Topology and Chaos of a Quantum Pump

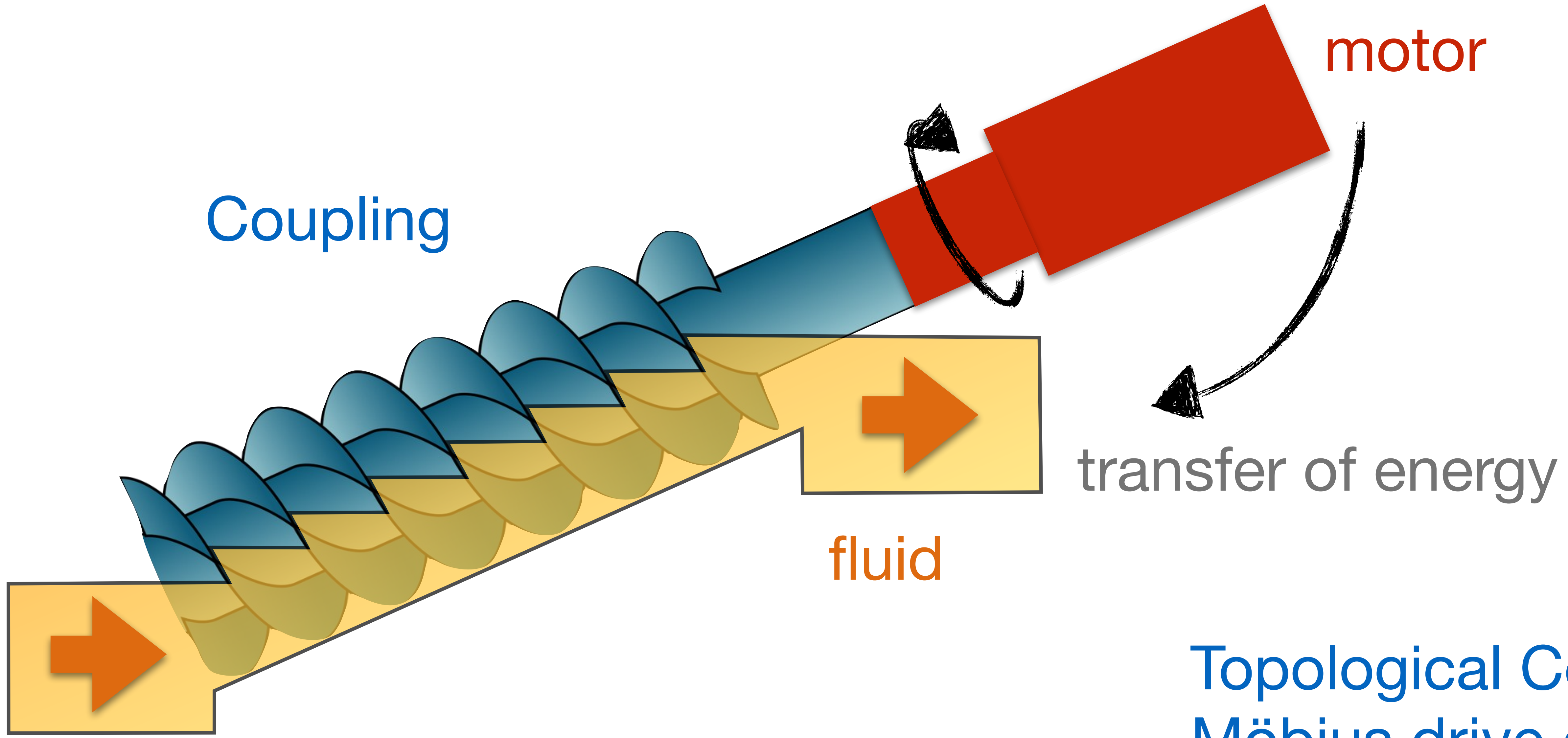
David Carpentier
(Ecole Normale Supérieure de Lyon)


Benoît Douçot

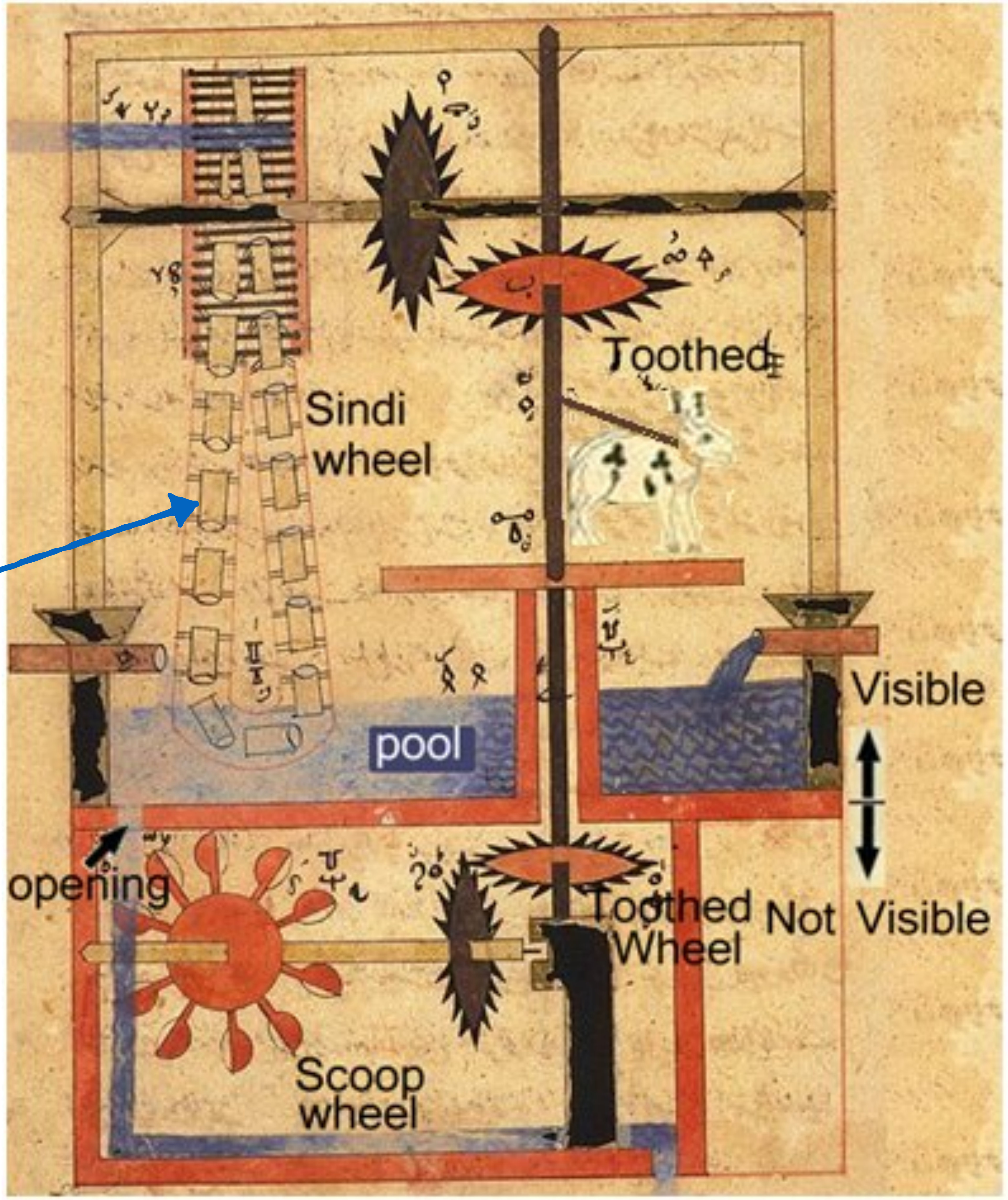

Jacquelin Luneau


Tommaso Roscilde

What is a (topological) pump ?



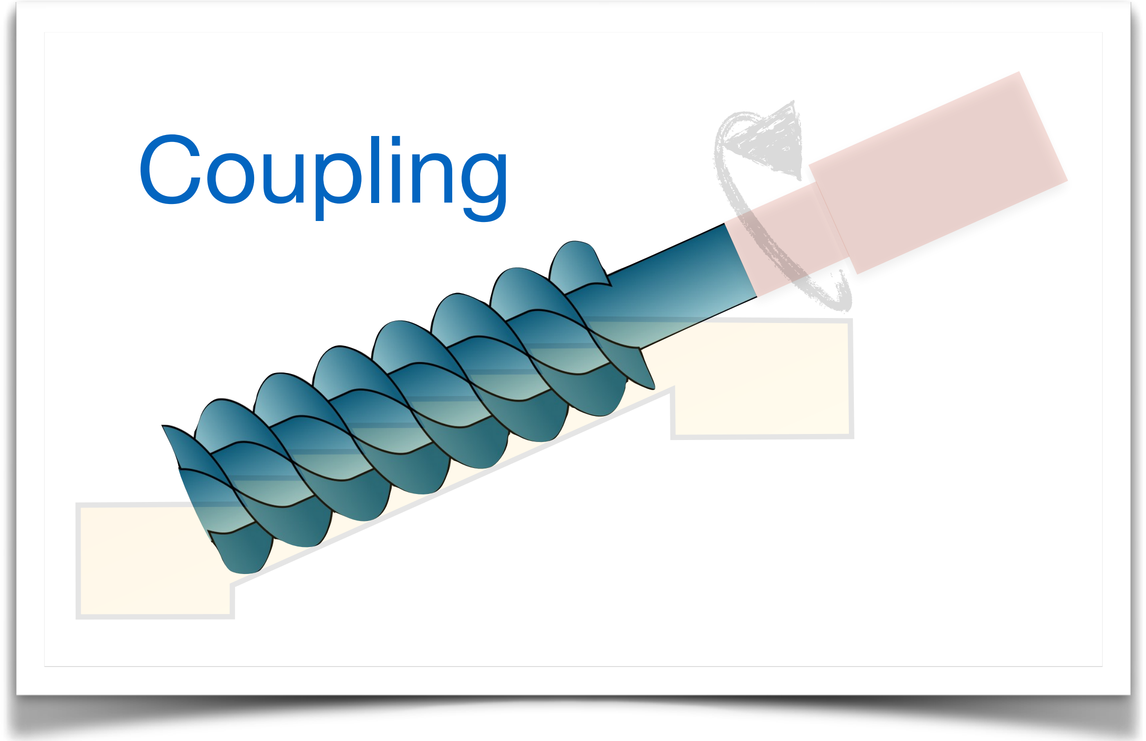
Ismail al-Jazari (1206)

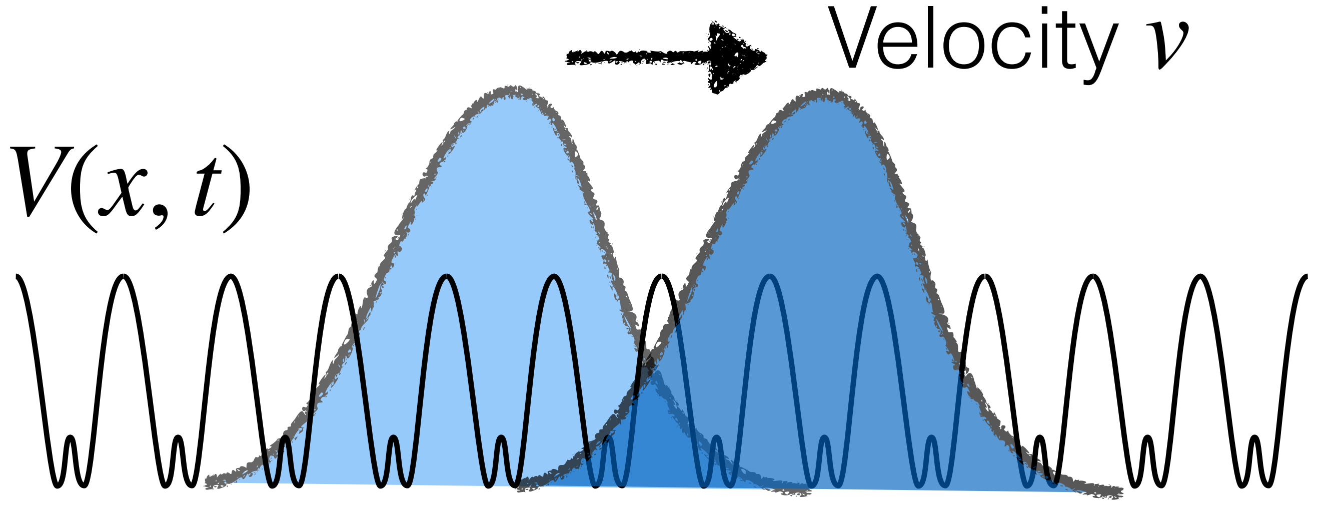


Topological Coupling:
Möbius drive chain

Twisted conveyor belt
in a machine shop

Topological quantum pumps: standard perspective



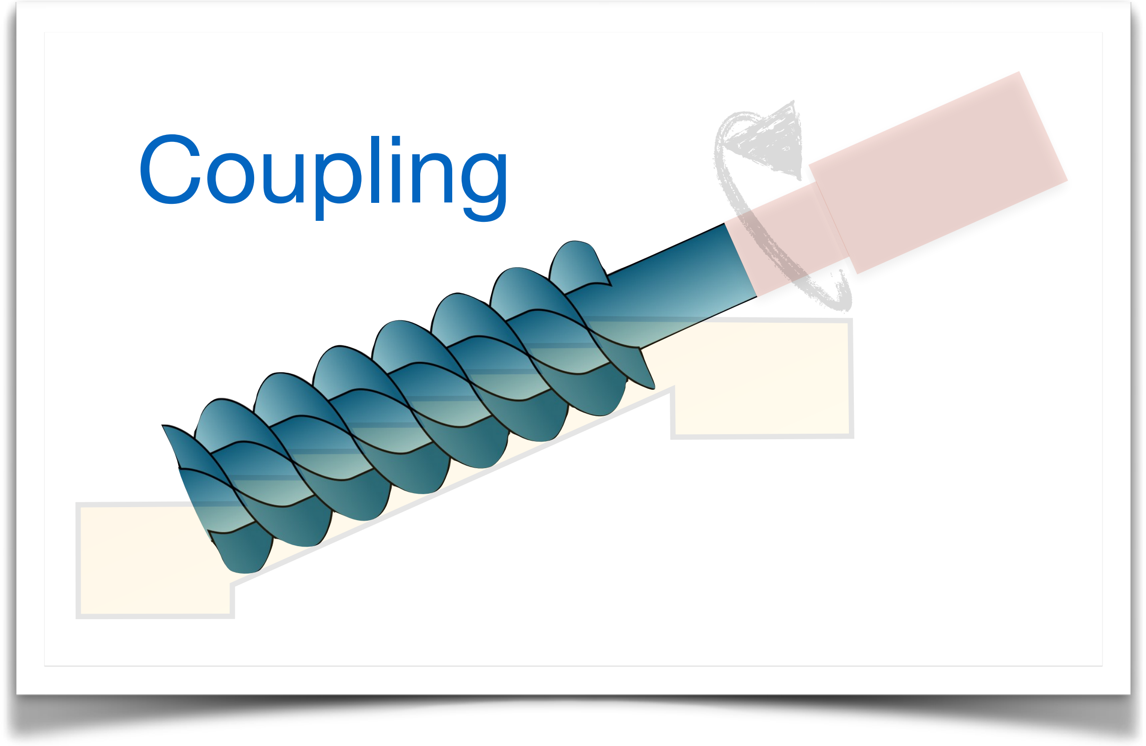


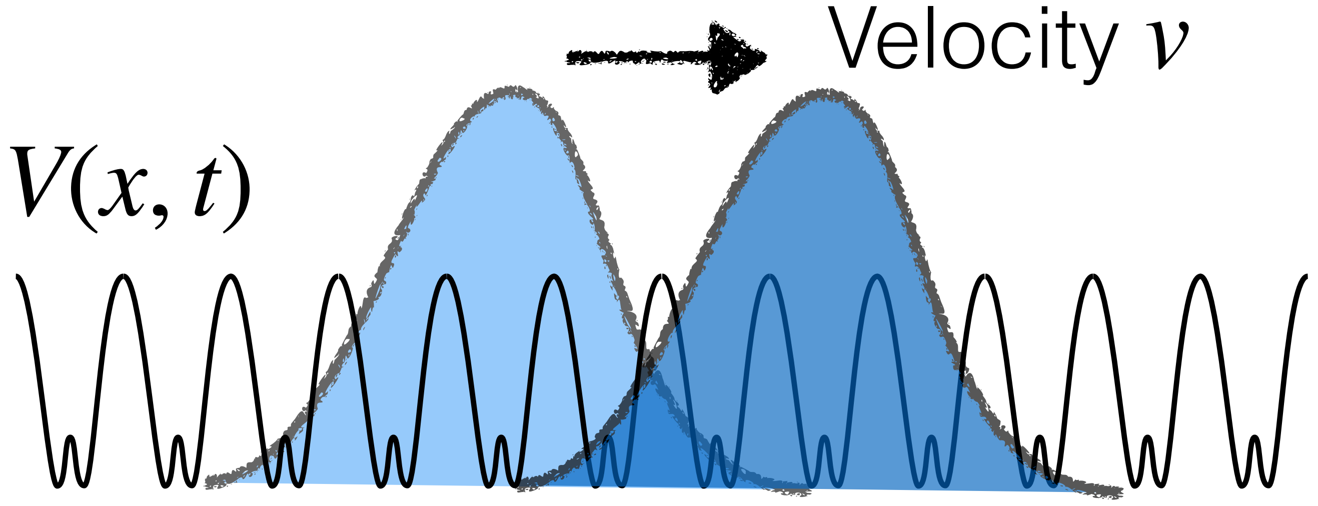
Thouless pump

- dimension **d=1**
- potential periodic in space and time $V(x, \phi_1(t))$
- velocity : $v = \mathscr{C}\dfrac{a}{T}$ ↔ topological (Chern) number $\mathscr{C}$

D. Thouless (1983)
Q. Niu, D. Thouless (1984)

Topological quantum pumps: standard perspective





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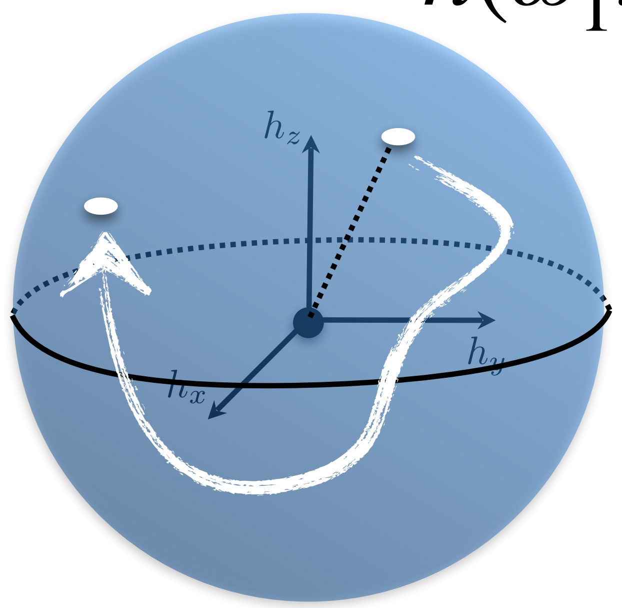


Topological frequency converter

I. Martin, G. Refael, B. Halperin (2017)

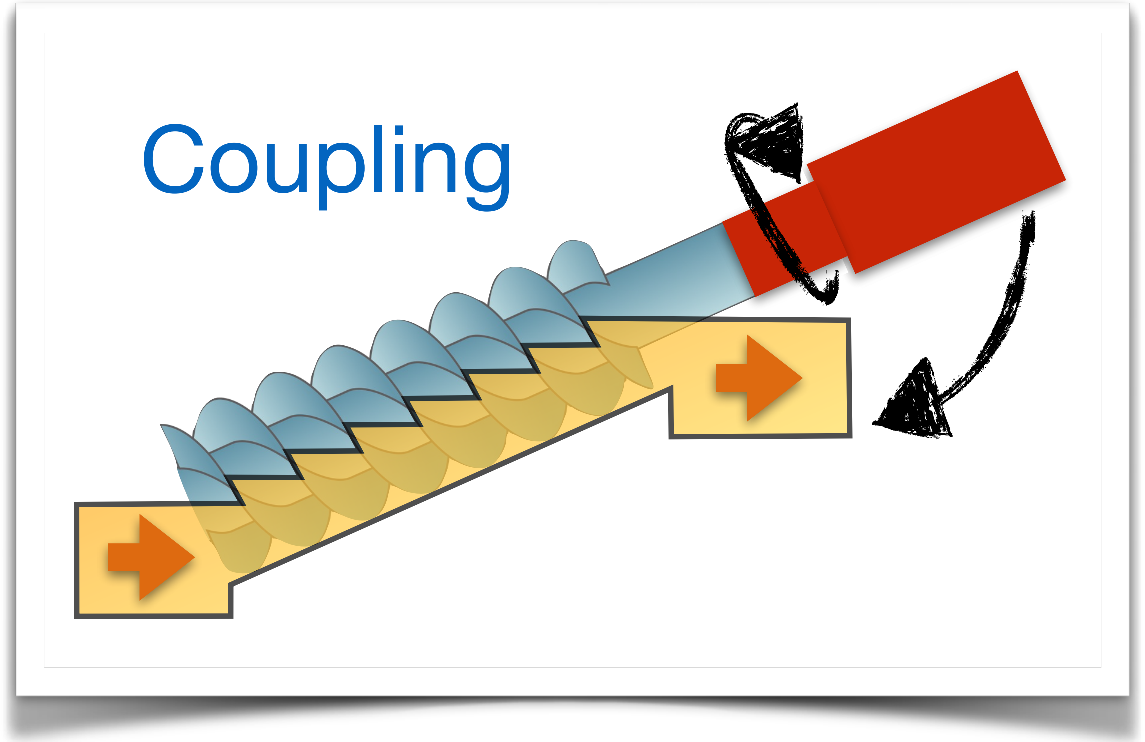
- dimension **d=0** (qubit), $H(\phi_1(t), \phi_2(t))$
- Transverse velocity in harmonic spaces (Floquet theory)

Multi-terminal Josephson junctions

R.-P. Riwar, M. Houzet, J.S. Meyer, and Y.V. Nazarov (2016)

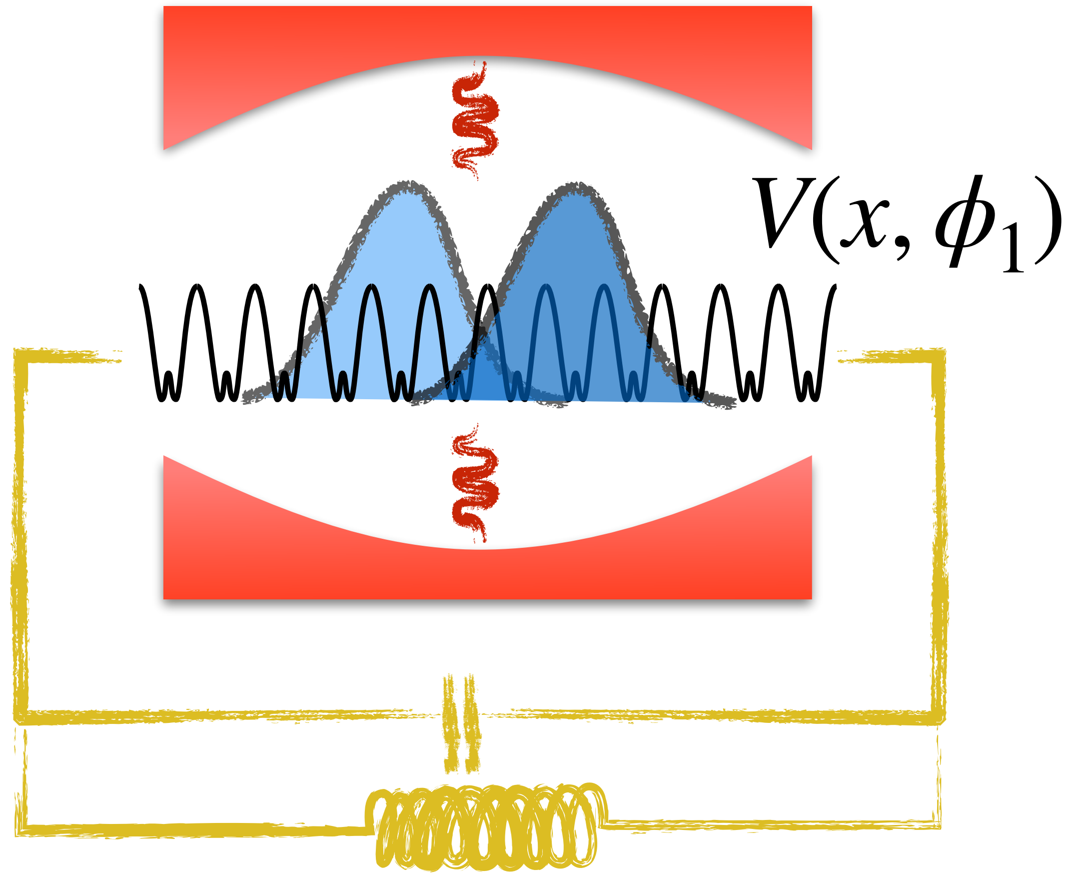
- dimension **d=0**
- quantized transconductance

Topological quantum pumps: coupling fast-slow d.o.f.



$n_1, \phi_1 = \omega_1 t$

Φ_2, Q_2



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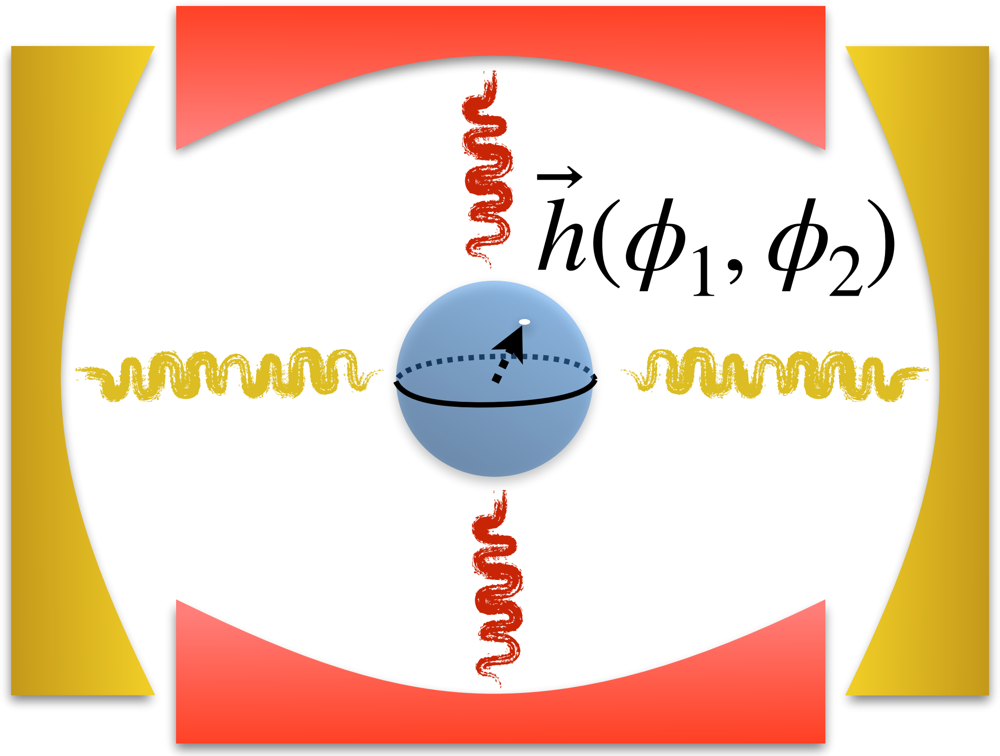
- potential periodic in space and time $V(x, \phi_1(t))$
- velocity : $v = \mathscr{C}\dfrac{a}{T}$ $\leftrightarrow$ topological (Chern) number $\mathscr{C}$
- $I = \dfrac{e}{\hbar}\dot{Q}_2 = \dfrac{e}{2\pi}\mathscr{C}\omega_1 = \mathscr{C}\dfrac{e}{2\pi}\dot{\phi}_1$

$n_1, \phi_1 = \omega_1 t$

$n_2, \phi_2 = \omega_2 t$



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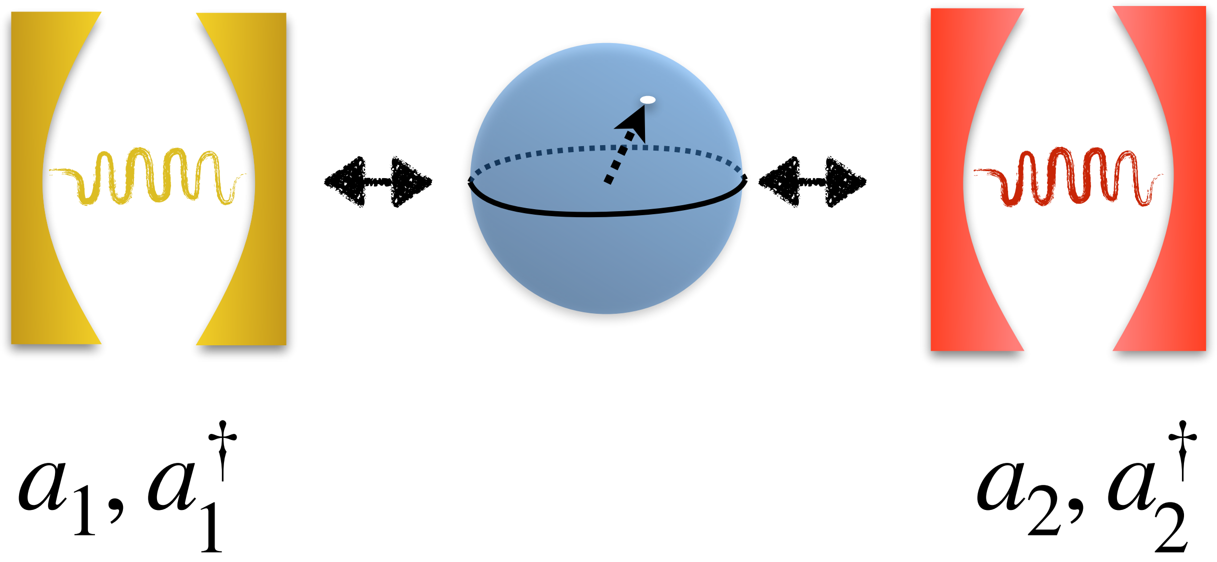
- dimension **d=0** (qubit), $H(\phi_1(t), \phi_2(t))$
- Transverse velocity in harmonic spaces (Floquet theory)
- $\hbar \dot{n}_2 = \mathscr{C}\dfrac{\hbar}{2\pi}\dot{\phi}_1$

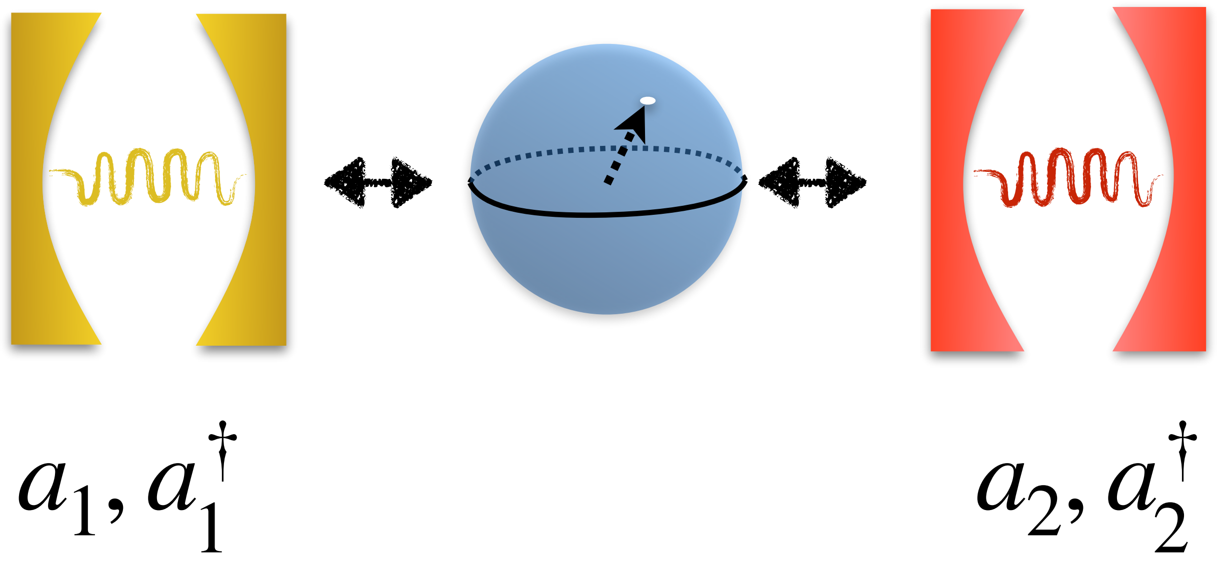
Topological quantum coupling



- Hamiltonian $\hat{H} = \hbar\omega_1\, a_1^\dagger a_1 + \hbar\omega_2\, a_2^\dagger a_2 + H_c$

$$H_c/\hbar = g_1(a_1 + a_1^\dagger)\sigma_x + g_2(a_2 + a_2^\dagger)\sigma_y + \left(\omega_q + ig_1(a_1 - a_1^\dagger) + ig_2(a_2 - a_2^\dagger)\right)\sigma_z$$

Topological quantum coupling



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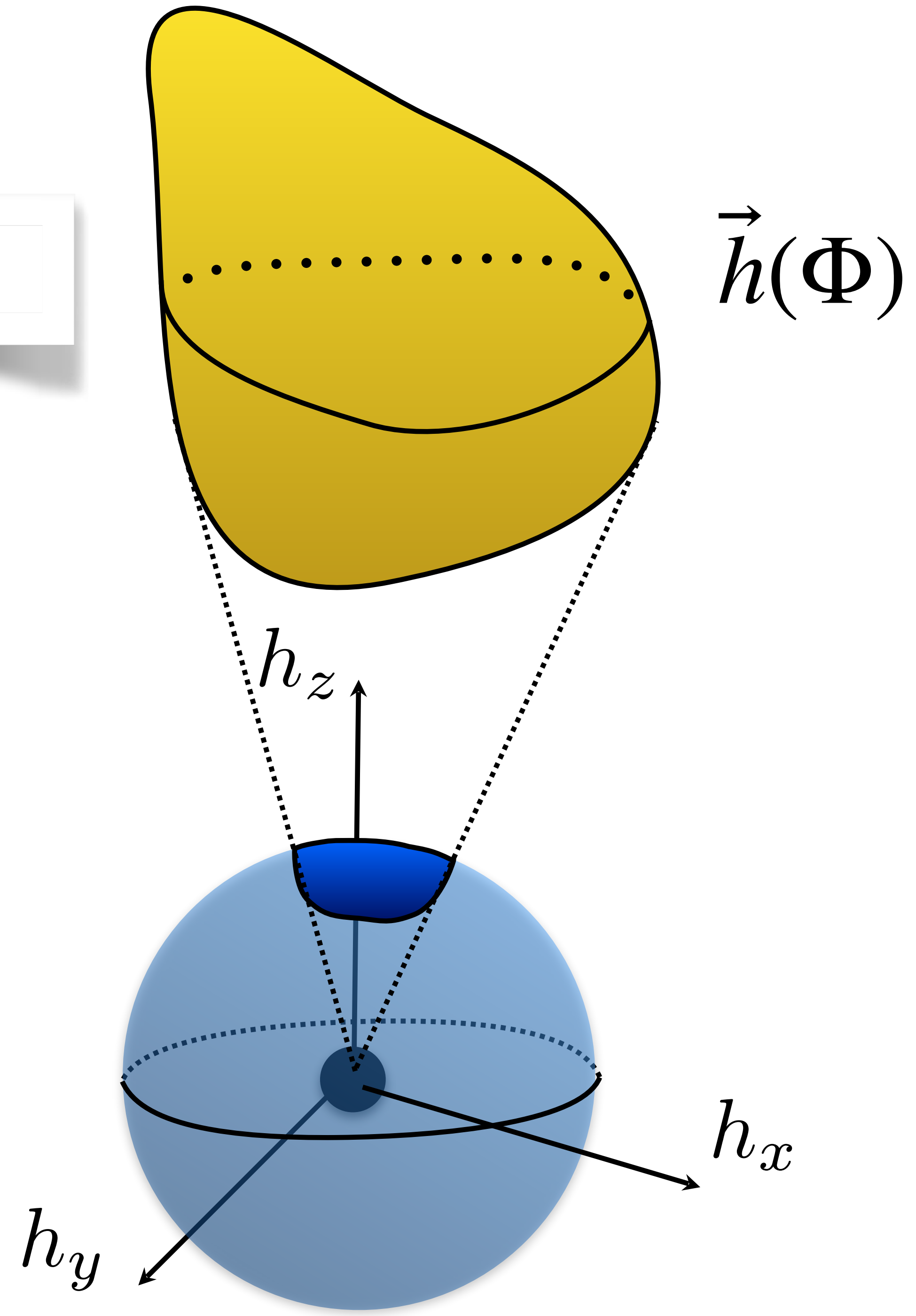
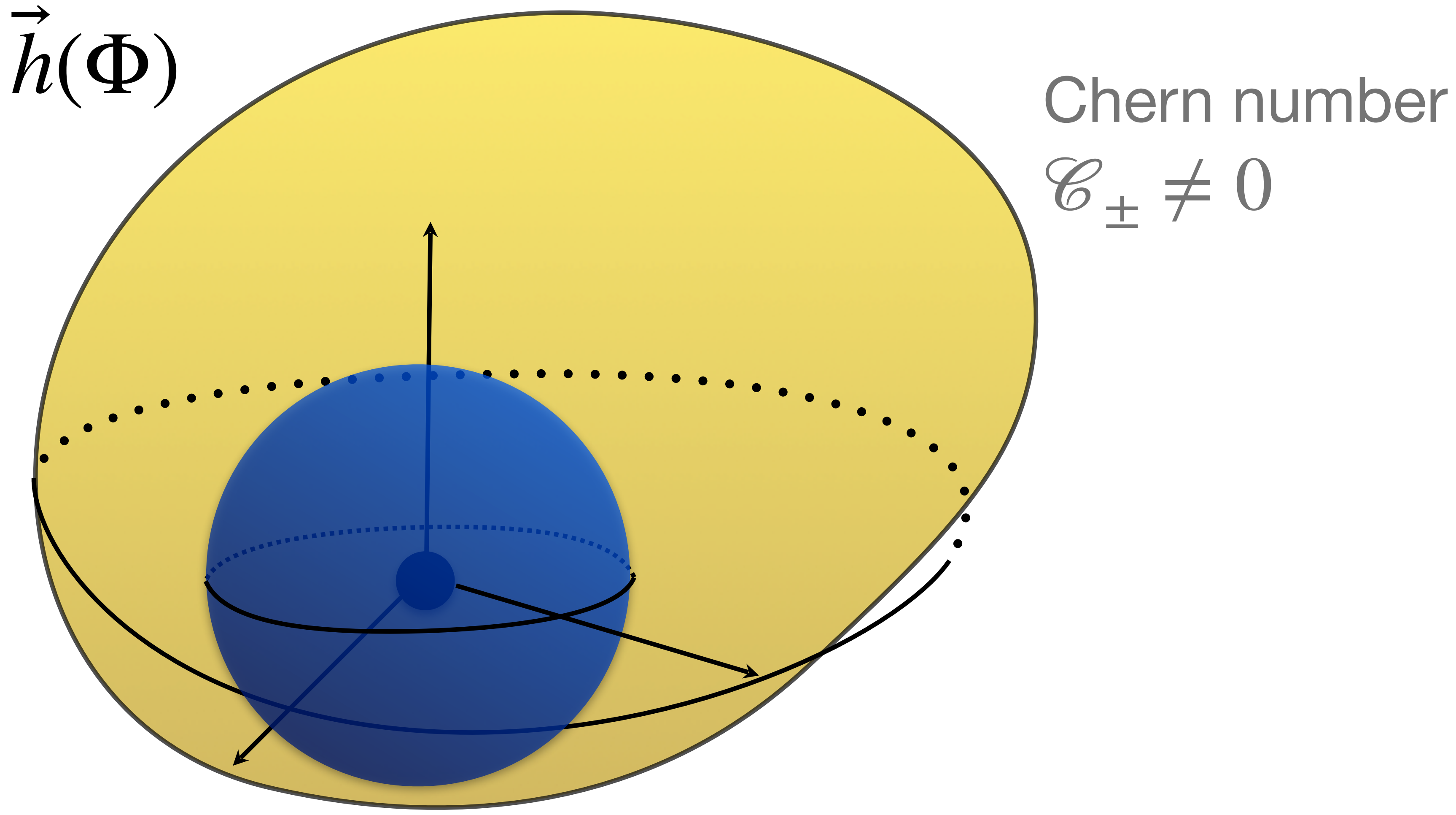
- Semiclassical analysis : $a_j \to \sqrt{n_i}e^{i\phi_j},\; H_c \to \hbar\, \vec{h}(n_1, n_2, \phi_1, \phi_2)\,.\,\vec{\sigma}$
- Topological coupling : fixed n_1, n_2, gapped $\vec{h}(\Phi) = \vec{h}(n_1, n_2, \phi_1, \phi_2)$

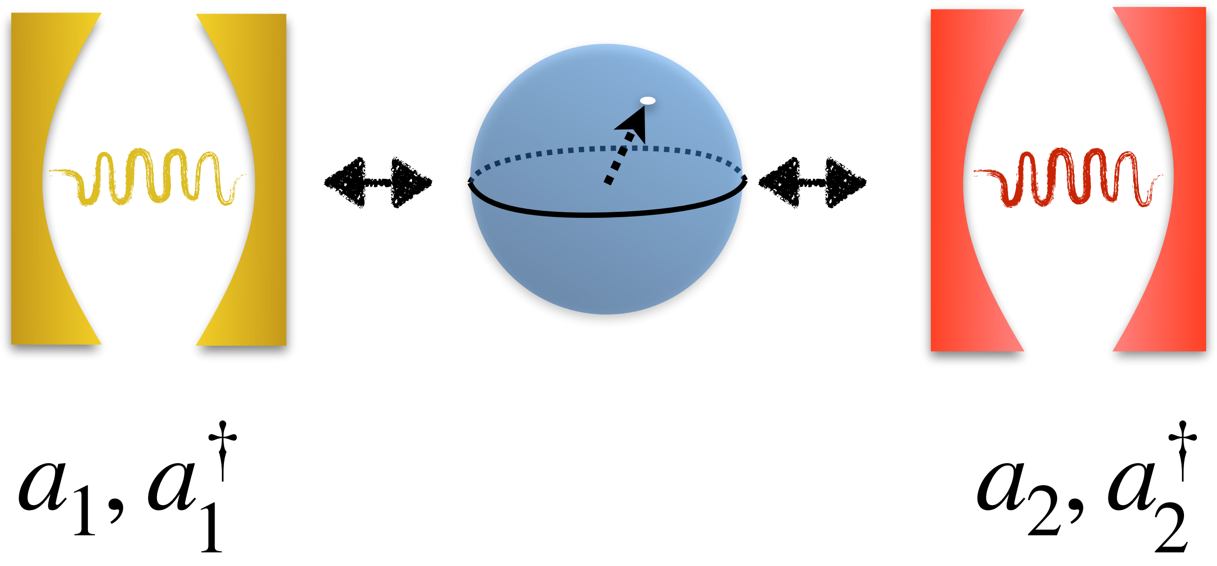
Trivial gap



Topological gap



Topological quantum coupling



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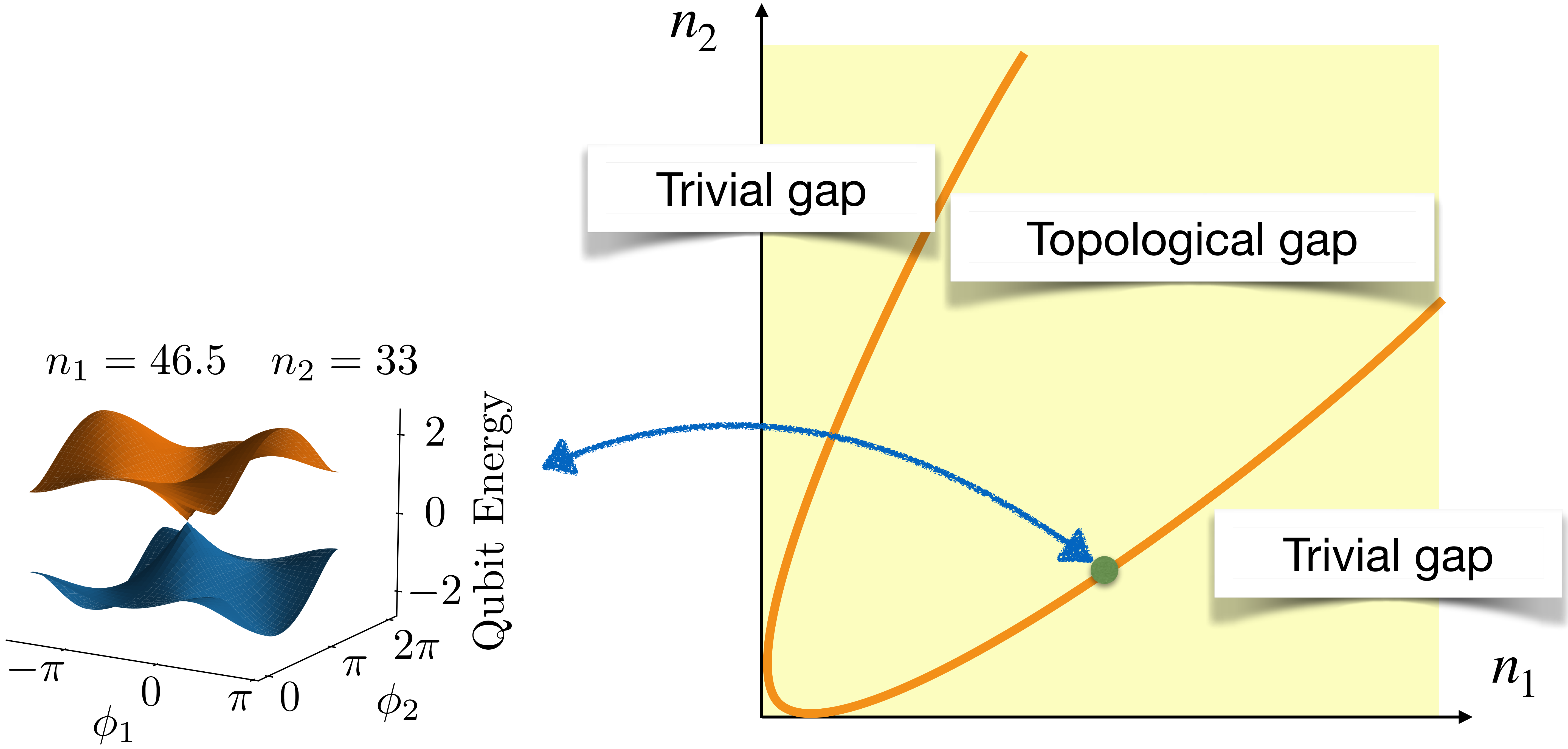
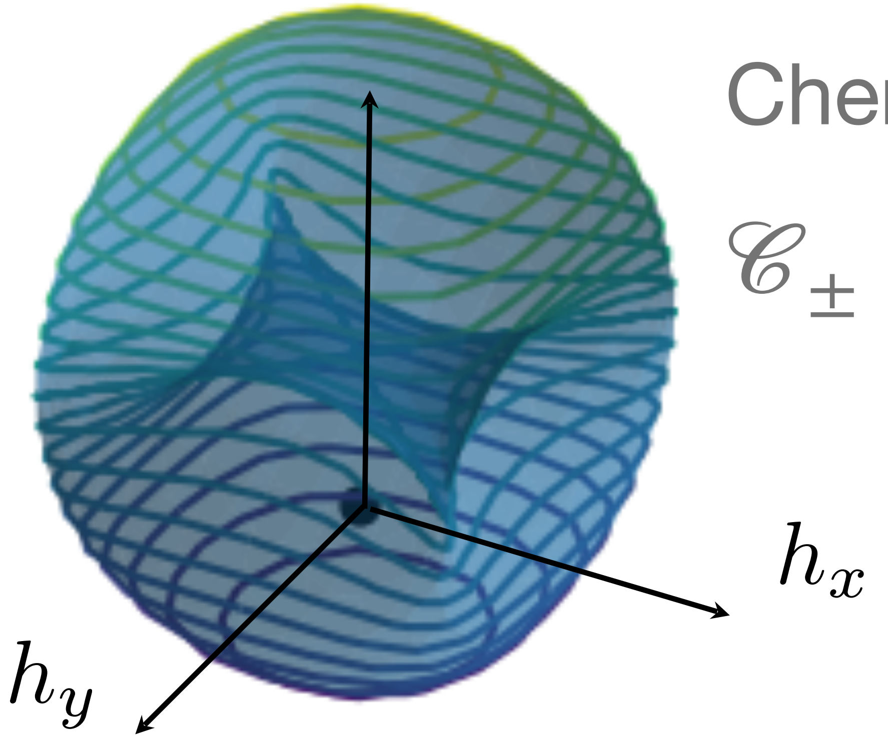


Half-BHZ model

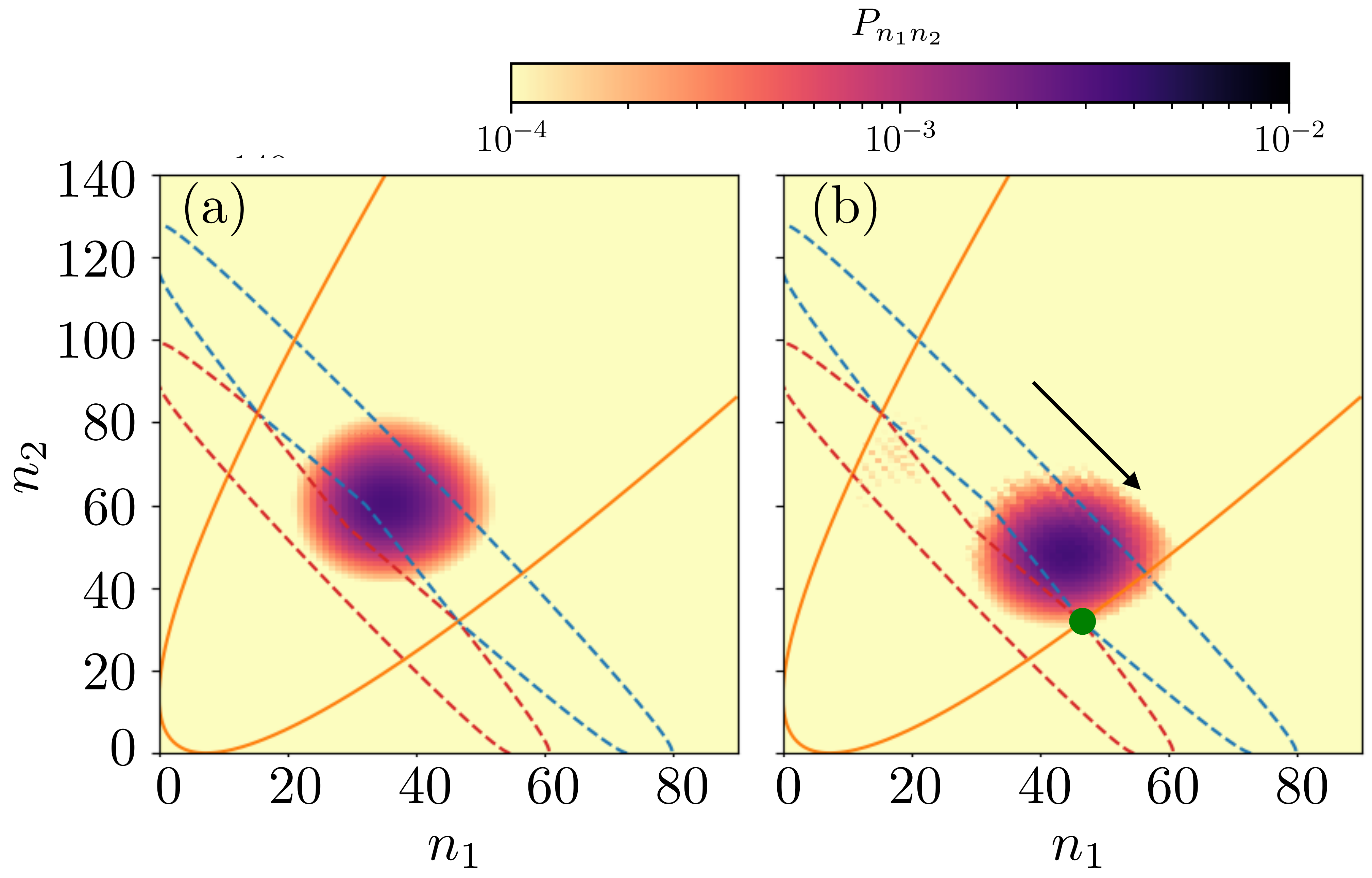
$$\vec{h} = \begin{pmatrix} v_1 \sin(\phi_1) \\ v_2 \sin(\phi_2) \\ m - b_1\cos(\phi_1) - b_2\cos(\phi_2) \end{pmatrix}$$



Chern number

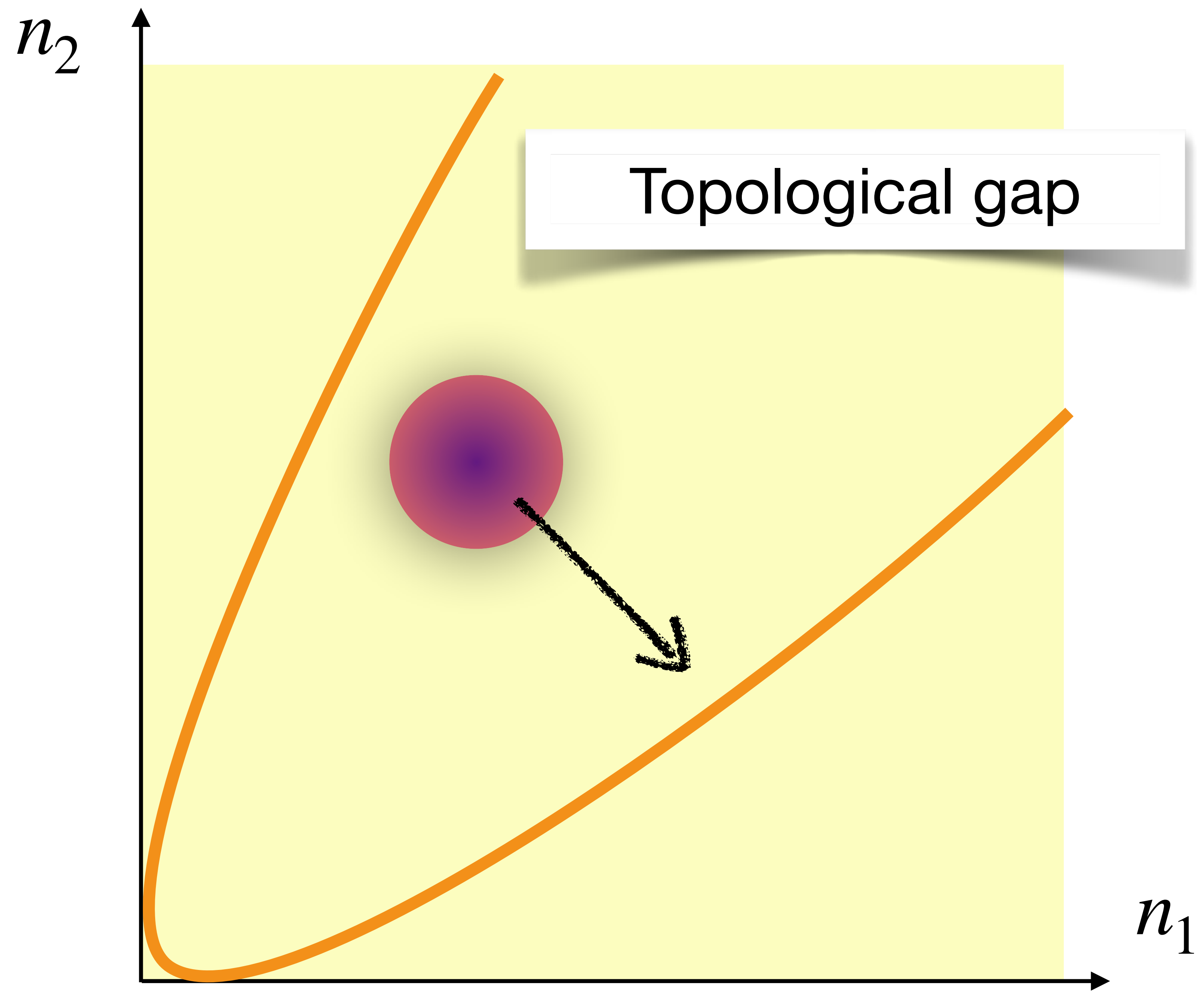
$\mathscr{C}_\pm = \mp 1$

Adiabatic topological pumping

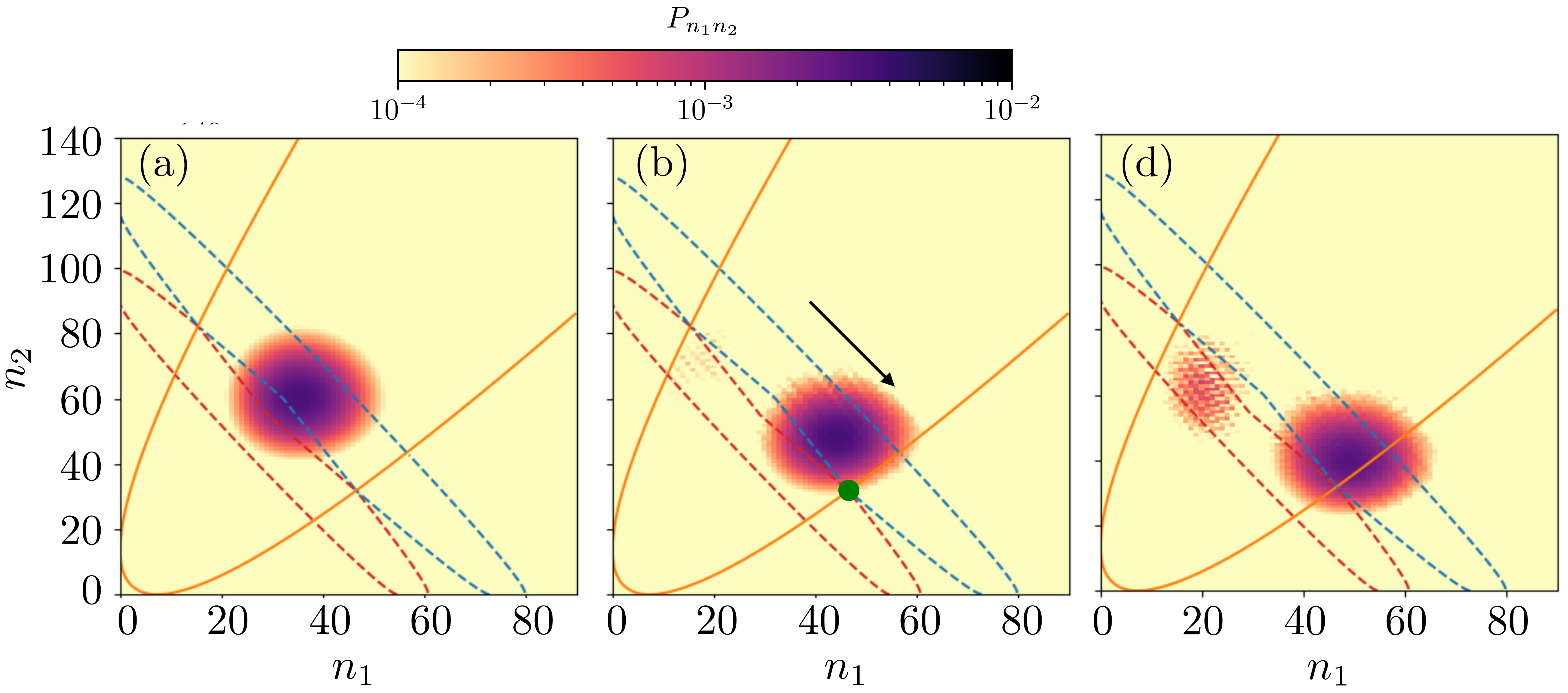


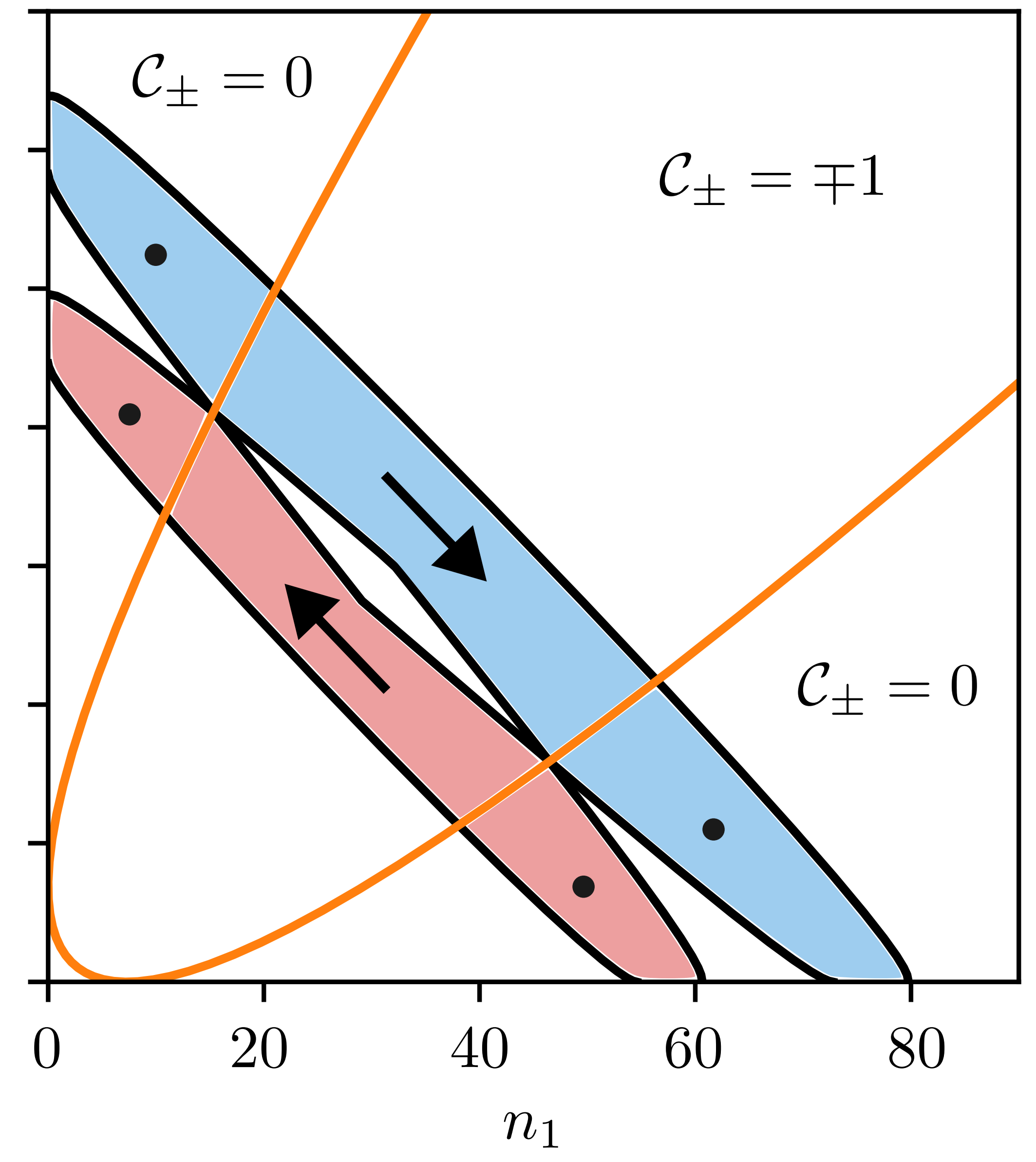
1. Topological (adiabatic) pumping

$$\hbar\omega_1\dot{n}_1 = \frac{\mathscr{C}}{2\pi}\hbar\omega_1\omega_2 = -\hbar\omega_2\dot{n}_2$$



Landau-Zener transitions





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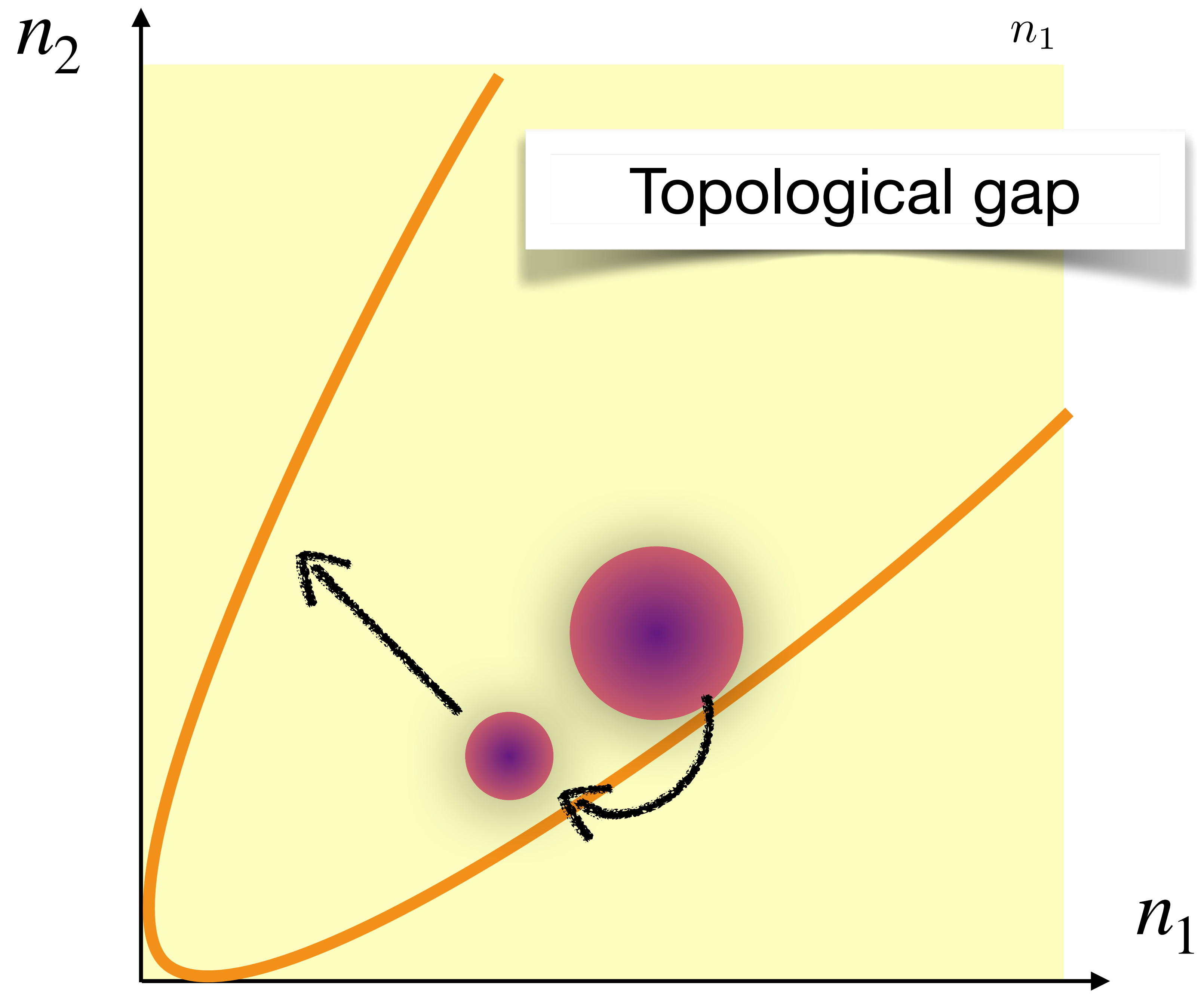
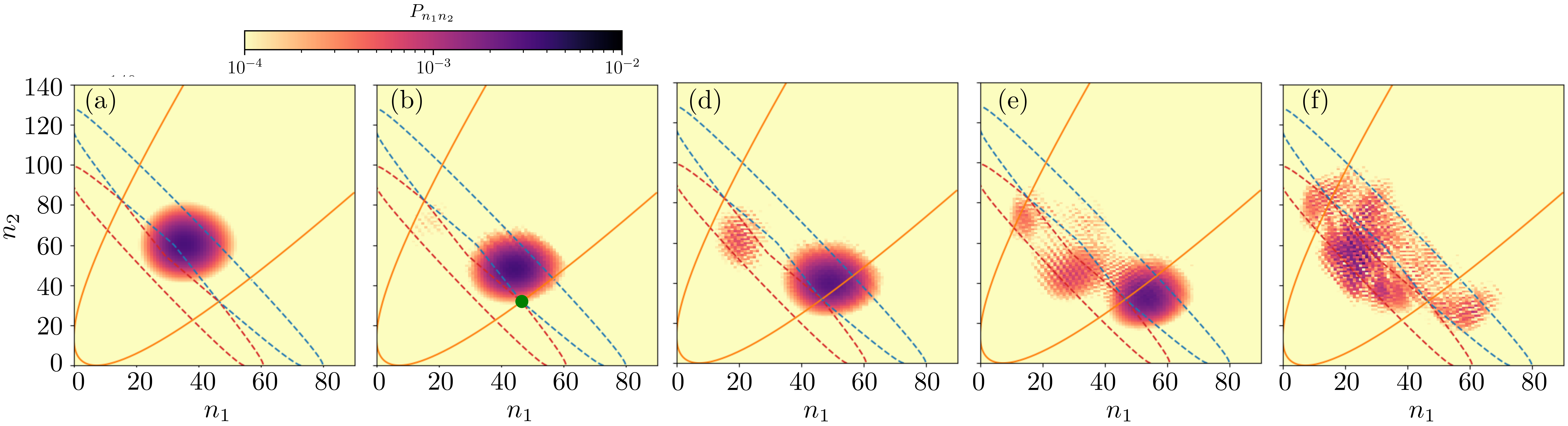
2. Landau Zener transition + pumping in reversed direction (qubit in excited state)



Chaotic Dynamics



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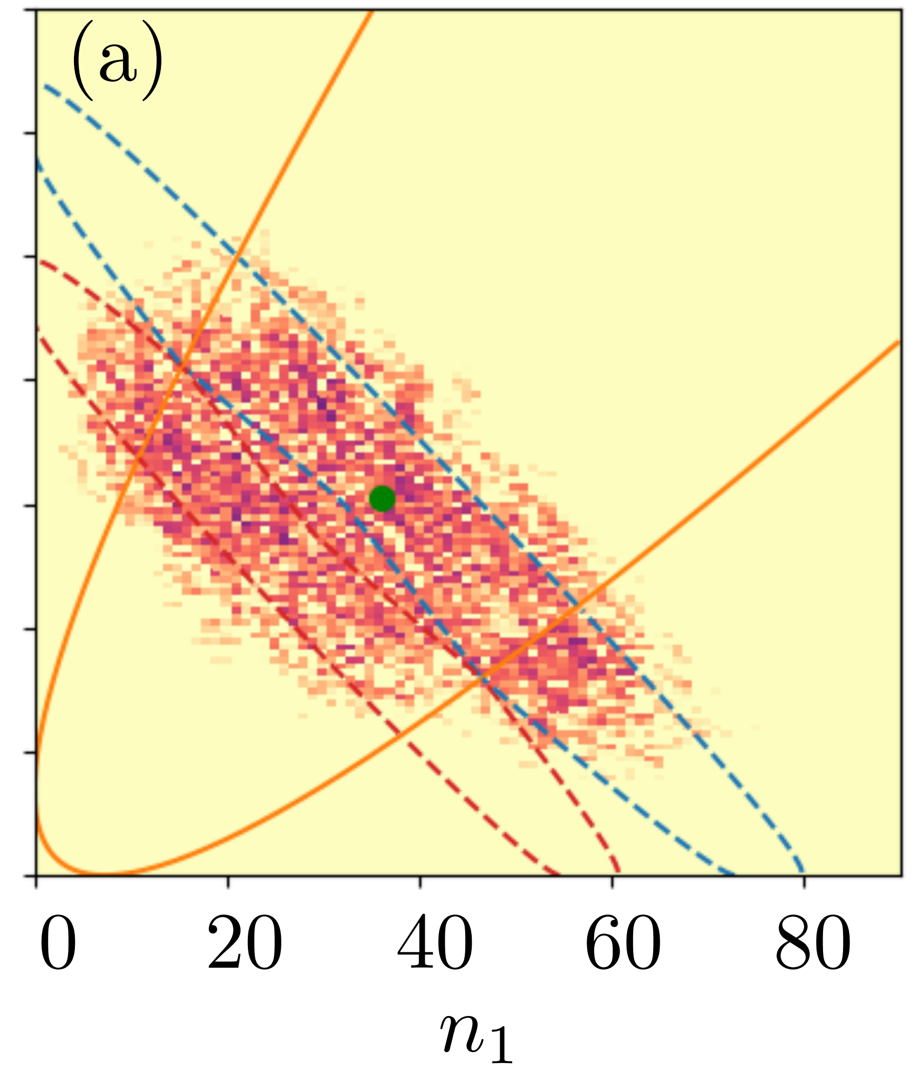
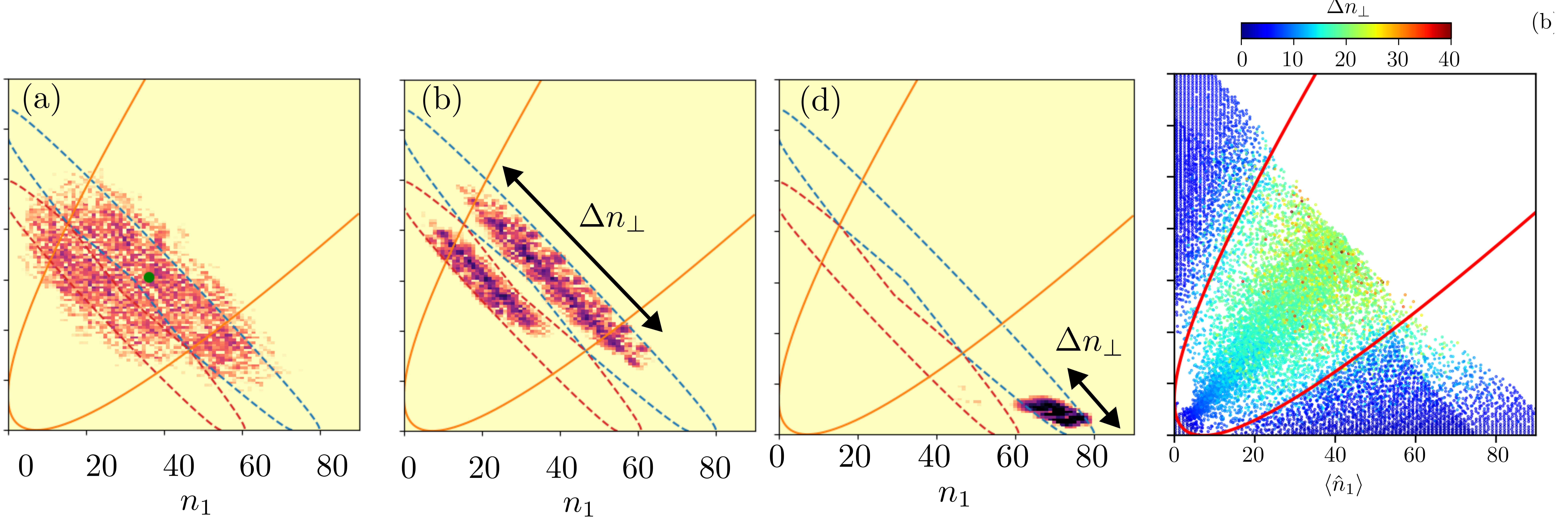
3. Stationnary state : characteristic of topological chaotic dynamics



Chaotic Dynamics and eigenstates

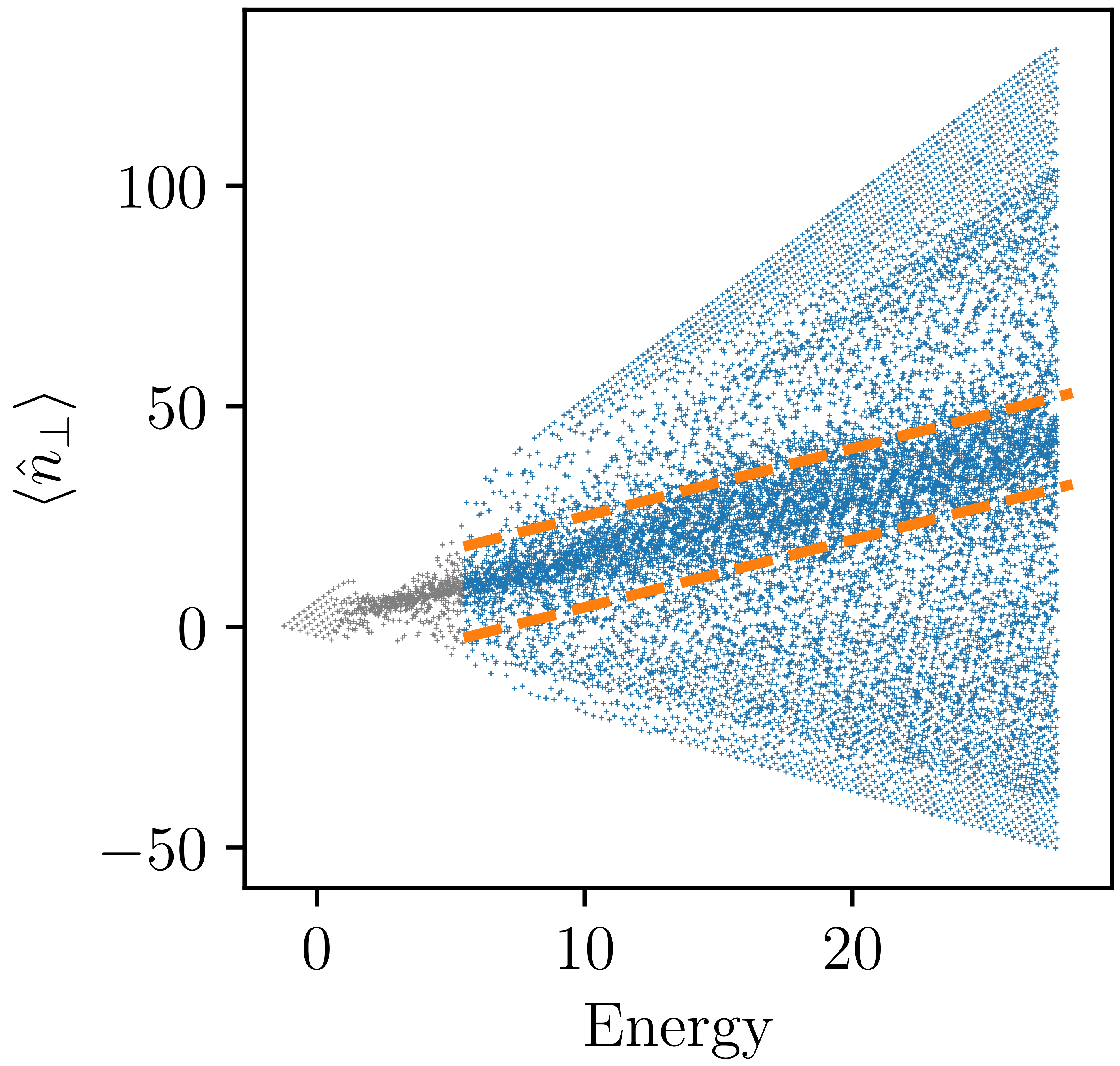


Stationnary state characteristic of topological chaotic dynamics

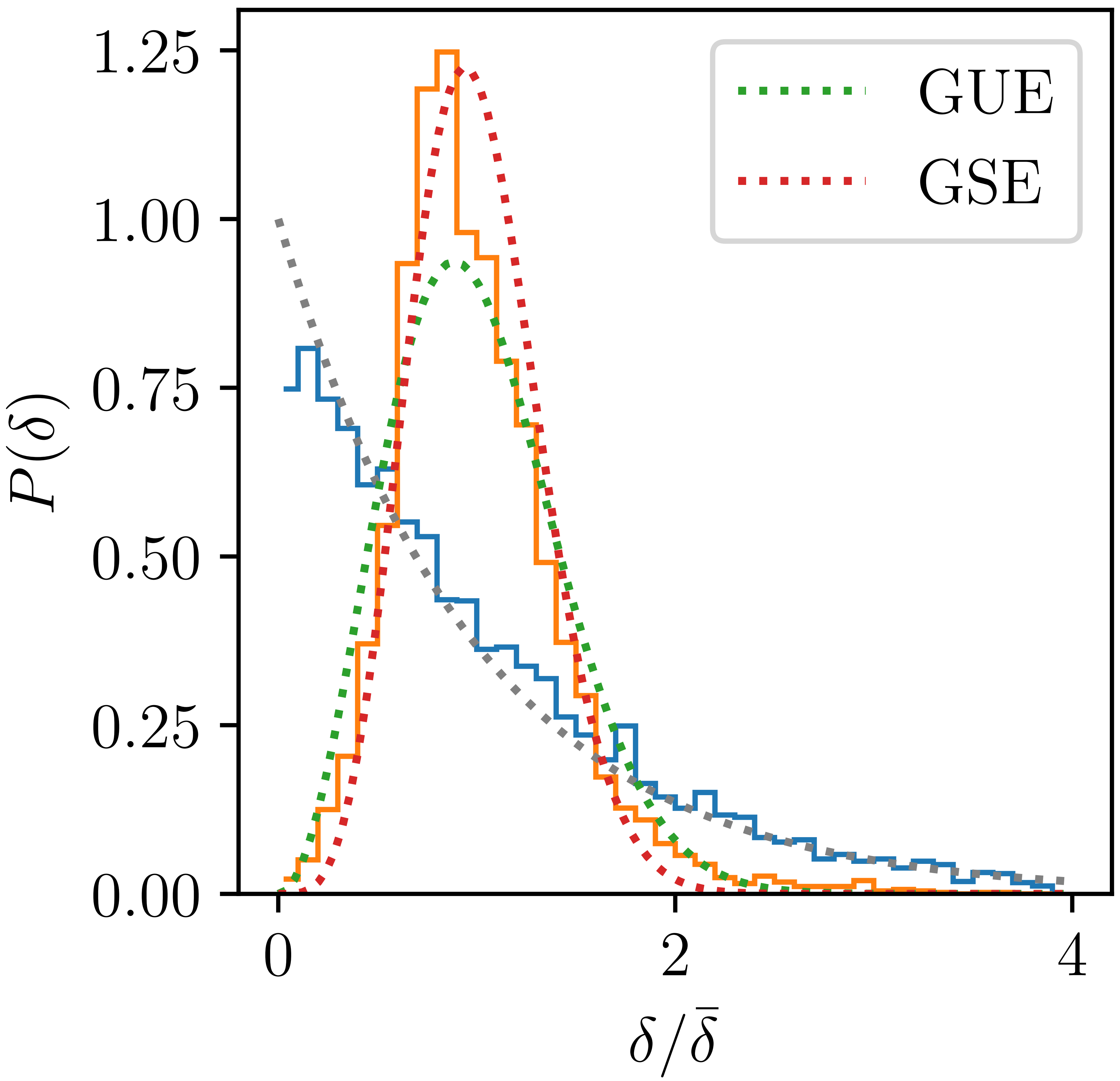
Eigenstate in « topological region »

Eigenstate in « trivial region »

Eigenstates



Uneven distribution of eigenstates :
« topological » vs « trivial »



2 level distributions of eigenenergies:

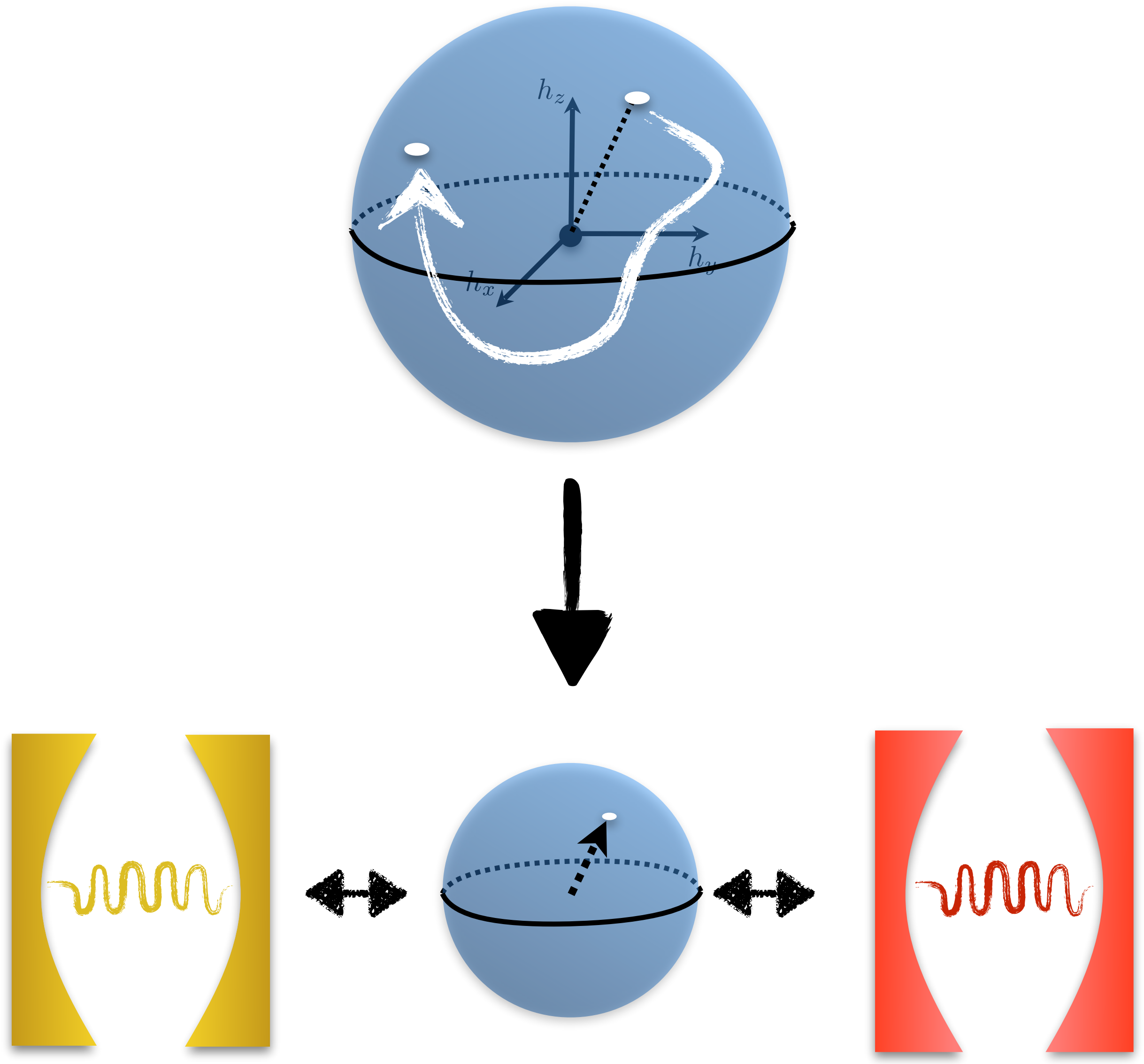
- Level repulsion for« topological »eigenstates
- No level repulsion for « trivial » ones

Summary

J. L., C. Dutreix, Q. Ficheux, P. Delplace, B. Douçot, B. Huard, D. Carpentier, Phys. Rev. Research 4, 013169 (2022).
J. L., B. Douçot, D. Carpentier, arXiv:2211.13502 (2022).
J. Luneau, T. Roscilde, B. Douçot, and D. Carpentier, in preparation.

Topological pump

$\vec{h}(\omega_1 t, \omega_2 t)$



Topological coupling

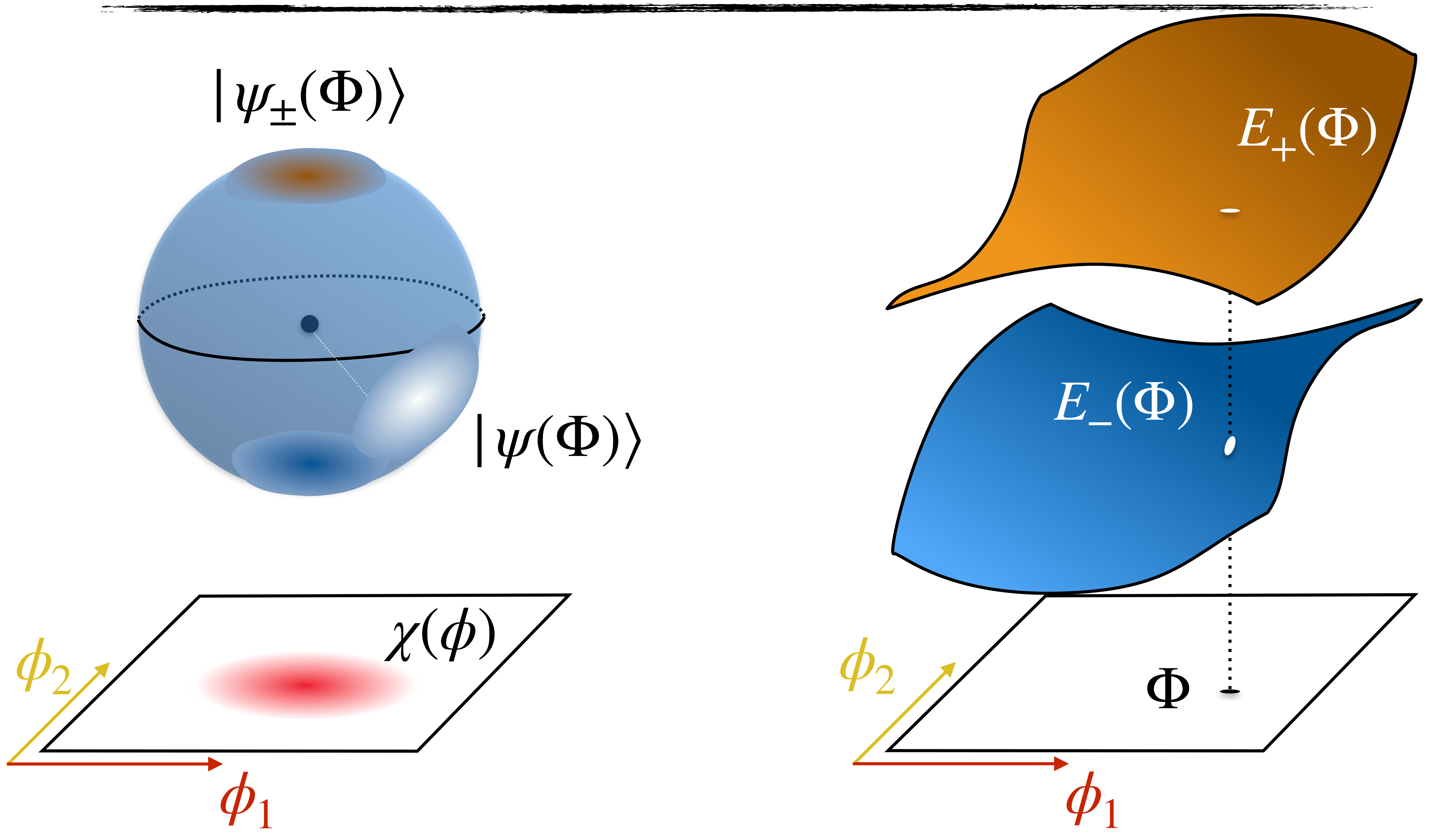
2 classes of dynamics / eigenstates <u>at same energy</u>

- Trivial state remain localized
- Topological delocalization and chaotic dynamics

Topology → discriminating filter on eigenstates

Ubiquitous mechanism

Adiabatic dynamics



Adiabatic projector

- Slow $\Phi = \Omega t$, adiabatic states $|\Phi(t)\rangle \otimes |\psi_\pm(\Phi(t))\rangle$
- Projector $\hat{P}_\pm = \int d\Phi \; |\Phi\rangle\langle\Phi| \otimes \pi_\pm(\Phi)$,
- $\pi_\pm = |\psi_\pm\rangle\langle\psi_\pm|$ perturb. in $\epsilon \simeq \hbar\Omega/\Delta E$

Adiabatic decomposition

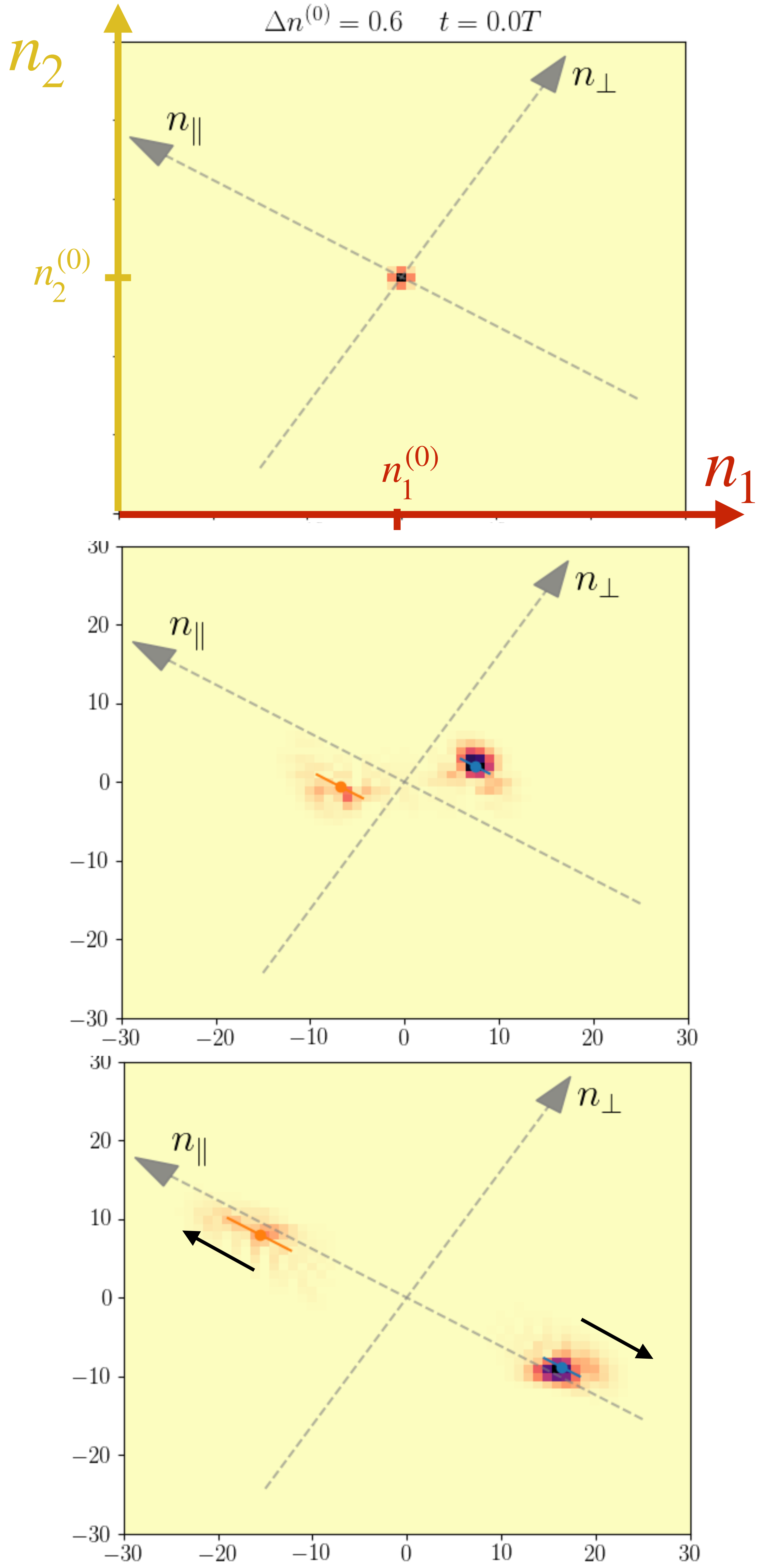
- Arbitrary state $|\Psi\rangle = \int d^2\Phi \; \chi(\Phi)|\Phi\rangle \otimes |\psi(\Phi)\rangle$
- Adiabatic decomposition $|\Psi\rangle = |\Psi_-\rangle + |\Psi_+\rangle$, $|\Psi_\pm\rangle = \hat{P}_\pm|\Psi\rangle$

Adiabatic dynamics

Prepare a gaussian separable state

$$|\Psi\rangle = \int d^2\Phi \; \chi(\Phi)\,|\Phi\rangle \otimes |\uparrow, z\rangle$$

Characterized by $\Delta\Phi \sim 1/\Delta n$



Topological pumping in direction:

$$n_\parallel = \frac{1}{|\Omega|}(-\omega_2 n_1 + \omega_1 n_2)$$

$$\frac{d}{dt}\langle \hat{n}_\parallel \rangle_{\Psi_\pm} = |\Omega| \frac{\mathscr{C}_\pm}{2\pi}$$

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Topological pumping

- For each adiabatic state

Berry curvature $F \simeq \frac{\mathscr{C}}{2\pi}$

$$\frac{d}{dt}\langle \hat{n}_1 \rangle_{\Psi_\pm} = \int d^2\Phi \; |\chi_\pm(\Phi)|^2 \left(\frac{1}{\hbar}\frac{\partial E_\pm}{\partial \phi_1}(\Phi(t)) + \omega_2 \, F_{\pm,12}(\Phi(t)) \right)$$