Measuring the spatially resolved rapidities distribution of a 1D Bose gas

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Homogeneous 1D Bose gas with contact repulsive interactions:

- Lieb Liniger Hamiltonian:
  \[ \hat{H} = -\frac{\hbar^2}{2m} \sum_i^N \frac{\partial^2}{\partial z_i^2} + g \sum_{i<j} \delta(z_i - z_j) \]

**Integrable system:** the eigenstates of \( \hat{H} \) are the Bethe Anzatz states \(|k_1, k_2, \ldots, k_N\rangle \) labelled by \( N \) numbers \((k_1, k_2, \ldots, k_N)\) called the **rapidities**.

**Rapidities distribution** \( \rho(k) \): \( L \rho(k) \delta k \) is the number of rapidities in the \([k, k + \delta k]\) range.

\( \rho(k) \) is conserved during the out-of-equilibrium dynamics of the system.

The relaxed state is described by the rapidities distribution \( \rho(k) \)!
Rapidities distribution can be measured: after a 1D longitudinal expansion of the cloud, the profile density converges towards the rapidities distribution.

\[ n(z, t = 0) \neq \rho(k) \]
\[ n(z, t = \text{t}_{\text{exp}}) \rightarrow +\infty \approx \tilde{n}\left(\frac{z}{t_{\text{exp}}}\right) = \rho\left(\frac{z}{t_{\text{exp}}}\right) \frac{L_0}{t_{\text{exp}}} \]

Density profile: \( n(z, t) \)
Rapidities distribution: \( \rho(k, t) \)

« Generalized hydrodynamics in strongly interacting 1d Bose gases », Malvania, N. & co, Science 373(6559)
Local Density Approximation (Euler scale):
A spatially resolved rapidity distribution \( \rho(z, k) \) can be defined.

**Objective:**
Measuring experimentally the spatially resolved rapidity distribution \( \rho(z, k) \)
Outline

Asymptotic regime of a 1D expansion
- Preparing initial conditions
- 1D expansion

Hydrodynamics for integrable systems
- Generalized HydroDynamics
- Dynamics of 1D expansion

Domain Wall Dynamics
- Domain Wall Dynamics protocol
- Domain Wall Dynamics reconstruction
PREPARING INITIAL CONDITIONS

How to reach the 1D regime?

Transverse Energies states

Excited states

\[ \Delta E_\perp = \hbar\omega \]

Ground state

If \( k_B T, \mu \ll \Delta E_\perp \), the gas is frozen in the transverse direction!
PREPARING INITIAL CONDITIONS

How to reach the 1D regime?

If $k_B T, \mu \ll \Delta E_\perp$, the gas is frozen in the transverse direction!

How do we produce a 1D Bose gas?

**Atoms used**: Rubidium 87 in the $|F = 2, m_F = 2\rangle$ atomic state.

**Atom chip experiment**: the atoms magnetically trapped thanks to micro-wires deposited on a chip.

**Properties**:
- Independent control of the longitudinal coupling and the transverse one
- Longitudinal trapping: $V(z) = \sum_{i=1}^{4} a_i z^i$
PREPARING INITIAL CONDITIONS

How do we produce a 1D Bose gas?

Atoms used: Rubidium 87 in the \( |F = 2, m_F = 2 \rangle \) atomic state.

Atom chip experiment: the atoms magnetically trapped thanks to micro-wires deposited on a chip.

Assumption: system described by a Gibbs ensemble

Density profile of the cloud in the harmonic trap before the selection

Properties:
- Independent control of the longitudinal coupling and the transverse one
- Longitudinal trapping:
  \[
  V(z) = \sum_{i=1}^{4} a_i z^i
  \]

⇒ \( T = 105 \, nK, \, \mu = 55 \, nK \)
Objective: measure the spatially resolved rapidities distribution

- Implementation of a selection spatial tool
- Use of the radiation pressure

Implementation of a DMD to shape the beam.

**Potential** $V(z) \propto z^2$

**Density** $n(z)$

DMD beam profile

remaining atoms
Experimental setup to obtain a homogeneous 1D gas before imaging it:

- CCD
- DMD
- Optical fiber
- Beam splitter 90/10
- Optical system
- High-resolution objective
- Vacuum chamber
- Atom chip
- Trapped atoms

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**PREPARING INITIAL CONDITIONS**
Experimental setup to obtain a homogeneous 1D gas before imaging it:

- **Vacuum chamber**
- **Trapped atoms**
- **Atom chip**
- **Beam splitter 90/10**
- **Optical system**
- **CCD**
- **DMD**
- **Optical fiber**
- **High-resolution objective**
1D EXPANSION

- The longitudinal confinement is removed,
- The transverse one is maintained

Density profile of a 1D gas after different times of expansion:

Expansion time = 0 ms

Pixels = 1.77 μm

Linear density (μm⁻¹)

Longitudinal axis (μm)  Time of expansion (ms)
Rescaled profile density:

- We look at the profile densities after 50 ms of expansion
- Profiles rescaled with the time

\[ n(z, t)_{t \to +\infty} \simeq \tilde{n} \left( \frac{z}{t_{\text{exp}}} \right) = \rho \left( \frac{z}{t_{\text{exp}}} \right) \frac{L_0}{t_{\text{exp}}} \]

！An asymptotic regime seems to be reached, it corresponds to the rapidities distribution
Conclusion:

- Differences between the data and the rapidities distribution: Non-thermal distribution?
- Do we reach an asymptotic regime? Look at the dynamics
**Generalised Hydrodynamics (GHD)**

**Hydrodynamic approach**: continuum of locally homogeneous cells of fluid of size $\Delta l$ with $L \gg \Delta l \gg d$ where

- $d$ is the microscopic length scale
- $L$ is the macroscopic one

\[
L \gg \Delta l \\
\Delta l \gg d \\
d
\]
**Generalised HydroDynamics (GHD)**

Classical HydroDynamics (CHD): a non integrable system can locally be described by a Gibbs state with $n(z, t)$, $n(z, t)p(z, t)$ and $u(z, t)$.

- **Conservation of the atom number**:
  \[
  \partial_t n(z, t) + \partial_z (v(z, t)n(z, t)) = 0
  \]
  Atomic density Hydrodynamic velocity

- **Conservation of total momentum**:
  \[
  \partial_t (np) + \partial_z (mnv^2 + P) = 0
  \]
  momentum Pressure

- **Conservation of total energy**:
  \[
  \partial_t \left( n \frac{mv^2}{2} + nc \right) + \partial_z \left( n \left( n \frac{mv^2}{2} + nc \right) + nP \right) = 0
  \]
  Energy per atom $u(z, t)$
**Generalised Hydrodynamics (GHD)**

Classical Hydrodynamics:

- $n(z,t)$
- $n(z,t)\rho(z,t)$
- $u(z,t)$

General Hydrodynamics:

- $\int dz \rho(k, z, t)\Delta k$ is conserved

General Hydrodynamics (GHD): an integrable system can locally be described by the spatially resolved rapidity distribution $\rho(z, k, t)$.

- Conservation of the rapidity distribution on a $\delta k$ slice:

$$\partial_t \rho(z, k, t) + \partial_z \left( v_{\text{eff}}[\rho](k) \rho(z, k, t) \right) = 0$$

with

$$v_{\text{eff}}[\rho](k) = k - \int_{-\infty}^{+\infty} \Delta(k' - k') \left[ v_{\text{eff}}[\rho](k') - v_{\text{eff}}[\rho](k''') \right]$$

and

$$\Delta(k) = \frac{2g}{g^2 + k^2}$$

Bertini et al. (2016), Castro-Alvaredo et al. (2016)
DYNAMICS OF 1D EXPANSION

Generalised HydroDynamics Equation:

\[ \partial_t \rho(z, k, t) + \partial_z \left( v_{\text{eff}}[\rho](k) \rho(z, k, t) \right) = 0 \]

- Equation invariant to \( \begin{cases} 
    z \rightarrow \alpha z \\
    t \rightarrow \alpha t 
\end{cases} \)

- With \( n(z, t) = \int_{-\infty}^{\infty} \rho(k, z, t) dk \)

For different initial sizes: the dynamics must be the same to within \( L_0/t \).

\[ \Rightarrow n(z, t) = \tilde{n}\left( \frac{z}{t}, \frac{t}{L_0} \right) \]
DYNAMICS OF 1D EXPANSION

Generalised HydroDynamics Equation:

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\[
\Rightarrow n(z, t) = \tilde{n} \left( \frac{z}{t}, \frac{t}{L_0} \right)
\]

Superimposition of profiles for different initial sizes \( L_0 \) but with the same \( t/L_0 \) ratio.
DYNAMICS OF 1D EXPANSION

Comparison between experimental profiles for different times and dynamics obtained by Generalised HydroDynamics:

$t = 5\text{ms}$

$n(z/t)/L_0$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$t = 10\text{ms}$

$n(z/t)/L_0$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$t = 30\text{ms}$

$n(z/t)/L_0$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$t = 40\text{ms}$

$n(z/t)/L_0$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$t = 50\text{ms}$

$n(z/t)/L_0$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

$-400$ $-300$ $-200$ $0$ $100$ $200$ $300$ $400$

Experimental profile

Profile from GHD with $T = 105nK$ and $\mu = 55nK$
DYNAMICS OF 1D EXPANSION

Comparison between experimental profiles for different times and dynamics obtained by Generalised HydroDynamics:

The asymptotic regime almost but not completely reached.
Domain Wall Dynamics: edge deformation dynamics of an initially homogeneous, semi-infinite gas

GHD theory was developed to solve the problem of Domain Wall Dynamics.

Bertini et al. (2016), Castro-Alvaredo et al. (2016)
**Domain Wall Dynamics Protocol**

**Domain Wall Dynamics**: edge deformation dynamics of an initially homogeneous, semi-infinite gas

**GHD theory** was developed to solve the problem of Domain Wall Dynamics.

**Generalised HydroDynamics Equation**:

\[ \partial_t \rho(z, k, t) + \partial_z \left( v_{\text{eff}}[\rho](k) \rho(z, k, t) \right) = 0 \]

For different initial sizes: the dynamics must be the same to within \( L_0/t \).

Translation-invariant system along z-axis

The edge profile is a function of \( \frac{z}{t} \Rightarrow n[\rho] \left( \frac{z}{t} \right) \)

*Bertini et al. (2016), Castro-Alvaredo et al. (2016)*
DYNAMICS

**Dynamics of the edge:**

**Experimentally** : The atoms are trapped in a quartic potential so that the quasi-homogeneous zone is large enough.

Then:
- The longitudinal **quartic** confinement is removed,
- The transverse one is maintained
**Conclusion**: after $8 \, ms$, a ballistic dynamics well described the dynamics of the edge.
Domain Wall Dynamics reconstruction

Reconstruction of the rapidities distribution:
In first approximation we suppose that we have a thermic cloud

⇒ **Fit with a Gibbs ensemble:** the $T$ temperature $\mu$ and the chemical potential are the fit parameters.

Fitting parameters:

\[
T = 450 \text{ nK} \\
\mu = 105 \text{ nK}
\]

Edge profile after $t_{\text{exp}} = 8 \text{ms}$
CONCLUSION

- Implementation of a spatial selection tool

- Longitudinal expansion of a homogeneous gas:
  - Asymptotic regime is almost reached
  - Expansion agrees with Generalized Hydrodynamics

- Domain wall dynamics protocol:
  - Ballistic evolution well observed
  - Extraction of rapidities distribution
THANK YOU FOR YOUR ATTENTION!
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Domain Wall Dynamics
- Domain Wall Dynamics protocol
- Domain Wall Dynamics reconstruction
**Outline**

**Asymptotic regime of a 1D expansion**
- Preparing initial conditions
- Radiation pressure
- 1D expansion

**Hydrodynamics for integrable systems**
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**Domain Wall Dynamics**
- Domain Wall Dynamics protocol
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DYNAMICS OF 1D EXPANSION

Generalised HydroDynamics Equation:

\[ \partial_t \rho(z, k, t) + \partial_z (\nu_{\text{eff}}[\rho](k)\rho(z, k, t)) = 0 \]

Let \( \rho(L_0, z, k, t) = \tilde{\rho}(L_0, z/L_0, k, t/L_0) \) a solution of the GHD equation

\[
\begin{align*}
  z &\rightarrow \alpha z \\
  t &\rightarrow \alpha t \\
\end{align*}
\]

\( \Rightarrow \tilde{\rho}(L_0, \alpha z/L_0, k, \alpha t/L_0) \)

is also a solution of the GHD equation.

With \( n(z, t) = \int_{-\infty}^{\infty} \rho(k, z, t)dk \)

\[ \Rightarrow n(z, t) = \tilde{n} \left( \frac{z}{t}, \frac{t}{L_0} \right) \]
Asymptotic regime of a 1D expansion
• Preparing initial conditions
• Radiation pressure
• 1D expansion

Hydrodynamics for integrable systems
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Domain Wall Dynamics
• Domain Wall Dynamics protocol
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Comparison between experimental profiles for different times and the dynamics obtained by Generalised HydroDynamics:
Comparison between experimental profiles for different times and the dynamics obtained by Generalised HydroDynamics:

The asymptotic regime is not completely reached.
Radiation pressure

Pushing away the atoms with radiation pressure:

- For one photon absorbed, $v_{\text{rec}} = \frac{\hbar k}{m}$.
- $E_{\text{rec}} = \frac{1}{2m} \left( \hbar k N_{\text{sc}}^{\text{th}} \right)^2 = \Delta E$

$\Rightarrow N_{\text{sc}}^{\text{th}} \approx 14$ photons.
RADIATION PRESSURE

Pushing away the atoms with radiation pressure:

- For one photon absorbed, $v_{\text{rec}} = \frac{\hbar k}{m}$.
- $E_{\text{rec}} = \frac{1}{2m} \left( \hbar k N_{\text{sc}}^{\text{th}} \right)^2 = \Delta E$
  $\Rightarrow N_{\text{sc}}^{\text{th}} \approx 14$ photons.

Measure of $N_{\text{exp}}^{\text{sc}}$ with fluorescence imaging:

What is the minimum power needed to remove all the atoms?

![Graph showing potential energy versus distance from the central wire](image)

- $I_{\text{sat}} = 16.6\, \text{W.m}^{-2}$
- $T_{\text{pulse}} = 20\, \mu\text{s}$
- $s = 0.16$
**Domain Wall Dynamics Reconstruction**

Reconstruction of the rapidities distribution:
In first approximation we suppose that we have a thermic cloud.

⇒ **Fit with a Gibbs ensemble**: the $T$ temperature $\mu$ and the chemical potential are the fit parameters.

*Edge profile after $t_{\text{exp}} = 8\text{ms}$*
Radiation pressure

Pushing away the atoms with radiation pressure:

- For one photon absorbed, \( n_{\text{rec}} = \frac{\hbar k}{m} \).
- \( E_{\text{rec}} = \frac{1}{2m} (\hbar k N_{\text{sc}}^{\text{th}})^2 = \Delta E \)
  \( \Rightarrow N_{\text{sc}}^{\text{th}} \simeq 14 \) photons.

**Measure of \( N_{\text{sc}}^{\text{exp}} \) with fluorescence imaging:**

**Experimental parameters**: \( T_{\text{pulse}} = 20 \mu s, s = 0.16 \)

**Fluorescence measurement**: \( N_{\text{sc}}^{\exp} \simeq 15 \pm 5 \) scattered photons / atoms
Take into account the level structure, the magnetic field and the beam polarisation, \( \sigma = \alpha \sigma_0 \) with \( \sigma_0 = \frac{3 \lambda^2}{2\pi} \).

**Experimentally** : \( \alpha = 0.25 \)

**Measure** of \( N_{sc}^{\text{exp}} \) with fluorescence imaging:

**Experimental parameters** : \( T_{\text{pulse}} = 20 \mu s, s = 0.18 \)

**Fluorescence measurement** :

\[ N_{sc}^{\text{exp}} \simeq 15 \pm 5 \text{ scattered photons / atoms} \]
LIMITATIONS

Limitations:

• Image of an edge is not perfect

What do we see:
Limitations:

- Image of an edge is not perfect

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Limitations:

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- Image of an edge is not perfect
- Photon reabsorption in the remaining cloud
- Diffusion of light by the optical system can lead to absorption photon in the remaining cloud

What do we see:

![Graph showing relative intensity vs. position (μm)]
**Limitations**

**Limitations:**

- Image of an edge is not perfect
- Photon reabsorption in the remaining cloud
- Diffusion of light by the optical system can lead to absorption photon in the remaining cloud

**What do we see:**

- The density of remaining atoms doesn’t change

![Graph showing relative intensity vs. position](graph1.png)

![Graph showing density vs. position](graph2.png)
**Generalised HydroDynamics (GHD)**

**General HydroDynamics (GHD):** An integrable system can locally be described by the spatial resolved rapidity distribution $\rho(z, k, t)$.

- Conservation of the rapidity distribution on a $\delta k$ slice:

  $$\partial_t \rho(z, k, t) + \partial_z \left( v_{\text{eff}}[\rho](k) \rho(z, k, t) \right) = 0$$

  with

  $$v_{\text{eff}}[\rho](k) = k - \int_{-\infty}^{+\infty} dk' \Delta(k - k') \left[ v_{\text{eff}}[\rho](k') - v_{\text{eff}}[\rho](k) \right]$$

  and

  $$\Delta(k) = \frac{2g}{g^2 + k^2}$$

Bertini et al. (2016), Castro-Alvaredo et al. (2016)
Limitations:

• Image of an edge is not perfect

• Photon reabsorption in the remaining cloud

• Diffusion of light by the optical system can lead to absorption photon in the remaining cloud

What do we see:

• The density of remaining atoms doesn’t change

• Phonon temperature doesn’t change before and after the selection

• Transverse size after a time of flight doesn’t change.
Domain Wall Dynamics Reconstruction

Occupation factor $\nu[\rho](k) \in [0, 1]$:

$$\partial_t \rho(z, k, t) + \partial_z \left( v_{\text{eff}}[\rho](k) \rho(z, k, t) \right) = 0 \Rightarrow n(z, t) = \int_{-\infty}^{\infty} \rho(k, z, t) dk$$

$$\Rightarrow \partial_t \nu[\rho](z, k, t) + v_{\text{eff}}[\rho](k) \partial_z \left( \nu(z, k, t) \right) = 0 \quad \text{Conservation along a trajectory}$$

Occupation factor at $t = 0$: there is no dependance on $z$.

$$\nu(k, z, t = 0) = \begin{cases} 
\nu(k) & \text{if } |z| < z_0 \\
0 & \text{otherwise.}
\end{cases}$$
Domain Wall Dynamics Reconstruction

Occupation factor $\nu[\rho](k) \in [0, 1]$: 

$$\partial_t \rho(z, k, t) + \partial_z (v_{\text{eff}}[\rho](k)\rho(z, k, t)) = 0 \Rightarrow n(z, t) = \int_{-\infty}^{\infty} \rho(k, z, t) dk$$

$$\Rightarrow \partial_t \nu[\rho](z, k, t) + v_{\text{eff}}[\rho](k)\partial_z (\nu(z, k, t)) = 0 \quad \text{Conservation along a trajectory}$$

Occupation factor $\nu[\rho]$ at $t > 0$: The edge dynamic is obtained with the effective velocities $v_{\text{eff}}[\nu^*](k^*)$ where $\nu^*(k)$ is the truncated initial occupation factor.

For each $k^*$, $v_{\text{eff}}[\nu^*](k^*)$ doesn’t depend on time $\Rightarrow \text{The edge profile is a function of } n[\rho](z, t)$. 

\[ \nu^*(k) = \begin{cases} \nu(k) & \text{if } k > k^* \\ v_{\text{eff}}(k) & \text{otherwise.} \end{cases} \]
Reconstruction of the occupation factor directly with the density profile

\[ \frac{d\nu}{dk} = D'[\nu] \frac{dn}{dv_{\text{eff}}} + A[\nu] D[\nu] \frac{d^2n}{dv_{\text{eff}}^2} \]

Little by little, in a discretized space

With \( \nu_N = 0, \nu_{N-1} = 0 \) and

\( \nu_{\text{eff}}^N = \hbar k_N/m, \nu_{\text{eff}}^{N-1} = \hbar k_{N-1}/m \)
Occupation factor $\nu[\rho](k) \in [0, 1]$:

$$\partial_t \rho(z, k, t) + \partial_z (v_{\text{eff}}[\rho](k)\rho(z, k, t)) = 0 \Rightarrow n(z, t) = \int_{-\infty}^{\infty} \rho(k, z, t) dk$$

$$\Rightarrow \partial_t \nu[\rho](z, k, t) + v_{\text{eff}}[\rho](k)\partial_z (\nu(z, k, t)) = 0 \quad \text{Conservation along a trajectory}$$

$\nu(k, z, t = 0) = \begin{cases} \nu(k) & \text{if } |z| < z_0 \\ 0 & \text{otherwise} \end{cases}$
**DYNAMICS OF 1D EXPANSION**

Occupation factor $\nu[\rho](k) \in [0, 1]$:

$$\partial_t \rho(z, k, t) + \partial_z \left( v_{\text{eff}}[\rho](k) \rho(z, k, t) \right) = 0 \Rightarrow n(z, t) = \int_{-\infty}^{\infty} \rho(k, z, t) \, dk$$

$$\Rightarrow \partial_t \nu[\rho](z, k, t) + v_{\text{eff}}[\rho](k) \partial_z \left( \nu(z, k, t) \right) = 0$$

*Conservation along a trajectory*

**Occupation factor at $t > 0$**: the edges dynamic are obtained with the effective velocities $v_{\text{eff}}[\nu^*](k^*)$ where $\nu^*$ is the initial occupation factor truncated between $k_{\text{min}}$ and $k_{\text{max}}$.

$$\nu^*(k) = \begin{cases} 
\nu(k) & \text{if } k_{\text{min}} < k < k_{\text{max}} \\
0 & \text{otherwise.}
\end{cases}$$
Objective: Measuring experimentally the rapidity distribution for a qBEC and recovering the half-circle distribution.

**Lieb parameter:** $\gamma = \frac{mg}{n}$ with $g$ the 1D interaction parameter and $n$ the density:

- $\gamma \to \infty$: hardcore regime
- $\gamma \to 0$: 1D quasi-condensate (qBEC)

**Half-circle equation**

$$\rho(k)^2 \sim \left(1 - \left(\frac{k}{K}\right)^2\right), \gamma \to 0$$
**Context**

**Lieb parameter:** \( \gamma = \frac{mg}{n} \) with \( g \) the 1D interaction parameter and \( n \) the density:

- \( \gamma \rightarrow \infty \): hardcore regime
- \( \gamma \rightarrow 0 \): 1D quasi-condensate (qBEC)

For \( T = 0 \) and \( \gamma \rightarrow 0 \):

\[
\rho(k)^2 \sim \left(1 - \left(\frac{k}{K}\right)^2\right)
\]

**Half-circle equation**

With \( K = 2\sqrt{gn} \).

“Exact analysis of an interacting Bose Gas”, Lieb & Liniger, 1963
1D EXPANSION

Rescaled profile density:
- We look at the profile densities after 40 ms of expansion
- Profiles rescaled with the time

![Graph showing rescaled densities with time]

-$L_0 = 54 \mu m$

An asymptotic regime seems to be reached, it corresponds to the rapidities distribution

[Graph showing half circle distribution expected for the ground state]
1D EXPANSION

For different initial sizes:
- \( t_{\text{asym}}/L_0 \) is constant
- Asymptotic regime seems to be reached
- The rapidities distribution obtained for \( L_0 = 93\mu m \) is in agreement with the other initial sizes
Approximation: the gas is thermic

\[ T = 150 \, nK, \, \mu = 70 \, nK \]

Density profile of the cloud in the harmonic trap before the selection:

- \( \omega_\parallel = 9Hz \), \( \omega_\perp = 4.1KHz \)

1D Expansion

Expected profile for \( T \to 0 \)
The two protocols concern homogeneous bose gas! Initially, the gas is in a quadratic / quartic trap.

- Implementation of a selection spatial tool
- Use of the radiation pressure

Implementation of a DMD to shape the beam.
Homogeneous 1D Bose gas with contact repulsive interactions:

- Lieb Liniger Hamiltonian: \( \hat{H} = -\frac{\hbar^2}{2m} \sum_i^N \frac{\partial^2}{\partial z_i^2} + g \sum_{i<j} \delta(z_i - z_j) \)

**Integrable system:** the eigenstates are known
Integrable system: the eigenstates of $\hat{H}$ are the Bethe Ansatz states $|k_1, k_2, ..., k_N\rangle$ labelled by N numbers $(k_1, k_2, ..., k_N)$ called the rapidities.

Wavefunction: $\psi\{k_i\}(z_1 < ... < z_N) = \sum_{\sigma} A_\sigma e^{(i(k_{\sigma(1)}z_1 + ... + k_{\sigma(N)}z_N))}$

Rapidities distribution $\rho(k)$: $L \rho(k) \delta k$ is the number of rapidities in the $[k, k + \delta k]$ range.

⚠️ $\rho(k)$ is conserved during the out-of-equilibrium dynamics of the system.

The relaxed state is described by the rapidities distribution $\rho(k)$!
**Domain Wall Dynamics protocol**

**Domain Wall Dynamics**: edge deformation dynamics of an initially homogeneous, semi-infinite gas

**GHD theory** was developed to solve the problem of Domain Wall Dynamics.

**Generalised HydroDynamics Equation**:

\[
\partial_t \rho(z_0, k, t_0) + \partial_z \left( v_{\text{eff}}[\rho](k) \rho(z_0, k, t_0) \right) = 0
\]

The edge profile is a function of \( \frac{z}{t} \Rightarrow n[\rho] \left( \frac{z}{t} \right) \)

**Experimentally**: The atoms are trapped in a quartic potential so that the quasi-homogeneous zone is large enough.
Occupation factor $\nu[\rho](k) \in [0, 1]$: 
\[
\partial_t \rho(z, k, t) + \partial_z \left( v_{\text{eff}}[\rho](k) \rho(z, k, t) \right) = 0 \\
\Rightarrow \partial_t \nu[\rho](z, k, t) + v_{\text{eff}}[\rho](k) \partial_z \left( \nu(z, k, t) \right) = 0 \quad \text{Conservation along a trajectory}
\]

\begin{align*}
\nu(k, z, t = 0) = \begin{cases} 
\nu(k) & \text{if } |z| < z_0 \\
0 & \text{otherwise.}
\end{cases} \\
\nu^*(k) = \begin{cases} 
\nu(k) & \text{if } k_{\text{min}} < k < k_{\text{max}} \\
0 & \text{otherwise.}
\end{cases}
\end{align*}

\[n(z, t) = \int_{-\infty}^{\infty} \rho(k, z, t) dk\]
**Obtaining a 1D Bose gaz**

**How do we produce a 1D Bose gas?**

*Atoms used*: Rubidium 87 in the $|F = 2, m_F = 2\rangle$ atomic state.

Atoms are trapped by a magnetic field created with wires deposited on a chip.

**Potential felt by the atoms** under a $\vec{B}$ magnetic field:

$$V = g_F m_F \mu_B |\vec{B}|$$

**If** $k_B T, \mu \ll \Delta E_\perp$, the gas is frozen in the transverse direction!

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**How to reach the 1D regime?**

**3D harmonic oscillator:**
- longitudinal trapping $\omega_\parallel$
- transverse trapping $\omega_\perp$

![Diagram](image)

Energy

$\Delta E_\perp = \hbar \omega_\perp$

Ground state

Excited states

Energy levels

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The atom chip
**PREPARING INITIAL CONDITIONS**

**Atom chip experiment:** the atoms magnetically trapped thanks to micro-wires deposited on a chip.

- \( N_{\text{atoms}} \sim 10000 \)
- \( \omega_\perp = 2.5 \text{ KHz} \)
- \( \omega_\parallel = 9.0 \text{ Hz} \)

**Longitudinally trapping:**

\[
V(z) = \sum_{i=1}^{4} a_i z^i 
\]

In the experiment, we are working with harmonic and quartic potential.
**OBTAINING A 1D BOSE GAZ**

The three wires: producing a quadrupole with a zero magnetic field above the central wire: *transverse confinement*

The ‘D’ and ‘d’ wires: creating a *longitudinal confinement.*

\[ V(z) = a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + \ldots \]

- Independent control of the longitudinal coupling and the transverse one
  - \( \Delta E_\perp \propto \omega_\perp = 2.5 \text{ KHz} \)
  - Typical longitudinal quadratic potential:
    - \( \Delta E_\parallel \propto \omega_\parallel = 9 \text{ Hz} \)
Yang-Yang entropy:

\[ S_{YY}[\rho] \simeq \int_{-\infty}^{\infty} \left( \rho_s \log(\rho_s) - \rho \log(\rho) - (\rho_s - \rho) \log(\rho_s - \rho) \right) \, dk \]

Construction of conserved quantities:

\[
\begin{align*}
Q_0 &= \int \rho(k) \, dk \\
Q_1 &= \int k \rho(k) \, dk \\
\vdots \\
Q_i &= \int \frac{k^i}{i!} \rho(k) \, dk
\end{align*}
\]

⇒ The moments of the rapidities distribution are conserved quantities
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Yang, Yang (1969)

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\[ \Rightarrow \text{The moments of the rapidities distribution are conserved quantities} \]

Entropy maximisation:

\[ \frac{\delta}{\delta \rho} \left( S_{YY}[\rho] - \sum_i \lambda_i Q_i \right) = 0 \]

Lagrange multipliers

Thermic distribution:

\[ \sum_i \lambda_i Q_i = \lambda_0 Q_0 + \lambda_2 Q_2 = -\frac{\mu}{T} N + \frac{E}{T} \]

\[ Q_0 = \int \rho(k) \, dk = N \Rightarrow \lambda_0 = -\frac{\mu}{T} \]

\[ Q_2 = \int \frac{k^2}{2} \rho(k) \, dk = E \Rightarrow \lambda_2 = \frac{1}{T} \]
Rescaled profile density:
- We look at the profile densities after 30ms of expansion
- Profiles rescaled with the time

The asymptotic regime seems to be reached, a rapidity distribution can be fitted
⇒ Good shape but high temperature?
**Occupation factor**

**System:**
- $N$: atoms number
- $L$: size of the system
- Periodic boundary conditions

**Bethe equations:**

\[
\frac{L}{2\pi} \left[ k_i + \frac{1}{L} \sum_{j \neq i} 2\arctan \left( \frac{k_i - k_j}{g} \right) \right] = I_i
\]

with $i \in [1, N]$

- $I_i \in \mathbb{Z}$ if $N$ is odd, $I_i \in \mathbb{Z}/2$ if $N$ is even.
- The $I$'s are called the Bethe integers.
- $g$ is the strength repulsion term.
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- The $I$'s are called the Bethe integers.
- $\varphi$ is the strength repulsion term.

$$\left\{ \begin{array}{l}
L \rho_s(k) \, dk = dI \\
L \rho(k) \, dk : \text{number of rapidities in } [k_i, k_i + dk_i]
\end{array} \right.$$  

**Occupation factor:**
$$\nu[\rho](k) = \frac{\rho(k)}{\rho_s(k)} \in [0, 1]$$
Occupation factor: \( \nu_{[\rho]}(k) = \frac{\rho(k)}{\rho_s(k)} \in [0, 1] \)

- Relation between the rapidity distribution and the occupation factor:
  \[
  2\pi \rho(k) = \nu_{[\rho]}(k) + \int_{-\infty}^{\infty} \Delta(k - k') \nu_{[\rho]}(k) \rho(k') dk' \quad \text{with} \quad \Delta(k) = \frac{2g}{g^2 + k^2}
  \]

\[
\frac{N}{L} = \int_{-\infty}^{\infty} \rho(k) dk
\]

- New hydrodynamics equation on the occupation factor:
  \[
  \partial_t \rho(z, k, t) + \partial_z (v_{\text{eff}}[\rho](k) \rho(z, k, t)) = 0
  \]

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Conservation along a trajectory
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Conservation along a trajectory

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Yang, Yang (1969)
Change of the height of the spatial selection beam:

Density profile of the cloud in the harmonic trap before the selection

The rapidity distribution is obtained with entropy maximization + thermic distribution ($T$ and $\mu$ are fit parameters).

$$\Rightarrow T = 200 \ nK, \ \mu = 80 \ nK$$
High temperatures

Change of the height of the spatial selection beam:

- Height = 10μm
  - $L_0 = 75μm$

- Height < 5μm
  - $L_0 = 75μm$

$\tau_{exp} = 40 ms$

Density profile of the cloud in the harmonic trap before the selection

The rapidity distribution is obtained with entropy maximization + thermic distribution (T and $\mu$ are fit parameters).

$\Rightarrow T = 200\ nK, \ \mu = 80\ nK$

Diffusion of photons with the optical system. Reducing this effect gives reasonable temperature.
Time of flight: longitudinal and transverse confinement are removed.
**Time of flight**:

Longitudinal and transverse confinement are removed.

**Density distribution**:

\[ \rho(z) = \delta \rho(z) + \langle \rho(z) \rangle \]

- Density fluctuations
- Mean profile

**Quasi BEC (in situ)**:

- Small density fluctuations
  \[ \Rightarrow \rho(z)_{t=0} \approx \langle \rho(z) \rangle_{t=0} = n_0(z) \]

- Phase fluctuations
  \[ \Rightarrow \langle |\theta(z) - \theta(0)|^2 \rangle = \frac{2|z|}{l_c}, l_c = \frac{2\hbar^2 n_0}{mk_B T} \]
  \[ \Rightarrow \langle |\rho(q)|^2 \rangle = 4n_0^2 \langle \theta_q^2 \rangle \sin^2 \left( \frac{\hbar q^2 \tau_{tof}}{2m} \right) e^{-\sigma^2 q^2} \]
  with \[ \langle \theta_q^2 \rangle = \frac{mk_B T}{\hbar^2 n_0 q^2} \]
**Time of Flight**

**Time of flight** : longitudinal and transverse confinement are removed.

**Density distribution** : \( \rho(z) = \delta \rho(z) + \langle \rho(z) \rangle \)

Density fluctuations ➔ Mean profile

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- phase fluctuations
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with \( \langle \theta_q^2 \rangle = \frac{mk_BT}{\hbar^2 n_0 q^2} \)
To trap the atoms, one needs:

- A current $I$ through a wire

The potential felt by the atoms under a $\vec{B}$ magnetic field is given by:

$$V = g_F m_F \mu_B |\vec{B}|$$
To trap the atoms, one needs:

- A current $I$ through a wire
- A homogeneous transverse field $B_{\perp}$ to have a minimum of potential

\[ d = \frac{\mu_0 I}{2\pi B_{\perp}} \]

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\text{Potential felt by the atoms under a } \vec{B} 	ext{ magnetic field:} \\
V &= g_F m_F \mu_B |\vec{B}| 
\end{align*} \]
To trap the atoms, one needs:

- A current $I$ through a wire
- A homogeneous transverse field $B_\perp$ to have a minimum of potential
- A longitudinal field $B_z$ to have a non-zero potential

Potential felt by the atoms under a $\vec{B}$ magnetic field:

$$V = g_F m_F \mu_B |\vec{B}|$$
Measurement of rapidity distribution for a homogeneous gas / spatially resolved rapidity distribution

- Longitudinal potential
- Selection spatial impulsion
- Transverse confinement
- Focalisation impulsion
- Image
- 1D expansion
- $n(k)$ measurement