Superfluid Fraction in an Interacting Spatially Modulated Bose-Einstein Condensate

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SFP 2023

CHAUVEAU Guillaume

7th July

Laboratoire Kastler Brossel - Paris

Some characteristics

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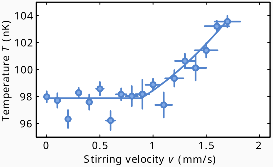

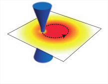

(Desbuquois et al., 2012), (Raman et al., 2001)

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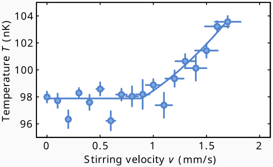

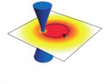

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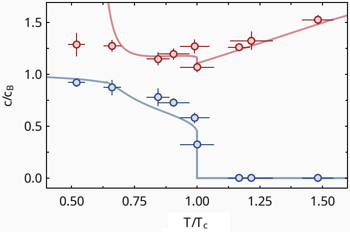
- Two sound modes



(Christodoulou et al., 2021)

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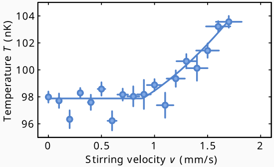

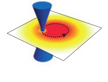

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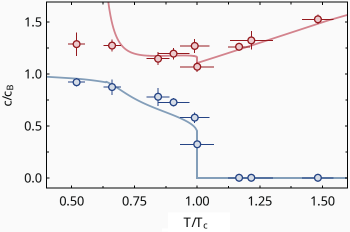
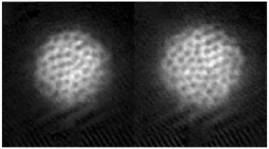


(Christodoulou et al., 2021)

- Quantized angular momentum: vortices



(Bretin et al., 2004)

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Total density $\rho = \rho_n + \rho_s$
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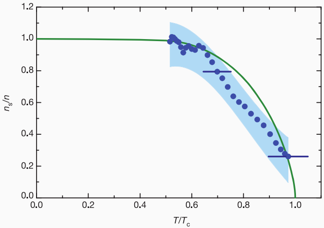

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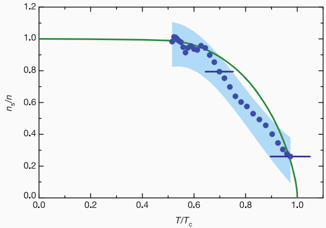

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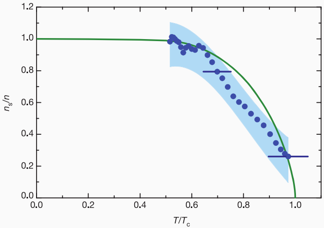

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Our work: $T = 0$

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⇒ In general, only an upper bound given by Leggett's formula (in *Can a solid be a superfluid?*) (Leggett, 1970):

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$$f_s \leq \left(\frac{1}{\langle \rho \rangle \langle 1/\rho \rangle} \right)$$

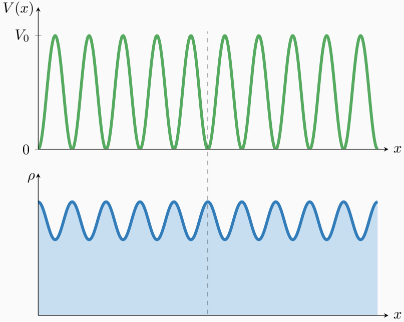
In the case of weakly interacting Bose gas at $T = 0$, with separable density, *i.e.* $\rho(x, y, z) = \rho_x(x)\rho_y(y)\rho_z(z)$, we showed:

Saturation of Leggett's formula

$$f_s = \frac{1}{\langle \rho \rangle \langle 1/\rho \rangle}$$

The superfluid fraction is reduced due to the density modulation

Break translational invariance along 1 direction with 1D periodic potential $V(x) = V_0 \cos(qx)$



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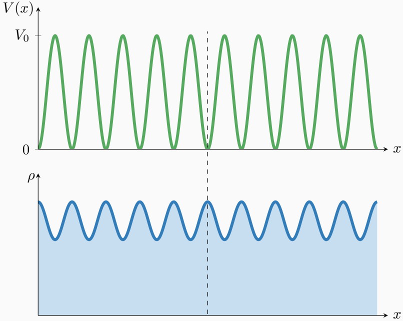
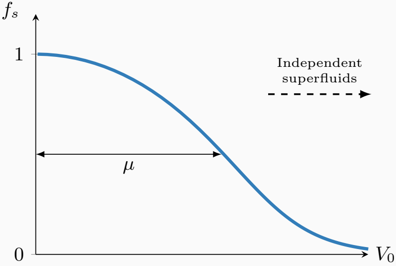


$\Rightarrow$ Expect reduction of f_s



μ: chemical potential

How to measure this reduction of superfluid fraction?

Superfluid fraction measured for different V_0, with two different methods:

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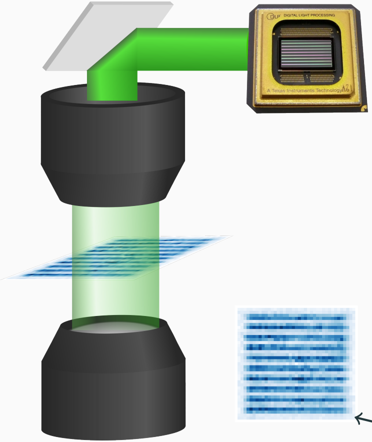
Superfluid fraction measured for different V_0, with two different methods:

- **Dynamic measurement**: excite the cloud and measure its response to density perturbation (oscillation at speed of sound frequency)
- **Static measurement**: measure density and apply Leggett's formula

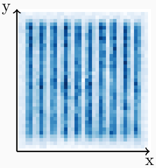
Our experimental platform



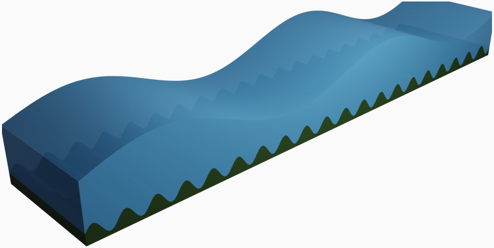
- ^{87}Rb atoms (bosons)
- Planar BEC
- In-plane confinement: box-like potential of size $L_x = L_y = 40\,\mu m$
- Periodic potential: optical lattice of period $d = 4\,\mu m \Rightarrow$ break translational invariance

Absorption image

Dynamic measurement of f_s



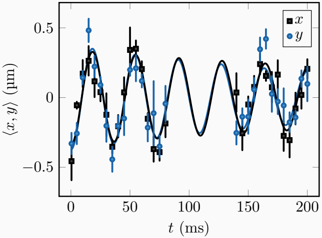
- For a given V_0, we shake the BEC along x
- We measure the frequency of the standing wave created by this perturbation ν_x
- Speed of sound $c_x = 2L_x\nu_x$
- We do the same but excitation along y and extract c_y
- We repeat these 2 measurements for another V_0



Speed of sound measurement of f_s

$$f_{s,x} = \frac{c_x^2}{c_y^2} = \frac{\nu_x^2}{\nu_y^2}$$

Center of mass oscillations



$V_0 = 0 : \nu_x \simeq \nu_y$

Oscillations fit by: $\langle x, y \rangle = e^{-\Gamma t}\left[A\cos(2\pi\nu_{x,y}t) + B\sin(2\pi\nu_{x,y}t)\right]$

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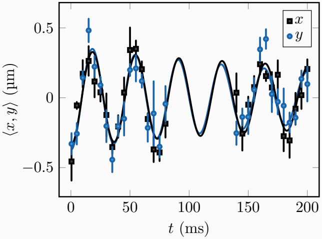

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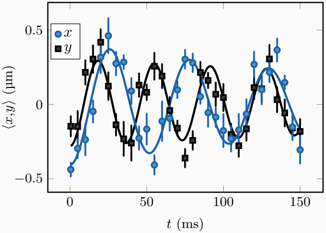

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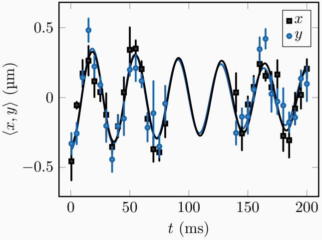

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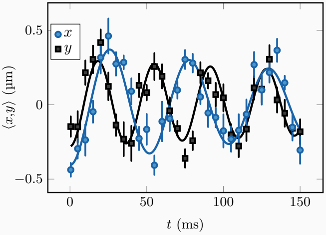

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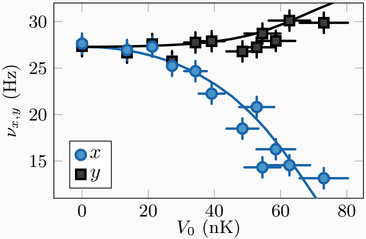
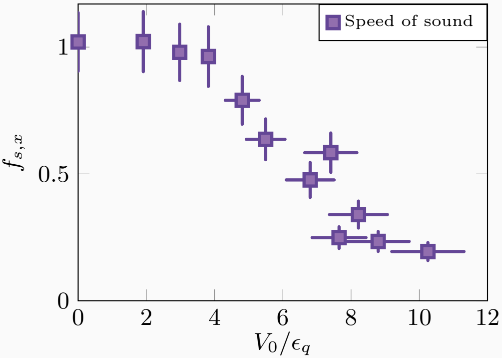


Solid lines: GP simulations

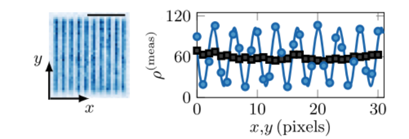
Dynamic measurement of f_s

$$f_{s,x} = \frac{\nu_x^2}{\nu_y^2}$$



$q = \frac{2\pi}{d}$, $\epsilon_q = \frac{\hbar^2 q^2}{2m}$

Naive way



Measure ρ, fit with a sine wave and deduce

$$f_s = \frac{1}{\langle \rho \rangle \langle 1/\rho \rangle} = \left(1 - \frac{\rho_1^2}{\rho_0^2}\right)^{\frac{1}{2}}$$

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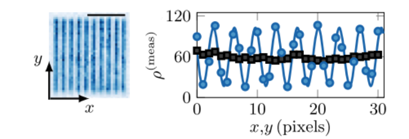

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More elaborate way

Higher harmonics take importance for large V_0 and we expand ρ in Fourier modes.

Ideal world infinite imaging resolution

$$\rho(x) = \rho_0 - \sum_{n>0} \rho_n \cos(nqx)$$

Static measurement of f_s

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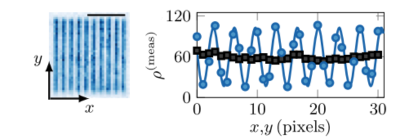

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More elaborate way

Higher harmonics take importance for large V_0 and we expand ρ in Fourier modes. Finite imaging resolution $\Rightarrow$ spatial filtering of the harmonics

Ideal world infinite imaging resolution

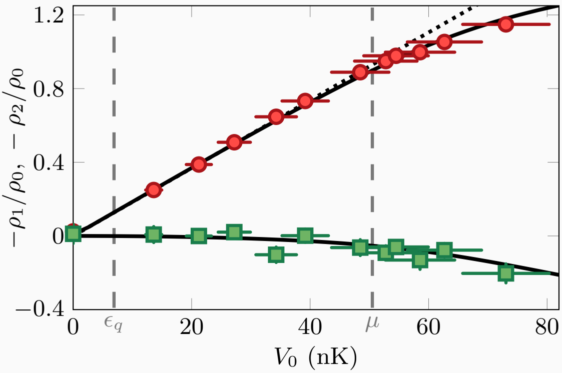
$$\rho(x) = \rho_0 - \sum_{n>0} \rho_n \cos(nqx)$$

Real world finite imaging resolution

$$\rho^{(\mathrm{meas})}(x) = \rho_0 - \sum_{n>0} \boldsymbol{\beta_n} \rho_n \cos(nqx)$$

$$\beta_1 = 0.73$$
$$\beta_2 = 0.27$$
$$\forall i \geq 3, \beta_i = 0$$

Reconstruction of $\rho(x)$



Red: first harmonic; green: second harmonic. Solid lines: GP predictions

Static measurement of f_s

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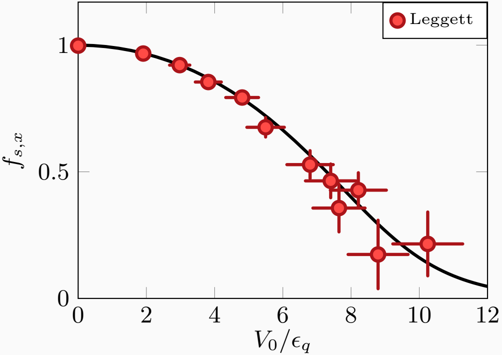

Solid line: GP prediction

Summarizing results

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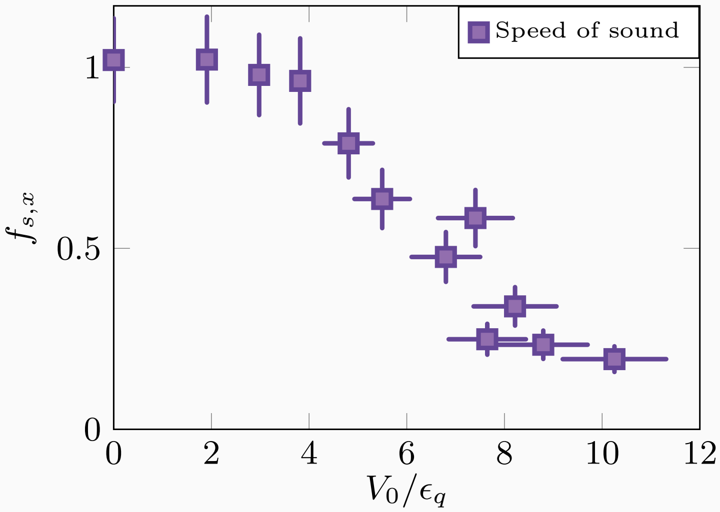

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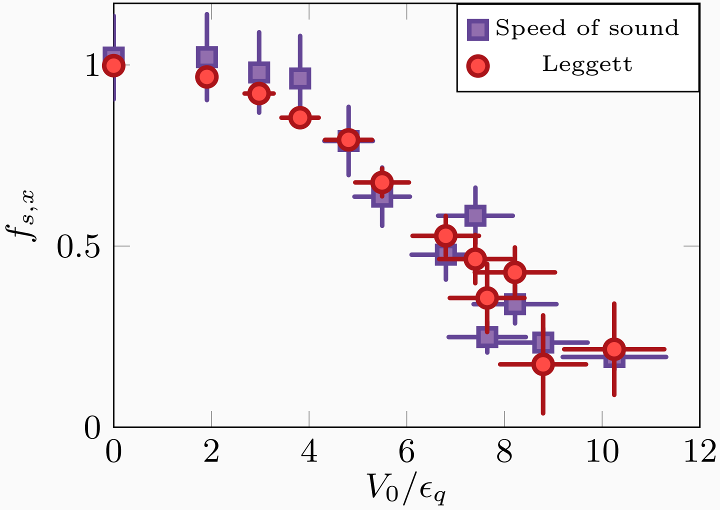
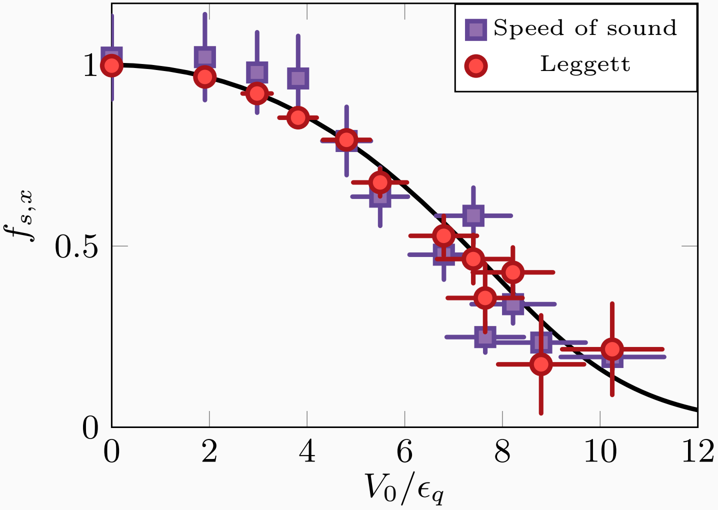
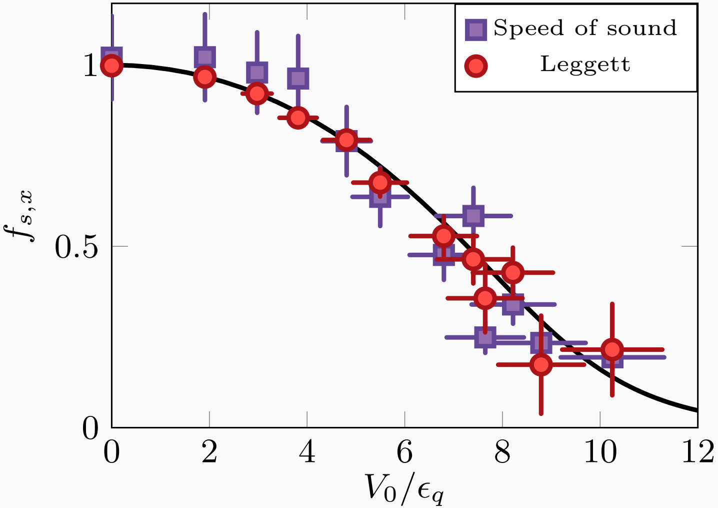



Speed of sound
Leggett
$f_{s,x}$
1
0.5
0
0
2
4
6
8
10
12
V_0/ϵ_q

Summarizing results



Outlook: Fermi gases, supersolids

Thanks!

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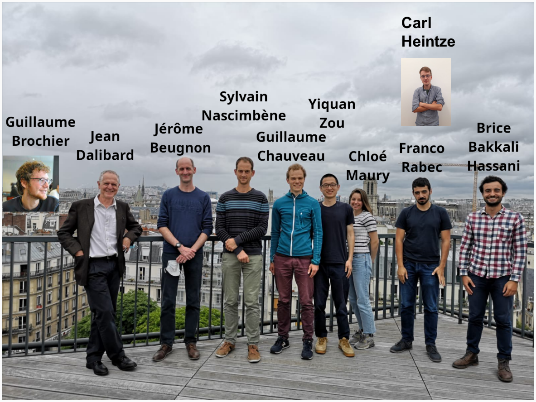

Santo Maria Roccuzzo

Sandro Stringari

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Theory - Saturation of Leggett's formula

- $f_{s,x}$ calculated by applying the perturbation $-v_0\hat{P}_x$ to the system

$$f_{s,x} = 1 - \lim_{v_0 \to 0} \frac{\langle \hat{P}_x \rangle}{Nmv_0} \quad (1)$$

- equation of continuity:

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial}{\partial x}[\rho(x,t)(v(x,t) - v_0)] = 0 \quad (2)$$

- stationary solution in the moving frame

$$\Rightarrow f_s = \frac{1}{\langle \rho \rangle \langle 1/\rho \rangle}$$

Equivalently, one can apply twisted boundary conditions and look at the evolution of energy:

- $$\psi(L) = \psi(0)e^{i\theta} \tag{3}$$
- $$E(\theta) \simeq E(0) + N f_{s,x} \frac{\hbar^2\theta^2}{2mL^2} \tag{4}$$

Look for solutions written as Bloch functions:

$$E(k) \simeq E(0) + N \frac{\hbar^2 k^2}{2m_x^*} \tag{5}$$

$$\Rightarrow f_{s,x} = \frac{m}{m_x^*}$$

Theory - Speed of sound

In the presence of $V(x)$, the velocity of a sound wave propagating along x is:

$$c_x^2 = \frac{1}{m_x^* \kappa} \tag{6}$$

with $\kappa = \frac{1}{\rho_0 \frac{\partial \mu(\rho_0)}{\partial \rho_0}}$.

And along y:

$$c_y^2 = \frac{1}{m\kappa} \tag{7}$$

- $$f_{s,x} = \frac{m}{m_x^*} = \frac{c_x^2}{c_y^2} \tag{8}$$
- $$f_{s,y} = \frac{m}{m_y^*} = 1 \tag{9}$$

Theory - Limit of small V_0

We expand the solution of the GPE in powers of V_0. It yields:

- $$\frac{\rho_1}{\rho_0} = \frac{2V_0}{2\mu_0 + \epsilon_q} + \mathcal{O}(V_0^3) \tag{10}$$
- $$f_{s,x} = 1 - \frac{2V_0^2}{(2\mu_0 + \epsilon_q)^2} + \mathcal{O}(V_0^4) \tag{11}$$
- $$\kappa = \mu_0^{-1}\left[1 - \frac{2V_0^2\epsilon_q}{(2\mu_0 + \epsilon_q)^3}\right] + \mathcal{O}(V_0^4) \tag{12}$$

Experiment - Full graph