Efficient estimation of trainability for Variational Quantum Algorithms

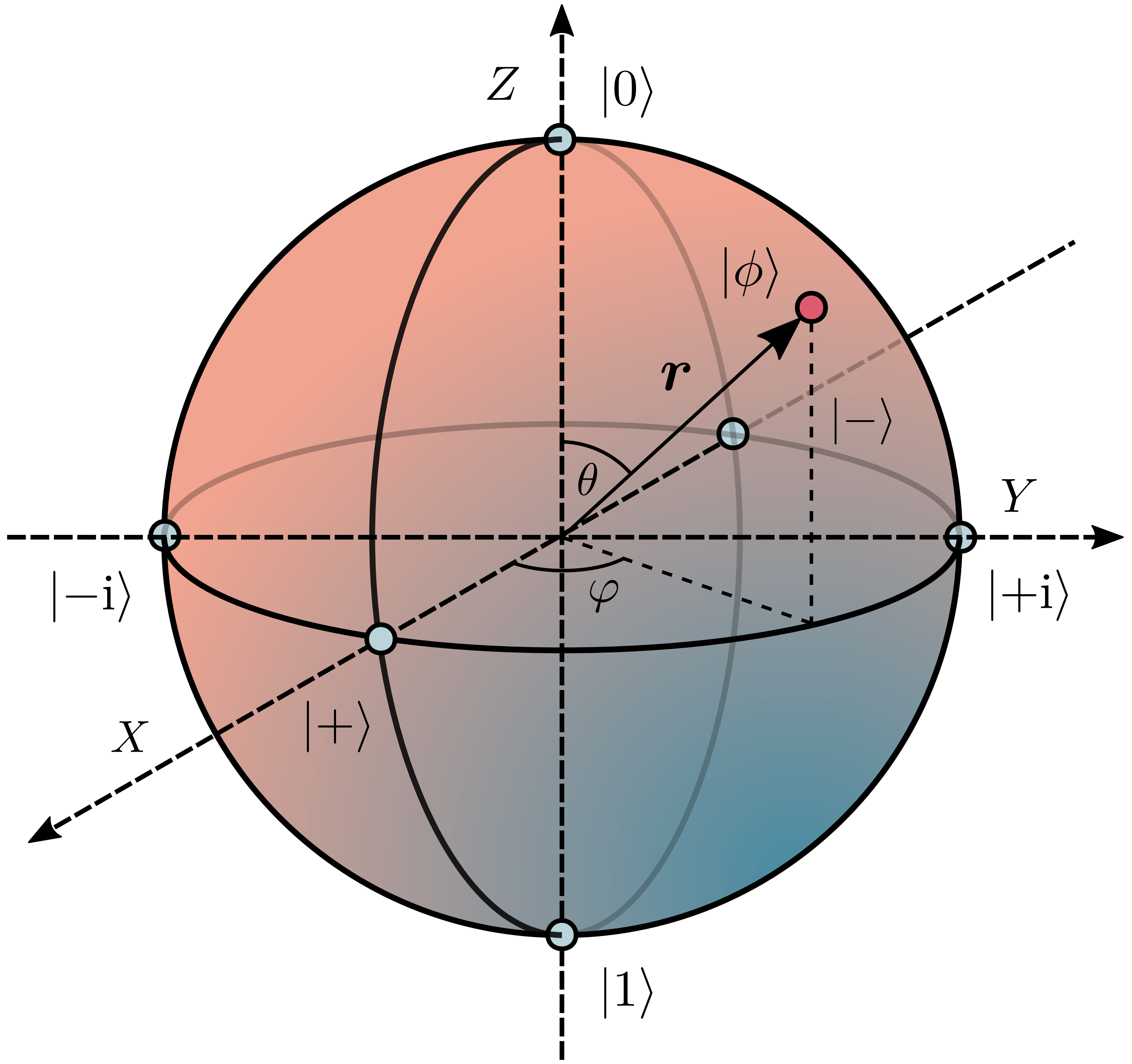
Congrès Général des 150 ans de la SFP
MC10 Physique et Intelligence Artificielle
06.07.2023

Valentin Heyraud

Lab. Matériaux et Phénomènes Quantiques - Université Paris Cité

"Efficient estimation of trainability for Variational Quantum Circuits",
V. Heyraud, Z. Li, K. Donatella, A. Le Boité, and C. Ciuti, arXiv:2302.04649

MPQ Université Paris Cité SIR TEQ cnrs



Summary

1. Variational Quantum Algorithms
2. Trainability and Barren Plateaus
3. Efficient estimation of the trainability
4. Perspectives

Variational Quantum Algorithms

Example: the Variational Quantum Eigensolver

McClean et al., “The theory of variational hybrid quantum-classical algorithms”, *New J. of Phys.* (2016)
Peruzzo et al., “A variational eigenvalue solver on a photonics quantum processor”, *Nat. Commun.* (2014)

Variational Quantum Algorithms

Example: the Variational Quantum Eigensolver

- Objective: Finding the ground state of a given N-spins Hamiltonian

$$\hat{H} = \sum_{\alpha} w_\alpha \hat{P}_\alpha \quad \text{with} \quad \hat{P}_\alpha \in \left\{ \hat{1}, \hat{X}, \hat{Y}, \hat{Z} \right\}^{\otimes N}$$

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- Variational state

$$|\phi_{\boldsymbol{\theta}}\rangle = \hat{U}\left(\boldsymbol{\theta}\right)|0\rangle^{\otimes N},$$

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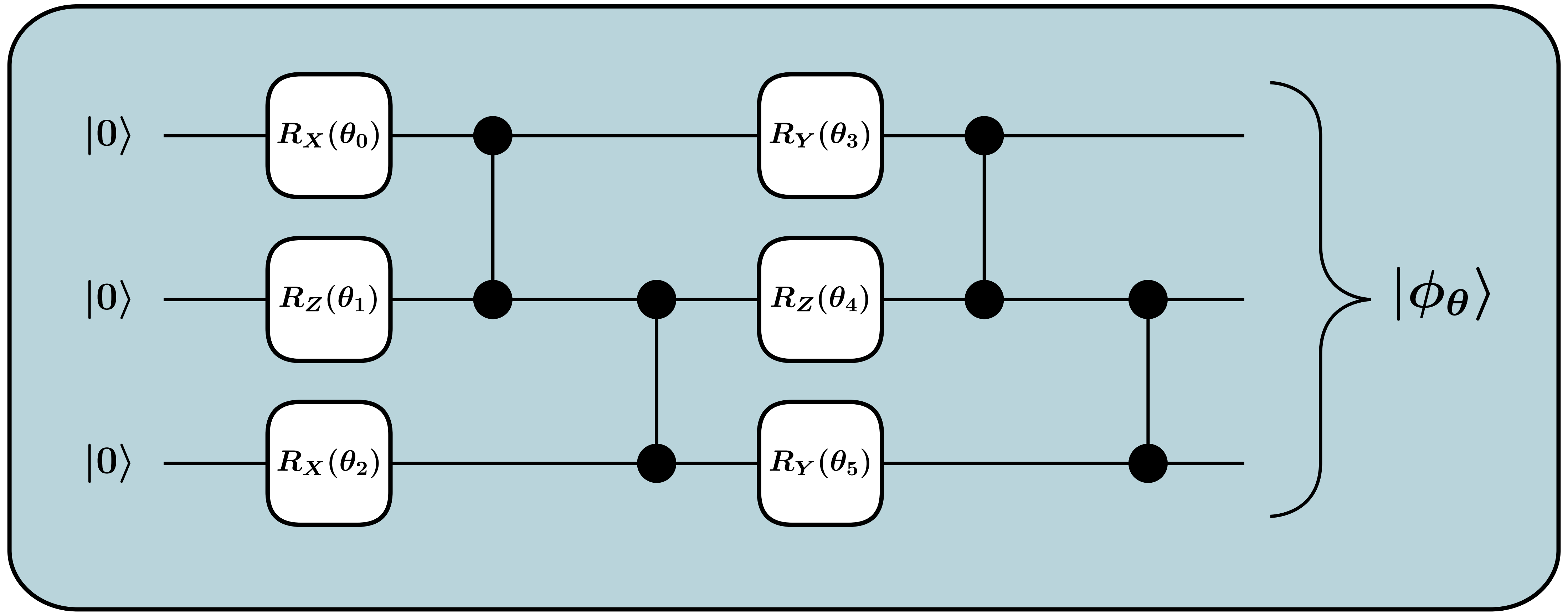

Hardware Efficient Ansatz circuit

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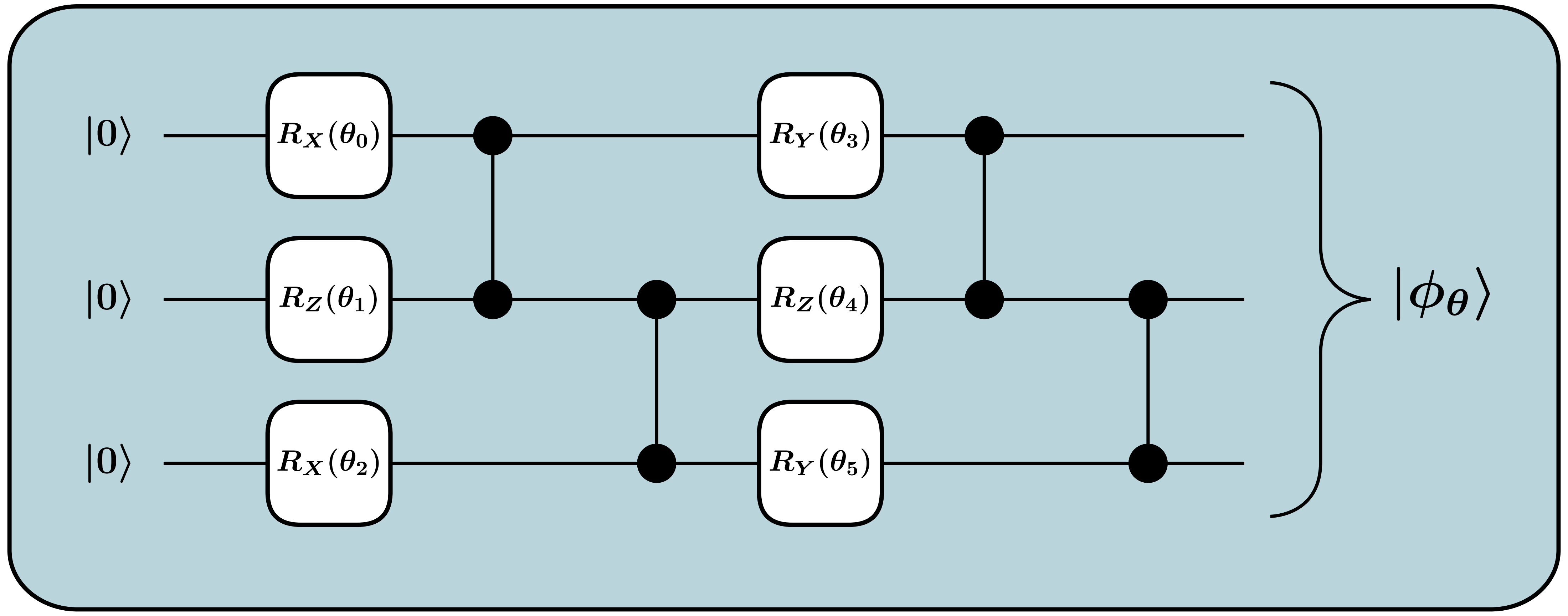
- Parameterized unitary

$$\hat{U}\left(\boldsymbol{\theta}\right) = \prod_{i=1}^{M} \left(e^{-\mathrm{i}\frac{\theta_\mathrm{i}}{2}\hat{\mathrm{P}}_\mathrm{i}} \right) \hat{W}_i \ , \qquad \hat{P}_i \in \left\{ \hat{X}, \hat{Y}, \hat{Z} \right\}$$



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- Minimization of the energy

$$E\left(\boldsymbol{\theta}\right) = \langle \phi_{\boldsymbol{\theta}} | \hat{H} | \phi_{\boldsymbol{\theta}} \rangle , \quad \boldsymbol{\theta}* = \text{argmin}\, E\left(\boldsymbol{\theta}\right)$$

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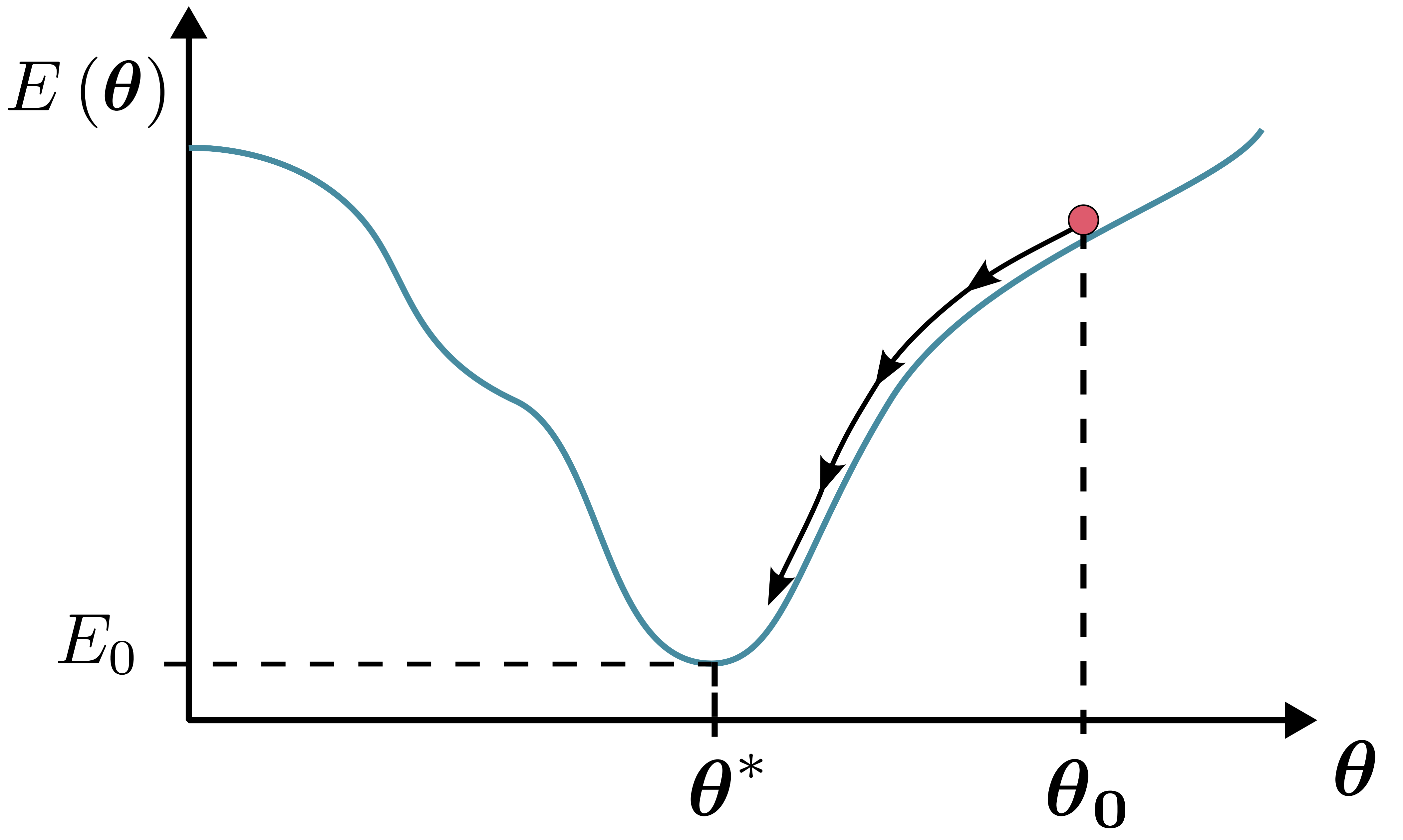

Gradient descent in the energy landscape

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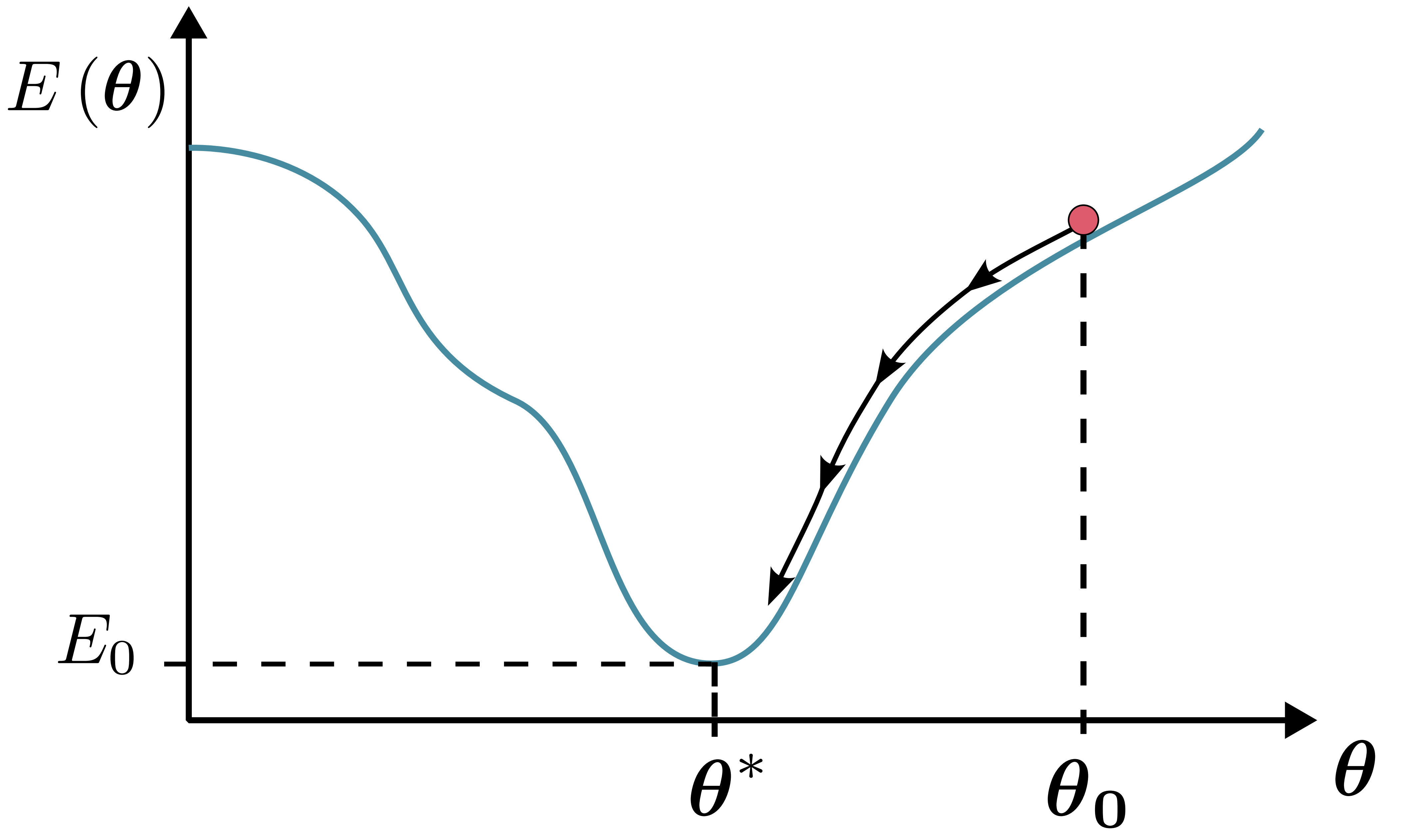
- Gradient descent

$$\boldsymbol{\theta}_{k+1} \leftarrow \boldsymbol{\theta}_k - \eta \nabla_{\boldsymbol{\theta}} E\left(\boldsymbol{\theta}_k\right),$$

$$\partial_i E\left(\boldsymbol{\theta}\right) = \frac{1}{2}\left(E\left(\boldsymbol{\theta} + \frac{\pi}{2}\boldsymbol{e}_i\right) - E\left(\boldsymbol{\theta} - \frac{\pi}{2}\boldsymbol{e}_i\right)\right)$$



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Trainability and Barren Plateaus

The Barren Plateaus phenomenon

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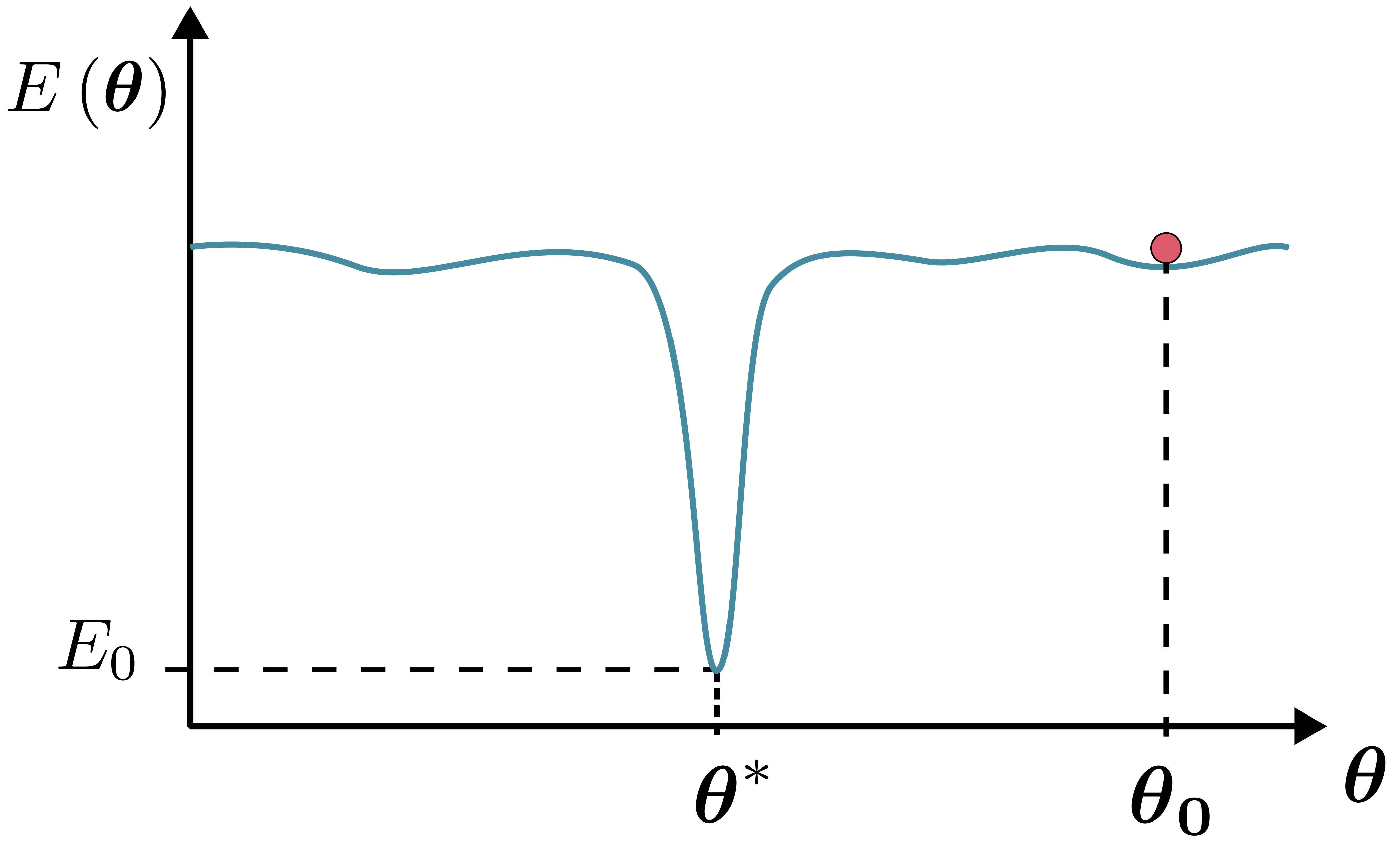

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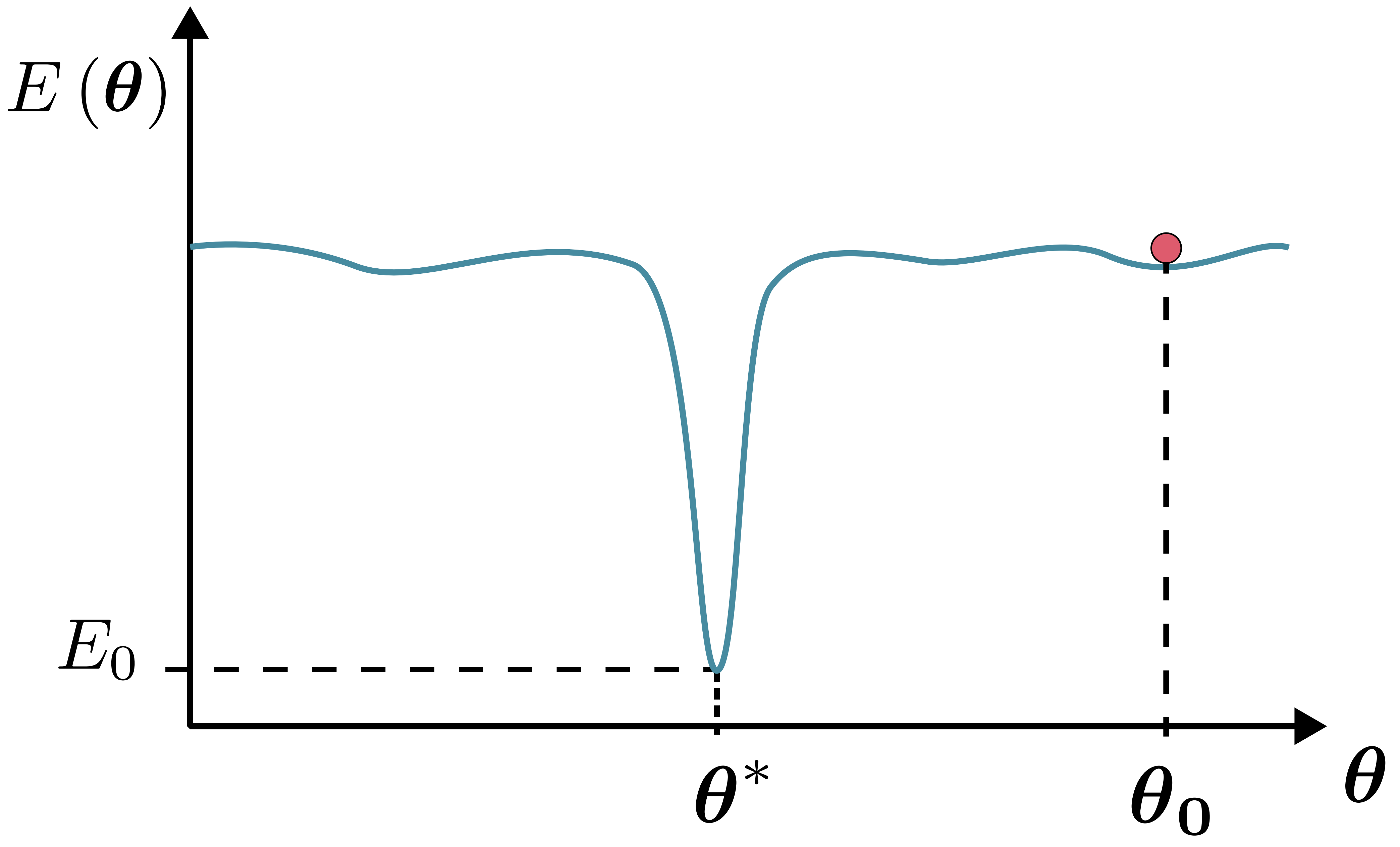
$$\mathbb{E}_{\boldsymbol{\theta}}\left[\partial_i E\left(\boldsymbol{\theta}\right)^2\right] \sim \mathcal{O}\left(\frac{1}{2^N}\right) \implies \mathbb{P}\left(\left|\partial_i E\left(\boldsymbol{\theta}\right)\right| \geq \epsilon\right) \leq \mathcal{O}\left(\frac{\epsilon^2}{2^N}\right)$$



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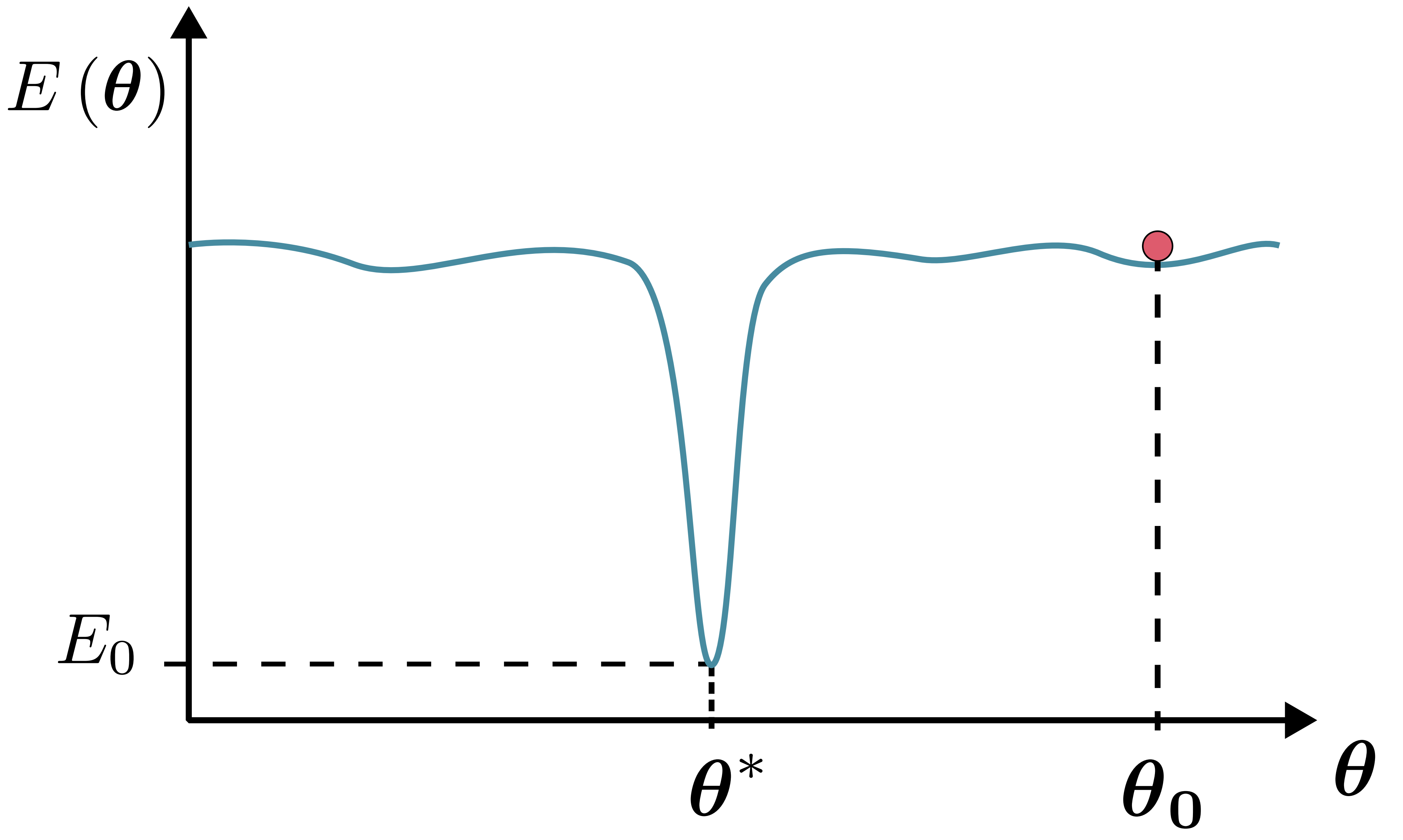

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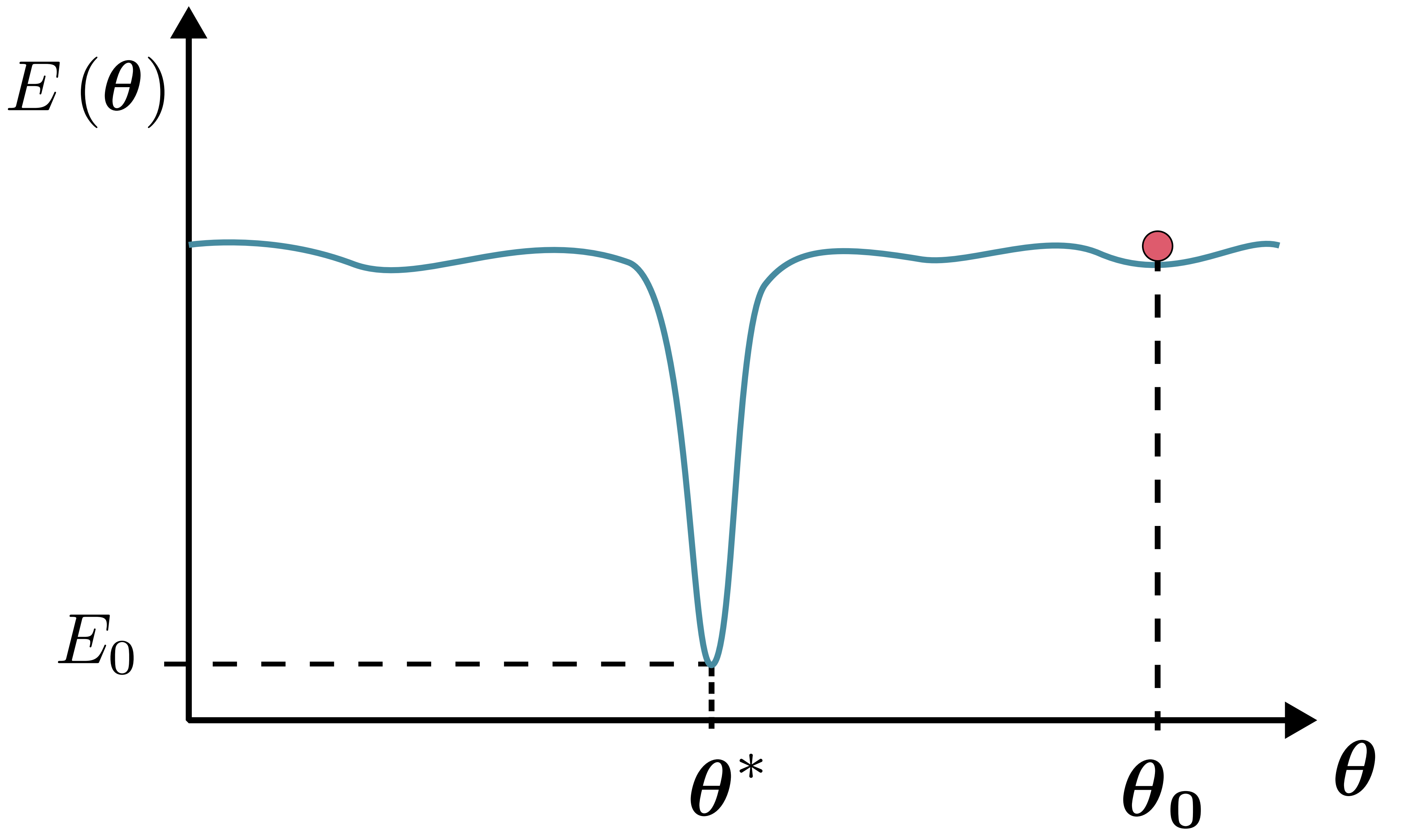

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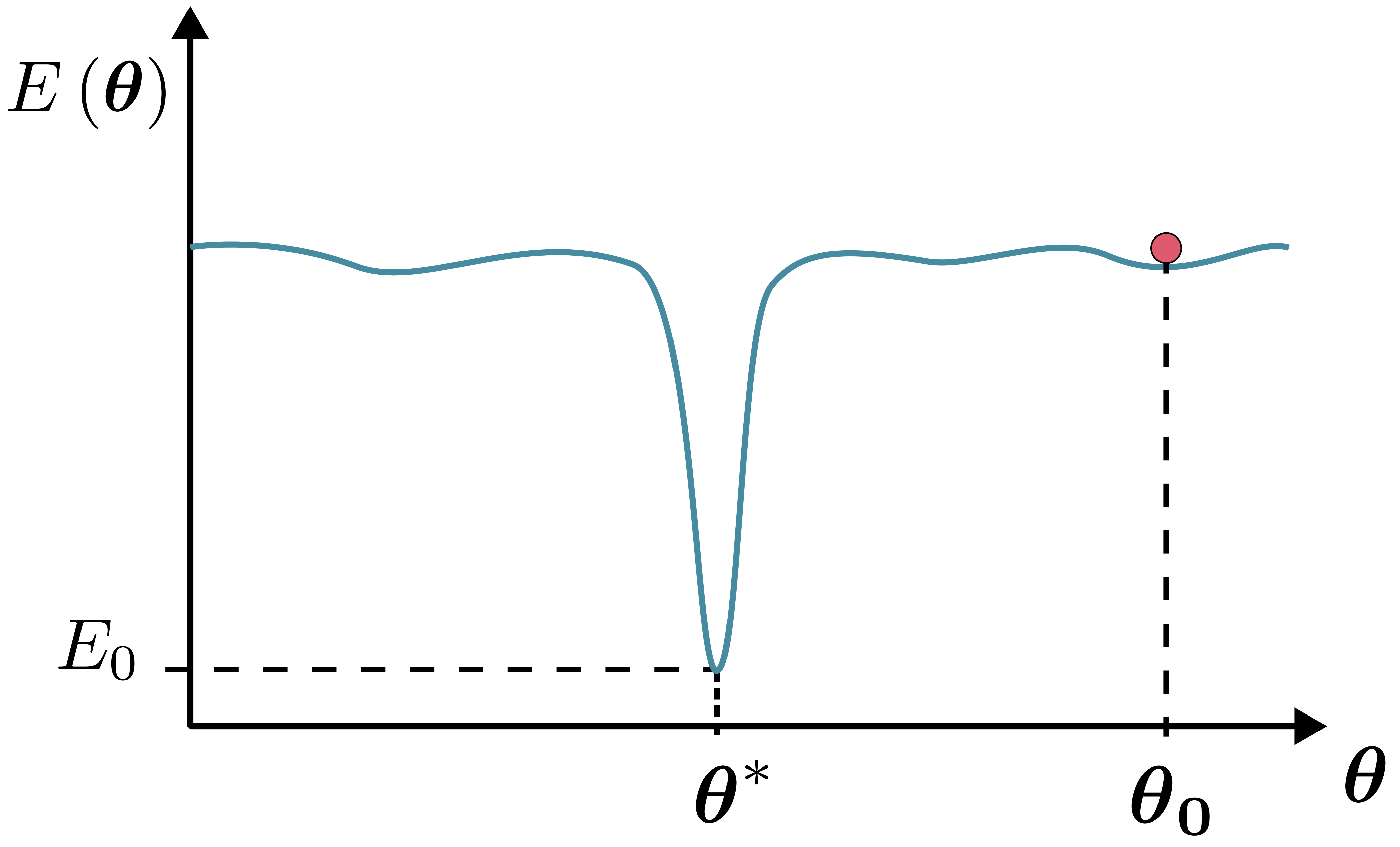
- Multiple origins:

  Noise, expressivity, entanglement, global cost functions …



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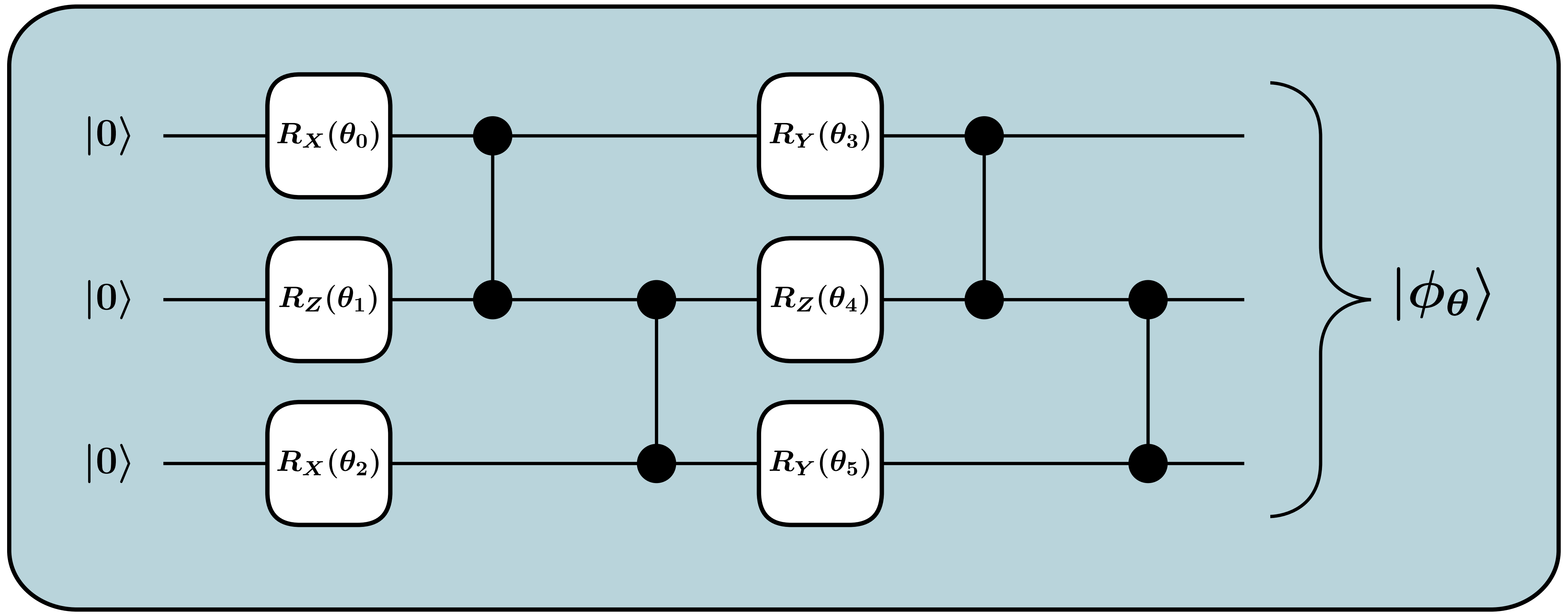

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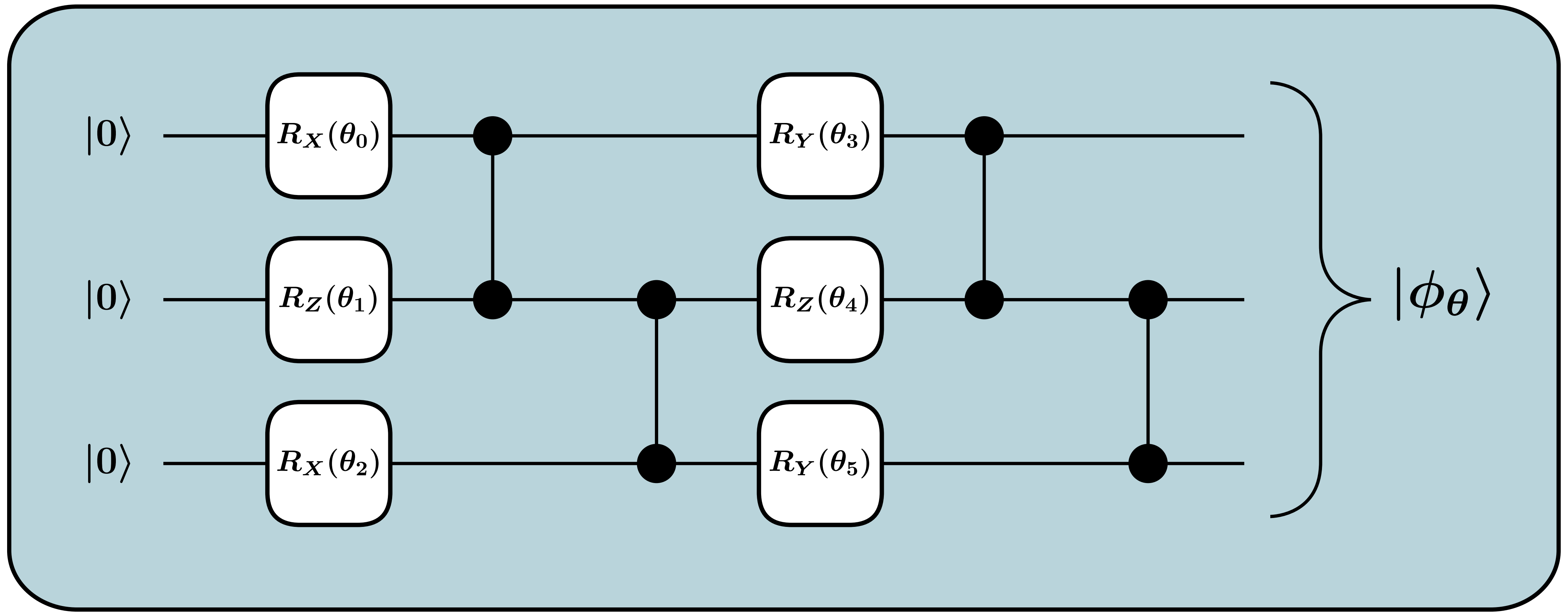
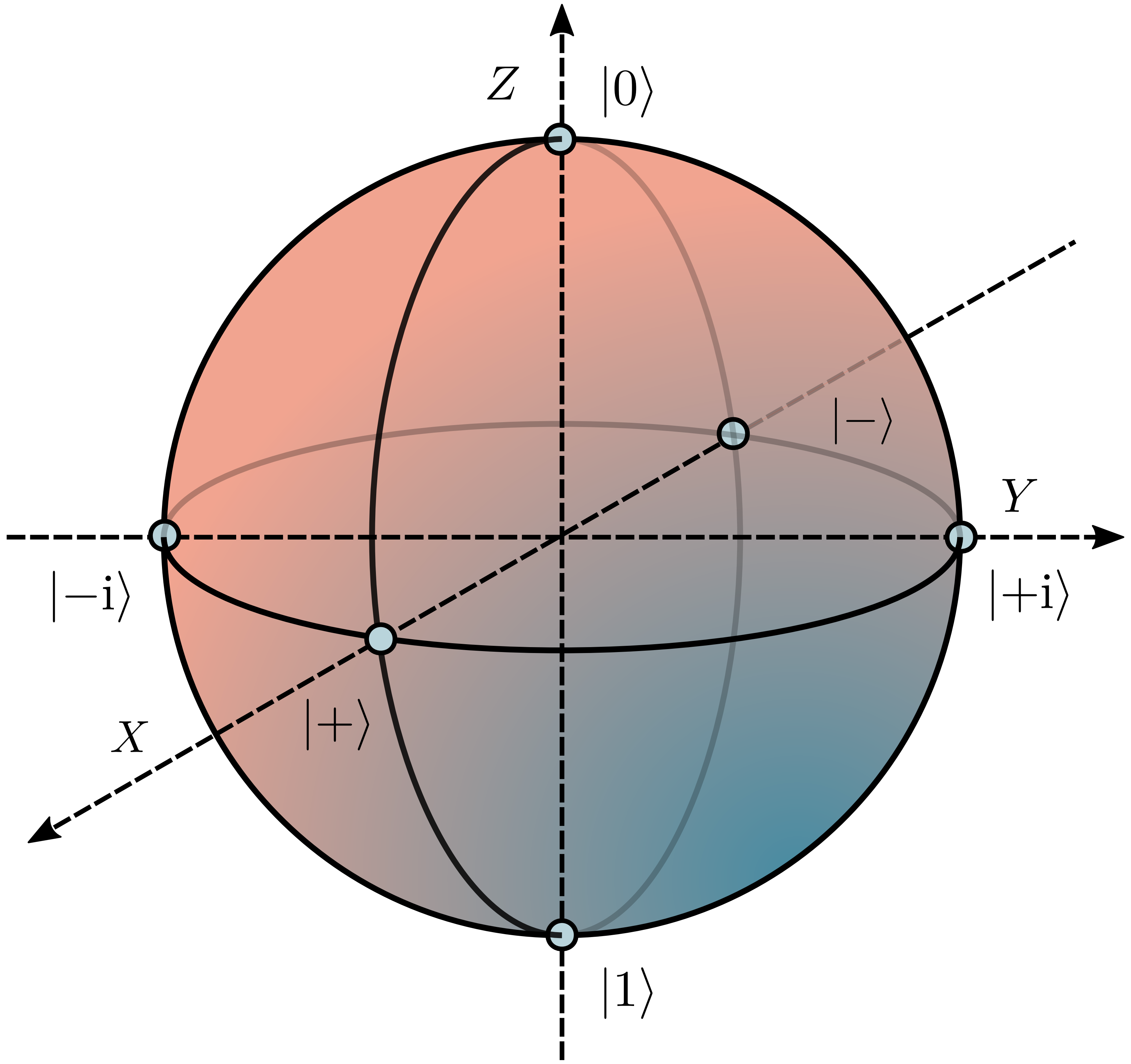


- Classical simulation complexity $\sim \mathcal{O}\left(2^N\right)$

$\implies$ Too costly!

Interlude: Clifford circuits



Bloch sphere

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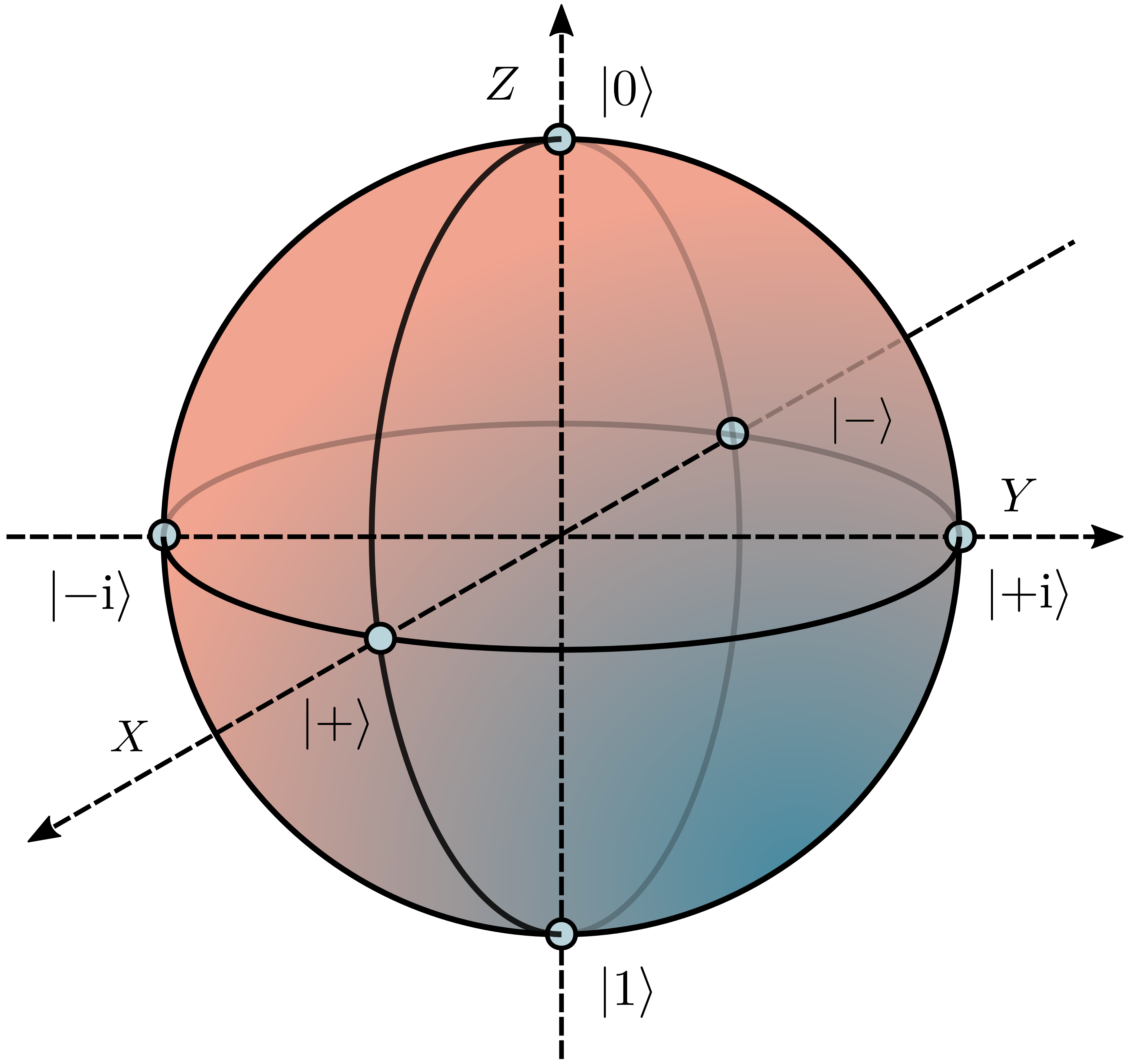

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Interlude: Clifford circuits

- Clifford gates: gates mapping Pauli operators to Pauli operators

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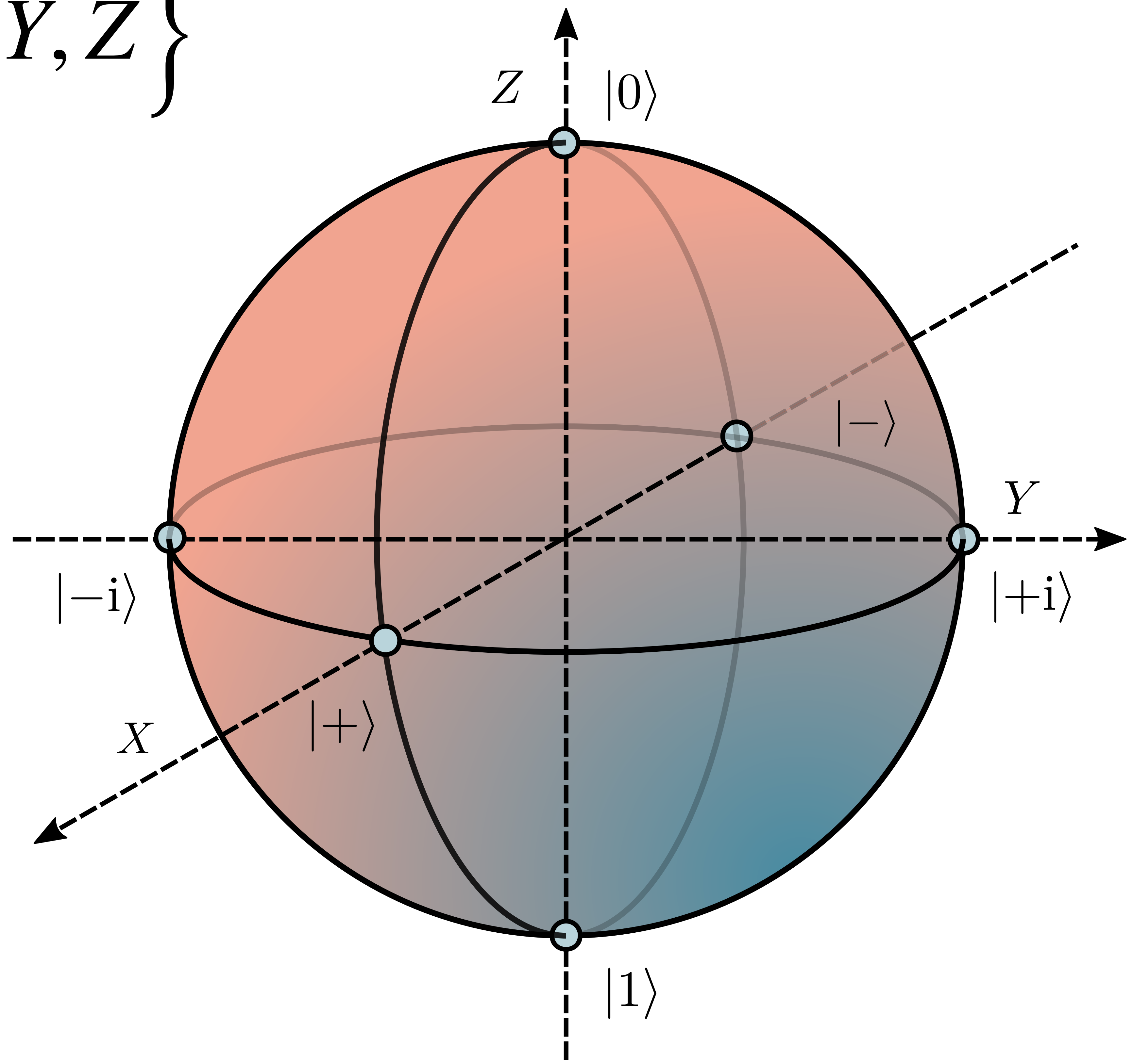

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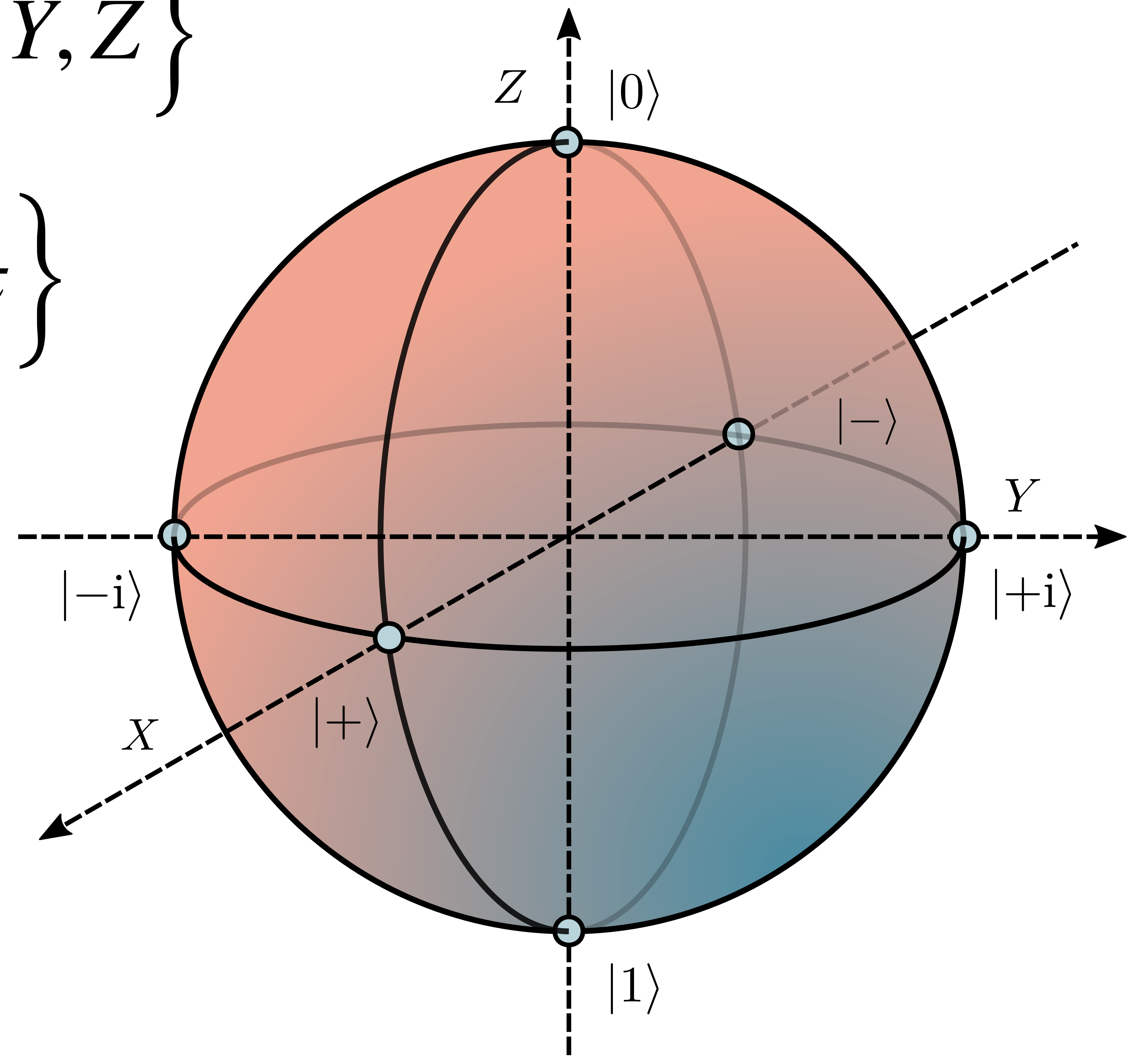
- Example: X, Y and Z rotations with $\theta \in \left\{ -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi \right\}$



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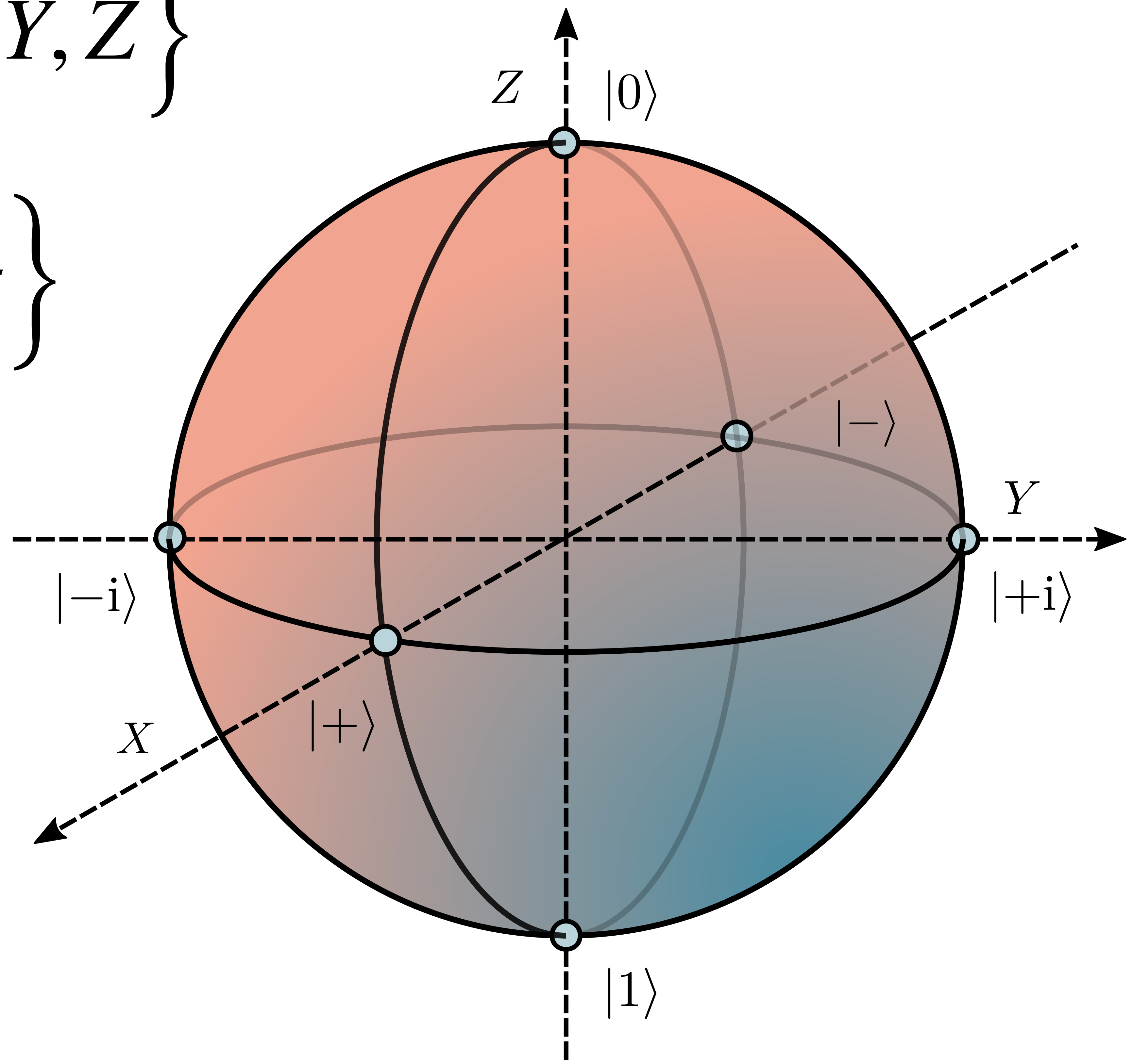
- Classical simulation of Clifford circuits:

  complexity $\sim \mathcal{O}\left(N^p\right)$



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Efficient estimation of the trainability

Using Clifford circuits to estimate averages

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- Under some conditions, sampling Clifford angles is enough to estimate average quantities:

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\partial_i E\left(\boldsymbol{\theta}\right)^2\right] \simeq \frac{1}{K}\sum_{j=1}^{K} \partial_i E\left(\boldsymbol{\theta}_j\right)^2$$

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Clifford circuits

- One can estimate the trainability with a complexity $\sim \mathcal{O}\left(N^p M^q\right)$

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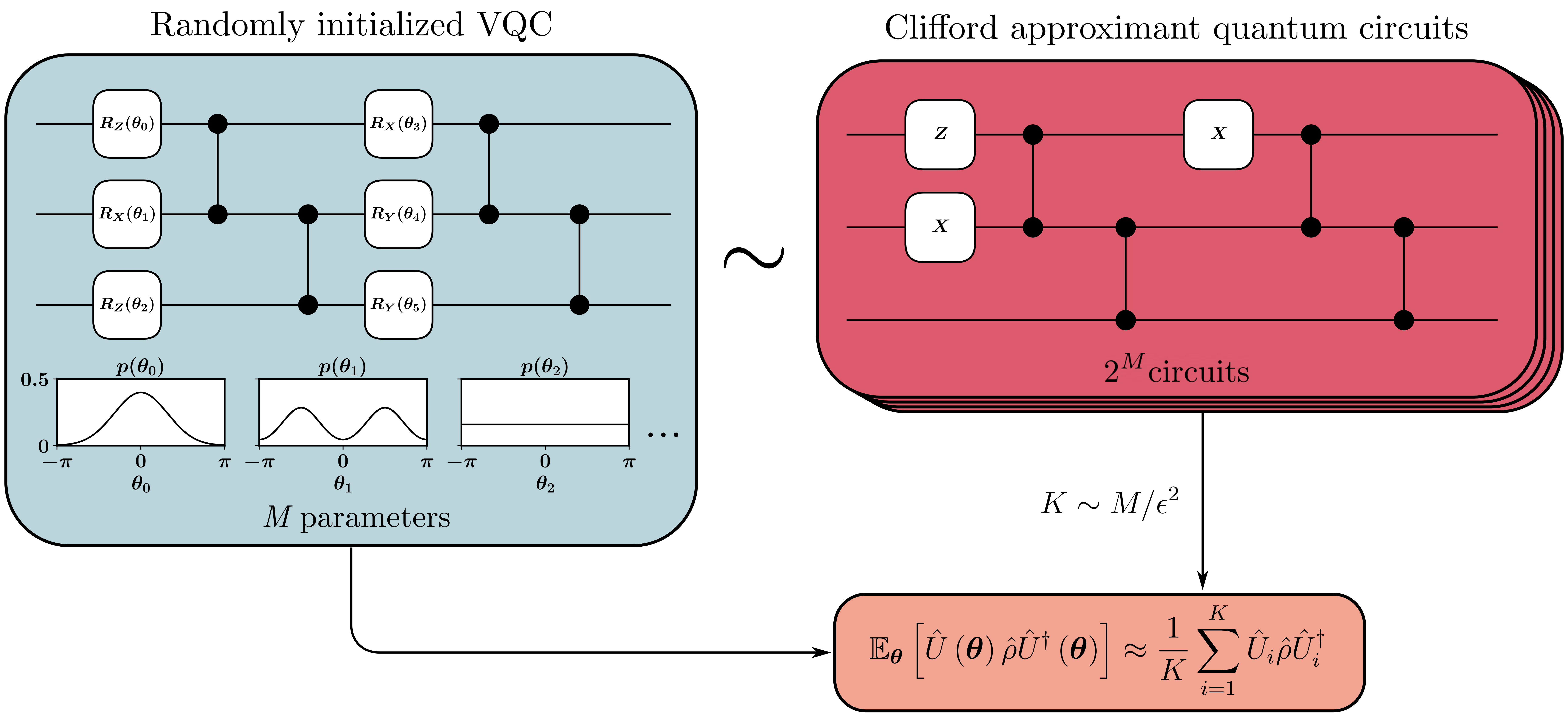

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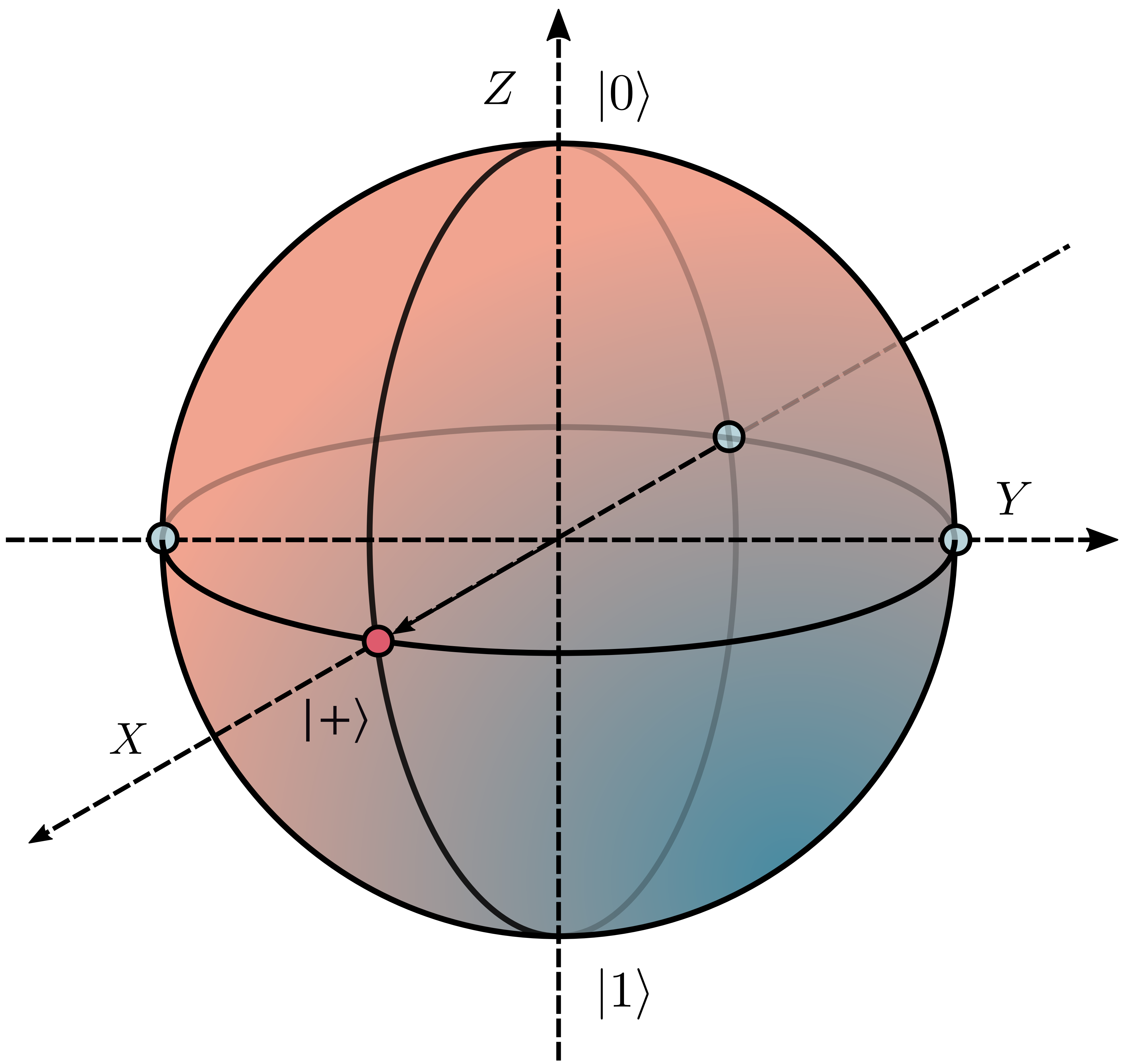
Physical intuition: decoherence



$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{\mathrm{i}\theta}|1\rangle\right)$$

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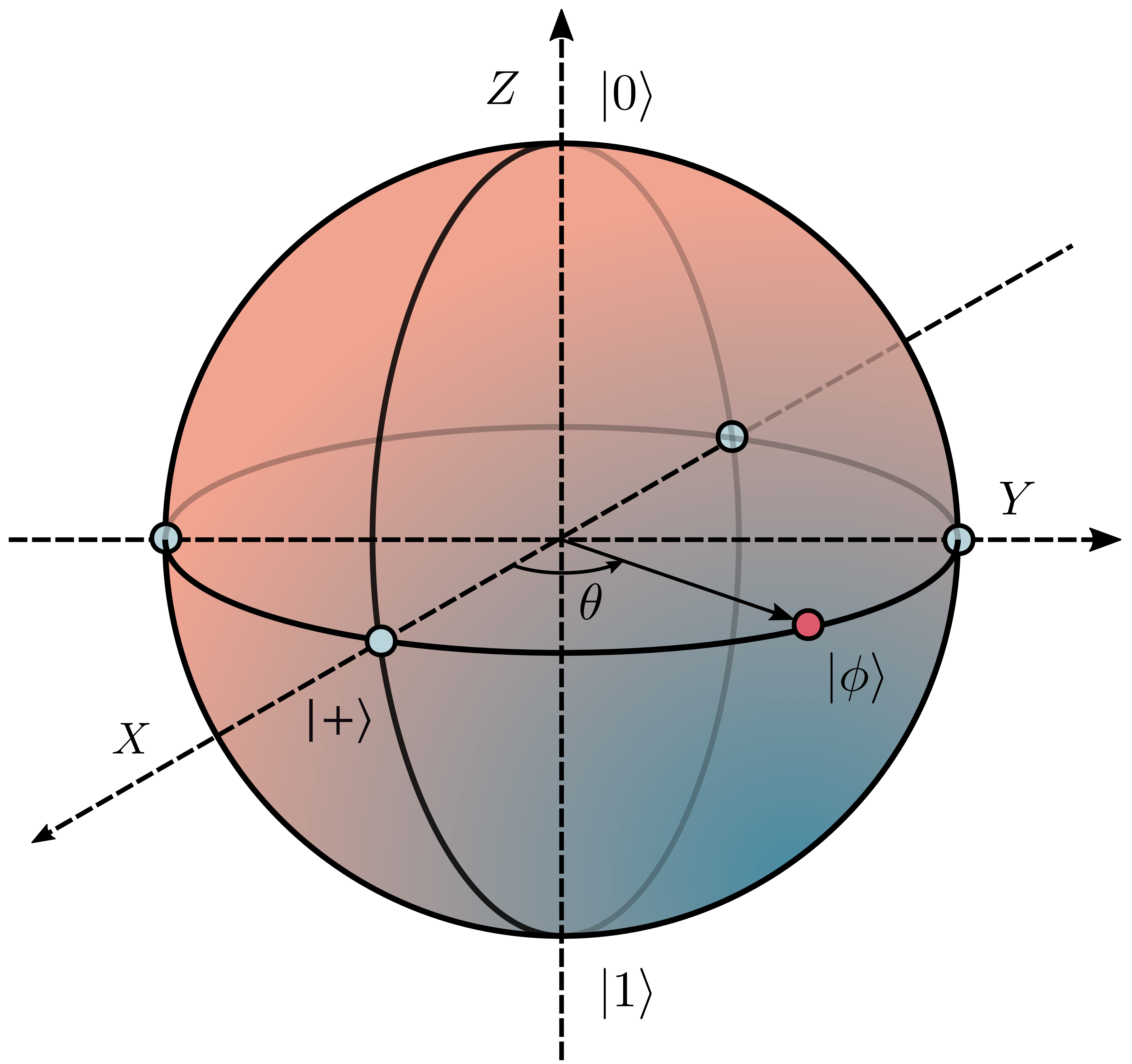

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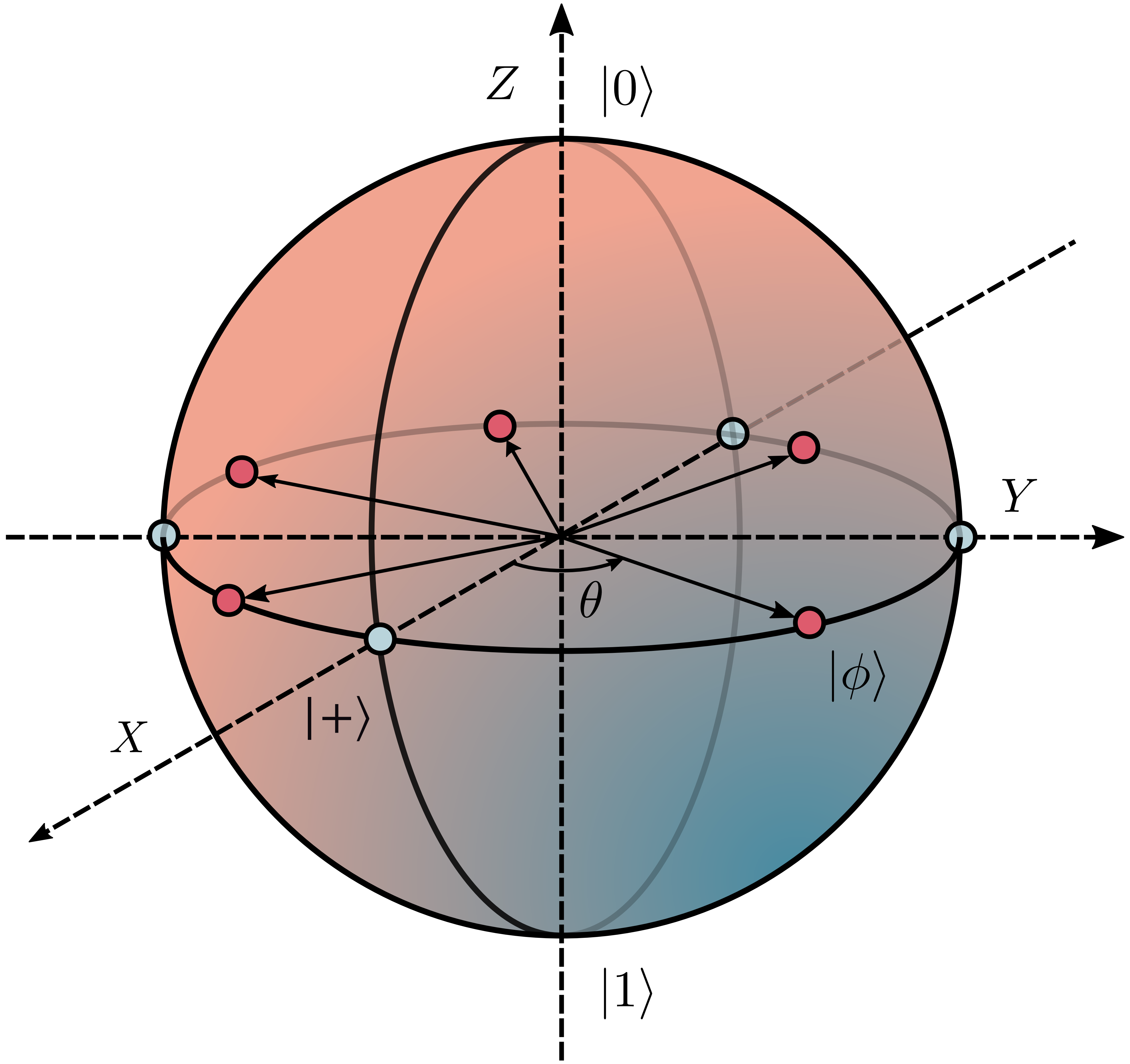

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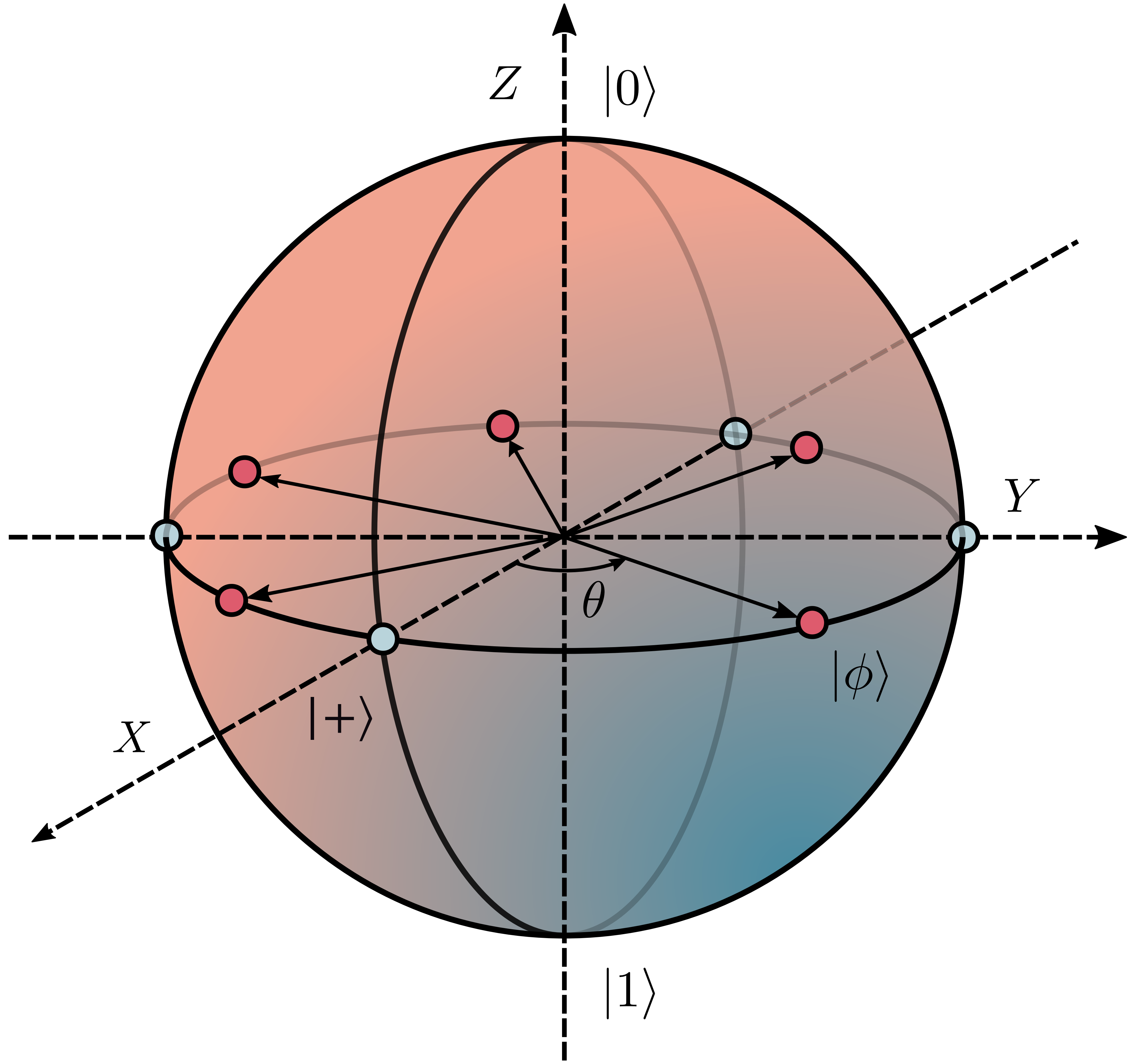

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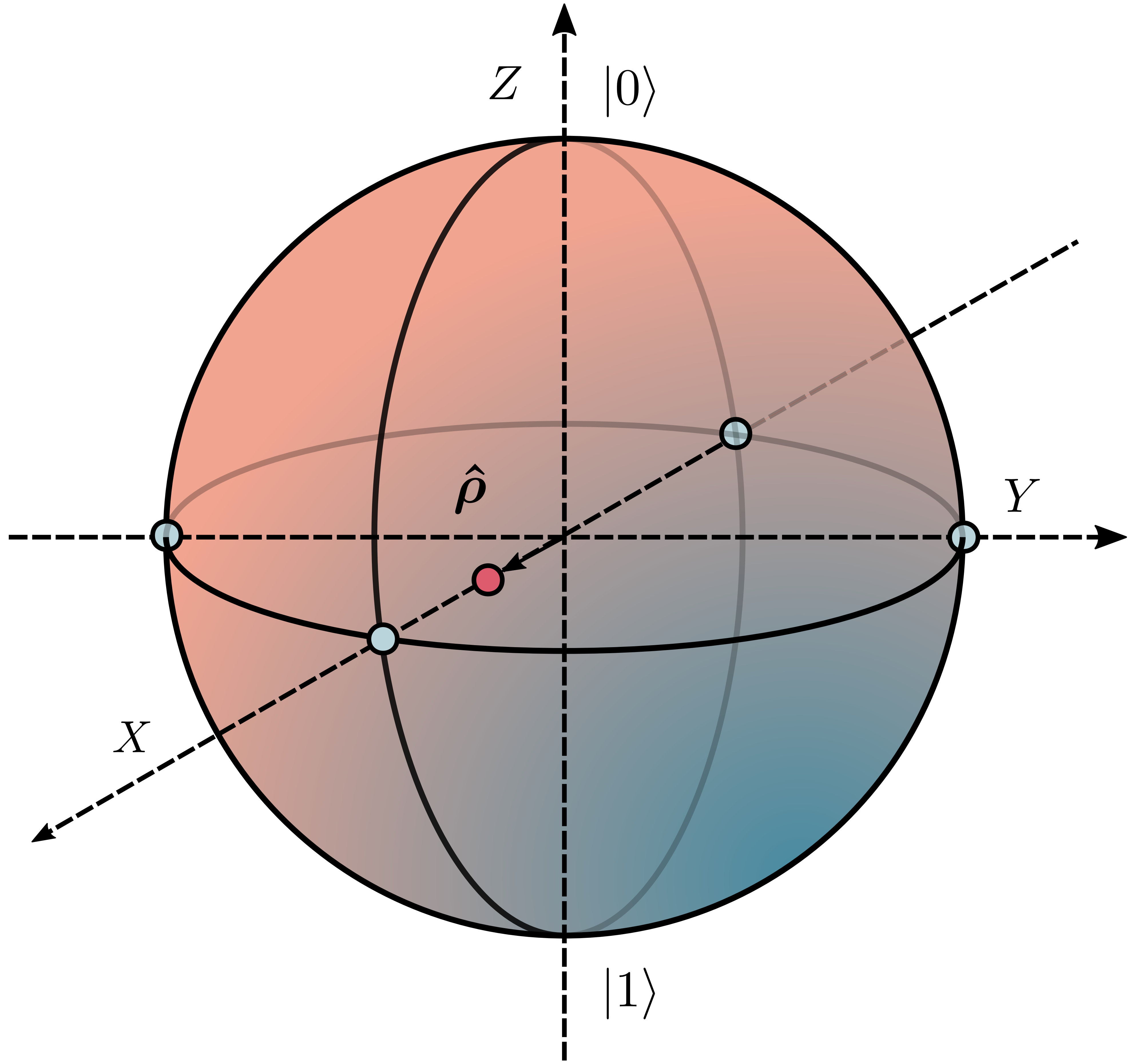

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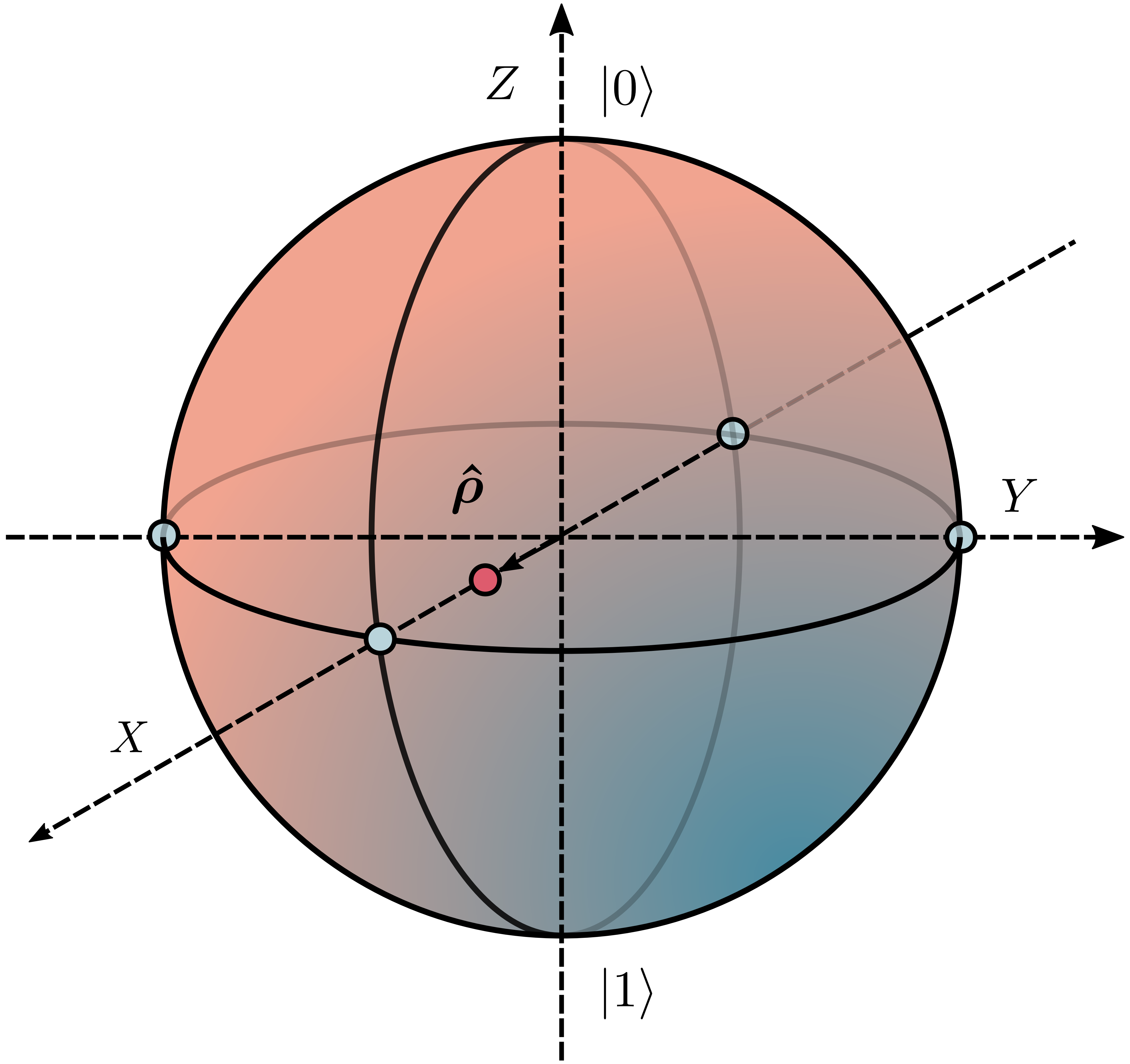

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$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \longrightarrow \hat{\rho} \simeq \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

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- Current investigations:
  Optimisation of the circuit architecture using reinforcement learning methods

Thank you !

Paper available on arXiv !

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