Projection methods as dimensionality reduction tool and application in neuroscience

Nicolas Levernier

I) Experimental data and analysis with dimension reduction tools

II) Theoretical analysis of this dimension reduction tool

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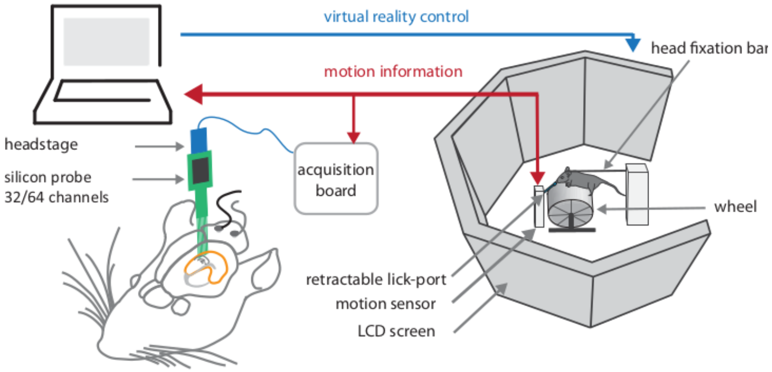
II) Theoretical analysis of this dimension reduction tool

Setup of J. Epsztein and J. Koenig

[Bourboulou et al. eLife 2019]



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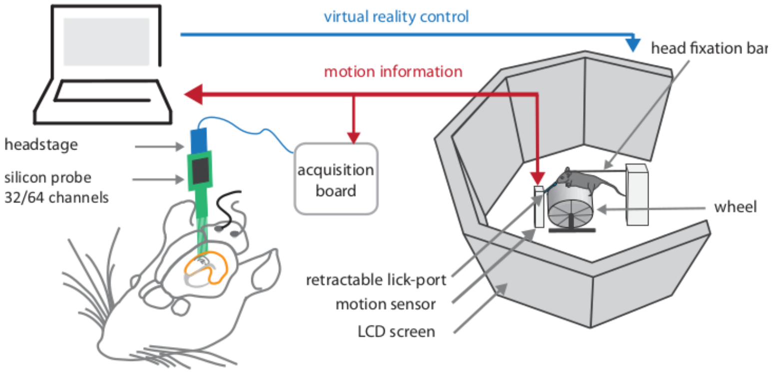

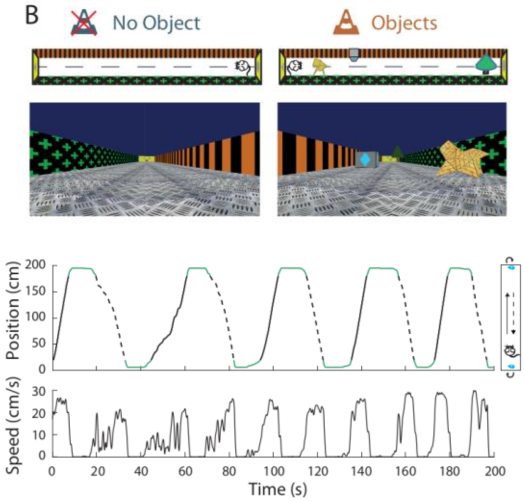

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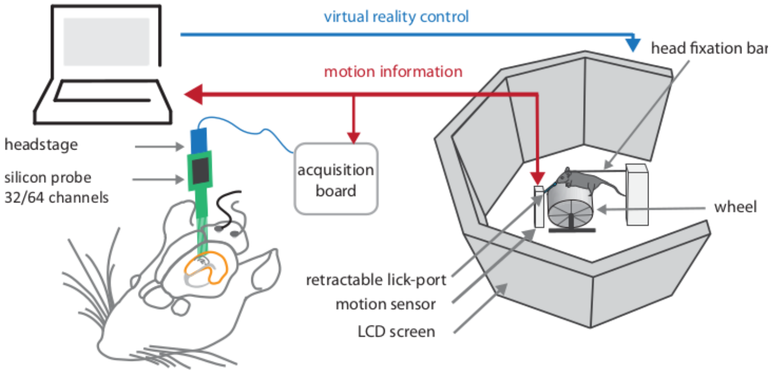

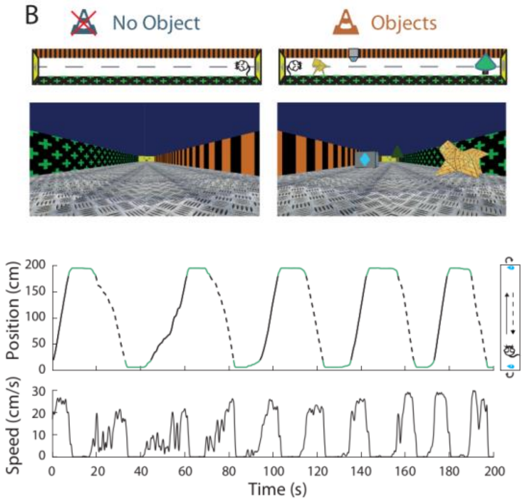

Obtained data :

N time series of activation of hippocampal neurons

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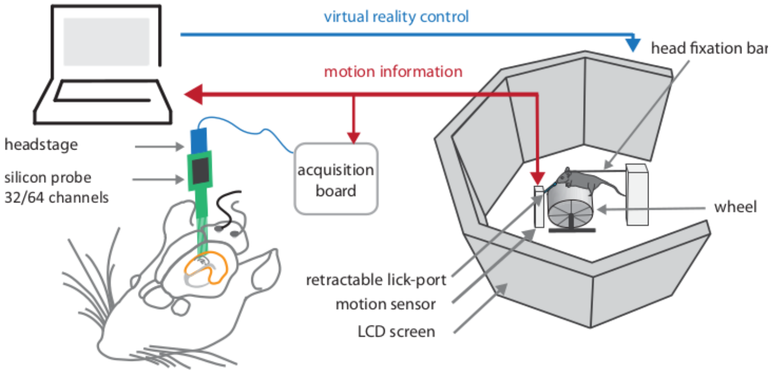

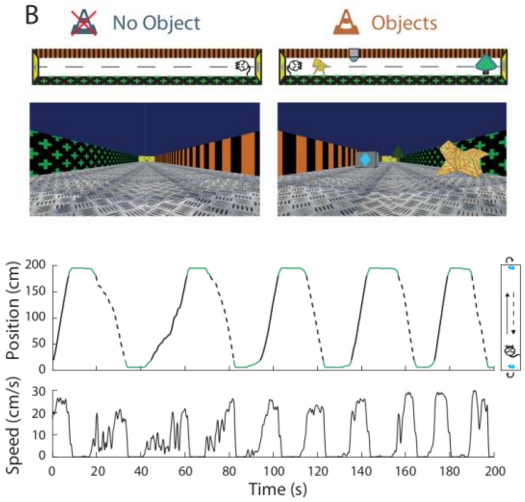

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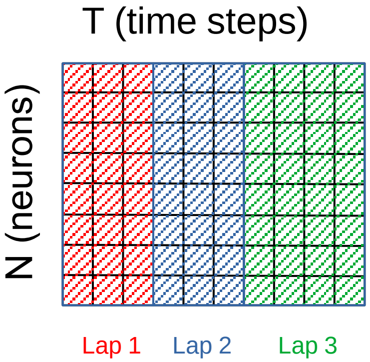
After binning and preprocessing :
matrix of firing rates, N * T



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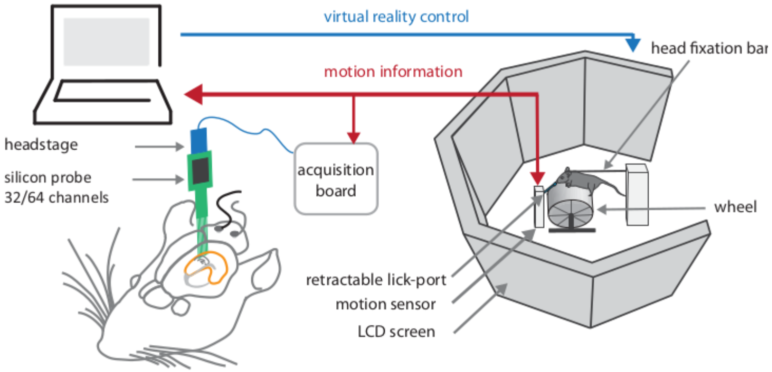

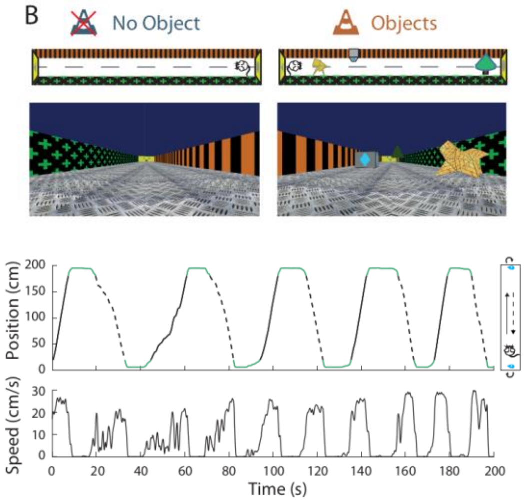

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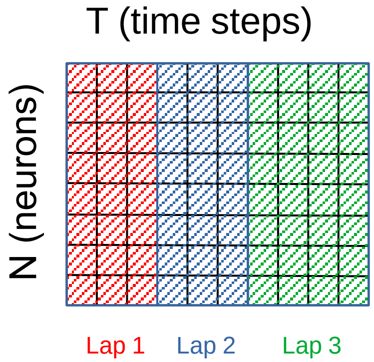
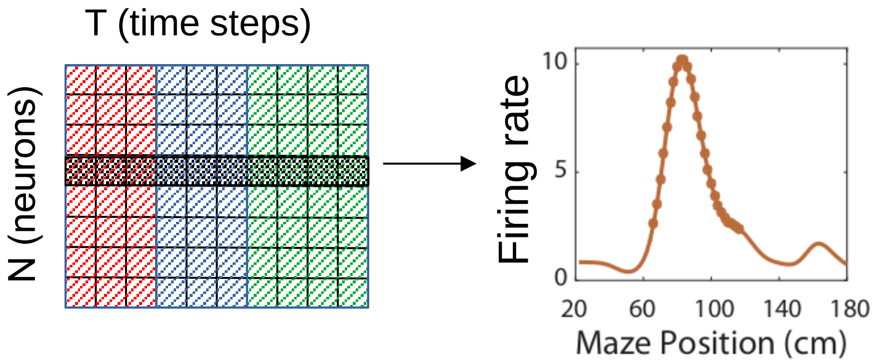


Does the activity state code for mouse's position in the corridor ?

I) Single cell analysis, averaging over laps



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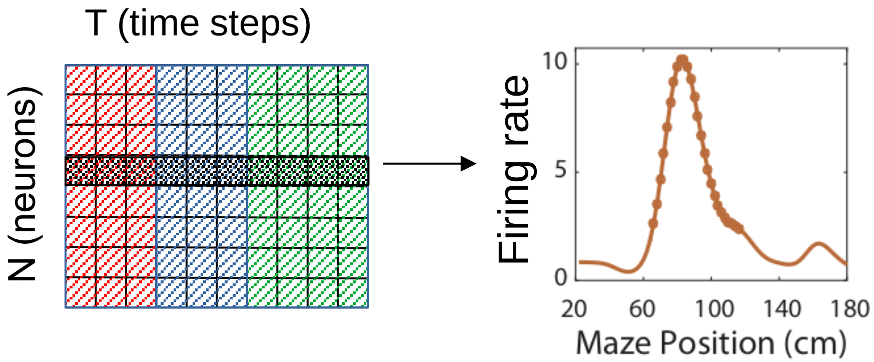

« Place cell »

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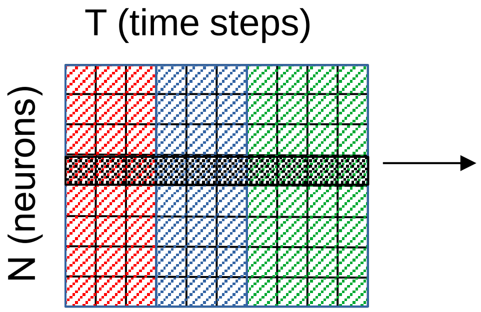
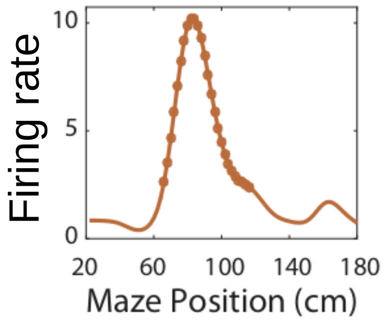
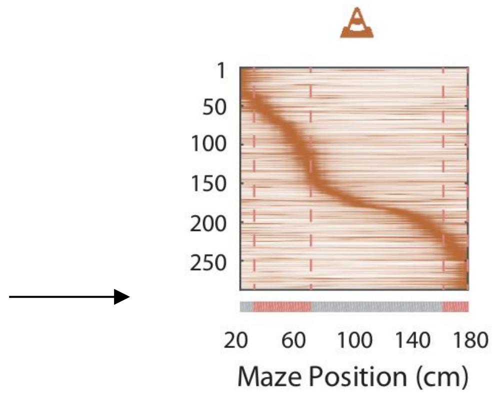




« Place cell »

N rate maps



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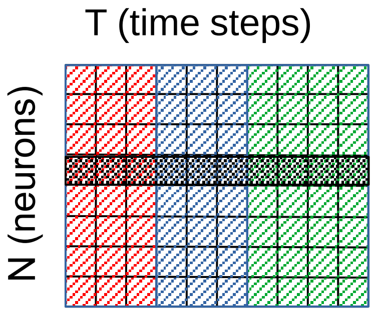

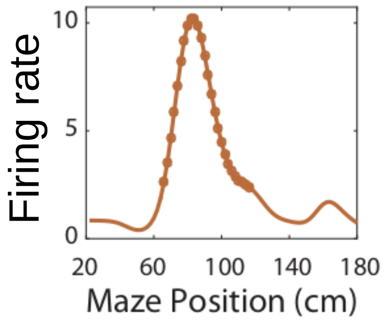

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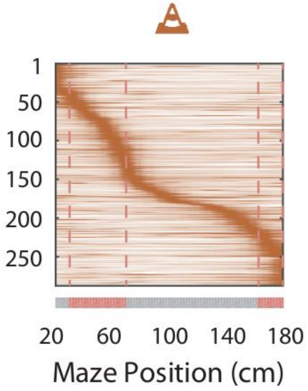


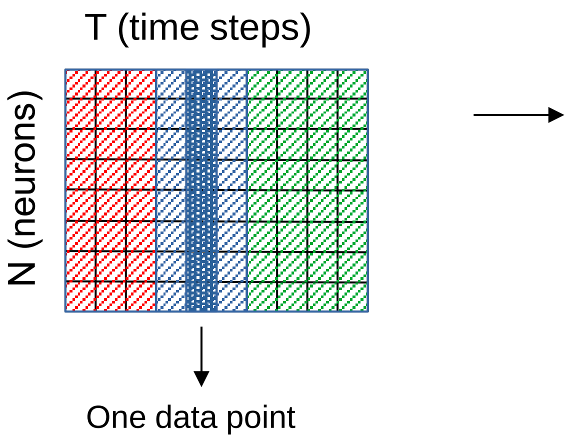
Advantages :

- can detect the most informative cells

Cons :

- assume a coding by place fields
- lose of a lot of information
- sensible to experimental issues

II) Population analysis, no averaging



T points in a N-dimensional space

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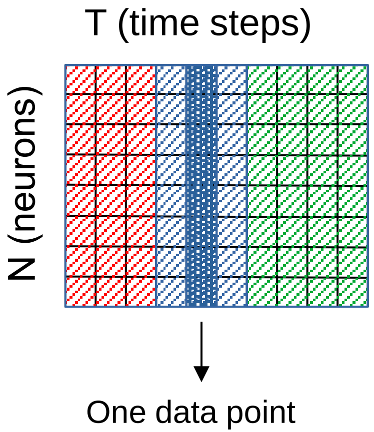


→ T points in a N-dimensional space

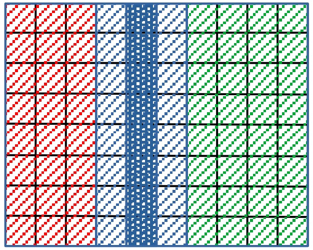
How are this points distributed ?
Can this distribution be matched on position in the maze ?

Two possible routes for analysis

II) Population analysis, no averaging

T (time steps)

N (neurons)



One data point

T points in a N-dimensional space

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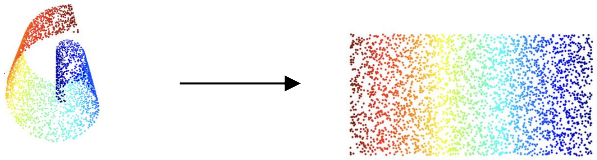
Dimensonality reduction tools



High dim

Low dim

II) Population analysis, no averaging

T (time steps)

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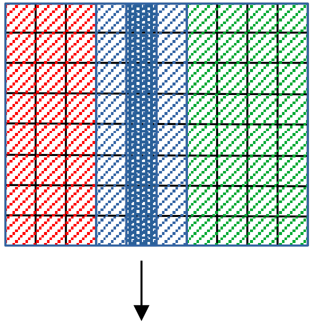

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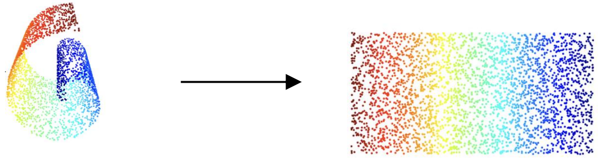


High dim → Low dim

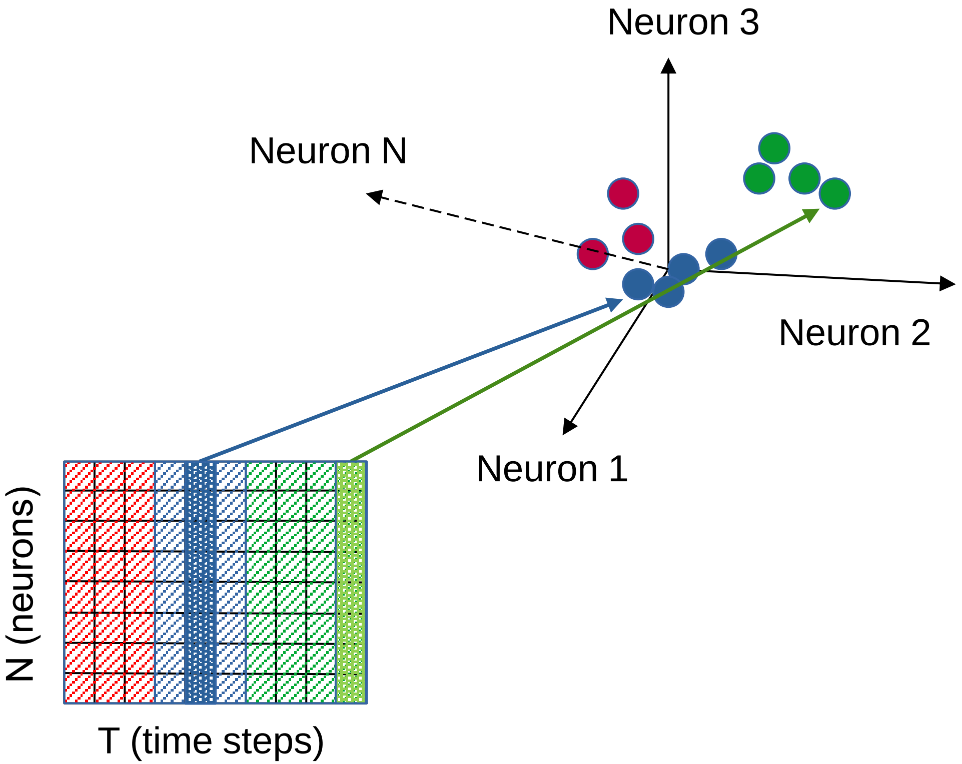
Advantages :

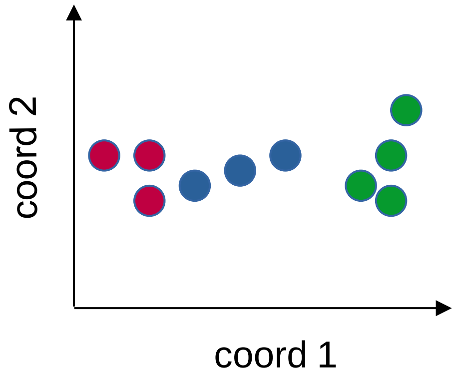
- no a priori about the coding
- use all the information
- no averaging over laps

Cons :

- signal could be lost if only a small fraction of cells are relevant

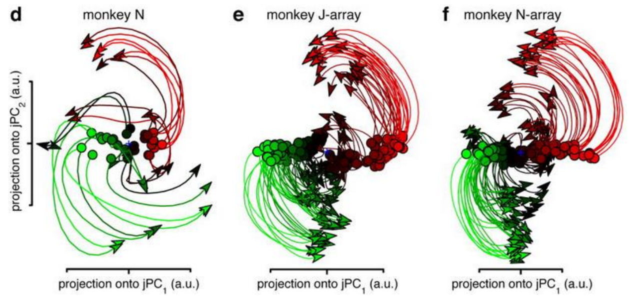
Find a projection in a low dim space preserving the « structure » of the data





- Usual methods : linear projection, such as PCA

Activity in motor cortex of reaching monkey
[Churchland et al, Nature 2013]



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But only works if the data lies on a low dimension vectorial space

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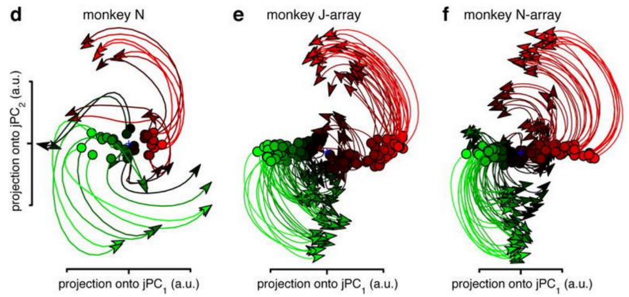

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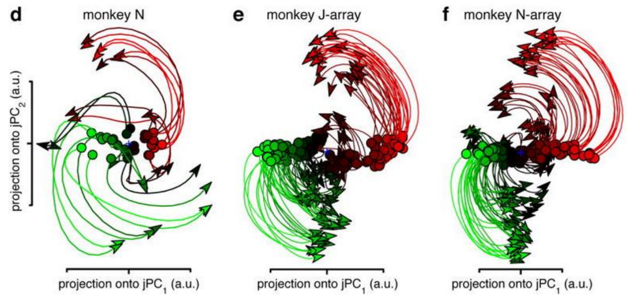

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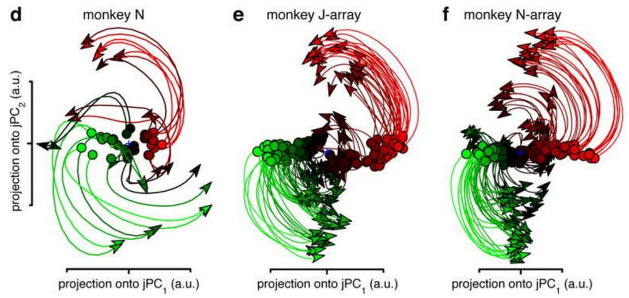

Head-direction cells activity
[Chandhuri et al, Nat Neurosci 2019]

Dimensionality reduction

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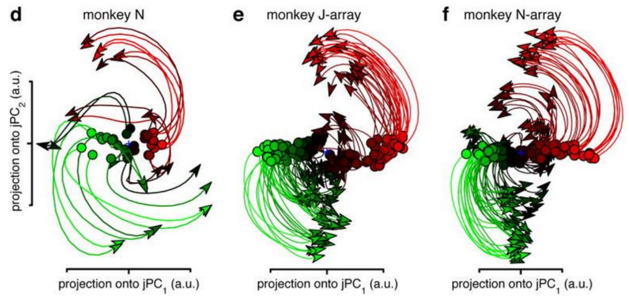

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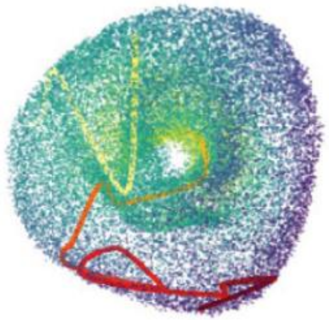
Same module grid-cells activity
[Gardner et al, Nature 2022]



[McInnes et al. arXiv 2020]

1) Compute all pair distances in the N-dim space

2) Find the best position in the low dim space such that the distribution of distances looks similar

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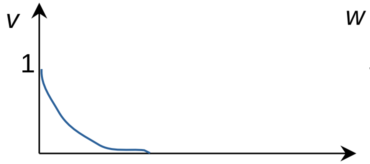
Mathematically : minimize the function

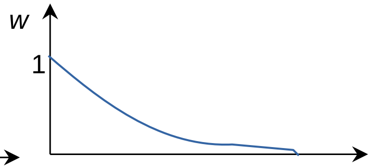
$$\mathcal{L}_{\mathrm{UMAP}} = -\sum_{i \neq j} v_{ij} \log(w_{ij}) + (1 - v_{ij}) \log(1 - w_{ij})$$

High dim space :



Distance in high dim space

Low dim space :



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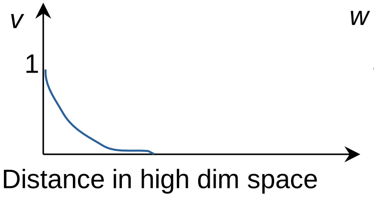
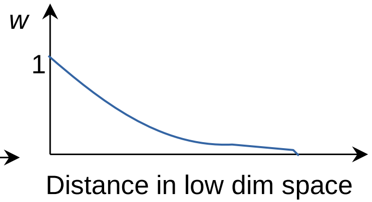
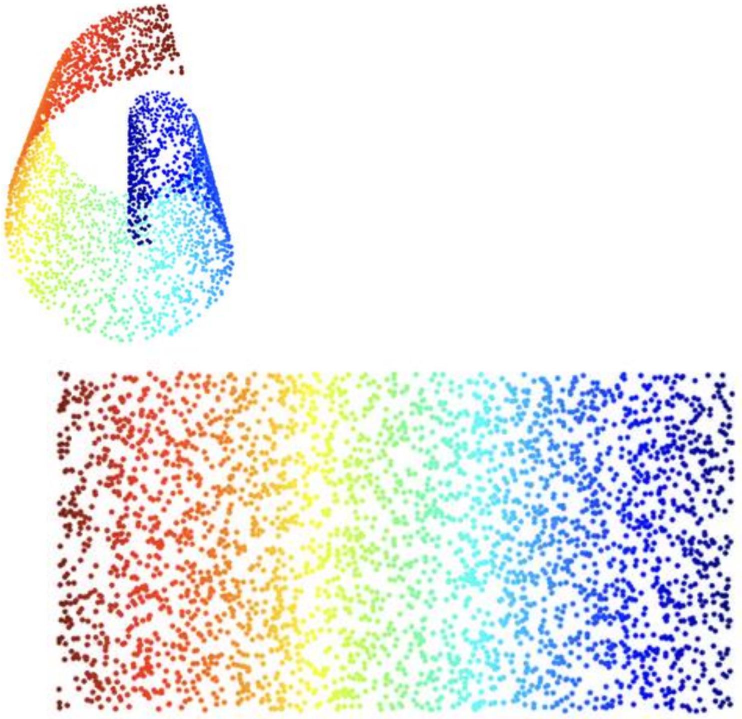


Low dim space :



Interpretation



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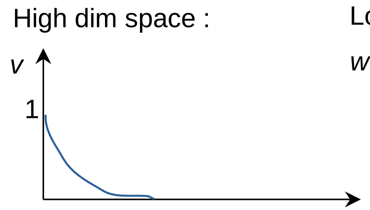

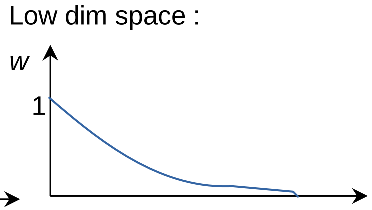

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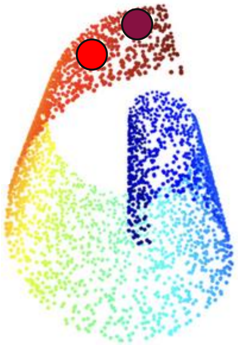
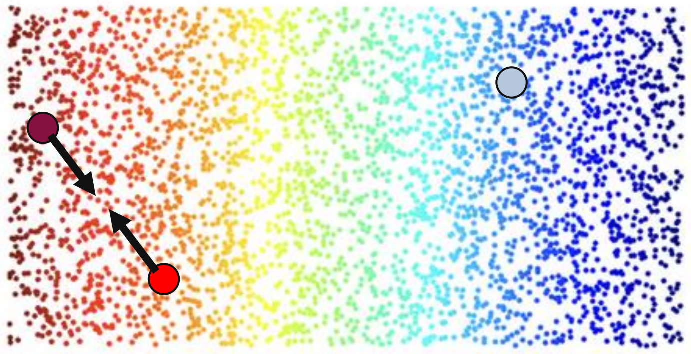


In the low dimension space :
- points that are close in the initial space will *attract*
- points that are far in the initial space will *repulse*



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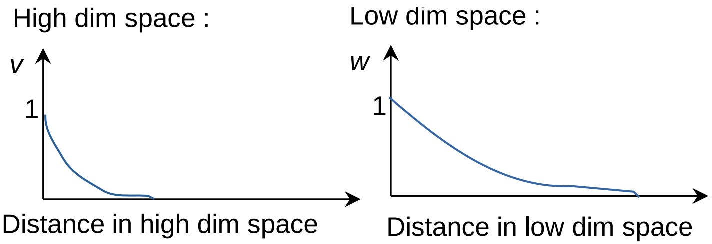

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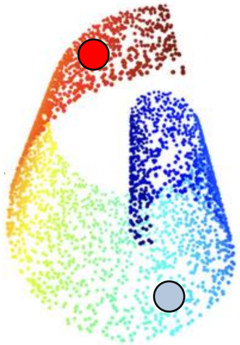

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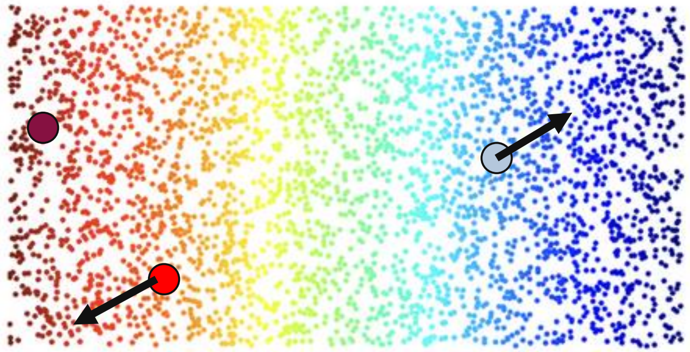
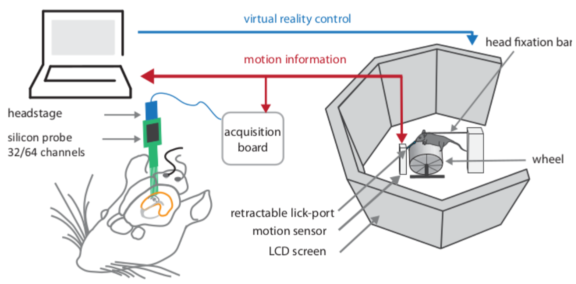
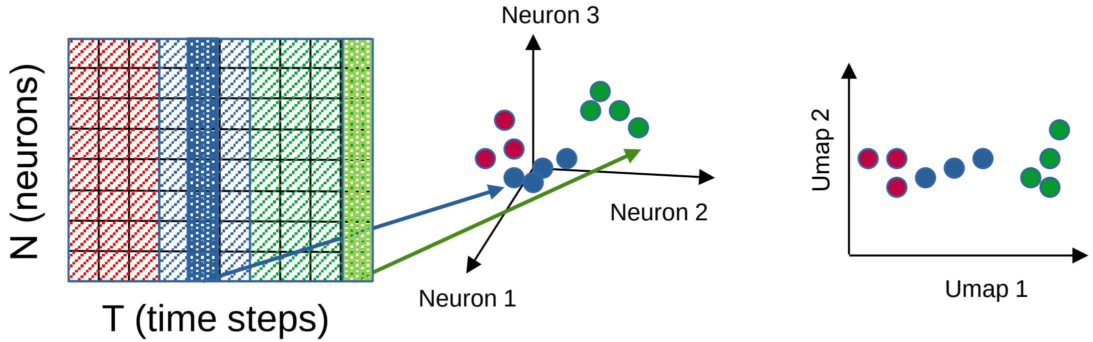


In the low dim space, points behave like a *gaz of particles* with pairwise interactions

Application to spatial coding without visual cues





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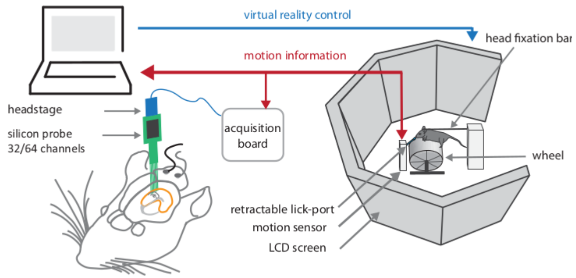
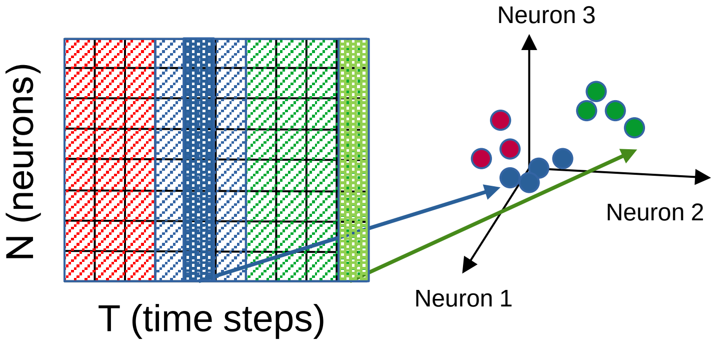
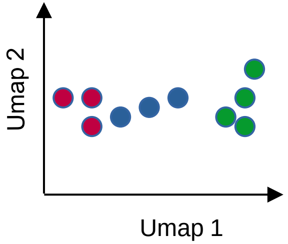
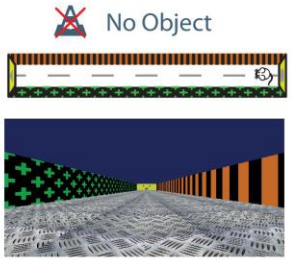
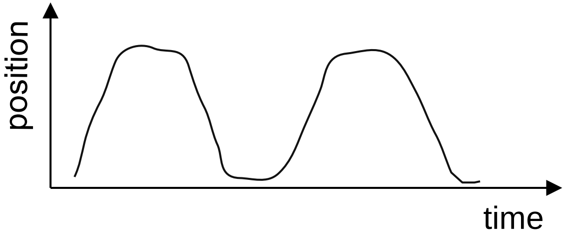






From dimension 64 to dimension 2





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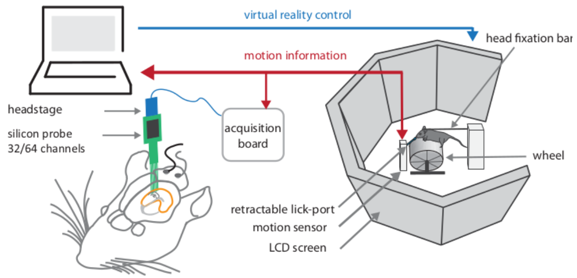

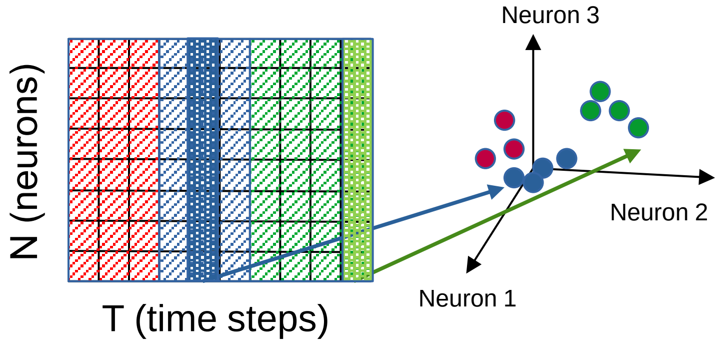

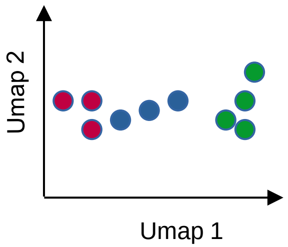

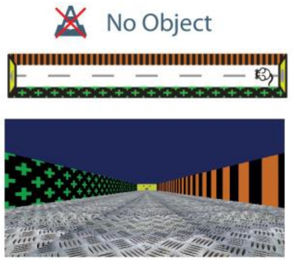

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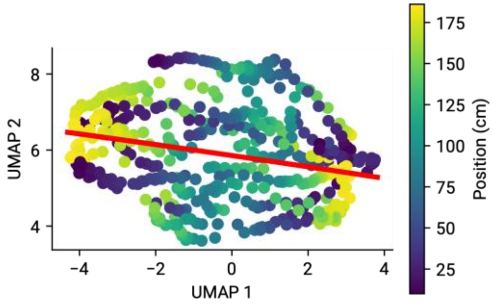

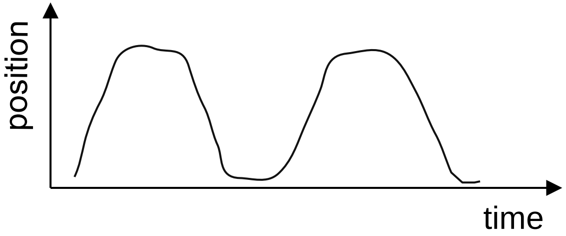

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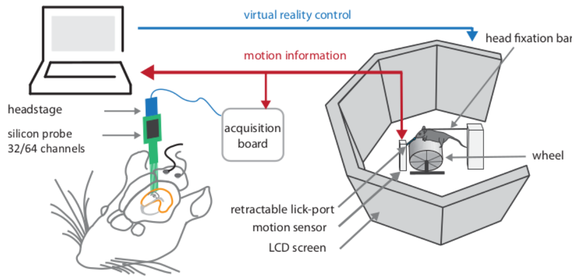

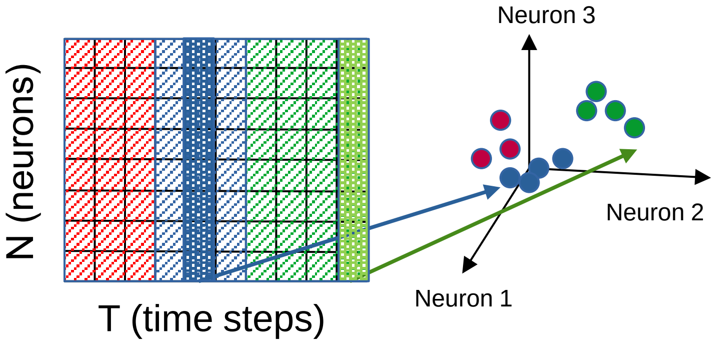

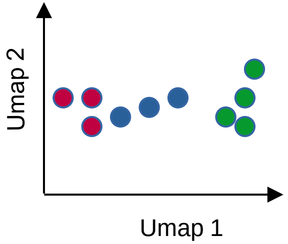

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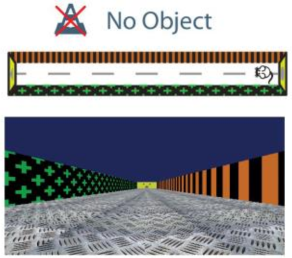
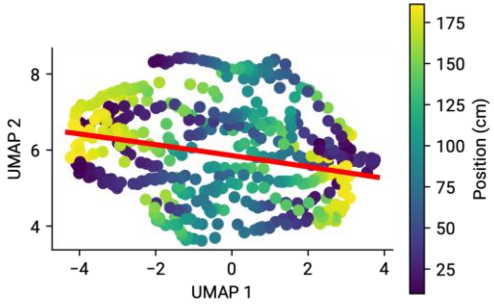
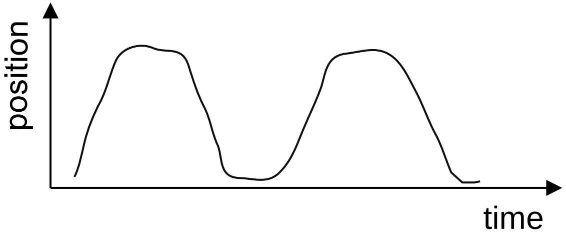
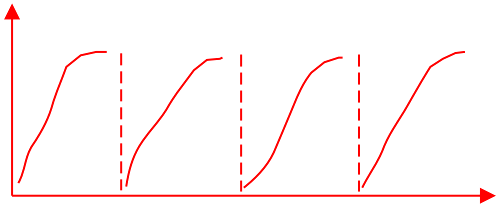
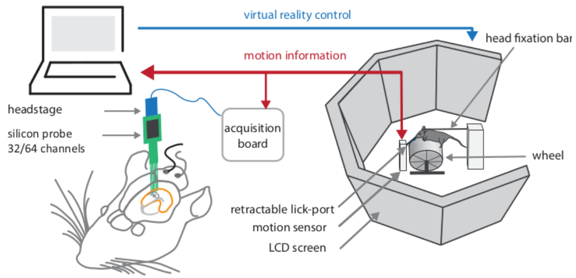


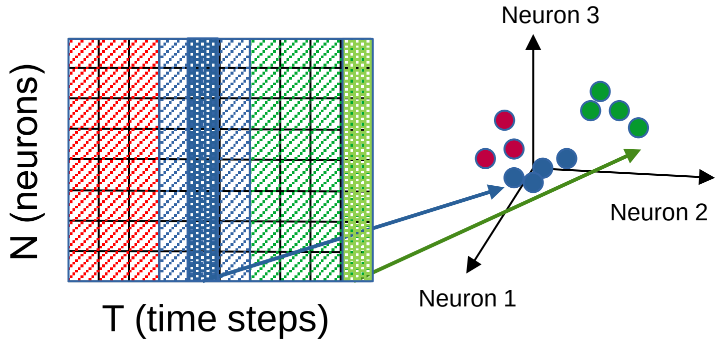


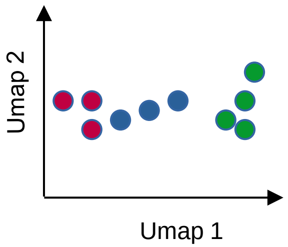




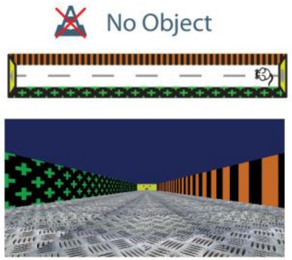
Application to spatial coding without visual cues

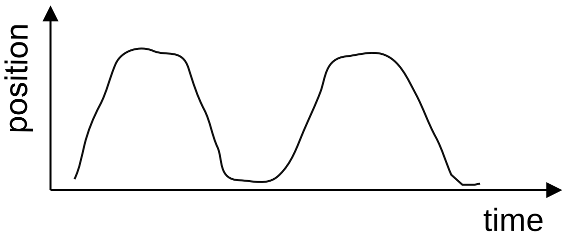


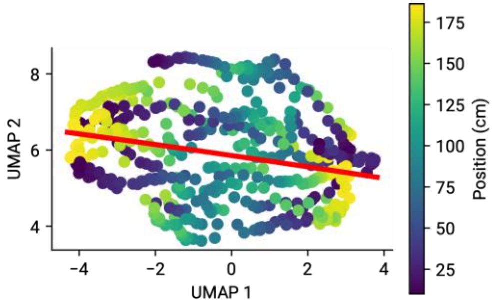


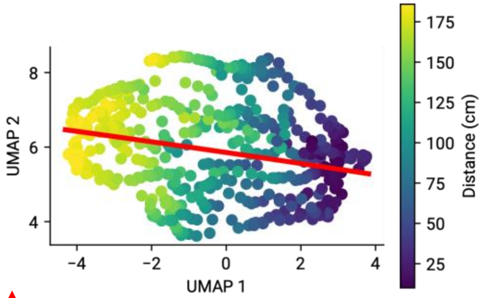


From dimension 64 to dimension 2

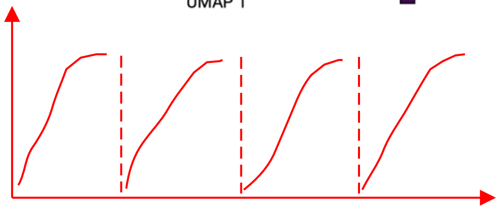


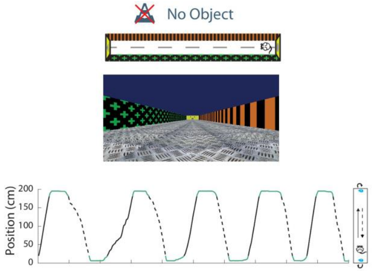


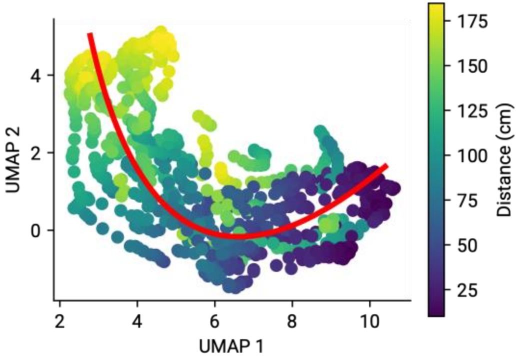




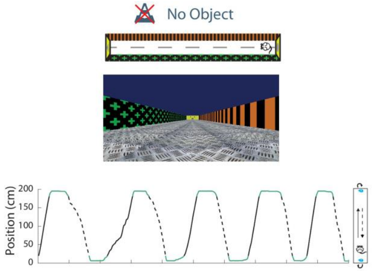
Path integration

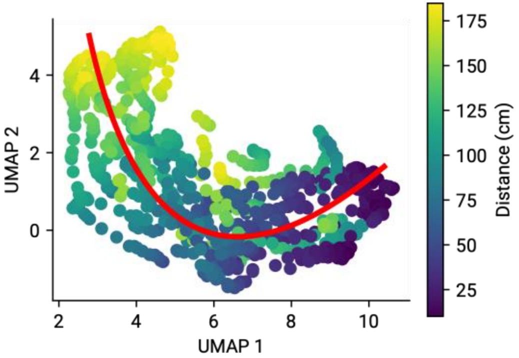


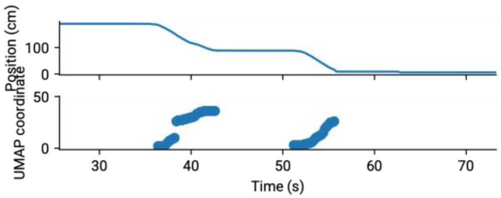

No Object
Position (cm)
200
150
100
50
0

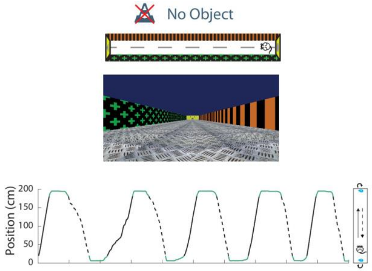

UMAP 2
4
2
0
2
4
6
8
10
UMAP 1
175
150
125
100
75
50
25
Distance (cm)

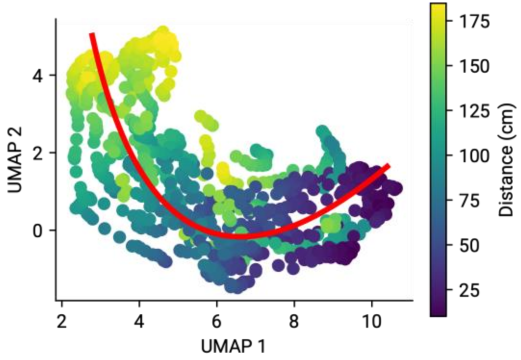
Application to spatial coding without visual cues

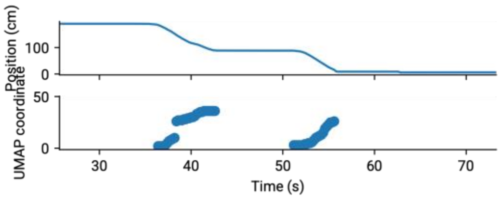


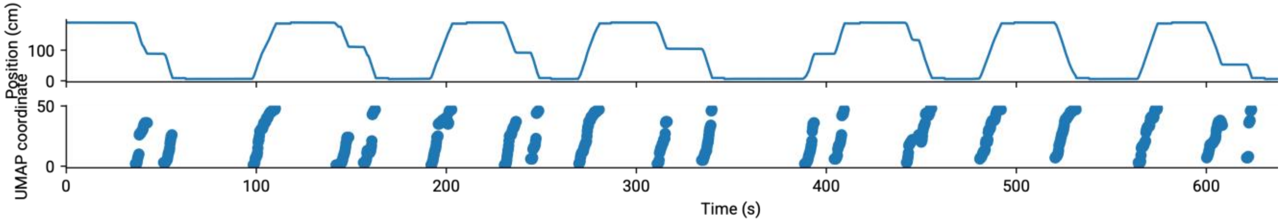




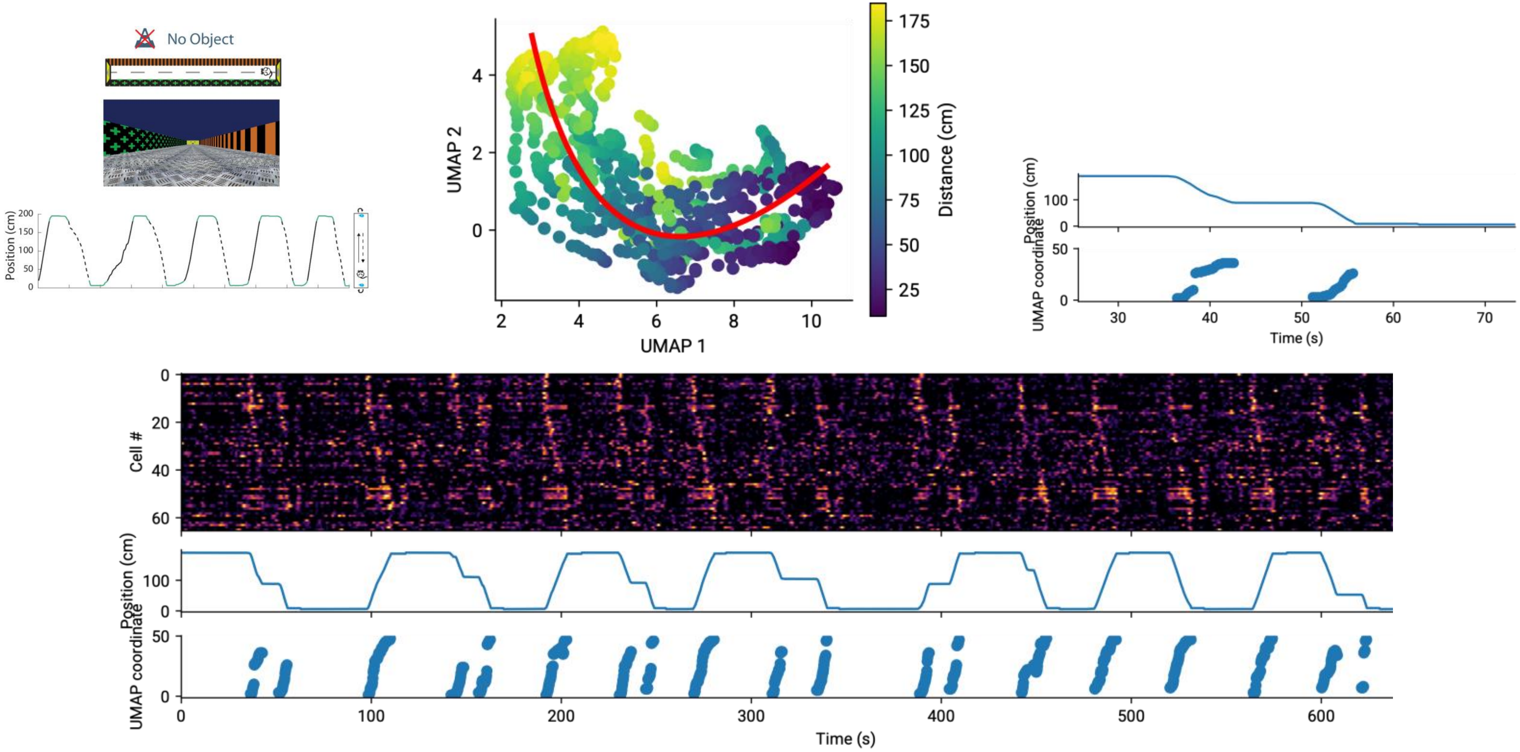






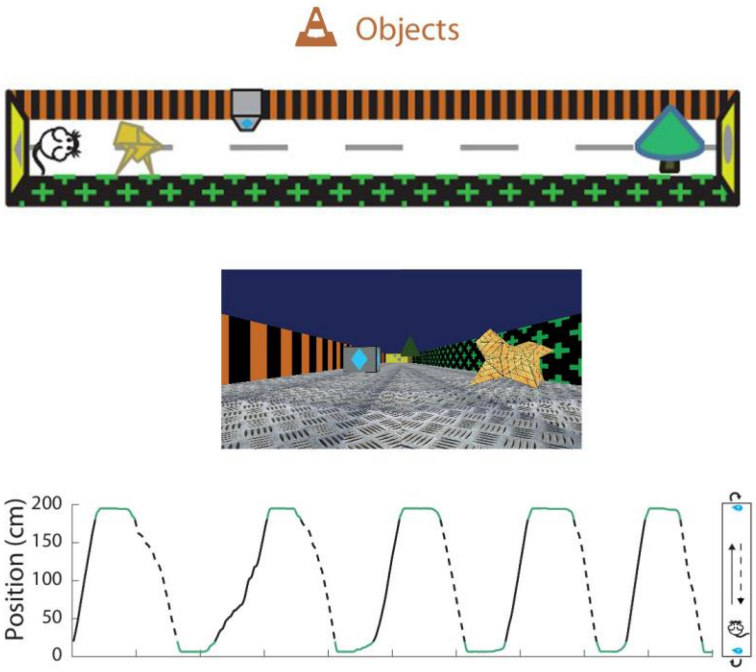


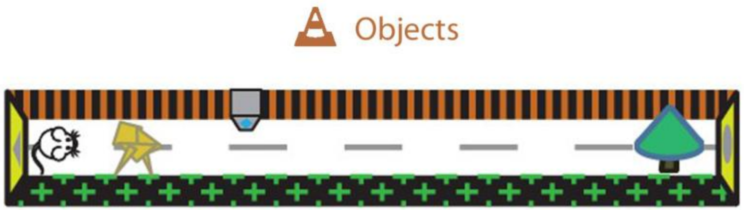
Stops : Resetting of path integration !

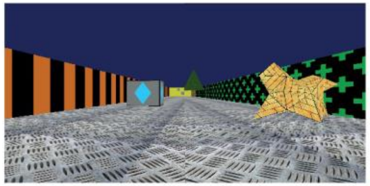


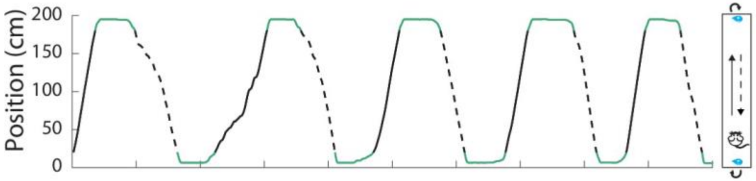
Stops : Resetting of path integration !

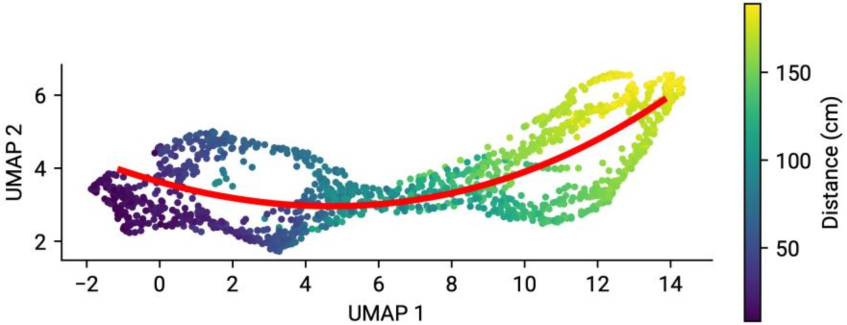
Application to spatial coding with visual cues



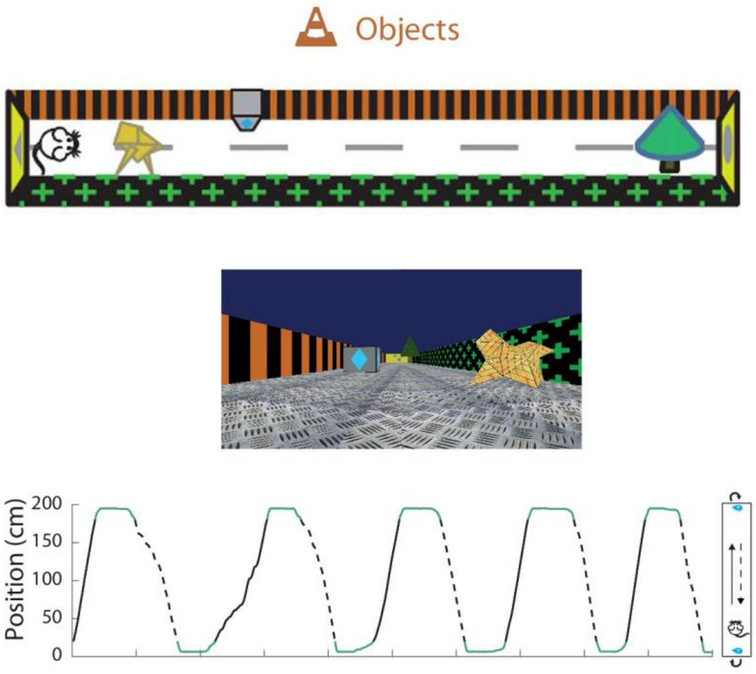

Objects

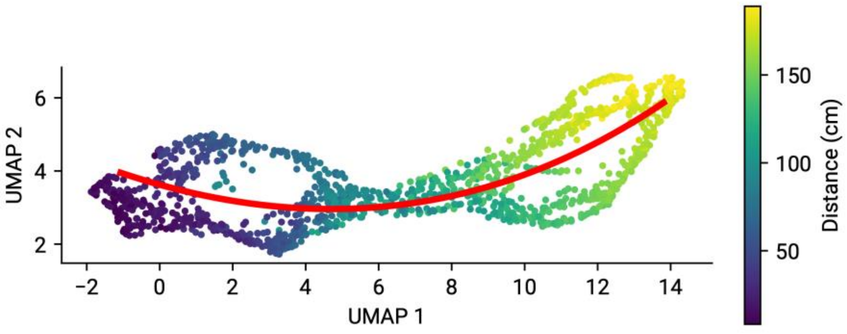


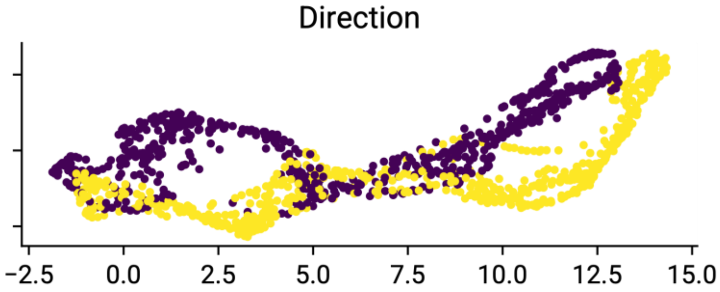

Position (cm)
200
150
100
50
0


UMAP 2
UMAP 1
Distance (cm)
150
100
50

Application to spatial coding with visual cues







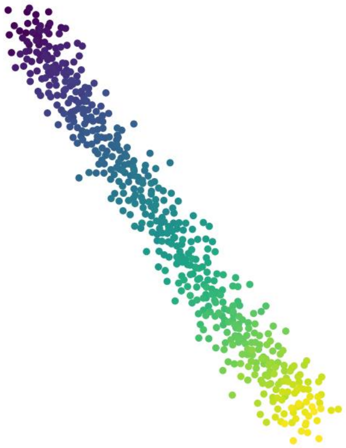
Cues affect the mental representation only locally

I) Experimental data and analysis with dimension reduction tools

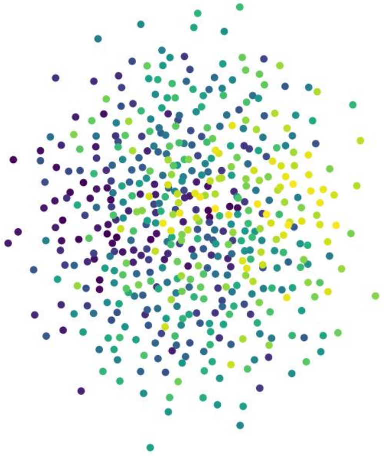
II) Theoretical analysis of this dimension reduction tool

Interpreting UMAP projection quantitatively ?

UMAP on a line in high dimension ***N***
+ noise of amplitude ***e***



Increase ***e***



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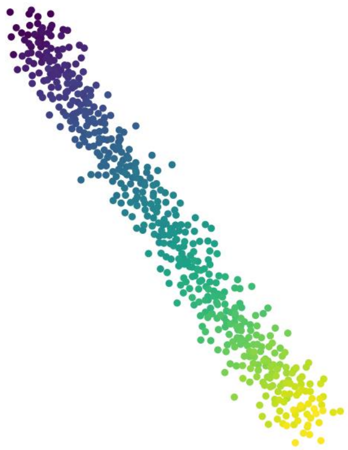
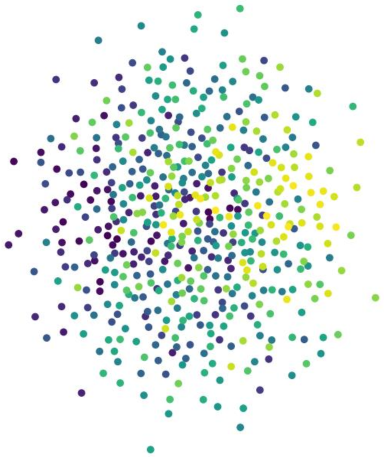


Increase ***e*** →



What is the critical noise **e** to detect signal ?

Can we infer signal-noise ratio from UMAP representation ?

Is UMAP optimal ?

Statistical physics tools !

Coulomb gas with quenched charges

$$\mathcal{L}_{\text{UMAP}} = -\sum_{i \neq j} v_{ij} \log(w_{ij}) + (1 - v_{ij}) \log(1 - w_{ij})$$

$$v_{ij} = \exp\left(-\frac{d_{ij} - \rho_i}{\sigma}\right) + \exp\left(-\frac{d_{ij} - \rho_j}{\sigma}\right) - \exp\left(-\frac{2d_{ij} - \rho_i - \rho_j}{\sigma}\right)$$

$$w_{ij} = \frac{1}{1 + a\|\mathbf{y}_i - \mathbf{y}_j\|^{2b}}$$

a, b, σ : Umap parameters

Well-defined data : pure noise

Coulomb gas with quenched charges

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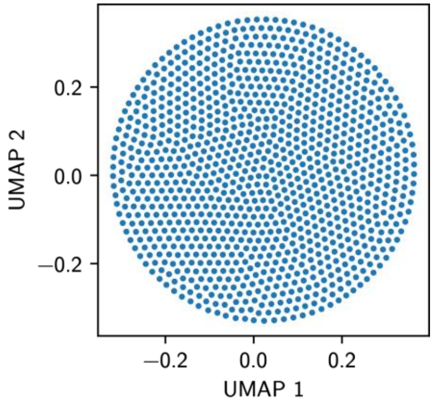
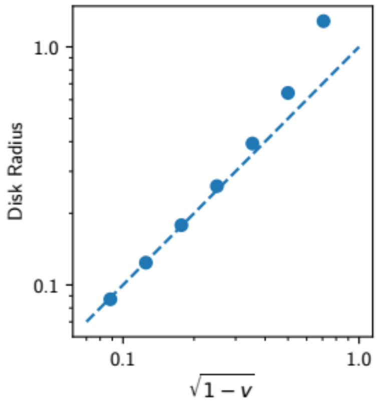
a, b, σ : Umap parameters

Uniform *v*





Girko-Ginibres’s circular law.

Coulomb gas with quenched charges

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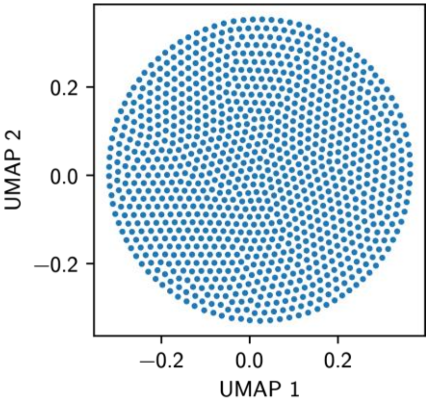
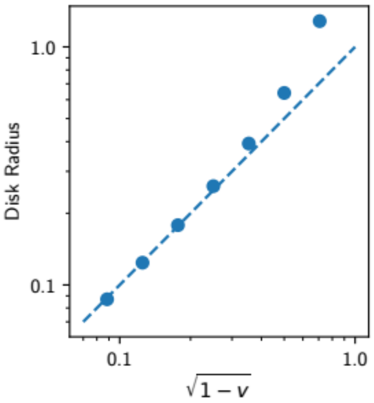
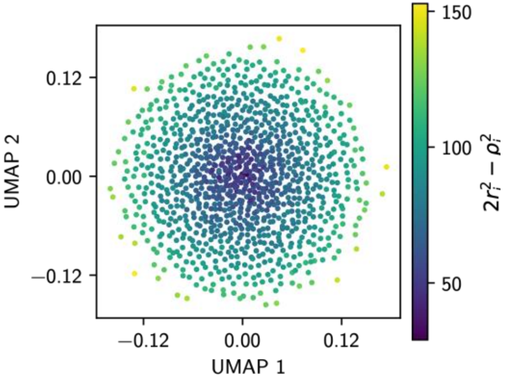
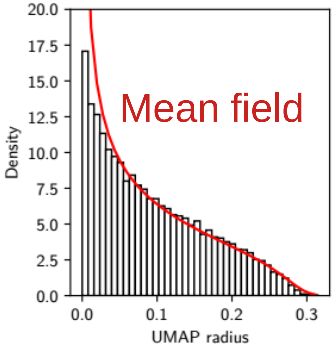




Girko-Ginibres’s circular law.

Normally iid points in high D space





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Binary signal

Two communities 1 and 2.

$$v_{11} = v_{22} = v$$

$$v_{12} = v - \delta v$$

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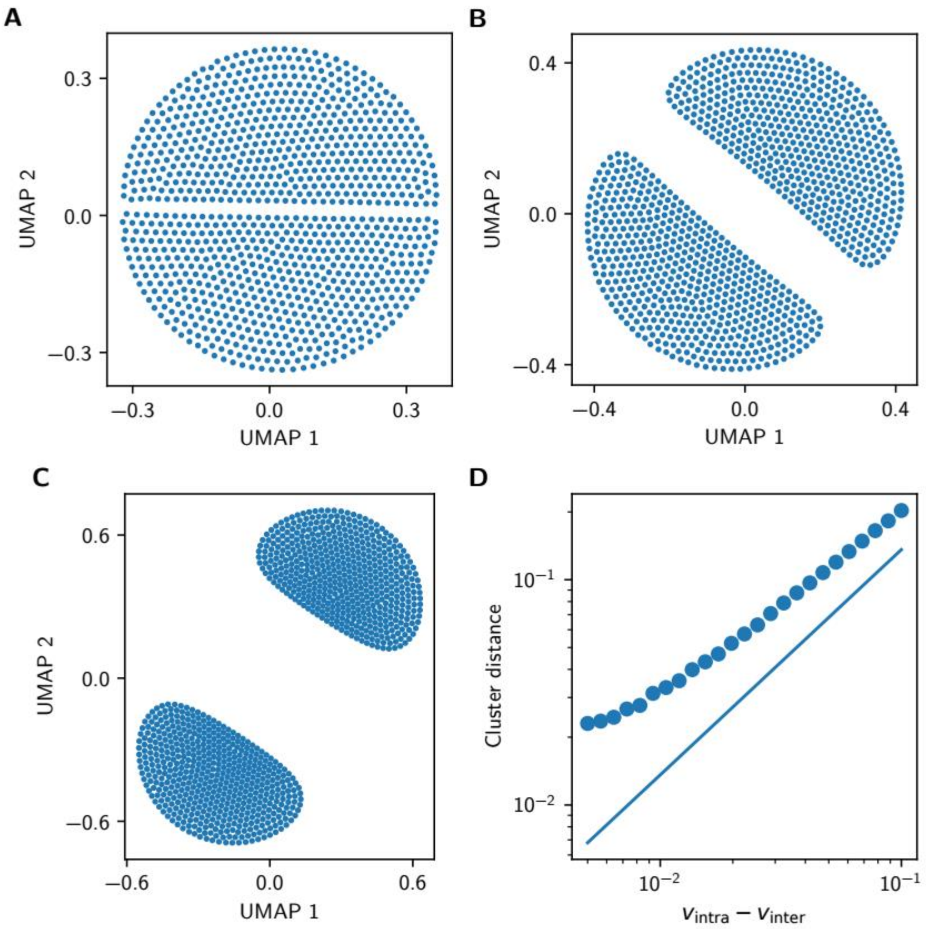

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Binary signal

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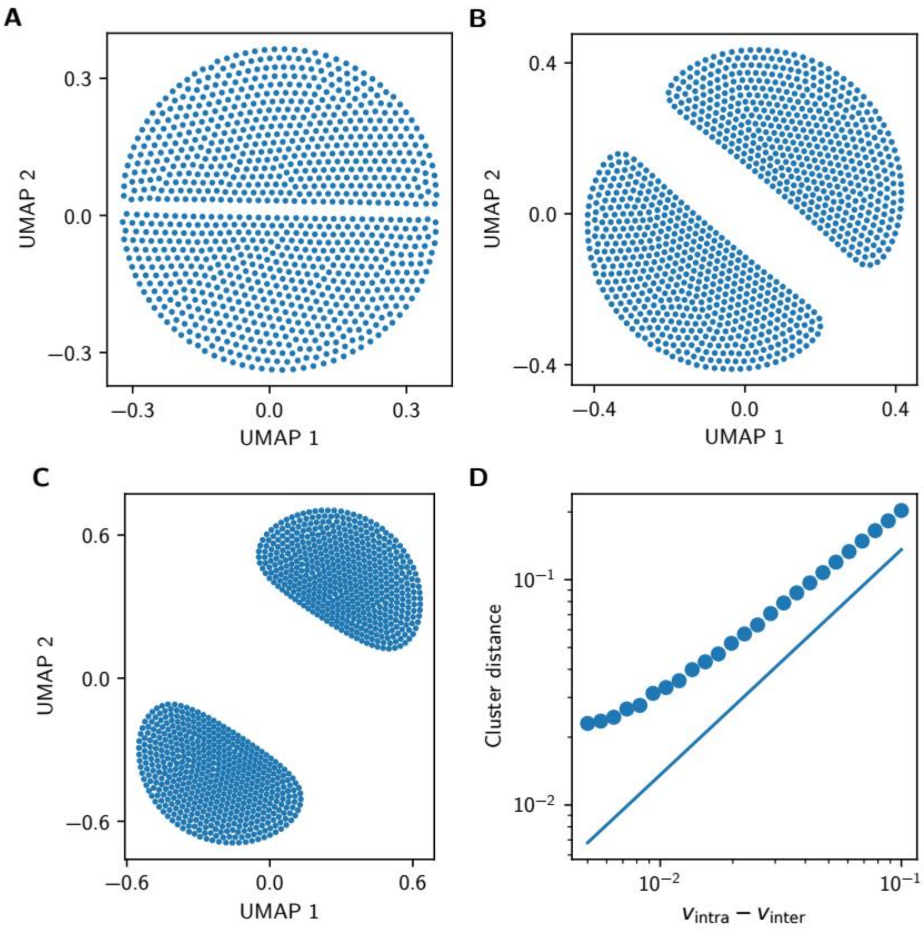
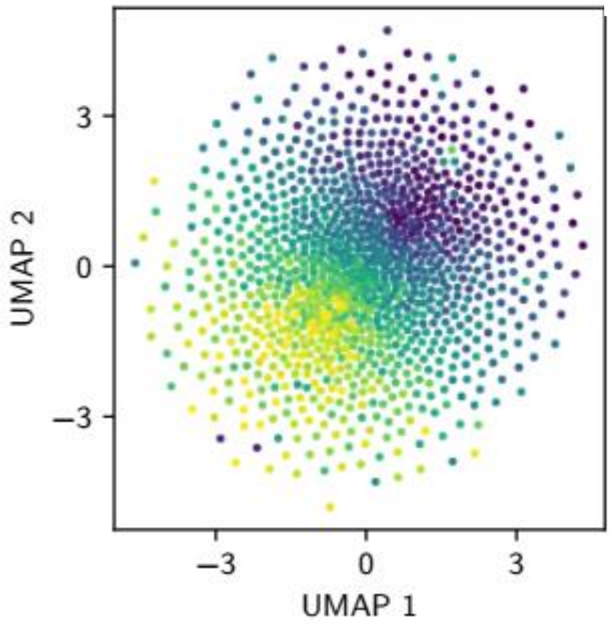
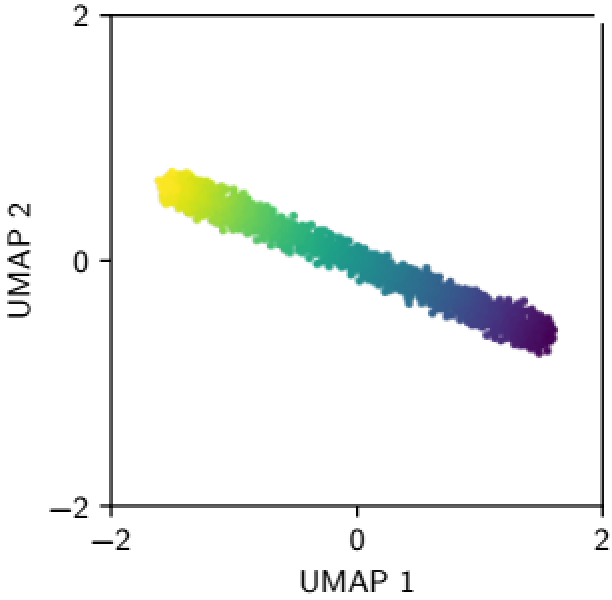
With noise, work in progress

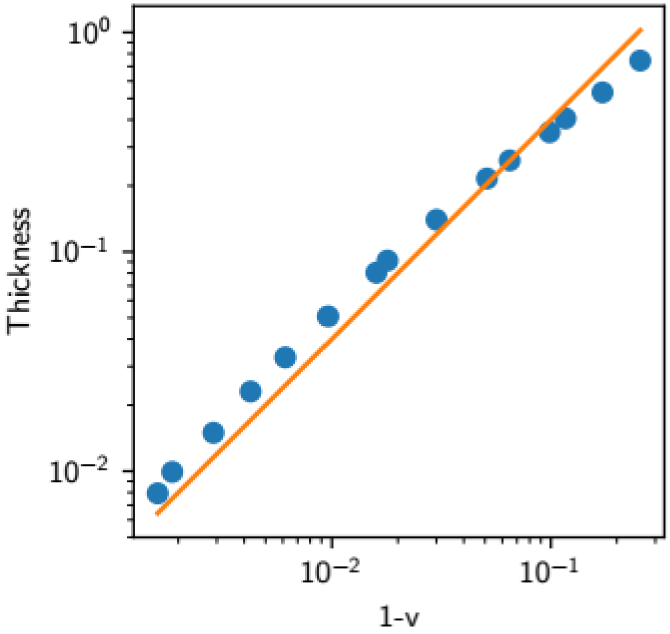


1d continuous signal

$$x_i^k = i/N\,\delta_{0k} + \epsilon\eta_i^k$$







Work in progress...

Conclusion

UMAP seems to be a good way to represent our data :

- use the **whole signal**
- **no a priori** about the coding scheme
- **no averaging**
- **can detect** some experimental issues

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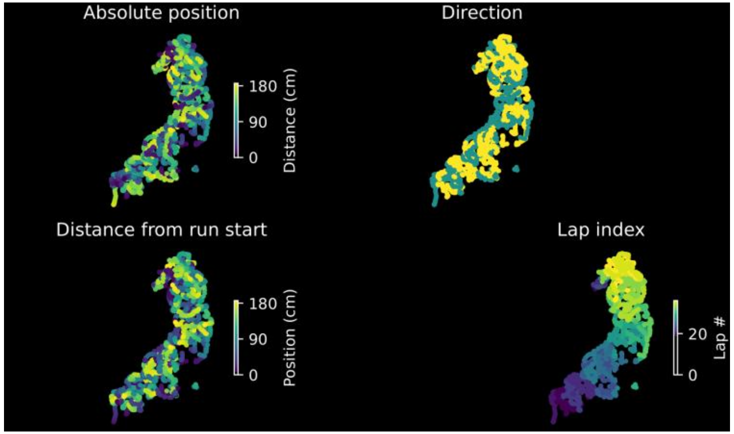

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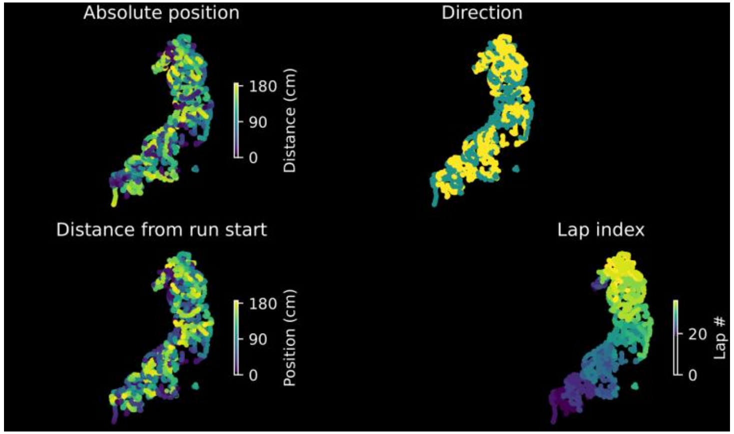

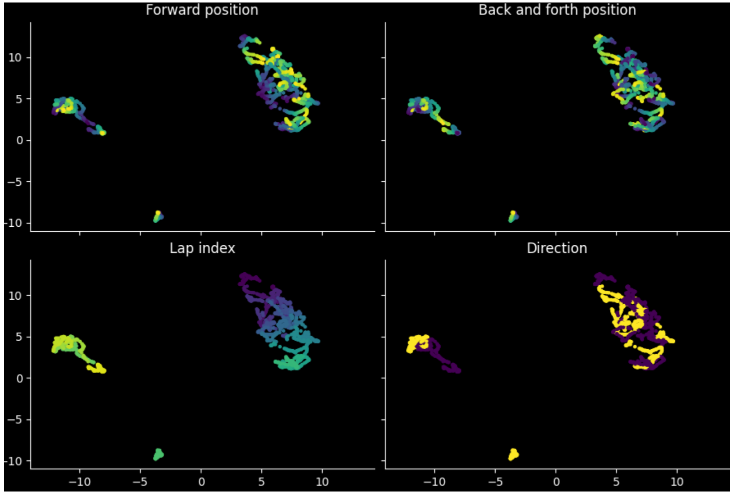

UMAP seems to be a good way to represent our data :

- use the **whole signal**
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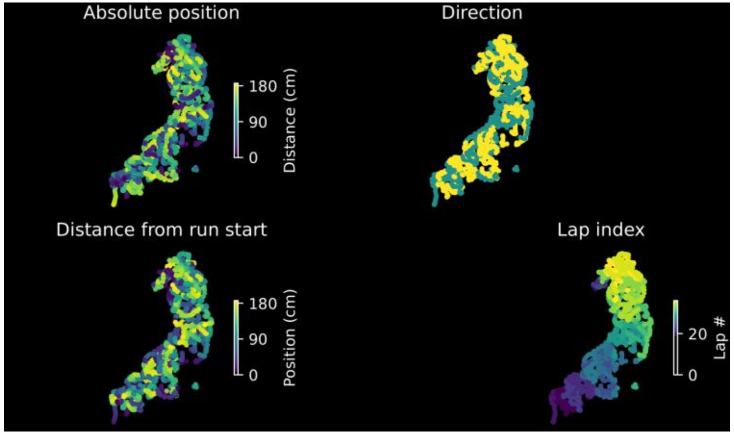
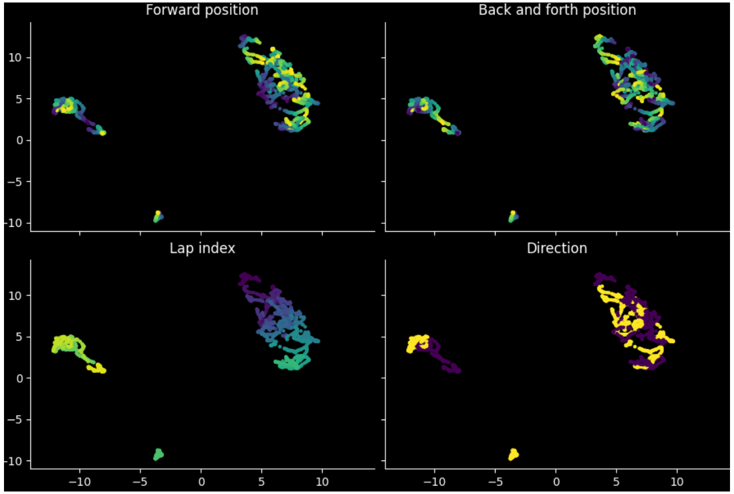
- **no averaging**
- **can detect** some experimental issues

We have highlighted the **robust path integration**.

On top of distance coding, some **position coding** emerge when objects are present.

Theoretical progess to use UMAP in a **quantitative** way, or to **improve** the alogrithm.





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Mathilde Nordlund, Jérôme Epsztein and Julie Koenig

Remi Monasson, Massimilliano Trippa