Measuring the spatially resolved rapidities distribution of a 1D Bose gas

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Société Française de Physique

CONTEXT

Homogeneous 1D Bose gas with contact repulsive interactions :

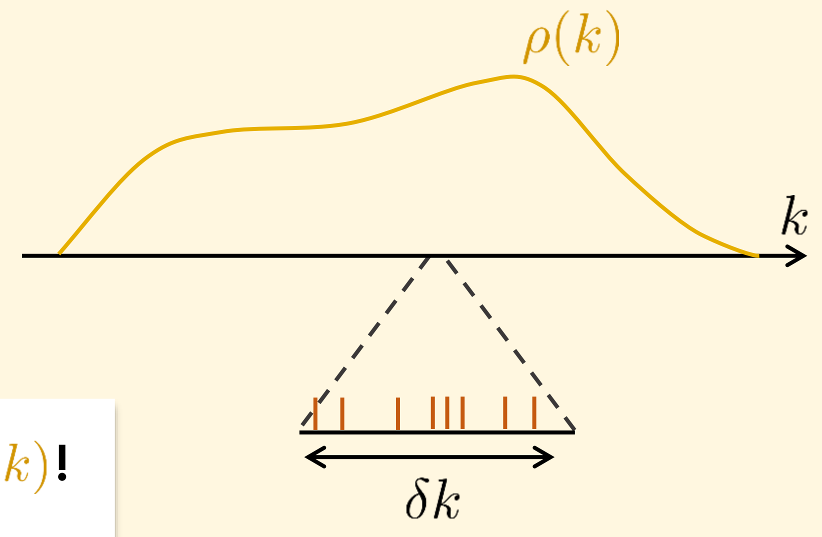
- Lieb Liniger Hamiltonian :

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i^N \frac{\partial^2}{\partial z^2} + g\sum_{i<j}\delta(z_i - z_j)$$

Integrable system: the eigenstates of $\hat{H}$ are the Bethe Anzatz states $|k_1, k_2, ..., k_N\rangle$ labelled by N numbers $(k_1, k_2, ..., k_N)$ called **the rapidities.**

Rapidities distribution $\rho(k)$ **:** $L\,\rho(k)\,\delta k$ is the number of rapidities in the $[k, k+\delta k]$range.

⚠ $\rho(k)$ is conserved during the out-of-equilibrium dynamics of the system

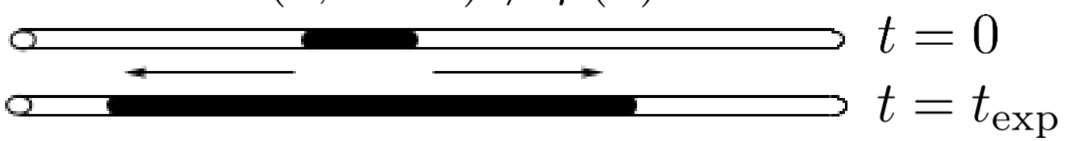


The relaxed state is described by the rapidities distribution $\rho(k)$!

Context

Rapidities distribution can be measured: after a 1D longitudinal expansion of the cloud, the profile density converges towards the rapidities distribution.

$$n(z, t=0) \neq \rho(k)$$



$$\Rightarrow\ n(z, t_{\mathrm{exp}})_{t_{\mathrm{exp}} \to +\infty} \simeq \tilde{n}\left(\frac{z}{t_{\mathrm{exp}}}\right) = \rho\left(\frac{z}{t_{\mathrm{exp}}}\right)\frac{L_0}{t_{\mathrm{exp}}}$$

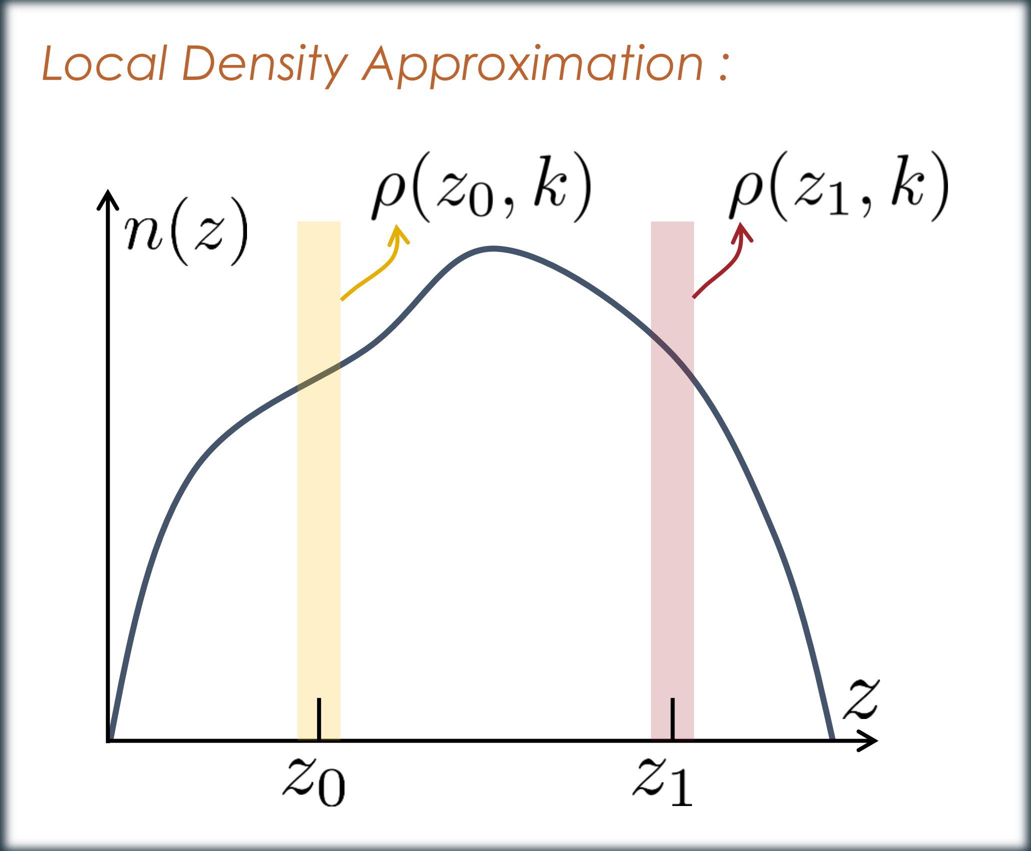
Density profile: $n(z,t)$
Rapidities distribution: $\rho(k,t)$

« Generalized hydrodynamics in strongly interacting 1d Bose gases », Malvania, N. & co, Science ***373****(6559)*

Objectives

Local Density Approximation (Euler scale):
A spatially resolved rapiditiy distribution $\rho(z,k)$ can be defined.

Objective :
Measuring experimentally the spatially resolved rapidity distribution $\rho(\mathbf{z},\mathbf{k})$



OUTLINE

Asymptotic regime of a 1D expansion

- Preparing initial conditions
- 1D expansion

Hydrodynamics for integrable systems

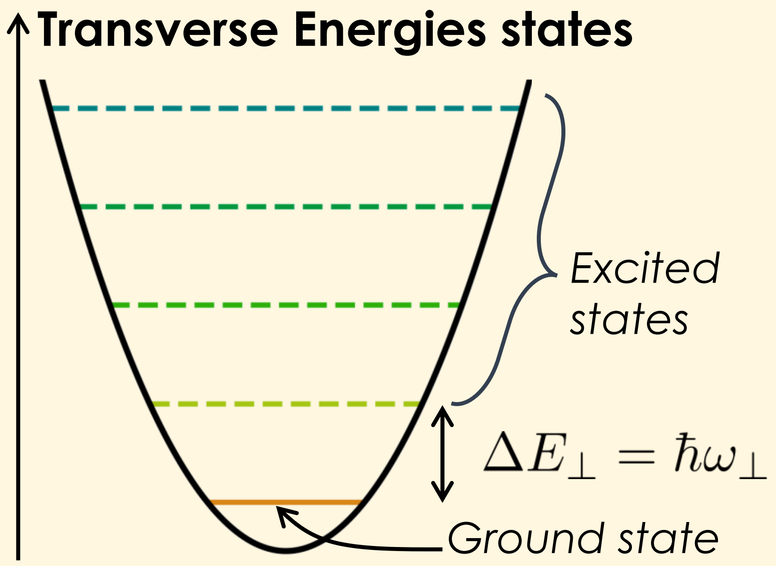
- Generalized HydroDynamics
- Dynamics of 1D expansion

Domain Wall Dynamics

- Domain Wall Dynamics protocol
- Domain Wall Dynamics reconstruction

PREPARING INITIAL CONDITIONS

How to reach the 1D regime?



If $k_B T, \mu \ll \Delta E_\perp$, the gas is frozen in the transverse direction !

PREPARING INITIAL CONDITIONS

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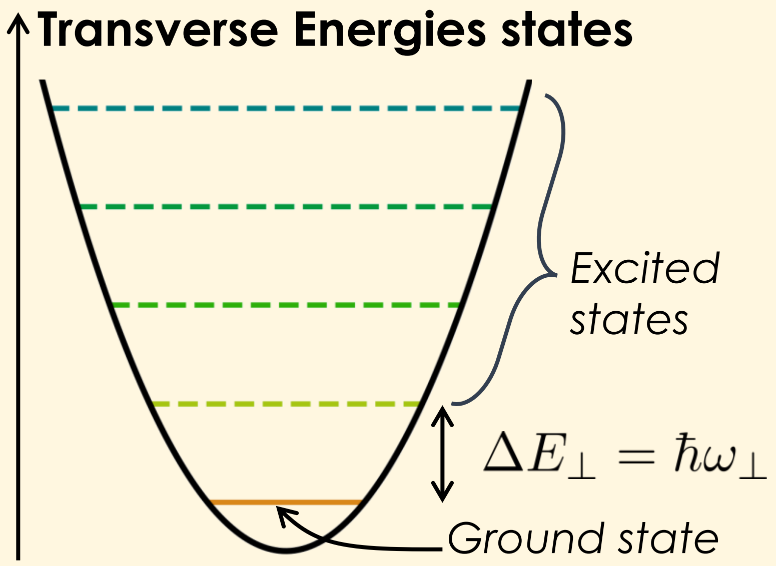
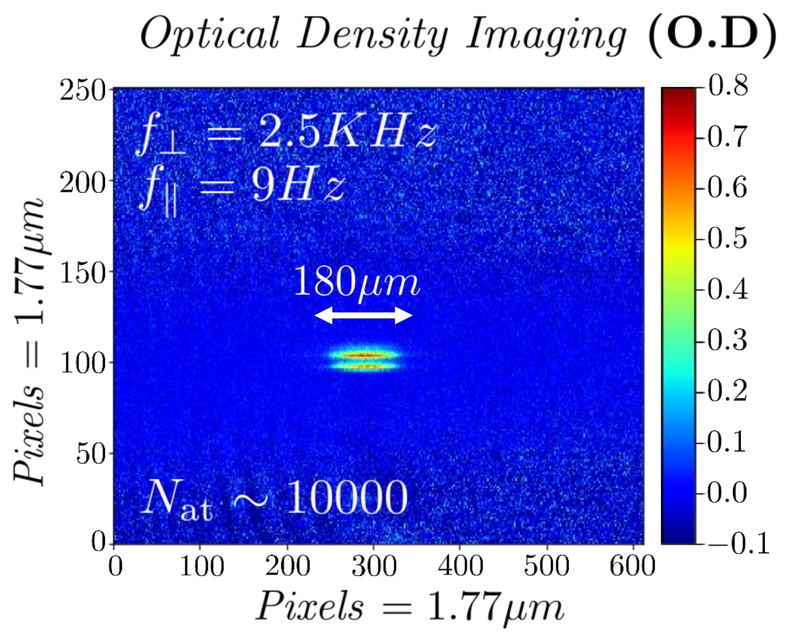


If $k_B T, \mu \ll \Delta E_\perp$, the gas is frozen in the transverse direction !

How do we produce a 1D Bose gas ?

Atoms used : Rubidium 87 in the $|F=2, m_F=2\rangle$ atomic state.

Atom chip experiment: the atoms magnetically trapped thanks to micro-wires deposited on a chip.



Properties :

- Independent control of the longitudinal coupling and the transverse one
- Longitunal trapping:

$$V(z) = \sum_{i=1}^{4} a_i z^i$$

Preparing initial conditions

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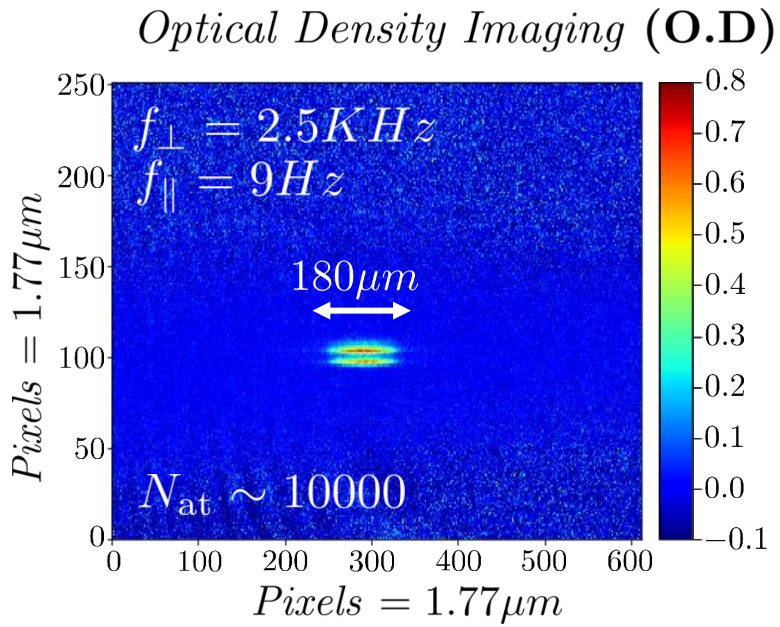


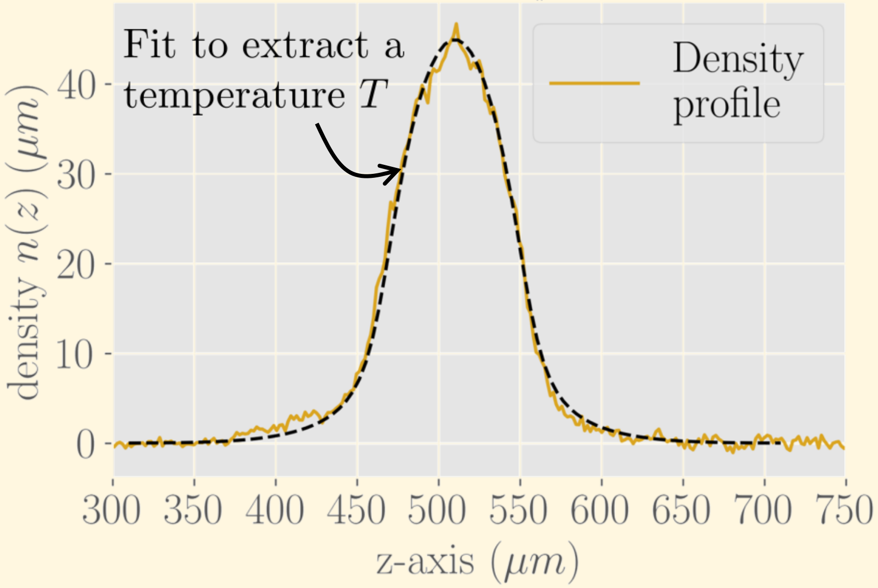
Properties :

- Independent control of the longitudinal coupling and the transverse one
- Longitunal trapping:

$$V(z) = \sum_{i=1}^{4} a_i z^i$$

Density profile of the cloud in the harmonic trap before the selection

Assumption : system described by a Gibbs ensemble



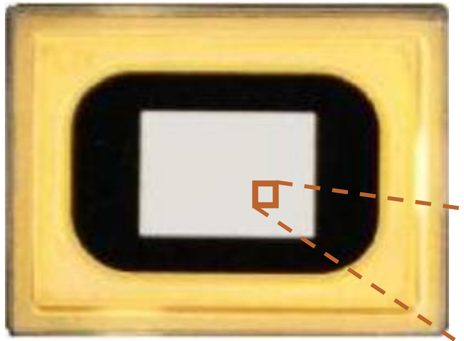
$$\Rightarrow T = 105\, nK,\ \mu = 55\, nK$$

Preparing initial conditions

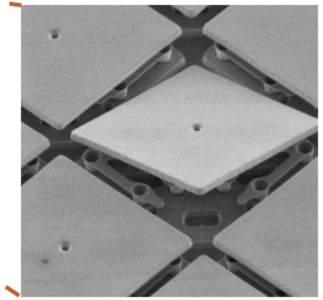
Objective: measure the spatially resolved rapidities distribution

- Implementation of a selection spatial tool
- Use of the radiation pressure

Implementation of a DMD to shape the beam.

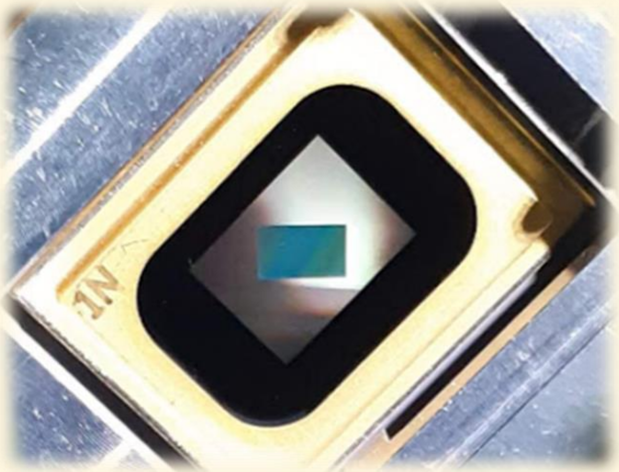


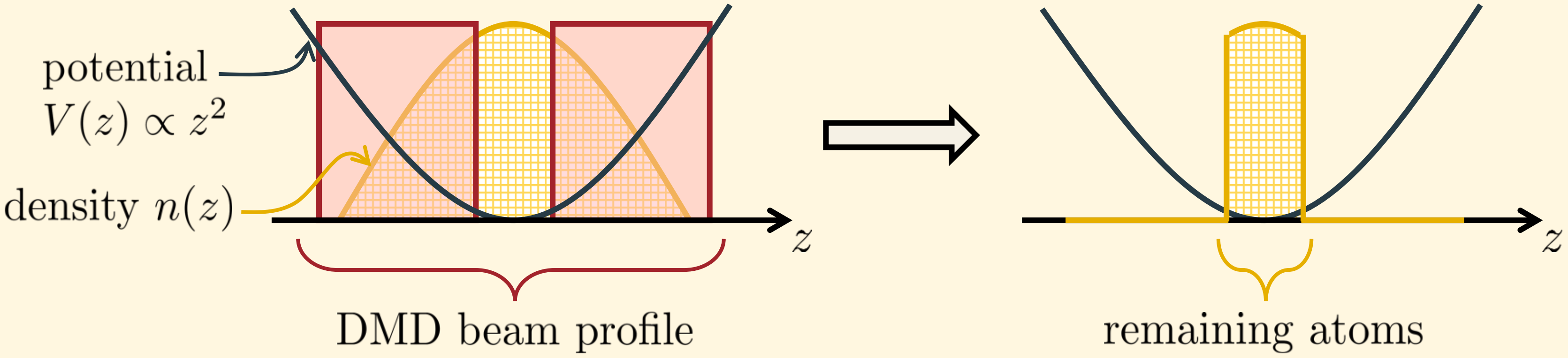
DMD
Digital Micromirrors Device



Features

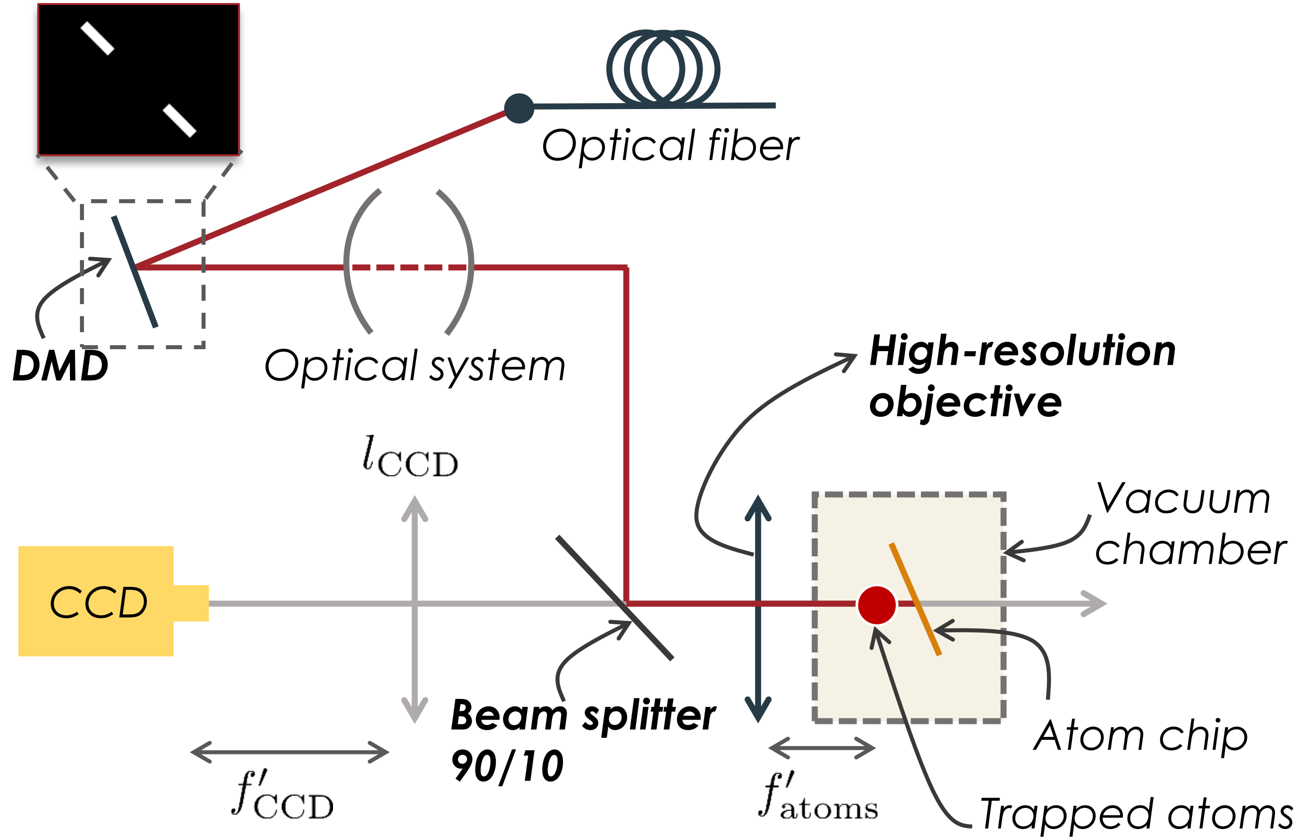
- 1024 x 768 mirrors
- 1 mirror : 14 µm x 14 µm

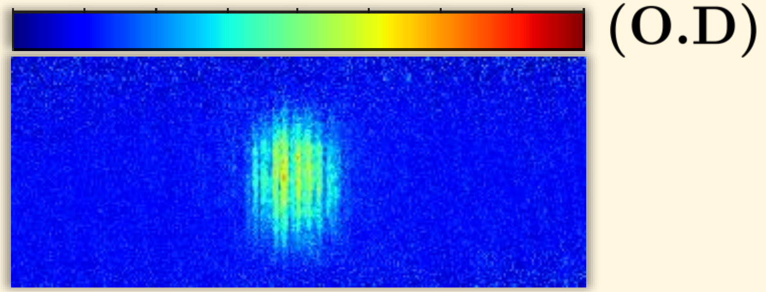


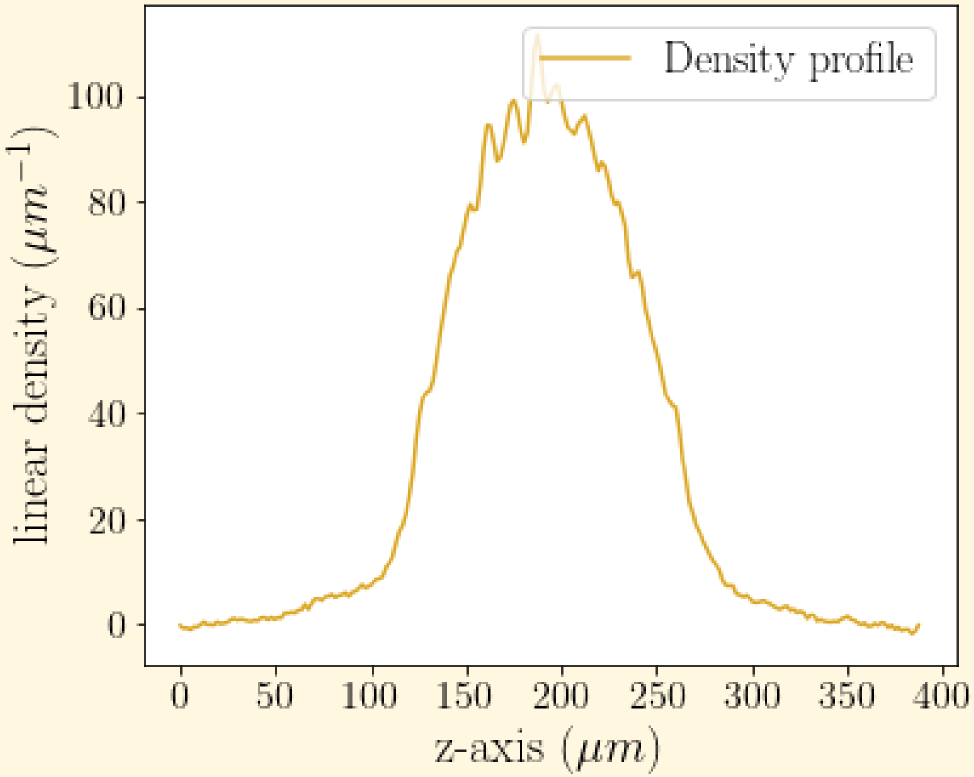


PREPARING INITIAL CONDITIONS

Experimental setup to obtain a homogeneous 1D gas before imaging it:







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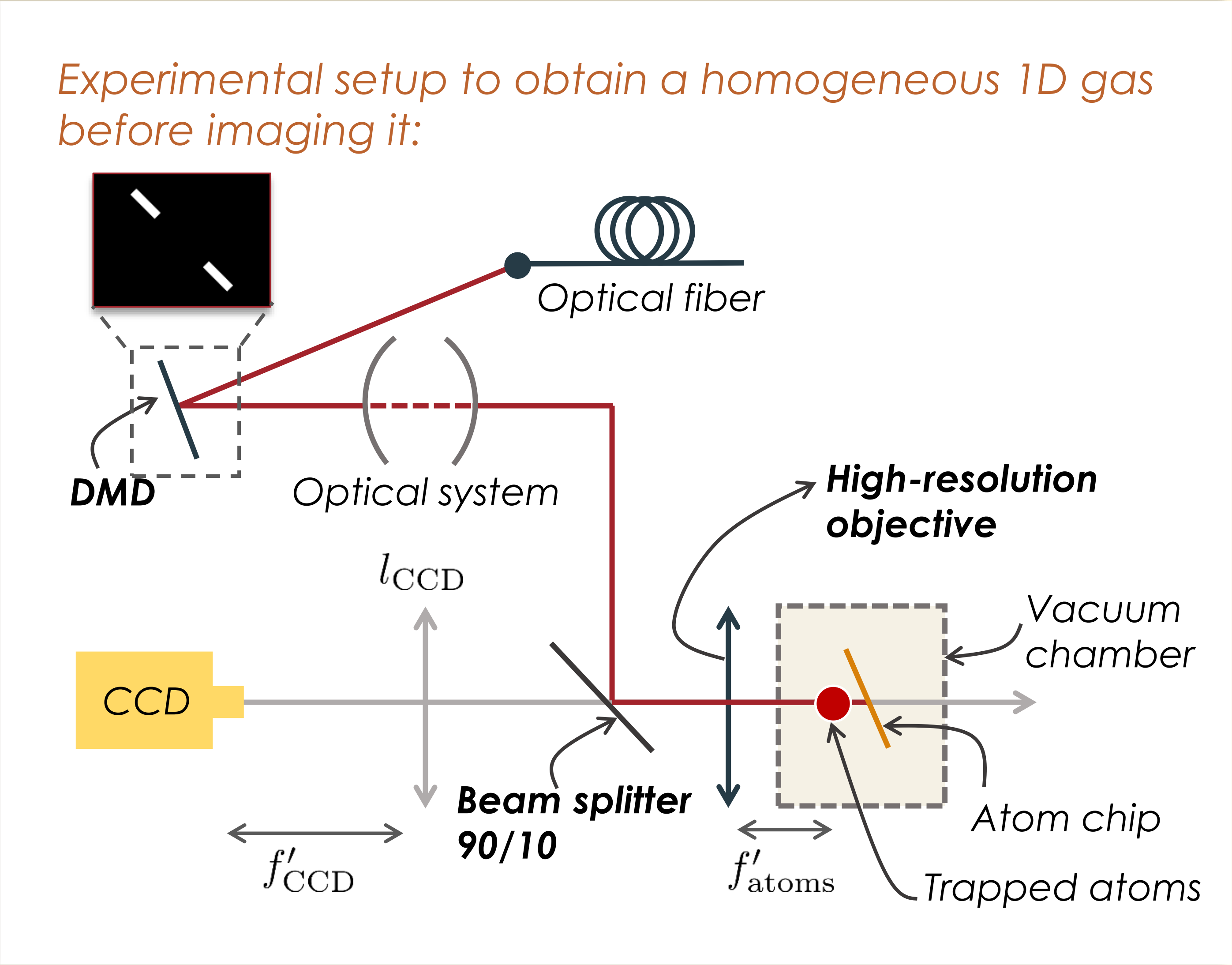
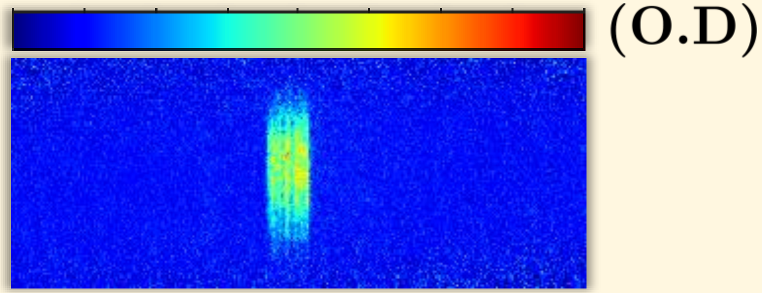
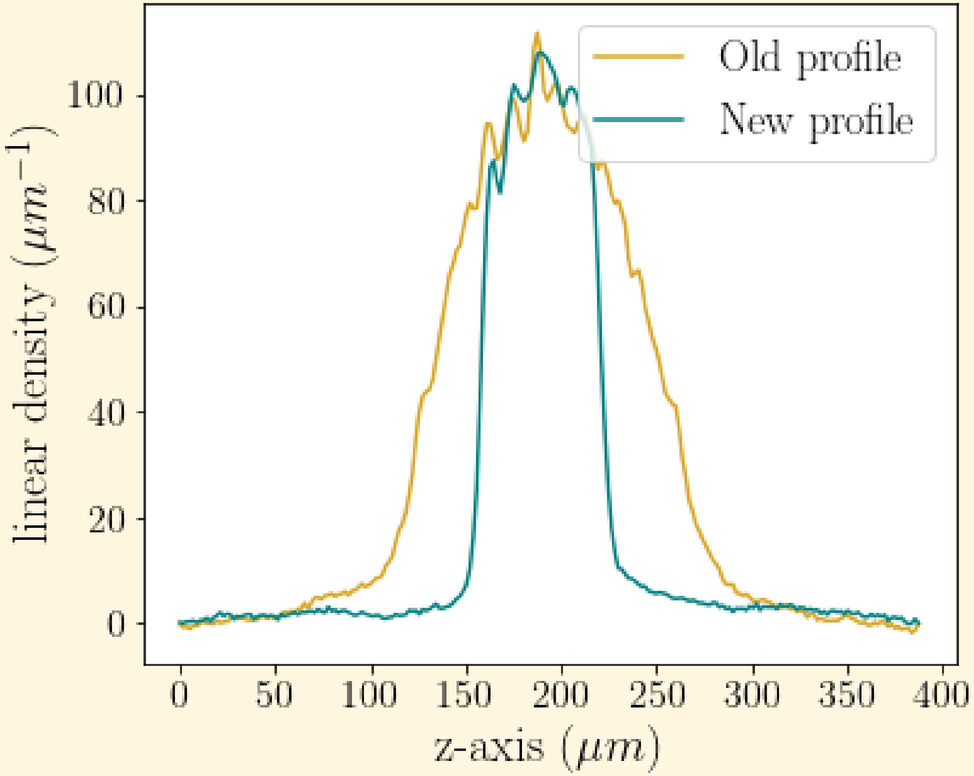
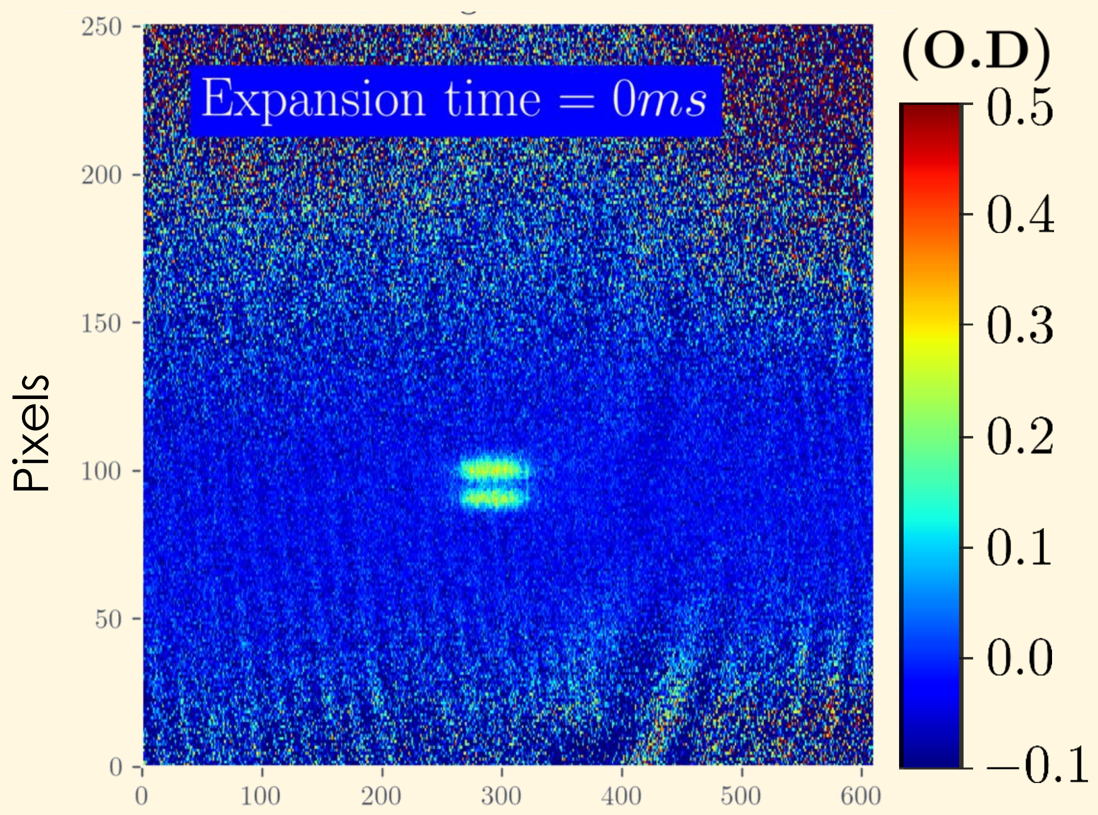






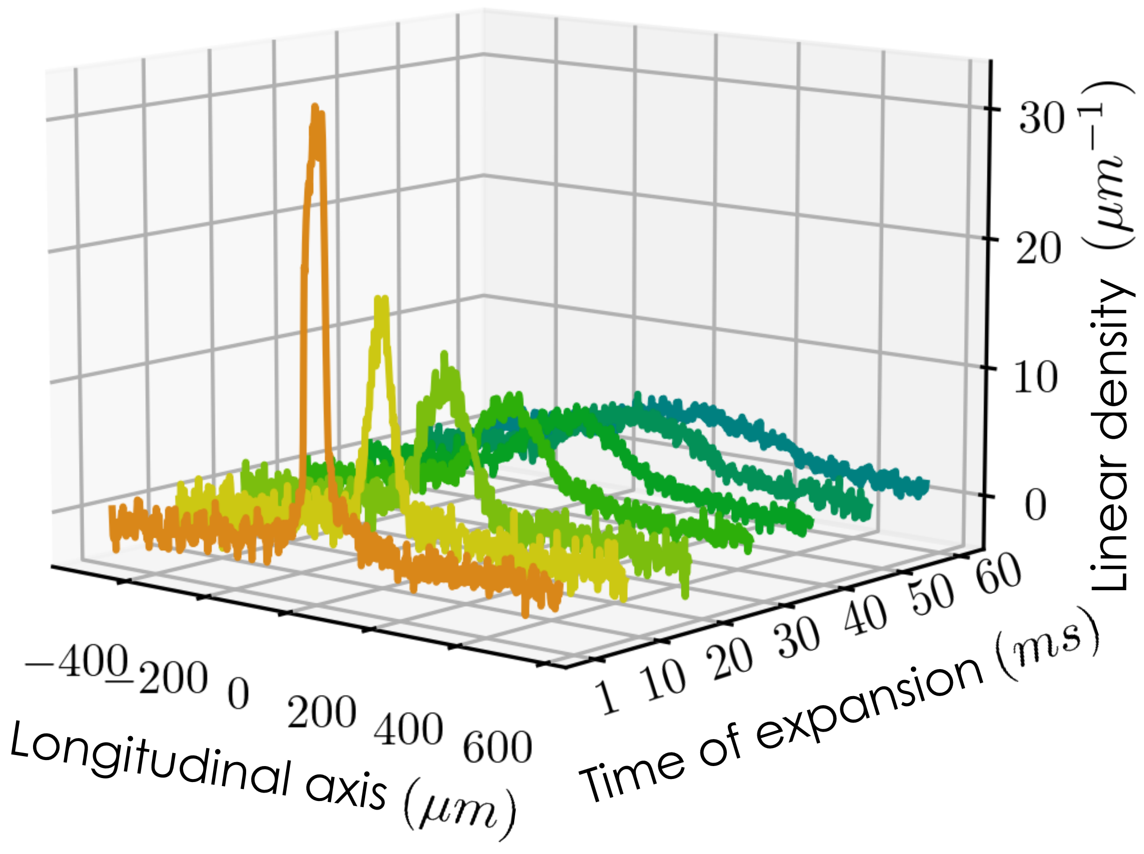
1D EXPANSION

- The longitudinal confinement is removed,
- The transverse one is maintained



Pixels $= 1.77\mu m$

Density profile of a 1D gas after different times of expansion:

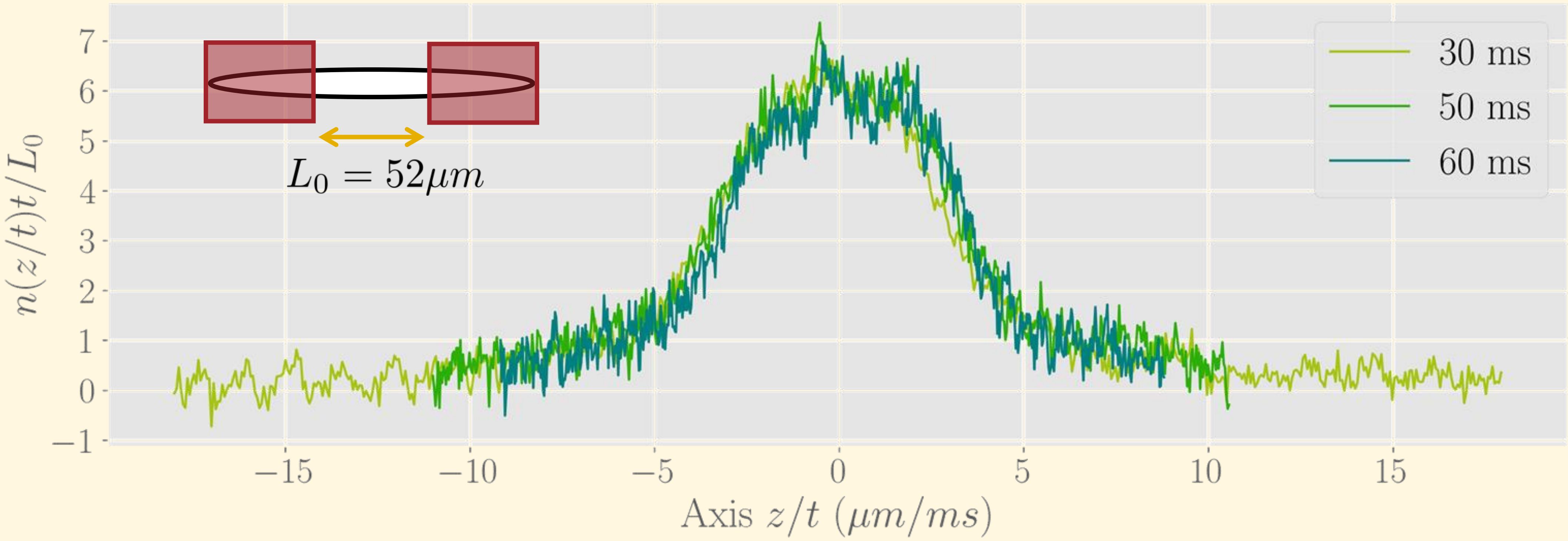


1D EXPANSION

Rescaled profile denstity:

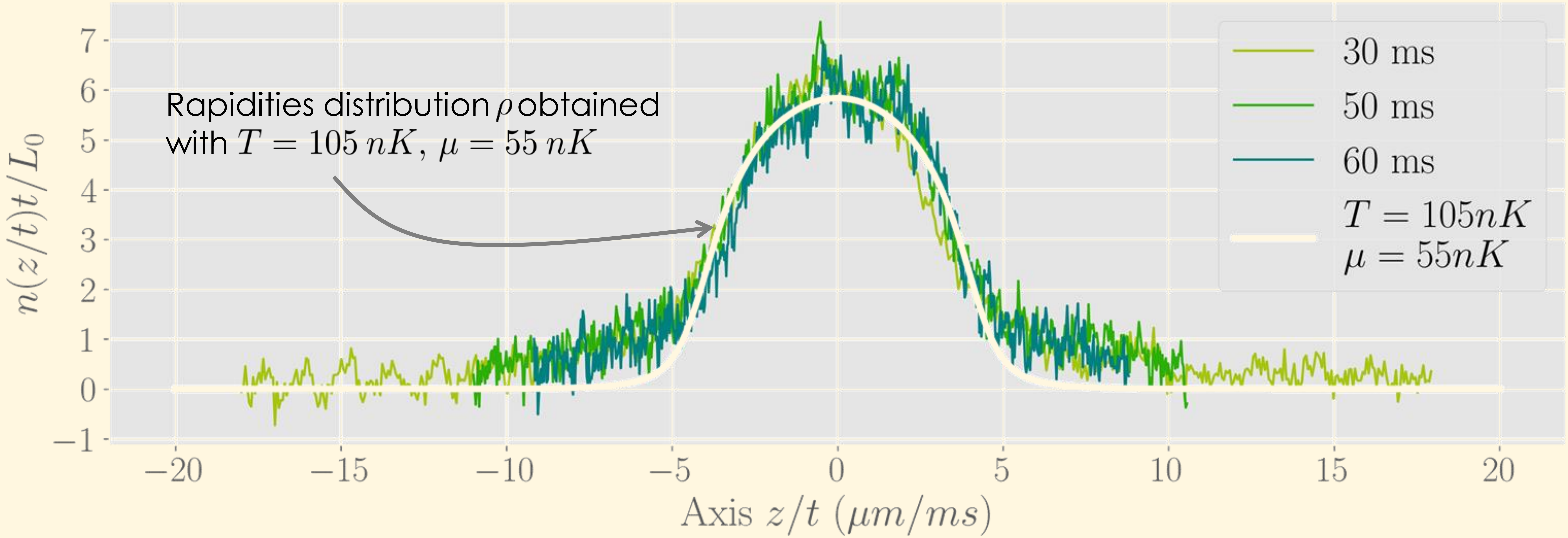
- We look at the profile densities after 50 ms of expansion
- Profiles rescaled with the time

$$n(z,t)_{t\to+\infty} \simeq \tilde{n}\left(\frac{z}{t_{\mathrm{exp}}}\right) = \rho\left(\frac{z}{t_{\mathrm{exp}}}\right)\frac{L_0}{t_{\mathrm{exp}}}$$



An **asymptotic regime** seems to be reached, it corresponds to the rapidities distribution

1D EXPANSION



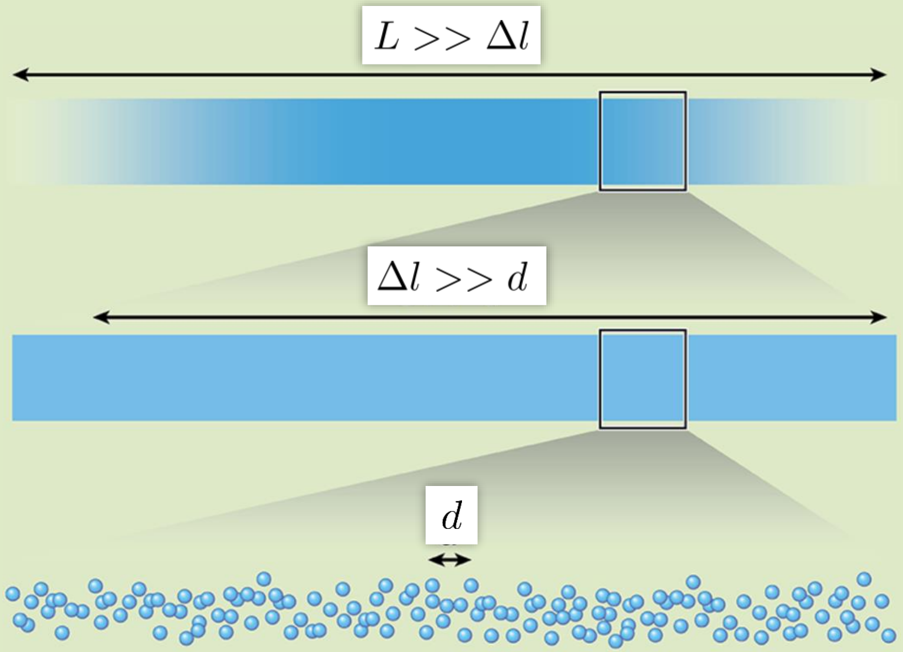
Conclusion:

- Differences between the datas and the rapidities distriution : **Non thermal distribution?**
- Do we reach an asymptotic regime ? **Look at the dynamics**

Generalised HydroDynamics (GHD)

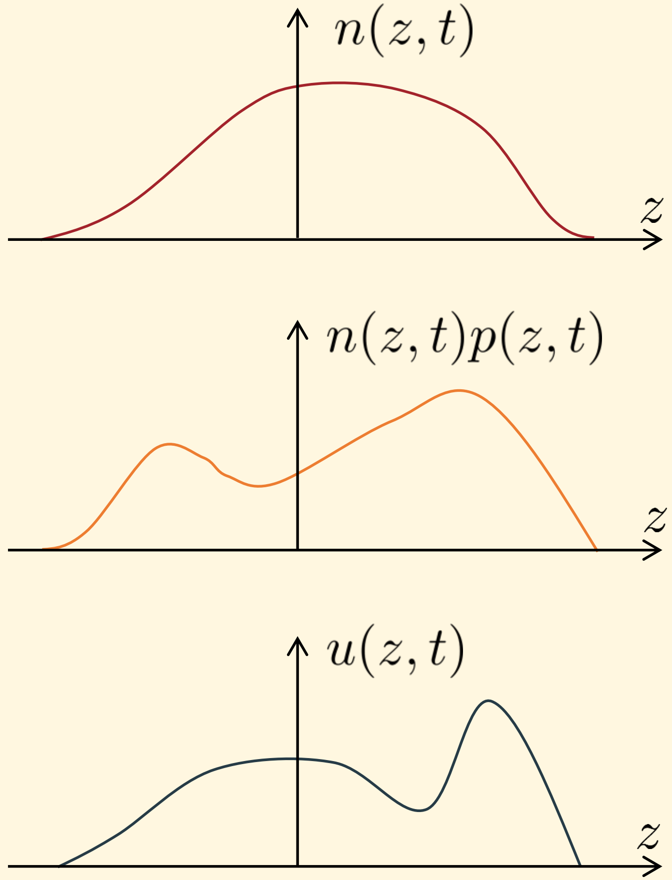
Hydrodynamic approach : continuum of locally homogeneous cells of fluid of size Δl with $L >> \Delta l >> d$ where

- d is the microscopic length scale
- L is the macroscopic one



GENERALISED HYDRODYNAMICS (GHD)

Classical Hydrodynamics :



Classical HydroDynamics (CHD) : a non integrable system can locally be described by a Gibbs state with $n(z,t)$, $n(z,t)p(z,t)$and $u(z,t)$.

- *Conservation of the atom number :*

$$\partial_t n(z,t) + \partial_z \left(v(z,t)n(z,t)\right) = 0$$

Atomic density *Hydrodynamic velocity*

- *Conservation of total momentum :*

$$\partial_t (np) + \partial_z \left(mnv^2 + P\right) = 0$$

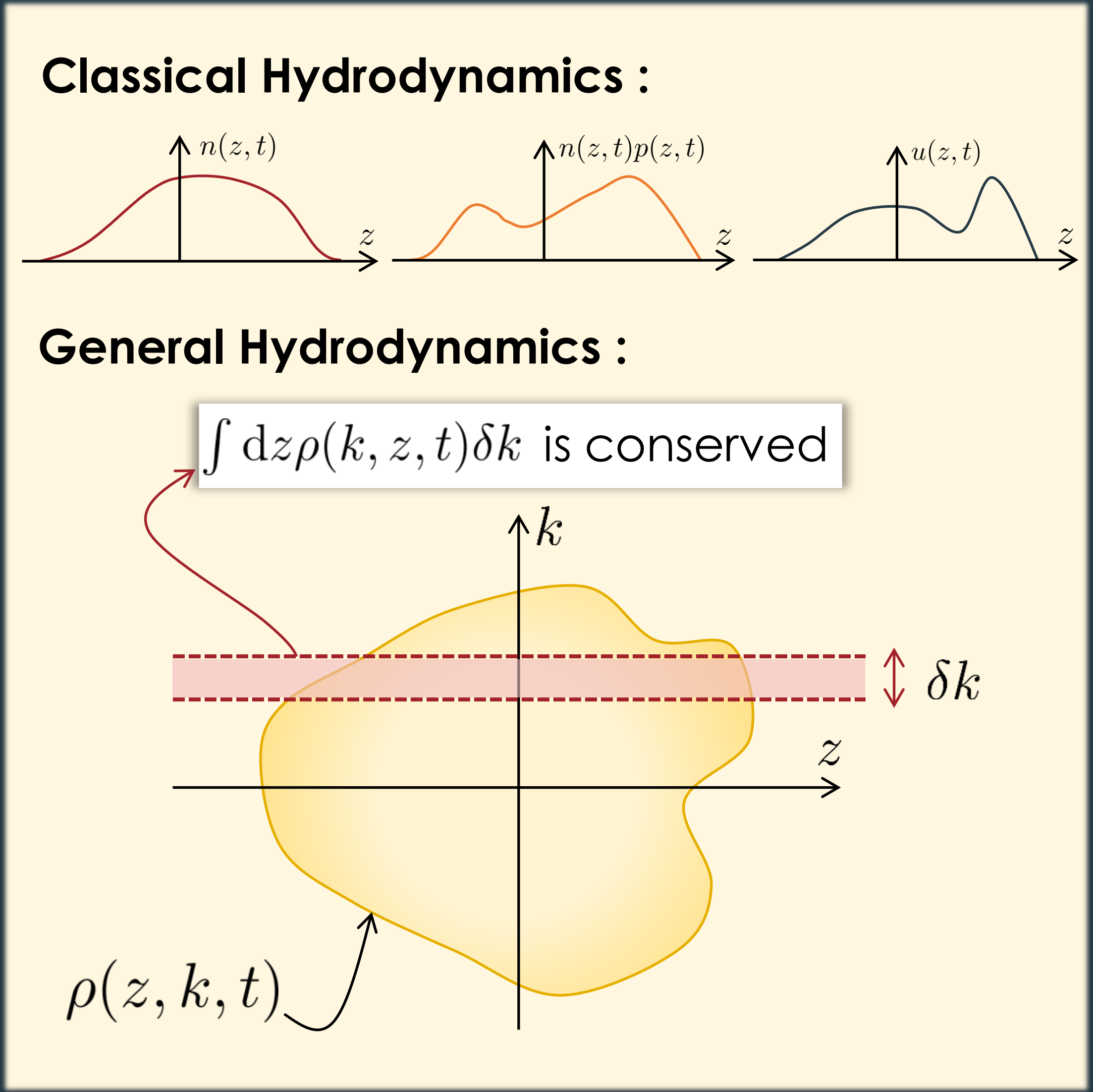
momentum *Pressure*

- *Conservation of total energy :*

$$\partial_t \left(\underbrace{n\frac{mv^2}{2} + ne}_{u(z,t)}\right) + \partial_z \left(n\left(n\frac{mv^2}{2} + ne\right) + nP\right) = 0$$

Energy per atom

GENERALISED HYDRODYNAMICS (GHD)



General HydroDynamics (GHD) : an integrable system can locally be described by the spatially resolved rapidity distribution $\rho(z,k,t)$.

- *Conservation of the rapidity distribution on a* δk *slice:*

$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z,k,t) \right) = 0$$

with

$$v_{\text{eff}}[\rho](k) = k - \int_{-\infty}^{+\infty} \mathrm{d}k' \Delta(k-k') \left[v_{\text{eff}}[\rho](k) - v_{\text{eff}}[\rho](k') \right]$$

and

$$\Delta(k) = \frac{2g}{g^2+k^2}$$

Bertini et al. (2016), Castro-Alvaredo et al. (2016)

Dynamics of 1D expansion

Generalised HydroDynamics Equation :

$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z,k,t)\right) = 0$$

- Equation invariant to $\begin{cases} z \to \alpha z \\ t \to \alpha t \end{cases}$
- With $n(z,t) = \int_{-\infty}^{\infty} \rho(k,z,t)\mathrm{d}k$

For different initial sizes: the dynamics must be the same to within $\mathrm{L_0/t}$.

$$\Rightarrow n(z,t) = \tilde{n}\left(\frac{z}{t}, \frac{t}{L_0}\right)$$

DYNAMICS OF 1D EXPANSION

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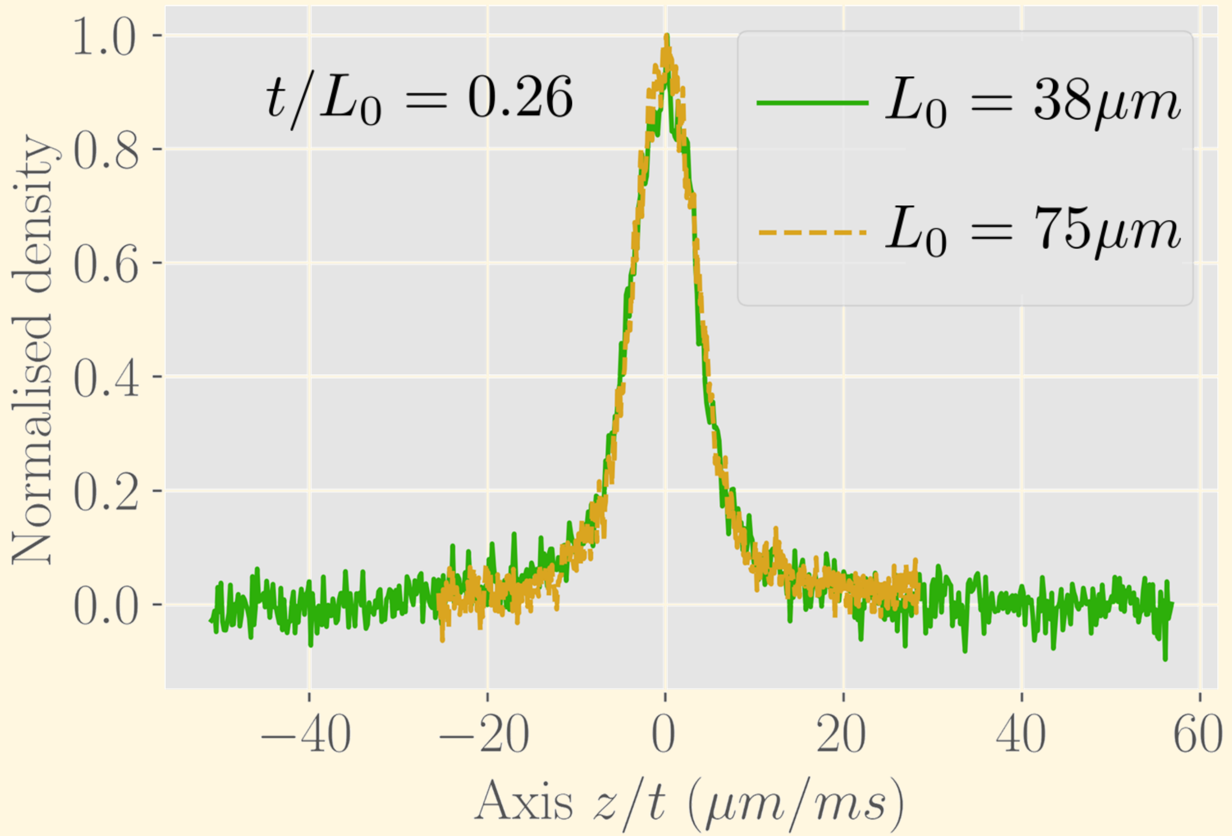
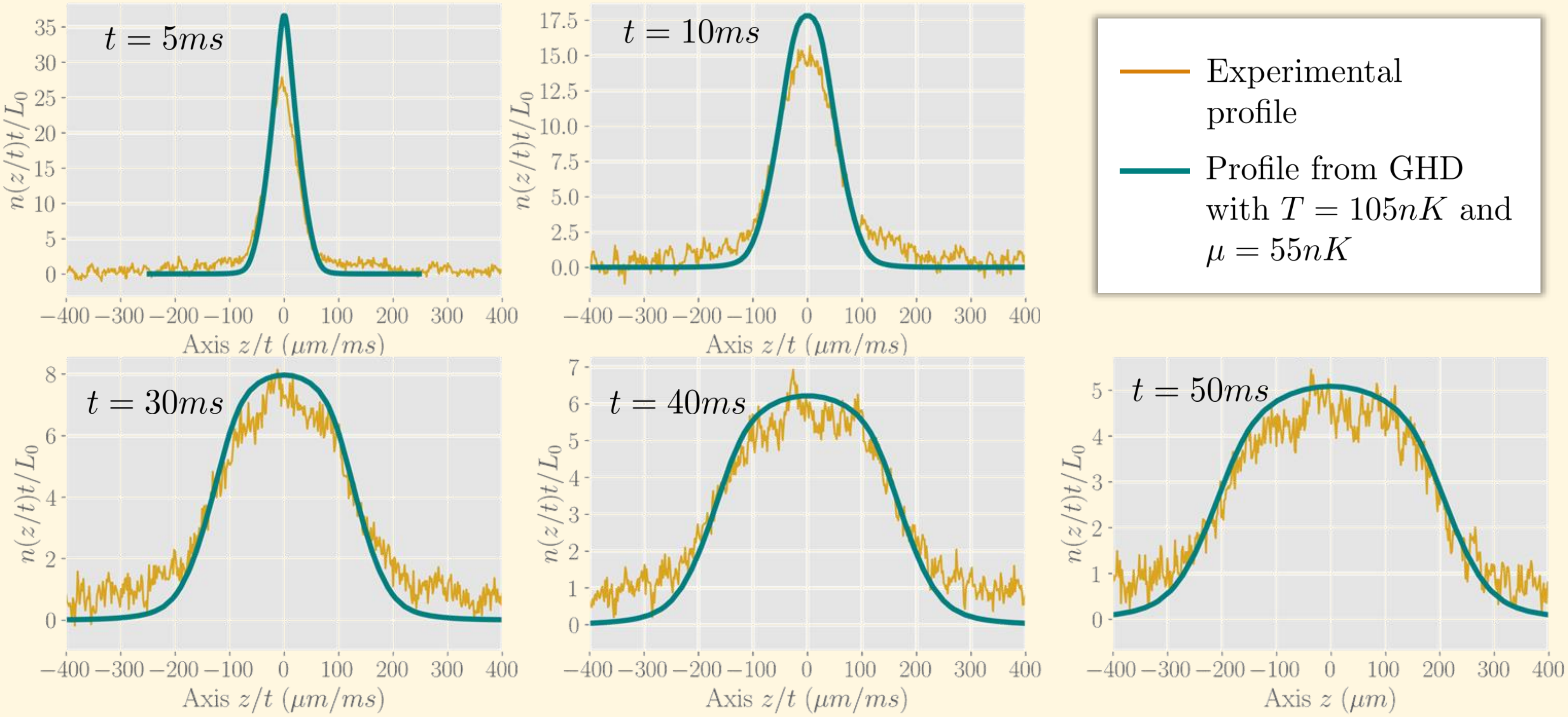
Superimposition of profiles for different initial sizes $\mathbf{L_0}$ but with the same $\mathbf{t/L_0}$ ratio.



Dynamics of 1D expansion

Comparison between experimental profiles for different times and dynamics obtained by Generalised HydroDynamics:



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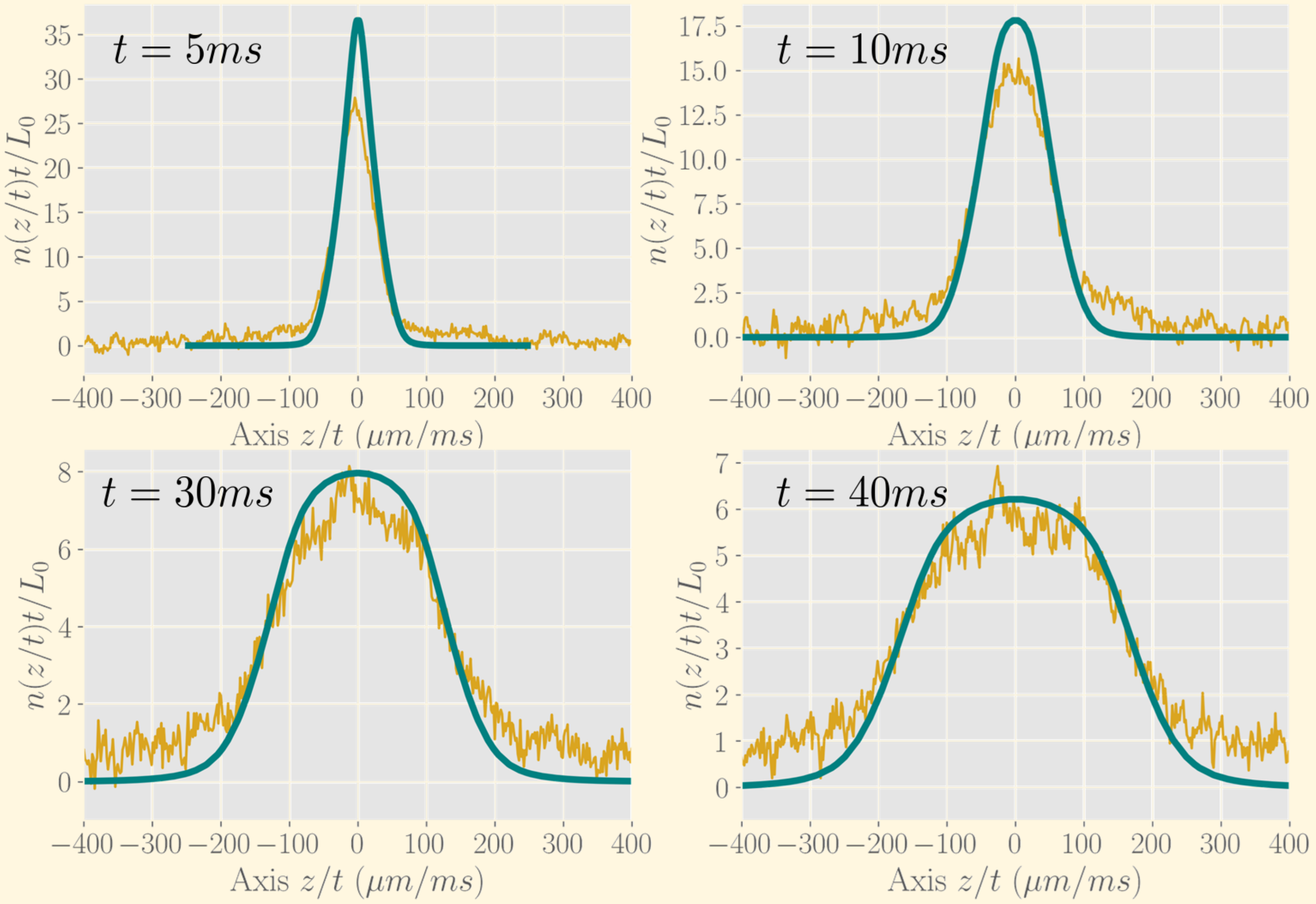
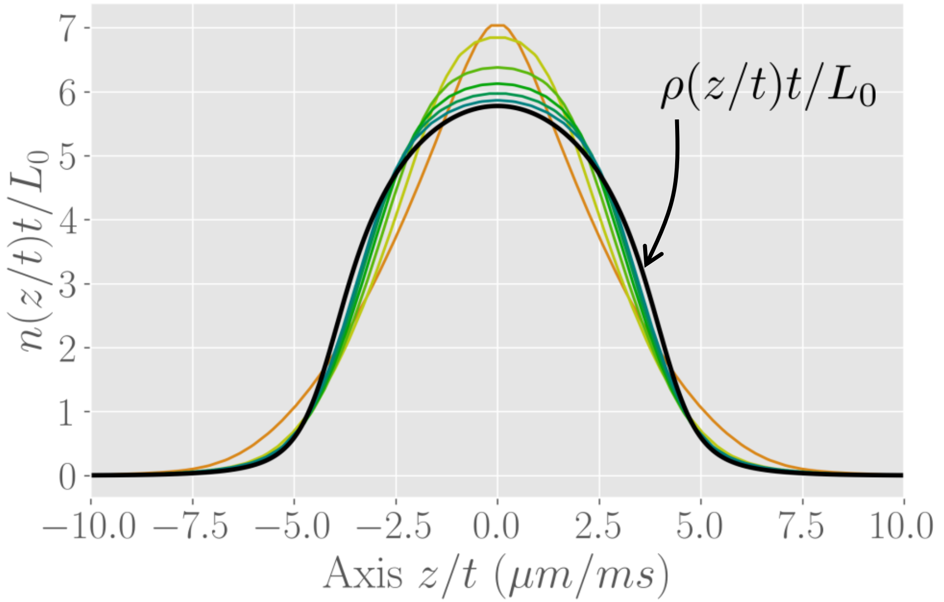


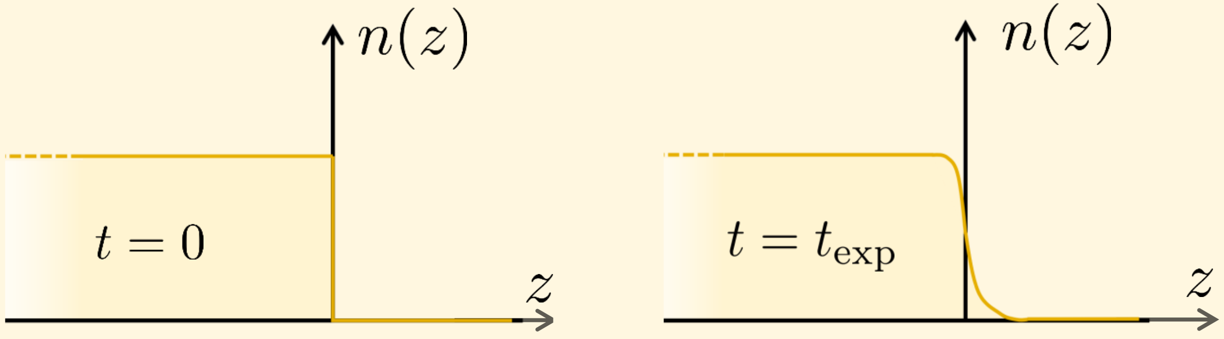


The asymptotic regime almost but not completely reached.

Domain Wall Dynamics protocol

Domain Wall Dynamics : edge deformation dynamics of an initially homogeneous, semi-infinite gas

GHD theory was developed to solve the problem of Domain Wall Dynamics.



Bertini et al. (2016), Castro-Alvaredo et al. (2016)

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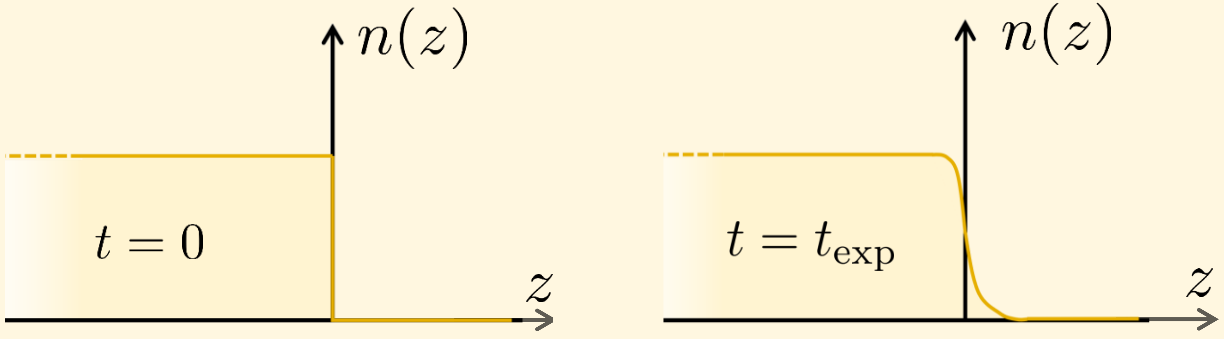

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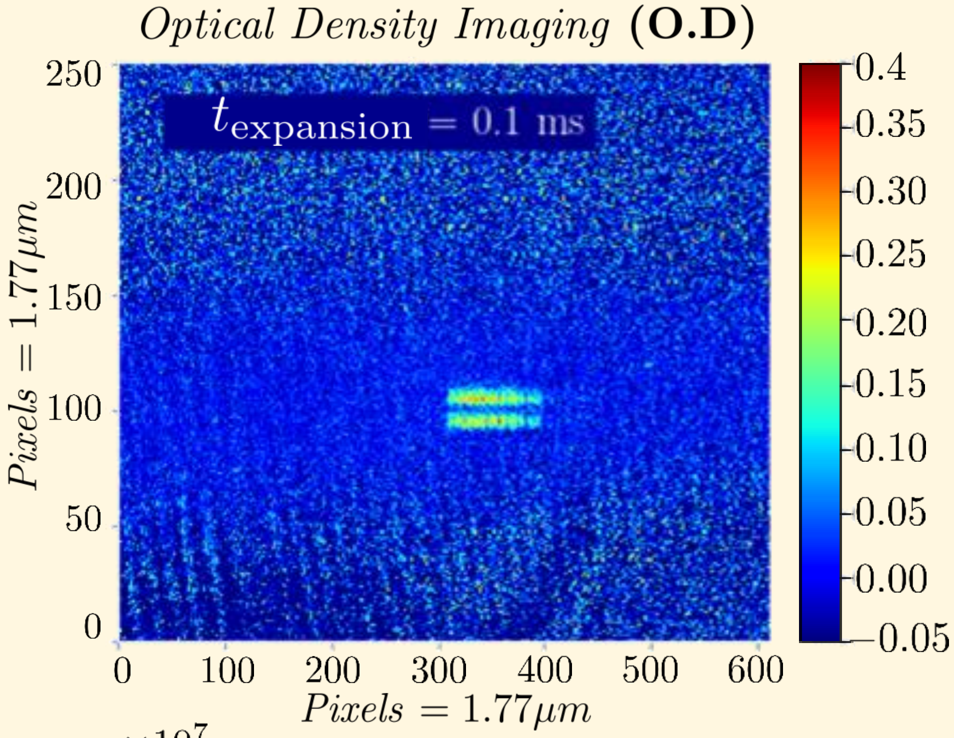
Generalised HydroDynamics Equation :

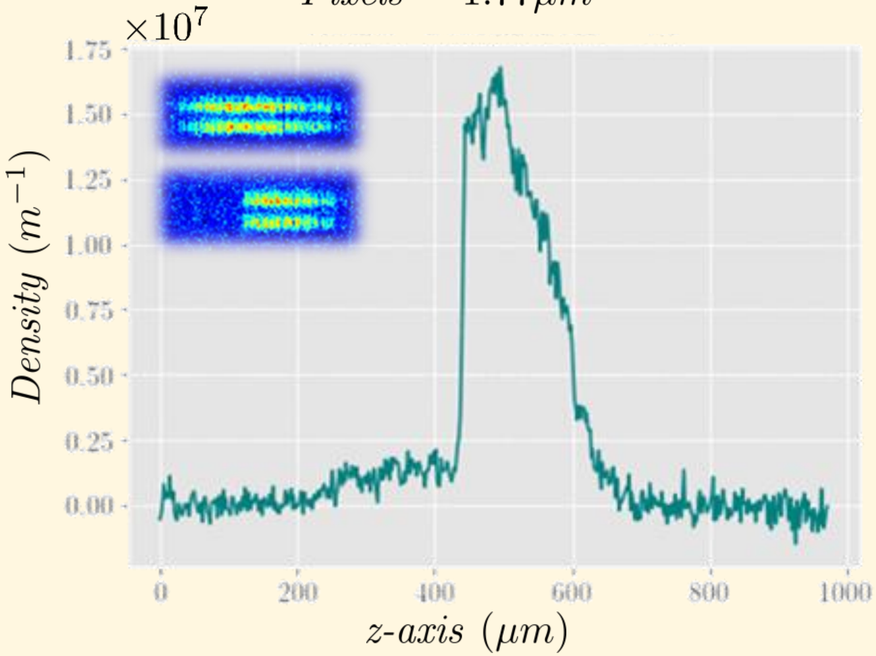
$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\mathrm{eff}}[\rho](k)\rho(z,k,t) \right) = 0$$

For different initial sizes: the dynamics must be the same to within $\mathrm{L_0/t}$.

Translation-invariant system along z-axis

The edge profile is a function of $\frac{\mathbf{z}}{\mathbf{t}} \Rightarrow \mathbf{n}[\rho]\left(\frac{\mathbf{z}}{\mathbf{t}}\right)$

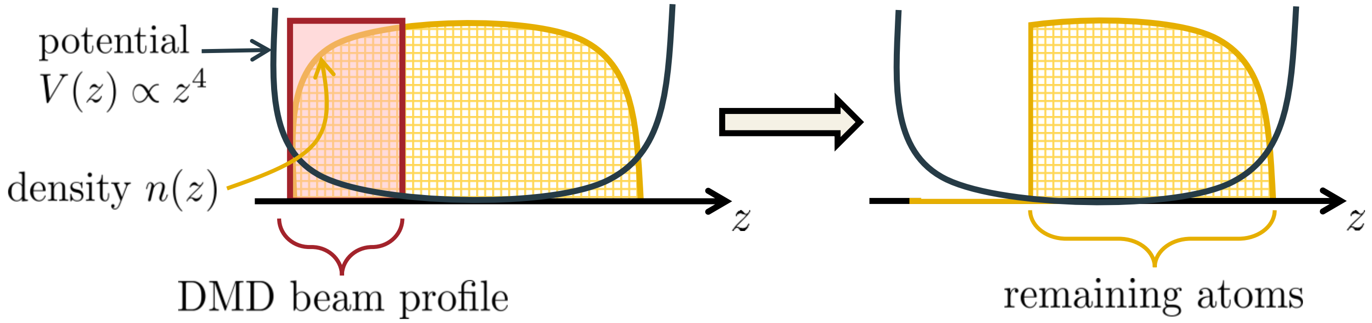




DOMAIN WALL DYNAMICS

Dynamics of the edge:

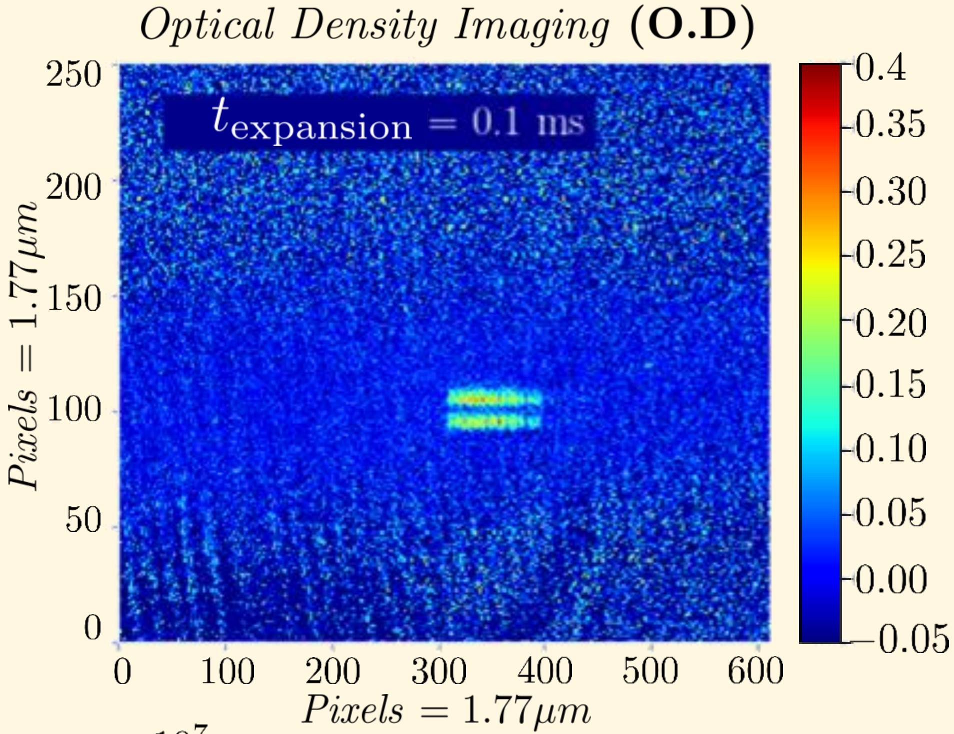
Experimentally : The atoms are trapped in a quartic potential so that the quasi-homogeneous zone is large enough.

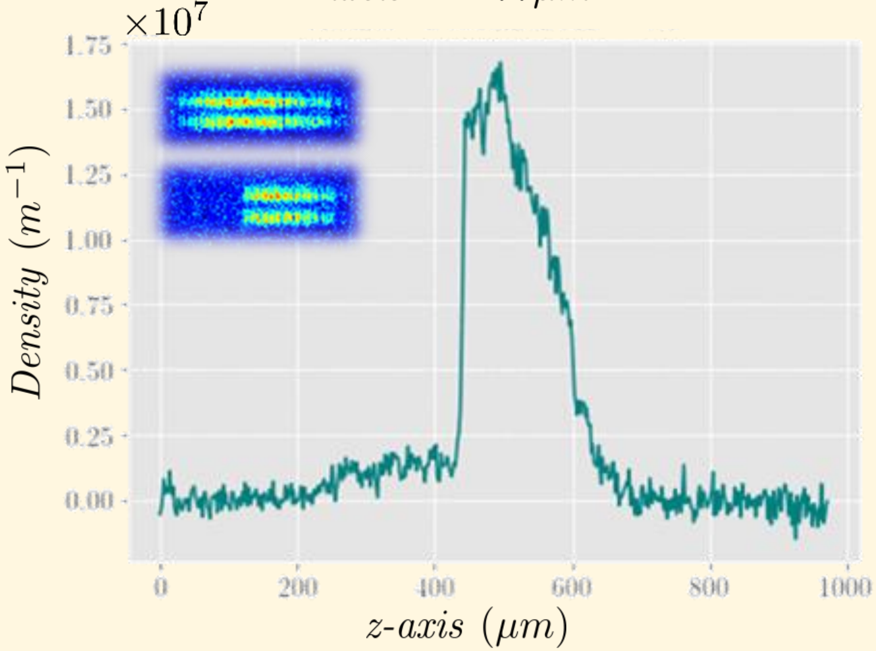


Then :

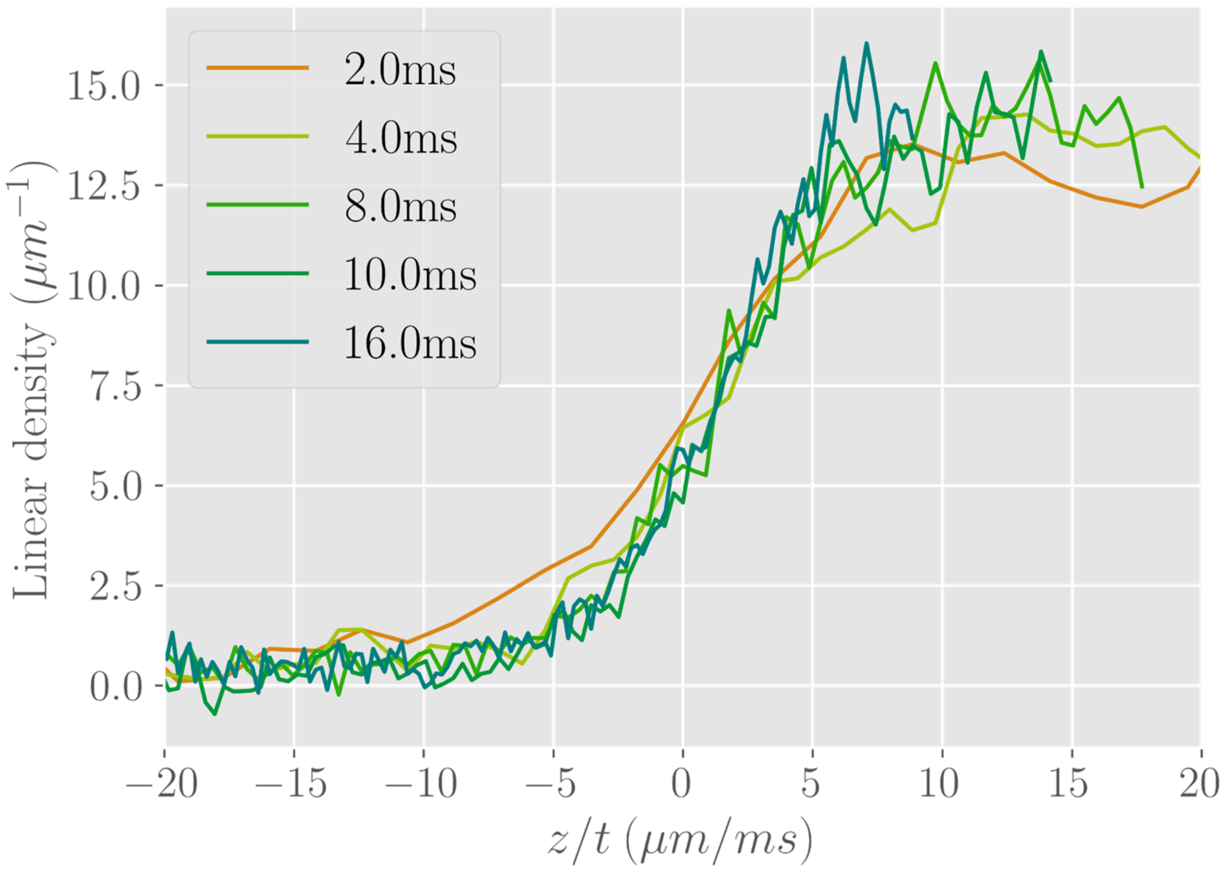
- The longitudinal **quartic** confinement is removed,
- The transverse one is maintained

DOMAIN WALL DYNAMICS





Dynamics of the edge:



Conclusion : after $8ms$, a ballistic dynamics well described the dynamics of the edge.

Domain Wall Dynamics reconstruction

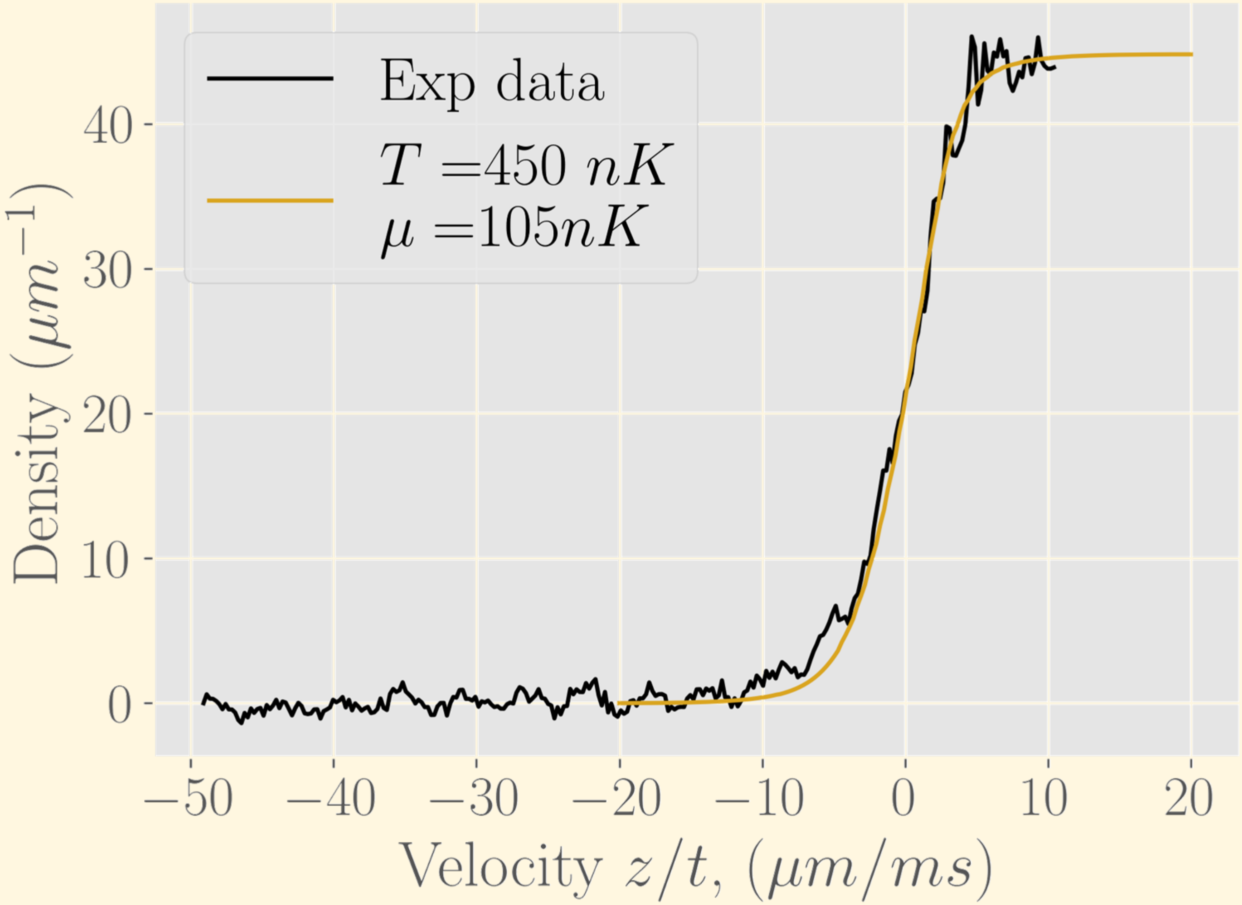
Reconstruction of the rapidities distribution :
In first approximation we suppose that we have a thermic cloud

$\Rightarrow$ **Fit with a Gibbs ensemble:** the T temperature μ and the chemical potential are the fit parameters.

Fitting parameters:

$$T = 450\,nK$$
$$\mu = 105\,nK$$

Edge profile after $\mathbf{t_{exp} = 8ms}$



CONCLUSION

- Implementation of a spatial selection tool

- Longitudinal expansion of a homogeneous gas :
    - Asymptotic regime is almost reached
    - Expansion agrees with Generalized Hydrodynamics

- Domain wall dynamics protocol :
    - Ballistic evolution well observed
    - Extraction of rapidities distribution

Thank you for your attention !

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Dynamics of 1D expansion

Generalised HydroDynamics Equation :

$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z,k,t)\right) = 0$$

Let $\rho(L_0, z, k, t) = \tilde{\rho}(L_0, z/L_0, k, t/L_0)$ a solution of the GHD equation

$$\begin{cases} z \to \alpha z \\ t \to \alpha t \end{cases} \Rightarrow \tilde{\rho}(L_0, \alpha z/L_0, k, \alpha t/L_0)$$

is also a solution of the GHD equation.

With $n(z,t) = \int_{-\infty}^{\infty} \rho(k,z,t)\mathrm{d}k$

$$\boxed{\Rightarrow n(z,t) = \tilde{n}\left(\frac{z}{t}, \frac{t}{L_0}\right)}$$

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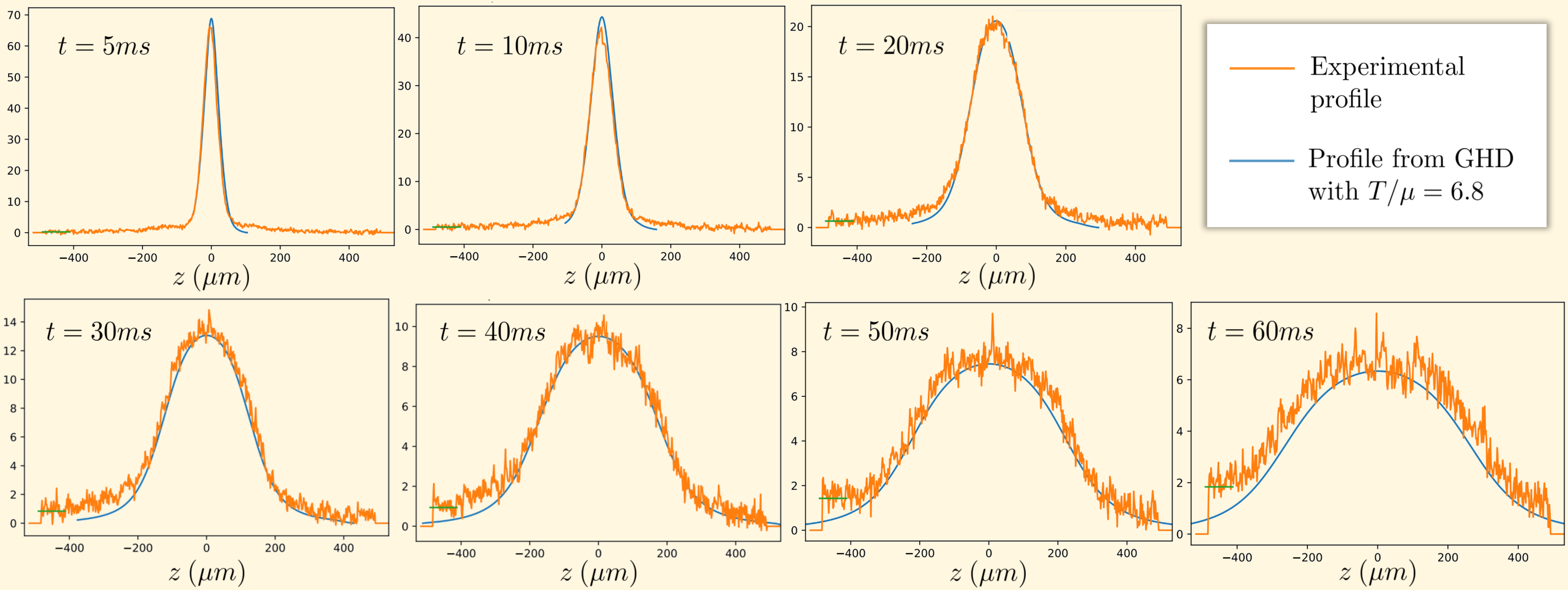
DYNAMICS OF 1D EXPANSION

Comparison between experimental profiles for different times and the dynamics obtained by Generalised HydroDynamics:



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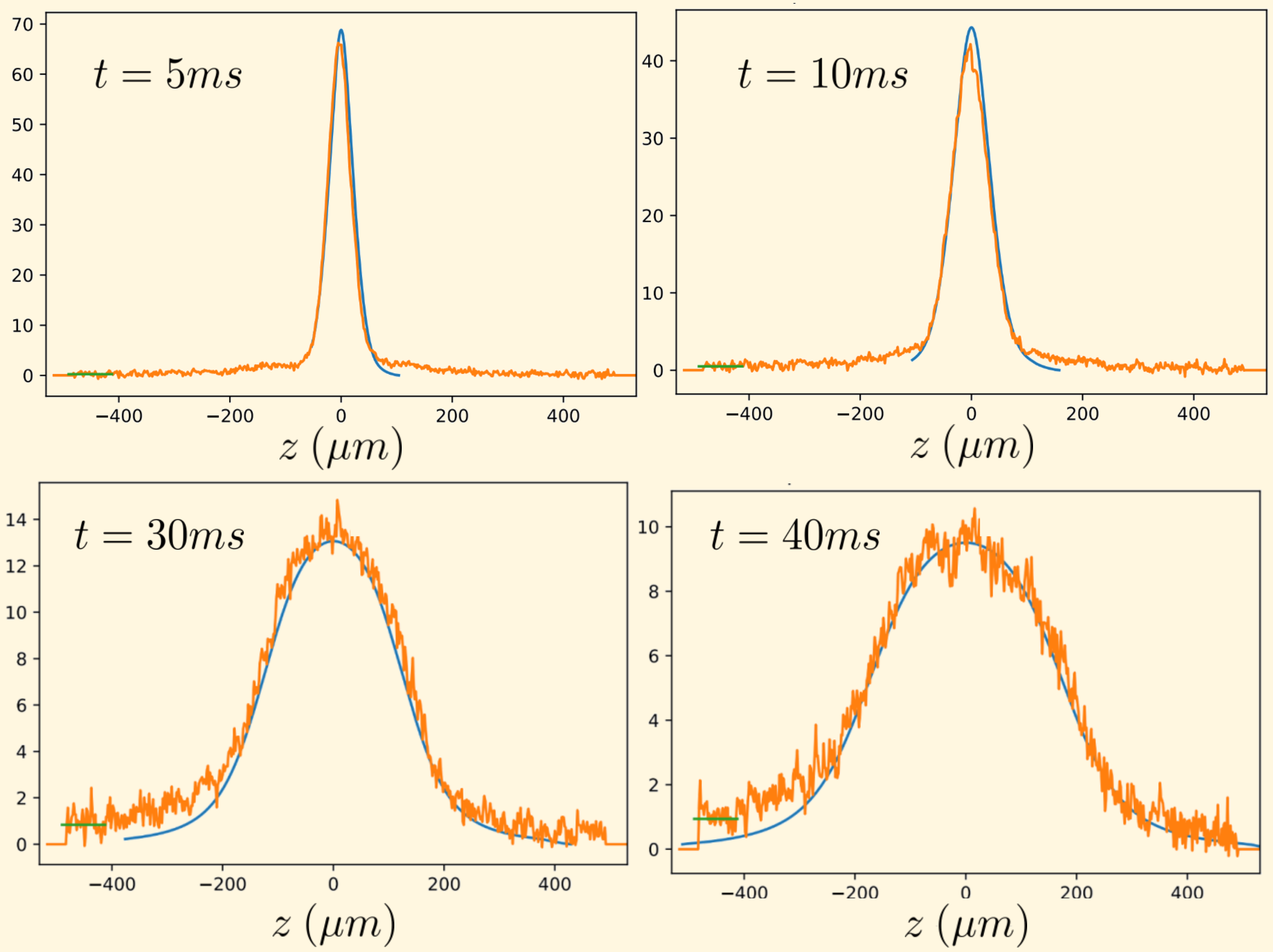
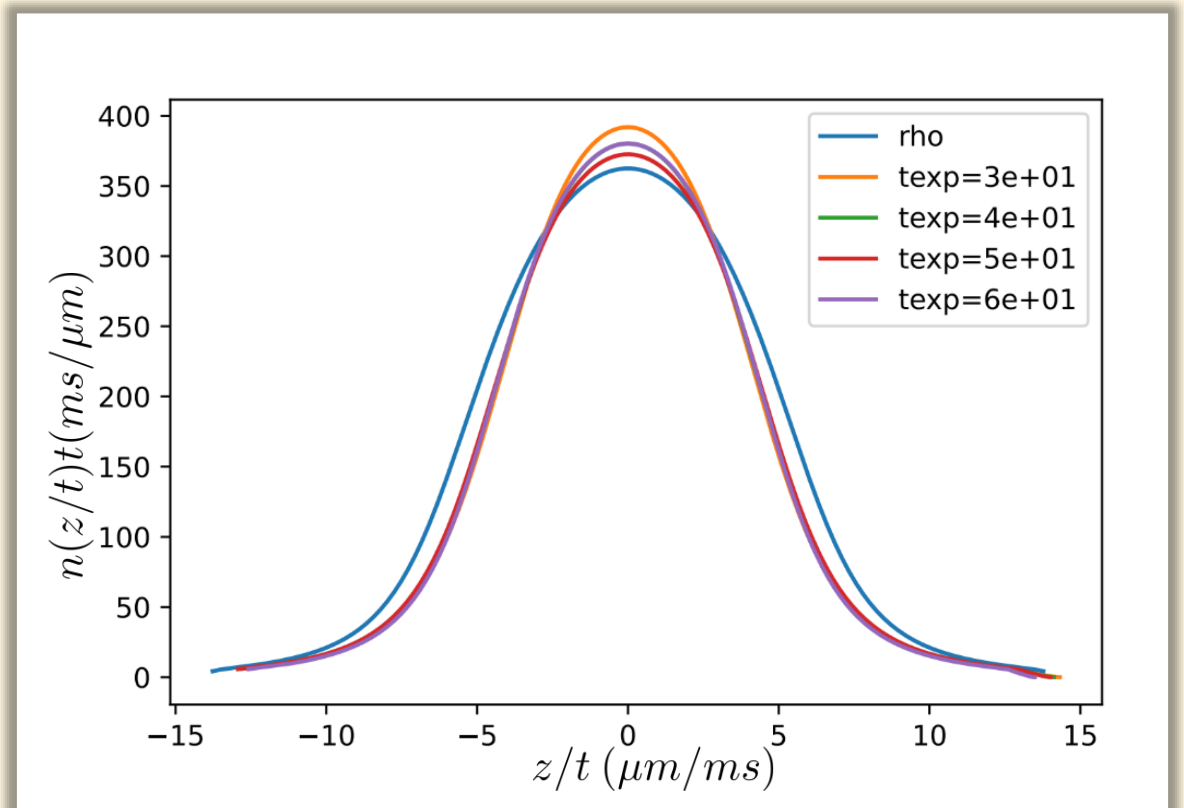




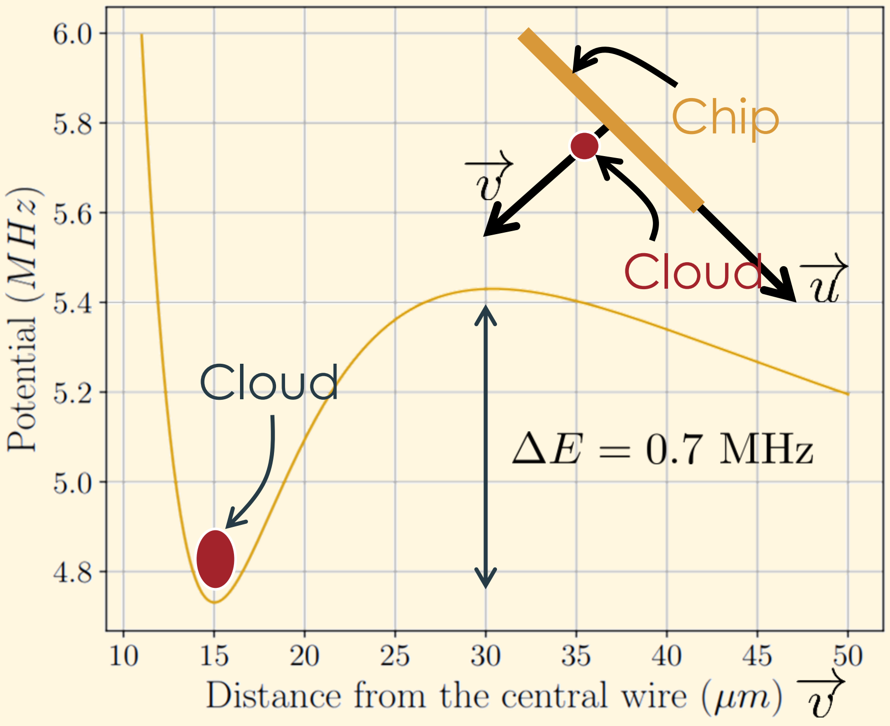
The asymptotic regime is not completely reached.

Radiation pressure

Pushing away the atoms with radiation pressure:

- For one photon absorbed, $v_{\text{rec}} = \frac{\hbar k}{m}$.
- $E_{\text{rec}} = \frac{1}{2m}\left(\hbar k N_{\text{sc}}^{\text{th}}\right)^2 = \Delta E$

$\Rightarrow N_{\text{sc}}^{\text{th}} \simeq 14$ photons.



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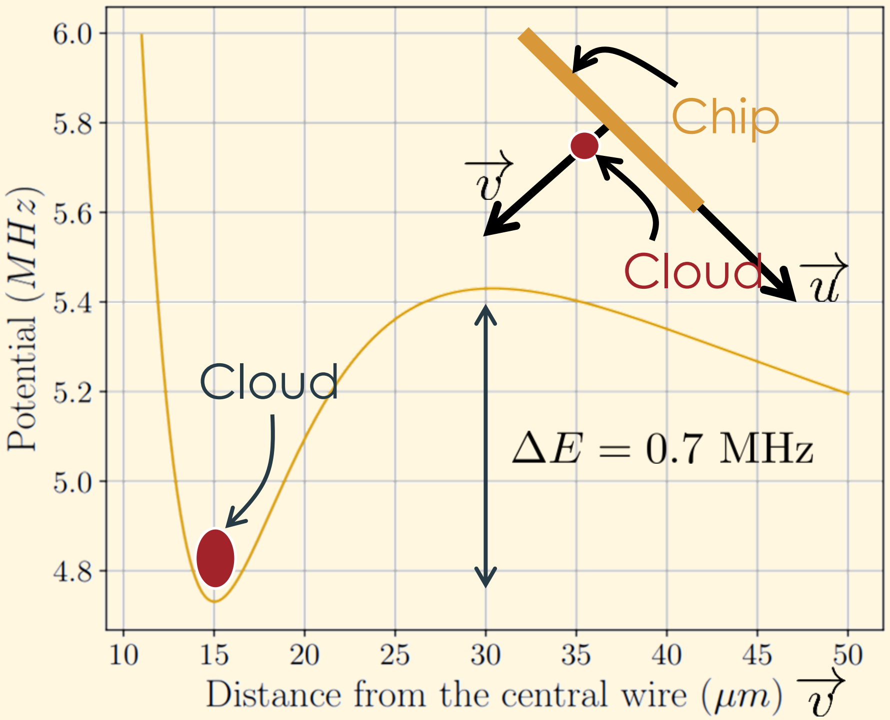
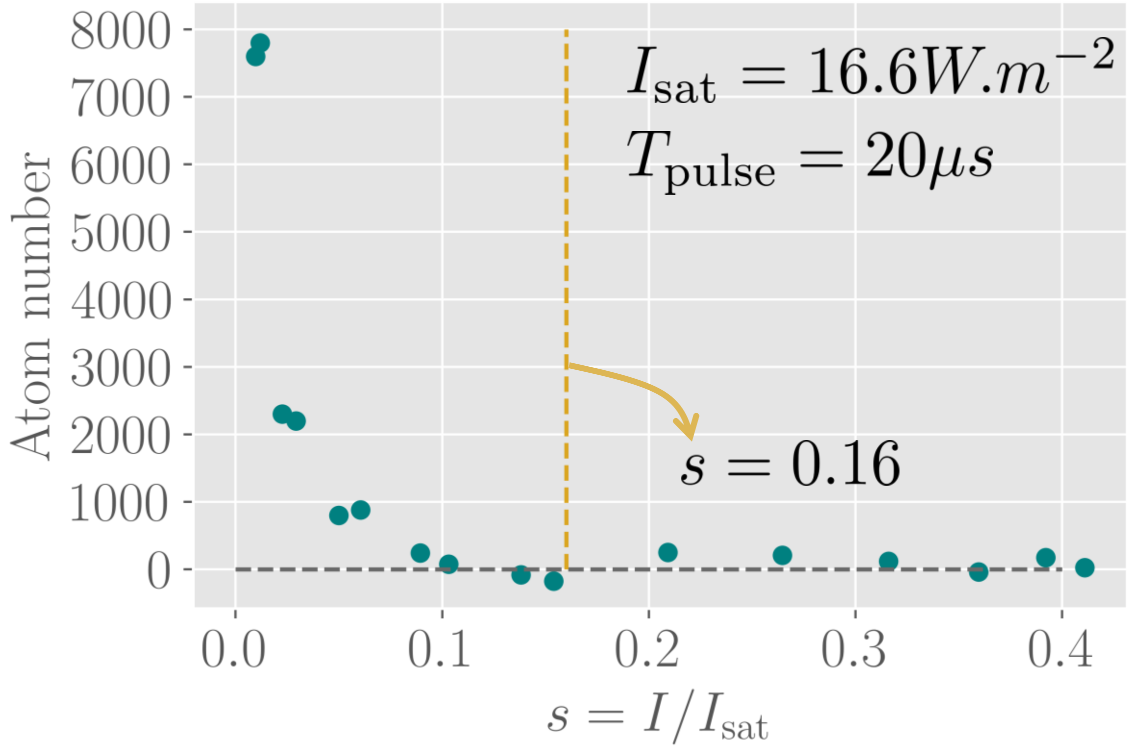


Measure of $N_{\text{sc}}^{\text{exp}}$ with fluorescence imaging:

What is the minimum power needed to remove all the atoms ?

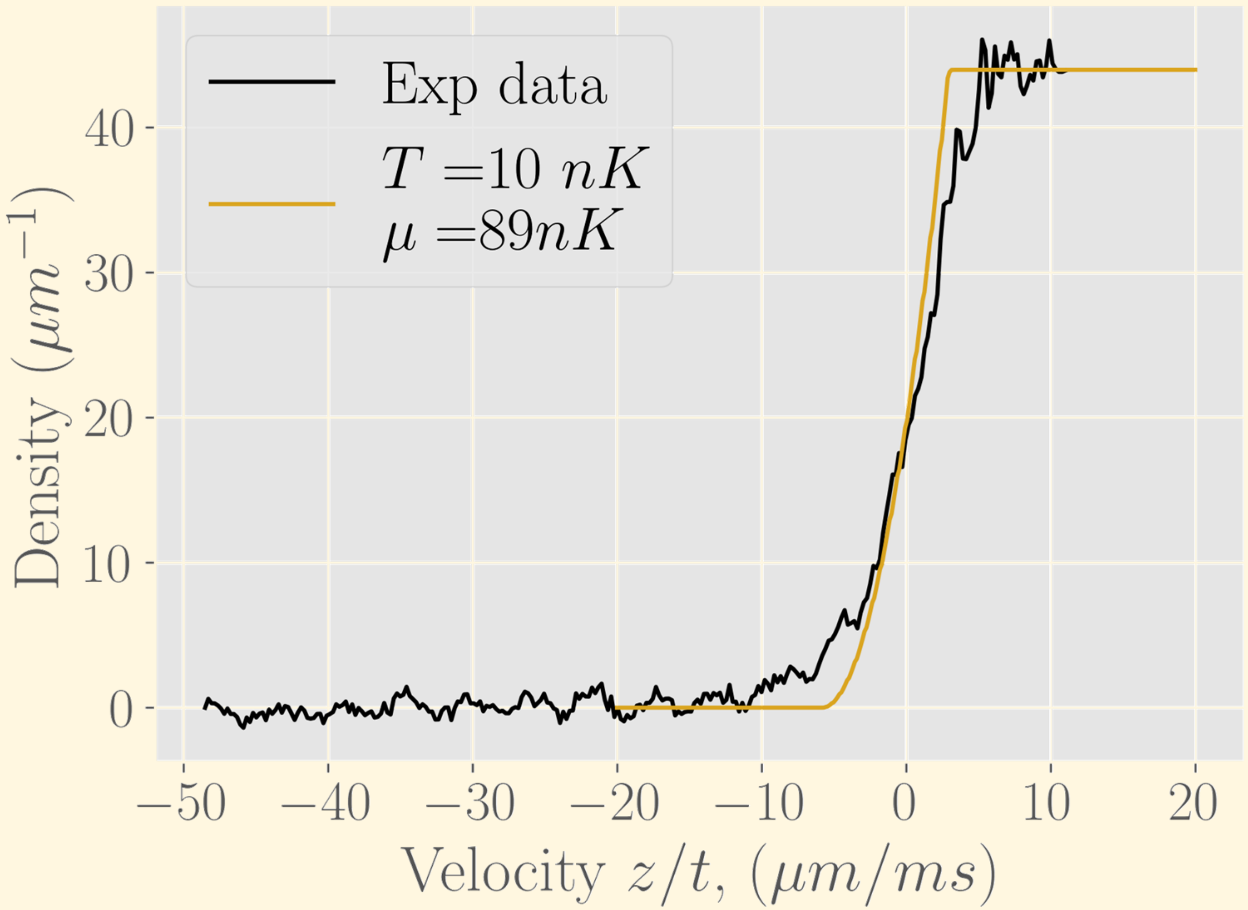


DOMAIN WALL DYNAMICS RECONSTRUCTION

Reconstruction of the rapidities distribution :
In first approximation we suppose that we have a thermic cloud

$\Rightarrow$ **Fit with a Gibbs ensemble:** the T temperature μ and the chemical potential are the fit parameters.

Edge profile after $\mathbf{t_{exp} = 8ms}$

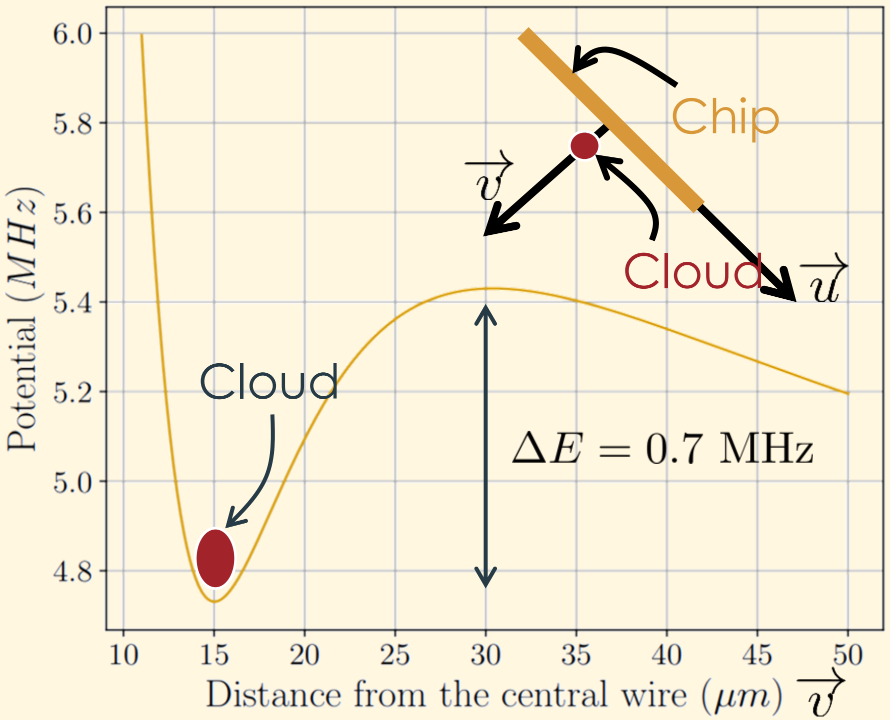


Radiation pressure

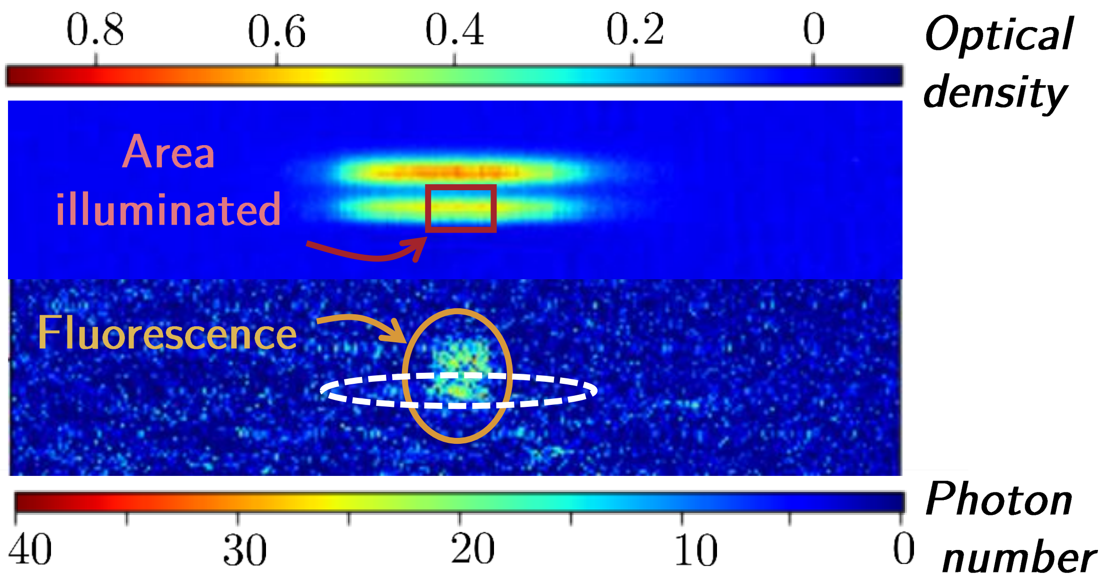
Pushing away the atoms with radiation pressure:

- For one photon absorbed, $v_{\text{rec}} = \frac{\hbar k}{m}$.
- $E_{\text{rec}} = \frac{1}{2m}\left(\hbar k N_{\text{sc}}^{\text{th}}\right)^2 = \Delta E$

$\Rightarrow N_{\text{sc}}^{\text{th}} \simeq 14$ photons.



Measure of $N_{\text{sc}}^{\text{exp}}$ with fluorescence imaging:



Experimental parameters : $T_{\text{pulse}} = 20\mu s\,, s = 0.16$

Fluorescence measurement :

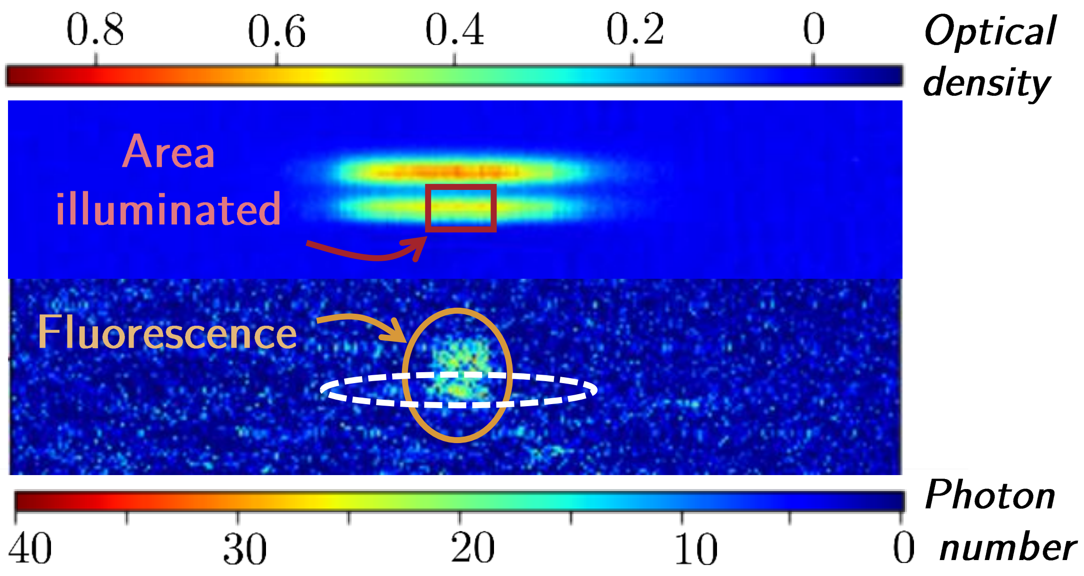
$N_{\text{sc}}^{\text{exp}} \simeq 15 \pm 5$ scattered photons / atoms

Radiation pressure

Take into account the level structure, the magnetic field and the beam polarisation, $\sigma = \alpha\sigma_0$ with $\sigma_0 = \frac{3\lambda^2}{2\pi}$

Experimentally : $\alpha = 0.25$

Measure of $N_{\mathrm{sc}}^{\mathrm{exp}}$ with fluorescence imaging:



Experimental parameters : $T_{\mathrm{pulse}} = 20\mu s\,, s = 0.18$

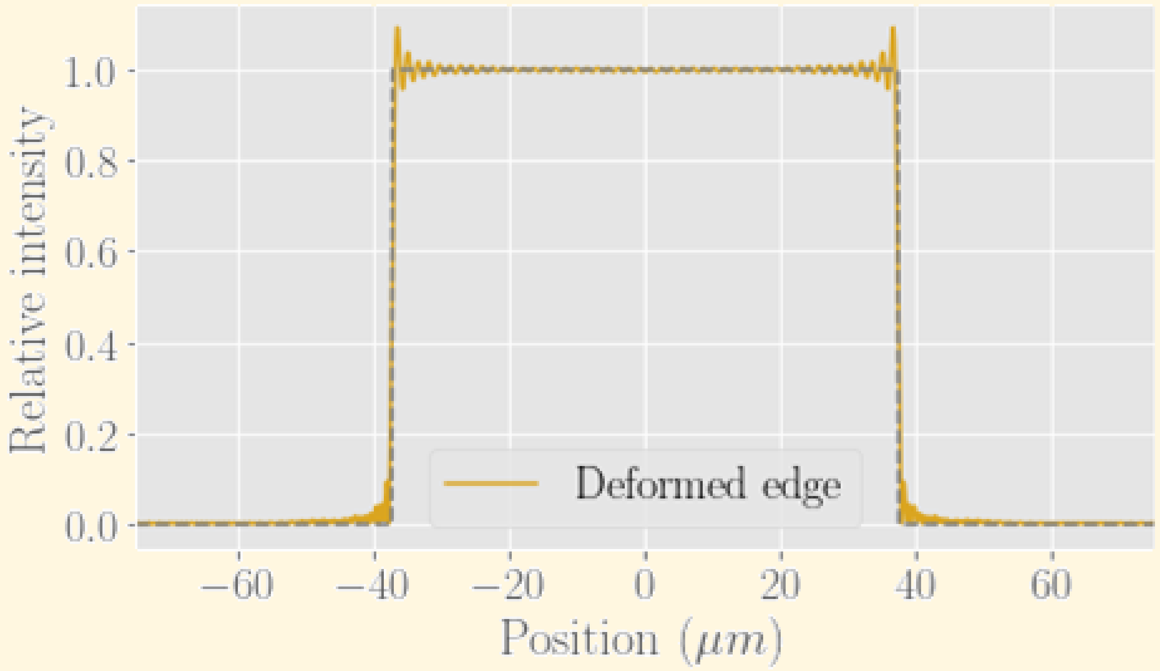
Fluorescence measurement :

$N_{\mathrm{sc}}^{\mathrm{exp}} \simeq 15 \pm 5$ scattered photons / atoms

Limitations

Limitations :

- Image of an edge is not perfect

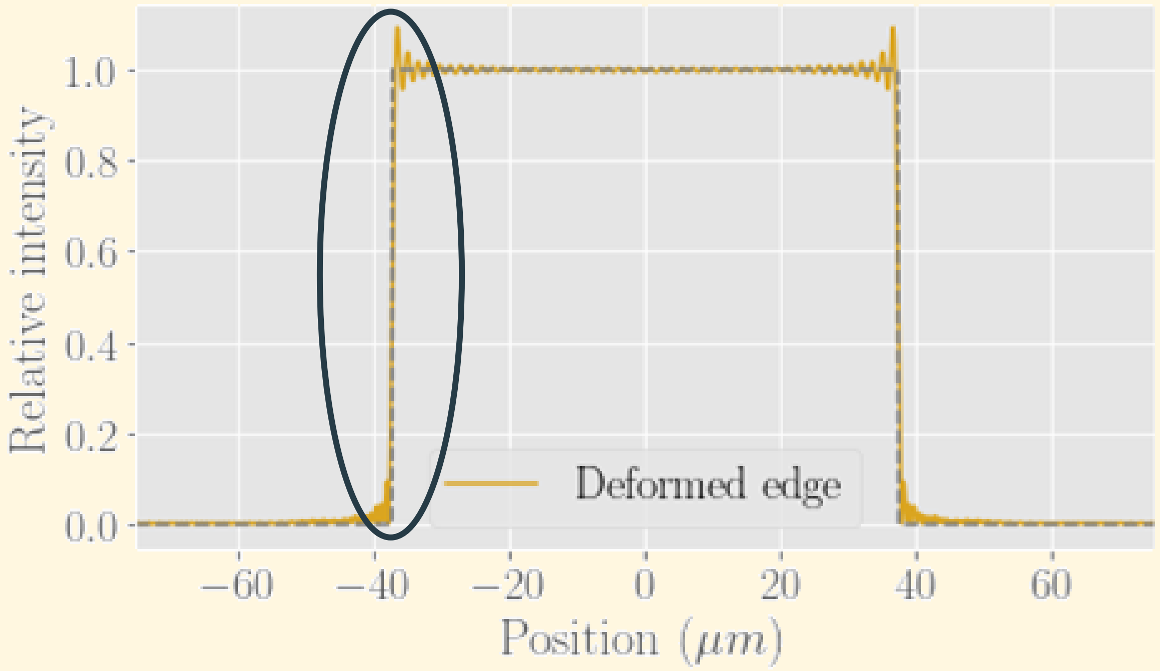


What do we see :

LIMITATIONS

Limitations :

- Image of an edge is not perfect

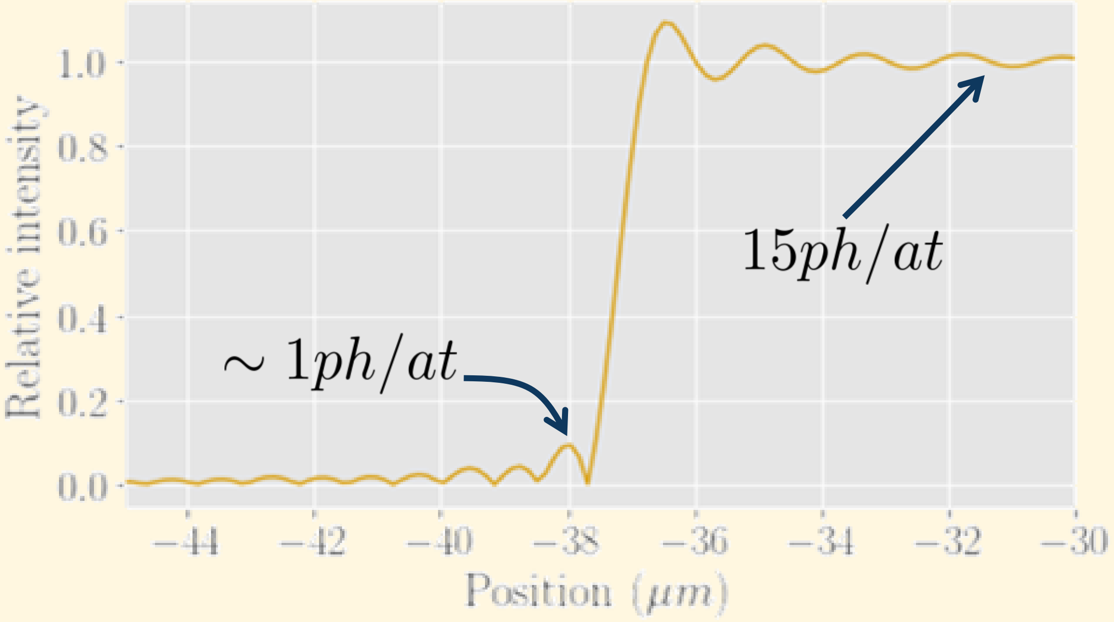


What do we see :

Limitations

Limitations :

- Image of an edge is not perfect



What do we see :

LIMITATIONS

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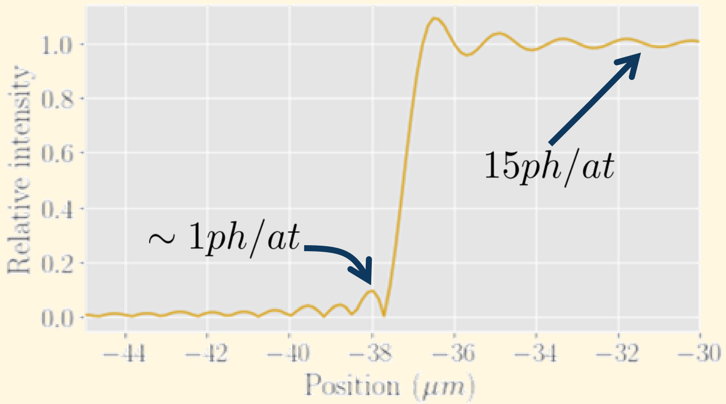


- Photon reabsorption in the remaining cloud

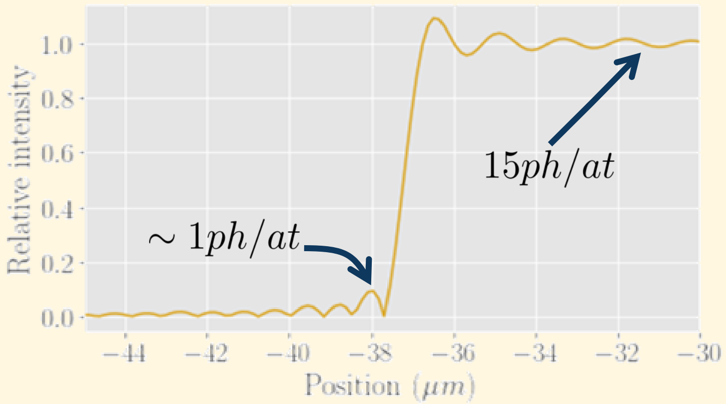
- Diffusion of light by the optical system can lead to absorption photon in the remaining cloud

What do we see :

LIMITATIONS

Limitations :

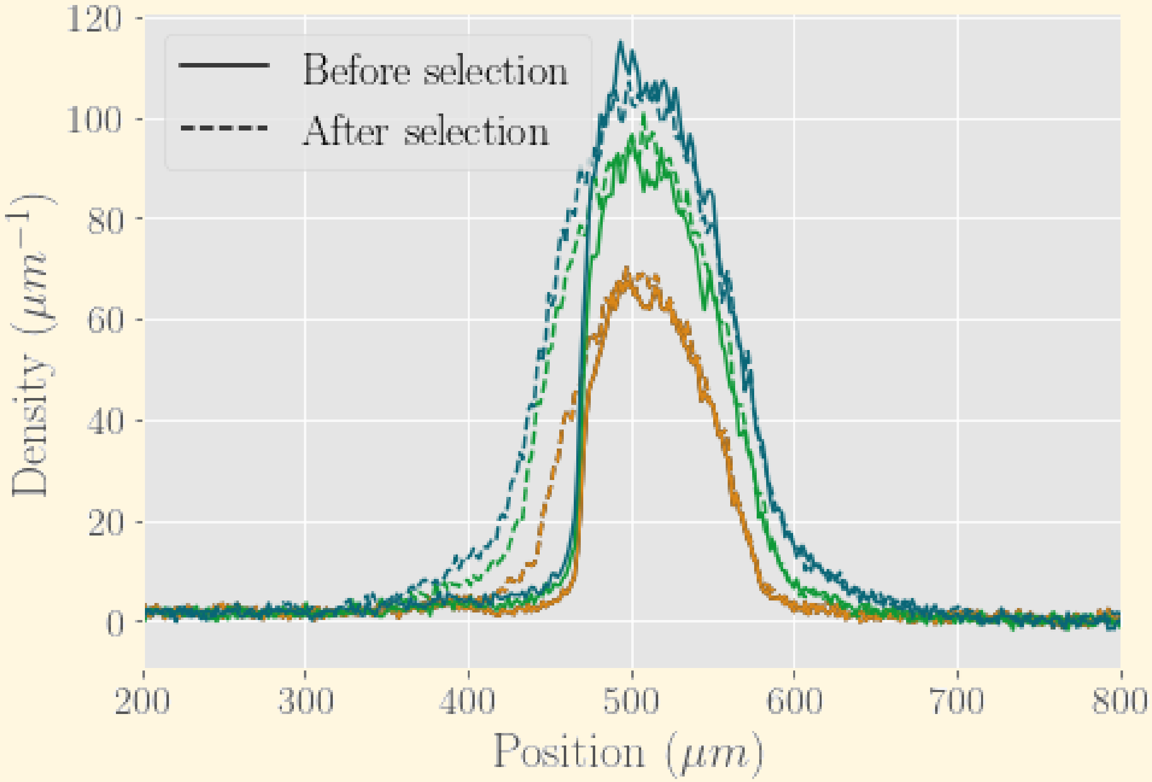
- Image of an edge is not perfect



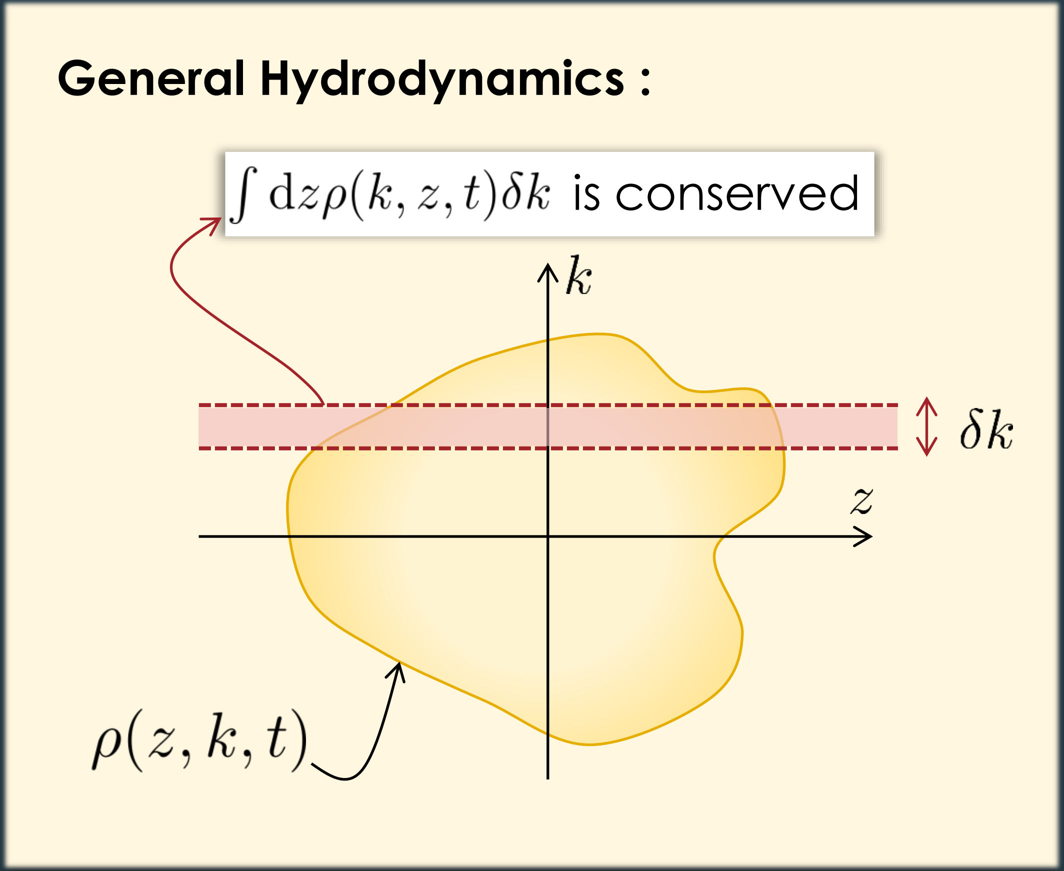
- Photon reabsorption in the remaining cloud
- Diffusion of light by the optical system can lead to absorption photon in the remaining cloud

What do we see :

- The density of remaining atoms doesn't change



Generalised HydroDynamics (GHD)



General HydroDynamics (GHD) : an integrable system can locally be described by the spatial resolved rapidity distribution $\rho(z,k,t)$.

- *Conservation of the rapidity distribution on a δk slice:*

$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k) \rho(z,k,t) \right) = 0$$

with

$$v_{\text{eff}}[\rho](k) = k - \int_{-\infty}^{+\infty} \mathrm{d}k' \Delta(k-k') \left[v_{\text{eff}}[\rho](k) - v_{\text{eff}}[\rho](k') \right]$$

and

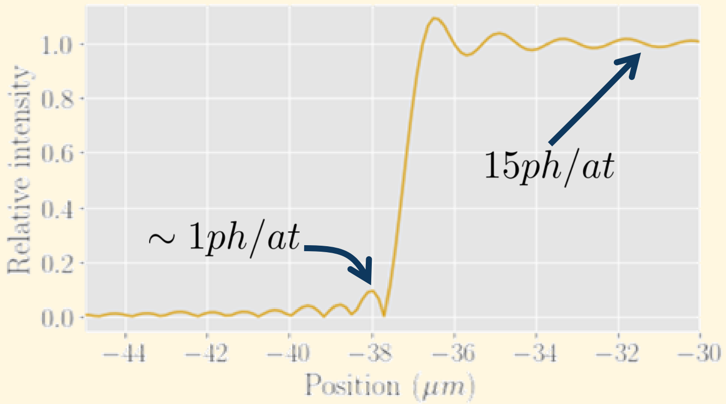
$$\Delta(k) = \frac{2g}{g^2+k^2}$$

Bertini et al. (2016), Castro-Alvaredo et al. (2016)

LIMITATIONS

Limitations :

- Image of an edge is not perfect



- Photon reabsorption in the remaining cloud

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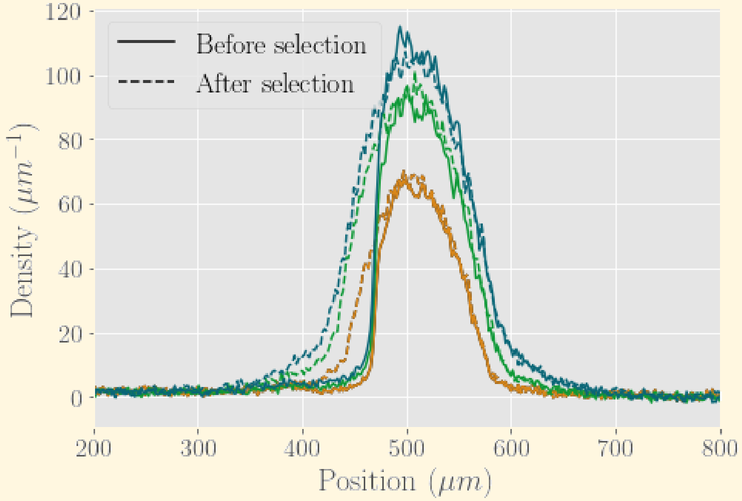
What do we see :

- The density of remaining atoms doesn't change



- Phonon temperature doesn't change before and after the selection

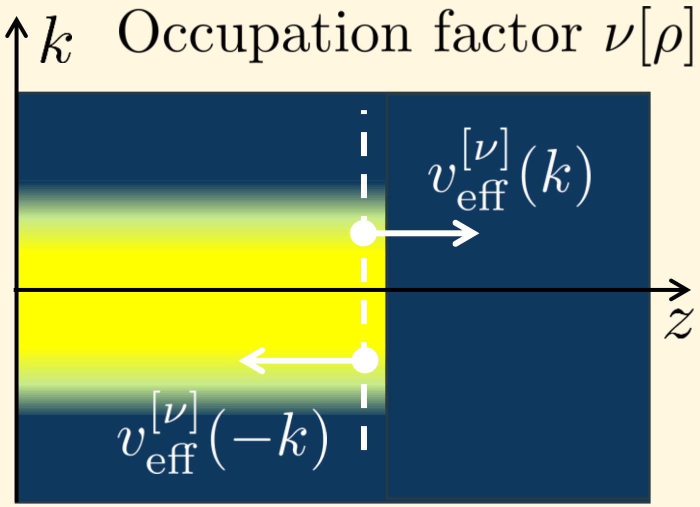
- Transverse size after a time of flight doesn't change.

DOMAIN WALL DYNAMICS RECONSTRUCTION

Occupation factor $\nu[\rho](k) \in [0,1]$ **:**

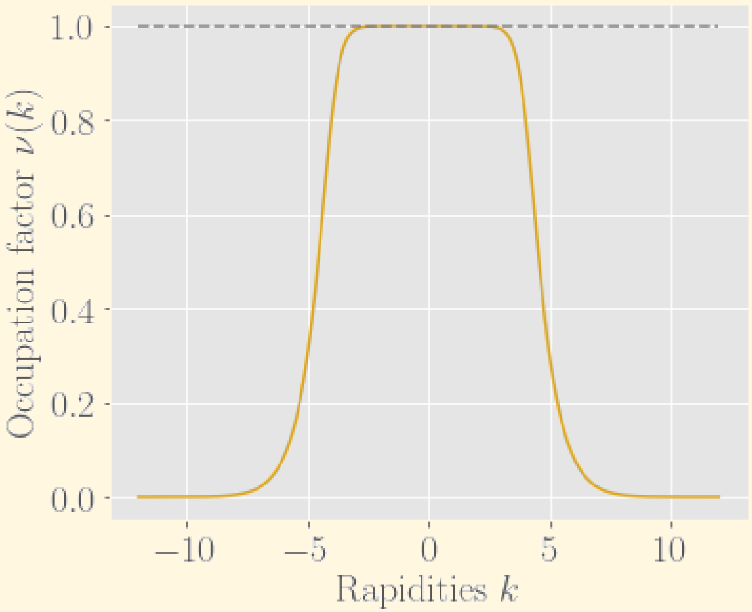
$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\mathrm{eff}}[\rho](k)\rho(z,k,t)\right) = 0 \Rightarrow n(z,t) = \int_{-\infty}^{\infty} \rho(k,z,t)\mathrm{d}k$$

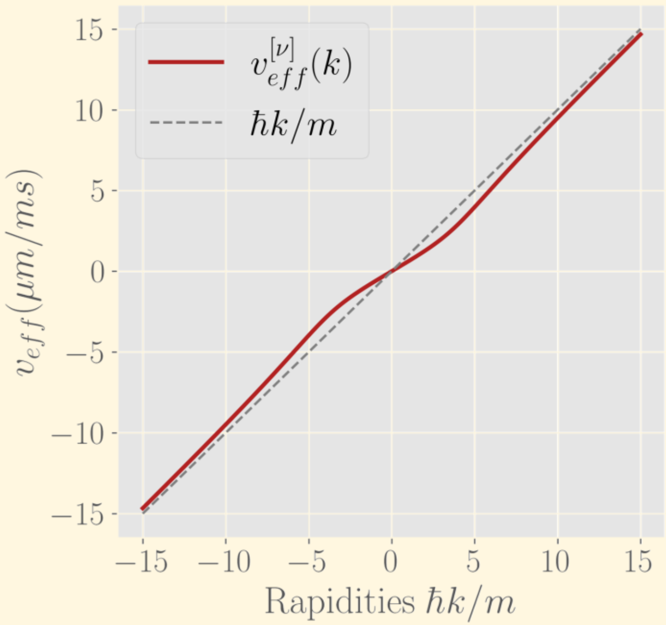
$$\Rightarrow \partial_t \nu_{[\rho]}(z,k,t) + v_{\mathrm{eff}}[\rho](k)\partial_z \left(\nu(z,k,t)\right) = 0$$ *Conservation along a trajectory*



$$\nu(k,z,t=0) = \begin{cases} \nu(k) & \text{if } |z| < z_0 \\ 0 & \text{otherwise.} \end{cases}$$

Occupation factor at $t=0$ **:** there is no dependance on z.





Domain Wall Dynamics reconstruction

Occupation factor $\nu[\rho](k) \in [0,1]$:

$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z,k,t)\right) = 0 \Rightarrow n(z,t) = \int_{-\infty}^{\infty} \rho(k,z,t)\mathrm{d}k$$

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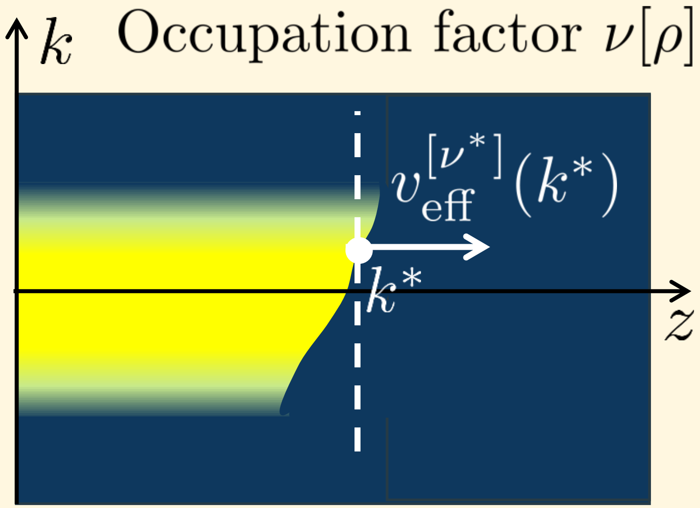
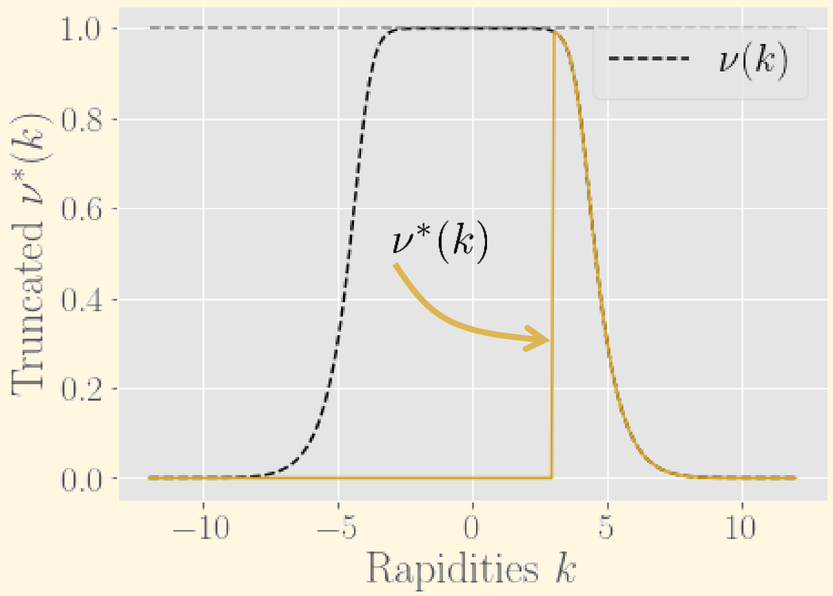


$$\nu^*(k) = \begin{cases} \nu(k) & \text{if } k > k^* \\ 0 & \text{otherwise.} \end{cases}$$

Occupation factor at $t > 0$:



The edge dynamic is obtained with the effective velocities $v_{\text{eff}}^{[\nu^*]}(k^*)$ where is the truncated initial occupation factor

For each k, $v_{\text{eff}}^{[\nu^*]}(k^*)$ doesn't depend on time $\Rightarrow$ **The edge profile is a function of** $\mathbf{n}[\rho]\left(\frac{\mathbf{z}}{\mathbf{t}}\right)$

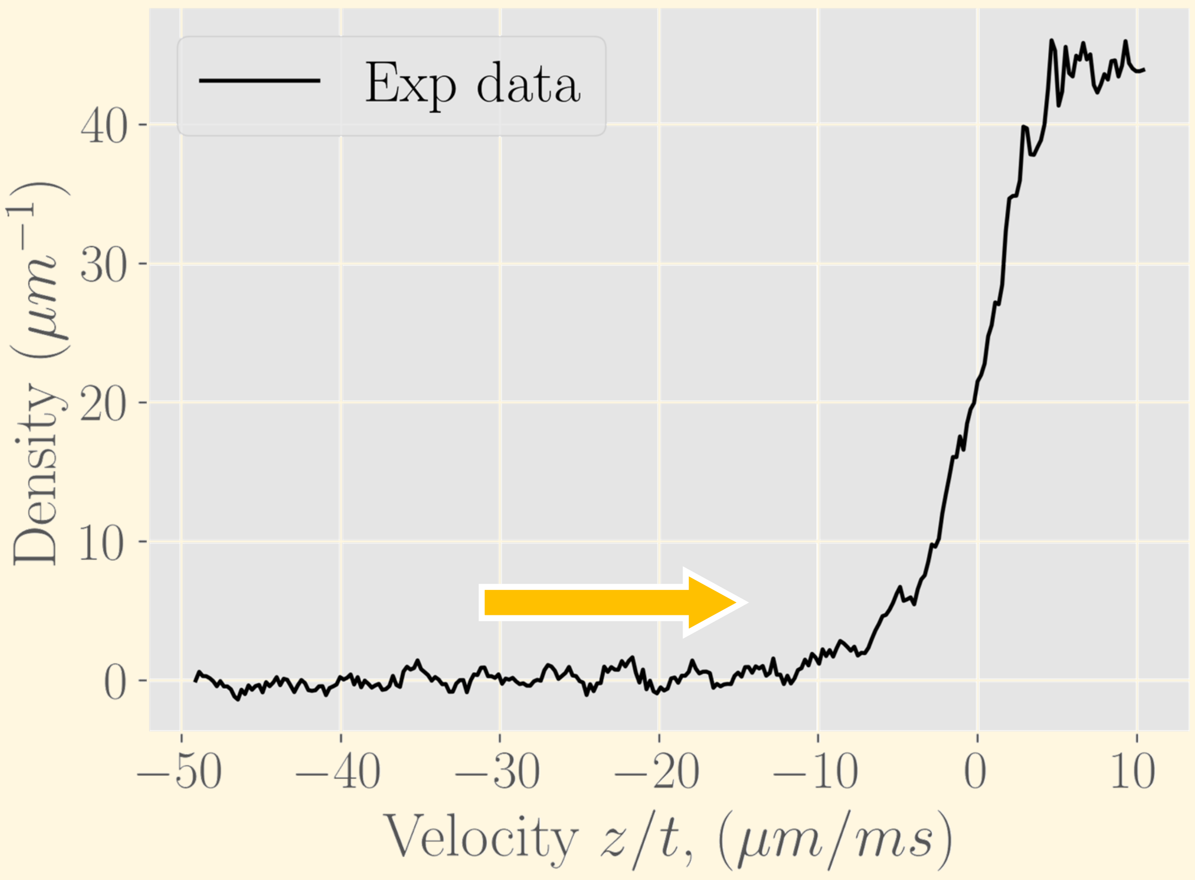
Domain Wall Dynamics reconstruction

Reconstruction of the occupation factor directly with the density profile

$$\frac{\mathrm{d}\nu}{\mathrm{d}k} = D'[\nu]\frac{\mathrm{d}n}{\mathrm{d}v_{\mathrm{eff}}} + A[\nu]D[\nu]\frac{\mathrm{d}^2 n}{\mathrm{d}v_{\mathrm{eff}}^2}$$

Little by little, in a discretized space

With $\nu_N = 0, \nu_{N-1} = 0$ and $v_N^{\mathrm{eff}} = \hbar k_N/m, v_{N-1}^{\mathrm{eff}} = \hbar k_{N-1}/m$

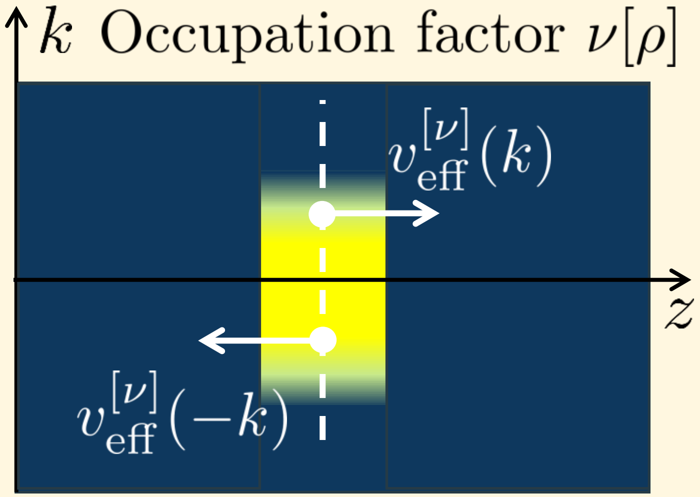


DYNAMICS OF 1D EXPANSION

Occupation factor $\nu[\rho](k) \in [0,1]$ **:**

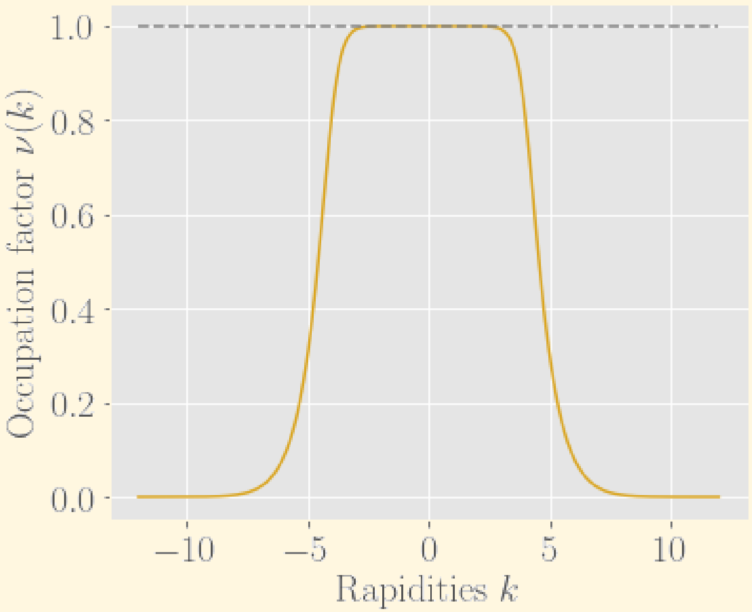
$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z,k,t)\right) = 0 \Rightarrow n(z,t) = \int_{-\infty}^{\infty} \rho(k,z,t)\mathrm{d}k$$

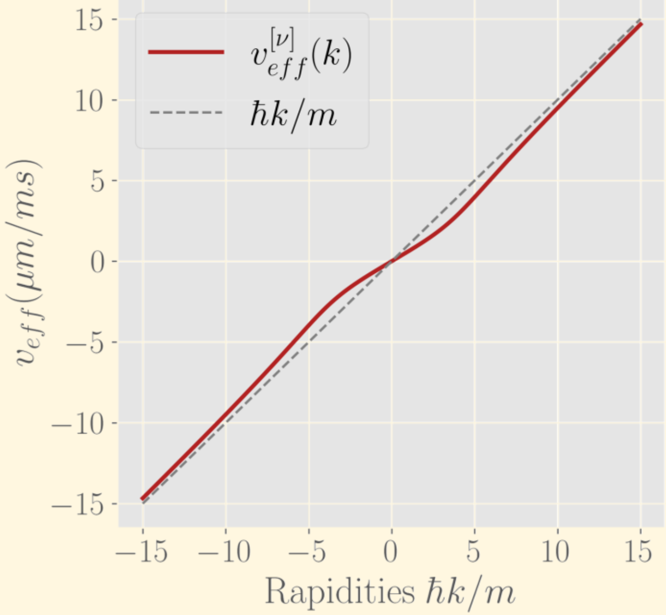
$$\Rightarrow \partial_t \nu_{[\rho]}(z,k,t) + v_{\text{eff}}[\rho](k)\partial_z\left(\nu(z,k,t)\right) = 0$$ *Conservation along a trajectory*



$$\nu(k,z,t=0) = \begin{cases} \nu(k) & \text{if } |z| < z_0 \\ 0 & \text{otherwise.} \end{cases}$$

Occupation factor at $t=0$ **:** there is no dependance on z.



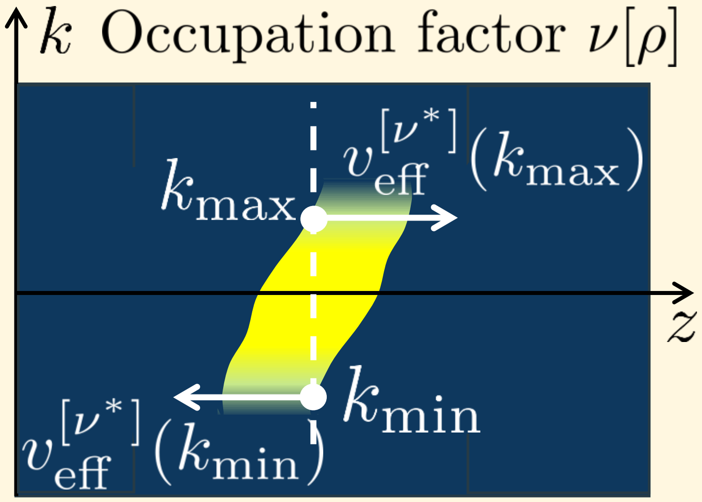


Dynamics of 1D expansion

Occupation factor $\nu[\rho](k) \in [0,1]$ **:**

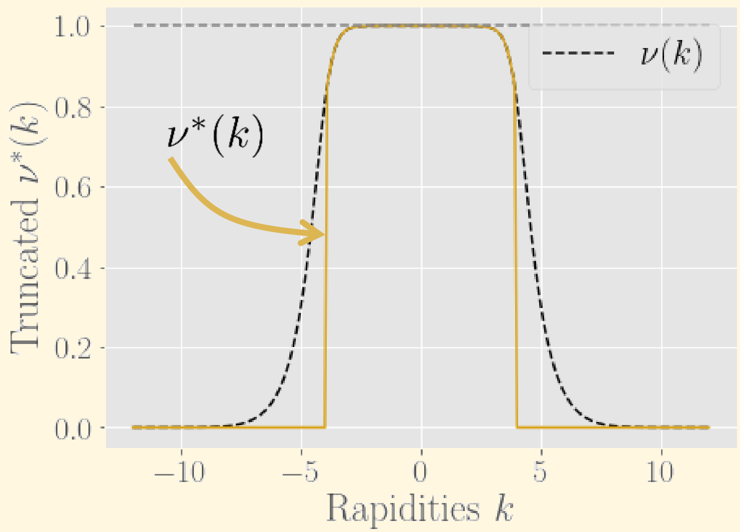
$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z,k,t)\right) = 0 \Rightarrow n(z,t) = \int_{-\infty}^{\infty} \rho(k,z,t)\mathrm{d}k$$

$$\Rightarrow \partial_t \nu_{[\rho]}(z,k,t) + v_{\text{eff}}[\rho](k)\partial_z\left(\nu(z,k,t)\right) = 0$$ *Conservation along a trajectory*



$$\nu^*(k) = \begin{cases} \nu(k) & \text{if } k_{\text{min}} < k < k_{\text{max}} \\ 0 & \text{otherwise.} \end{cases}$$

Occupation factor at $t > 0$ **:** the edges dynamic are obtained with the effective velocities $v_{\text{eff}}^{[\nu^*]}(k^*)$ where ν^* is the initial occupation factor truncated between k_{min} and k_{max}.



CONTEXT

Lieb parameter: $\gamma = \frac{mg}{n}$ with g the 1D interaction parameter and n the density:

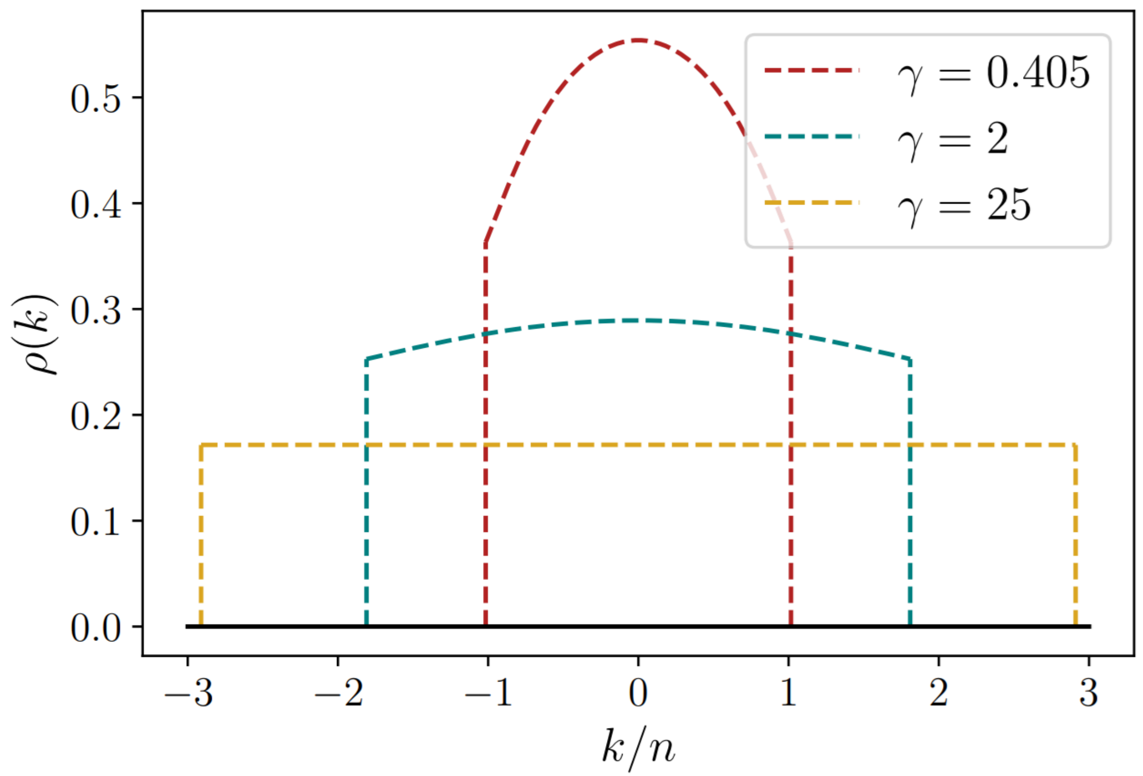
- $\gamma \to \infty$: hardcore regime
- $\gamma \to 0$: 1D quasi-condensate (qBEC)

$$\rho(k)^2 \sim \left(1 - \left(\frac{k}{K}\right)^2\right), \gamma \to 0$$

Half-circle equation

Objective : Measuring experimentally the rapidity distribution for a qBEC and recovering the half-circle distribution.

Rapidity distribution for a homogeneous gas



CONTEXT

Lieb parameter: $\gamma = \frac{mg}{n}$ with g the 1D interaction parameter and n the density:

- $\gamma \rightarrow \infty$: hardcore regime
- $\gamma \rightarrow 0$: 1D quasi-condensate (qBEC)

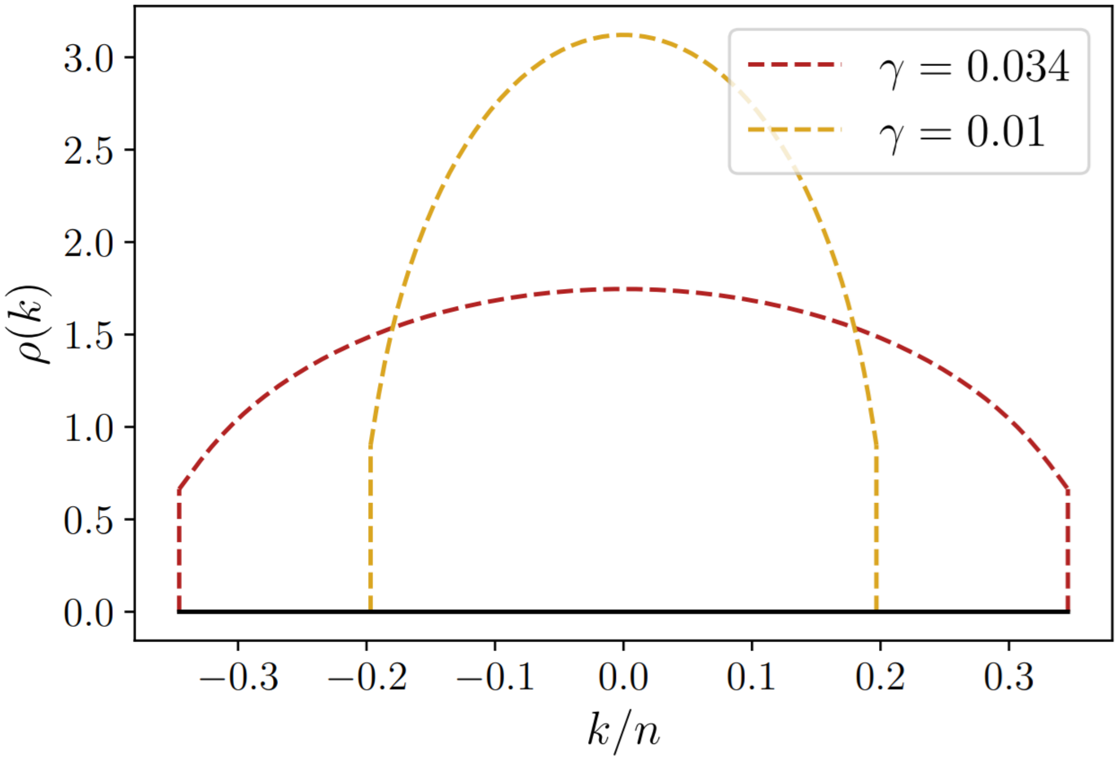
For $T = 0$ and $\gamma \rightarrow 0$:

$$\rho(k)^2 \sim \left(1 - \left(\frac{k}{K}\right)^2\right)$$

Half-circle equation

With $K = 2\sqrt{gn}$.

Rapidity distribution for a homogeneous gas for $T = 0$:

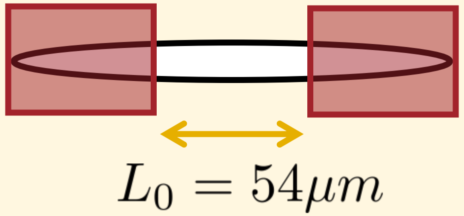


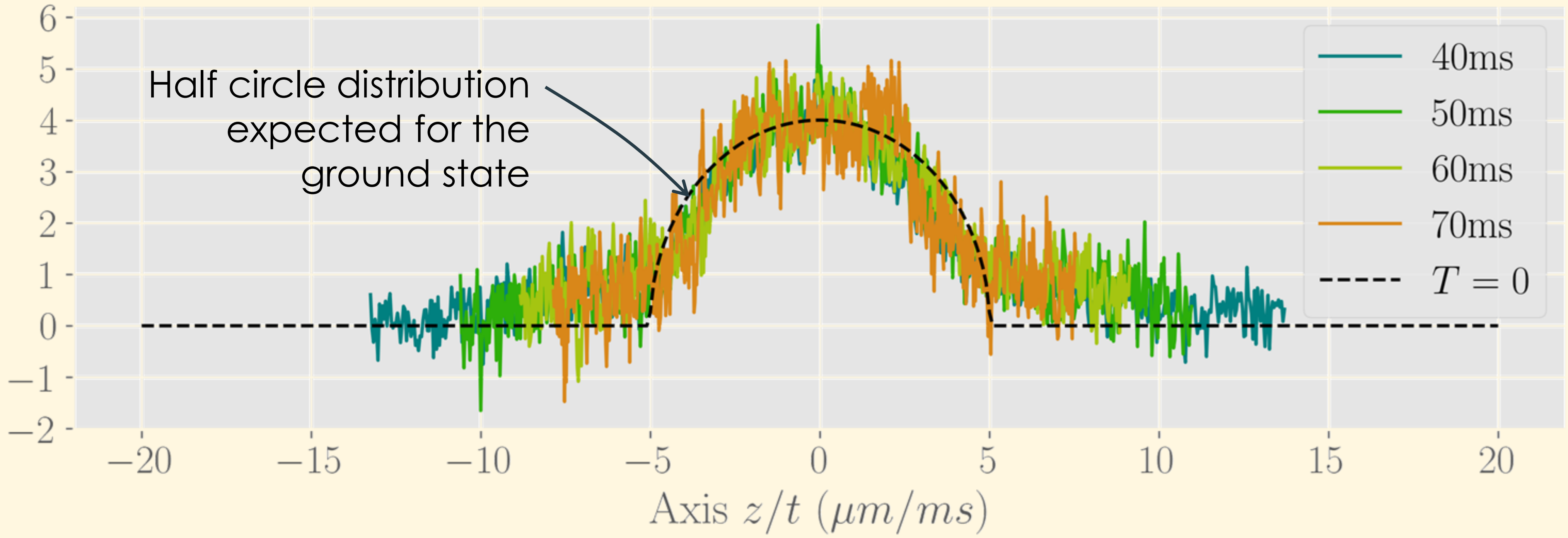
" Exact analysis pf an interacting Bose Gas ", Lieb & Liniger, 1963

1D EXPANSION

Rescaled profile denstity:

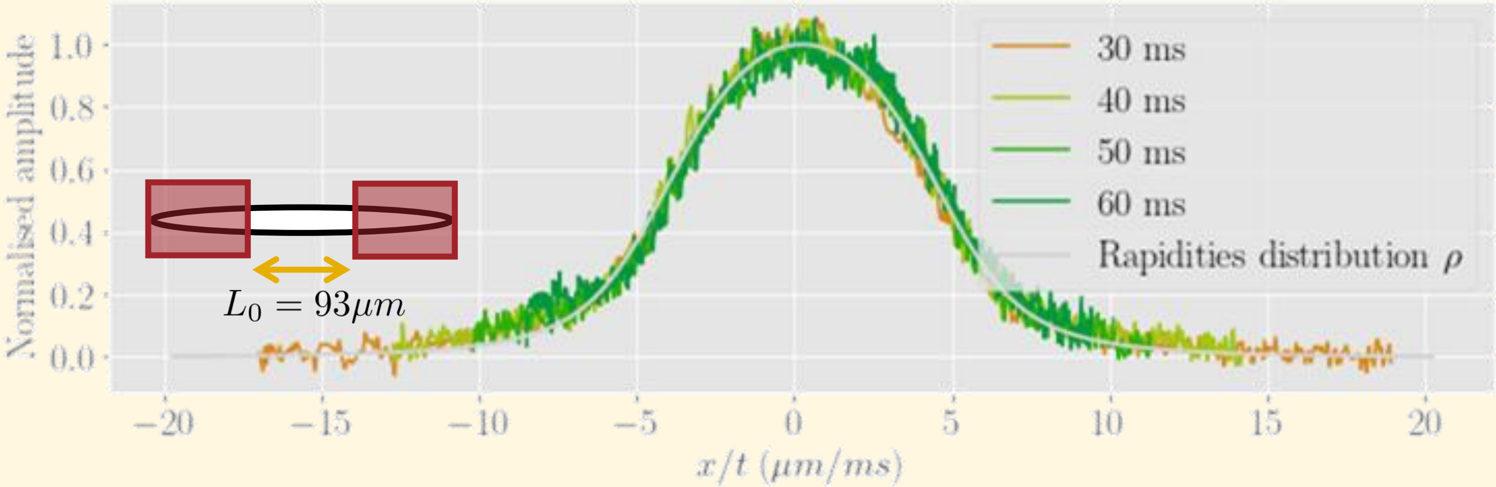
- We look at the profile densities after 40 ms of expansion
- Profiles rescaled with the time





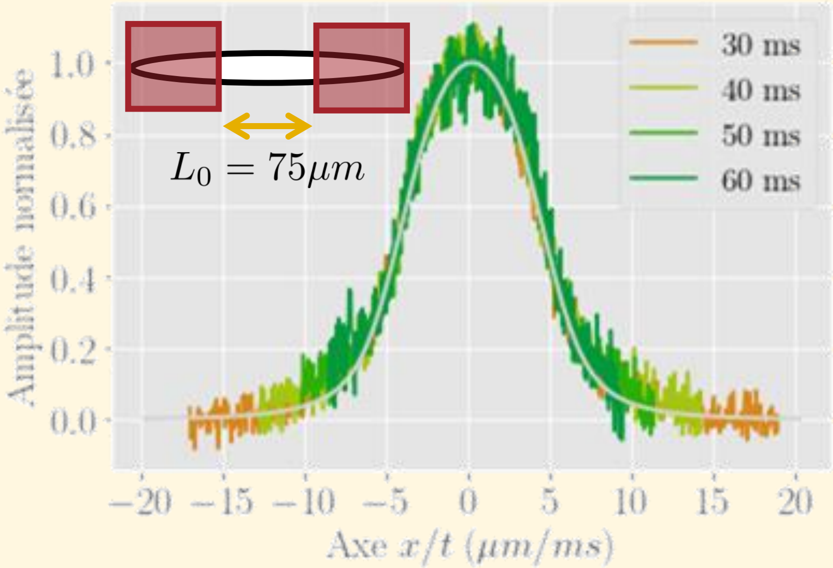
An **asymptotic regime** seems to be reached, it corresponds to the rapidities distribution

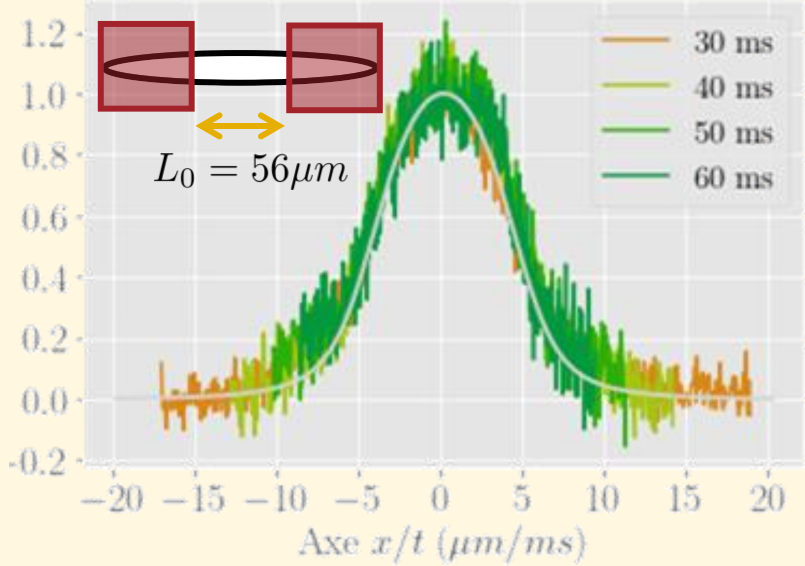
1D EXPANSION

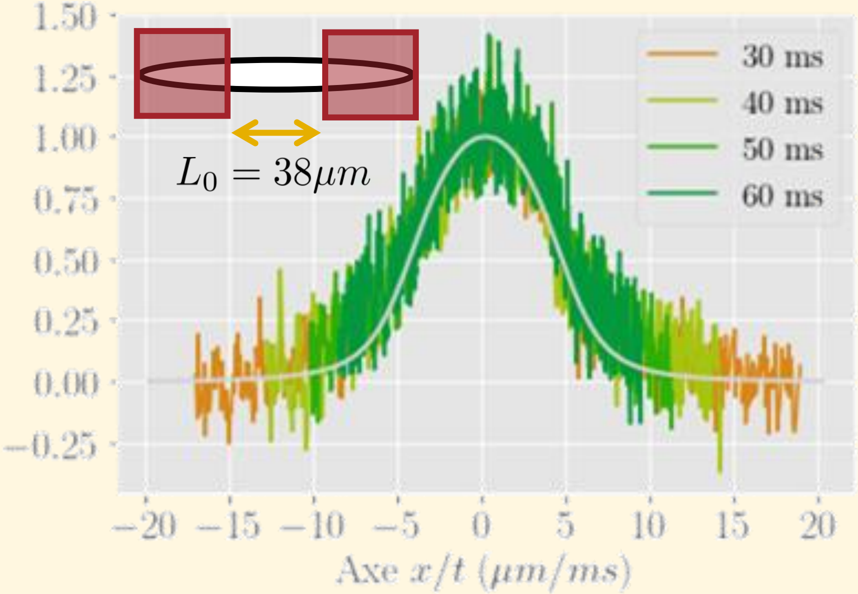


For different initial sizes:

- t_{asymp}/L_0 is constant
- Asymptotic regime seems to be reached
- The rapidities distribution obtained for $L_0 = 93\mu m$ is in agreement with the other initial sizes



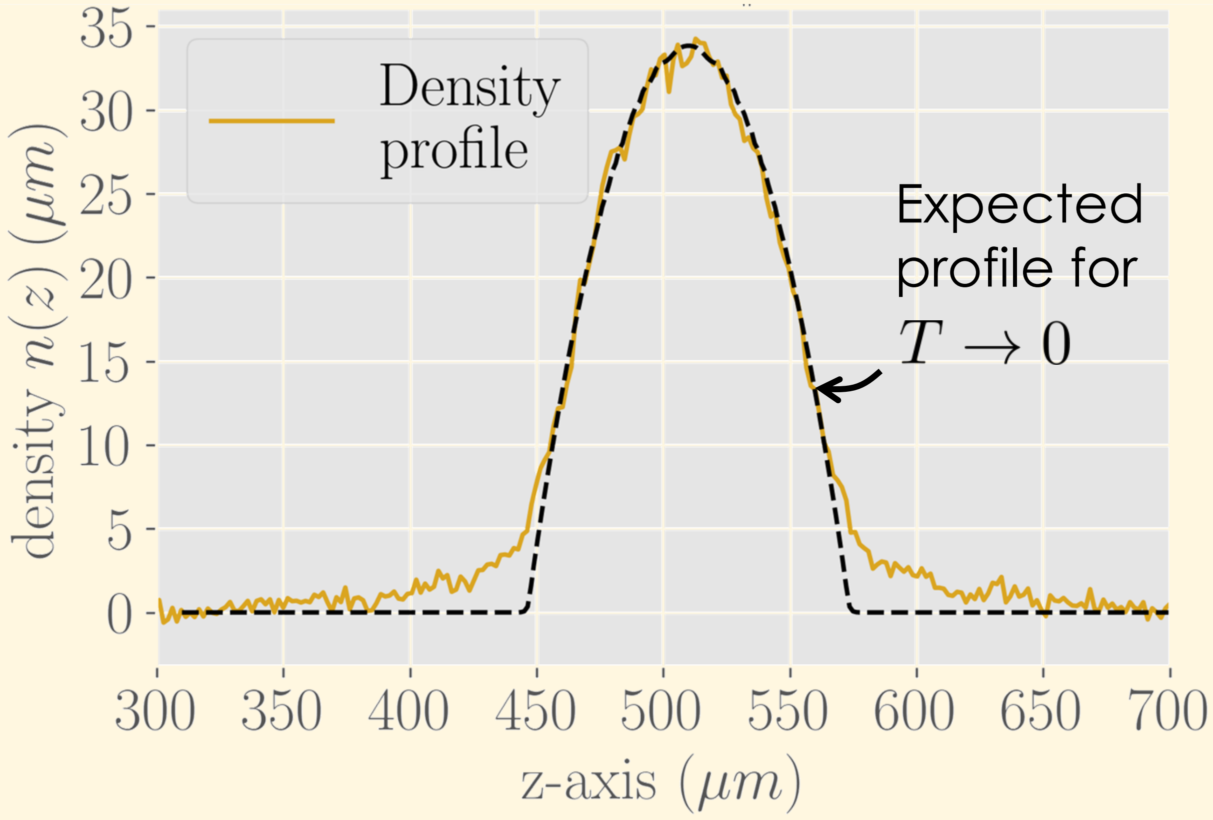




1D EXPANSION

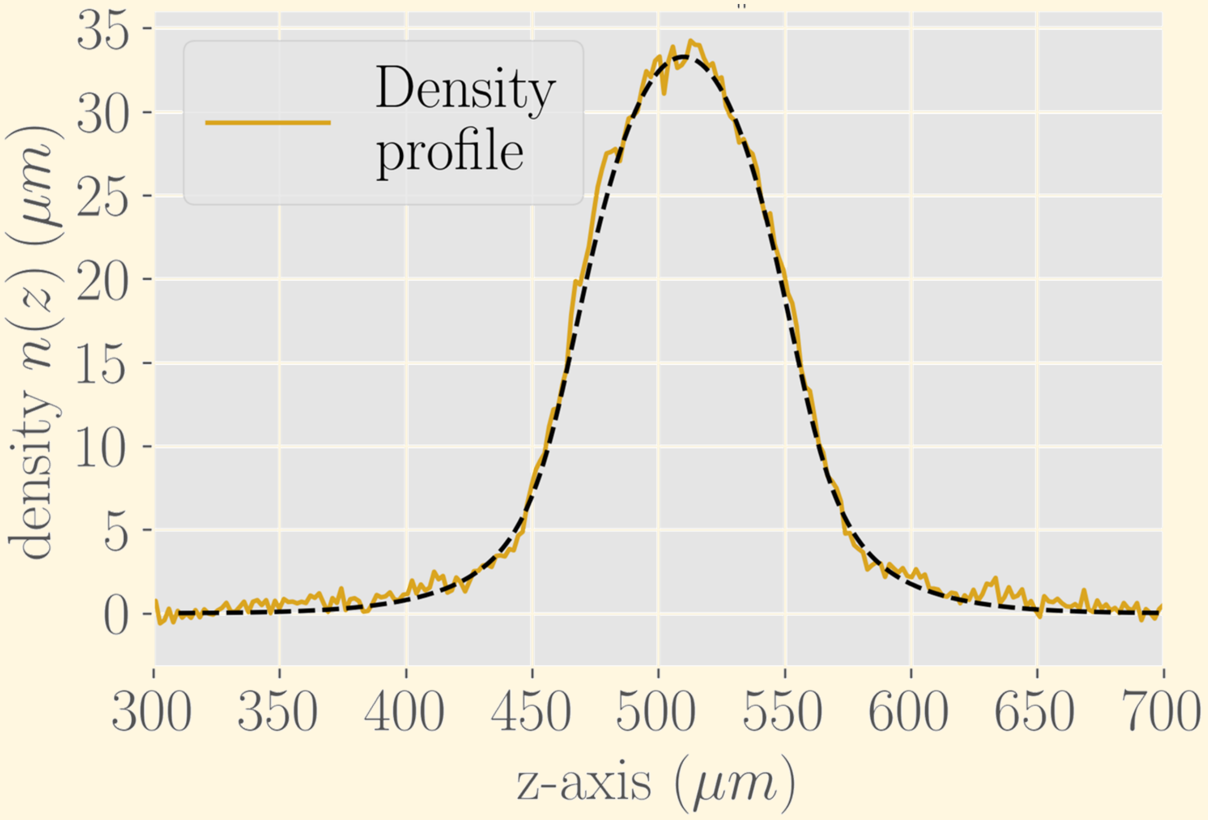
Density profile of the cloud in the harmonic trap before the selection :

- $\omega_{\parallel} = 9Hz, \omega_{\perp} = 4.1KHz$



Approximation : the gas is thermic

$$\Rightarrow T = 150\, nK,\ \mu = 70\, nK$$



PREPARING INITIAL CONDITIONS

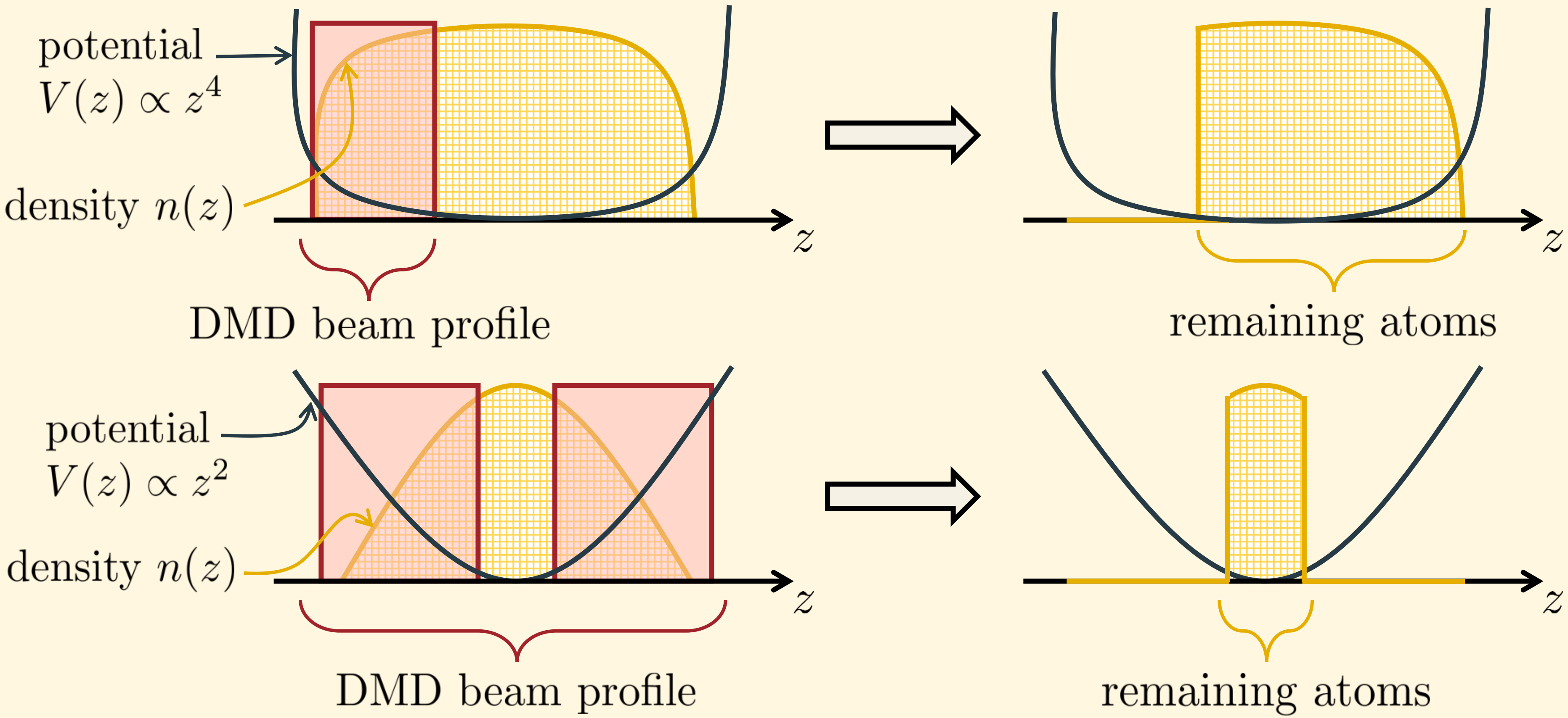
The two protocols concern homogeneous bose gas ! Initially, the gas is in a quadratic / quartic trap.

- Implementation of a selection spatial tool
- Use of the radiation pressure

Implementation of a DMD to shape the beam.

Domain Wall Dynamics

1D homogeneous expansion



CONTEXT

Homogeneous 1D Bose gas with contact repulsive interactions :

- Lieb Liniger Hamiltonian : $\hat{H} = \underbrace{-\frac{\hbar^2}{2m}\sum_i^N \frac{\partial^2}{\partial z^2}}_{\textit{Kinetic part}} + \underbrace{g\sum_{i<j}\delta(z_i - z_j)}_{\textit{Interaction term}}$

Integrable system: the eigenstates are known

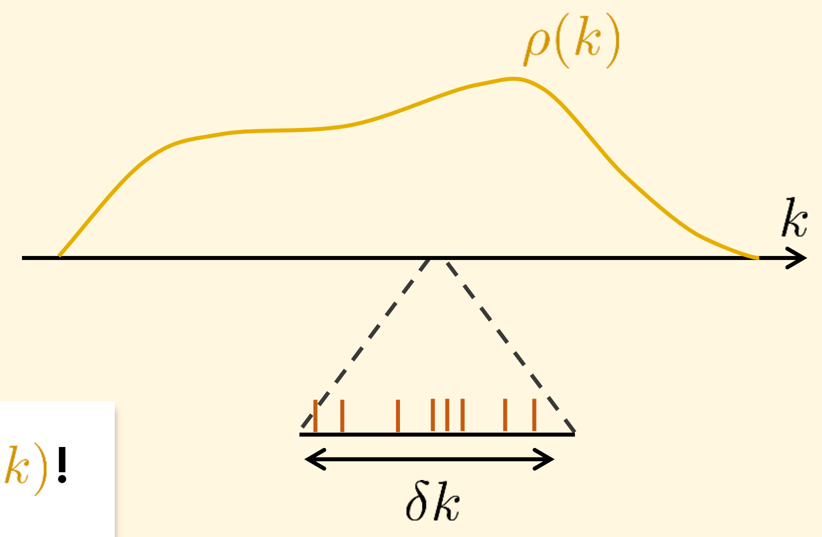
CONTEXT

Integrable system: the eigenstates of $\hat{H}$ are the Bethe Anzatz states $|k_1, k_2, ..., k_N\rangle$ labelled by N numbers $(k_1, k_2, ..., k_N)$ called **the rapidities.**

Wavefunction : $\psi_{\{k_i\}}(z_1 < ... < z_N) = \underbrace{\sum_\sigma}_{\text{Sum over the permutations}} \underbrace{A_\sigma}_{\text{Prefactor}} e^{\left(i(k_{\sigma(1)} z_1 + ... + k_{\sigma(N)} z_N)\right)}$

Rapidities distribution $\rho(k)$: $L\,\rho(k)\,\delta k$ is the number of rapidities in the $[k, k+\delta k]$ range.

⚠ $\rho(k)$ is conserved during the out-of-equilibrium dynamics of the system

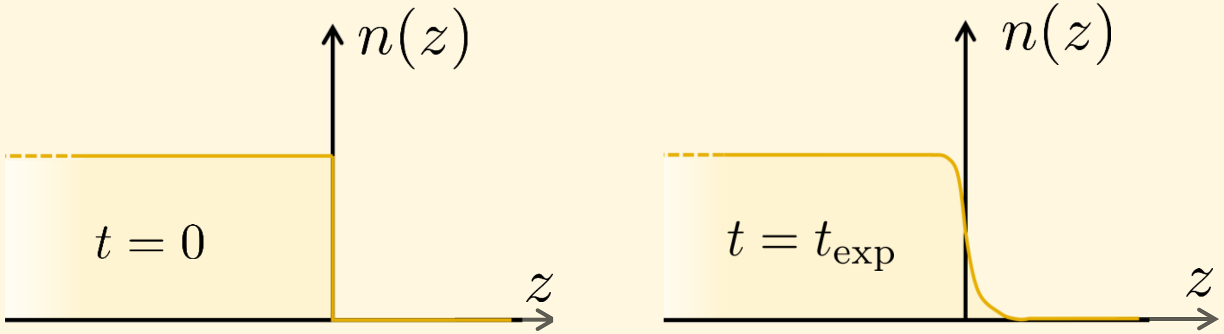


The relaxed state is described by the rapidities distribution $\rho(k)$!

Domain Wall Dynamics protocol

Domain Wall Dynamics : edge deformation dynamics of an initially homogeneous, semi-infinite gas

GHD theory was developed to solve the problem of Domain Wall Dynamics.

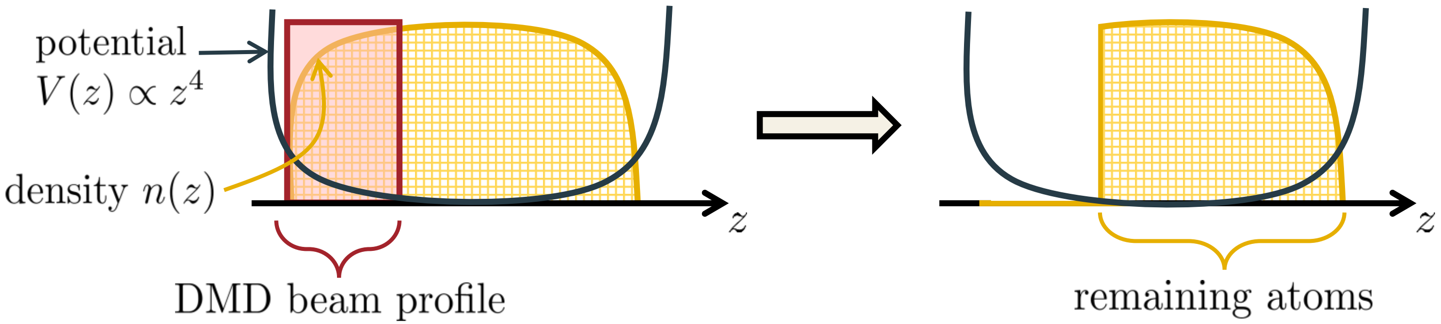


Generalised HydroDynamics Equation :

$$\partial_t \rho(z_0, k, t_0) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z_0, k, t_0)\right) = 0$$

The edge profile is a function of $\frac{\mathbf{z}}{\mathbf{t}} \Rightarrow \mathbf{n}[\rho]\left(\frac{\mathbf{z}}{\mathbf{t}}\right)$

Experimentally : The atoms are trapped in a quartic potential so that the quasi-homogeneous zone is large enough.

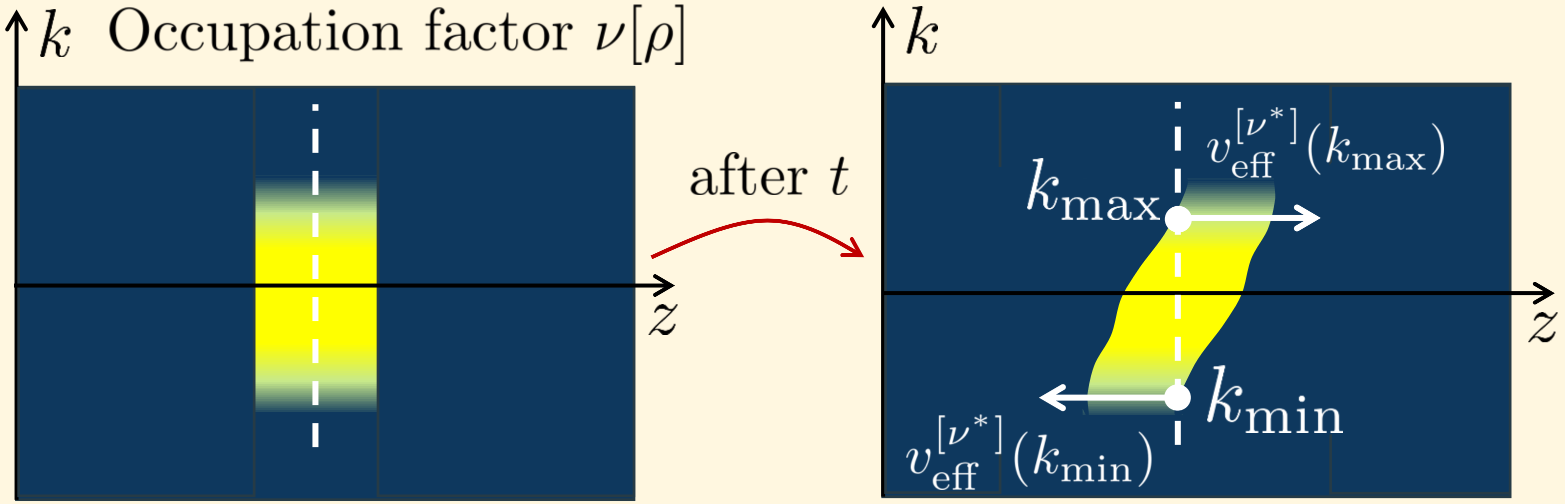


1D EXPANSION

Occupation factor $\nu[\rho](k) \in [0,1]$:

$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z,k,t)\right) = 0$$

$$\Rightarrow \partial_t \nu_{[\rho]}(z,k,t) + v_{\text{eff}}[\rho](k)\partial_z \left(\nu(z,k,t)\right) = 0$$ *Conservation along a trajectory*



$$n(z,t) = \int_{-\infty}^{\infty} \rho(k,z,t)\mathrm{d}k$$

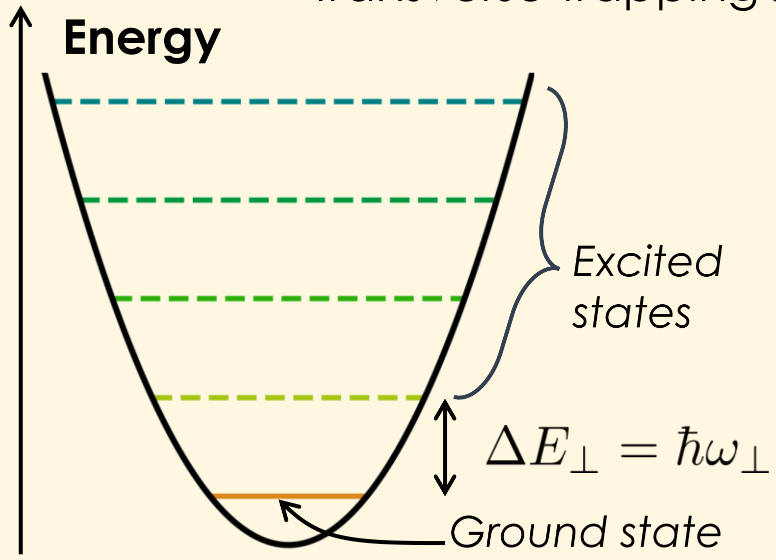
$$\nu(k,z,t=0) = \begin{cases} \nu(k) & \text{if } |z| < z_0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\nu^*(k) = \begin{cases} \nu(k) & \text{if } k_{\text{min}} < k < k_{\text{max}} \\ 0 & \text{otherwise.} \end{cases}$$

OBTAINING A 1D BOSE GAZ

How to reach the 1D regime?

3D harmonic oscillator:
- longitudinal trapping $\omega_{||}$
- transverse trapping $\omega_{\perp}$



If $k_B T, \mu \ll \Delta E_{\perp}$, the gas is frozen in the transverse direction !

How do we produce a 1D Bose gas ?

Atoms used : Rubidium 87 in the $|F=2, m_F=2\rangle$ atomic state.

Atoms are trapped by a magnetic field created with wires deposited on a chip.

Potential felt by the atoms under a $\vec{B}$ magnetic field :

$$V = g_F \, m_F \, \mu_B \, |\vec{B}|$$

Landé factor

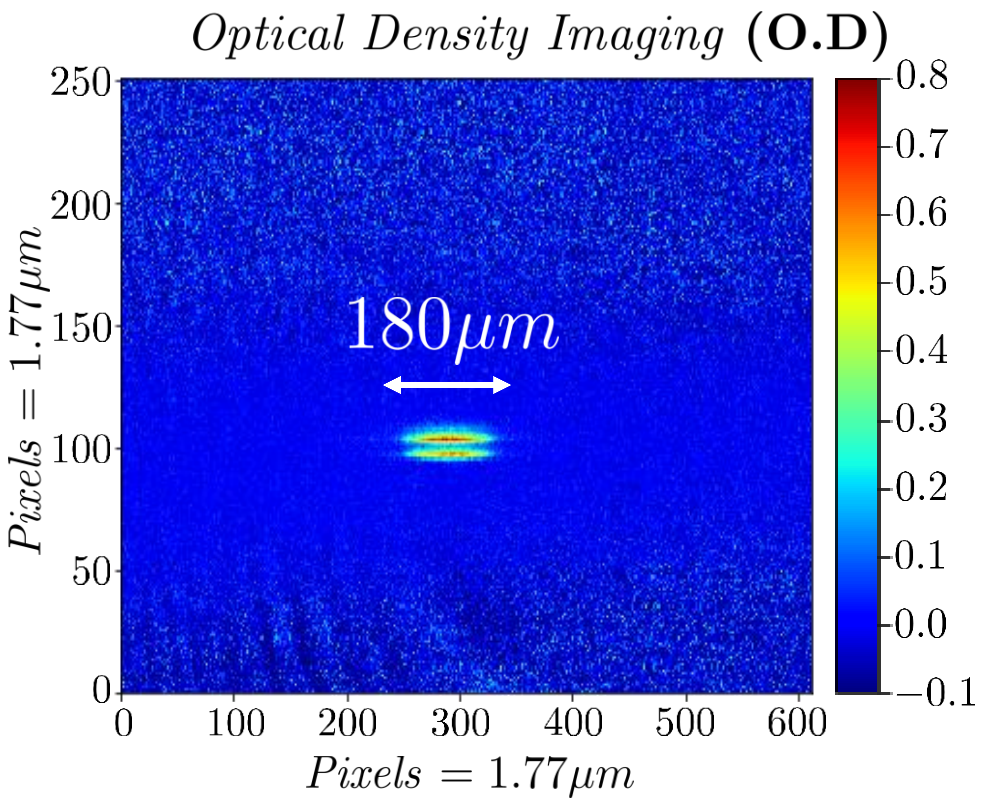
Bohr Magneton

The atom chip

PREPARING INITIAL CONDITIONS

Atom chip experiment: the atoms magnetically trapped thanks to micro-wires deposited on a chip.

1D quasi-BEC



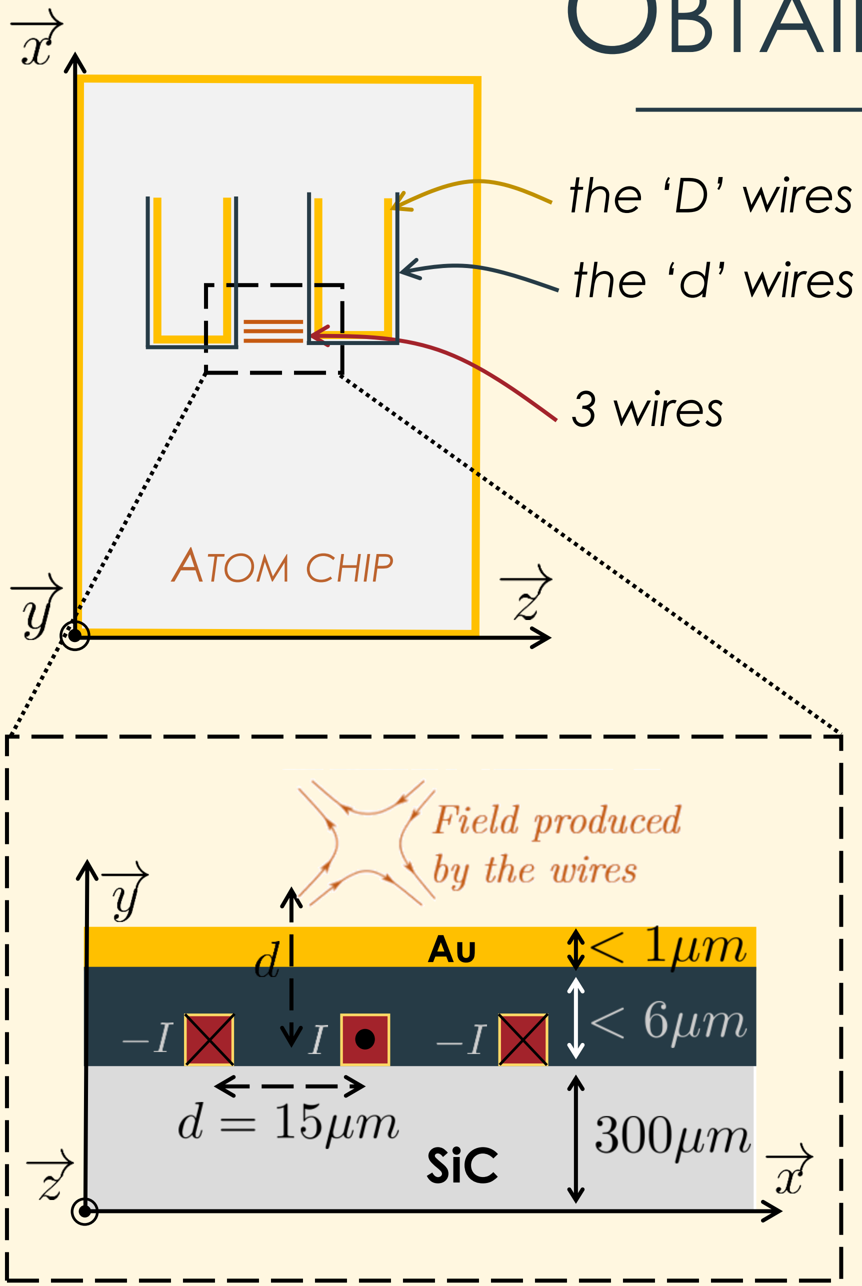
$$\text{Natoms} \sim 10000$$
$$\omega_\perp = 2.5 \text{ KHz}$$
$$\omega_\parallel = 9.0 \text{ Hz}$$

Longitudinnaly trapping :

$$V(z) = \sum_{i=1}^{4} a_i z^i$$

In the experiment, we are working with harmoninc and quartic potential.

Obtaining a 1D Bose gaz



The three wires : producing a quadrupole with a zero magnetic field above the central wire: *transverse confinement*

The 'D' and 'd' wires : creating a *longitudinal confinement.*

$$\Rightarrow V(z) = a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + \ldots$$

- *Independent control of the longitudinal coupling and the transverse one*
- $\Delta E_\perp \propto \omega_\perp = 2.5\,\mathrm{KHz}$
- *Typical longitudinal quadratic potential :*

$$\Delta E_\parallel \propto \omega_\parallel = 9\,\mathrm{Hz}$$

Extracting rapidities distribution

Yang-Yang entropy:

$$S_{\mathrm{YY}}[\rho] \simeq \int_{-\infty}^{\infty} \left(\rho_s \log(\rho_s) - \rho \log(\rho) - (\rho_s - \rho) \log\left(\rho_s - \rho\right)\right) \mathrm{d}k$$

Yang, Yang (1969)

Construction of conserved quantities:

$$\begin{cases} Q_0 = \int \rho(k)\mathrm{d}k \\ Q_1 = \int k\rho(k)\mathrm{d}k \\ \quad \cdots \\ Q_i = \int \frac{k^i}{i}\rho(k)\mathrm{d}k \end{cases}$$

$\Rightarrow$ The moments of the rapidities distribution are conserved quantities

EXTRACTING RAPIDITIES DISTRIBUTION

Yang-Yang entropy:

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Entropy maximisation:

$$\frac{\delta}{\delta\rho}\left(S_{\mathrm{YY}}[\rho] - \sum_i \lambda_i Q_i\right) = 0$$

Lagrange multpliers

Thermic distribution:

$$\sum_i \lambda_i Q_i = \lambda_0 Q_0 + \lambda_2 Q_2 = -\frac{\mu}{T}N + \frac{E}{T}$$

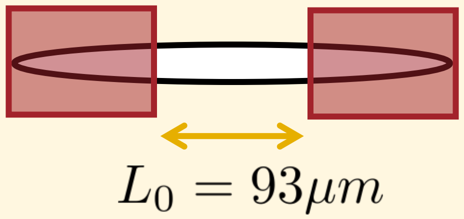
$$Q_0 = \int \rho(k)\mathrm{d}k = N \Rightarrow \lambda_0 = -\frac{\mu}{T}$$

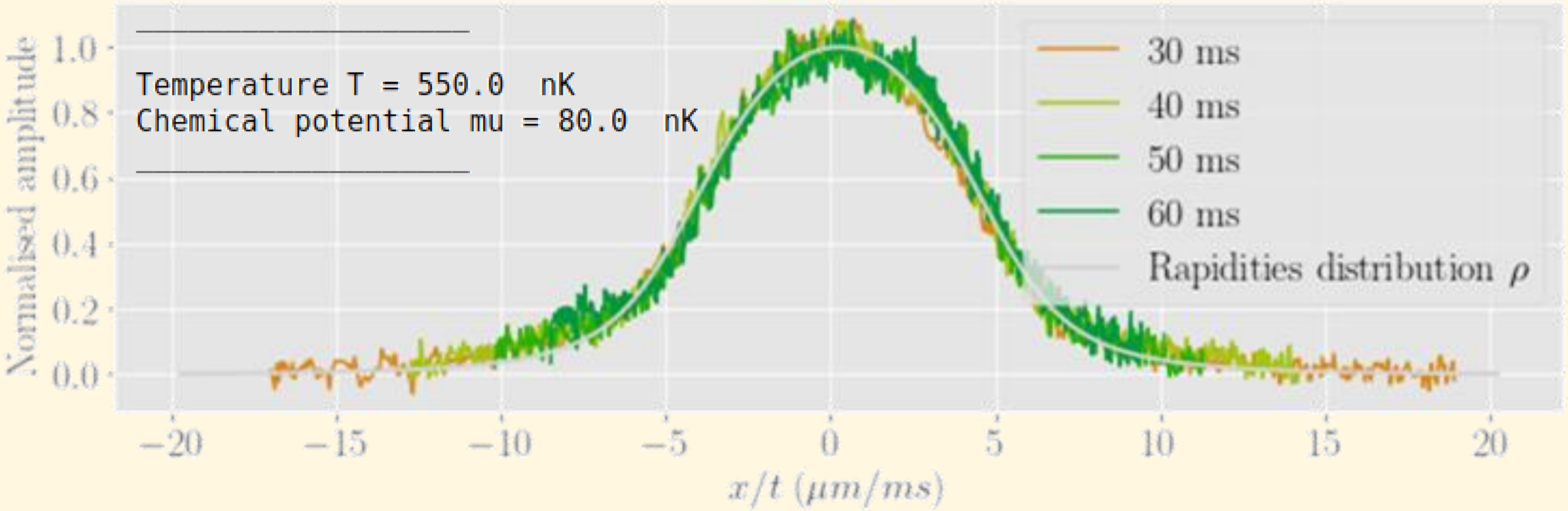
$$Q_2 = \int \frac{k^2}{2}\rho(k)\mathrm{d}k = E \Rightarrow \lambda_2 = \frac{1}{T}$$

1D EXPANSION

Rescaled profile denstity:

- We look at the profile densities after 30ms of expansion
- Profiles rescaled with the time





The **asymptotic regime** seems to be reached, a rapidity distribution can be fitted
$\Rightarrow$ **Good shape but high temperature ?**

Occupation factor

System:

- N : atoms number
- L : size of the system
- Periodic boundary conditions

Bethe equations :

$$\frac{L}{2\pi}\left[k_i + \frac{1}{L}\sum_{j\neq i} 2\arctan\left(\frac{k_i - k_j}{g}\right)\right] = I_i$$

with $i \in [1, N]$

- $I_i \in \mathbb{Z}$ if N is odd, $I_i \in \frac{\mathbb{Z}}{2}$ if N is even.
- The I's are called the Bethe integers.
- g is the strength repulsion term.

OCCUPATION FACTOR

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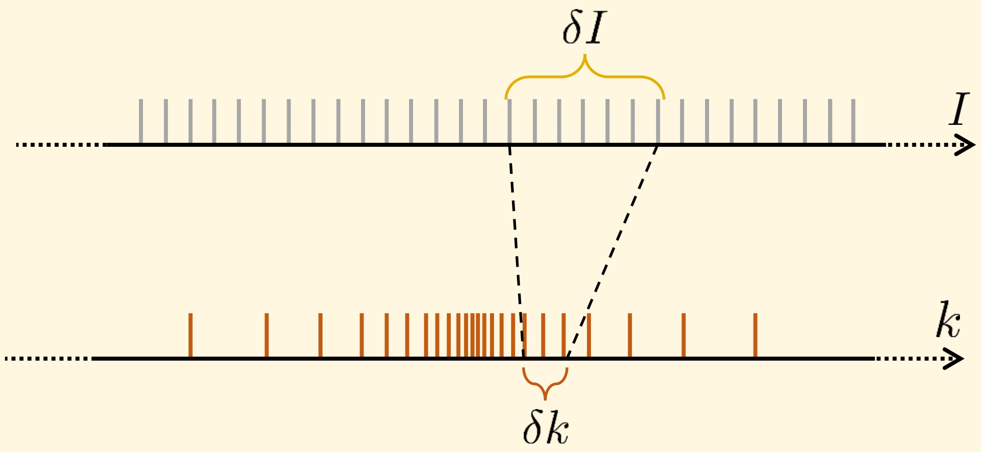
Density of state: $\rho_s(k) = \frac{1}{L}\frac{\mathrm{d}I}{\mathrm{d}k}$



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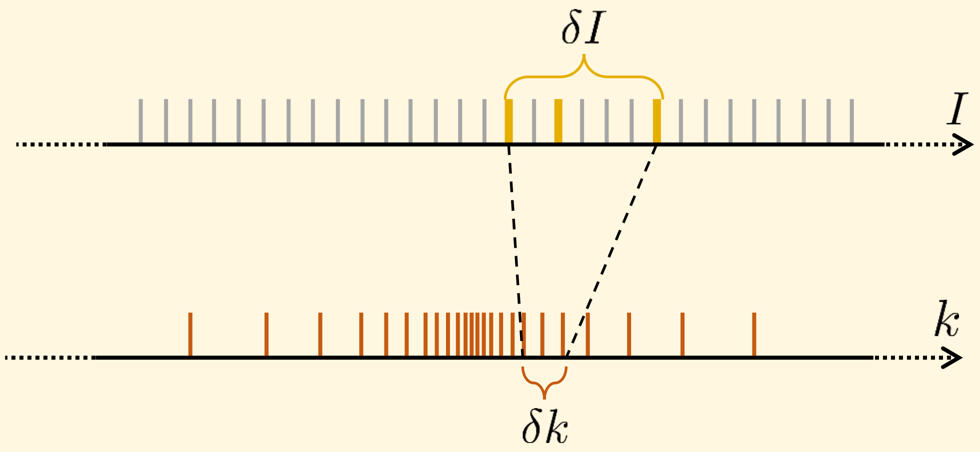

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$$\begin{cases} L\rho_s(k)\mathrm{d}k = \mathrm{d}I \\ L\rho(k)\mathrm{d}k : \text{number of rapidities in } [k, k+\mathrm{d}k] \end{cases}$$

Occupation factor : $\nu_{[\rho]}(k) = \frac{\rho(k)}{\rho_s(k)} \in [0, 1]$

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$$2\pi\rho(k) = \nu_{[\rho]}(k) + \int_{-\infty}^{\infty} \Delta(k-k')\nu_{[\rho]}(k)\rho(k')\mathrm{d}k' \text{ with } \Delta(k) = \frac{2g}{g^2+k^2}$$

$$\boxed{N/L = \int_{-\infty}^{\infty} \rho(k)\mathrm{d}k}$$

- New hydrodynamics equation on the occupation factor :

$$\partial_t \rho(z,k,t) + \partial_z \left(v_{\text{eff}}[\rho](k)\rho(z,k,t)\right) = 0$$

$$\boxed{\partial_t \nu_{[\rho]}(z,k,t) + v_{\text{eff}}[\rho](k)\partial_z \left(\nu(z,k,t)\right) = 0}$$

Conservation along a trajectory

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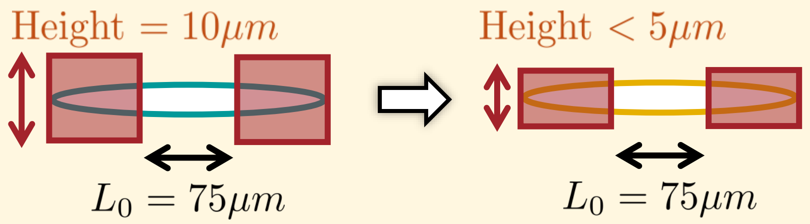
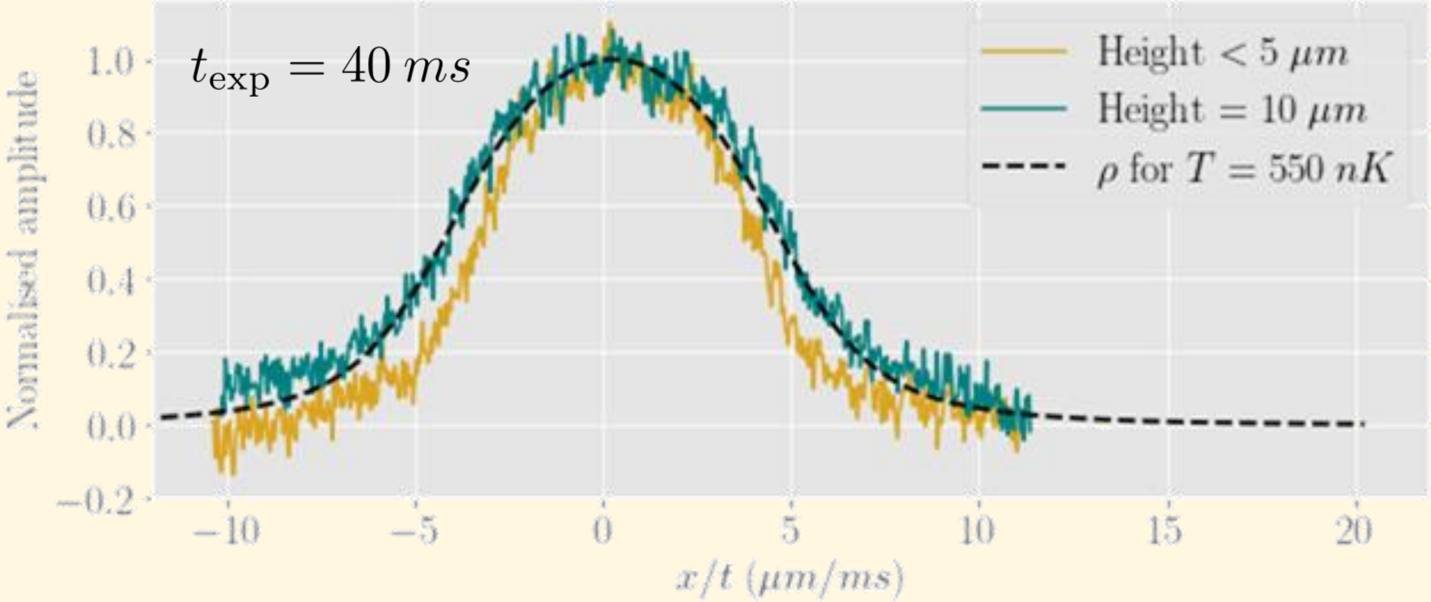
Yang, Yang (1969)

High temperatures

Change of the height of the spatial selection beam :

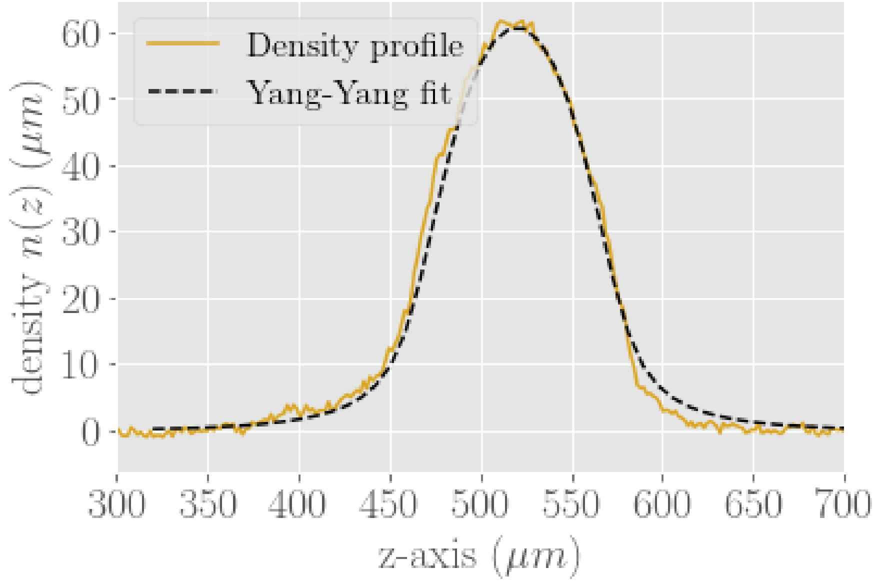




Density profile of the cloud in the harmonic trap before the selection

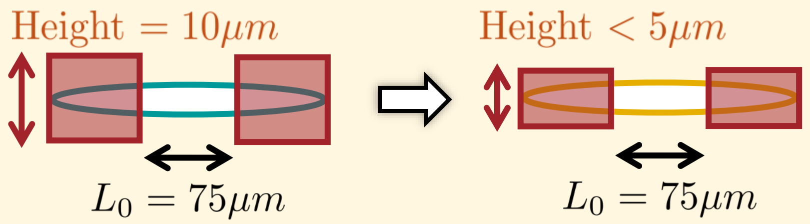
The rapidity distribution is obtained with entropy maximization + thermic distribution (T and μ are fit parameters).

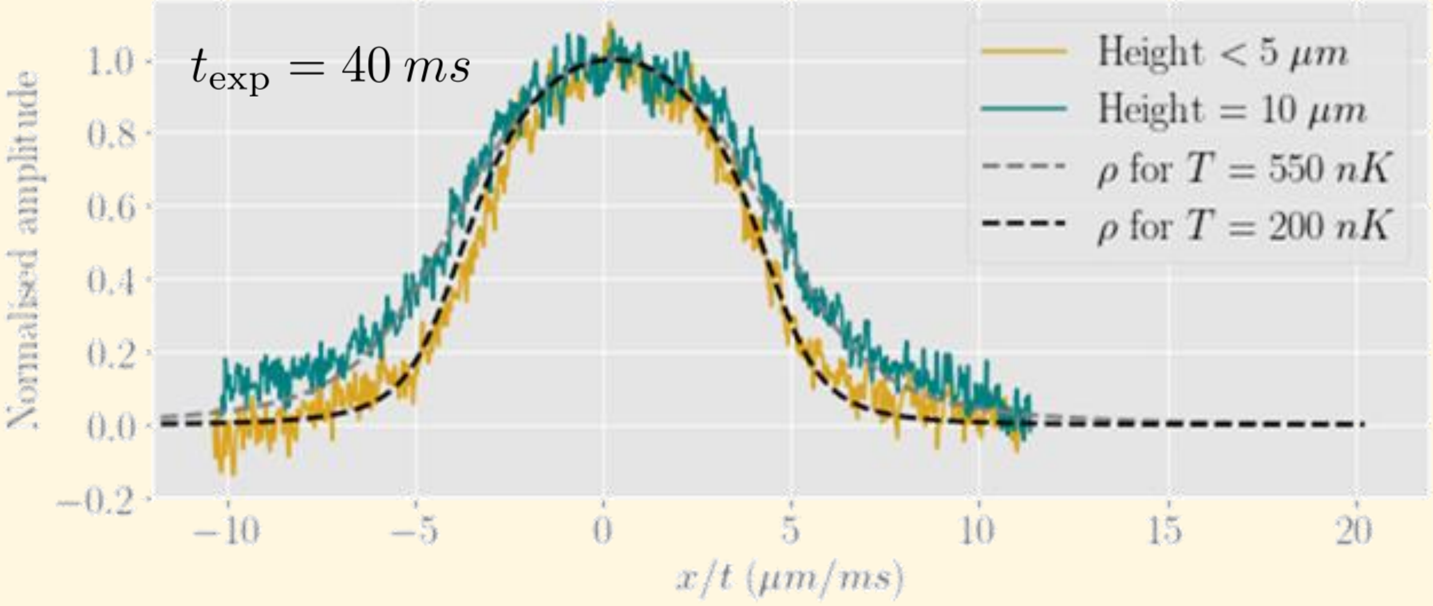
$$\Rightarrow T = 200\,nK,\ \mu = 80\,nK$$



HIGH TEMPERATURES

Change of the height of the spatial selection beam :





Diffusion of photons with the optical system.
Reducing this effect gives reasonnable temperature

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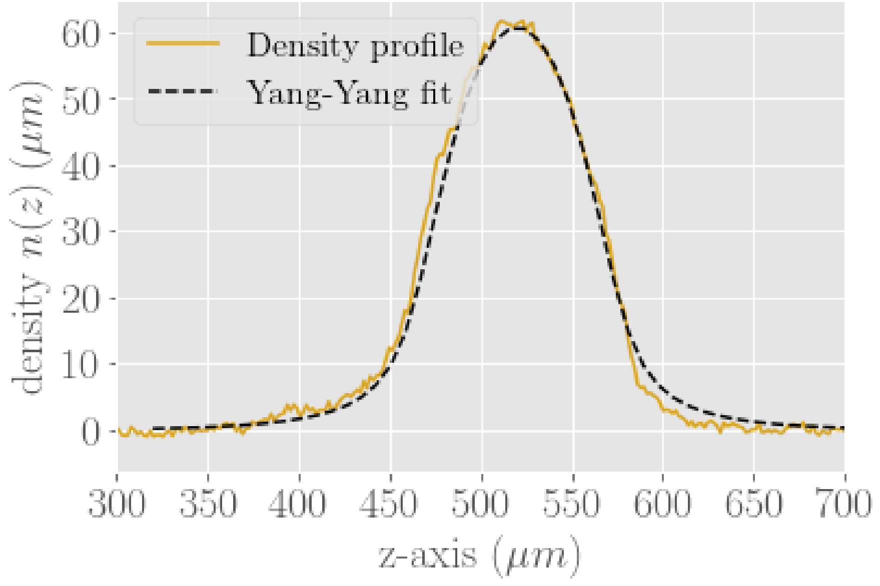
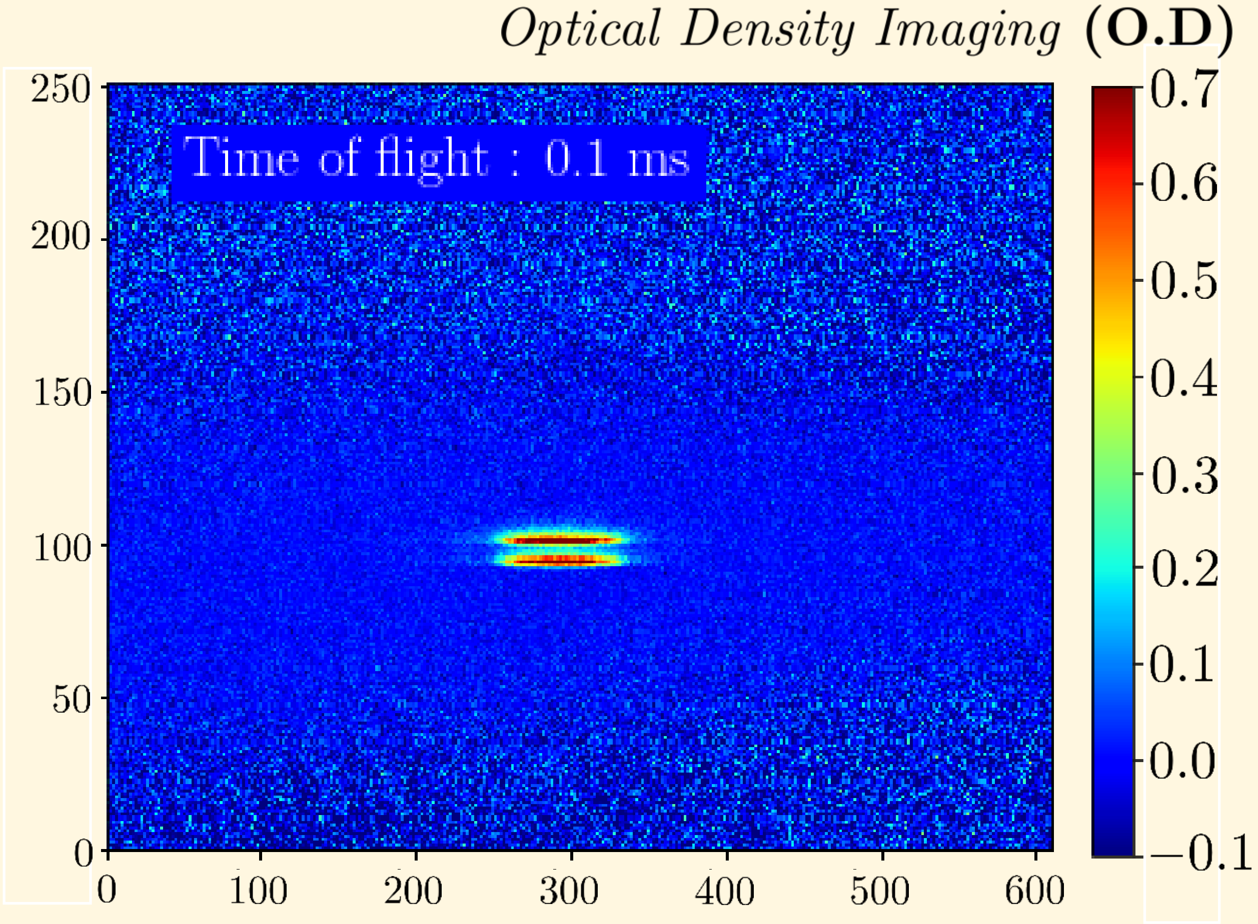


Time of flight

Time of flight : longitudinal and transverse confinement are removed.



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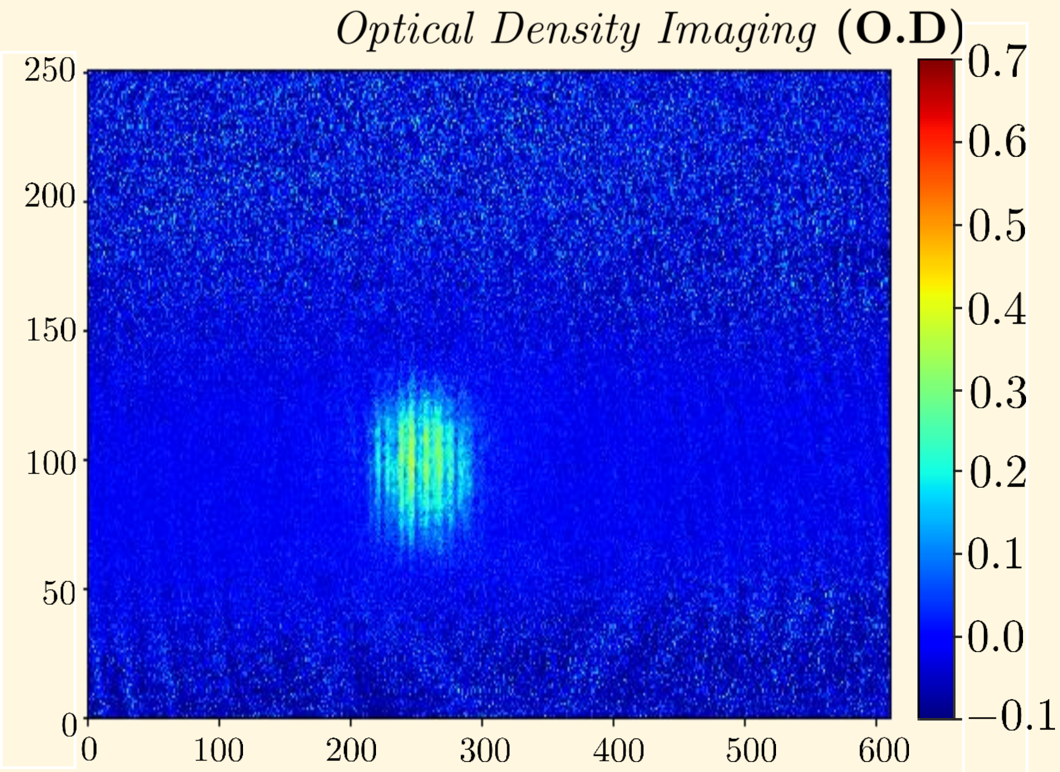
Density distribution : $\rho(z) = \delta\rho(z) + \langle \rho(z) \rangle$

Density fluctuations　　Mean profile



Quasi BEC (*in situ*) :

- **small density fluctuations**
  $\Rightarrow \rho(z)_{\mathrm{t=0}} \simeq \langle \rho(z) \rangle_{\mathrm{t=0}} = n_0(z)$
- **phase fluctuations**
  $\Rightarrow \langle |\theta(z) - \theta(0)|^2 \rangle = \frac{2|z|}{l_c}, l_c = \frac{2\hbar^2 n_0}{m k_b T}$

$$\Rightarrow \langle |\rho(q)|^2 \rangle = 4 n_0^2 \langle \theta_q^2 \rangle \sin^2 \left(\frac{\hbar q^2 t_{\mathrm{tof}}}{2m} \right) \mathrm{e}^{-\sigma^2 q^2}$$

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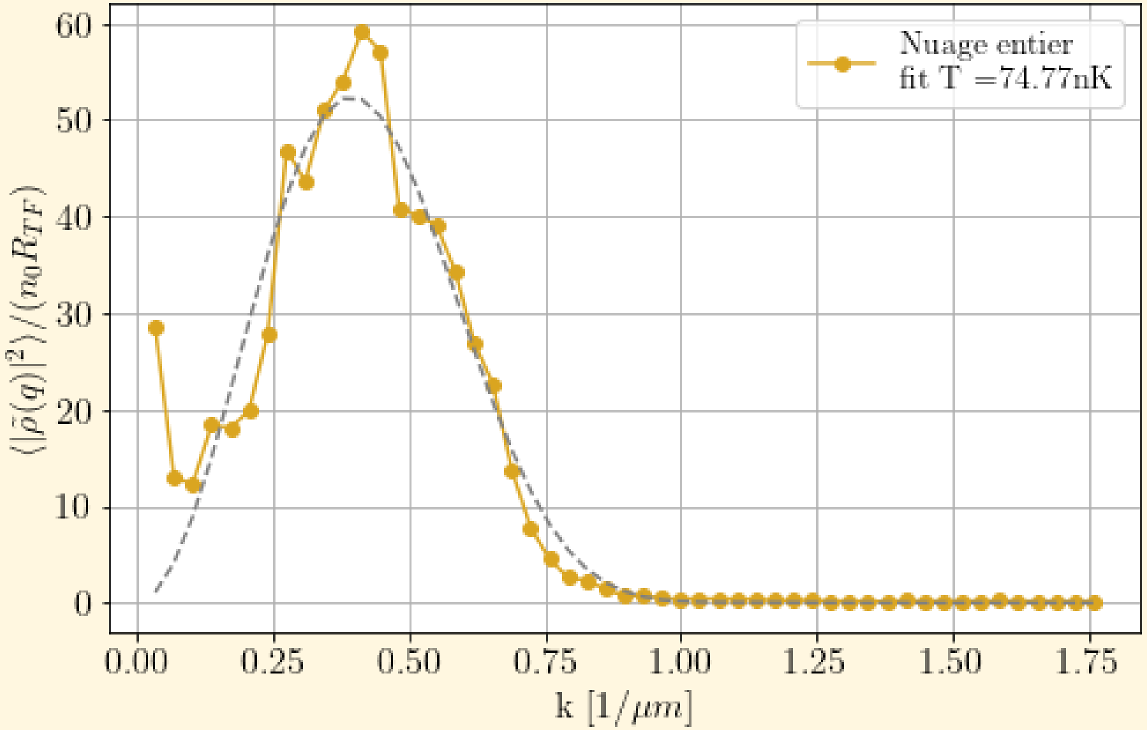

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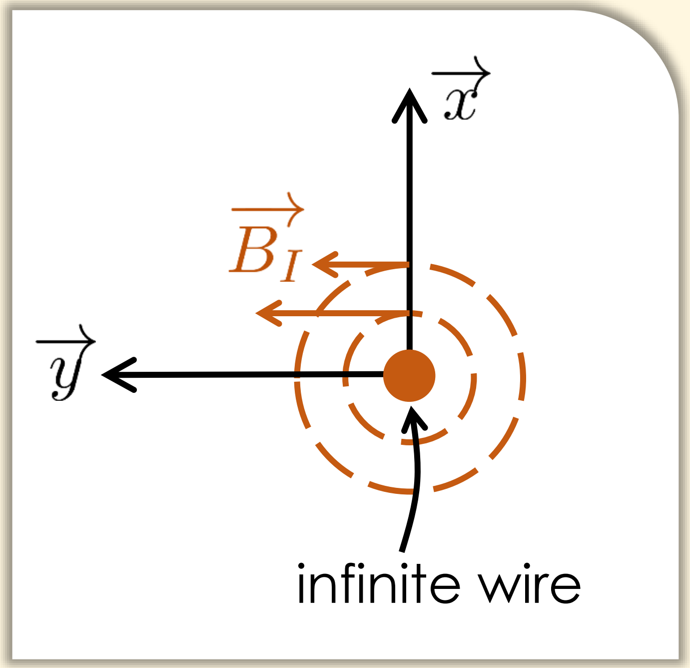
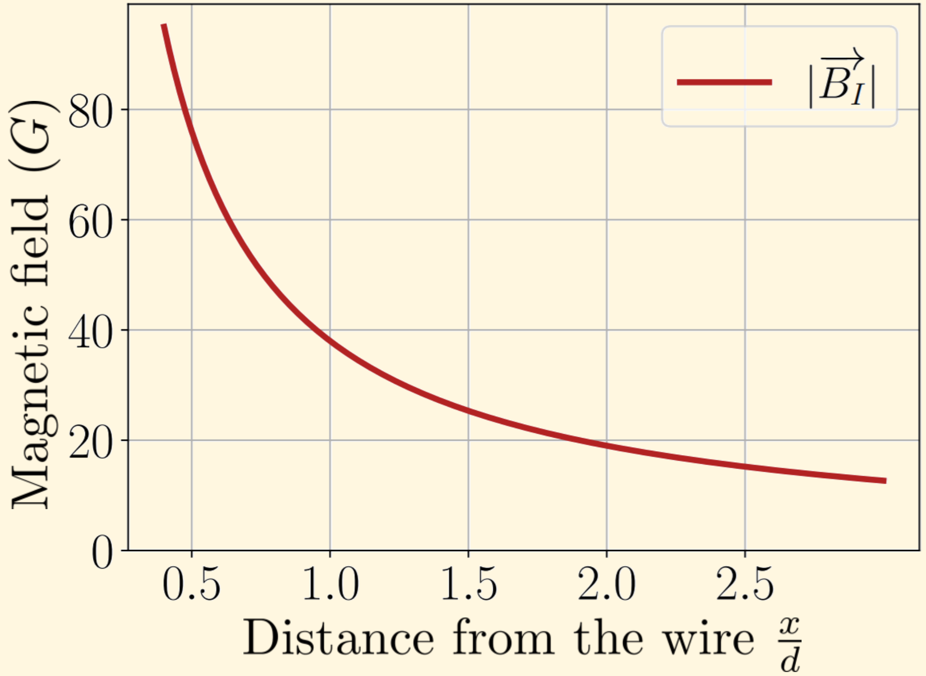
Obtaining a 1D Bose gaz

To trap the atoms, one needs:

- A current I through a wire





The atom chip

Potential felt by the atoms under a $\vec{B}$ magnetic field :

$$V = g_F \, m_F \, \mu_B \, |\vec{B}|$$

OBTAINING A 1D BOSE GAZ

To trap the atoms, one needs:

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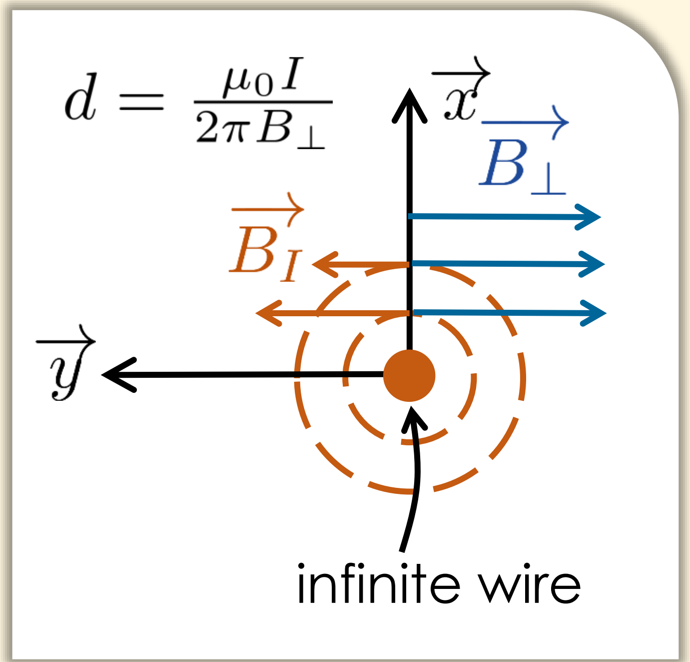
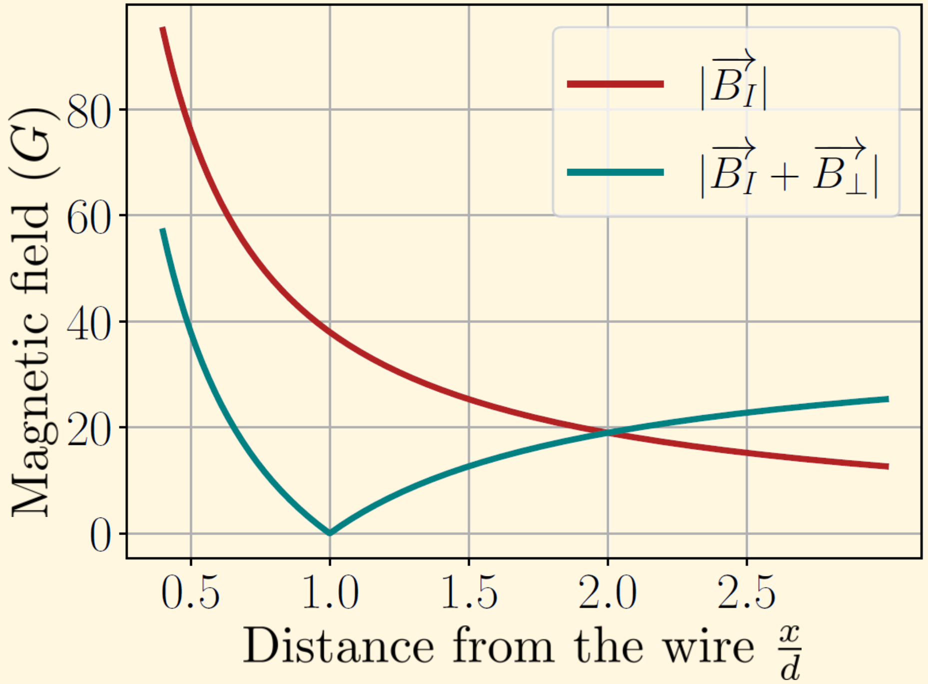
- A homogeneous transverse field $\overrightarrow{B_\perp}$ to have a minimum of potentiel





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OBTAINING A 1D BOSE GAZ

To trap the atoms, one needs:

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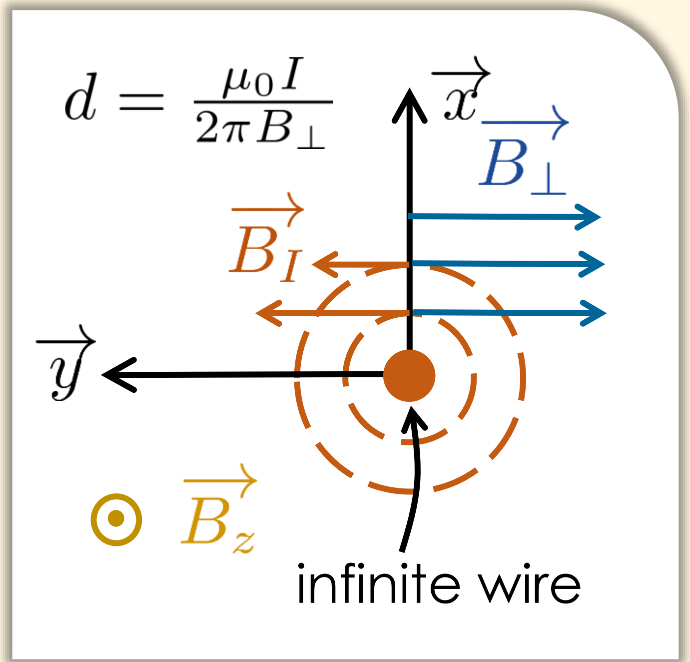
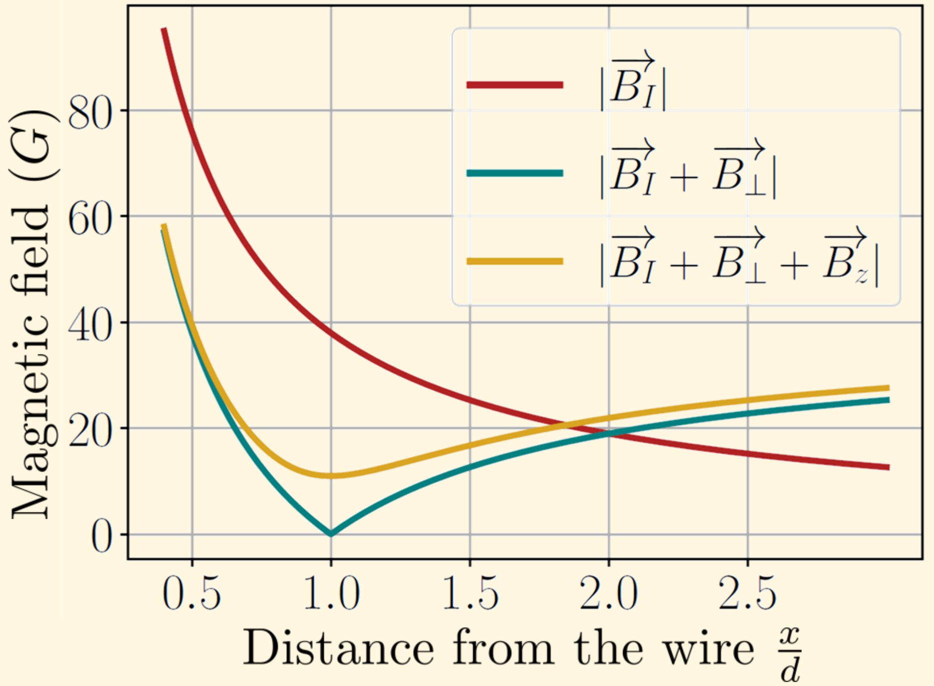
- A homogeneous transverse field $\overrightarrow{B_\perp}$ to have a minimum of potentiel
- A longitudinal field $\overrightarrow{B_z}$ to have a non zero of potentiel





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PERSPECTIVES