The proto-neutron-star inner crust in a multi-component plasma approach

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July 4, 2023

26ème Congrès Général de la SFP

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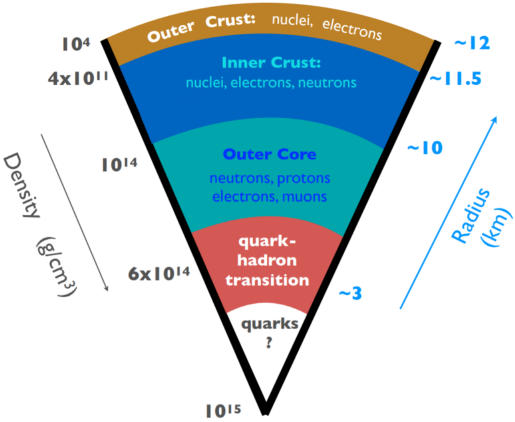
1.1 Neutron stars

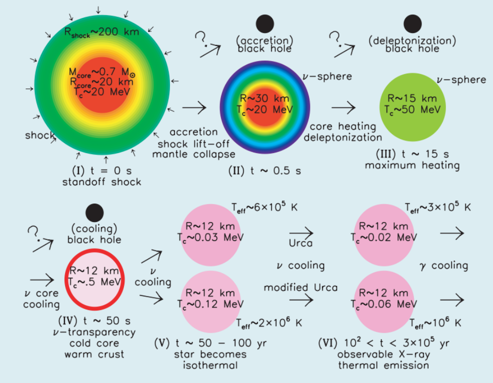
Figure taken from 3G Science White Paper



- Mass: $\sim 1-2\ M_\odot$
- Radius: ~ 10 km

1.2 NS formation

Lattimer & Prakash, Science(2004)



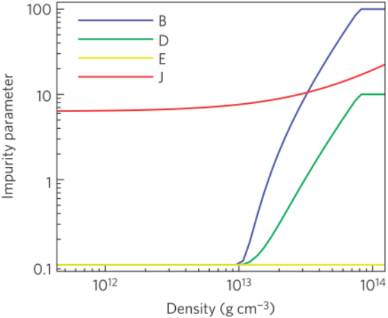
$\star$ NS are born hot with initial temperature exceeding 10^{10}K.

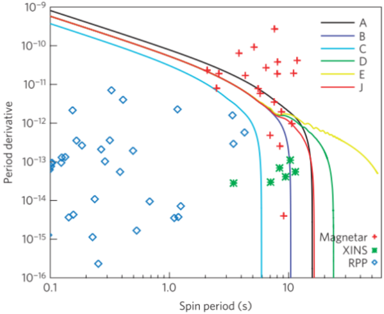
$\rightarrow$ Liquid multi-component plasma crust composed of different nuclear species $\rightarrow$ impurities

Gulminelli & Raduta, PRC(2015); Fantina et al, A&A(2020); Carreau et al., A&A(2020)

1.3 NS crust and implications

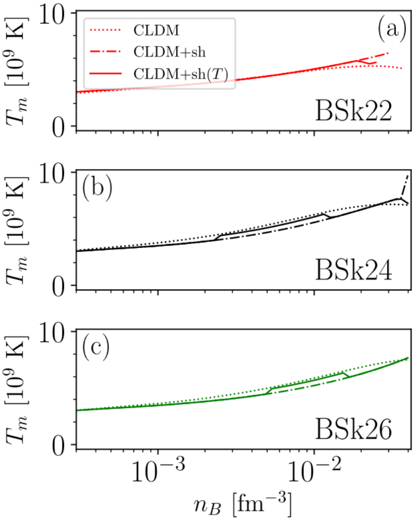
Impurities of the crust could impact timing properties (Pons, Nature(2013)), magnetothermal evolution (Vigano, MNRAS(2013)).





1.4 Crystallization temperature of the crust

T. Carreau et al. A&A 635, A84 (2020)



- Liquid crust, i.e., crustal ions are put into collective motion.
  → How does the center-of-mass motion influence the composition of the crust?
- Coexistence of different nuclear species.
  → How does the nuclear distribution, hence impurity parameter, evolve with n_B and T?

2.1 One-component plasma (OCP) approximation

OCP approximation

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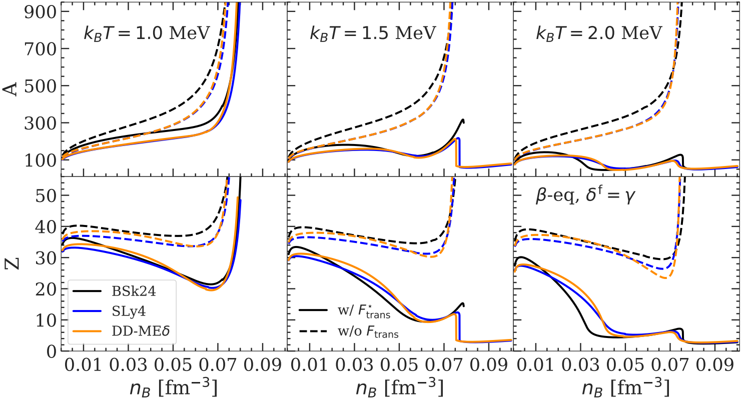
→ Minimizing $\mathcal{F}$ under **2** constraints: **charge neutrality** and **baryon number conservation.**

2.2 OCP results

- ◇ Finite-size effect: $V_{\rm f} = \frac{4}{3}\pi(r_{\rm WS} - r_N)^3$
- ◇ In-medium effect: $M_i^\star = M_i(1-\gamma)$

$$\boxed{F^\star_{\rm trans} = k_{\rm B}T \ln\left(\frac{1}{V_{\rm f}}\frac{\lambda_i^{\star 3}}{g_s}\right) - k_{\rm B}T}$$



Dinh Thi H. et al., A&A 672, A160 (2023)

3.1 Multi-component plasma (MCP) approach

MCP approach

At each given thermodynamic condition, matter is hypothesized to be composed of electrons, free nucleons, and an ensemble of different nuclear species.

Gulminelli & Raduta, PRC(2015); Grams et al., PRC(2018); Fantina et al, A&A(2020); Carreau et al., A&A(2020)

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⋆ Free energy density:

$$\mathcal{F}^{\mathrm{MCP}} = \mathcal{F}_g + \mathcal{F}_e + \sum_j n_N^{(j)} \left(F_i^{(j)} - V_N^{(j)} \mathcal{F}_g \right) .$$

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⋆ Minimizing $\mathcal{F}^{\mathrm{MCP}}$ under the constraints of charge neutrality and baryon number conservation $\rightarrow n_N^{(j)}$:

$$n_N^{(j)} = \bar{u}_f \exp\left\{ -\tilde{\Omega}_i^{(j)} / (k_B T) \right\}$$

3.2 Nuclear distribution

Occurence density

$$n_N^{(j)} = \bar{u}_f \exp\left\{-\tilde{\Omega}_i^{(j)}/(k_B T)\right\}$$

$$\tilde{\Omega}_i^{(j)} = F_i^{(j)} - F_{\text{trans}}^{\star,(j)} + k_B T \ln\left(\frac{(\lambda_i^{\star,(j)})^3}{g_s^{(j)}}\right) - V_N^{(j)} \mathcal{F}_g + \mathcal{R}^{(j)} + \mu_e Z^{(j)} - \mu_n A_e^{(j)}$$

$\star$ $\mathcal{R}^{(j)}$: Rearrangement term arising from the dependence of the Coulomb energy on the electron density.

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$\star$ $\mathcal{R}^{(j)}$: Rearrangement term arising from the dependence of the Coulomb energy on the electron density.

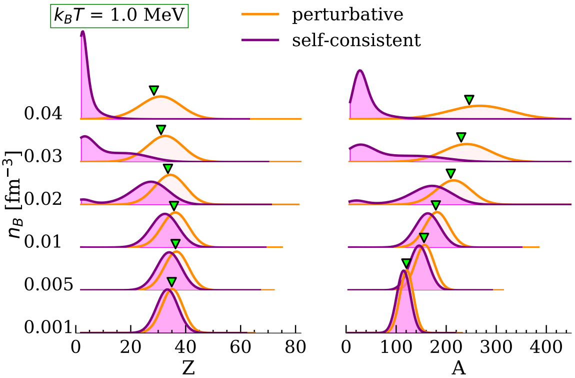
$\star$ Chemical potentials:

+ Perturbative MCP ($\mu \approx \mu_{\rm OCP}$): good approximation for the outer crust and outer part of the inner crust at crystallization temperature .

+ Self-consistent MCP ($\mu = \mu_{\rm MCP}$): calculated from the charge neutrality and baryon number conservation equations.

3.3 Nuclear distribution vs. n_B

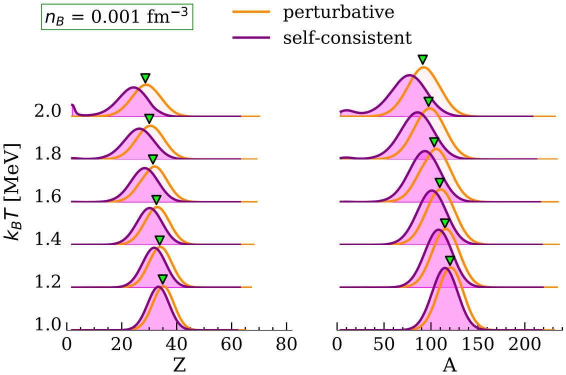
(A&A, submitted)



- ⋆ At low densities, large clusters dominate and:
  - The OCP prediction coincides with the most probable configuration in the MCP distribution.
  - Perturbative MCP results in similar distribution as the self-consistent one.
- ⋆ At high densities, small clusters dominate.

3.4 Nuclear distribution vs. T

(A&A, submitted)



→ The contribution from light nuclei becomes more important at higher T.

3.5 Impurity parameter

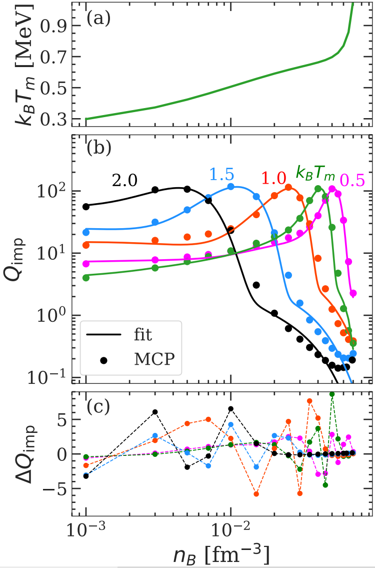
⋆ $Q_{\rm imp}$ has important impacts on NS phenomena, e.g., timing (Pons, Nature(2013)), magneto-thermal evolution (Vigano, MNRAS(2013)), and transport properties (Schmitt, & Shternin (2018)).

Impurity parameter

$$Q_{\rm imp} = \langle Z^2 \rangle - \langle Z \rangle^2$$

⋆ $Q_{\rm imp}$ is consistently calculated throughout the whole inner crust.
→ We provide tables and an analytical fit for $Q_{\rm imp}$.

(A&A, submitted)



4. Conclusions and outlooks

- The translational free energy has an important effect on the crust composition.
- For the inner crust of PNS, the OCP approximation is no longer reliable, especially at high densities and temperatures.

☐ Coexistence with pasta phases → could impact the impurity parameter

☐ Transport properties