Dynamo action sustaining turbulence: a subcritical transition

F. Daniel[1] L. Petitdemange[2] C. Gissinger[1,3]

[1]Laboratoire de Physique de l'ENS
ENS Paris, France

[2]Laboratoire d'Etudes du Rayonnement et de la Matière en Astrophysique et Atmosphères
Observatoire de Paris, France

[3]Institut Universitaire de France

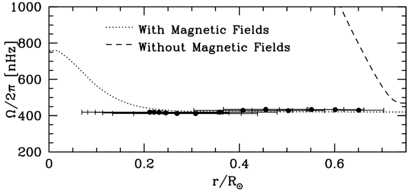
Congrès général de la SFP, July 2023

Table of contents

Table of contents

Radiative stars

Astrophysics :

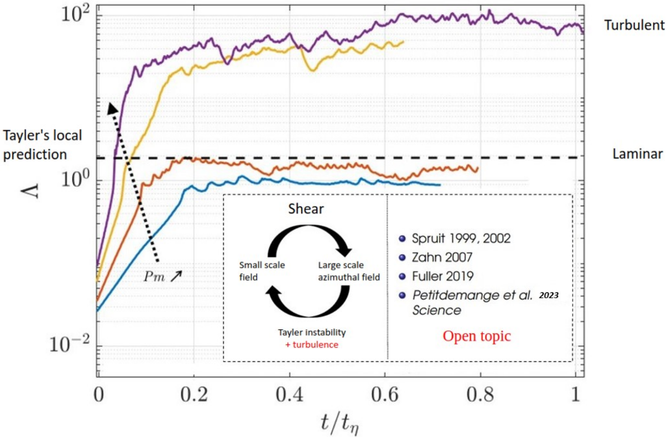


Numerical simulation for the radiative zone of the sun, with and without magnetic field, compared to observations (*Eggenberger et al. 2005*)

Schematic view of the convective (outer) and radiative (inner) zones of the sun (Wikipedia)

Physics : Subcritical transition to turbulence scenario

The Tayler-Spruit dynamo

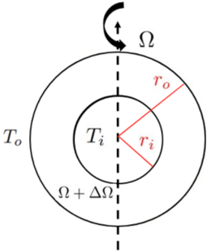


- Spruit 1999, 2002
- Zahn 2007
- Fuller 2019
- *Petitdemange et al.* ***2023*** *Science*

Open topic

Figure 1 – Time evolution of the dimensionless magnetic energy $\Lambda = \int B^2/\rho\mu\Omega\eta dV$ for different values of fluid conductivity (*Petitdemange et al. 2023*)

Simulations



Solve for $\mathbf{u}, \mathbf{B}, \Theta$

$\chi = \frac{r_i}{r_o} = 0.35$

Boussinesq

$\Delta T = T_o - T_i$

ν, κ, η

Figure 2 – Geometry

- BC for u : no slip
- BC for B : insulating at $r = r_o$, conducting at $r = r_i$
- Initial B : arbitrary weak
- Imposed (and maintained) inner rotation $\Delta\Omega$

MHD equations for a stably stratified incompressible fluid in rotation under Boussinesq approximation (PaRoDy code)

$$\boxed{\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\boldsymbol{\nabla})\mathbf{u} = -\frac{\boldsymbol{\nabla} P}{\rho_0} - 2\boldsymbol{\Omega} \times \mathbf{u} + \nu \Delta \mathbf{u} + \frac{1}{\rho_0 \mu_0}(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} \\ + \alpha \Theta g_0 r \mathbf{e_r} \\ \nabla.\mathbf{u} = 0 \\ \frac{\partial \Theta}{\partial t} + (\mathbf{u}.\boldsymbol{\nabla})\Theta = \kappa \Delta \Theta - (\mathbf{u}.\boldsymbol{\nabla}) T_s \\ \frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}) + \eta \boldsymbol{\Delta} \mathbf{B} \\ \nabla.\mathbf{B} = 0 \end{aligned}} \quad (1)$$

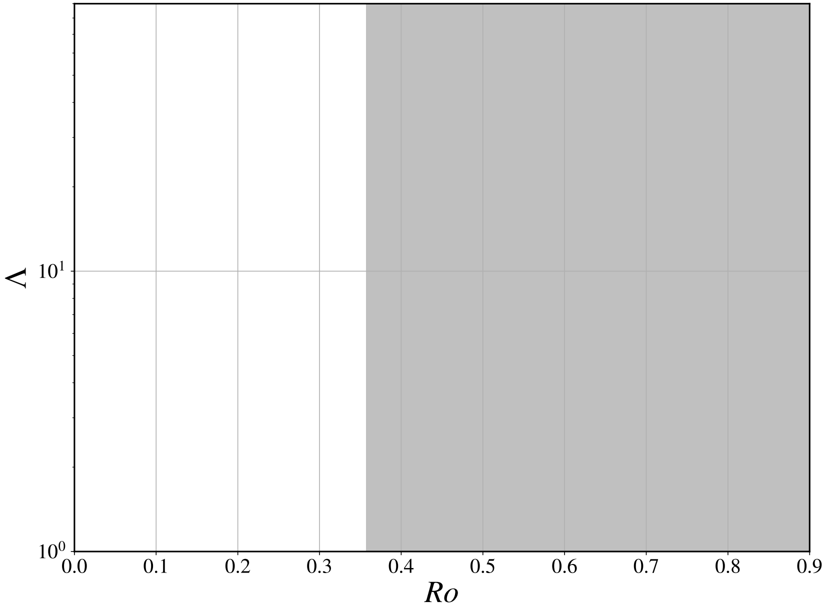
$$Ro = \frac{\Delta\Omega}{\Omega},\ Ek = \frac{\nu}{r_o^2 \Omega},\ \frac{N}{\Omega} = \sqrt{\frac{\alpha g \Delta T}{(r_o - r_i)\Omega^2}},\ P_r = \frac{\nu}{\kappa},\ P_m = \frac{\nu}{\eta} \quad (2)$$

Table of contents

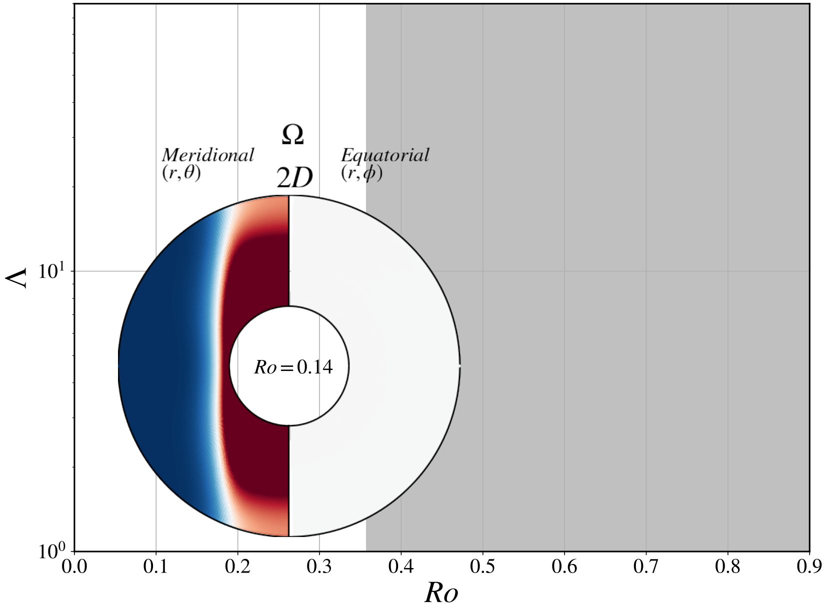
Results

- Fixed $Ek = 10^{-5}$, $Pr = 0.1$ and $\frac{N}{\Omega} = 1.24$
- Varying Ro and Pm
- Look at $\Lambda = \int \frac{B^2}{\rho\mu\eta\Omega} dV$

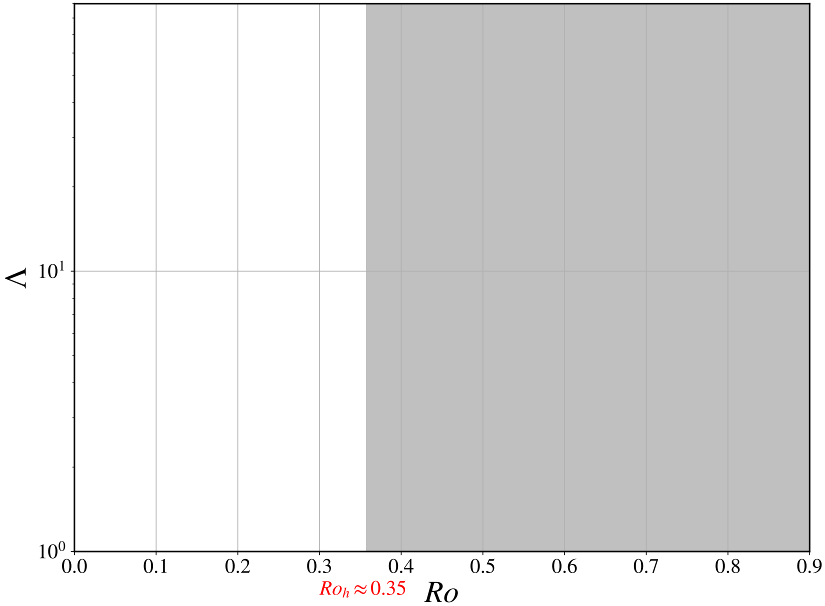
Bifurcations



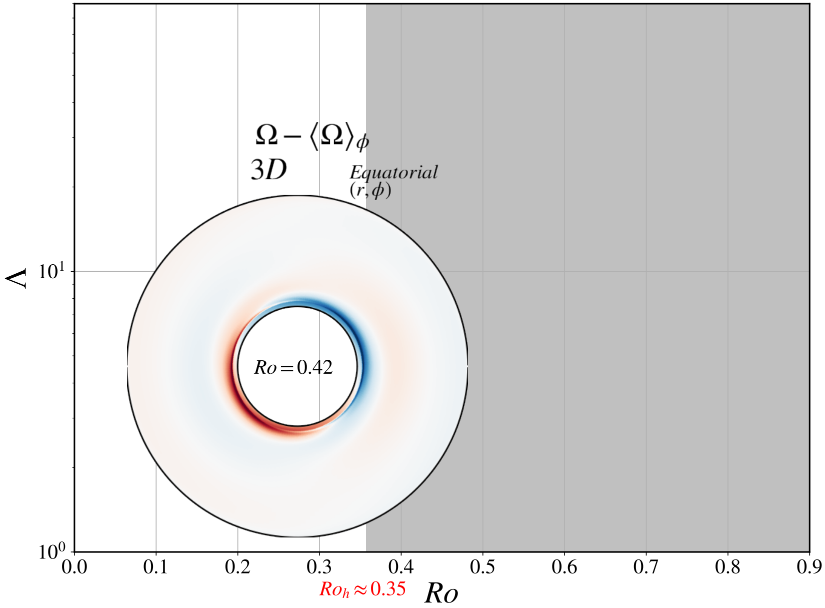
Bifurcations



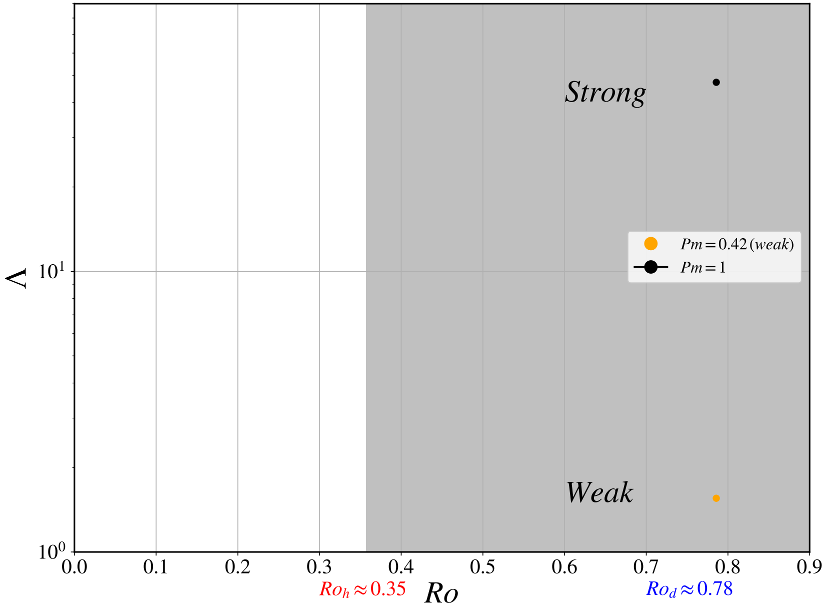
Bifurcations



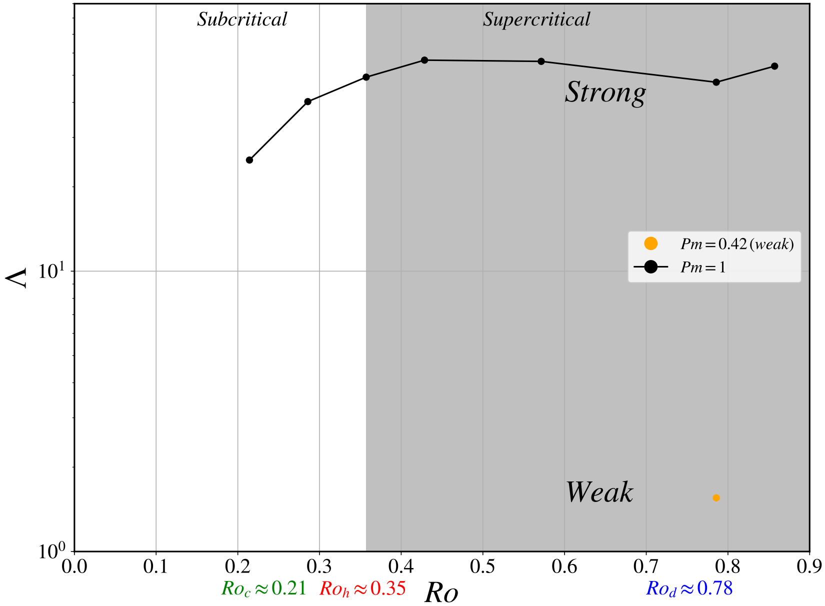
Bifurcations



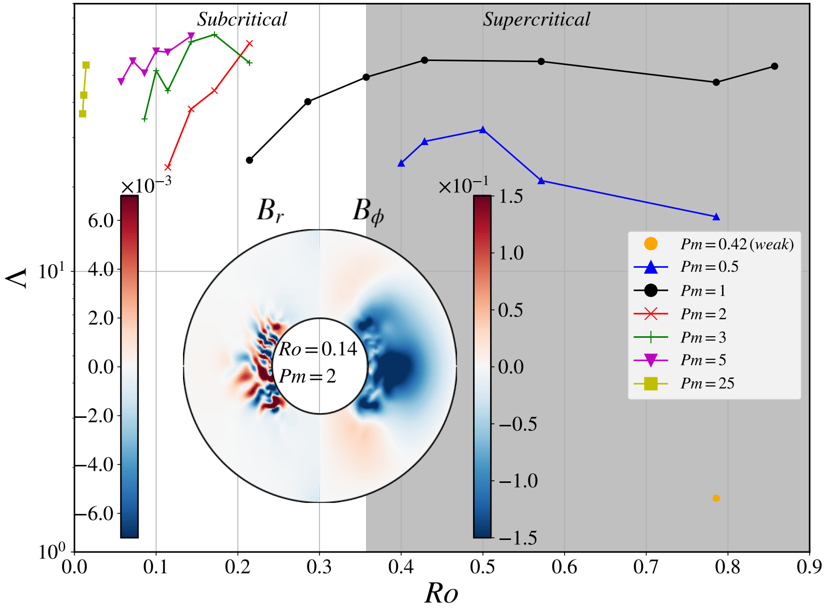
Bifurcations



Bifurcations



Bifurcations



Bifurcations

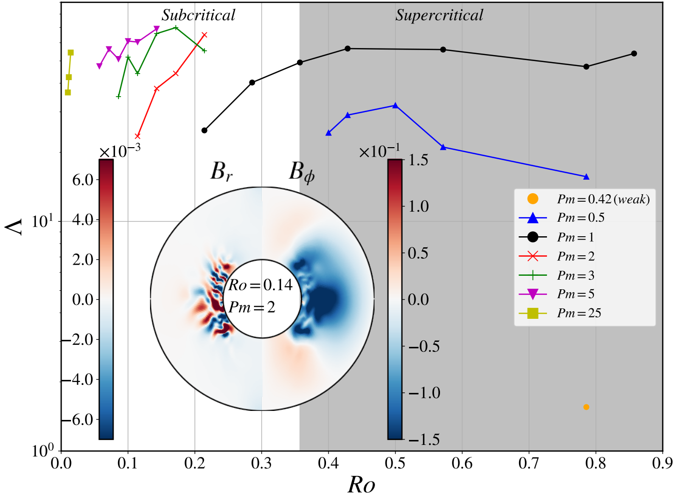


Figure 3 – Bifurcation diagram of the Elsasser number $\Lambda = \int B^2/\rho\mu\Omega\eta dV$ versus Ro for $Ek = 10^{-5}$, $N/\Omega = 1.24$, $Pr = 0.1$ (*Daniel et al. 2023*).

Bifurcations

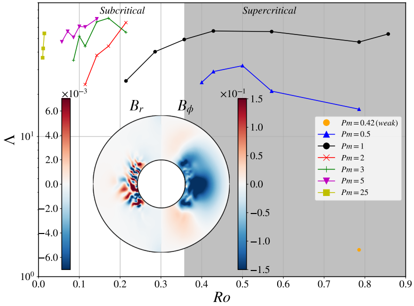


Figure 4 – Bifurcation diagram of the Elsasser number $\Lambda = \int B^2/\rho\mu\Omega\eta dV$ versus Ro for $Ek = 10^{-5}$, $N/\Omega = 1.24$, $Pr = 0.1$

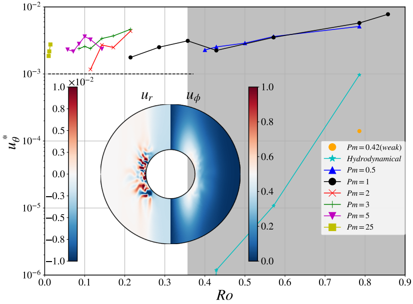


Figure 5 – Bifurcation diagram of the fluctuations of the velocity field u_θ^* versus Ro for $Ek = 10^{-5}$, $N/\Omega = 1.24$, $Pr = 0.1$

Spectra

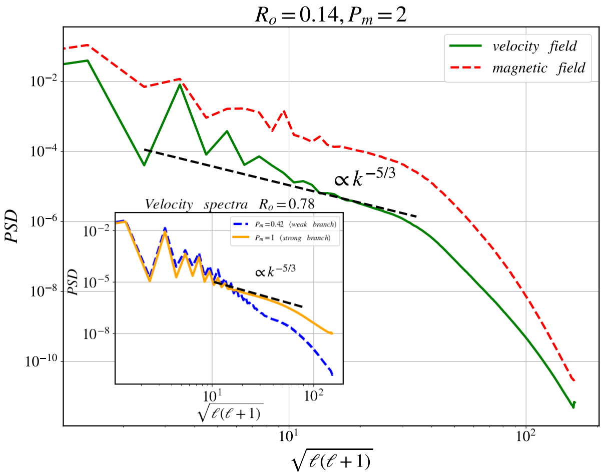


Figure 6 – *Main* : Power Spectrum Density for the run $Ro = 0.14,\ Pm = 2$, both for the velocity and magnetic field, computed in the active dynamo region ($0.4 < r < 0.6$) and time-averaged over the saturated phase. *Inset* : PSD for two runs at the same $Ro = 0.78$, but for different Pm (strong and weak branches).

Effect of the turbulence on Angular momentum transport

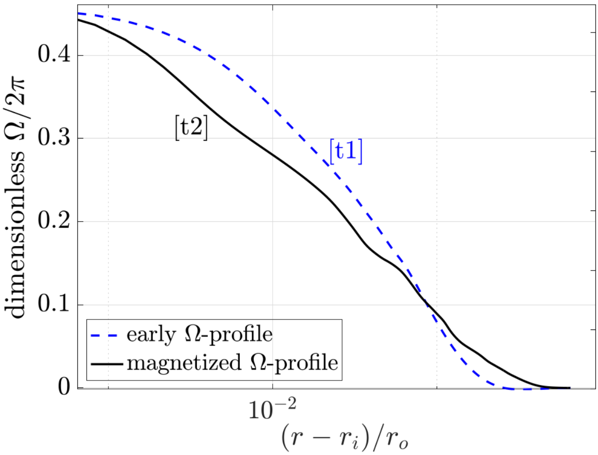


Figure 7 – Angular velocity profile Ω before the turbulence sets in (t1) and right after (t2) (*Petitdemange et al. 2023*)

Table of contents

Derivation

Reduced model involving (only) 3 modes :

- $m = 0$ Magnetic mode B (Dominant part of B)
- $m \neq 0$ Magnetic mode P
- $m \neq 0$ Hydrodynamic mode V (coupling via induction term)

Symmetries

- $(B, P) \rightarrow (-B, -P)$

Rotational invariance

- $V \rightarrow Ve^{im\phi}$, $P \rightarrow Pe^{im\phi}$

Reduced model

V grows

$$\begin{cases} \dot{B} = (-\mu \qquad\qquad)B \\ \dot{P} = (-\nu \qquad\qquad)P \\ \dot{V} = (\quad \lambda \qquad\qquad)V \end{cases} \tag{3}$$

Reduced model

V grows, B grows

$$\begin{cases} \dot{B} = (-\mu + |V|^2) B \\ \dot{P} = (-\nu \qquad\qquad) P \\ \dot{V} = (\ \ \lambda \qquad\qquad) V \end{cases} \tag{4}$$

Reduced model

V grows, B grows, P grows

$$\begin{cases} \dot{B} = (-\mu + |V|^2)B \\ \dot{P} = (-\nu + |B|^2)P \\ \dot{V} = (\quad \lambda \qquad\qquad)V \end{cases} \tag{5}$$

Reduced model

V grows, B grows, P grows

$$\begin{cases} \dot{B} =(-\mu + |V|^2)B \\ \dot{P} =(-\nu + |B|^2)P + VB \\ \dot{V} =(\quad \lambda \qquad\qquad)V + \gamma BP \end{cases} \tag{6}$$

Reduced model

V grows, B grows, P grows, saturation

$$\begin{cases} \dot{B} =(-\mu + |V|^2)B - |B|^2 B \\ \dot{P} =(-\nu + |B|^2)P + VB - |P|^2 P \\ \dot{V} =(\quad \lambda \qquad\qquad)V + \gamma BP - |V|^2 V \end{cases} \tag{7}$$

System

$$\begin{cases} \dot{B} = (-\mu + c_1|V|^2)B - \alpha_1|B|^2B \\ \dot{P} = (-\nu + \beta_2|B|^2)P + c_2VB - \beta_1|P|^2P \\ \dot{V} = \lambda V + \gamma BP - c_3|V|^2V \end{cases} \tag{8}$$

Time series

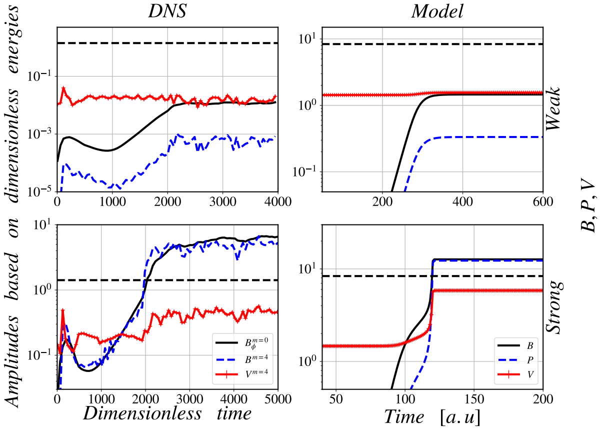


Figure 8 – Timeseries from our DNS (left) compared to the ones from the model derived here (right) for the weak (top) and the strong (bottom) dynamo branches. The horizontal dashed lines corresponds to Tayler instability threshold.

Bifurcation

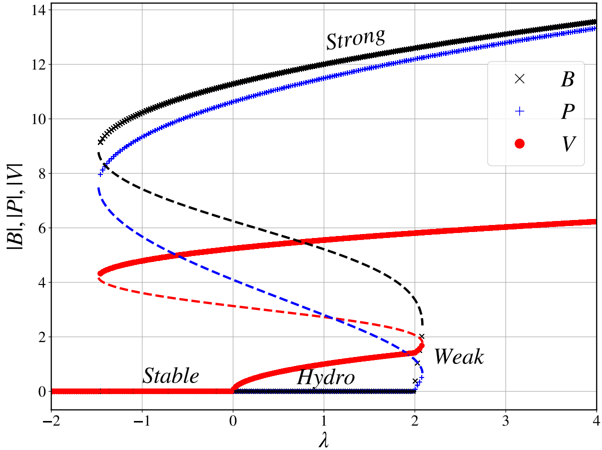
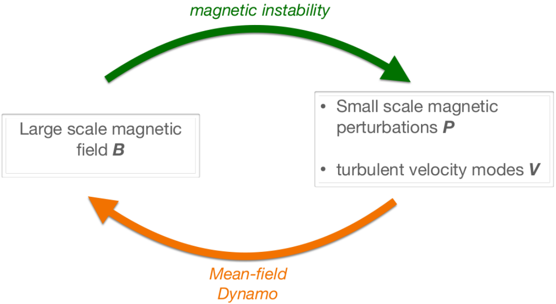


Figure 9 – Bifurcation diagram for the amplitude of each mode as λ varies. The other parameters are fixed.

Table of contents

Take-home message



magnetic instability

Large scale magnetic field ***B***

- Small scale magnetic perturbations ***P***
- turbulent velocity modes ***V***

Mean-field Dynamo

References

- Spruit H, *Dynamo action by differential rotation in a stably stratified stellar interior* AA 2002
- Petitdemange L, Marcotte F, Gissinger, *Spin-down by dynamo action in simulated radiative stellar layers* Science 2023
- Daniel F, Petitdemange L, Gissinger C, *Subcritical transition to turbulence triggered by a magnetic dynamo*, Under consideration for publication in PRF
- Dormy E, Cardin P, Jault D, *MHD flow in a slightly differentially rotating spherical shell,with conducting inner core, in a dipolar magnetic field*, EPSL 1998
- Aubert J, Aurnou J, Wicht J, *The magnetic structure of convection-driven numerical dynamos* GJI, 2007