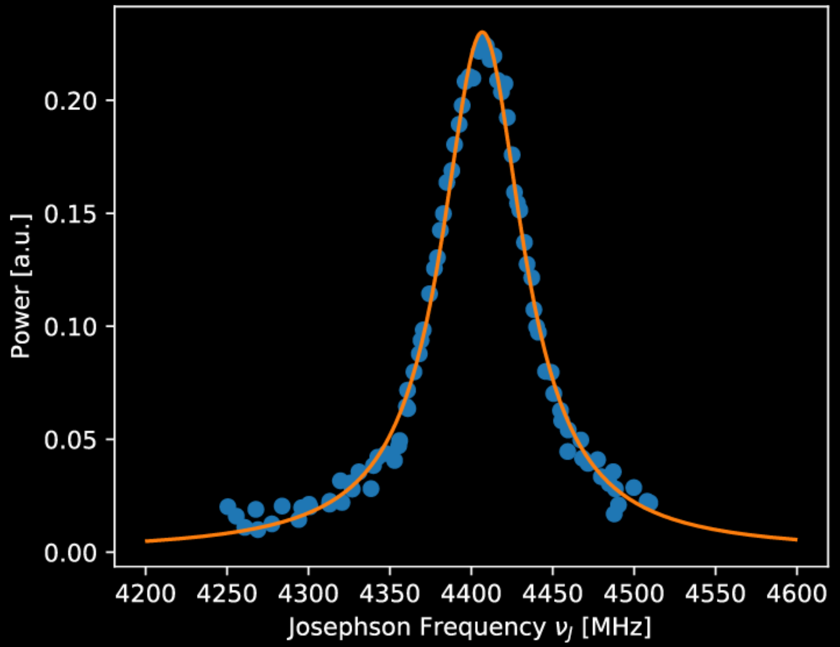


Sweeping voltage around resonance.

Parameters of resonator:

$Q = 72$

$\nu_R = 4{,}4\ \mathrm{GHz}$



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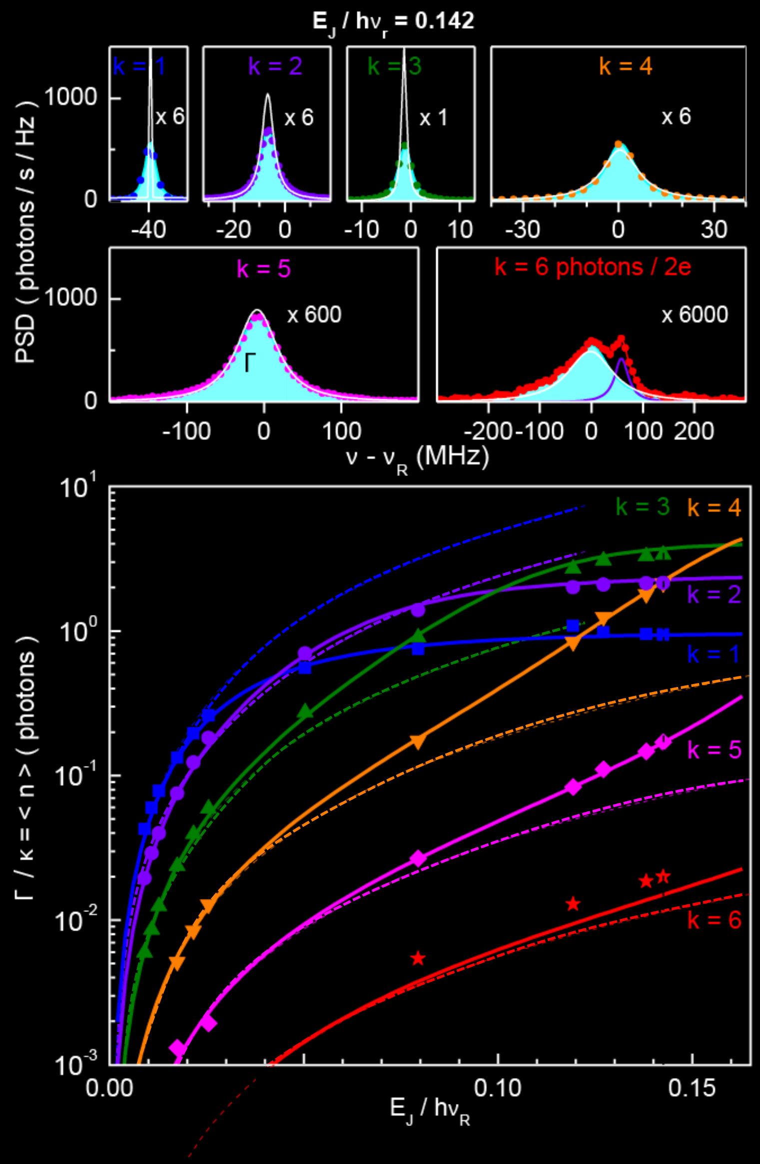
Power measurements



Low E_J:

(on resonance $2eV = kh\nu_R$, Purcell relaxation rate):

$$\gamma_k = \frac{\Gamma_k}{k} = -\left(\frac{E_j}{h\nu_R}\right)^2 \frac{\alpha^k e^{-\alpha}}{kk!} Q\nu_R$$

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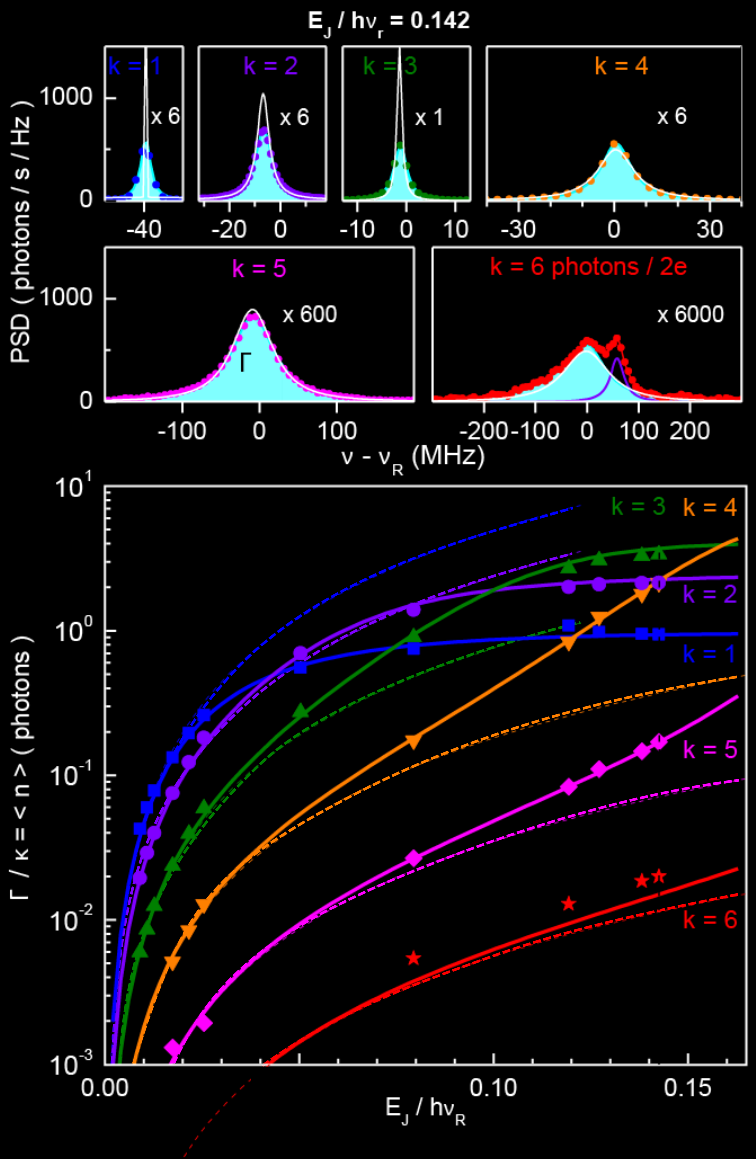
Power measurements



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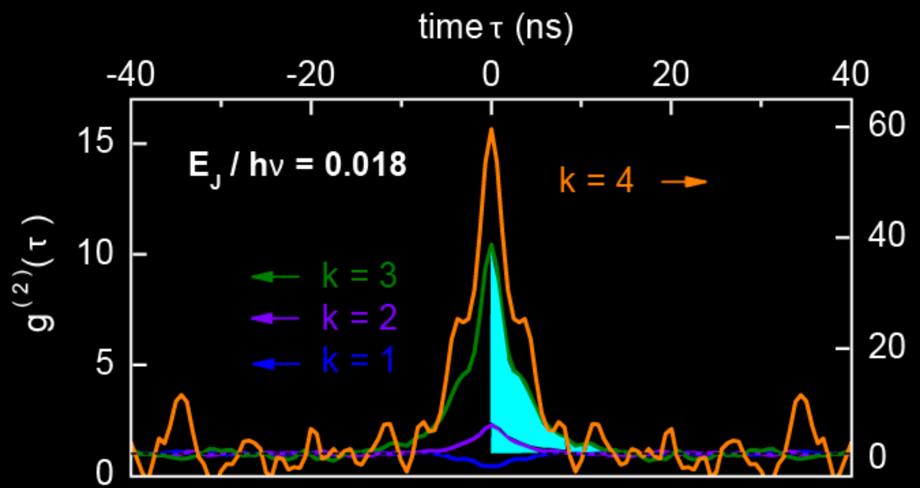
Strong E_J: Feedback of the resonator on emission

Perfect agreement between theory and experiment, taking into account full Hamiltonian.

Fano factor

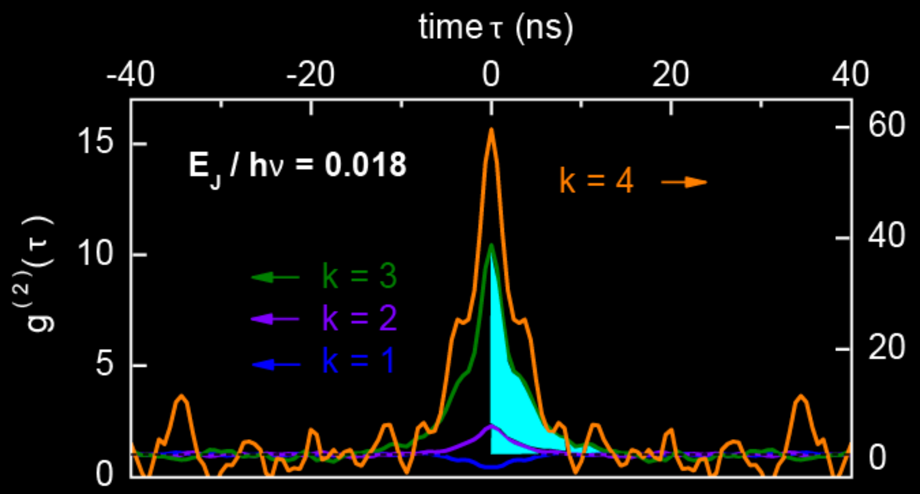


Fano factor

$$F_k = 1 + 2\Gamma_k \int_0^\infty [g^{(2)}(\tau) - 1] d\tau$$

Measures field statistics, variance of photons divided by number of photons emitted during time $t > \Gamma_k^{-1}$.

Fano factor



Fano factor

$$F_k = 1 + 2\Gamma_k \int_0^\infty \left[g^{(2)}(\tau) - 1\right] d\tau$$

Measures field statistics, variance of photons divided by number of photons emitted during time $t > \Gamma_k^{-1}$.

Second order correlator:

$$g^{(2)}(\tau) = \frac{\langle \hat{a}_{\mathrm{out}}^\dagger(0)\hat{a}_{\mathrm{out}}^\dagger(\tau)\hat{a}_{\mathrm{out}}(\tau)\hat{a}_{\mathrm{out}}(0)\rangle}{\langle \hat{a}_{\mathrm{out}}^\dagger \hat{a}_{\mathrm{out}}\rangle^2}$$

→ probability for two photons separated by τ to leak in the same electro-magnetic mode

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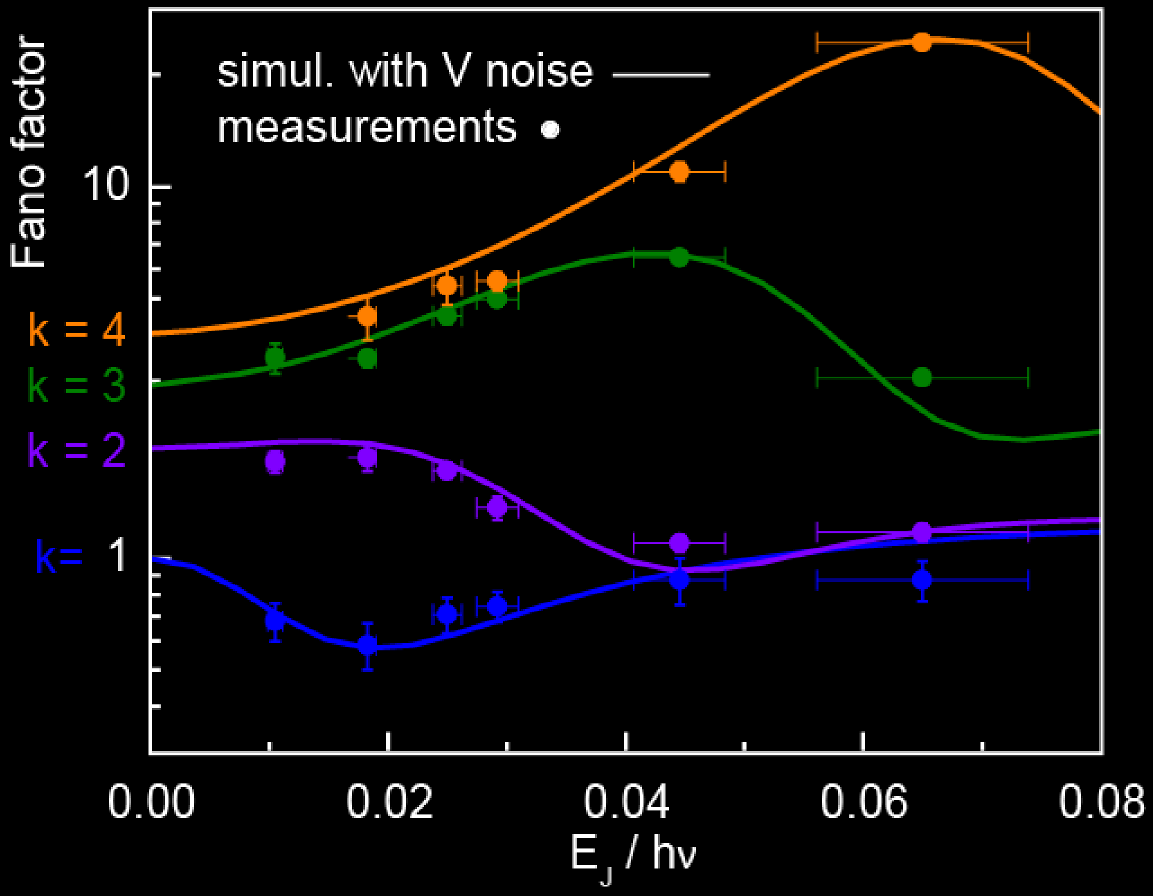
Fano factor



$F_k = k$ at low E_J up to $k = 4$

(limited by noise on $g^{(2)}(\tau)$)

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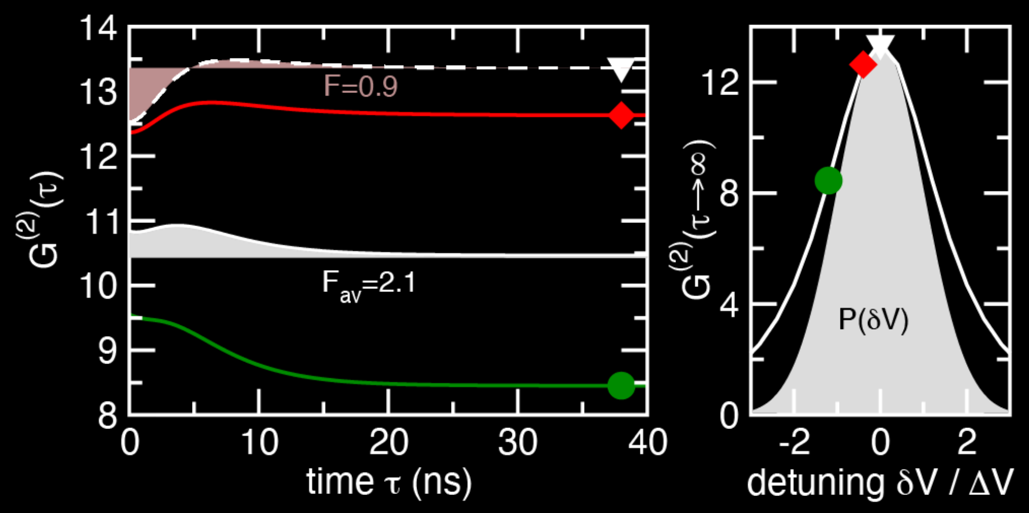
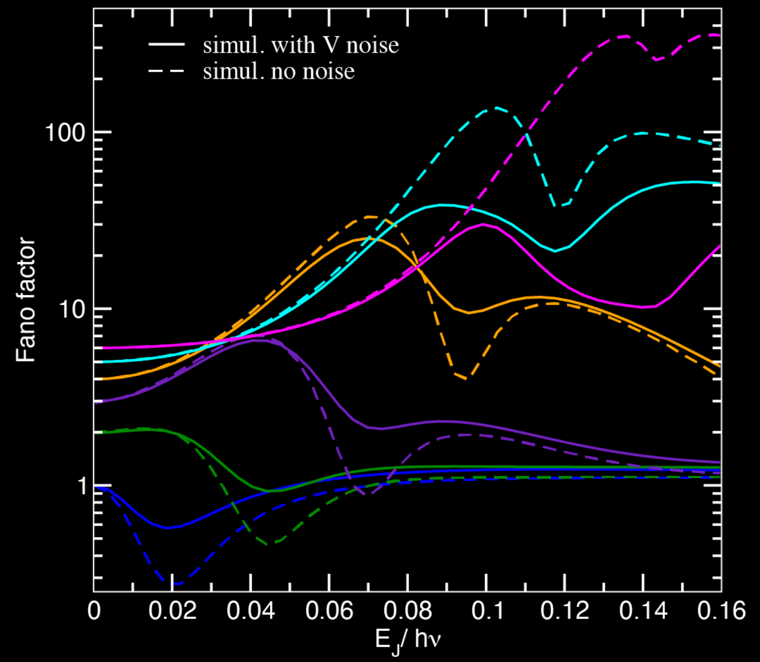
Effect of the voltage noise



Averaging various $g^{(2)}(\tau)$ around the central emission frequency.

Noise → dampening of Fano factor behavior



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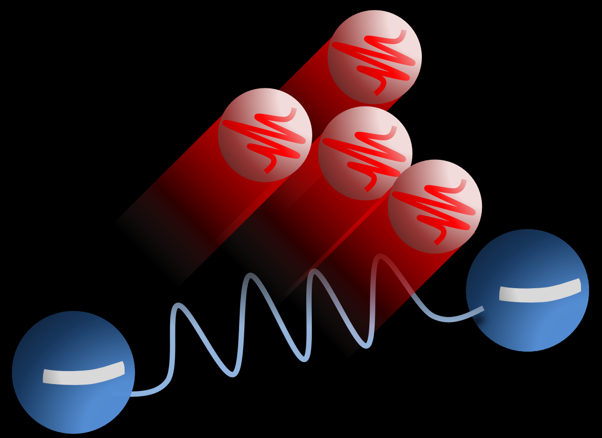
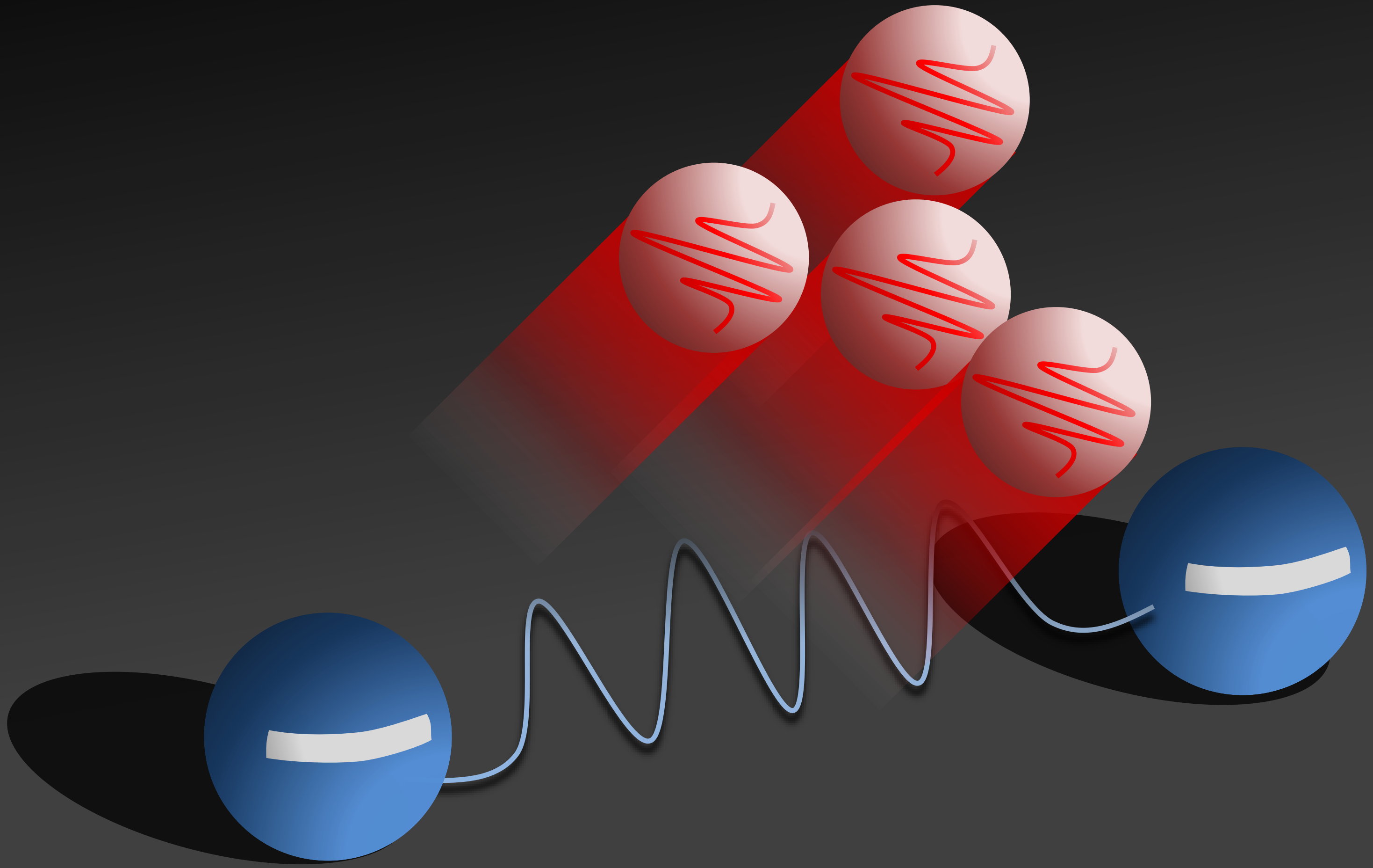
Conclusion

- Observation of photonic emission up to $k = 6$ (in power & frequency measurements)
- Measurement of Fano factor $= k$ for low E_J
- Complete theoretical understanding of behavior of Fano factor
- Emission of photon multiplets in high impedance electromagnetic environment



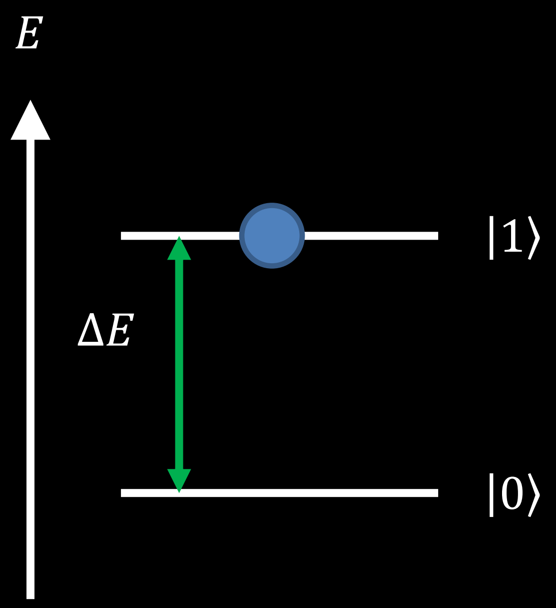
Thank your for your attention



Spontaneous emission of a two-level system

Simple two-level system

$$\hat{H} = \hat{H}_0 + \hat{V}_{\text{em}} = E_1|1\rangle\langle 1| + E_0|0\rangle\langle 0|$$

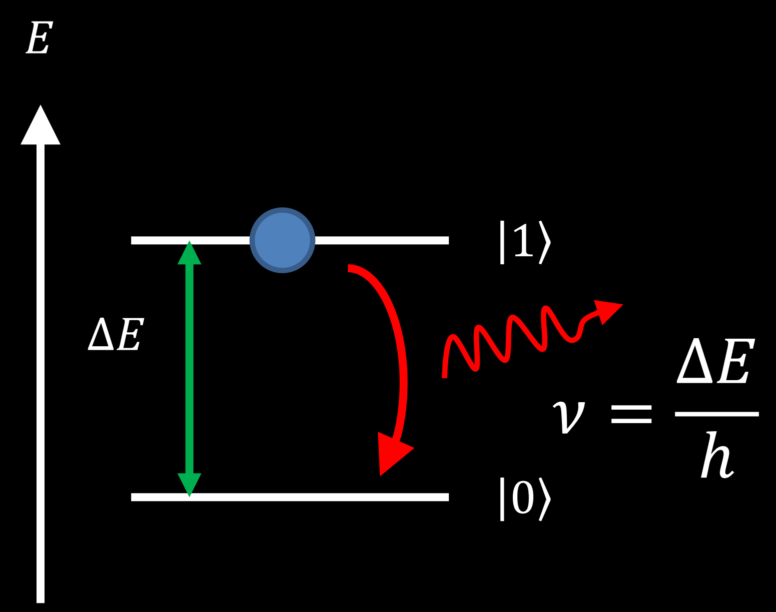


Without coupling, system stays in state $|1\rangle$

Spontaneous emission of a two-level system

Simple two-level system with external electromagnetic perturbation

$$\hat{H} = \hat{H}_0 + \hat{V}_{\mathrm{em}} = E_1|1\rangle\langle 1| + E_0|0\rangle\langle 0| + \hat{V}_{\mathrm{em}}$$



Coupling to the electromagnetic field →
Spontaneous emission of photons +
electronic relaxation

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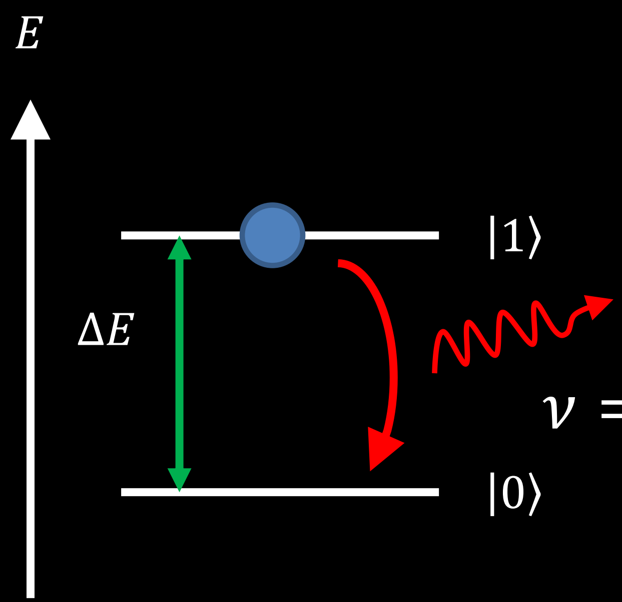

$$\nu = \frac{\Delta E}{h}$$

Coupling to the electromagnetic field → Spontaneous emission of photons + electronic relaxation

Emission rate:

$$\Gamma_{rad} \simeq \frac{2\pi}{\hbar}\left|\langle f|\hat{V}|i\rangle\right|^2 \delta\left(E_f - E_i \pm h\nu\right) \propto \alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

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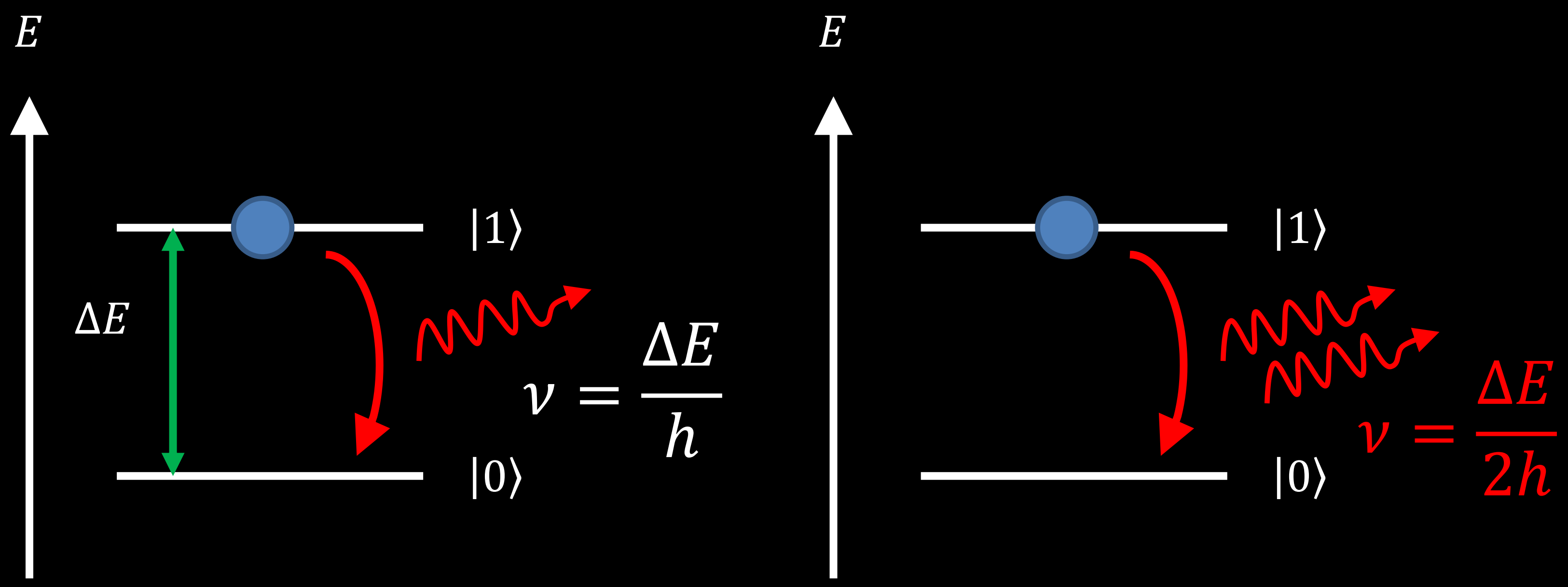
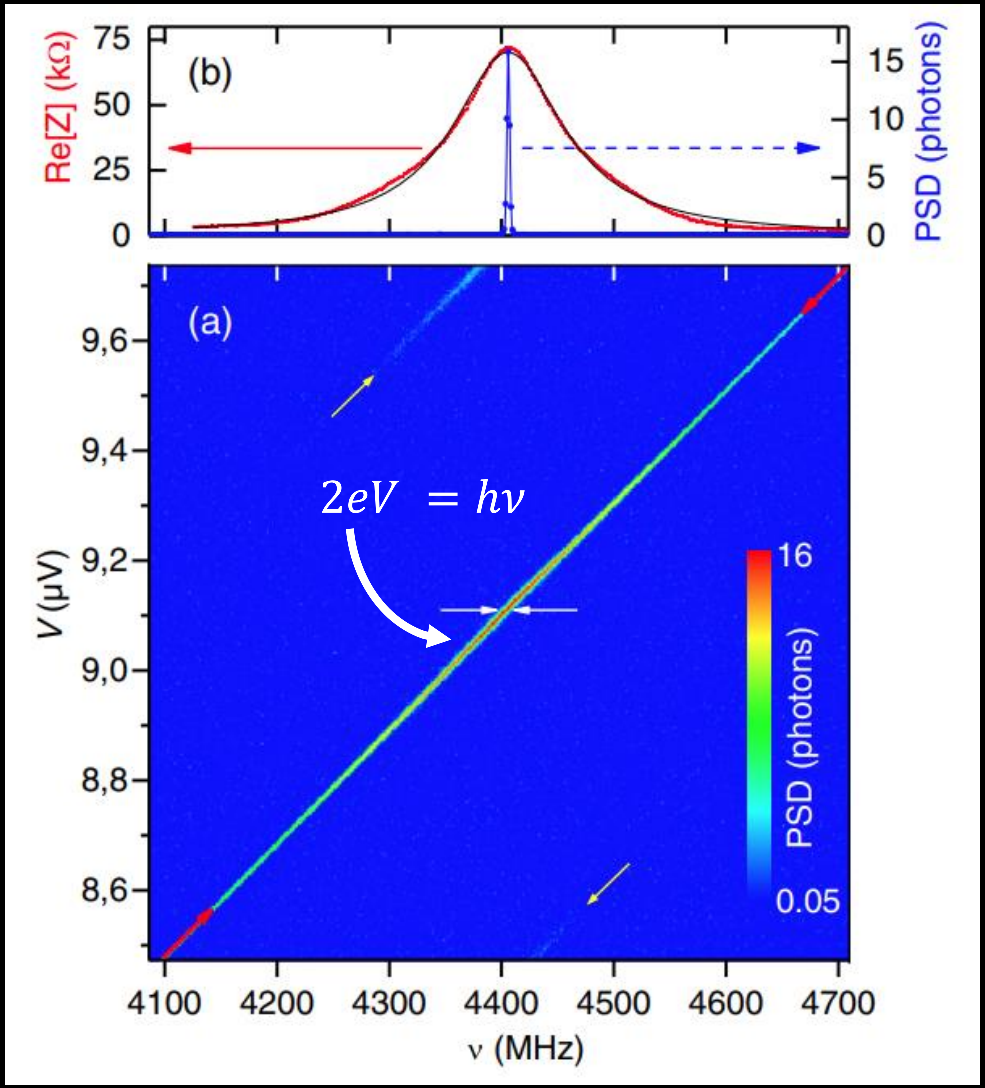


For n photons emission: $\Gamma_{rad}^{(k)} \propto \alpha^k$.
Rapidly tends to zero with increasing k.

Inelastic tunneling of Cooper pair



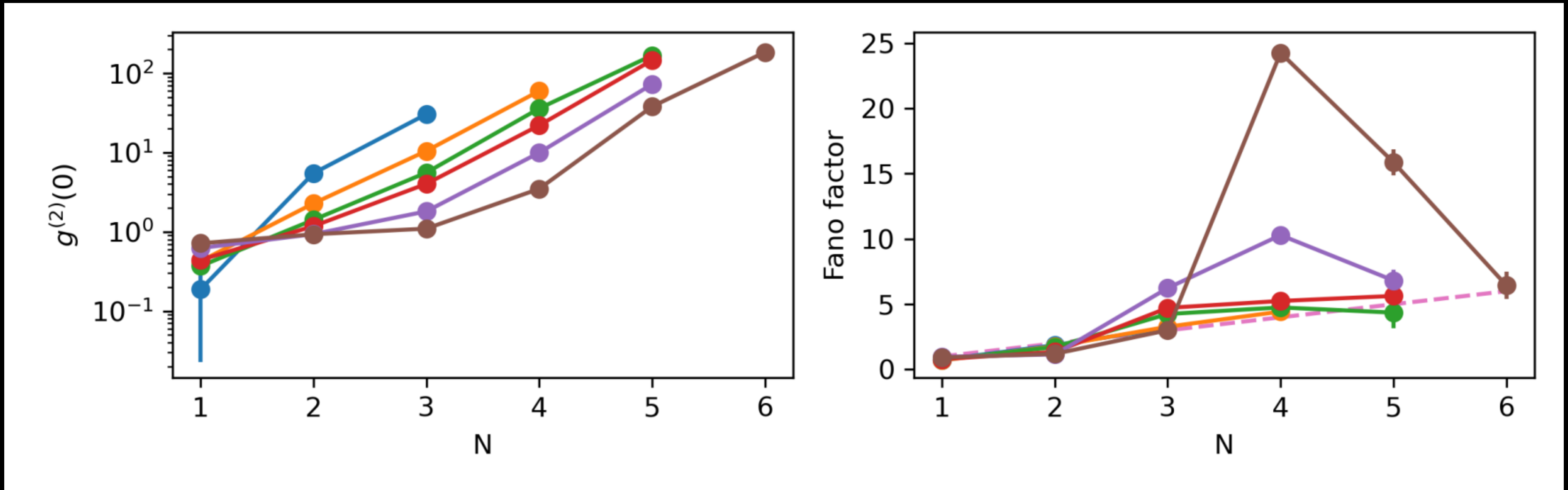
Main emission line follows the relation:

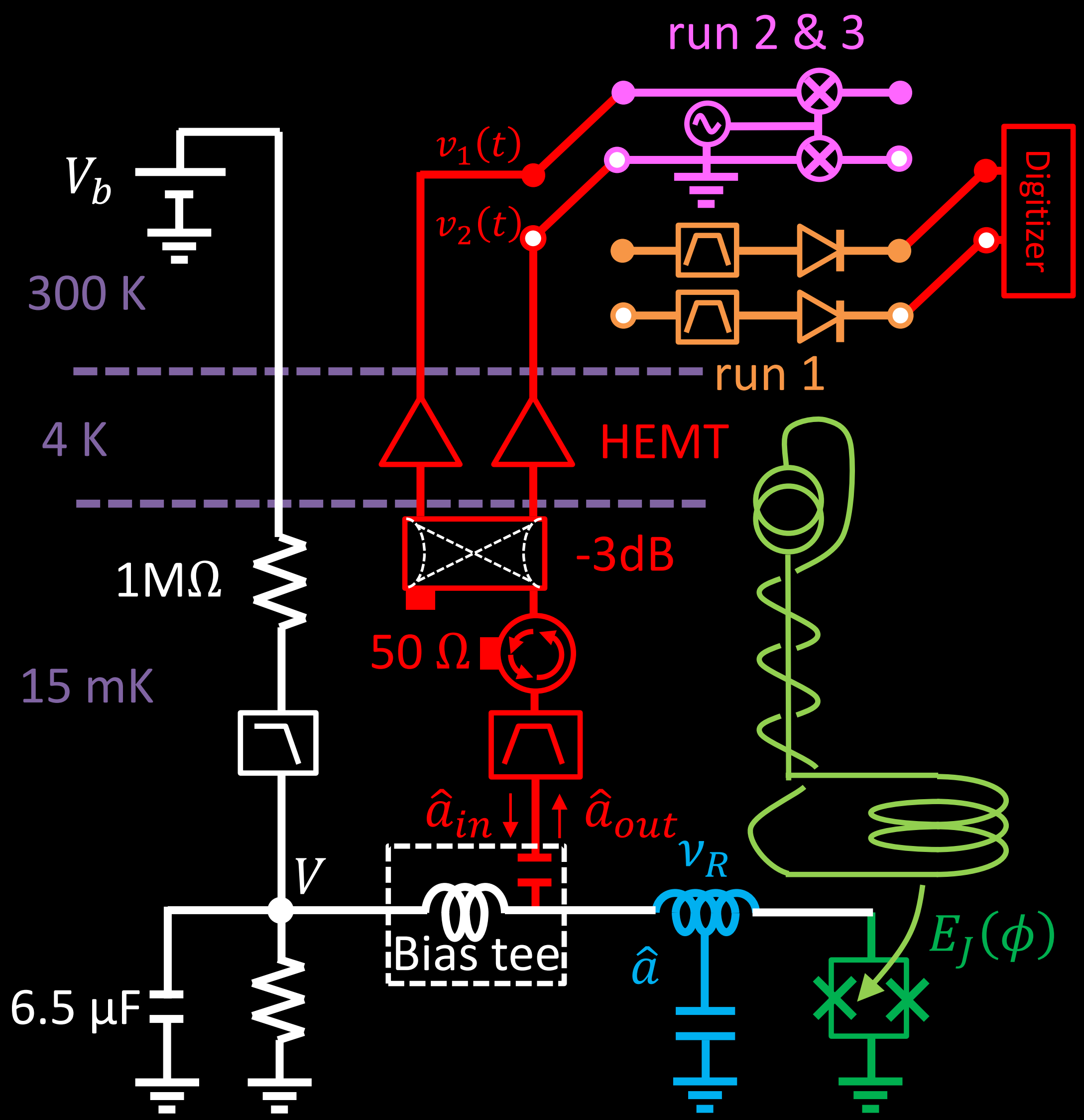
$2eV = h\nu$

C. Rolland et al. Phys. Rev. Lett. **122**, 186804 (2019)

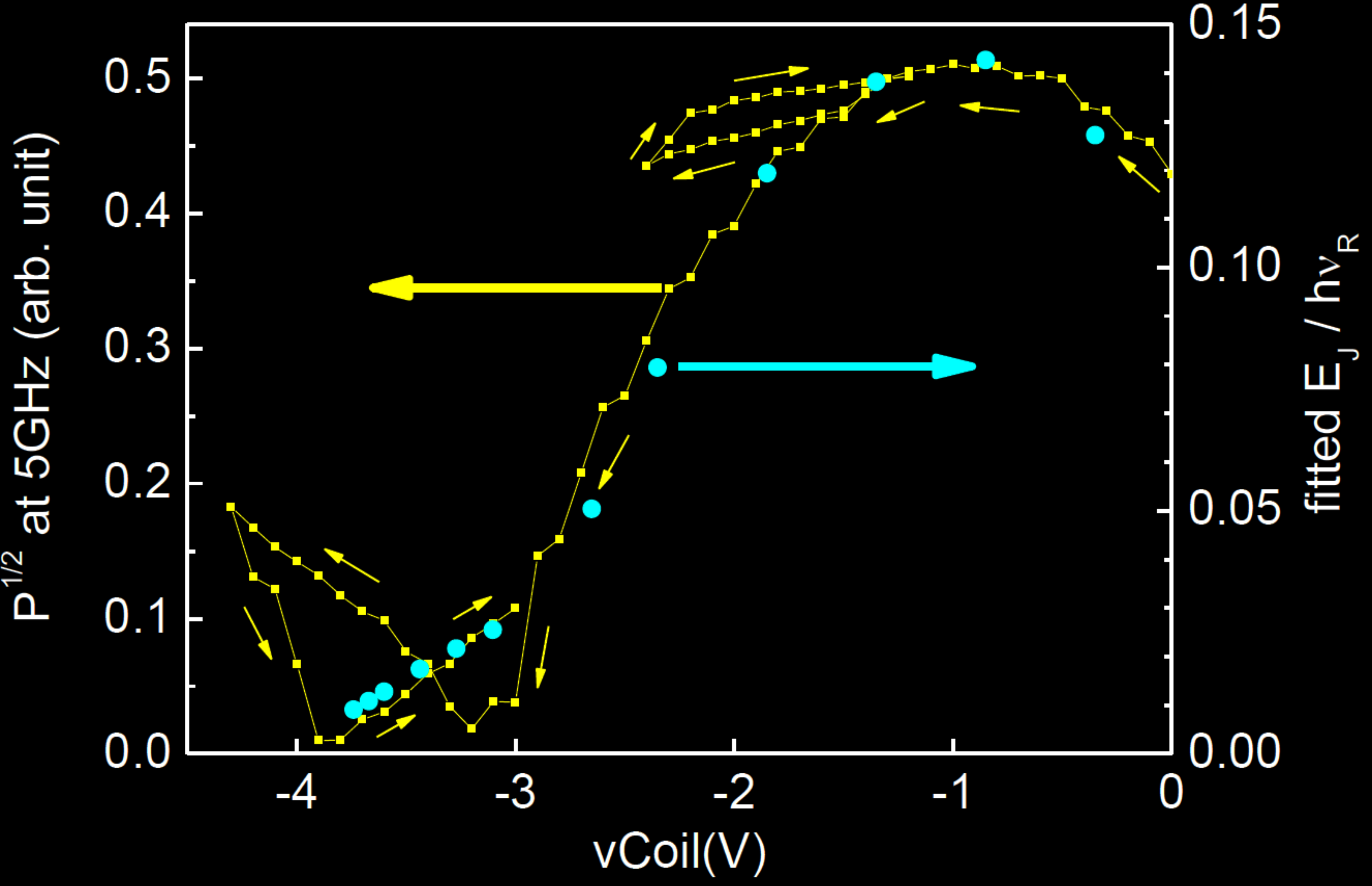
Fano factor

Fano factors for all k orders

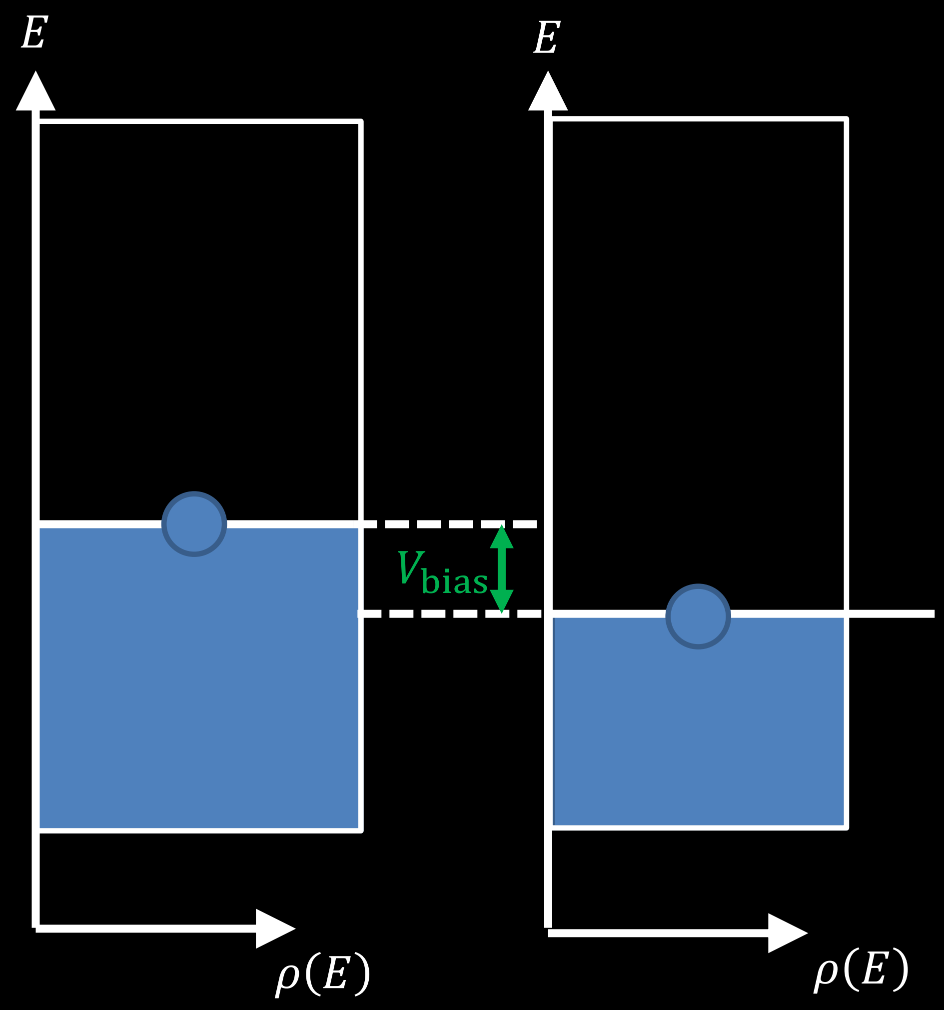



run 2 & 3
$v_1(t)$
$v_2(t)$
Digitizer
V_b
300 K
run 1
4 K
HEMT
1MΩ
-3dB
50 Ω
15 mK
$\hat{a}_{in}$
$\hat{a}_{out}$
V
ν_R
Bias tee
$\hat{a}$
$E_J(\phi)$
6.5 μF

Field hysteresis

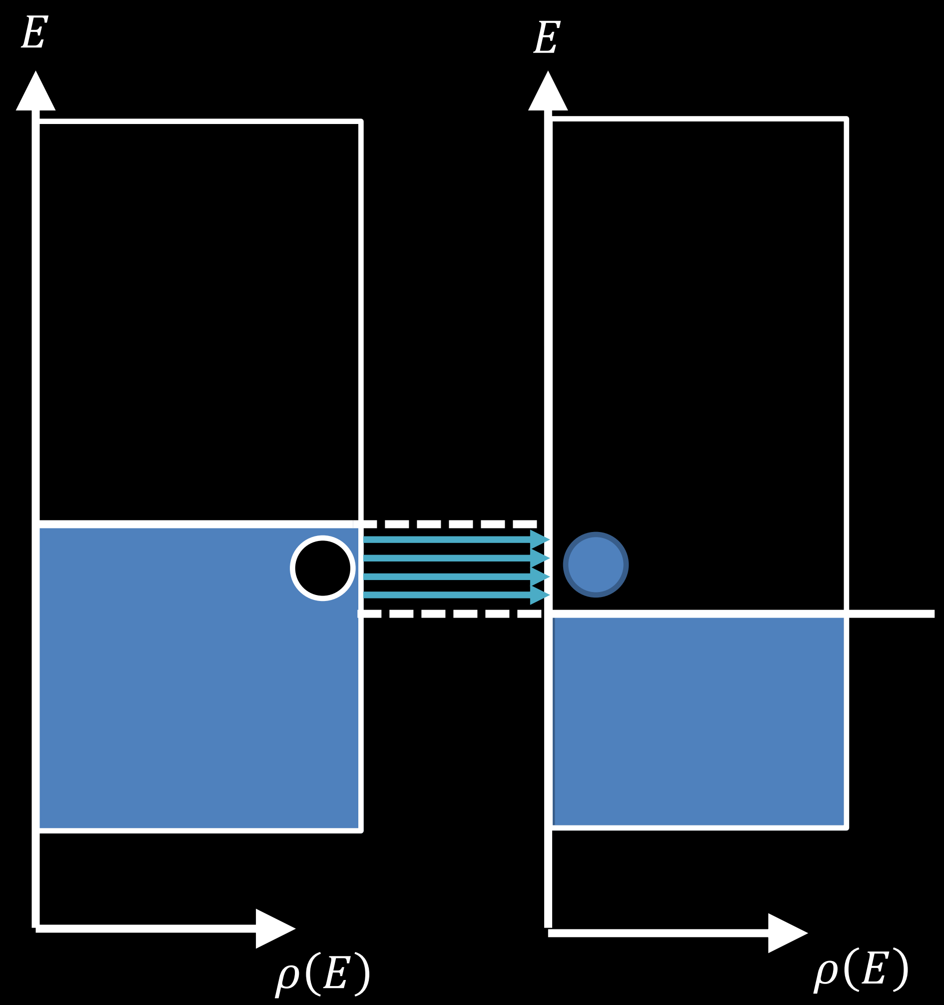


Normal metal comparison (T=0)



Tunneling can occur over the whole V_{bias} range without control on the energy of the transmitted electron.

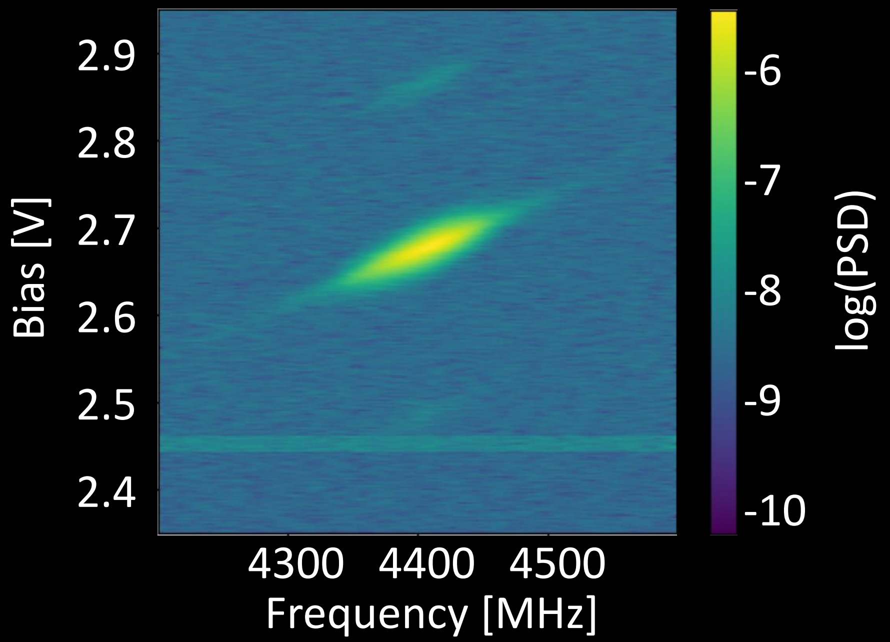
Normal metal comparison (T=0)



Tunneling can occur over the whole V_{bias} range without control on the energy of the transmitted electron.

No inelastic process favored compared to elastic ones.

Effect of the voltage noise



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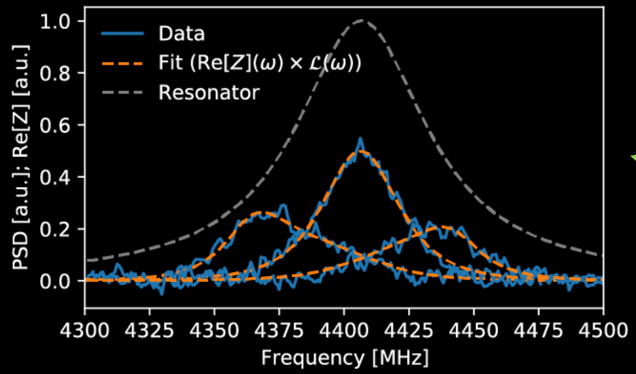
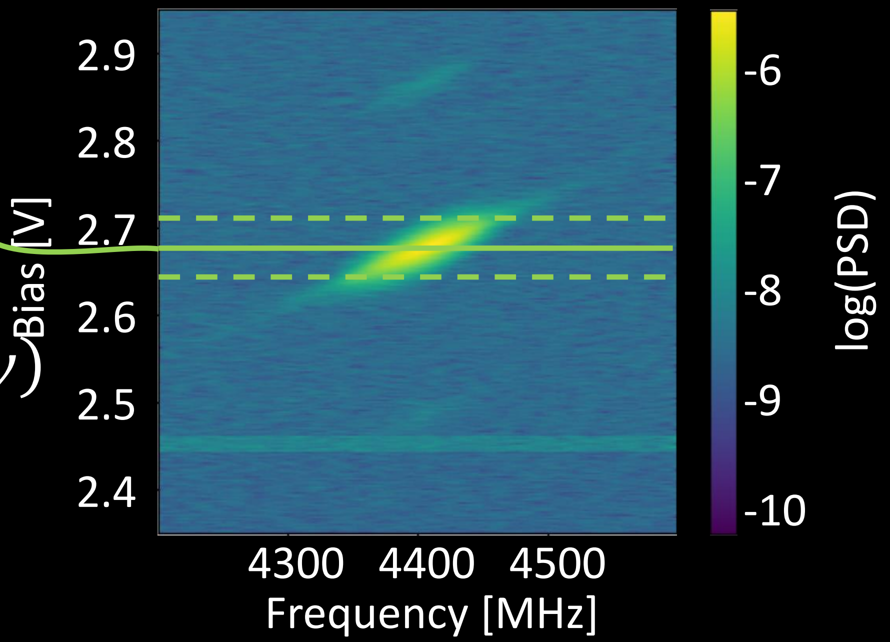
Effect of the voltage noise



$$\mathrm{PSD} \propto \mathrm{Re}\big(Z(\nu)\big) \times P(E = \mathrm{cte}, \nu)$$



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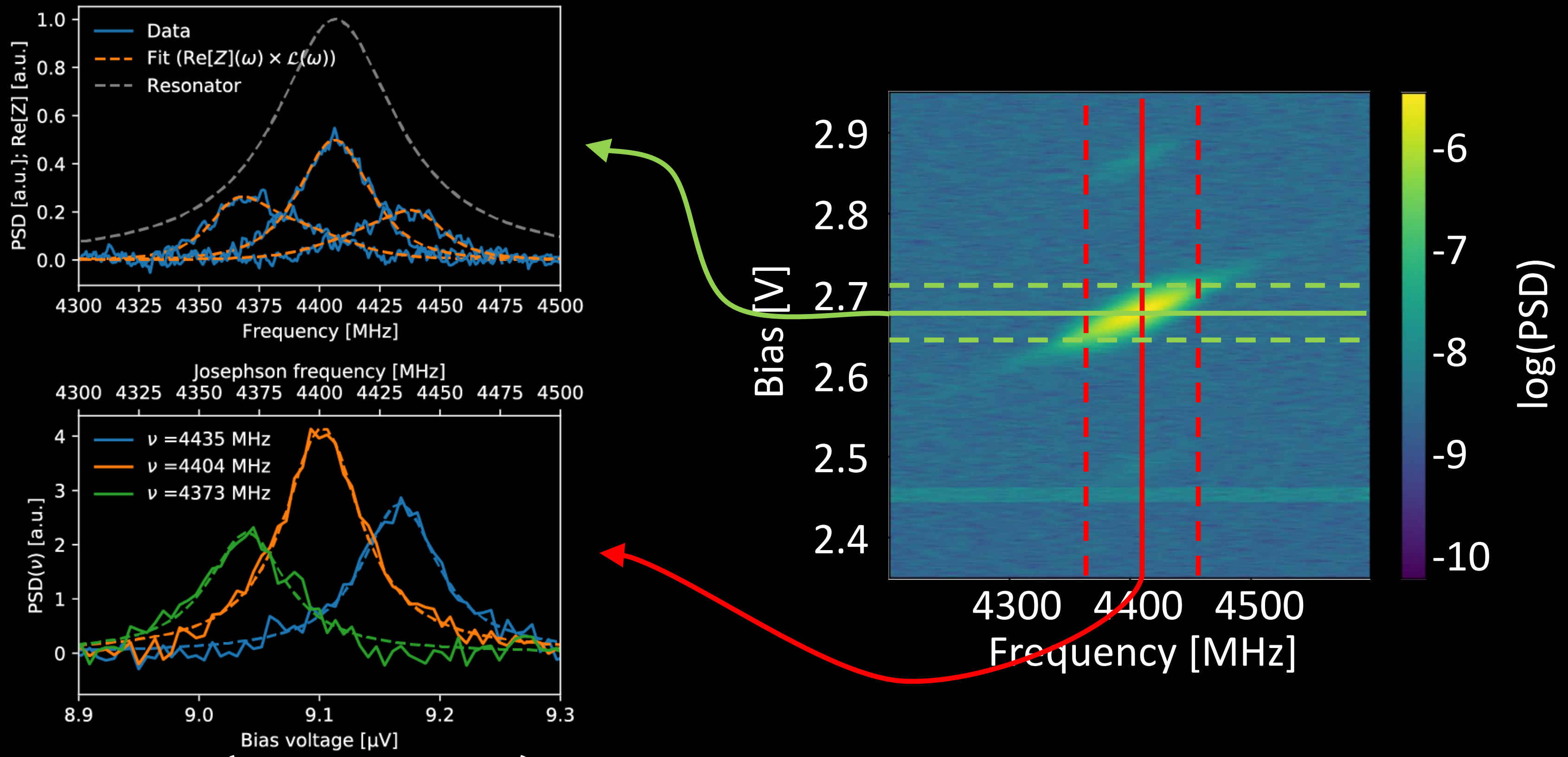
Effect of the voltage noise



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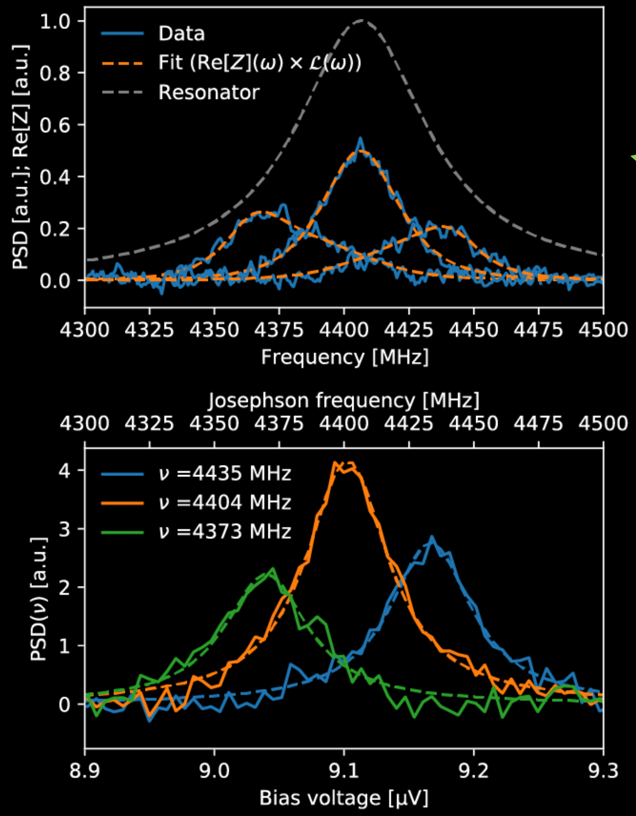
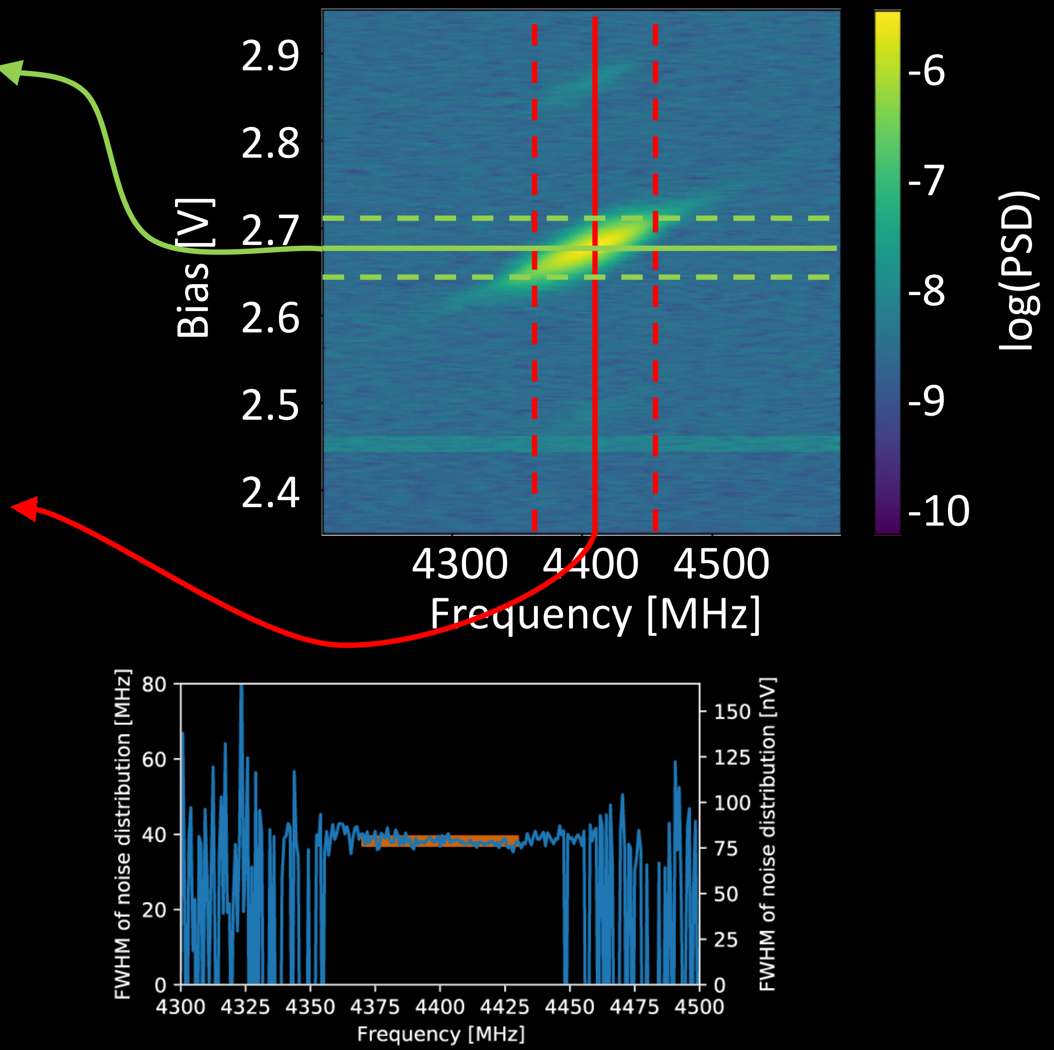
Effect of the voltage noise





Voltage noise

40 MHz ≡ 82 nV

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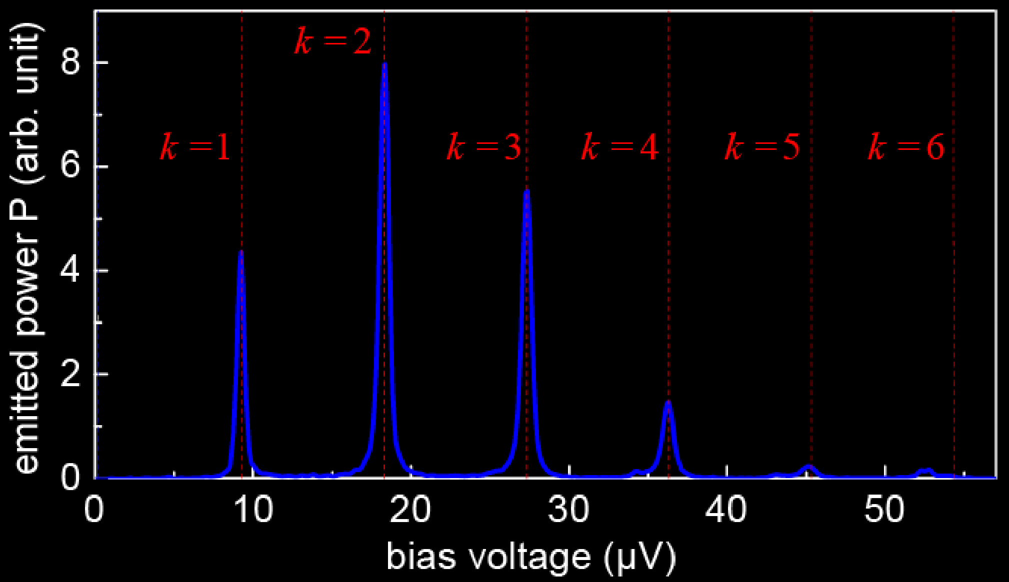
Power measurements

Frequency integrated power



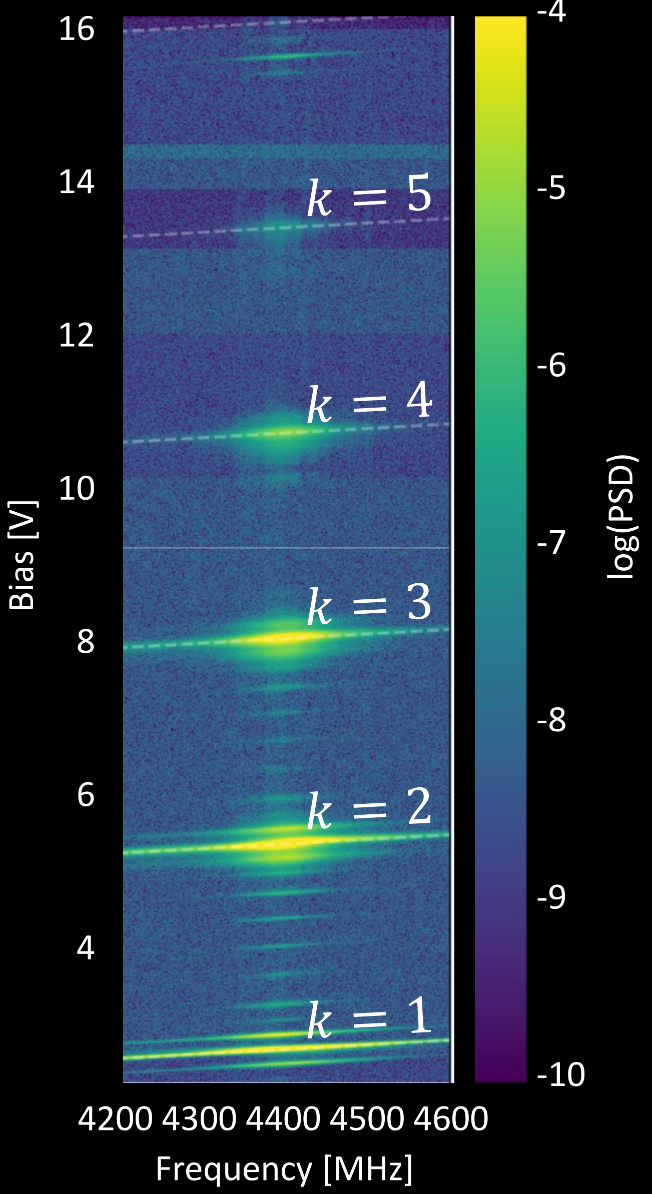
Increasing voltage:

Emission at $2eV = kh\nu_R$

Up to $k = 6$ photon pairs

($k = 6$ has an extra spurious resonance close by)

Full band spectrum



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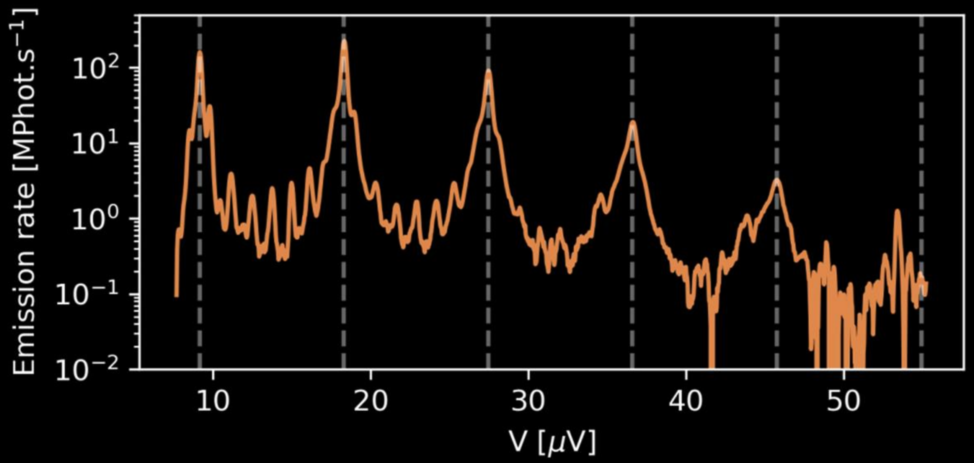
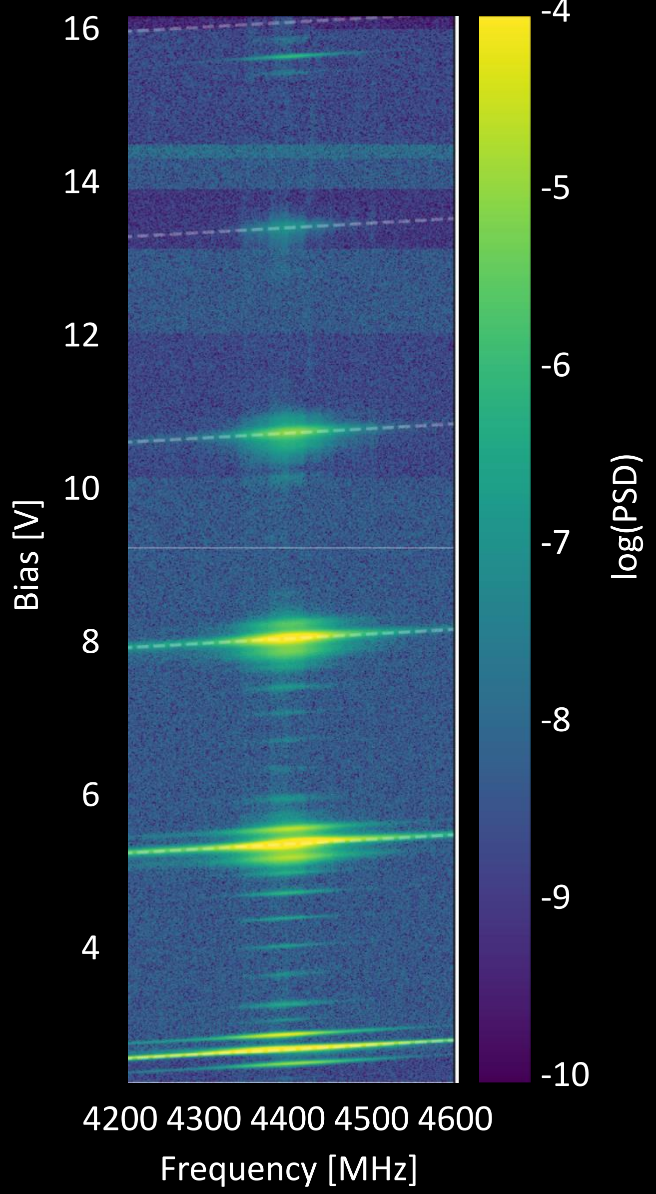
Stokes & antistokes processes



Full band spectrum



Additional intermediate peaks at
$2eV \neq kh\nu_0$

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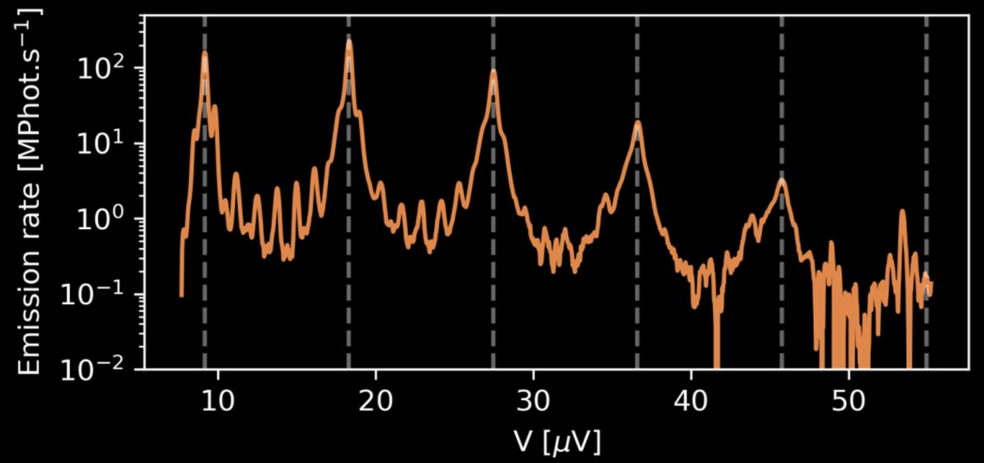
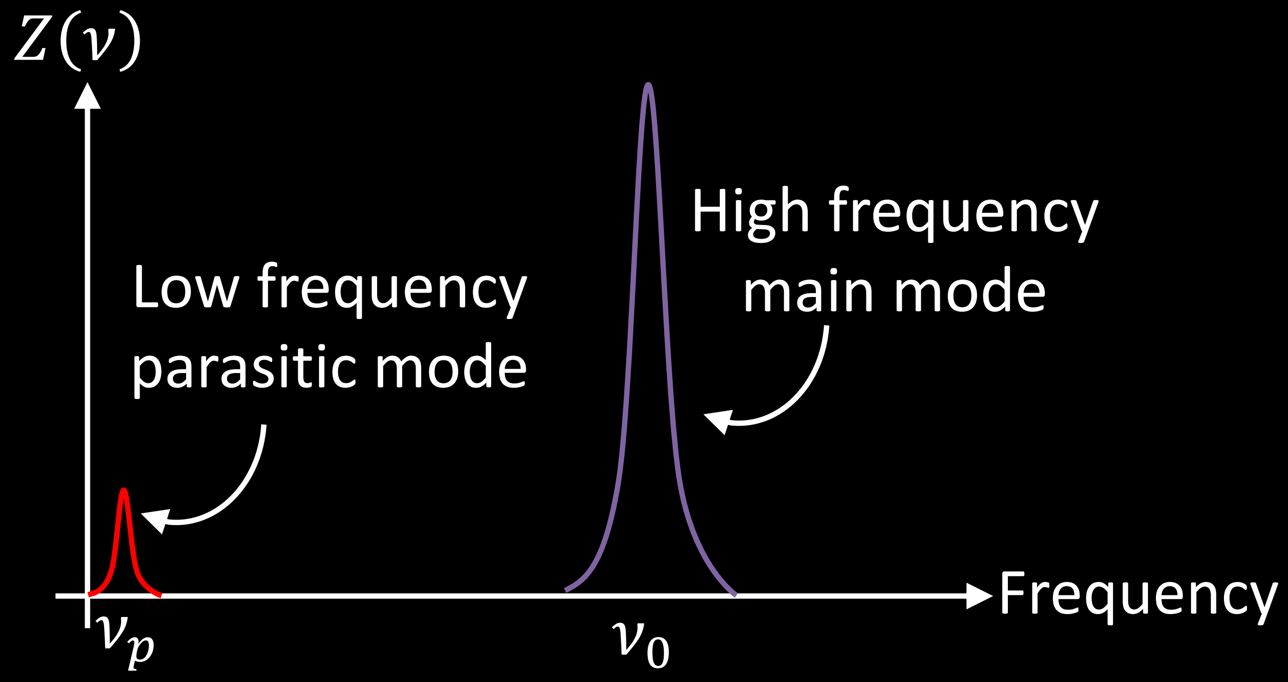
Stokes & antistokes processes





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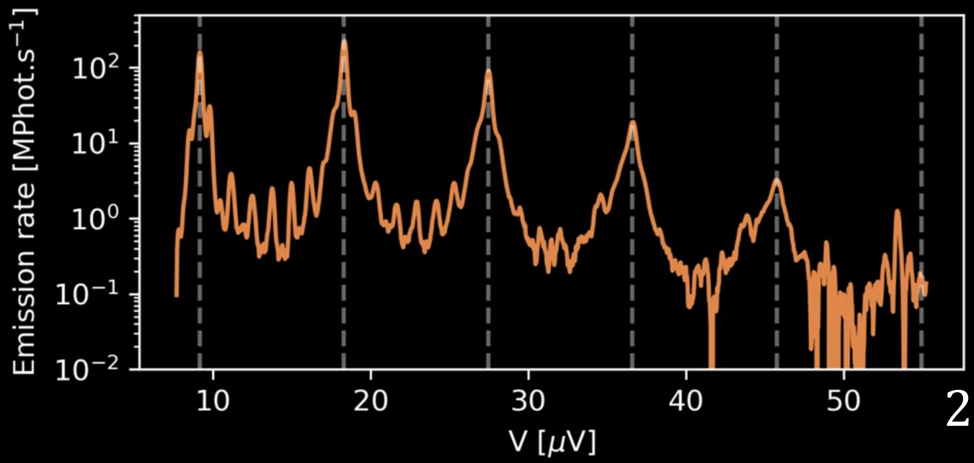
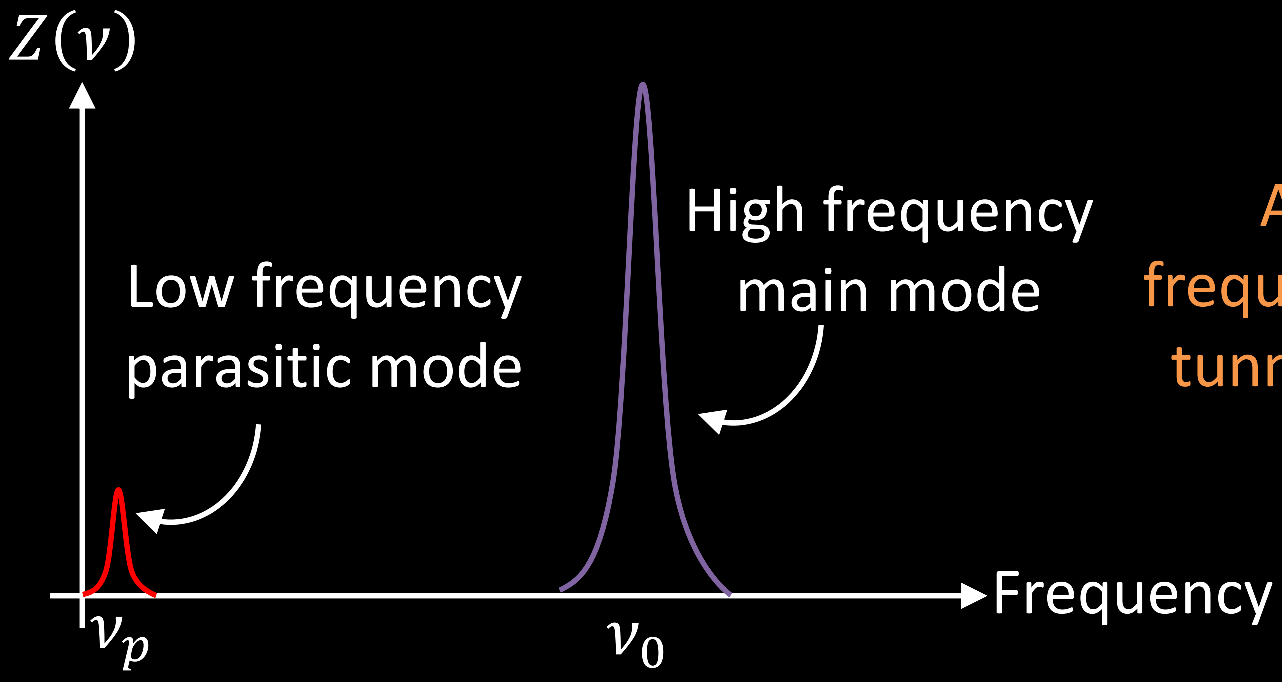
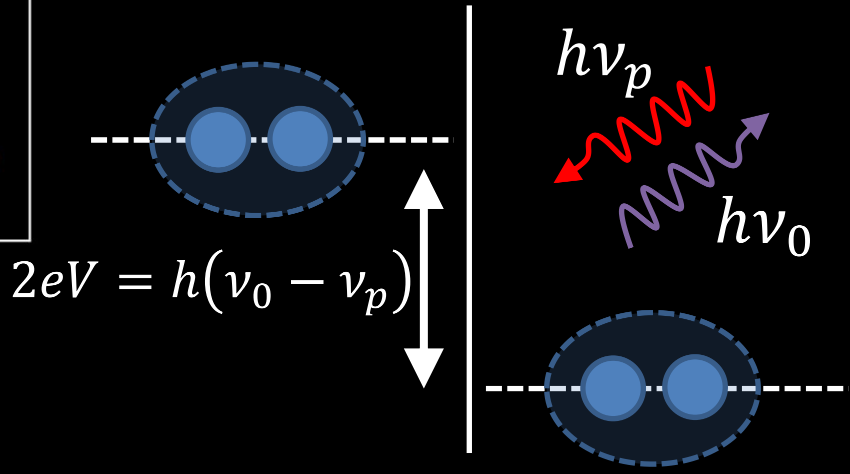
Stokes & antistokes processes







Absorption of low frequency photon to allow tunneling of Cooper pair (Stokes)

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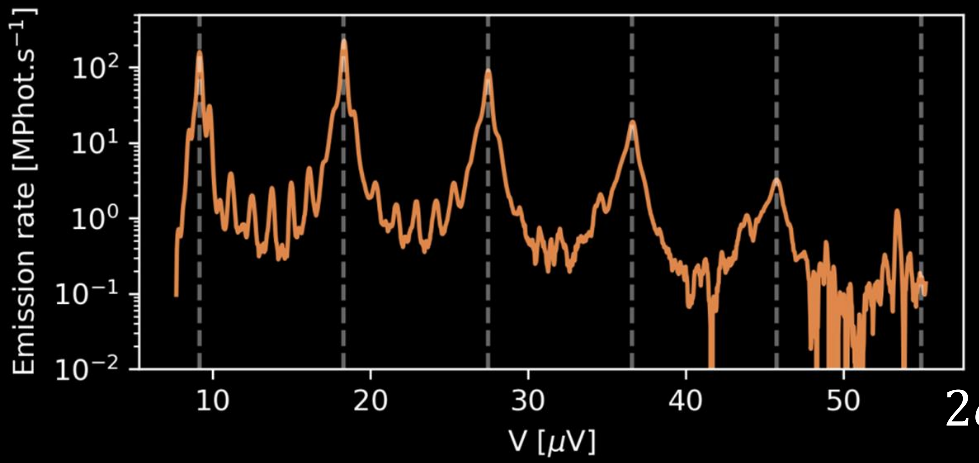
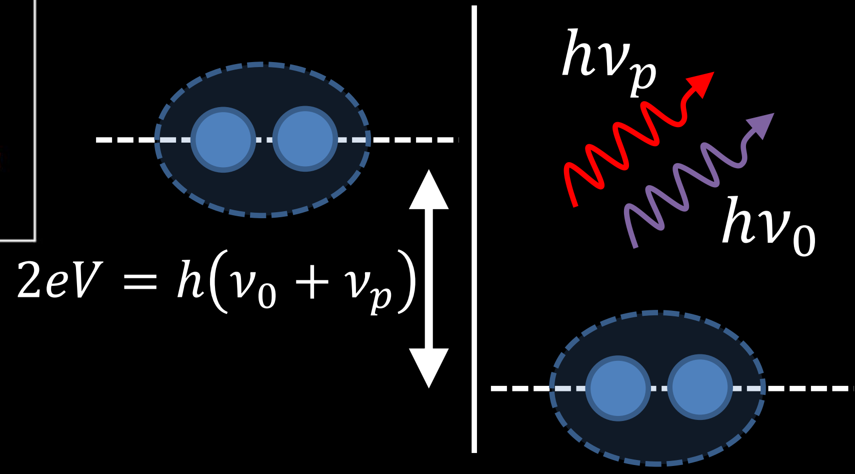
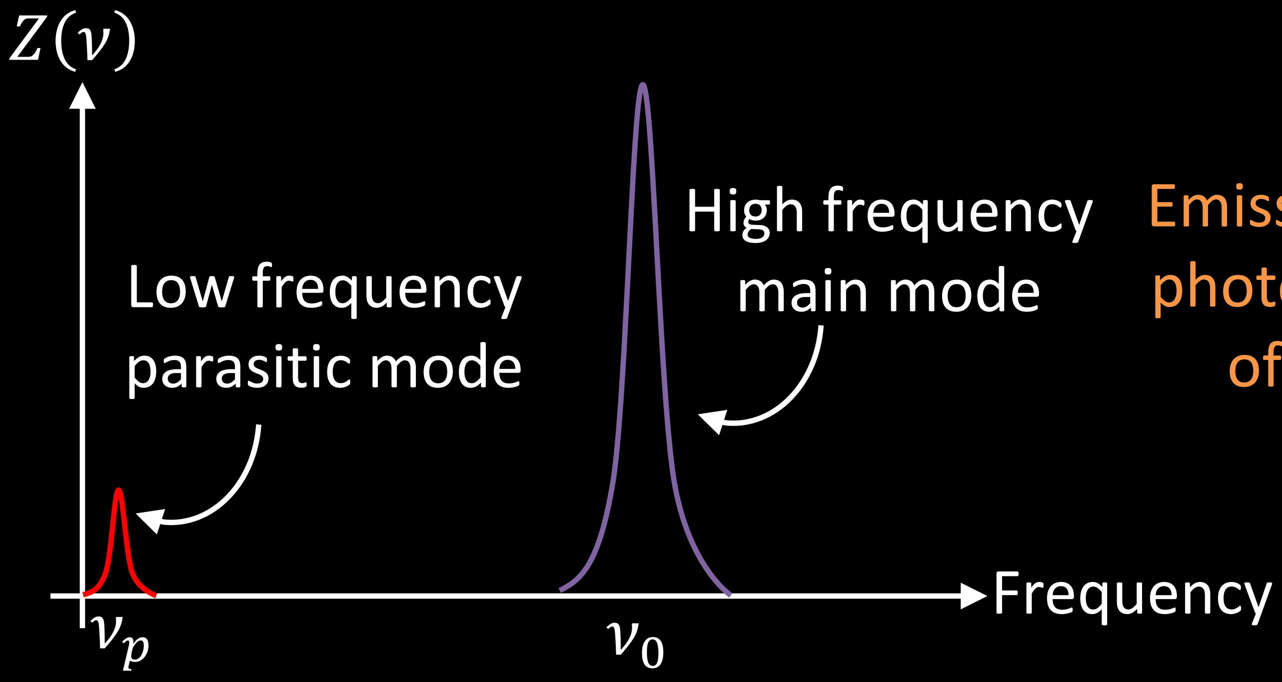
Stokes & antistokes processes







Emission of low frequency photon to allow tunneling of Cooper pair (anti-Stokes)

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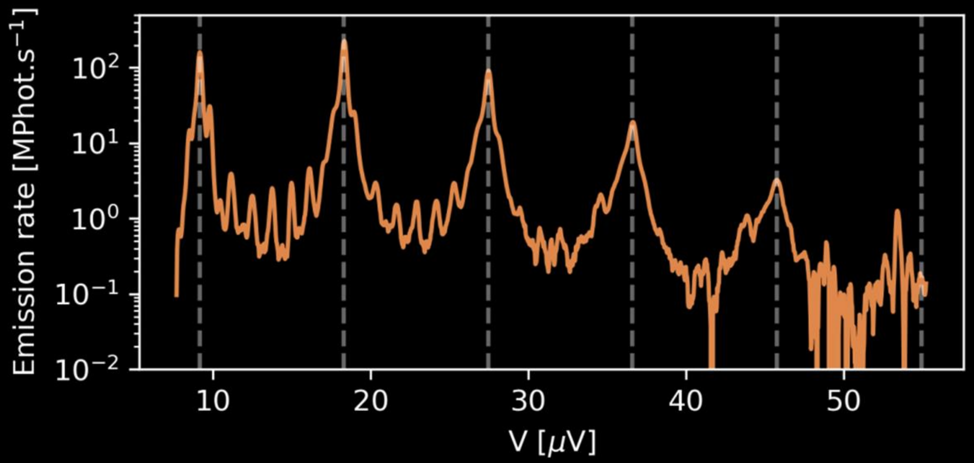
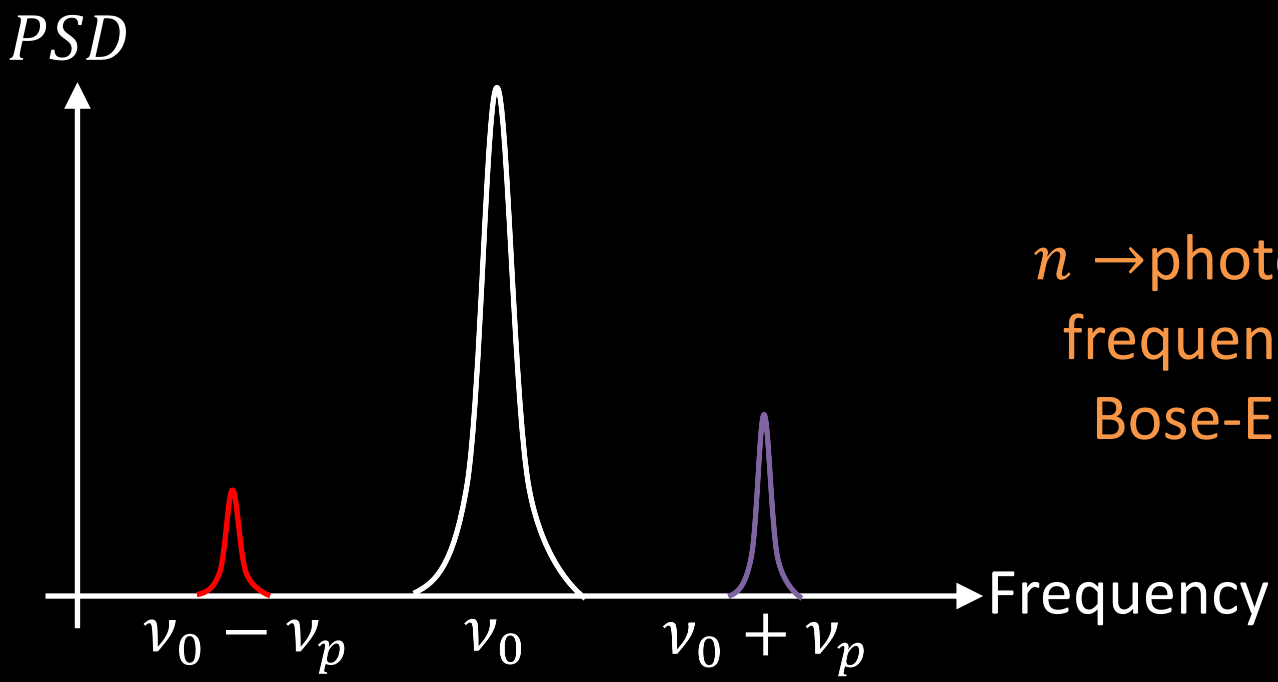
Stokes & antistokes processes





Amplitude of emission depends on temperature, allows for calibration of electronic temperature

$$\frac{A_S}{A_{AS}} = \frac{n}{n+1}$$

n →photon occupation of low frequency mode depend on Bose-Einstein distribution

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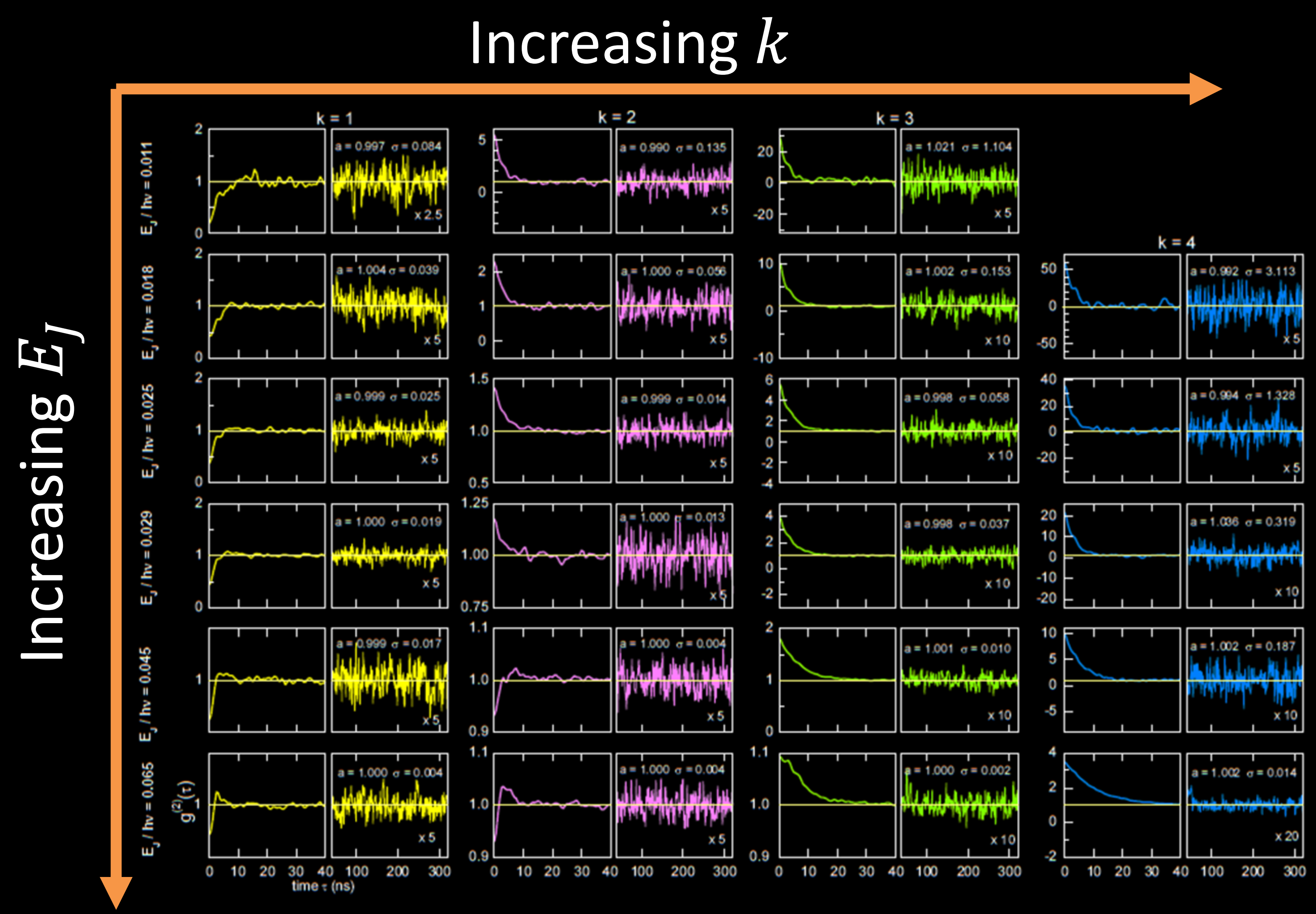
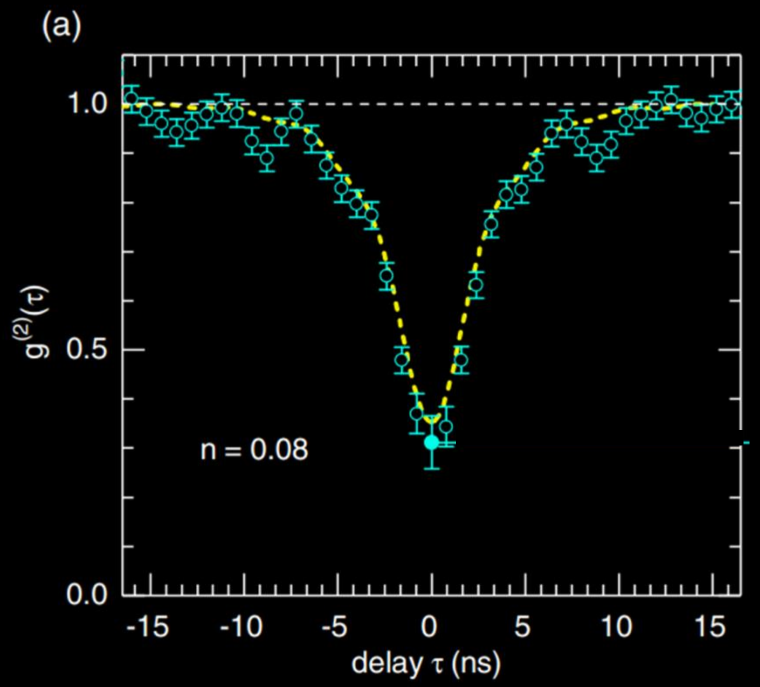
Fano factor: calculation from $g^{(2)}(\tau)$



Antibunching

$g^{(2)} < 1$ signals antibunching of photons

$k = 1$ (previous exp.)



C. Rolland et al. Phys. Rev. Lett. **122**, 186804 (2019)

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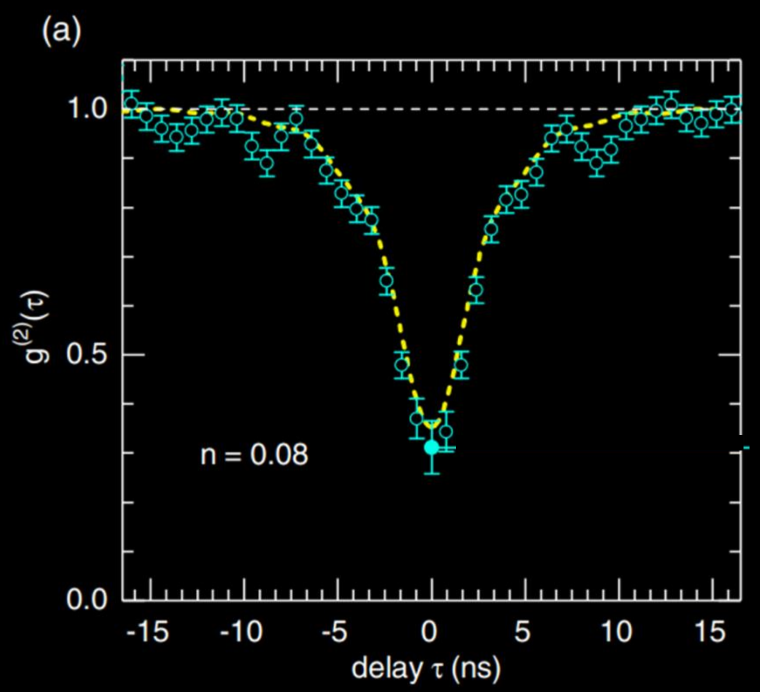
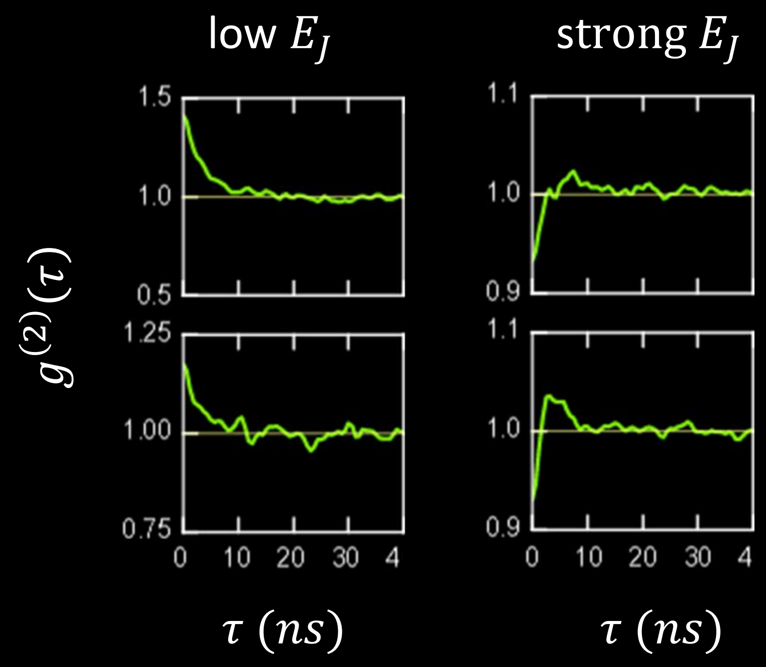


$k = 2$

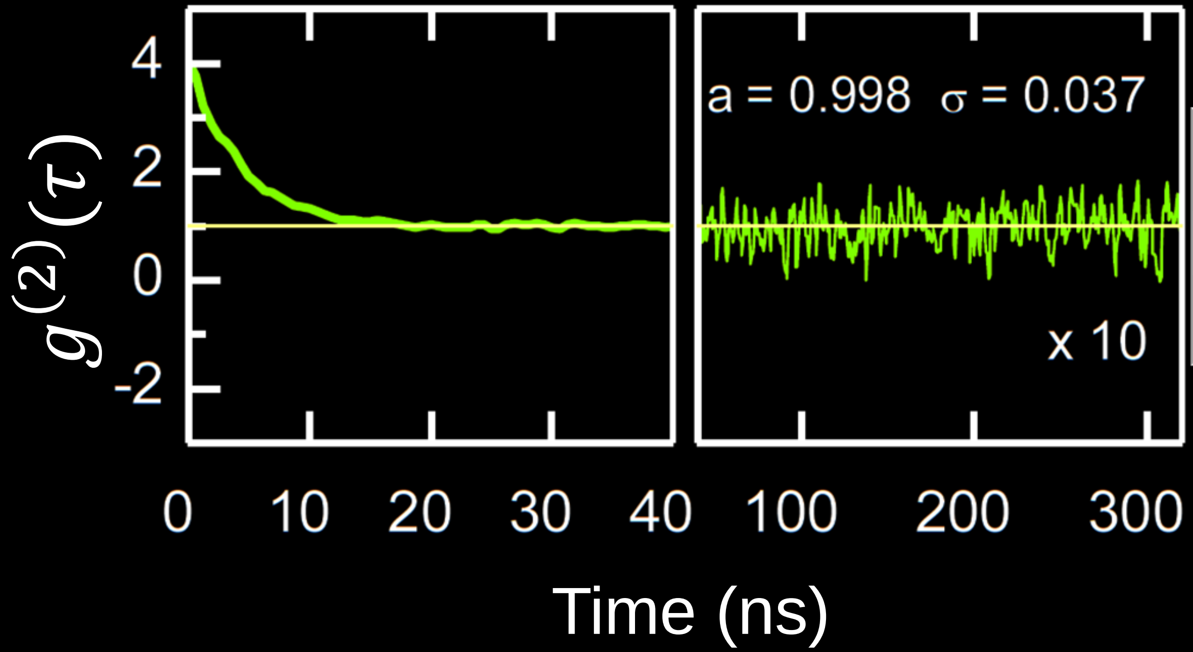


Observation of antibunching for strong E_J at $k = 2$

Fano factor: calculation from $g^{(2)}(\tau)$

Fano factor

$$F_k = 1 + 2\Gamma_k \int_0^\infty [g^{(2)}(\tau) - 1] d\tau$$



$$\frac{E_J}{h\nu} = 0{,}029$$

$$k = 3$$

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Fano factor: calculation from $g^{(2)}(\tau)$

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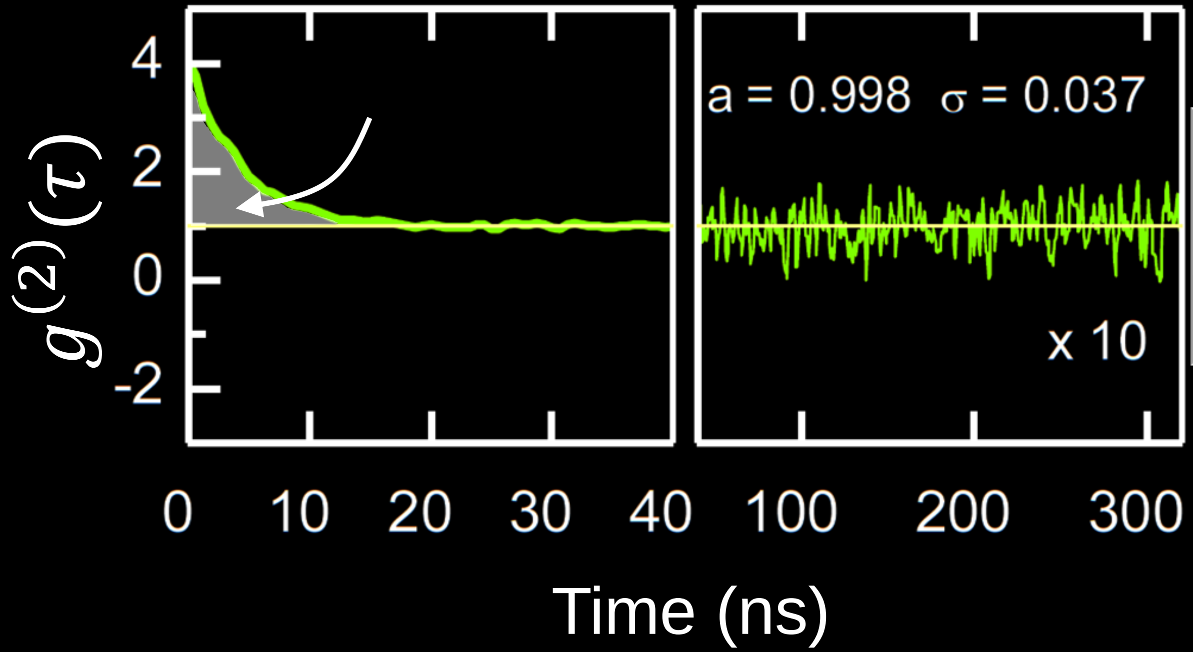

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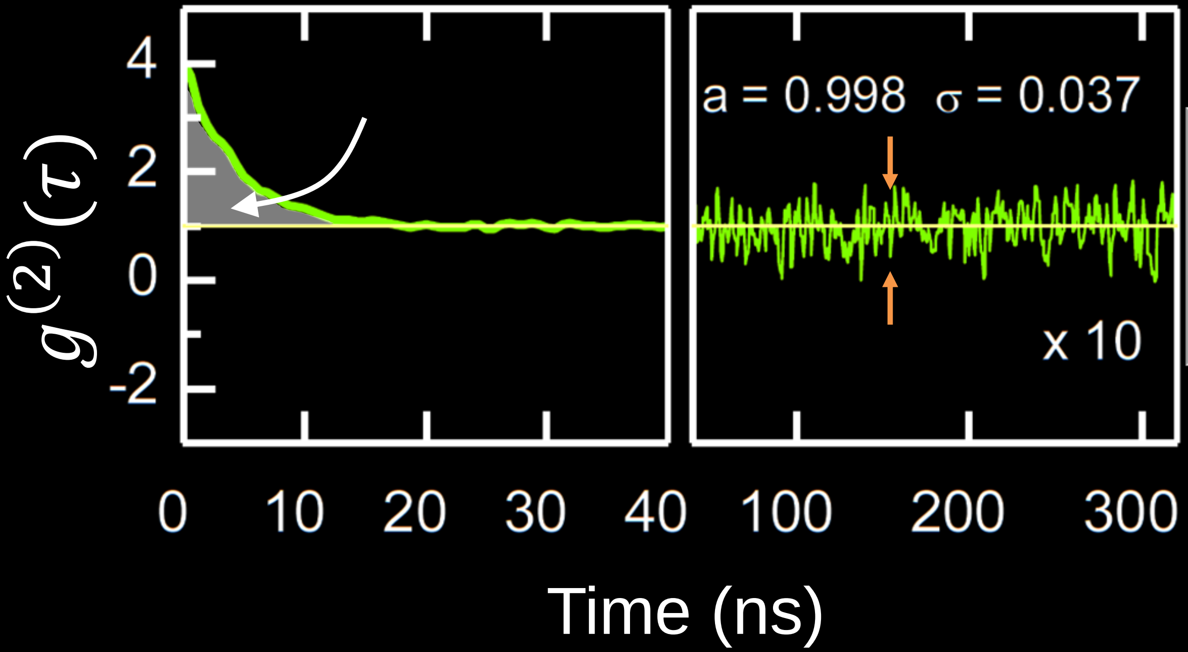

$$\frac{E_J}{h\nu} = 0{,}029$$
$$k = 3$$

Error estimated from long time fluctuations of $g^{(2)}(\tau)$

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