

Kardar Parisi Zhang universal scaling in the coherent emission of polariton condensates

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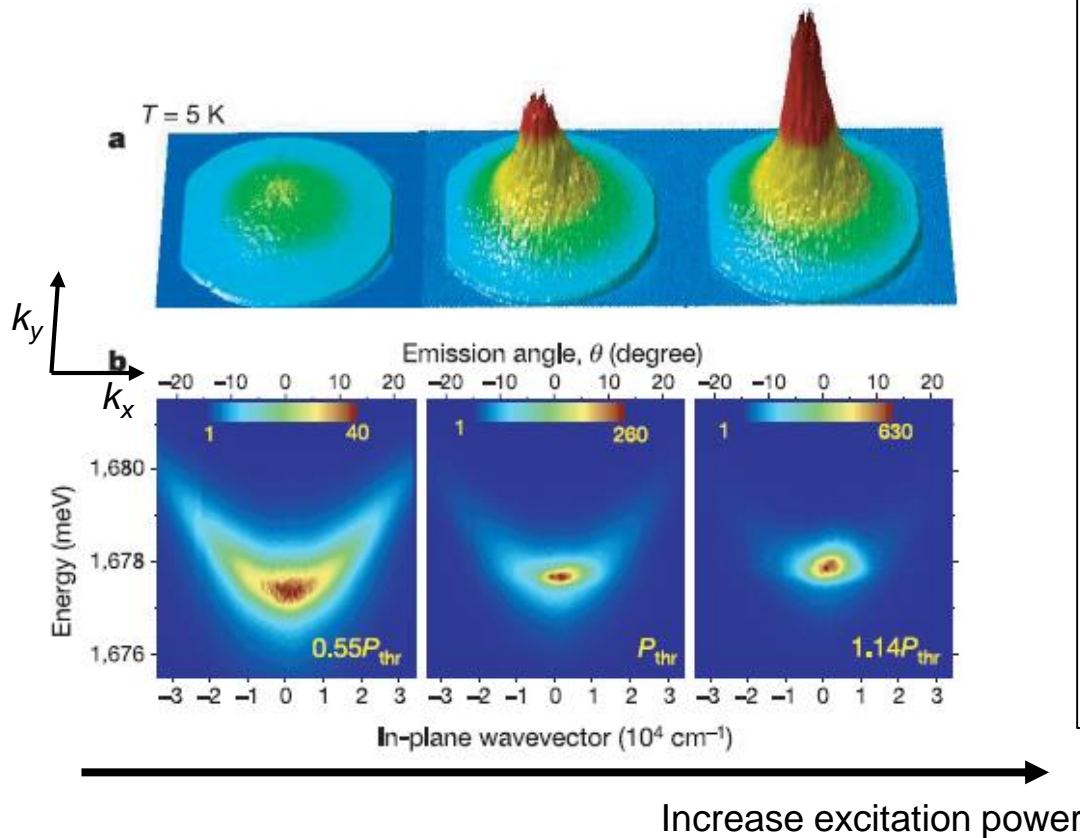
I. Carusotto
I. Amelio



M. Wouters

Bose-Einstein condensation of exciton polaritons

J. Kasprzak¹, M. Richard², S. Kundermann², A. Baas², P. Jeambrun², J. M. J. Keeling³, F. M. Marchetti⁴, M. H. Szymańska⁵, R. André¹, J. L. Staehli², V. Savona², P. B. Littlewood⁴, B. Deveaud² & Le Si Dang¹



Similarities with atomic BEC

BUT

Driven dissipative system
Out of equilibrium

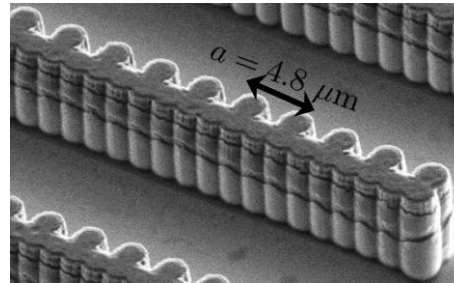
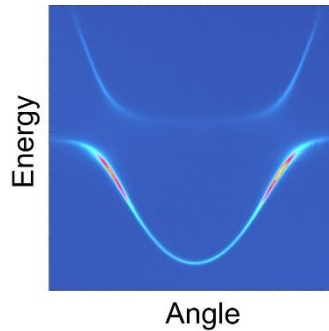
\Rightarrow Different universality class
than their equilibrium
counterpart

Kasprzak *et al.* Nature, **443**, 409 (2006)

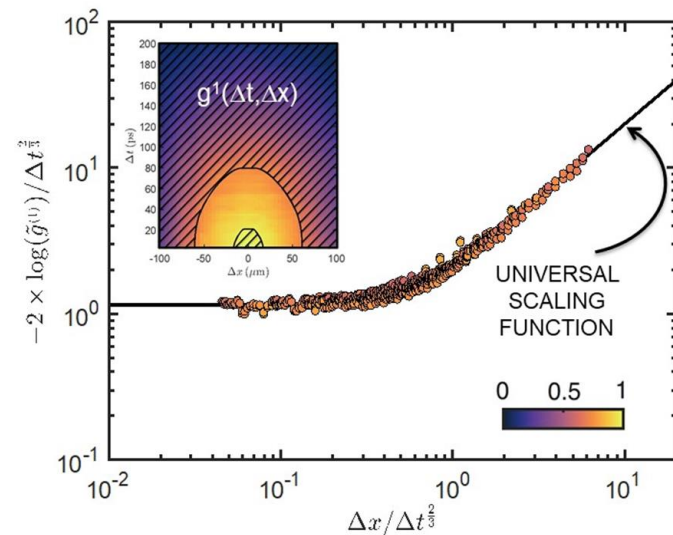
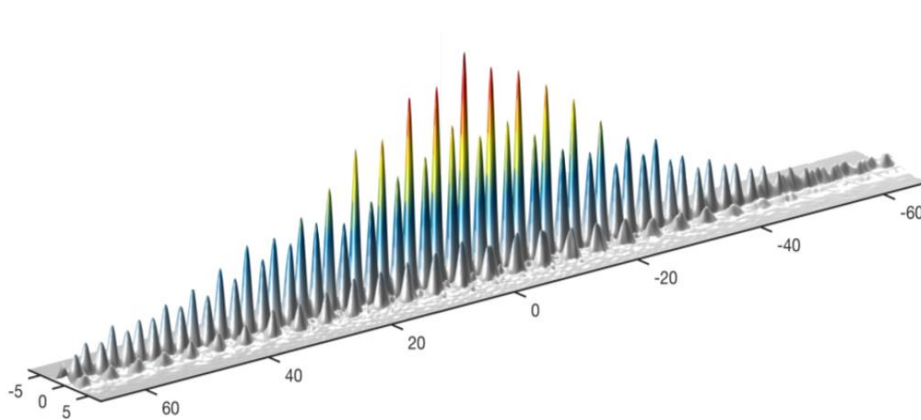
Also H. Deng *et al.* Science (2002), R. Balili *et al.*, Science (2007),.....

Outline of the talk

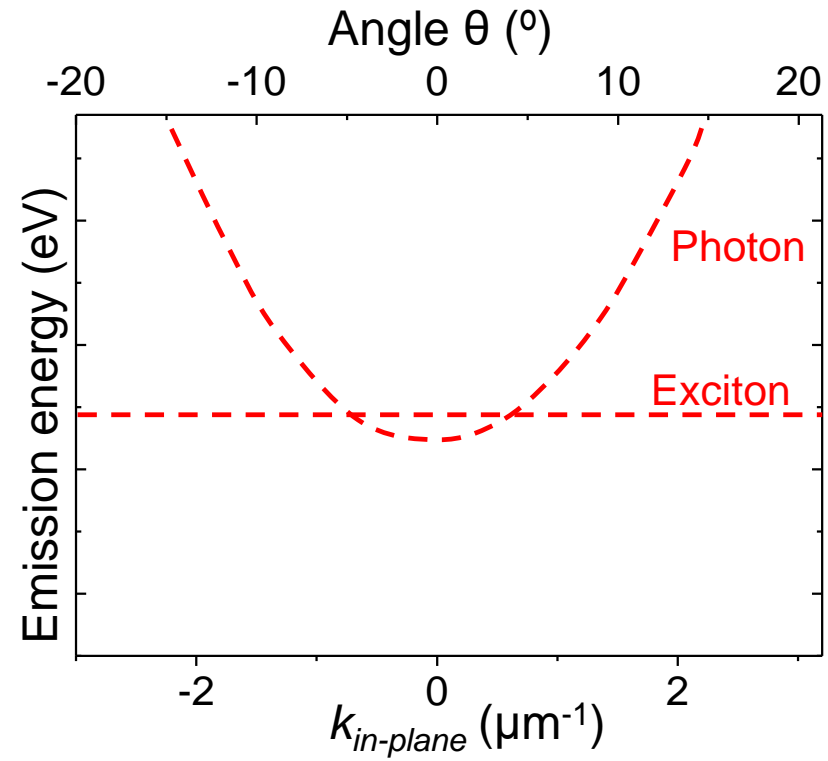
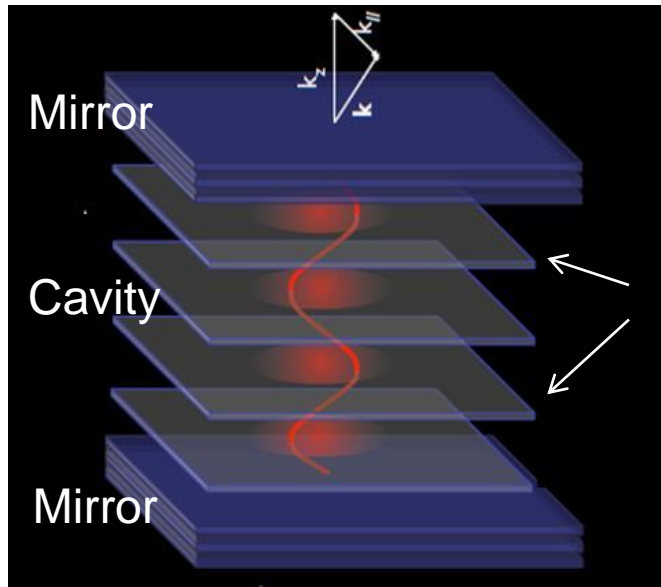
- Introduction : synthetic polariton matter



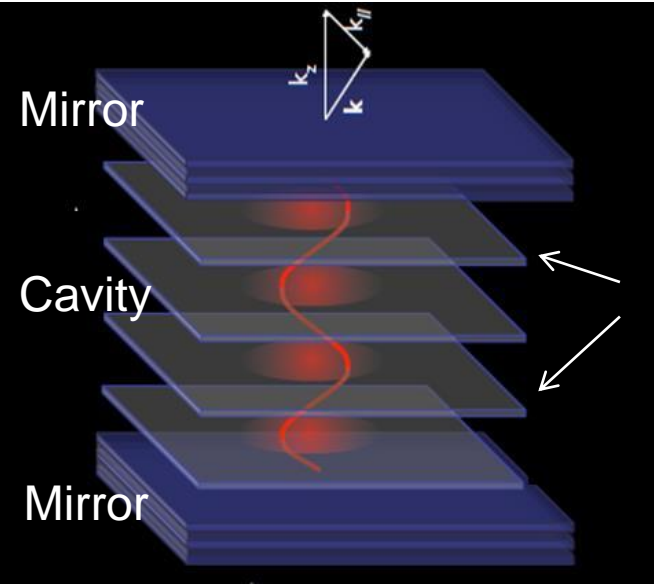
- Polariton condensates belong to the Kardar Parisi Zhang universality class



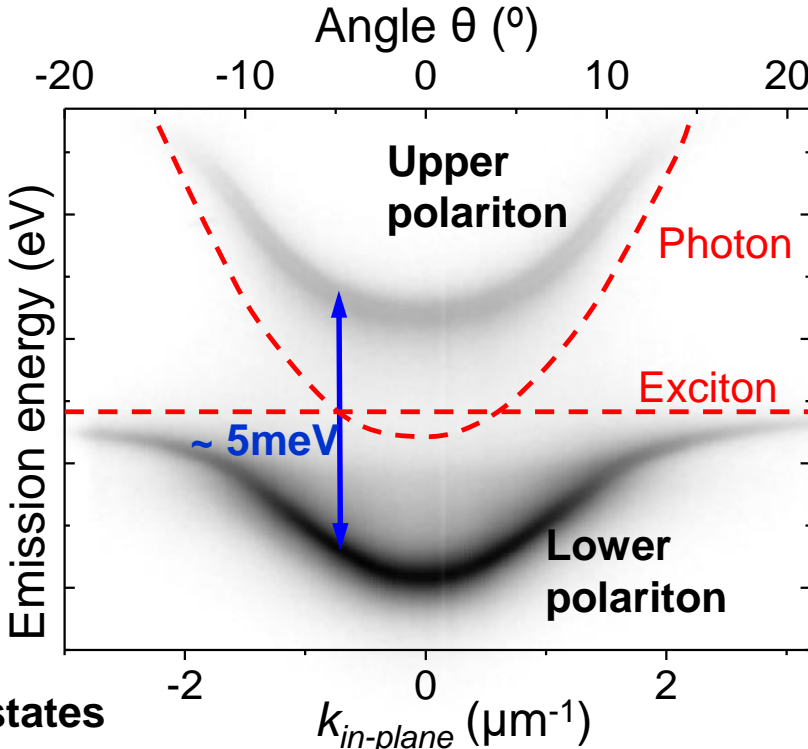
Microcavity polaritons



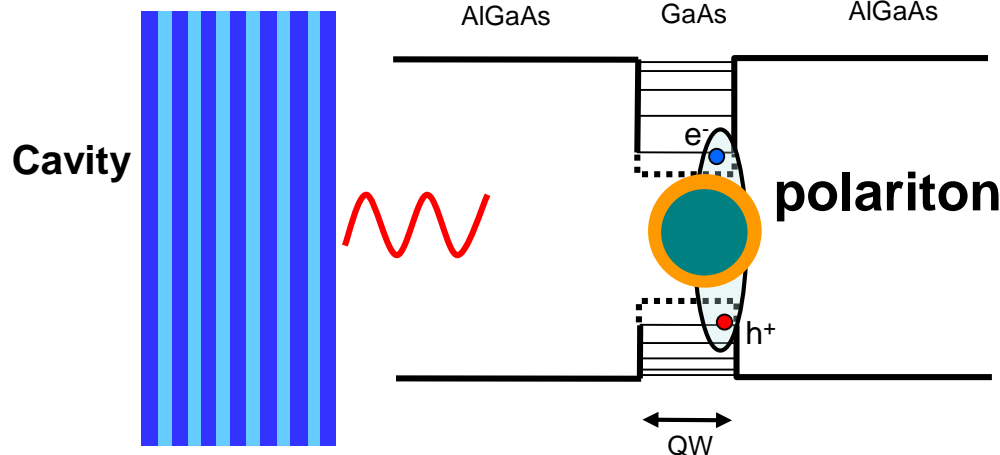
Microcavity polaritons



Quantum wells



Microcavity polaritons : mixed exciton-photon states

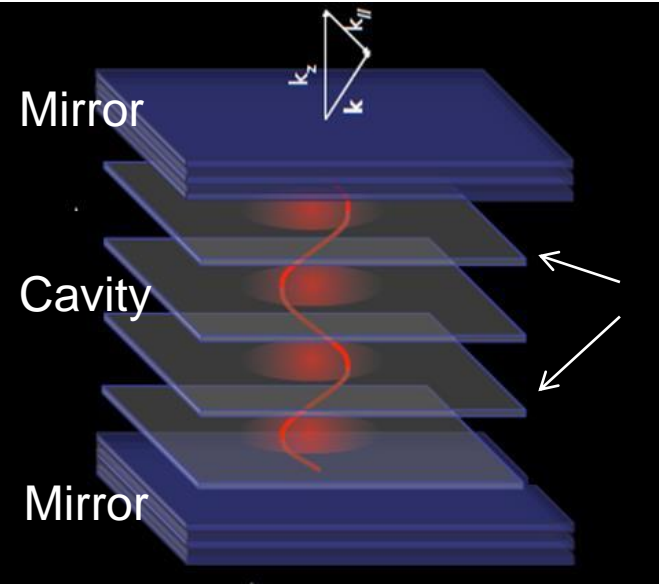


Courtesy D.Sanvitto

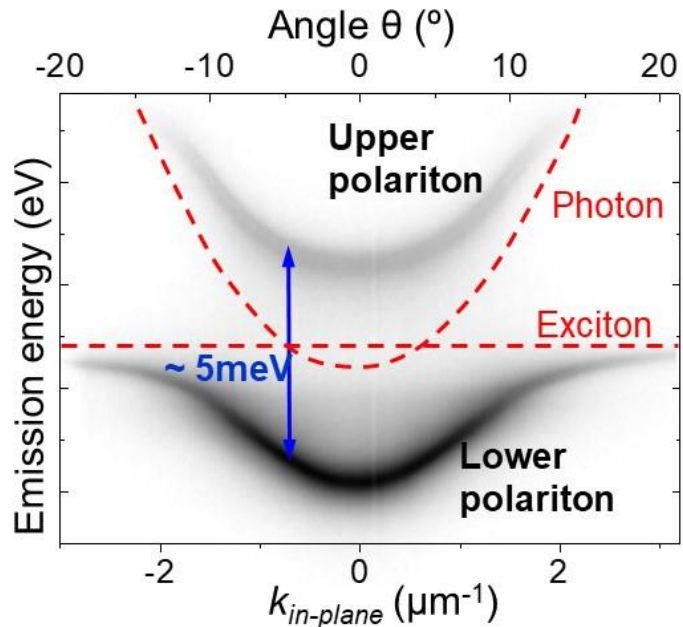


Claude Weisbuch
PRL **69**, 3314 (1992)

Microcavity polaritons



Quantum wells

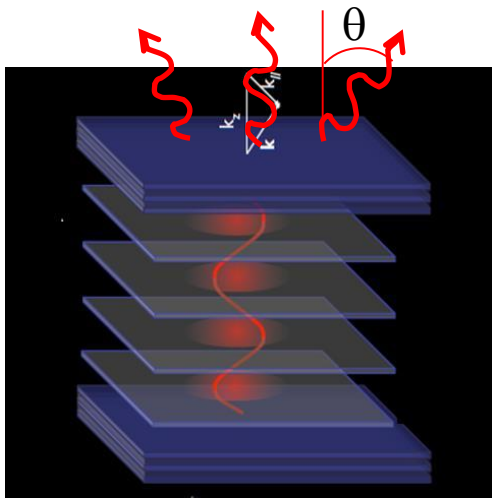


Microcavity polaritons : mixed exciton-photon states

Properties $|1LP_{\mathbf{k}}, 0_{\mathbf{k}}\rangle = (\underbrace{\cos \theta_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger}_{\text{Photons}} + \underbrace{\sin \theta_{\mathbf{k}} \hat{b}_{\mathbf{k}}^\dagger}_{\text{Excitons}}) |0_{\mathbf{k}}, 0_{\mathbf{k}}\rangle$

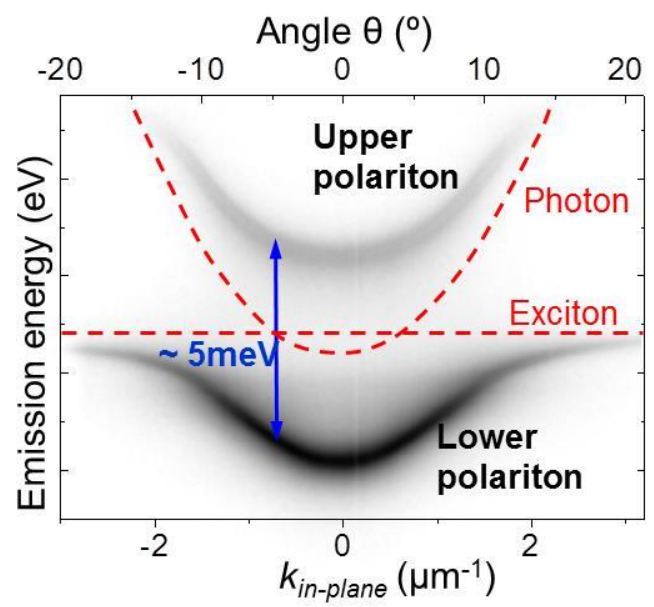
- Photonic component \Rightarrow Confinement in microstructures
Dissipation
- Excitonic component \Rightarrow • Interactions - $\chi^{(3)}$ (dominated by exchange)
• Gain (lasing)
• Sensitivity to magnetic field

Probing polariton states

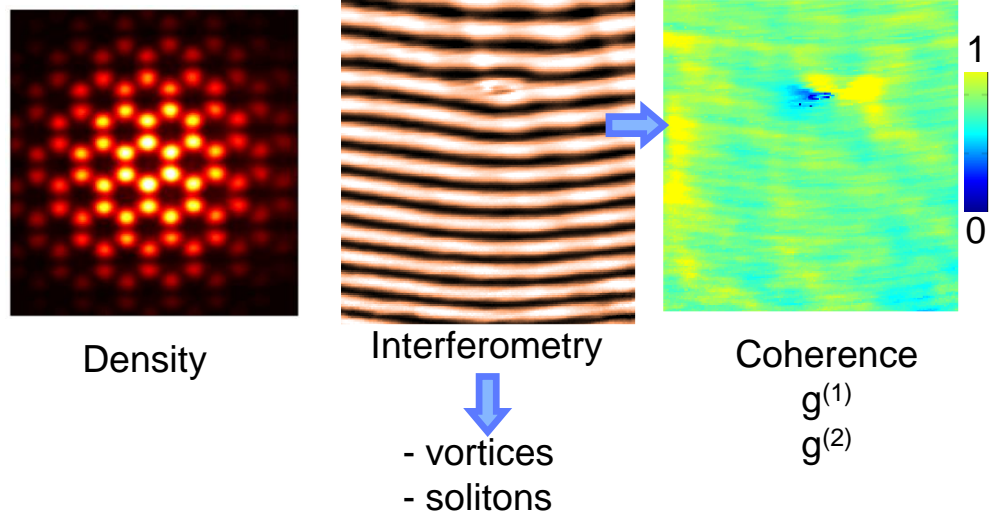


$$k_{||} = \omega/c \sin(\theta)$$

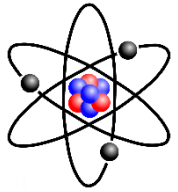
Imaging of k-space



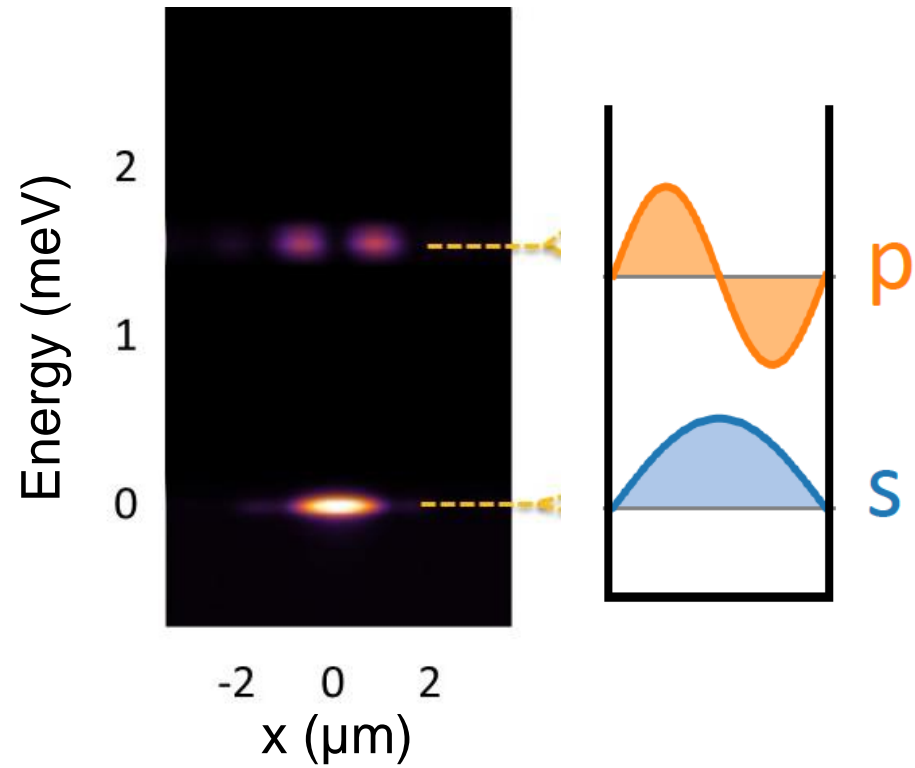
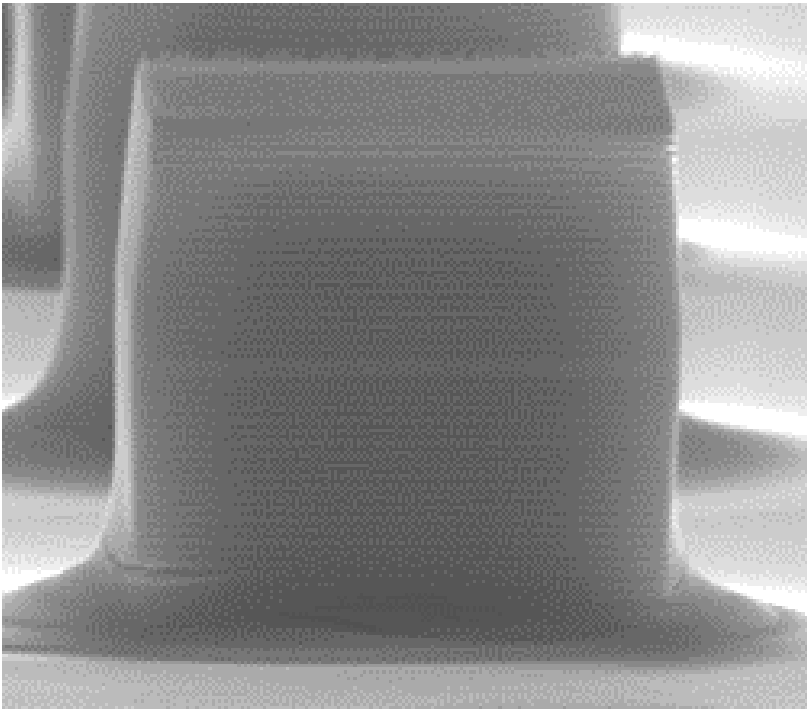
Imaging of real space



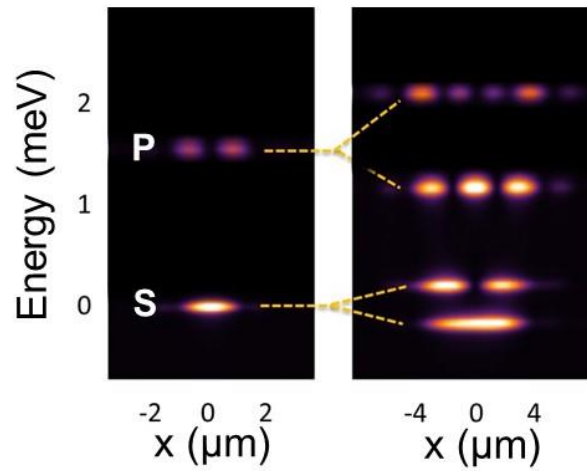
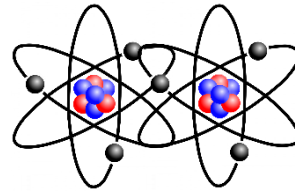
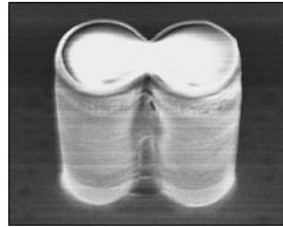
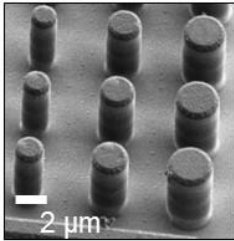
Lattices of coupled micropillars



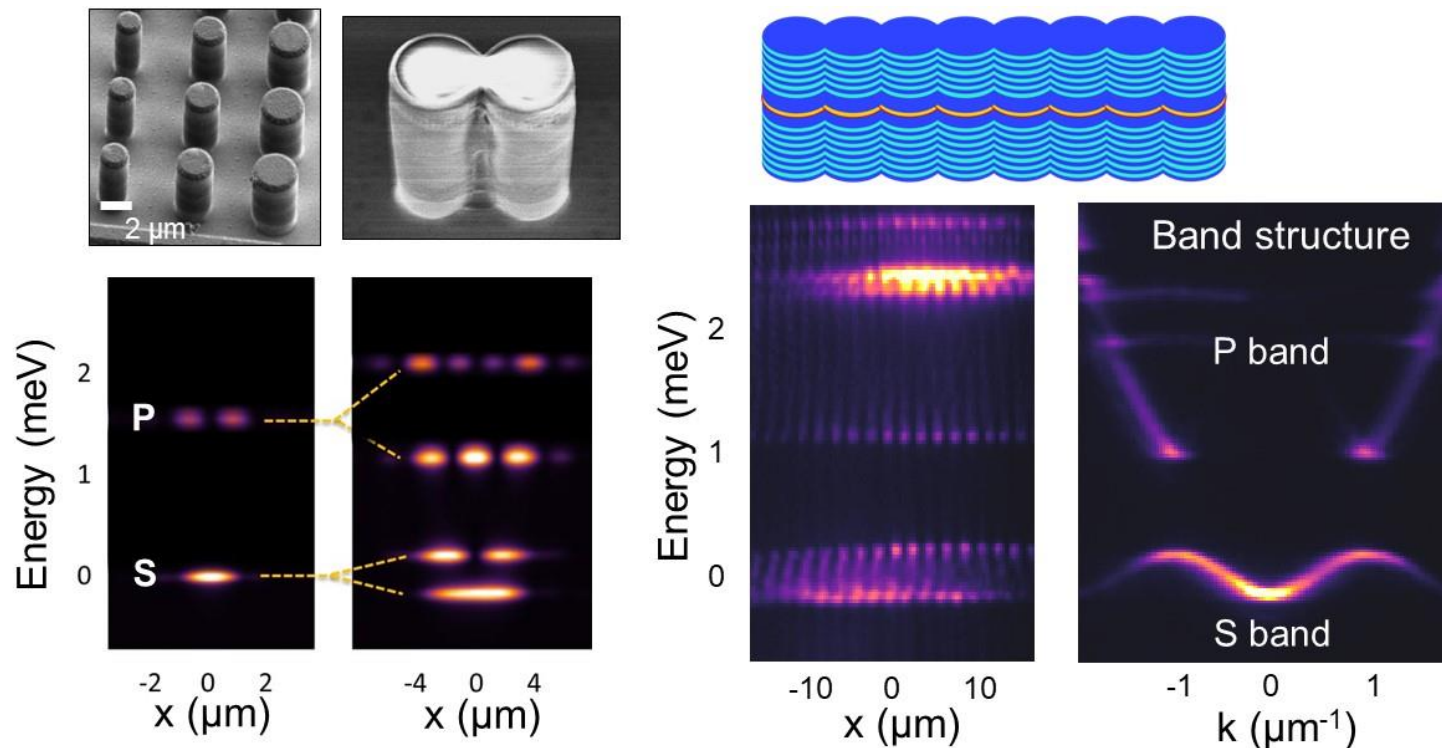
Building block



Lattices of coupled micropillars



Lattices of coupled micropillars



Correspondance : Wavefunction = electric field
Spin = Polarisation

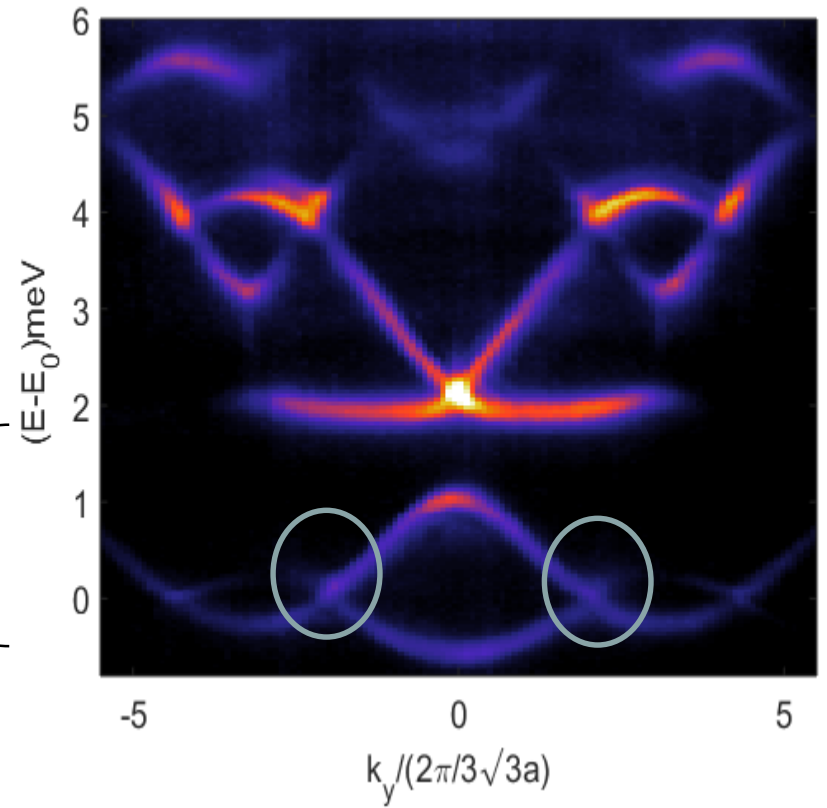
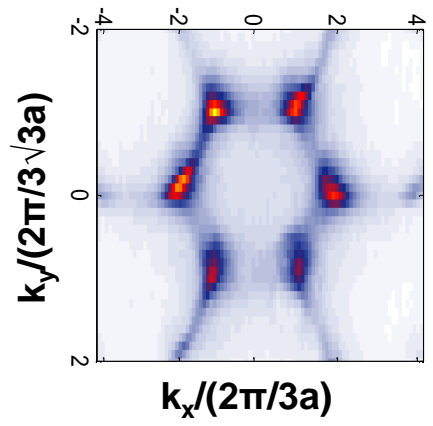
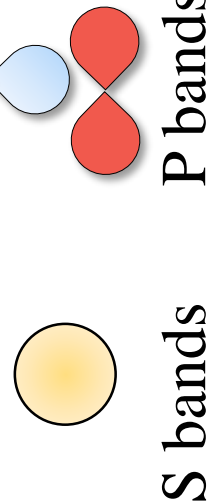
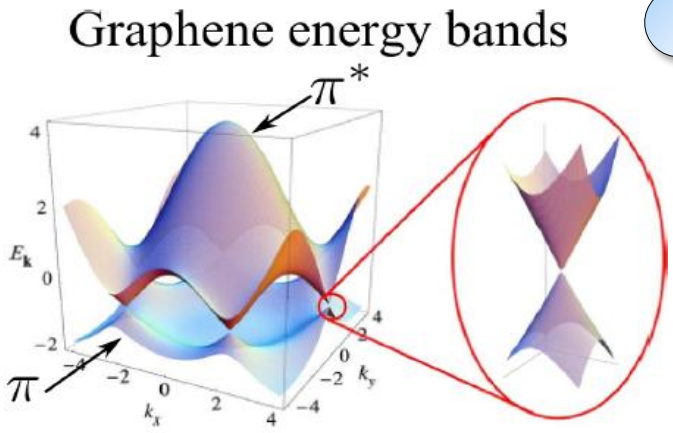
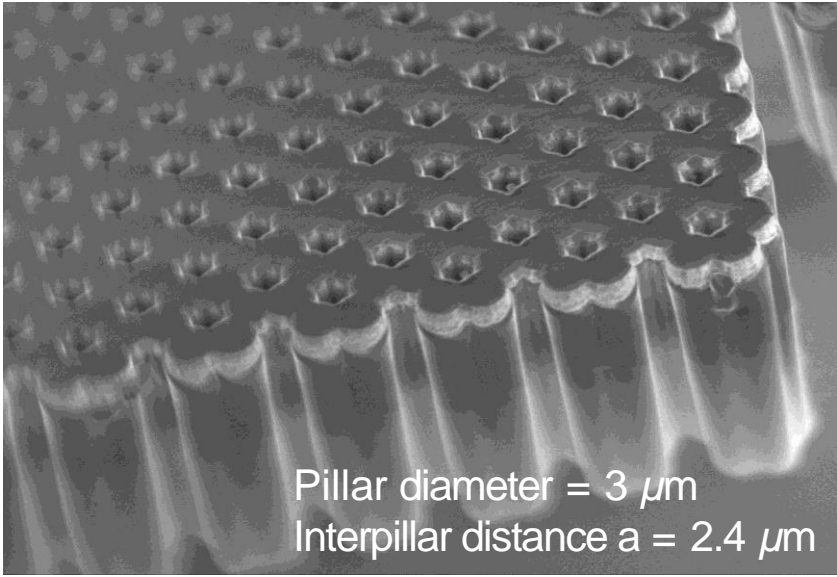
$$i\hbar\dot{\psi}_n = \left(\hbar\omega_n - i\frac{\gamma_n}{2} \right) \psi_n + U |\psi_n|^2 \psi_n - J\psi_m + F_n e^{-i\omega t}$$

C. Ciuti & I. Carusotto, Rev. Mod. Phys. **85**, 299 (2013)

Compte Rendus Physique Vol. 17, Issue 8, Pages 805-956 (2016)

Physique des polaritons: Edité par A. Amo, J. Bloch and I. Carusotto

Polariton honeycomb lattice

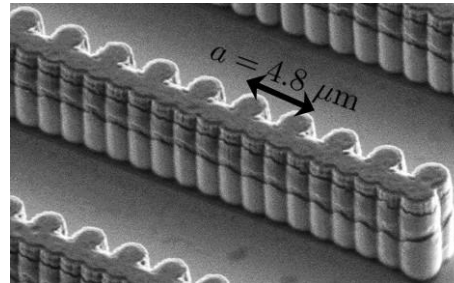
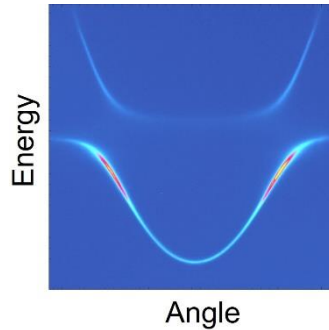


Castro Neto et al., Rev. Mod. Phys. 81 (2009)

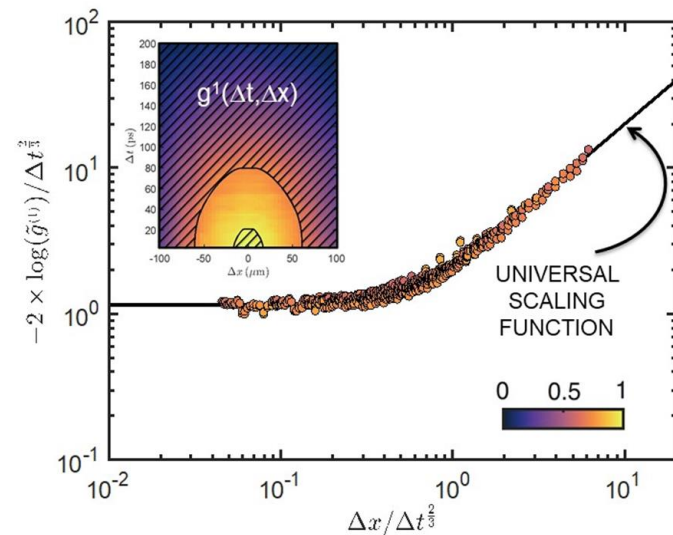
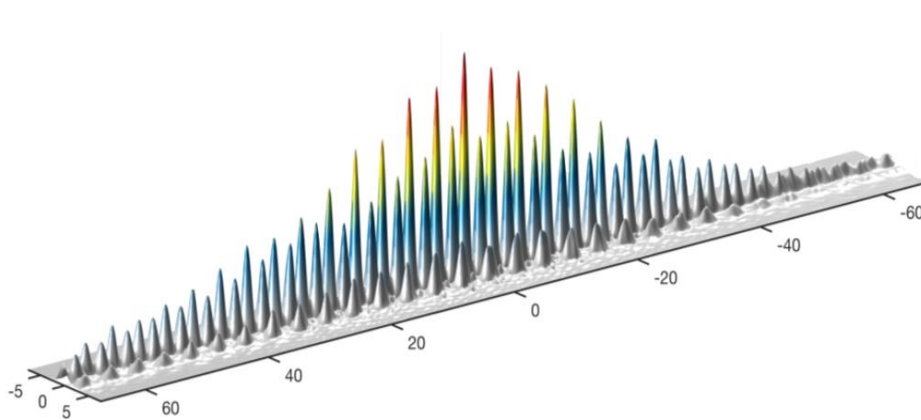
Jacqmin *et al.*, PRL **112**, 116402 (2014)
 M. Milićević et al., Phys. Rev. X **9**, 31010 (2019)
 B. Real et al., Phys. Rev. Lett. **125**, 186601 (2020)

Outline of the talk

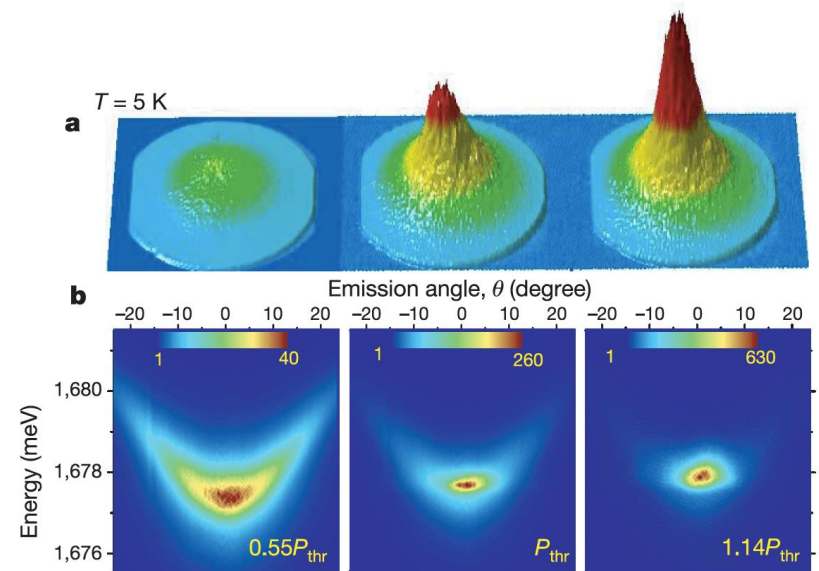
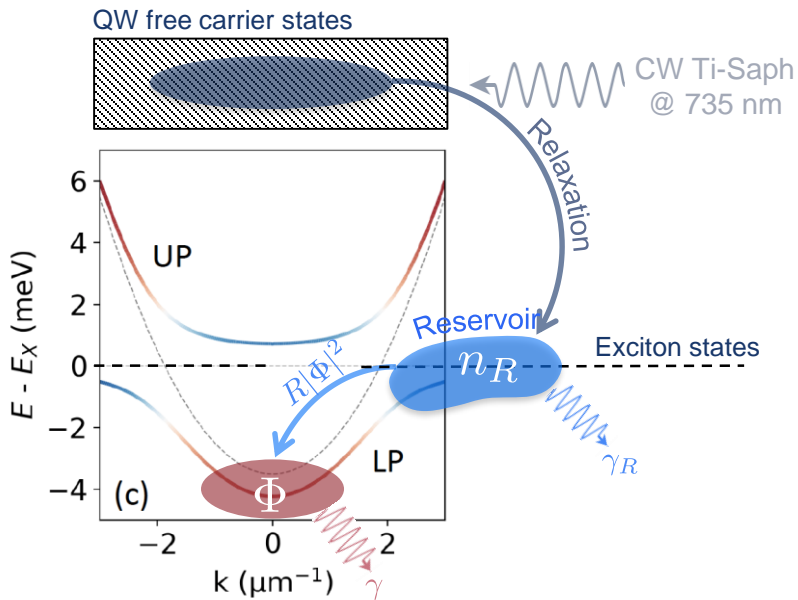
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- Polariton condensate belong to the Kardar Parisi Zhang universality class



Polariton Bose Einstein condensation

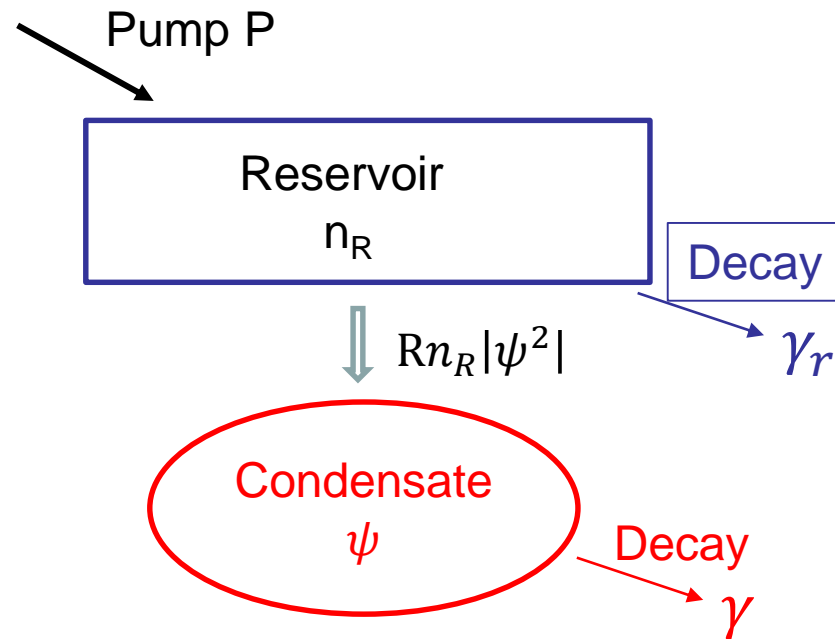
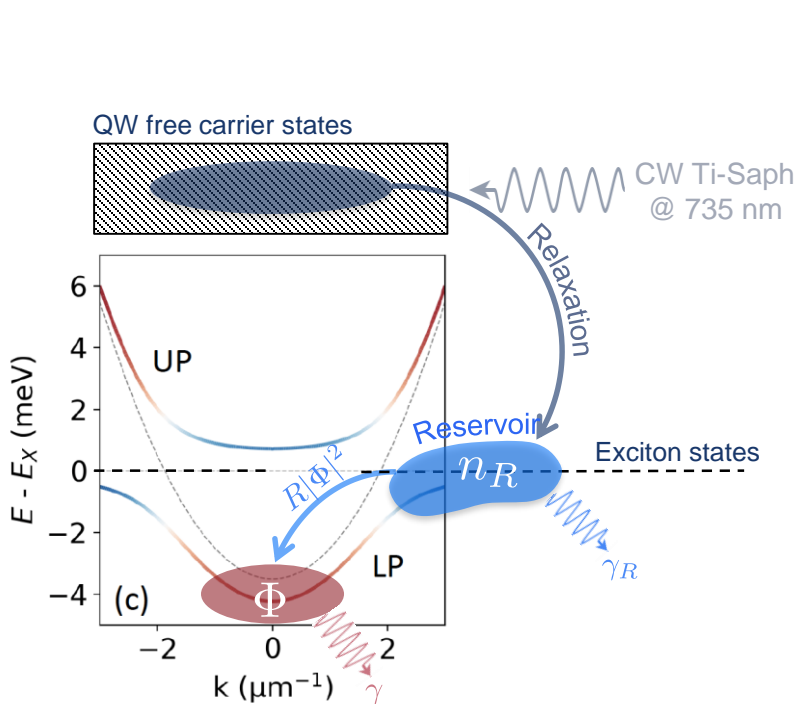


Kasprzak *et al.* Nature, **443**, 409 (2006)

See also H. Deng *et al.* Science (2002), R. Balili *et al.*, Science (2007)

J. Bloch, I. Carusotto and M. Wouters, *Spontaneous coherence in spatially extended photonic systems: Non-Equilibrium Bose-Einstein condensation*, Nature Review Physics (2022)
 (<https://doi.org/10.1038/s42254-022-00464-0>)

Polariton Bose Einstein condensation



MEAN FIELD DESCRIPTION OF THE POLARITON FLUID

(Incoherent Pumping = GPE + Reservoir)

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*} \nabla^2 + g |\psi|^2 + 2g_R n_R + i\frac{\hbar}{2} (R n_R - \gamma(\mathbf{k})) \right] \psi + \xi$$

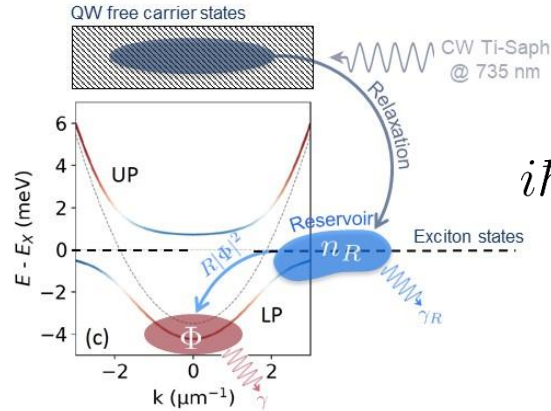
noise

$$\frac{\partial n_R}{\partial t} = P - (\gamma_R + R |\psi|^2) n_R$$

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Phase coherence in a polariton condensates

E. Altman, S. Diehl, M. Wouters 2015



$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*} \nabla^2 + g |\psi|^2 + 2g_R n_R + i\frac{\hbar}{2} (R n_R - \gamma(\mathbf{k})) \right] \psi + \xi$$

$$\frac{\partial n_R}{\partial t} = P - (\gamma_R + R |\psi|^2) n_R$$

- Density-phase representation : $\psi(\mathbf{x}, t) = \sqrt{\rho(\mathbf{x}, t)} e^{-i\omega_0 t + i\theta(\mathbf{x}, t)}$
- Assume different time scales for density and phase fluctuations :

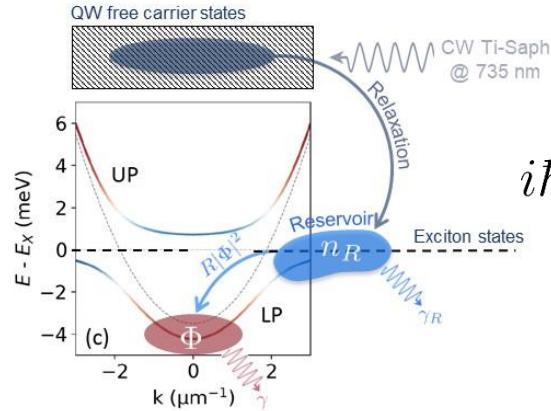
E. Altman, *et al.*, PRX **5**, 011017 (2015)

K. Ji, *et al.*, PRB **91**, 045301 (2015)

L. He, *et al.*, PRB **92**, 155307 (2015)

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- Assume different time scales for density and phase fluctuations :

$$\partial_t \theta = \nu \nabla^2 \theta + \frac{\lambda}{2} (\nabla \theta)^2 + \sqrt{D} \eta$$

This is the famous Kardar Parisi Zhang equation!!

E. Altman, *et al.*, PRX **5**, 011017 (2015)

K. Ji, *et al.*, PRB **91**, 045301 (2015)

L. He, *et al.*, PRB **92**, 155307 (2015)

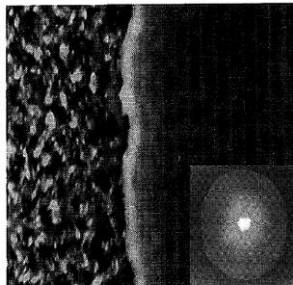
Kardar-Parisi-Zhang theory of interface stochastic growth



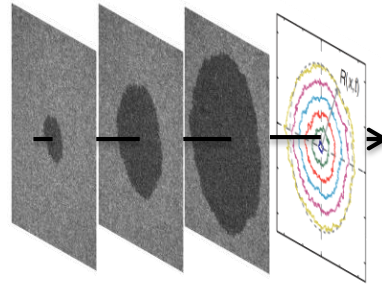
Kardar, Parisi and Zhang, *PRL* (1986)



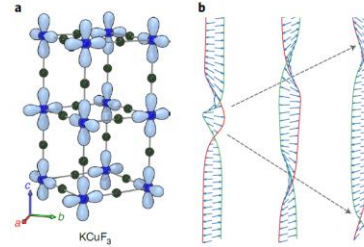
Frost on a window



Bacteria



Liquid crystals

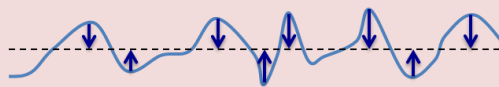


1D antiferromagnet (*Nature Phys.* 2021)
See also D. Wei, *Science* 376 716 (2022)

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(\mathbf{x}, t)$$

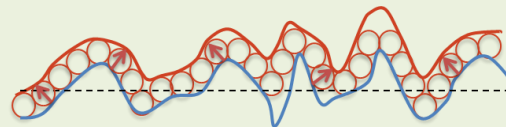
Diffusion term

➤ *smoothens the surface*



Growth term (nonlinear)

- *Orthogonal to the surface*
- *Non-equilibrium*



Stochastic term (fluctuations)

➤ *White Gaussian noise*

$$\langle \eta(\mathbf{x}, t) \rangle = 0$$

$$\langle \eta(\mathbf{x}) \eta(\mathbf{x}') \rangle = \delta(\mathbf{x} - \mathbf{x}')$$

Landmark signatures of KPZ physics



Kardar, Parisi and Zhang, *PRL* (1986)

➤ Self-organized **scale invariance**

⇒ Critical exponents (universal)

$$C(\mathbf{x}, t) = \langle h(\mathbf{x}, t)h(0, 0) \rangle - \langle h(\mathbf{x}, t) \rangle \langle h(0, 0) \rangle \propto \begin{cases} t^{2\beta} & (\mathbf{x} = 0) \\ x^{2\chi} & (t = 0) \end{cases}$$

$$C(\mathbf{x}, t) \propto t^{2\beta} \mathcal{F}_{\text{KPZ}} \left(\kappa \frac{|\mathbf{x}|}{t^{1/\mathcal{Z}}} \right)$$

└─→ **KPZ universal scaling function** (tabulated)

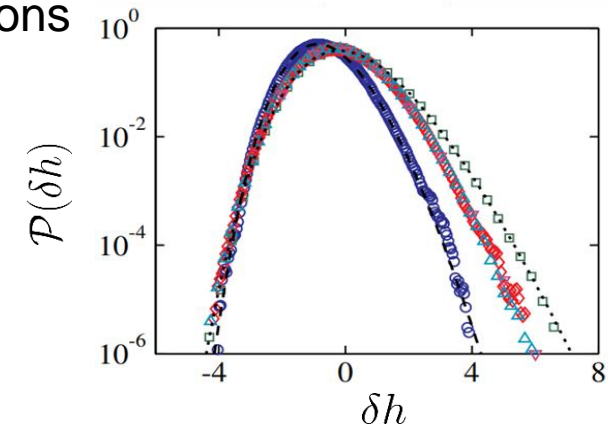
1D KPZ universality class

$$\beta = 1/3 \quad \chi = 1/2$$

$$\mathcal{Z} = \chi/\beta = 3/2$$

➤ Non-Gaussian probability distribution of height fluctuations

$$\delta h(t) = \frac{h(\mathbf{x}_0, t) - v_\infty t}{(\Gamma t)^{1/3}}$$



T. Halpin-Healy, & Y.-C. Zhang, *Phys. Rep.* **254**, 215 (1995)

J. Krug, *Adv. Phys.* **46**, 139 (1997)

K. A. Takeuchi, *Physica A* **504**, 77 (2018)

K. A. Takeuchi, *PRL* **110**, 210604 (2013)

Polariton condensates

$$\partial_t \theta = \nu \nabla^2 \theta + \frac{\lambda}{2} (\nabla \theta)^2 + \sqrt{D} \eta$$

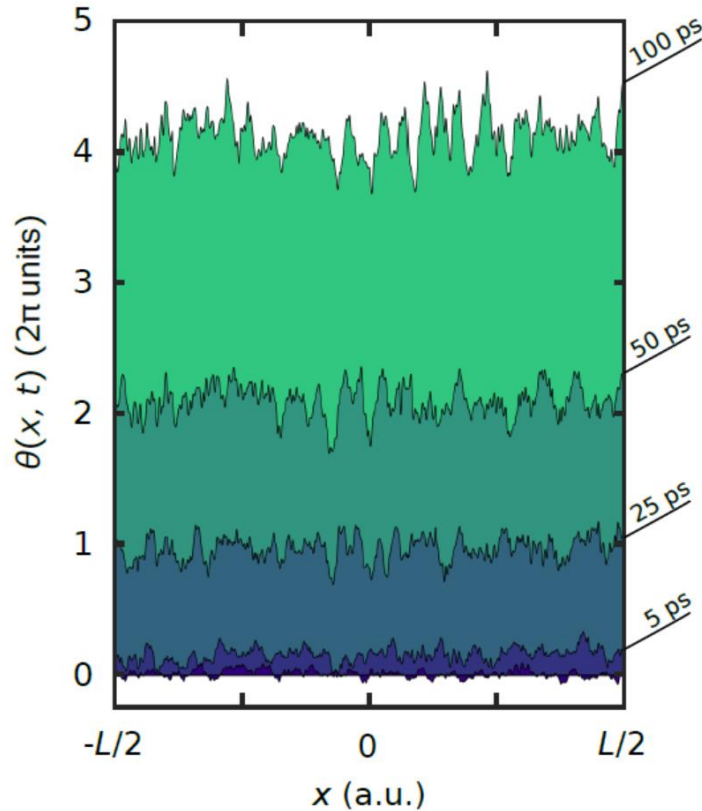
- The phase front behaves as a growing interface

KPZ scaling expected in the spatio-temporal correlations of the phase

$$\text{Var} [\Delta \theta(\Delta x, \Delta t)]$$

Instantaneous phase : difficult to access in the experiment ...

How to probe KPZ scaling ???



Polariton condensates

We can measure amplitude amplitude correlations of the field (first order coherence) :

$$g^{(1)}(\Delta x, \Delta t) = \frac{\langle \psi^*(x, t_0) \psi(-x, t_0 + \Delta t) \rangle}{\sqrt{\langle \rho(x, t_0) \rangle} \sqrt{\langle \rho(-x, t_0 + \Delta t) \rangle}}$$

- If phase fluctuations are independent of density fluctuations and for small density fluctuations :
$$g^{(1)}(\Delta x, \Delta t) = \left\langle \exp [i\Delta\theta(\Delta x, \Delta t)] \right\rangle$$

Polariton condensates

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- If phase fluctuations are independent of density fluctuations and for small density fluctuations :
$$g^{(1)}(\Delta x, \Delta t) = \left\langle \exp [i\Delta\theta(\Delta x, \Delta t)] \right\rangle$$

- For small phase fluctuations :

$$|g^{(1)}(\Delta x, \Delta t)|^2 \simeq \exp(-\langle \Delta\theta(\Delta x, \Delta t)^2 \rangle + \langle \Delta\theta(\Delta x, \Delta t) \rangle^2) \equiv \exp(-\text{Var} [\Delta\theta(\Delta x, \Delta t)])$$

$$\text{Var} [\Delta\theta] \simeq -2 \log \left(\left| g^{(1)} \right| \right)$$

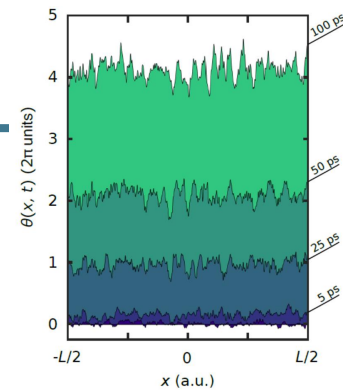
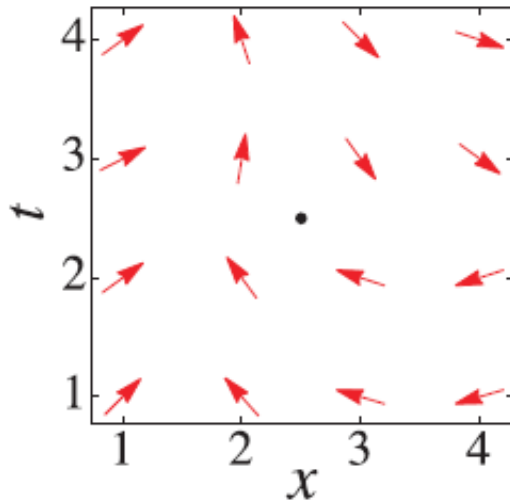
$$\text{➤ In 1D:} \quad -2 \log \left(\left| g^{(1)}(\Delta x, \Delta t) \right| \right) \sim \begin{cases} \Delta t^{2\beta} & (\Delta x = 0) \\ \Delta x^{2\chi} & (\Delta t = 0) \end{cases}$$

$$\beta = 1/3 \quad \chi = 1/2$$

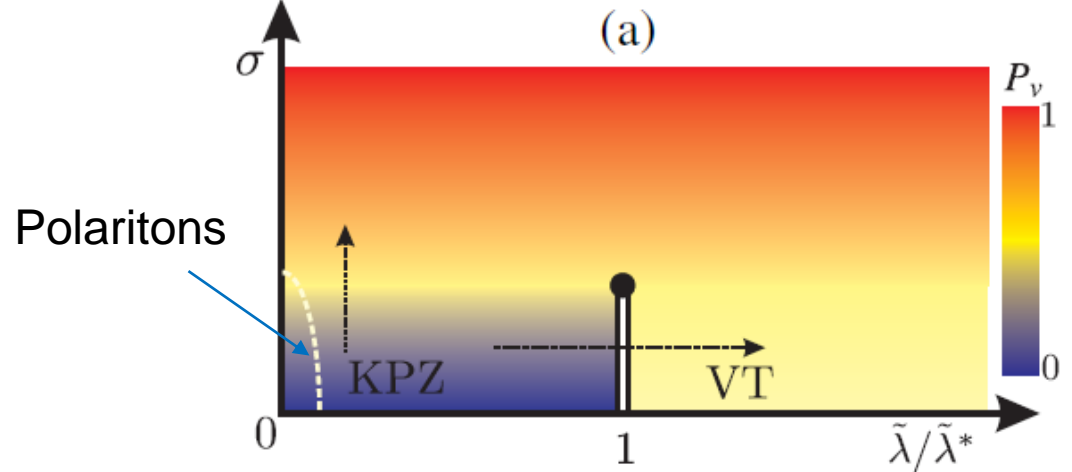
KPZ physics in polariton condensates

The phase is a compact variable : $\theta \in [0, 2\pi]$

➤ Even in **1D** : Space time vortex



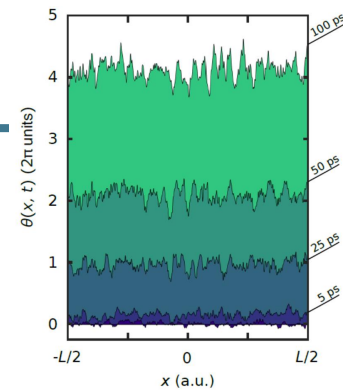
Effective 2D system: 1st order phase transition



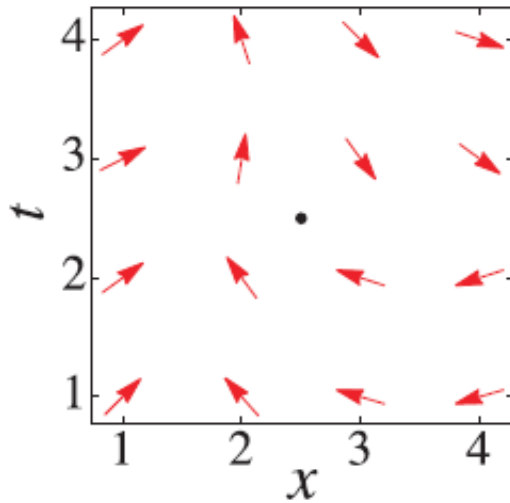
L. He, L.M. Sieberer and S. Diehl, Phys. Rev. Lett. 118, 085301 (2017)

KPZ physics in polariton condensates

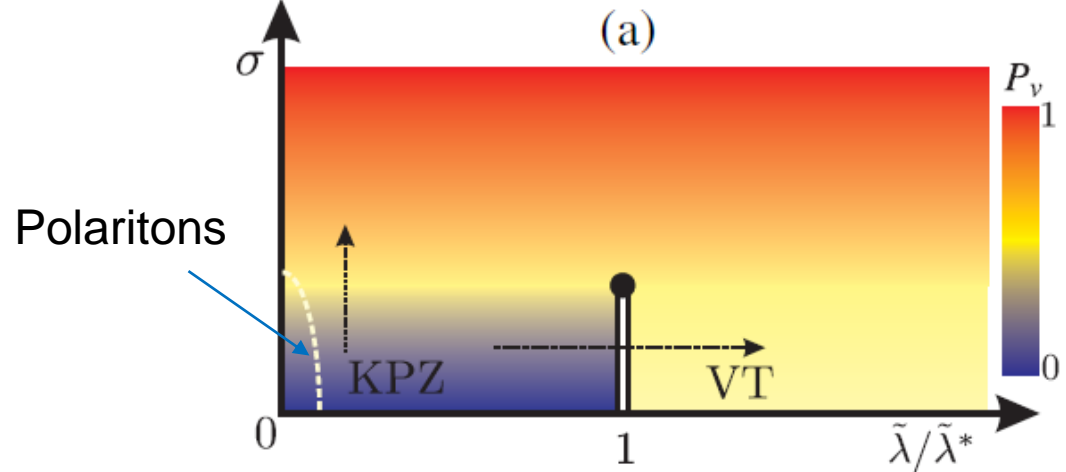
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L. He, L.M. Sieberer and S. Diehl, Phys. Rev. Lett. 118, 085301 (2017)

- In **2D**: Space time AND spatial vortices

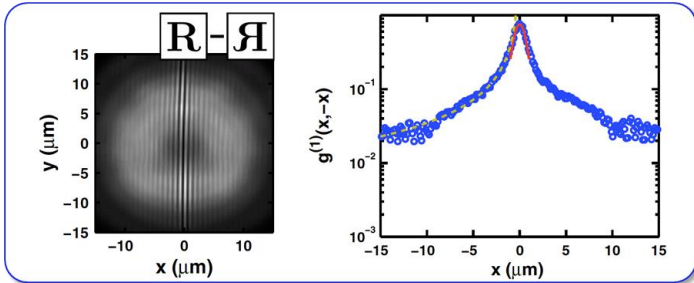
KPZ scaling in 2D open condensates? \Rightarrow Still actively debated

E. Altman, et al., PRX 5, 011017 (2015)
A. Zamora, et al., PRX 7, 041006 (2017)

Q. Mei, et al., PRB 103, 045302 (2021)
A. Ferrier, et al., PRB 105, 205301 (2022)

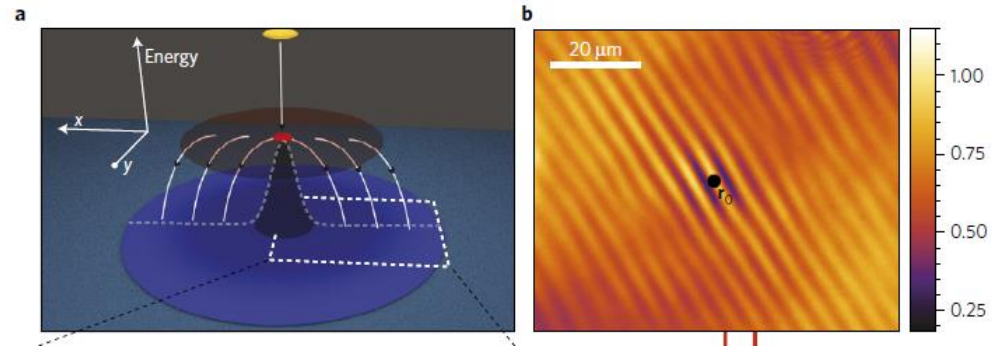
Experimental observation?

Small size effects



G. Roumpos et al., PNAS 109 (17) 6467 (2011)
See also J. Fischer et al., Phys. Rev. Lett. 113, 203902 (2014)

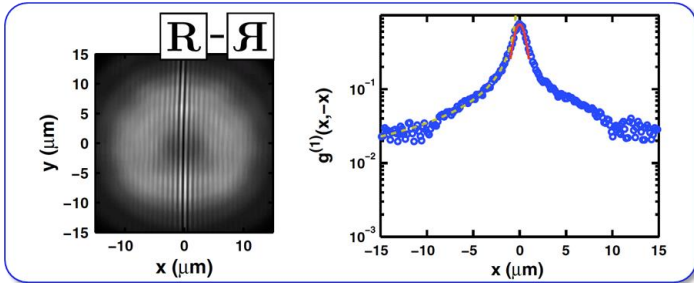
Weak overlap with the reservoir



D. Caputo et al., Nature Materials 17, 145 (2018)
D. Ballarini et al., Phys. Rev. Lett. 118, 215301 (2017)

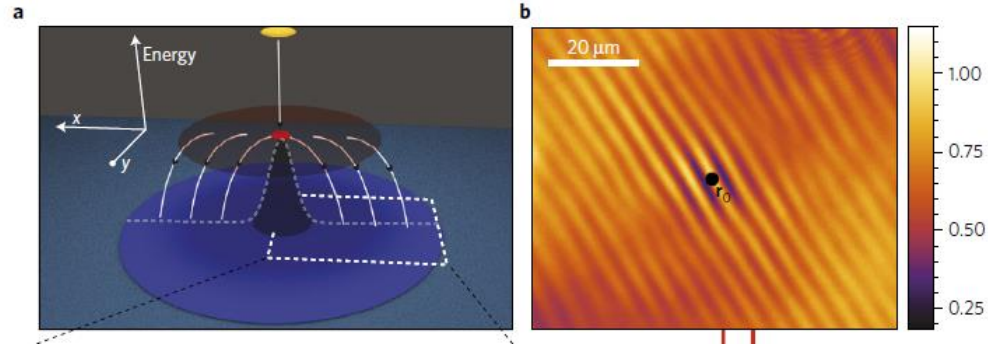
Experimental observation?

Small size effects



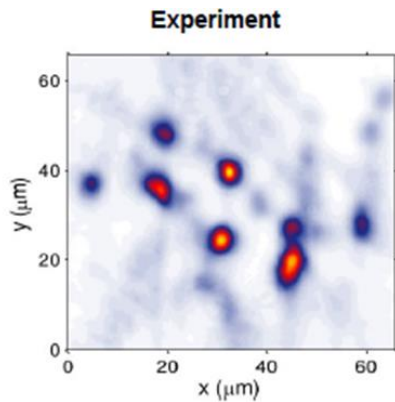
G. Roumpos et al., PNAS 109 (17) 6467 (2011)
See also J. Fischer et al., Phys. Rev. Lett. 113, 203902 (2014)

Weak overlap with the reservoir

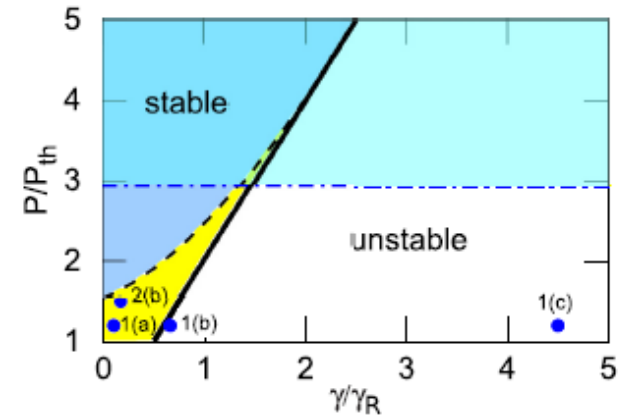


D. Caputo et al., Nature Materials 17, 145 (2018)
D. Ballarini et al., Phys. Rev. Lett. 118, 215301 (2017)

Condensation within an large excitation spot



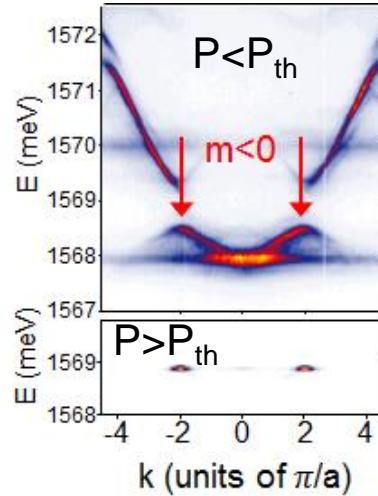
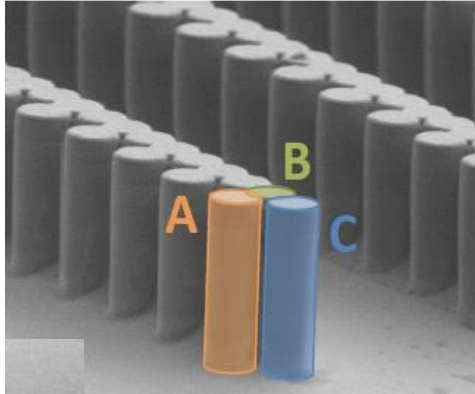
Modulation instability



F. Baboux et al., Optica 5, 1163 (2018)

M. Wouters et al., Phys. Rev. B 77, 115340 (2008)
N. Bobroska et al., PRB 90, 205304 (2014),
N. Bobrovski and M. Matuszewski, PRB 92, 035311 (2015)

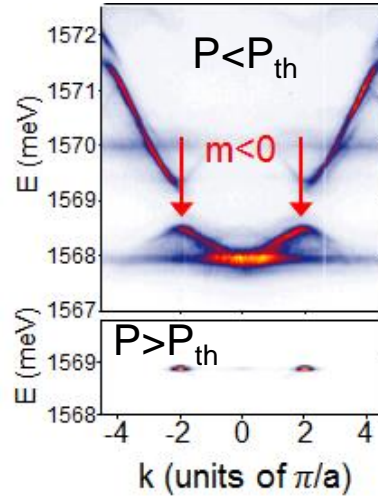
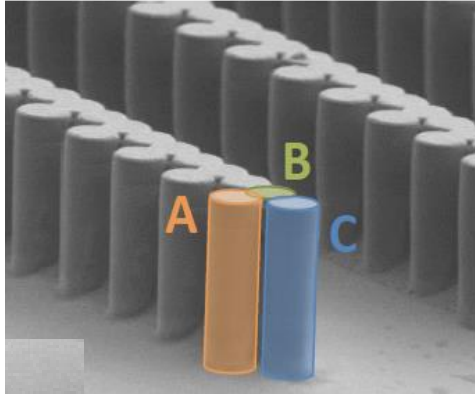
Taming the instability : condensation in a negative mass band



F. Baboux *et al.*, *Optica* **5**, 1163 (2018)

Review on polariton lattice engineering: C. Schneider *et al.*, *Rep. Prog. Phys.* **80**, 16503 (2017)

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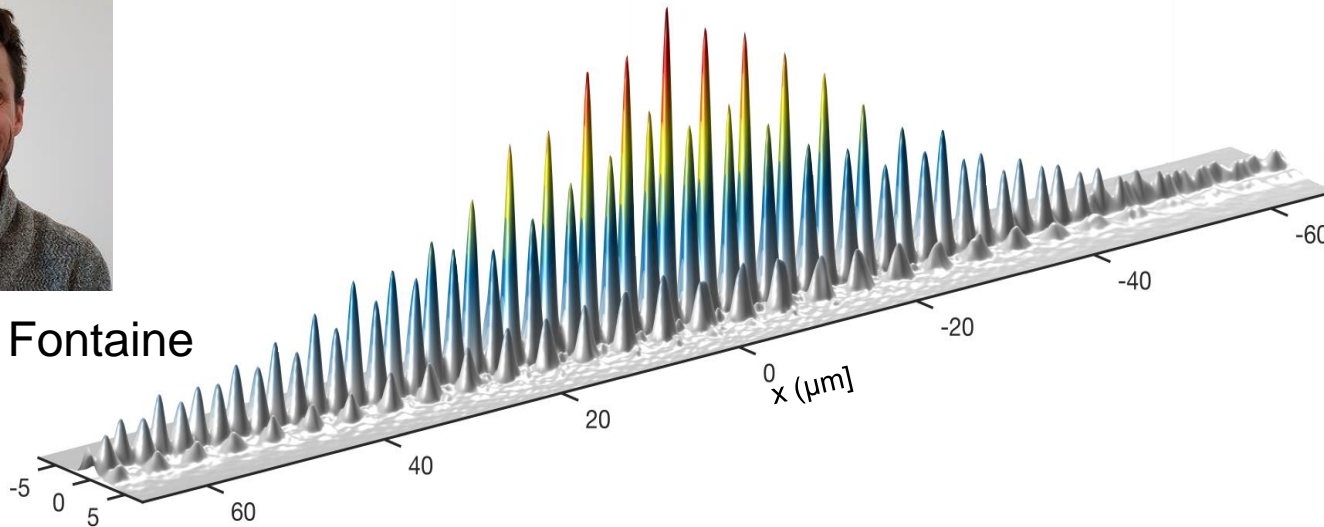


F. Baboux *et al.*, *Optica* **5**, 1163 (2018)

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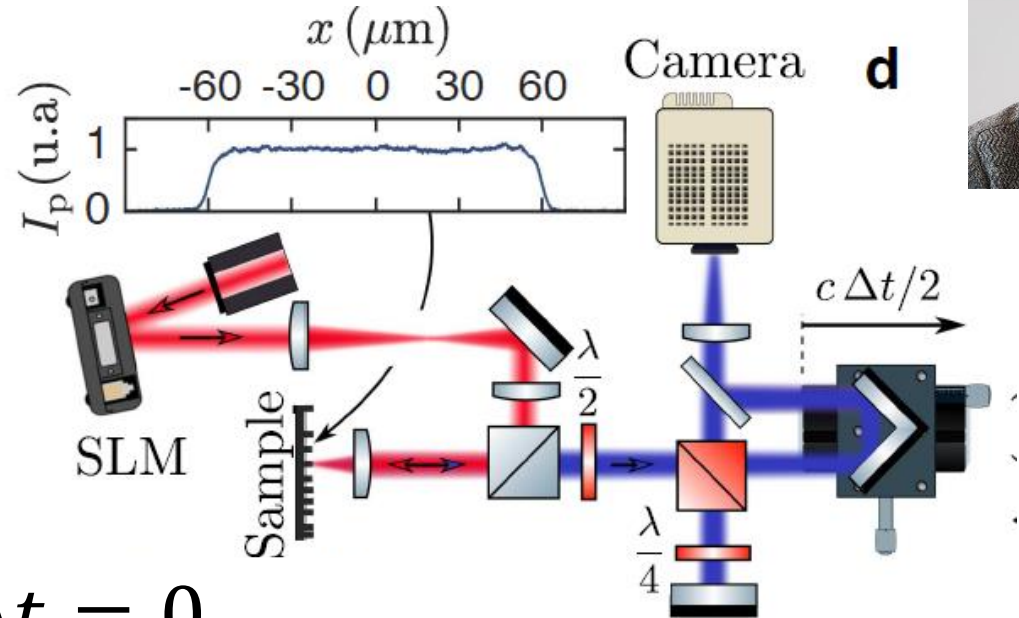
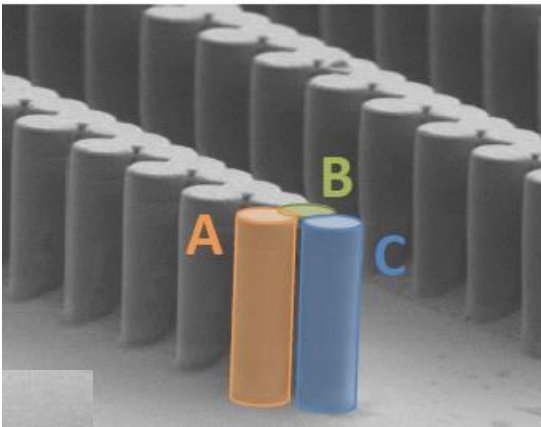


Quentin Fontaine

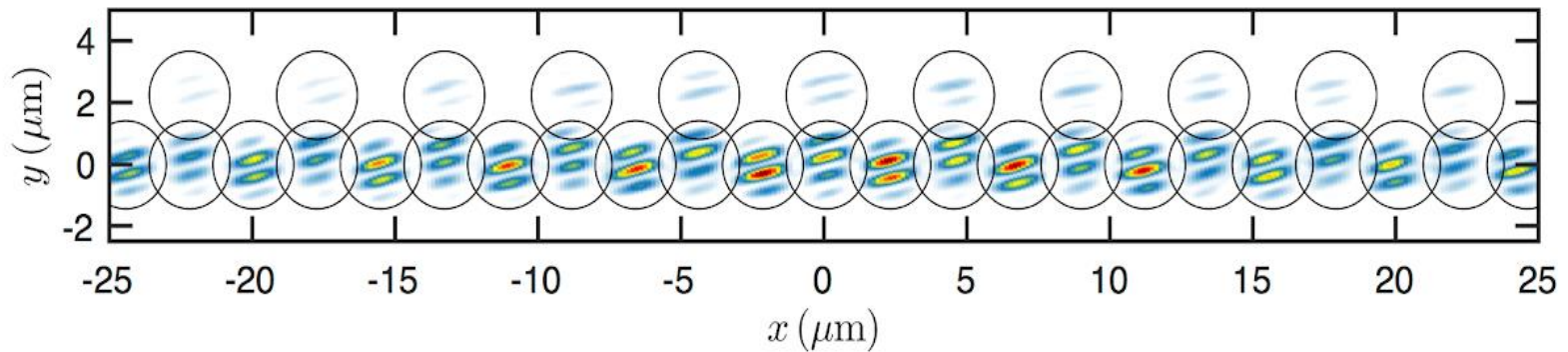


Q. Fontaine, *et al.*, *Nature* **608**, 687 (2022)

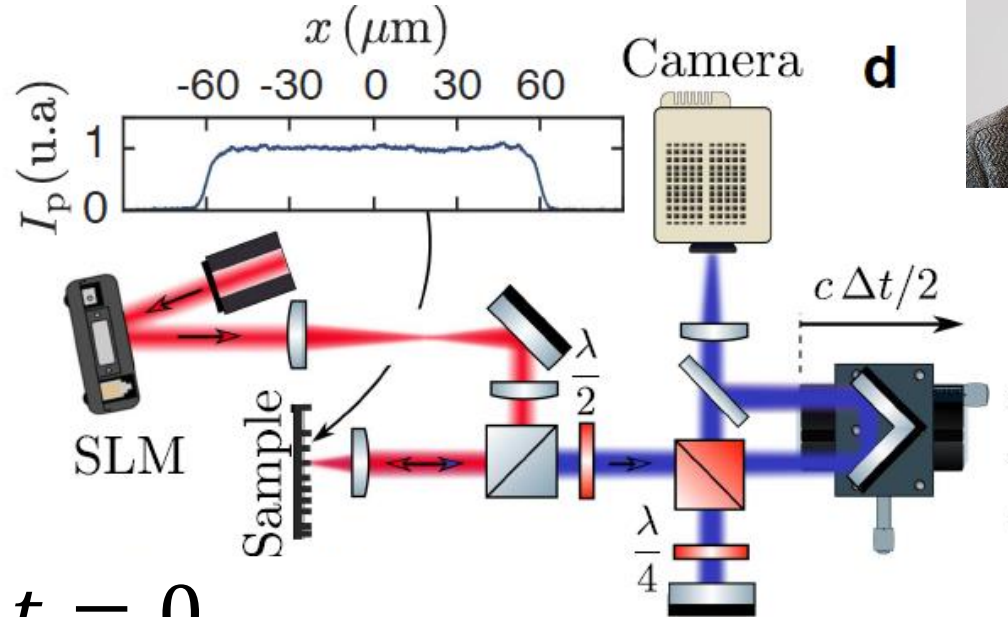
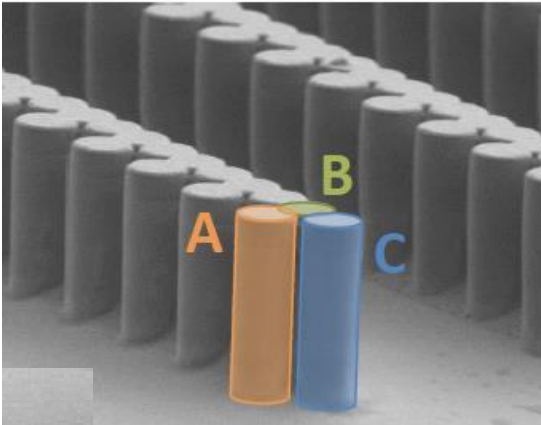
KPZ physics in 1D polariton condensates



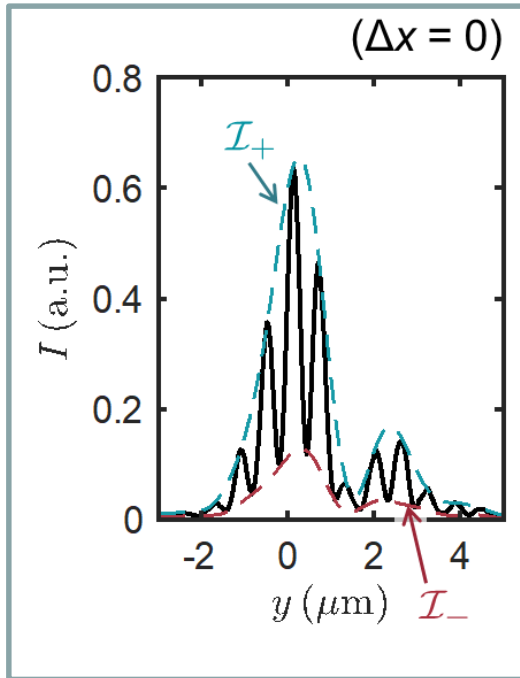
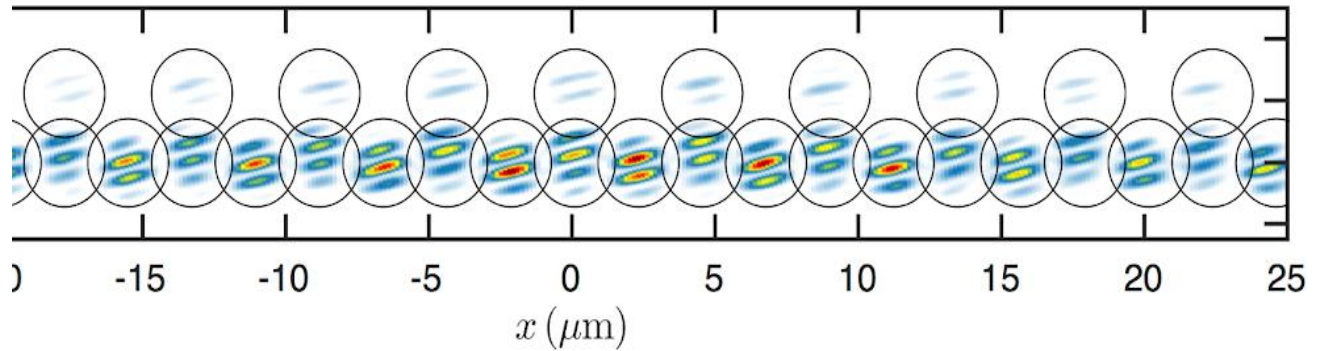
$$\Delta t = 0$$



KPZ physics in 1D polariton condensates

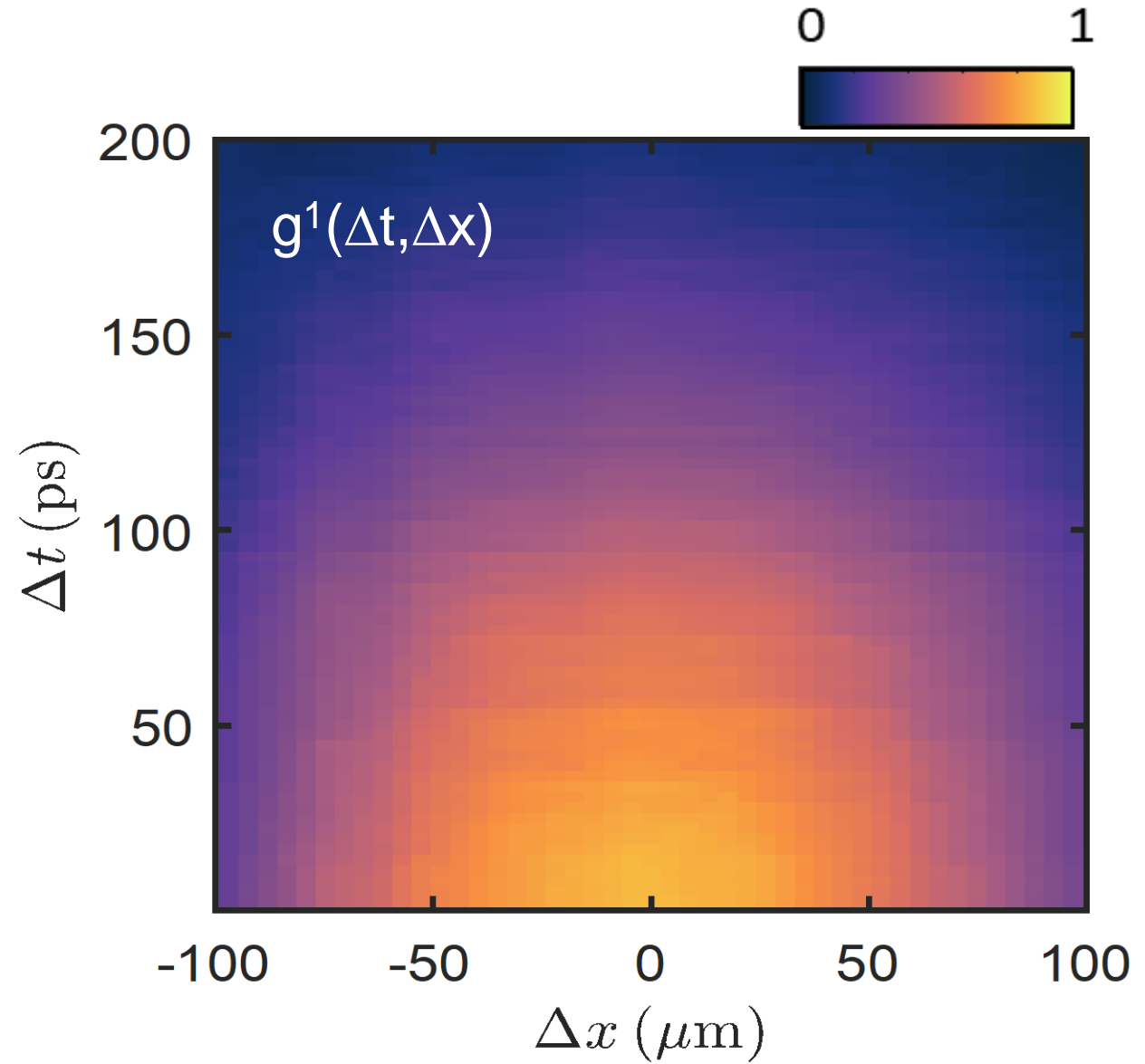
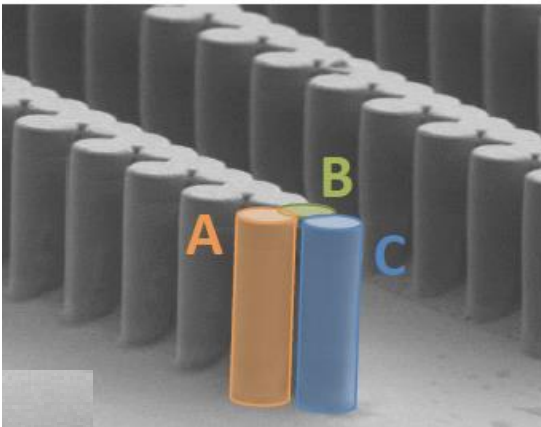


$\Delta t = 0$



$$\Rightarrow \frac{I_+ - I_-}{I_+ + I_-} = \frac{2\sqrt{I(\mathbf{r})I(-\mathbf{r})}}{I(\mathbf{r}) + I(-\mathbf{r})} |g^{(1)}(\Delta\mathbf{r}, \Delta t)|$$

KPZ physics in 1D polariton condensates

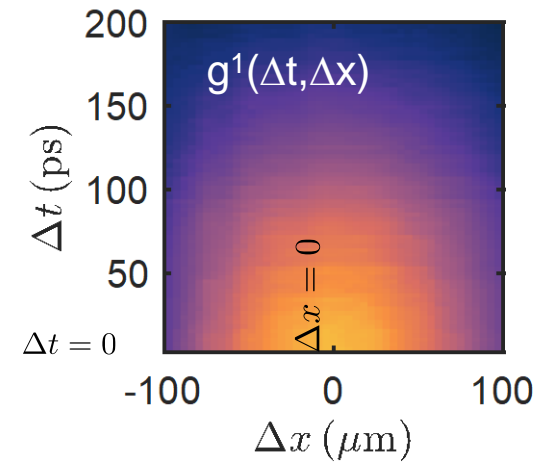


KPZ physics in 1D polariton condensates

➤ “SURFACE ROUGHNESS” \leftrightarrow $\text{Var} [\Delta\theta] \simeq -2 \log \left(g^{(1)} \right)$

➤ WE EXPECT:

$$-2 \ln \{g^{(1)}(\Delta x, \Delta t)\} \sim \begin{cases} \Delta t^{2\beta} & \text{for } \Delta x = 0 & \beta = 1/3 \\ \Delta x^{2\chi} & \text{for } \Delta t = 0 & \chi = 1/2 \end{cases}$$

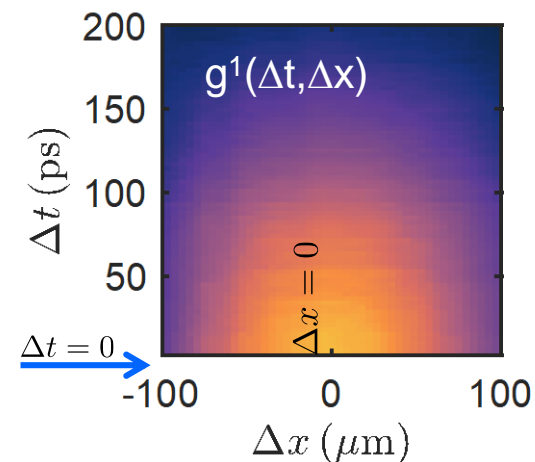
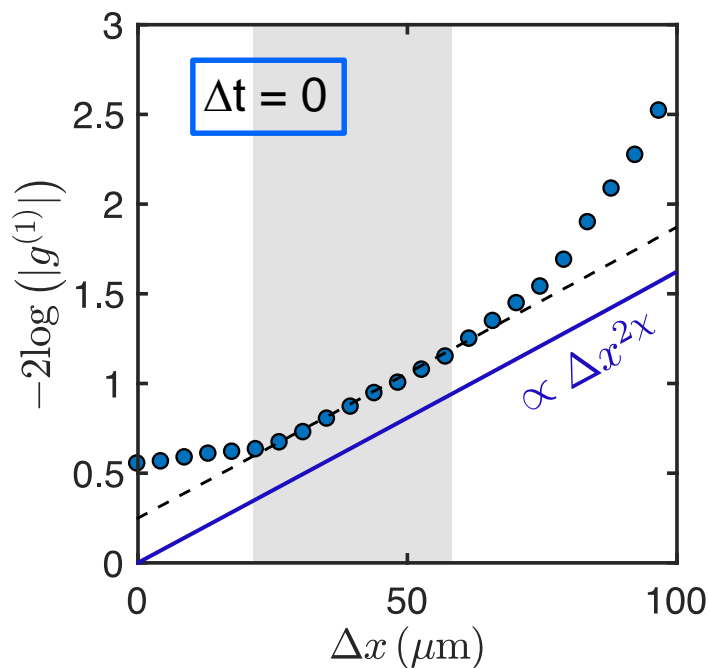


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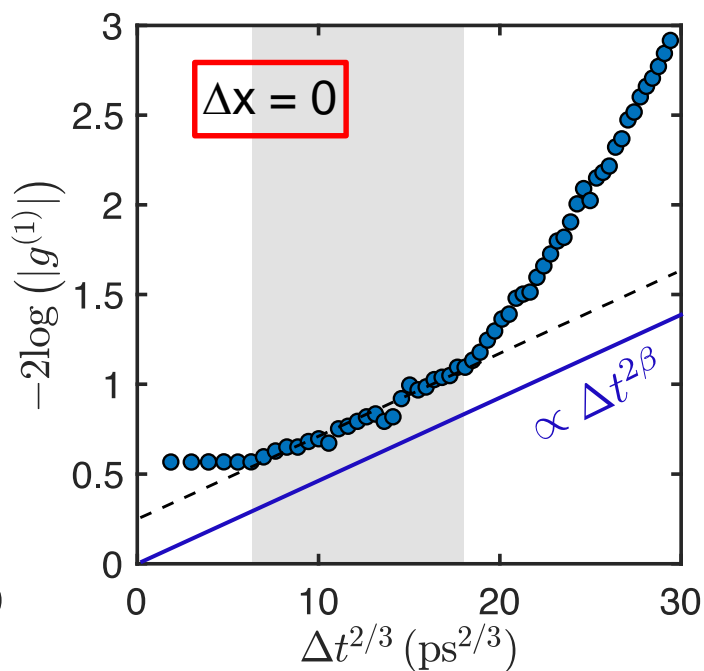
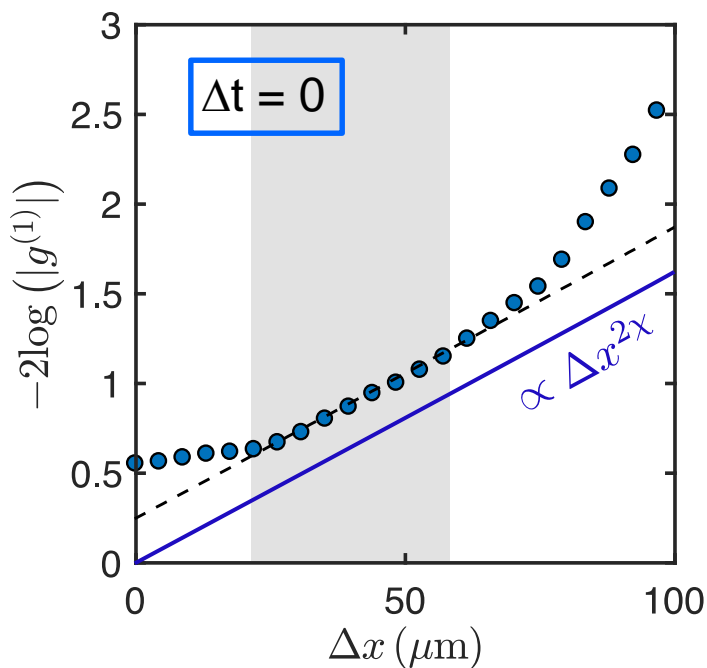
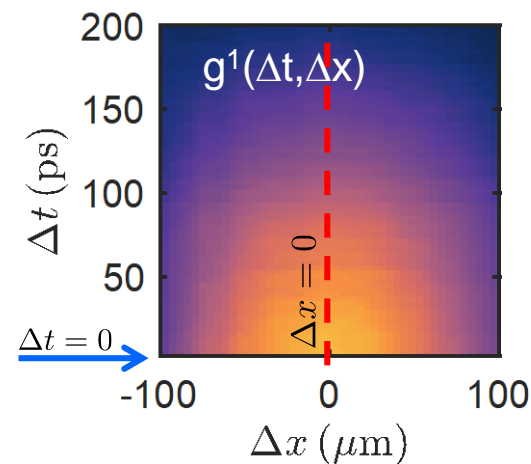


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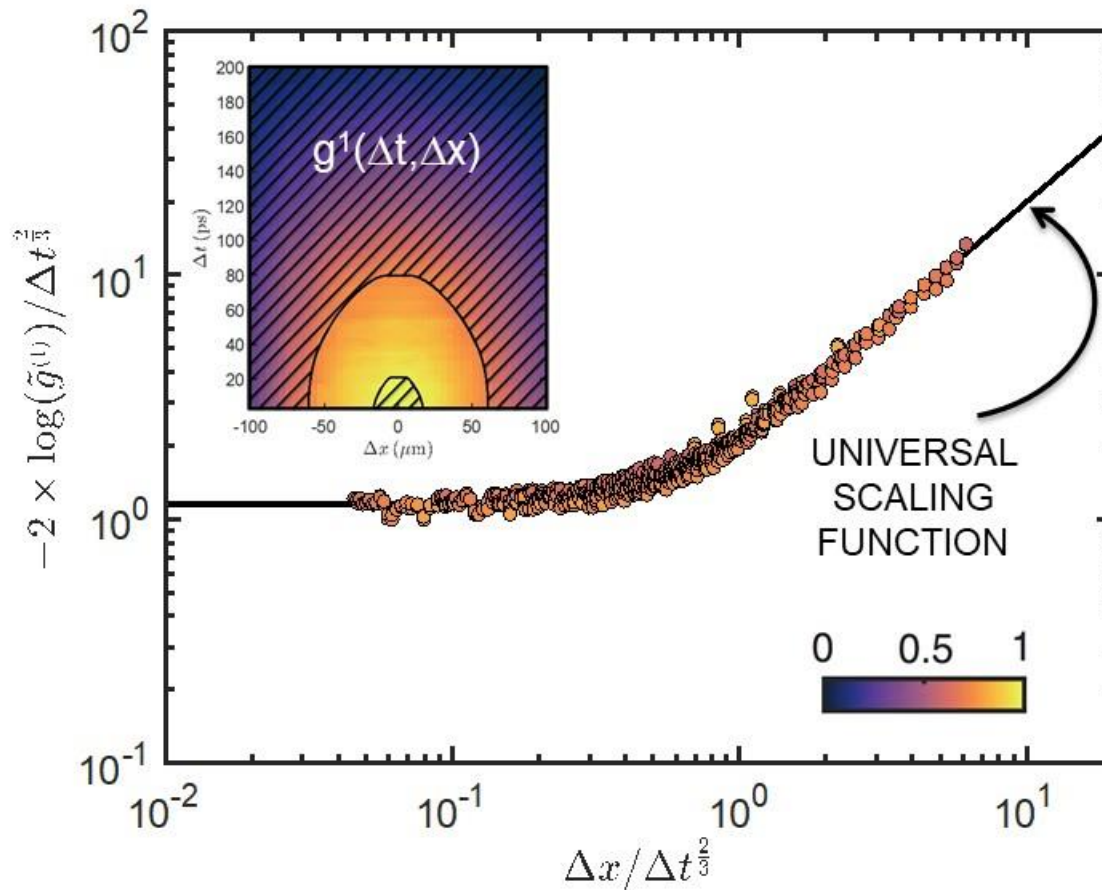
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KPZ scaling laws in 1D polariton condensates

KPZ scaling
$$-2 \ln \{g^{(1)}(\Delta x, \Delta t)\} = A \times \Delta t^{2\beta} \mathcal{F} \left[B \times \frac{\Delta x}{\Delta t^{1/z}} \right]$$

where $\mathcal{F}(y) = \begin{cases} c_0, & y \rightarrow 0 \\ y, & y \rightarrow \infty \end{cases}$ is the UNIVERSAL KPZ SCALING FUNCTION.

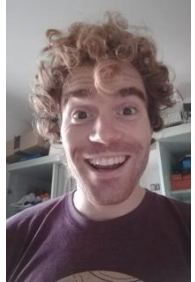
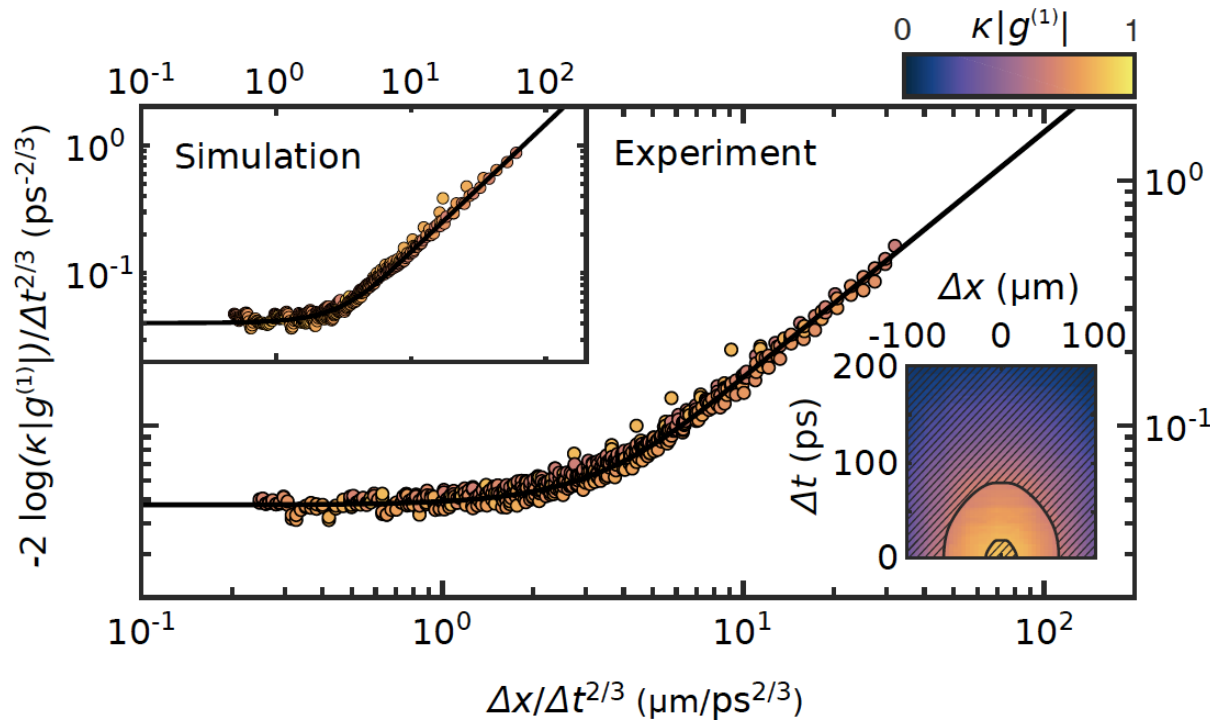


SIMULATIONS - COMPARISON WITH EXPERIMENTS

- Integrate numerically the two coupled equations model

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*} \nabla^2 + g |\psi|^2 + 2g_R n_R + i\frac{\hbar}{2} (R n_R - \gamma(\mathbf{k})) \right] \psi + \xi$$

$$\frac{\partial n_R}{\partial t} = P - (\gamma_R + R |\psi|^2) n_R$$



D. Squizzato

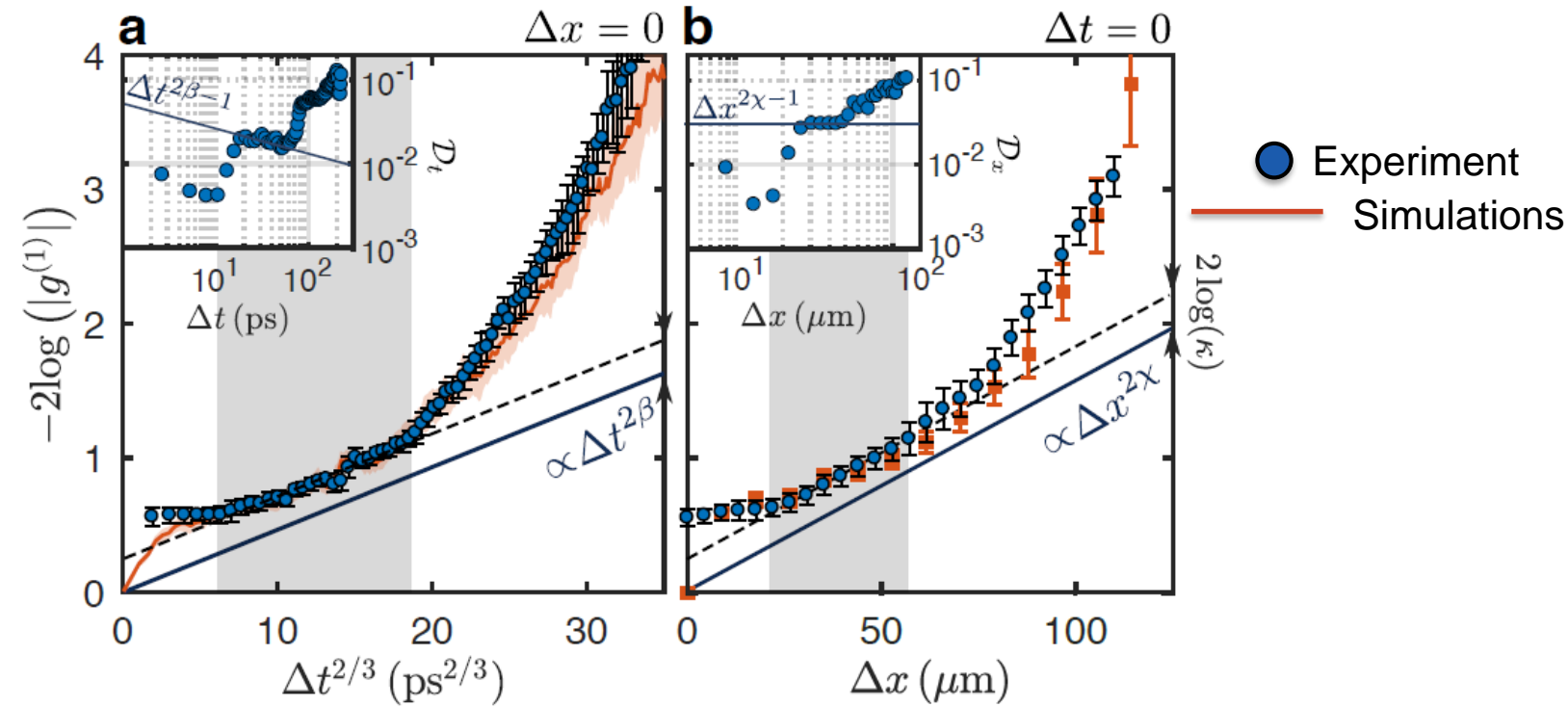


A. Minguzzi



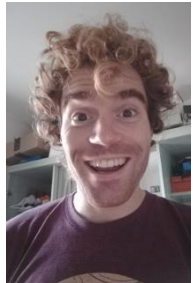
L. Canet

SIMULATIONS - COMPARISON WITH EXPERIMENTS



$$\chi_{\text{exp}} = 0.51 \pm 0.08$$

$$\beta_{\text{exp}} = 0.36 \pm 0.11$$



D. Squizzato



A. Minguzzi



L. Canet

Phase dynamics (simulations)

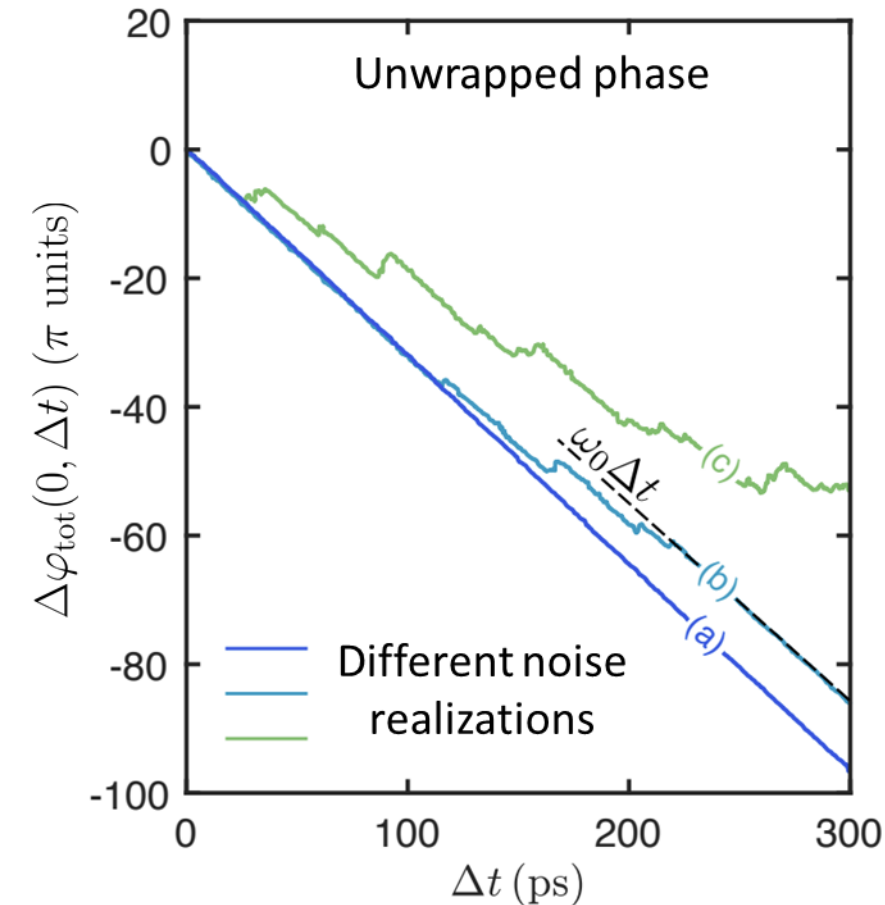
- Calculate total phase (unwrapped) difference for several noise realisations:

$$\Delta\varphi_{\text{tot}}(0, \Delta t) = \varphi_{\text{tot}}(0, \Delta t) - \varphi_{\text{tot}}(0, 0) = -\omega_0\Delta t + \Delta\theta(0, \Delta t)$$

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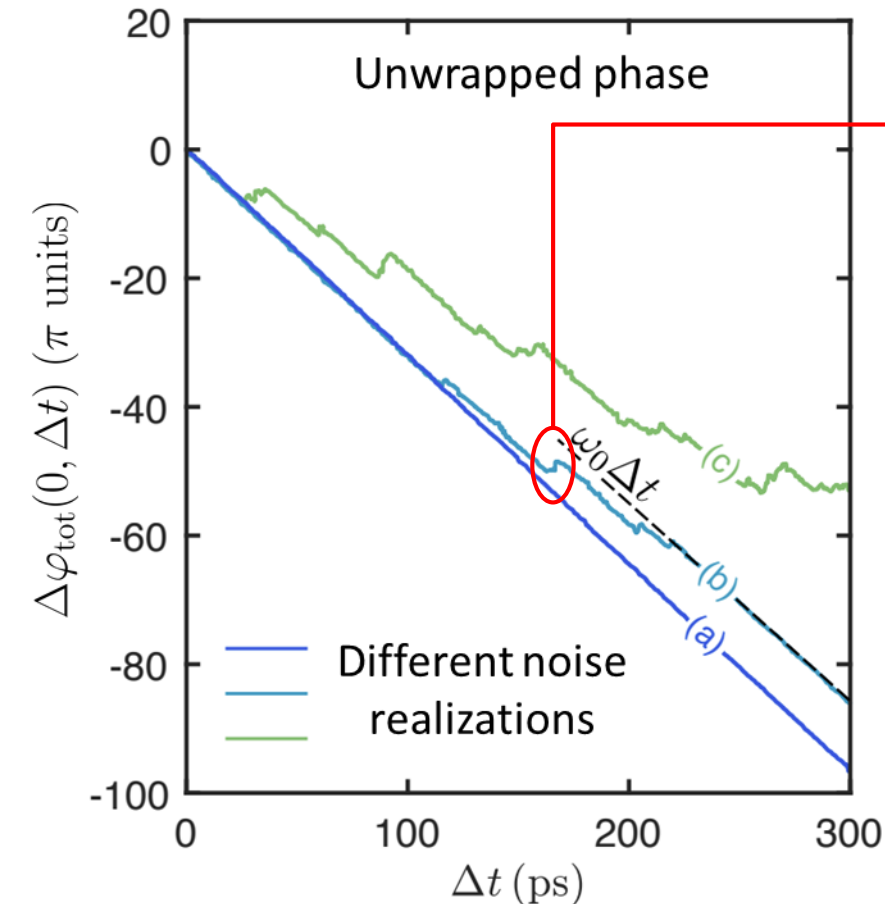
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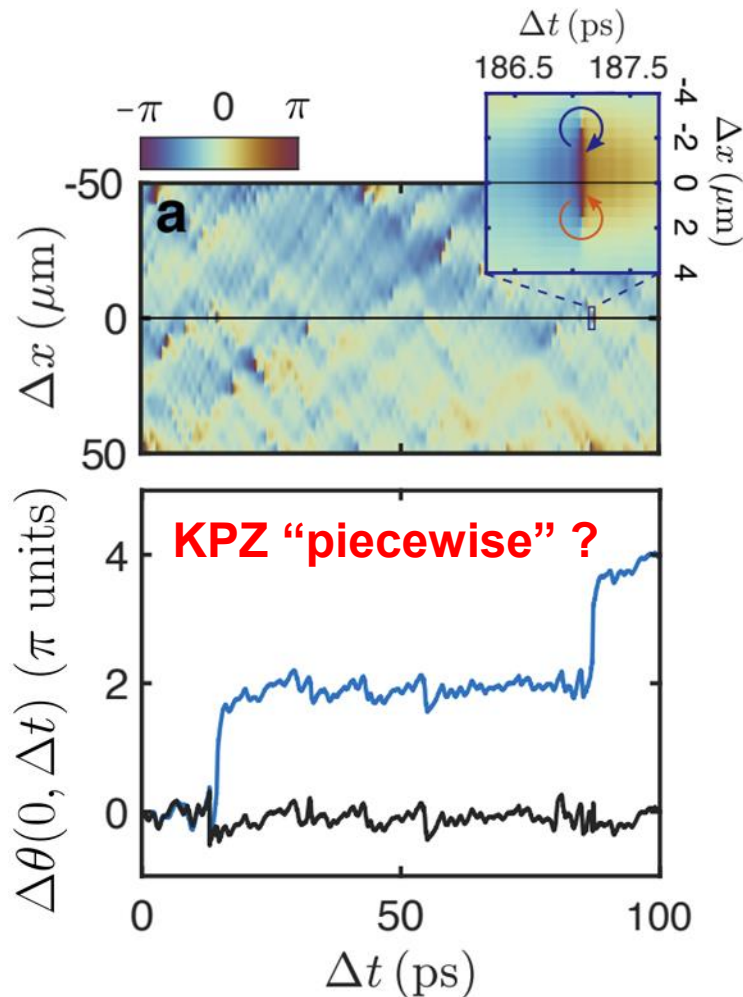


- Occasional 2π phase jumps.

Phase dynamics (simulations)

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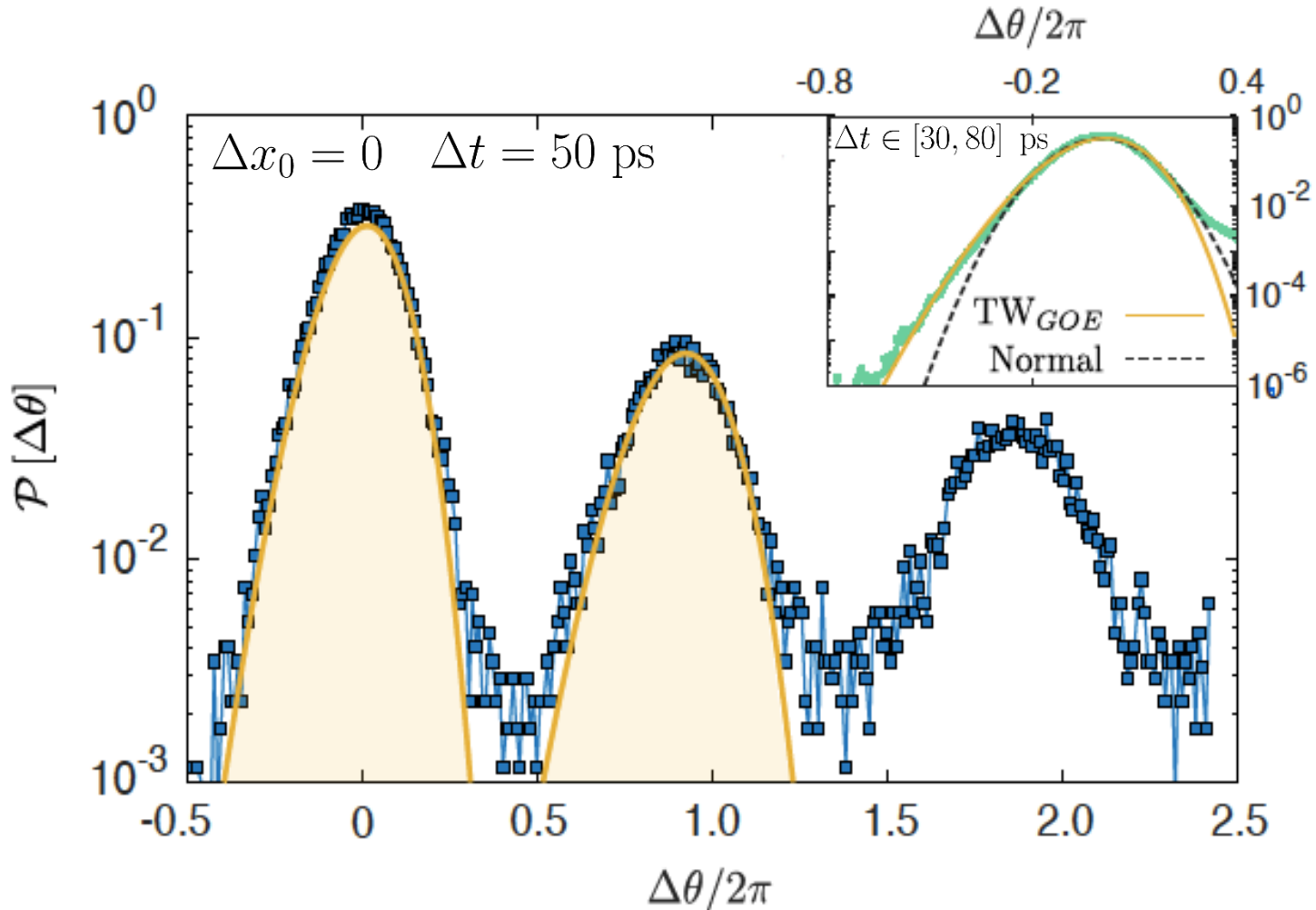


- Occasional 2π phase jumps.

- Pairs of vortex and antivortex appear in effective 2D space ($\Delta x, \Delta t$).

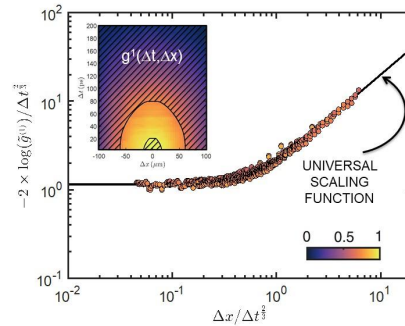
Amplitude distribution of phase fluctuations

- For Δx and Δt within KPZ window $\Delta\theta(\Delta x_0, \Delta t)/(|\Gamma|\Delta t^{2/3})$ is a random variable expected to obey Tracy-Widom statistics (non-Gaussian).

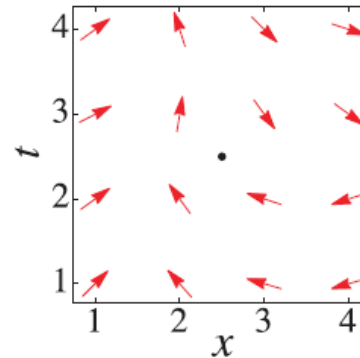


Conclusion and prospects

- 1D driven-dissipative condensates belong to the KPZ universality class



- Compact version of KPZ with a phase variable
⇒ topological defects



- KPZ scaling can be resilient to these defects

Turbulent phase for higher noise / pump power?

Conclusion and prospects

- In 2D: Space time **AND** spatial vortices
Vortex proliferation kills KPZ correlations?

Debated topics!

E. Altman, *et al.*, PRX **5**, 011017 (2015)

A. Zamora, *et al.*, PRX **7**, 041006 (2017)

Q. Mei, *et al.*, PRB **103**, 045302 (2021)

A. Ferrier, *et al.*, PRB **105**, 205301 (2022)

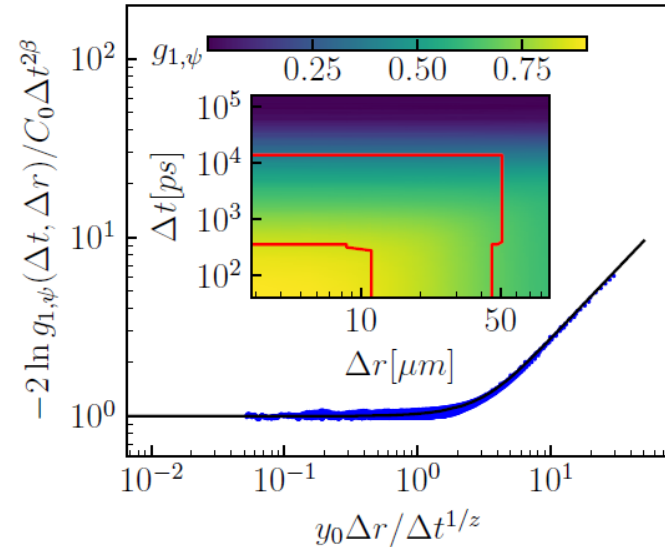
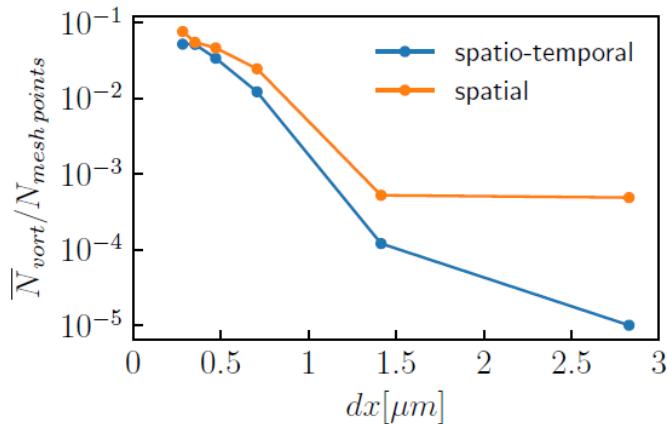
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- KPZ predicted in recent simulations using 2D discrete (lattice) model

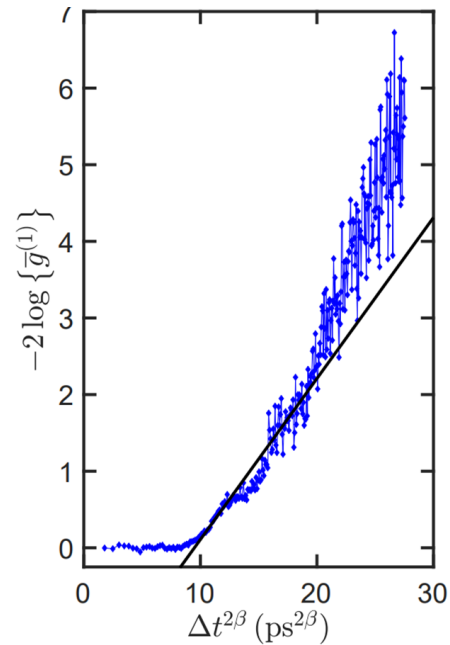
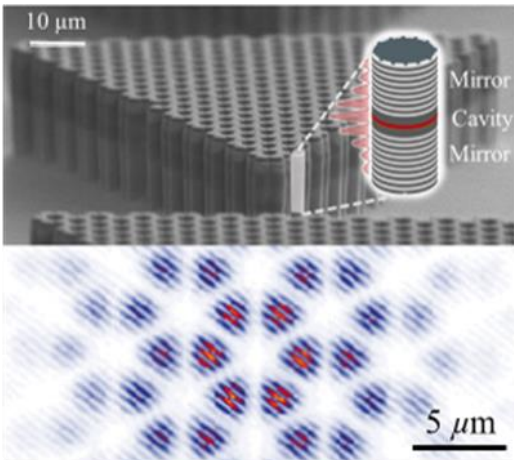


K. Deligiannis, *et al.*, Phys. Rev. Research **4**, 043207 (2022)

Conclusion and prospects

Negative mass condensates in 2D

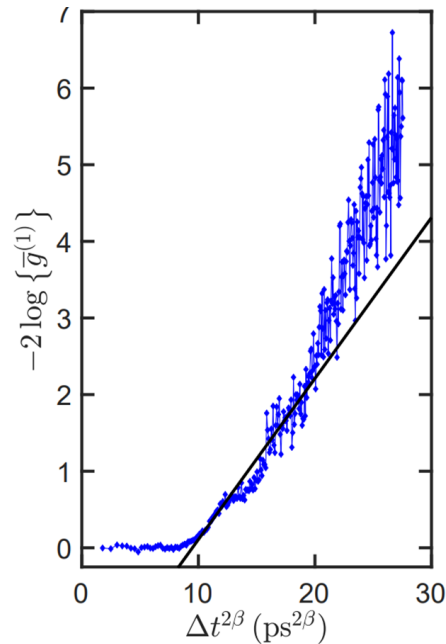
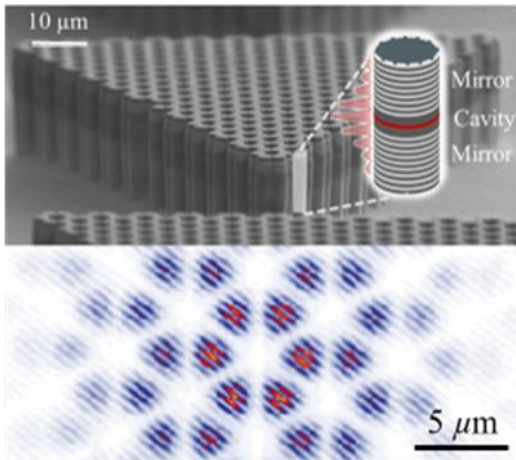
Preliminary results
in 2D



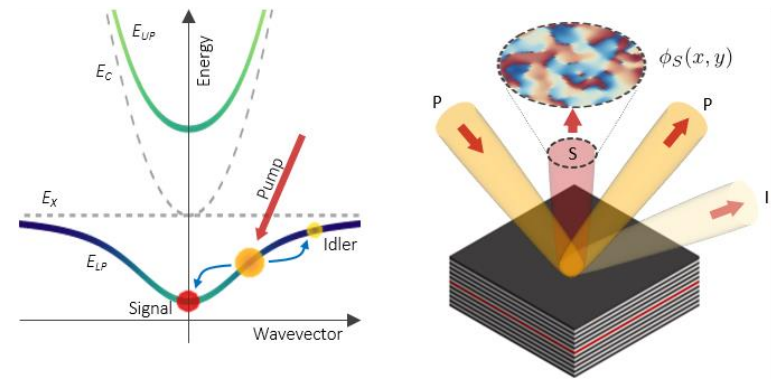
Conclusion and prospects

Negative mass condensates in 2D

Preliminary results
in 2D



KPZ scaling predicted in the OPO regime



A. Zamora, *et al.*, PRX 7, 041006 (2017)

Polariton lattices may provide the first experimental platform to probe KPZ physics in 2D

