A fast rotating superfluid on a curved surface

Romain Dubessy & BEC group

Laboratoire de physique des lasers, CNRS UMR 7538
Université Sorbonne Paris Nord, Villetaneuse, France

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MC16 – Fluides classiques et quantiques hors équilibre
Dynamics of quantum gases: superfluidity
Specific dynamical properties

Quantum gases with weak repulsive interactions are superfluid. Superfluidity is a dynamic property with subtle effects.

- absence of viscosity, leading to persistent currents in a circular guide
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- **critical velocity** $v_c = c$ for excitations (Landau criterion) [data: 2D gas, Desbuquois, Nat. Phys. 2012, picture: polaritons, Amo Nat. Phys. 2009]
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Interest of cold atoms for studying superfluidity: great **flexibility** in the control of the **confinement geometry** (harmonic traps, rings, lattices, low dimensions. . . )
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In this talk: a superfluid rotating in a bubble trap.
Physics in a bubble

rf-induced adiabatic potentials — the dressed quadrupole trap

**Adiabatic potentials** for rf-dressed atoms

Ingredients: inhomogeneous $B$ field + strong rf field, coupling $\Omega_{\text{rf}}$

Here: quadrupole field, magnetic gradient $b'$

- local $B$ and rf fields: atomic spin follows **adiabatically** a local eigenstate
- local eigenenergy acts as a **potential**
- atoms are **strongly confined** to a resonant isomagnetic surface

$$\mu B(r) = \hbar \omega$$

- **smooth** surface potentials
- cooling with an **rf knife**.

For a quadrupole field: ellipsoidal **isomagnetic surface**

$$x^2 + y^2 + 4z^2 = r_0^2$$

with $r_0 \propto \omega / b'$.

Trapping atoms on a surface
A smooth two-dimensional trap

\[ \omega_z \propto \frac{b'}{\sqrt{\Omega_{rf}}} \sim 1 \text{ kHz} \]

\[ \omega_x, \omega_y \propto \sqrt{\frac{g}{r_0}} \sim 20-50 \text{ Hz} \]

- very flat \( \omega_z \gg \omega_x, \omega_y \)
- in-plane anisotropy \( \eta = \frac{\omega_x}{\omega_y} \) controlled through rf polarization:
  - rotationally invariant \( (\eta = 1) \) for a \( \sigma^+ \) polarization along \( z \)
  - anisotropic \( (\eta \neq 1) \) for linear horizontal polarization
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- very flat \( \omega_z \gg \omega_x, \omega_y \)
- in-plane anisotropy \( \eta = \frac{\omega_x}{\omega_y} \) controlled through rf polarization:
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  - anisotropic (\( \eta \neq 1 \)) for linear horizontal polarization
- geometry can be modified dynamically
- ideal for the study of the 2D trapped gas dynamics

[Dubessy NJP 2014]

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Why rotations?
Quantum Hall effect with atoms

ideal 2D trapped rotating atomic gas

$H = \hbar \omega_r \left( \hat{a}_x^{\dagger} \hat{a}_x + \hat{a}_y^{\dagger} \hat{a}_y \right) - \Omega \hat{L}_z$

$E = n \hbar \omega_r$

Landau level structure

- Highly degenerate groundstate
- Small energy gap

$\Omega = 0$

$\Omega = \omega_r^2 \hbar \omega_r$

$\Omega = \omega_r / 2$

$\Omega = \omega_r^2 / 2 \hbar \omega_r$

Lattices
Why rotations?
Quantum Hall effect with atoms

ideal 2D trapped rotating atomic gas  $\leftrightarrow$  2D electron gas with a uniform magnetic field

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Landau level structure $\Rightarrow$ highly degenerate groundstate

Small energy gap $\Rightarrow$ increased role of temperature
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<td>0</td>
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Landau level structure \( \Rightarrow \) highly degenerate groundstate


Small energy gap \( \Rightarrow \) increased role of temperature
Vortex crystals
Low energy modes & melting transition

- for $\Omega \leq \omega_r$ groundstate is a large Abrikosov lattice
- with well defined modes:
  - longitudinal (Kelvin)
  - in plane: elasticity of the lattice
- Thermal population of the modes can melt the lattice...
  never observed in BEC!
Vortex crystals
Low energy modes & melting transition

\[ \omega_r/\omega_z = 4.2 \]

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- Thermal population of the modes can melt the lattice...

- To lower $T_m/T_c$: decrease $\omega_r/\omega_z \Rightarrow$ go 2D !  
- bubble: $\omega_r/\omega_z \leq 0.1$

$\omega_r/\omega_z = 4.2$
$T_m/T_c \sim 0.78$
$T_m/T_c \sim 5.89$

[Abo-Shaer Science 2001]
[Coddington PRL 2003]
[Bretin PRL 2004]
Vortex lattice in fast rotating bubble trap
A first quick look

- Start from a degenerate cloud at rest at the bottom of the bubble, $\omega_r = 2\pi \times 34$ Hz
- Induce an in-plane elliptic deformation
  \[ V(r) = M\omega_r^2/2 \times [(1 - \epsilon)x'^2 + (1 + \epsilon)y'^2] + ... \]
- Rotate the trap main axes $x'$, $y'$ at frequency $\Omega_{\text{rot}}$
- Restore the rotationally invariant trap

Increasing $\Omega_{\text{rot}}$...

Blue dashed circle: Thomas-Fermi radius after 27 ms time-of-flight

$\Omega_{\text{Rot}}/(2\pi) = 20$ Hz

$\epsilon = 0.18$
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We need to measure $N$, $\Omega$, $T$ and compare to $T_m(N, \Omega)$...
Rotation control
Stirring and evaporating

Effective rotation $\Omega/\omega_r$

- with post-stirring rf ramp $80 \rightarrow 60$ kHz
  - allows to reach $\Omega \sim \omega_r$
  - at constant $T \sim 18$ nK
Rotation control

Stirring and evaporating

Effective rotation $\Omega/\omega_r$

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Counting vortices

- top view, 27 ms tof
Rotation control
Stirring and evaporating

Counting vortices
- top view, 27 ms tof
- enhance visibility

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Counting vortices
- top view, 27 ms tof
- enhance visibility
- detect positive curvature

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Quantitative study of the vortex lattice

Vortex-vortex correlations
Thermal melting of the vortex lattice?

Studying a quasi-2D crystal

quasi 2D rotating Bose gas $T_m \leq 0.23 T_{BKT}$

[Giord PRA 2008]

upper bound on melting temperature

(computed using low energy modes of the crystal lattice & KTHNY theory)

studied in many systems: supraconductors, colloids, ...

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Remember: rf-knife sets the temperature $T \simeq 18$ nK,

$\tau = T/T_{BKT}$

Qualitative agreement: change in $g(r)$ as $T / T_m$ varies.
Can we rotate even faster?
Fighting the centrifugal force

To restore the trapping potential, add a quartic term to $V(r)$:

$$V_{\text{eff}}(r) = \frac{M}{2}(\omega_r^2 - \Omega^2)r^2 + \lambda r^4.$$  

[ Bretin PRL 2004 ]

$\Omega = 0$
$\Omega = \omega_r$
$\Omega = 1.15 \omega_r$

$\Rightarrow$ the bubble trap has higher order terms.
Theoretical predictions
Rotating beyond the trapping frequency

Giant vortex in a harmonic + quartic trap:

vortex lattice
dynamical ring

Singly quantized vortex array
Singly quantized array with hole
Multiple quantization

Interaction strength
Rotation rate

50 µm

GP simulation for the bubble (quartic approximation)

[Kavoulakis NJP 2003, Fetter PRA 2005]
Creating a dynamical ring
Using the spin-up evaporation

\[ \varepsilon = 0.18 \]

stirring at \( \Omega_{\text{stir}} \)

\[ \omega_y < \Omega_{\text{stir}} < \omega_x \]

\[ \varepsilon = 0.01 \]

\( \Omega_{\text{exc}} \)

(spectroscopy)

\[ \omega_{kn}/2\pi \]

65 kHz

forced evaporation

58 kHz

\[ \varepsilon = 0 \]

\( \varepsilon = 0.01 \)

stirring at \( \Omega_{\text{stir}} \)

\( \Omega_{\text{exc}} \)

(spectroscopy)

In-situ pictures

-0.2 s

-0.048 s

3 s

20 s

24 s

38 s

71 s

[Guo PRL 2020]

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A fast rotating superfluid on a curved surface
A supersonic flow
Measuring the rotation from time-of-flight expansion

- size$^2$ scales as $t_{TOF}^2$ (ballistic expansion)
- fit gives: $\Omega \sim 1.05\omega_r$, i.e. $v = 7.4$ mm/s
- peak density $n_0 \sim 15 \, \mu m^{-2}$
  $\Rightarrow c_0 = 0.4$ mm/s

A degenerate gaz flowing at Mach 18!

[see also Pandey Nature 2019]
Summary & prospects

Fast rotations on a shell

A very smooth and tunable shell trap to study fast rotations

- Fine control of the effective rotation
- Vortex lattice melting for $\Omega \sim \omega_r$
- Formation of a long-lived dynamical ring flowing at Mach 18 for tens of second for $\Omega > \omega_r$

⇒ investigate the decay mechanisms
⇒ test the melting scenario (KTHNY)
⇒ play with the curvature in the rotating frame
Thanks for your attention!

The BEC group at Villetaneuse

ANR funded postdoc position available [2 years]

Former PhDs: M. de Goër de Herve, Y. Guo, V. Bagnato, M. Olshanii, A. Minguzzi, S. Nazarenko

Collaborations (on going)

www-lpl.univ-paris13.fr/bec