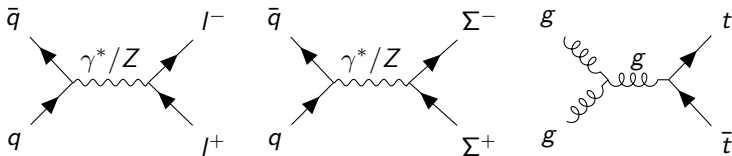


Precision predictions for (B)SM studies

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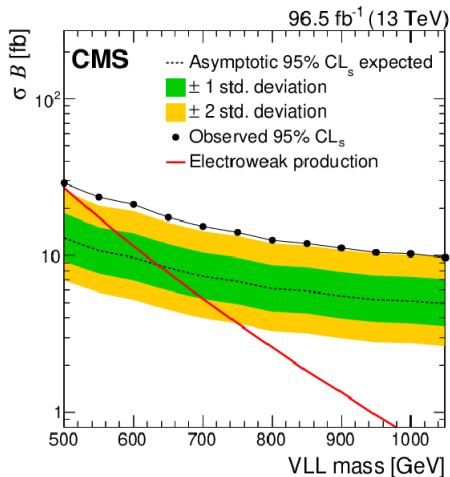
Content

- 1 Why (and how) more precision?
- 2 Fixed order and resummation predictions
- 3 Conclusion and take away message

SM and BSM searches

Motivations

- Precision era of LHC: also theoretical predictions !
- Many BSM models and potential new particles to be tested: VLL (4321), Type-III seesaw ...
- SM precision tests also for indirect searches: top, high multiplicity...



Accurate, **precise** and kinematically correct predictions → higher orders

Theoretical predictions: two complementary approaches

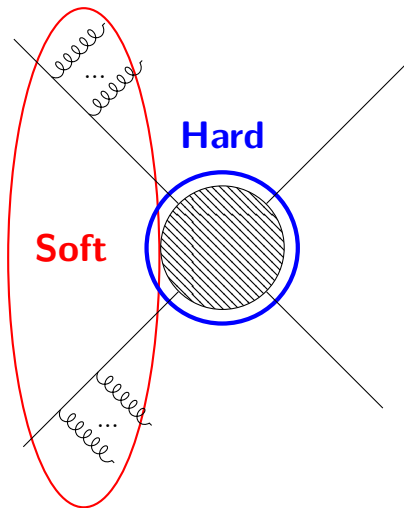
Fixed order perturbation theory

- Perturbative series in coupling (QED / EW / QCD)
- High order loops / real emission, each order increase complexity
- Automatic tools for NLO: MADGRAPH5_AMC@NLO
- → Model implementation needed (UFO)

Resummation and logarithms

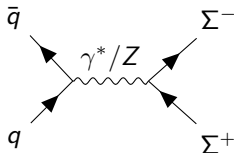
- Increase asymptotic convergence by resumming generic all-orders towers of logarithms
- Reducing scale dependence of fixed order with **resummation** at N^k Large Logarithm accuracy

About soft gluon threshold resummation



- Factorization theorem:
Soft scale vs. **Hard** scale
- Large logarithms arising from **soft gluon emissions**
- Threshold: $z = \frac{M^2}{\hat{s}} \rightarrow 1$
- $z \rightarrow 1 \leftrightarrow N \rightarrow \infty$ in Mellin space, need to resum $\log(1 - z)$ or $\log(N)$ terms

Threshold resummation for Drell-Yan like processes



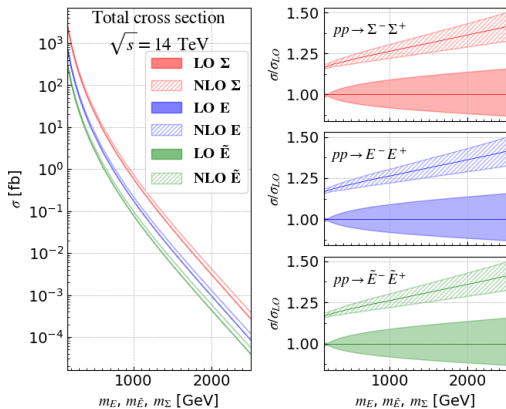
- Only massless initial states quarks emitting gluons
 → apply to all similar processes: Drell-Yan like
- Universal *soft part* known up to N^3LL

$$\Delta_{q\bar{q}}^{\text{res}}(N, M^2, \mu_F^2) \Big|_{N^kLL} = \tilde{g}_{0,q\bar{q}}(M^2, \mu_F^2, \mu_R^2) \Big|_{N^kLO} \\
\times \exp \left(\mathbf{g}_{1,q\bar{q}}(\omega) \ln N + \sum_{j=2}^{k+1} a_s^{j-2}(\mu_R^2) \mathbf{g}_{j,q\bar{q}}(\omega) \right)$$

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NLO impact on total cross sections



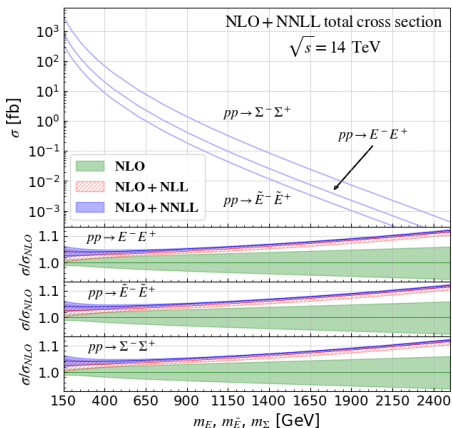
Remarkable features

- Important impact of NLO corrections = $O(15 - 50\%)$
- Outside of LO error bars: scale variation not trustworthy for LO
- K factor not constant
- Same behaviour for all processes: Drell-Yan like

Figure: Total cross sections NLO/LO comparison

See [Ajjath, Fuks, Shao & Y.S., P.R.D. 107, 075011]

Resummation improvement

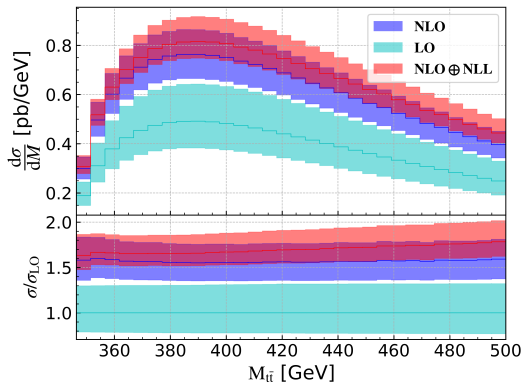


Remarkable features

- Decrease of scale uncertainties:
 NLO = $O(5\%)$
 NLO + NLL = $O(1\%)$
- Significant increase: NLO \rightarrow
 NLO + NNLL = $O(10\%)$
- α_s expansion convergence improved

Figure: Scale uncertainties reduction

Top pair production: a fruitful example of SM prediction



Remarkable features

- Also applicable to SM predictions
- Reduction of scale-induced error bars (precision)
- Shift of the central value (accuracy)

Figure: Differential cross section w.r.t. $M_{t\bar{t}}$

See also [Pecjak, Scott, Wang, Yang, P.R.L. **116**, 202001]

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In conclusion

- Higher orders: *accuracy*, **precision** and correct kinematics
- Fixed order is necessary but can be further improved by **resummation** (observable dependant)
- Automated NLO computation are on the market, not yet resummed
- Many more results to come (top physics & high multiplicity processes, Higgs proprieties, high luminosity LHC...)

Thank you for your attention!



Backup

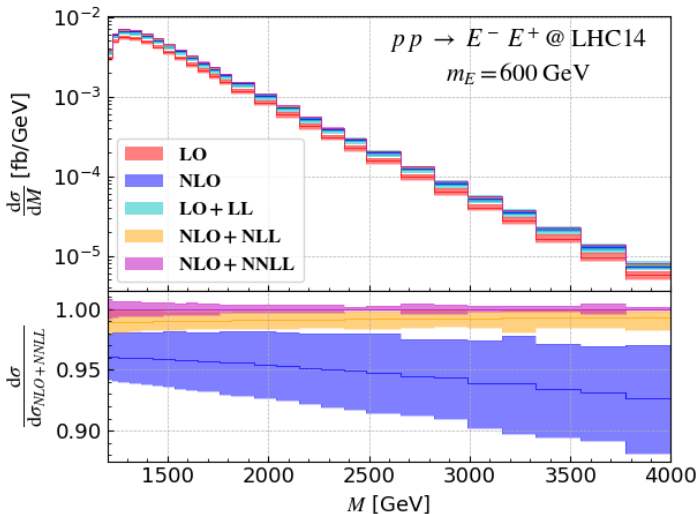


Figure: Differential cross section w.r.t. invariant mass for VLL doublet

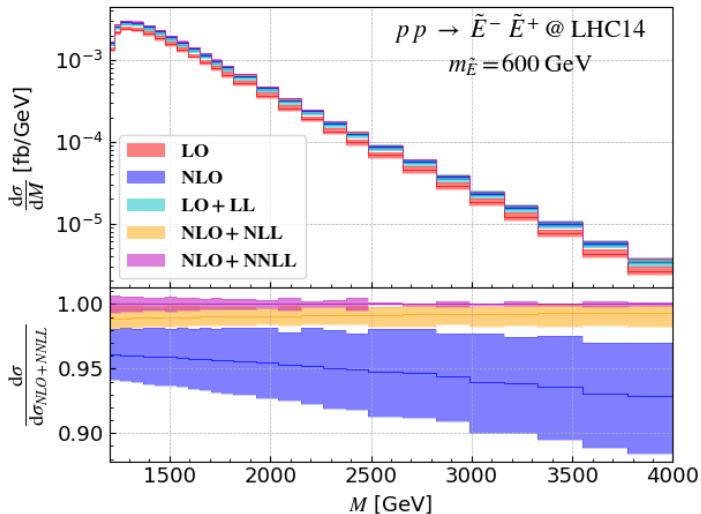


Figure: Differential cross section w.r.t. invariant mass for VLL singlet

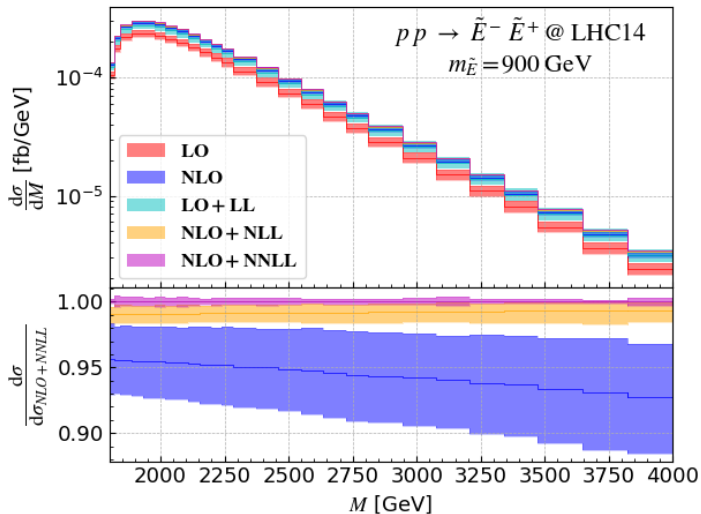


Figure: Differential cross section w.r.t. invariant mass for VLL singlet

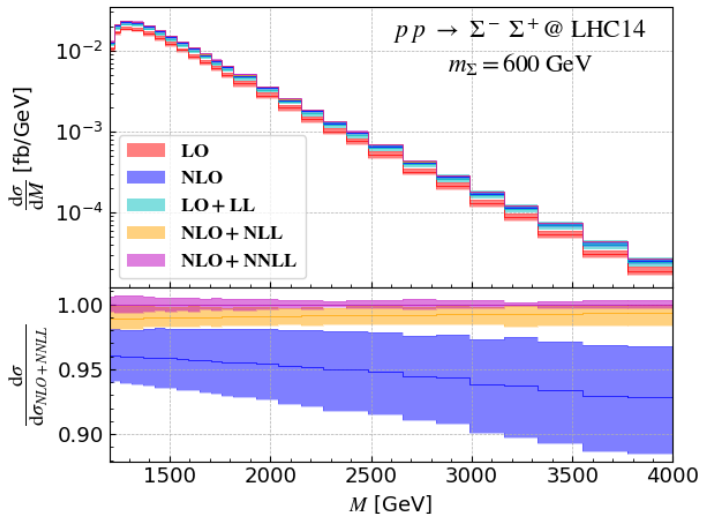


Figure: Differential cross section w.r.t. invariant mass for Type-III triplet

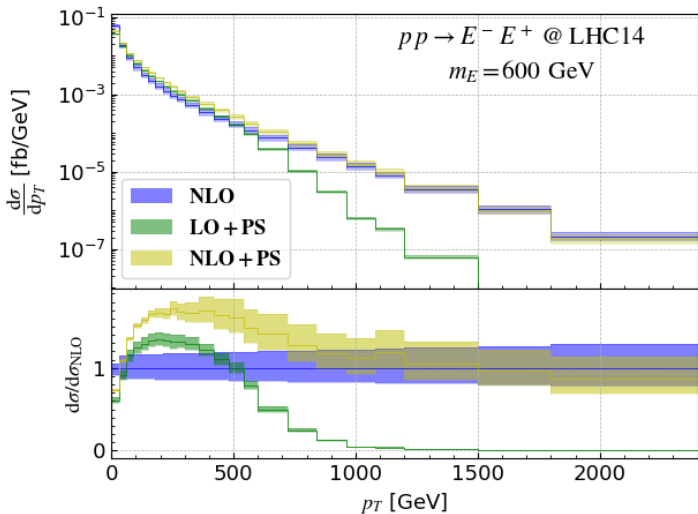


Figure: Differential cross section w.r.t. p_T for VLL doublet

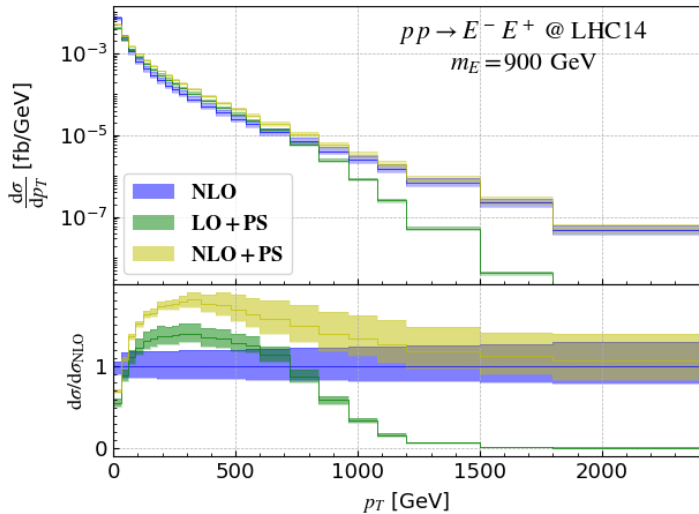


Figure: Differential cross section w.r.t. p_T for VLL doublet

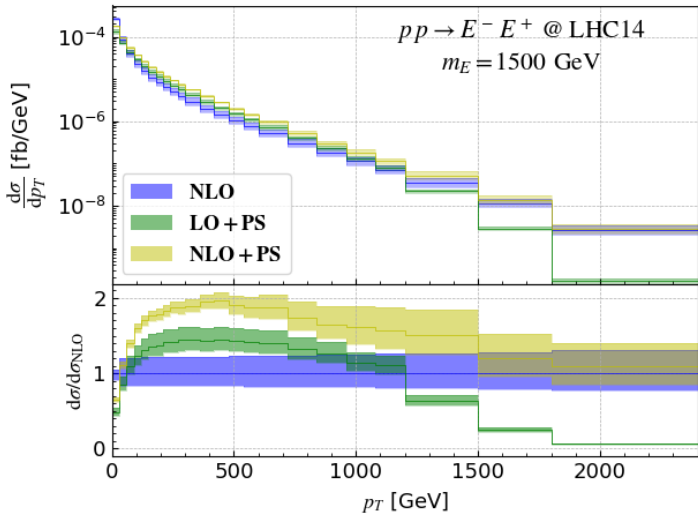


Figure: Differential cross section w.r.t. p_T for VLL doublet

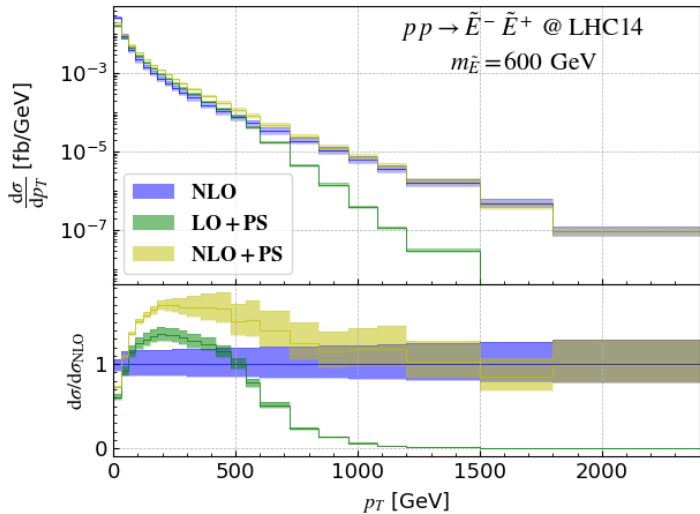


Figure: Differential cross section w.r.t. p_T for VLL singlet

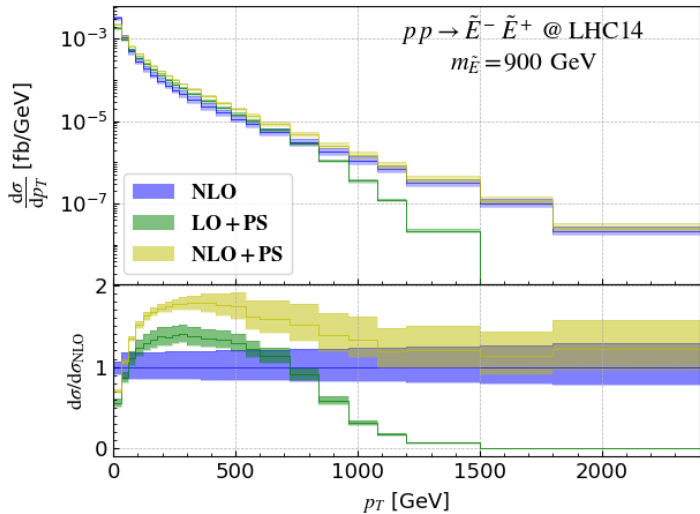


Figure: Differential cross section w.r.t. p_T for VLL singlet

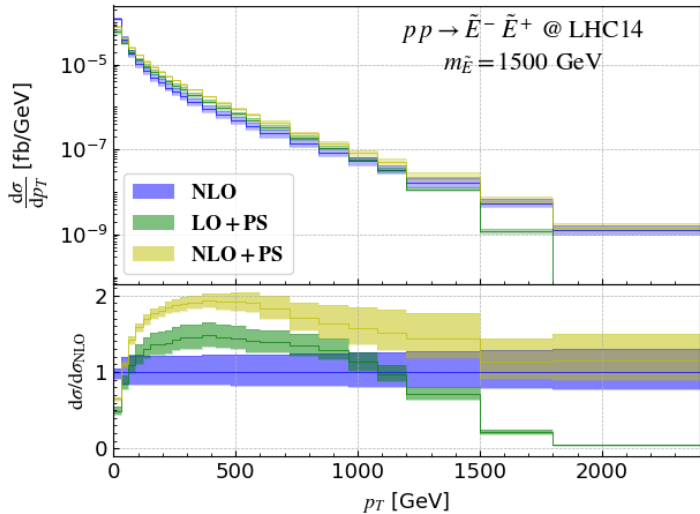


Figure: Differential cross section w.r.t. p_T for VLL singlet

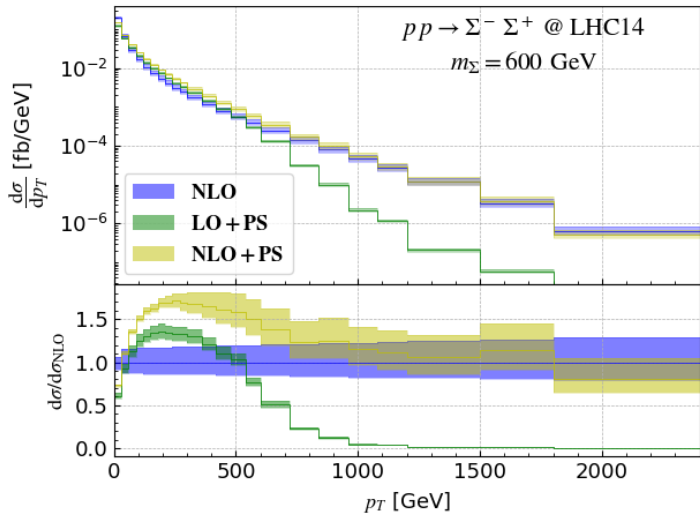


Figure: Differential cross section w.r.t. p_T for Type-III triplet

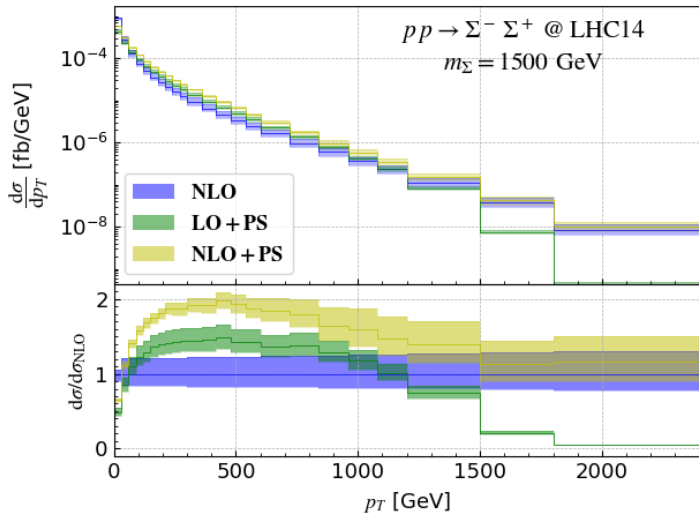


Figure: Differential cross section w.r.t. p_T for Type-III triplet

Large logs

$$d\hat{\sigma}_{p_1 p_2 \rightarrow \dots}^N = \frac{d\Phi_n}{F} \sum \overline{|\mathcal{M}_{p_1 p_2 \rightarrow \dots}^N|^2}$$

For the $q\bar{q} \rightarrow Z$ process at NLO:

$$d\hat{\sigma}_{q\bar{q} \rightarrow Z}^N = d\sigma_0 \left(1 + \frac{\alpha_s}{2\pi} \left[4C_F \ln^2(\bar{N}) - 4C_F \ln(\bar{N}) \ln(m_Z^2/\mu_F^2) + \tilde{C}_{q\bar{q} \rightarrow Z}^{(1)} \right] \right) \quad (1)$$

$$N \text{ Mellin conjugate to } z = \frac{Q^2}{\hat{s}}, \quad |N| \rightarrow +\infty \longleftrightarrow z \rightarrow 1$$

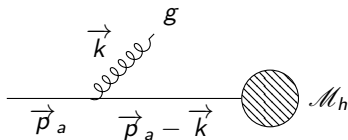
Generic logarithms coming from real emissions of (*collinear*-)soft gluons appear to all orders: can be *resummed*.

Expansion of logarithms

Resumming and exponentiation of logarithms produces a new power series in $\alpha_s L$ at large $L = \ln(N)$: Leading-Logarithms (LL), NLL, N^k LL

$\alpha_s^m L^k$	LO	NLO	...	N^m LO
LL	$m = k = 0$	$k = 2$...	$m + 1 \leq k \leq 2m$
NLL	\emptyset	$m = k = 1$...	$m \leq k \leq 2m - 1$
...
N^p LL	\emptyset	\emptyset	...	$m + 1 - p \leq k \leq 2m - p$

Eikonal approximation

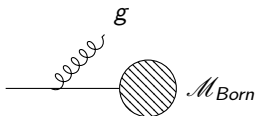


$$\mathcal{M}_e = \mathcal{M}_h \frac{i(\not{p}_a - \not{k} + m)}{(p_a - k)^2 - m^2 + i\epsilon} (-ig_s \mathbf{T}^a \gamma^\mu) u(p_a) \epsilon_\mu^*(k)$$

$$\mathcal{M}_e \xrightarrow{k \ll p} \mathcal{M}_h u(p_a) g_s \mathbf{T}^a \frac{-\not{p}_a^\mu}{p_a \cdot k + i\epsilon} \epsilon_\mu^*(k)$$

Effective Feynman rules for soft ($k \ll p$) gluon radiation and generators depending of particle nature and if it's incoming/outgoing.

△ building



$$\mathcal{M}_{e \text{ Eikonal}} = \mathcal{M}_{\text{Born}} u(p_a) g_s \mathbf{T}^a \frac{-p_a^\mu}{p_a \cdot k + i\epsilon} \epsilon_\mu^*(k)$$

$$d\sigma_e \propto \frac{2p_a \cdot p_b}{E_a E_b k_0^2 (1 - \cos^2(\theta))} d\sigma_{\text{Born}}$$

Phase space factorization:

$$d\Phi_2 = \frac{d^{d-1}k}{(2\pi)^{d-1} 2k_0} \frac{2\pi}{\hat{s}} \delta\left(\frac{2k}{\sqrt{\hat{s}}} - 1 + z\right)$$

Regularization and resummation

When we add the virtual contribution in the same eikonal approximation to regulate $z \rightarrow 1$ divergences and integrate over phase space we get for each massless leg:

$$I = 2C_i \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_{-1}^1 \frac{d \cos(\theta)}{1 - \cos^2(\theta)} \frac{\alpha_s}{\pi}$$

In the collinear limit $\cos(\theta) \approx 1 - \frac{\theta^2}{2}$ and we can approximate $\frac{d\theta^2}{\theta^2} \approx \frac{dk^2}{k^2}$ where k represents the momentum taken away by the gluon.

$$I = C_i \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_{\mu_F^2}^{(1-z)^2 Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{\pi}$$

We can extrapolate this for multiple emissions, decoupled for this LL integral:

$$\Delta_i^{LL} = \sum_{n=0}^{+\infty} \frac{I^n}{n!} = e^I$$

General formalism

In a general way, we can write the factorized formula:

$$d\hat{\sigma}_{ij \rightarrow \dots}^{N, res.} \propto \text{Tr} \left(\mathbf{H} e^{\int \Gamma^\dagger} \mathbf{S} e^{\int \Gamma} \right) \Delta_i \Delta_j \quad (2)$$

- **H**: “hard” matrix, high energy process part
- **S**: “soft” matrix, low energy emissions and color structure
- Γ : soft anomalous dimension color matrix controlling the evolution over RG of **S**
- Δ_i : (colinear-)soft radiations from initial state massless partons

$$\Delta_i = e^{g_1(\alpha_s \ln(\bar{N})) \ln(\bar{N}) + g_2(\alpha_s \ln(\bar{N})) + \dots}$$

Matching

We can also expand $d\hat{\sigma}^{N, res.}$ to NLO in α_s and compare it to the usual NLO cross section (obtained with MADGRAPH5_AMC@NLO).

Fixed order

$$d\sigma|_{NLO}$$

Valid away from
threshold

Resummed

$$d\sigma^{res.}$$

Valid near threshold

Resummed @ f. o.

$$d\sigma_{|NLO}^{res.}$$

Double counting

Matching: $d\sigma|_{NLO} + d\sigma^{res.} - d\sigma_{|NLO}^{res.}$ valid everywhere

Expected behaviours

- We expect the ratio $\frac{1}{d\sigma^0/dM^2} \left(\frac{d\sigma^{res.}}{dM^2} - \frac{d\sigma^{res.}}{dM^2} \Big|_{NLO} \right) \xrightarrow{M^2 \ll S_h} 0$

Away from threshold the logarithmic terms are not important and the behaviour is captured by the first orders of the expansion.

- We expect also $\frac{1}{d\sigma^0/dM^2} \left(\frac{d\sigma^{NLO}}{dM^2} - \frac{d\sigma^{res.}}{dM^2} \Big|_{NLO} \right) \xrightarrow{M^2 \rightarrow S_h} 0$

In the threshold regime, the resummed expanded reproduces the behaviour of original cross section.

To obtain a sensible cross section in all ranges we may consider the combination: $\sigma|_{NLO} + \sigma^{res.} - \sigma|_{NLO}^{res.}$

Some Feynman rules

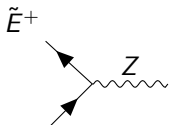


Diagram showing a vertex where a Z boson (wavy line) couples to a selectron (\tilde{E}^+ , incoming) and an anti-selectron (\tilde{E}^- , incoming).

$$ie\gamma^\mu \tan(\theta_w)$$

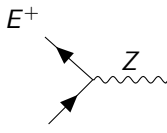


Diagram showing a vertex where a Z boson (wavy line) couples to an electron (E^+ , incoming) and a positron (E^- , incoming).

$$\frac{-ie\gamma^\mu}{\tan(2\theta_w)}$$

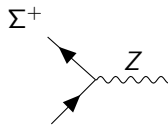


Diagram showing a vertex where a Z boson (wavy line) couples to a sigma plus (Σ^+ , incoming) and a sigma minus (Σ^- , incoming).

$$\frac{-ie\gamma^\mu}{\tan(\theta_w)}$$

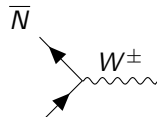


Diagram showing a vertex where a W^\pm boson (wavy line) couples to an anti-neutrino (\bar{N} , incoming) and a charged lepton (E^\pm , incoming).

$$\frac{-ie\gamma^\mu}{\sqrt{2} \sin(\theta_w)}$$

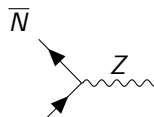


Diagram showing a vertex where a Z boson (wavy line) couples to an anti-neutrino (\bar{N} , incoming) and a neutrino (N , incoming).

$$\frac{ie\gamma^\mu}{\sin(2\theta_w)}$$

g functions

$$\begin{aligned}
 \tilde{g}_{0,q\bar{q}}^{(1)} &= \frac{-64}{3} + \frac{64}{3}\zeta_2 - 8L_{fr} + 8L_{qr}, \\
 \tilde{g}_{0,q\bar{q}}^{(2)} &= \frac{-1291}{9} + \frac{64\zeta_2}{9} + \frac{368\zeta_2^2}{3} + \frac{4528\zeta_3}{27} \\
 &\quad + \frac{188L_{fr}^2}{3} + \frac{4L_{qr}^2}{3} \\
 &\quad + L_{fr} \left(\frac{1324}{9} - \frac{1888\zeta_2}{9} + \frac{32\zeta_3}{3} \right) \\
 &\quad + L_{qr} \left(\frac{148}{9} - 64L_{fr} + \frac{416\zeta_2}{9} - \frac{32\zeta_3}{3} \right),
 \end{aligned} \tag{3}$$

with $L_{qr} = \ln \frac{M^2}{\mu_R^2}$, $L_{fr} = \ln \frac{\mu_F^2}{\mu_R^2}$ and ζ_n being the Riemann zeta function.

Lagrangians

$$\begin{aligned}
 \mathcal{L}_{\text{VLL}} = & \mathcal{L}_{\text{SM}} + i\bar{L}\not{D}L - m_N\bar{N}N - m_E\bar{E}E + i\bar{\tilde{N}}\not{\partial}\tilde{N} - m_{\tilde{N}}\bar{\tilde{N}}\tilde{N} + i\bar{\tilde{E}}\not{\partial}\tilde{E} - m_{\tilde{E}}\bar{\tilde{E}}\tilde{E} \\
 & + \sum_{\Psi=E,\tilde{E}} \left[h\bar{\Psi} \left(\hat{\kappa}_L^\Psi P_L + \hat{\kappa}_R^\Psi P_R \right) \ell + \frac{g}{\sqrt{2}} \bar{\Psi} \not{W}^- \kappa_L^\Psi P_{L\nu\ell} \right. \\
 & \left. + \frac{g}{2c_W} \bar{\Psi} \not{Z} \left(\tilde{\kappa}_L^\Psi P_L + \tilde{\kappa}_R^\Psi P_R \right) \ell + \text{H.c.} \right] \\
 & + \sum_{\Psi=N,\tilde{N}} \left[h\bar{\Psi} \hat{\kappa}_L^\Psi P_{L\nu\ell} + \frac{g}{2c_W} \bar{\Psi} \not{Z} \tilde{\kappa}_L^\Psi P_{L\nu\ell} + \frac{g}{\sqrt{2}} \bar{\Psi} \not{W}^+ \left(\kappa_L^\Psi P_L + \kappa_R^\Psi P_R \right) \ell + \text{H.c.} \right], \quad (4)
 \end{aligned}$$

$$\mathcal{L}_{\text{TypeIII}} = \overline{\mathcal{L}}_{\text{SM}} + \mathcal{L}_{\text{kin}} + \left(y_\ell \Phi^\dagger L_L \cdot E_R + 2y_\Sigma \Phi \cdot \left[\Sigma^k T^k L_L \right] + \text{H.c.} \right). \quad (5)$$

Bibliography



Roel Aaij et al.

Measurement of the ratio of branching fractions

$$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau) / \mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \mu^- \bar{\nu}_\mu).$$

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