

# Throwing objects with the superpropulsion effect

G. Giombini, C. D'Angelo, F. Celestini, C. Raufaste

*Université Côte d'Azur, CNRS, Institut de Physique de Nice (INPHYNI),  
06200 Nice, France*

*Institut Universitaire de France (IUF), 75005 Paris, France*

# A fascinating ability

## Throwing

an action which consists in accelerating a projectile and then releasing it so that it follows a ballistic trajectory. (From Wikipedia)

## Throw and records

>12000 occurrences in Guinness World Records  
distance, speed, precision, frequency

“Longest throw of an object with no tail” (427.2 m)

“Fastest Jai-Alai (Pelota) throw” (305.77 km/h)

“Most basketball free throws in three minutes” (201)

“Furthest distance to throw and catch an egg” (98.51 m)

“Farthest throw of a washing machine” (4.45 m)

“Most tea bags thrown into mugs in 30 seconds” (30)

# Evolution of throwing in humans

## □ Humans

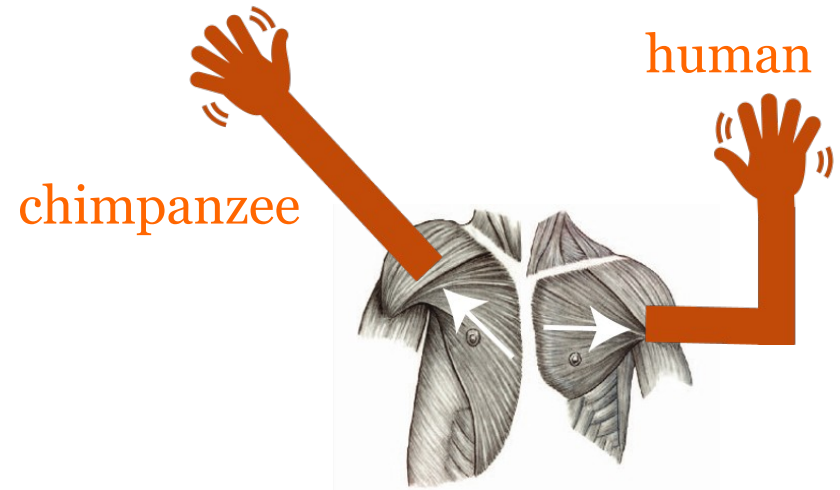
slow, weak, lack natural weapons  
unique abilities among primates  
hunting 2 Myr ago

## □ Anatomical features

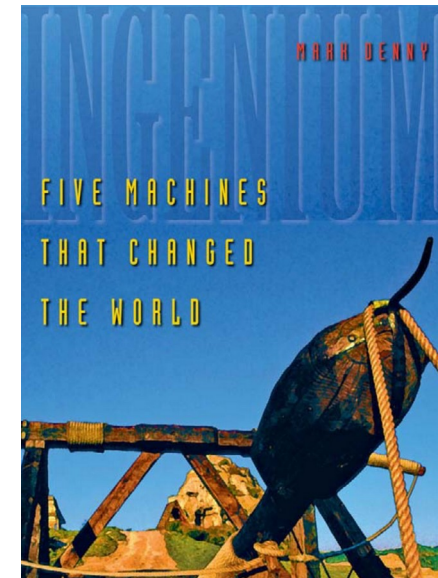
rotation of the shoulder  
elbow flexion

## □ Later development of tools/weapons

context: hunting, warfare, sports  
spear - 0.5 Myr ago  
bow - 70000 yr ago  
counterweight trebuchet 900 yr ago



Roach et al., Nature (2013)



# Hand throwing

## □ Biomechanical aspects

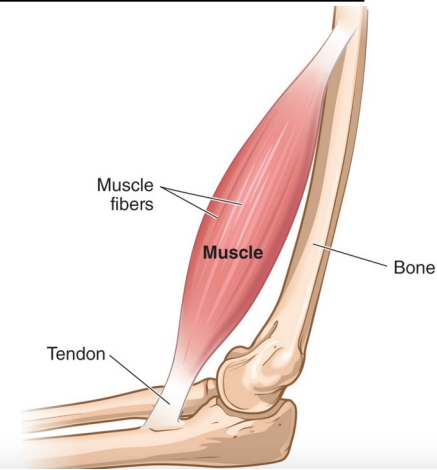
sequential activation of many muscles

legs, hips, torso, shoulder, elbow, wrist

role of tendons

elastic energy storage and release

accumulation and transmission of kinetic energy



## □ Available energy in shot putters

muscle power  $\sim 100$  W/kg

muscle weight  $\sim 25$  kg (20% of body mass)

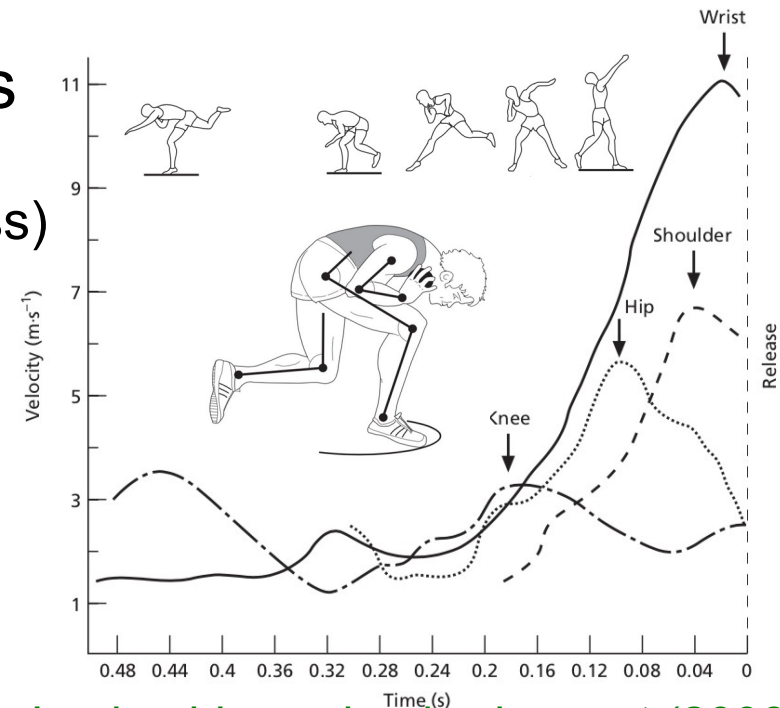
activation time  $\sim 200$  ms

available energy  $\sim 500$  J

## □ Kinetic energy of the shot

$v_{\text{shot}} \sim 10$  m/s,  $m_{\text{shot}} \sim 10$  kg

$KE_{\text{shot}} = 1/2 m_{\text{shot}} v_{\text{shot}}^2 \sim 500$  J



# Always efficient ? A simple experiment

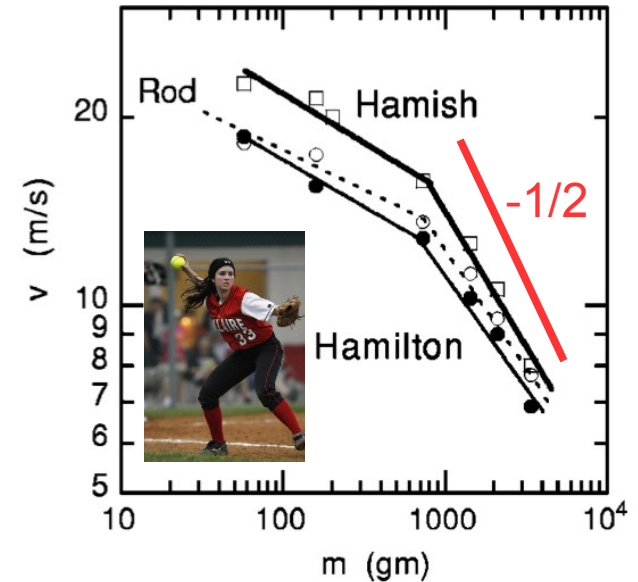
- Effect of the projectile mass  
example in overarm throw  
not efficient with light projectiles

- Simple model

kinetic energy  
of the projectile

$$E_0 = KE = \frac{1}{2}mV^2 \text{ or } V = \sqrt{2E_0/m}$$

available energy  
in muscles



Cross, Am. J. Phys. (2008)

# Always efficient ? A simple experiment

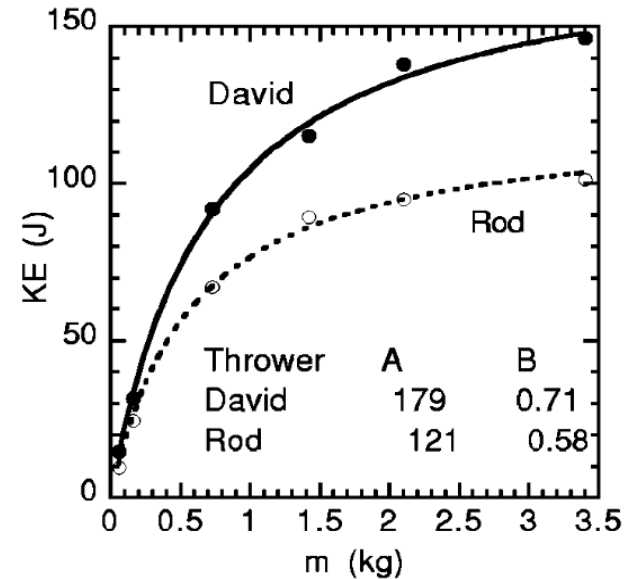
- Effect of the projectile mass  
example in overarm throw  
not efficient with light projectiles

- Simple model

kinetic energy  
of the projectile

$$E_0 = KE = \frac{1}{2}mV^2 \text{ or } V = \sqrt{2E_0/m}$$

available energy  
in muscles



Cross, Am. J. Phys. (2008)

# Always efficient ? A simple experiment

- Effect of the projectile mass  
example in overarm throw  
not efficient with light projectiles

- Simple model

kinetic energy (KE)  
of the projectile

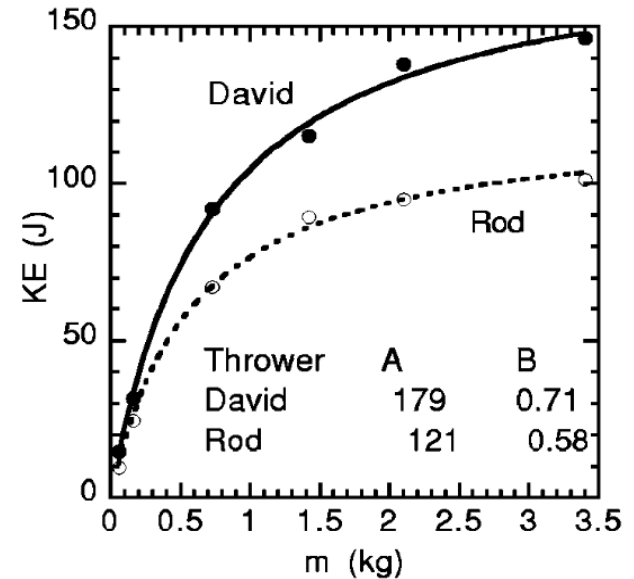
$$E_0 = \frac{1}{2}mV^2 + \frac{1}{2}MV^2 \quad \text{or} \quad KE = \frac{E_0}{1 + M/m}$$

available energy  
in muscles

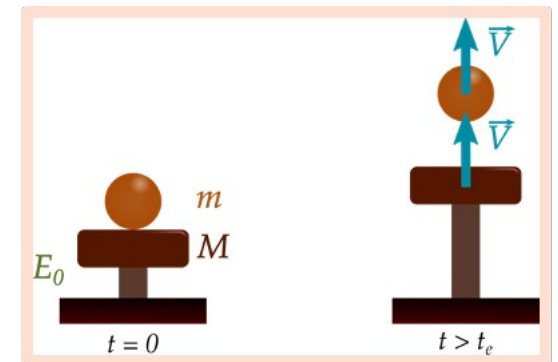
kinetic energy remaining  
in the body through the motion  
of a virtual mass  $M$

- Implications

$M \approx 2$  kg corresponds to hand and forearm  
Difficulties to throw at large distances  
Higher risk of injuries with light objects



Cross, Am. J. Phys. (2008)



# Light projectiles: need for a tool

## □ Efficiency $KE/E_0$

Limit	Projectile	Handthrow $M \sim 2 \text{ kg}$	Throw with instrument
$m \gg M$	shot put ( $\sim 7.3 \text{ kg}$ )	$KE/E_0 \sim 80 \%$	-
$m \ll M$	basque pelota ( $\sim 150 \text{ g}$ )	$KE/E_0 \sim 7 \%$	chistera $V \approx 35 \text{ m/s}$ , $KE \approx 150 \text{ J}$ $KE/E_0 \sim 30\text{-}35 \%$



# Light projectiles: need for a tool

## □ Efficiency $KE/E_0$

Limit	Projectile	Handthrow $M \sim 2 \text{ kg}$	Throw with instrument
$m \gg M$	shot put ( $\sim 7.3 \text{ kg}$ )	$KE/E_0 \sim 80 \%$	-
$m \ll M$	basque pelota ( $\sim 150 \text{ g}$ )	$KE/E_0 \sim 7 \%$	chistera $V \approx 35 \text{ m/s}$ , $KE \approx 150 \text{ J}$ $KE/E_0 \sim 30\text{-}35 \%$

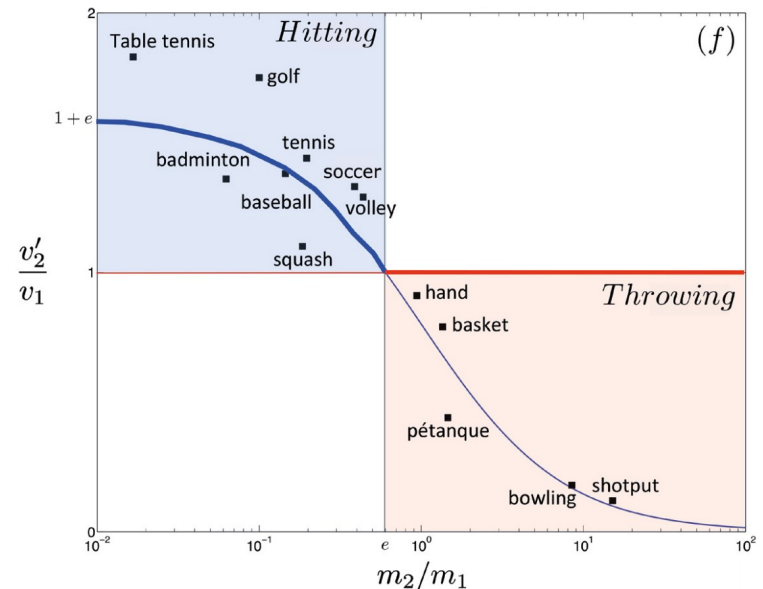
## □ Other strategies with tools

hitting (golf, tennis, ...)

spinning (sling, hammer throw ...)

loading (bow, slingshot ...)

Cohen & Clanet, Europhys. News 2016

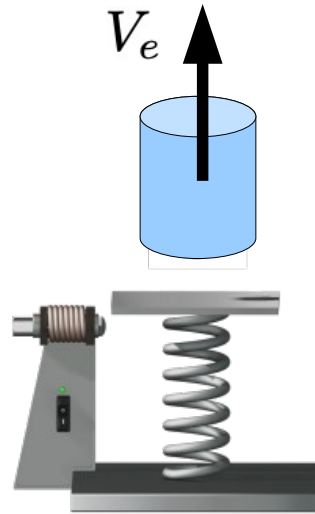
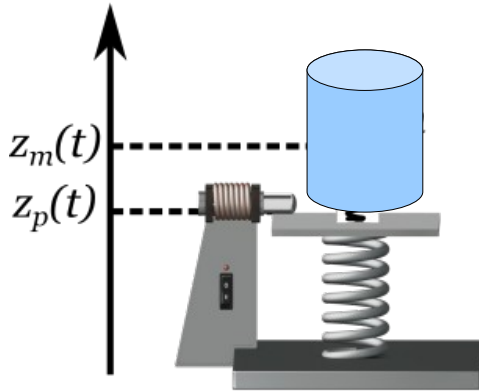


# Scientific questions

- How to increase the throw efficiency of light objects ?
- Can we find other strategies than the use of a tool ?
  - Mimic the action of tendons
- What input from soft matter and materials physics ?
  - Find the good materials and geometries to reach relevant time scales

# Main idea

## □ Basic geometry



Perfect throwing engine ( $M \gg m$ )

Harmonic motion

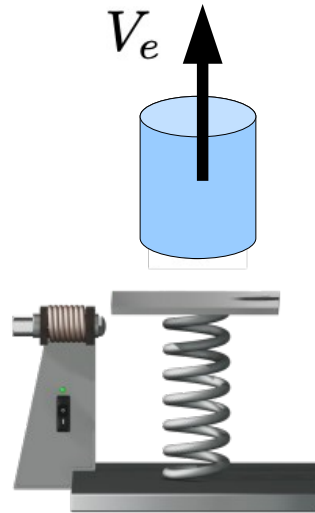
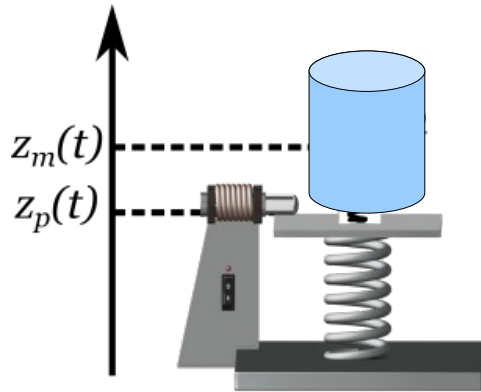
Amplitude  $A$ , frequency  $f$

$$z_p(t) = A[1 - \cos(2\pi ft)]$$

$$V_p^* = 2\pi fA \text{ maximum speed}$$

# Main idea

## □ Basic geometry



Perfect throwing engine ( $M \gg m$ )

Harmonic motion

Amplitude  $A$ , frequency  $f$

$$z_p(t) = A[1 - \cos(2\pi ft)]$$

$$V_p^* = 2\pi fA \text{ maximum speed}$$

## □ Case of a rigid object

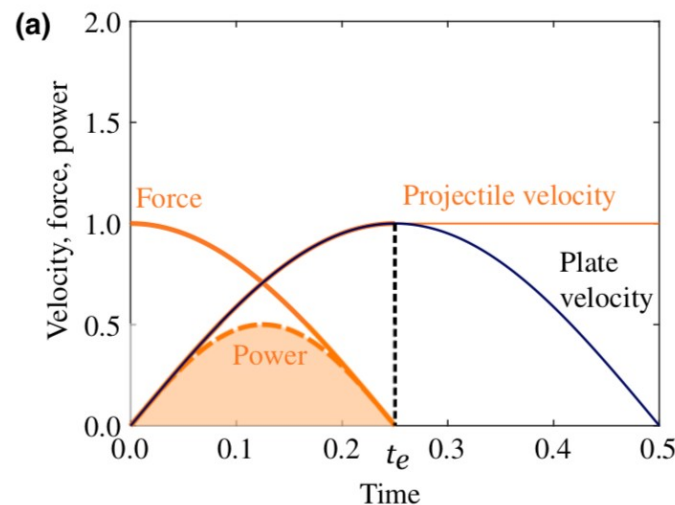
rigid object

ejection speed

$$V_e = V_p^*$$

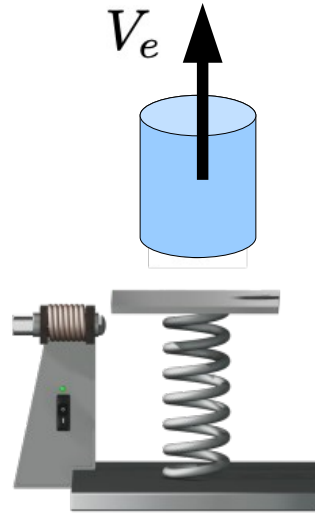
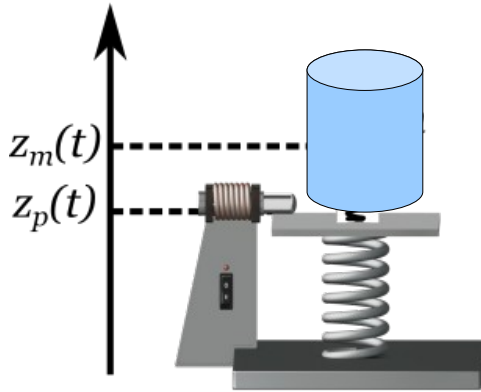
transferred energy

$$\int_0^{t_e} F(t) \dot{z}_p(t) dt$$



# Main idea

## Basic geometry



Perfect throwing engine ( $M \gg m$ )

Harmonic motion

Amplitude  $A$ , frequency  $f$

$$z_p(t) = A[1 - \cos(2\pi ft)]$$

$$V_p^* = 2\pi fA \text{ maximum speed}$$

## Case of a rigid object

rigid object

ejection speed

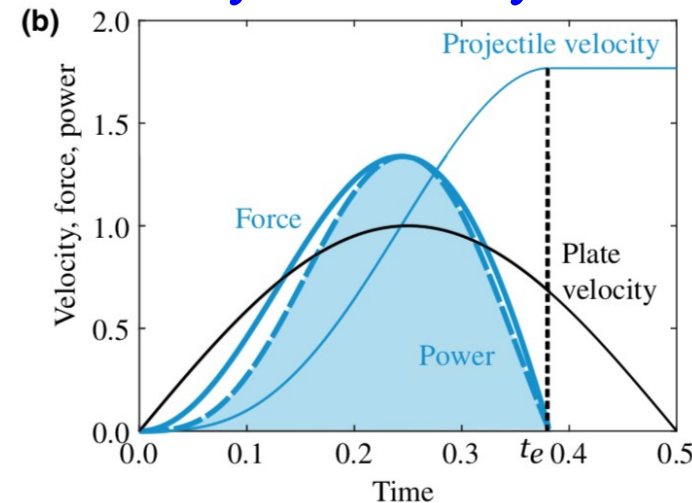
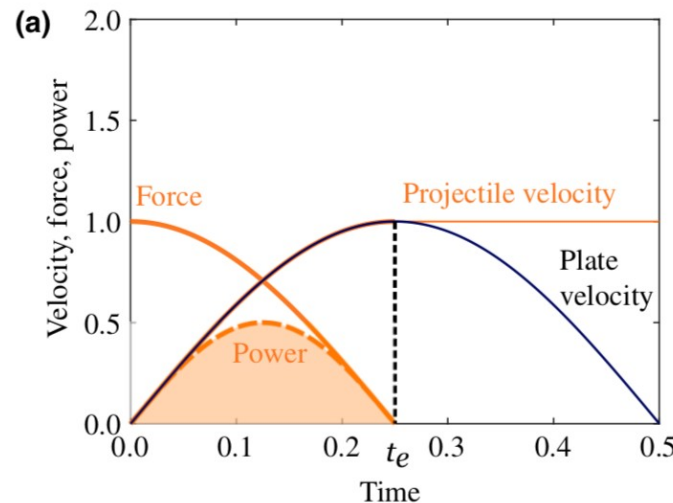
$$V_e = V_p^*$$

transferred energy

$$\int_0^{t_e} F(t) \dot{z}_p(t) dt$$

optimal case

delayed force system



# Solution: soft elastic projectiles

## □ Requirements

Delayed response and tunable time scale

Good elastic restitution

## □ Examples of quasi-1D gelatin hydrogels

Young modulus 12 kPa

Deformation wave speed  $c = 3.4$  m/s

Typical length  $L$ : 3 - 30 mm

Eigenfrequency  $f_0 = c/(2L)$ : 60 - 600 Hz



# Solution: soft elastic projectiles

## □ Requirements

Delayed response and tunable time scale

Good elastic restitution

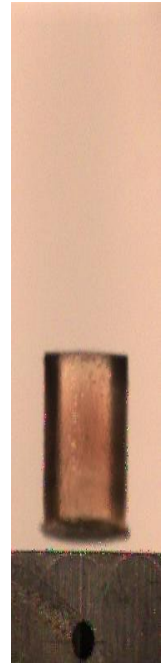
## □ Examples of quasi-1D gelatin hydrogels

Young modulus 12 kPa

Deformation wave speed  $c = 3.4$  m/s

Typical length  $L$ : 3 - 30 mm

Eigenfrequency  $f_0 = c/(2L)$ : 60 - 600 Hz



$A \sim 1$  mm

$f \sim 50$  Hz

# Solution: soft elastic projectiles

## □ Requirements

Delayed response and tunable time scale

Good elastic restitution

## □ Examples of quasi-1D gelatin hydrogels

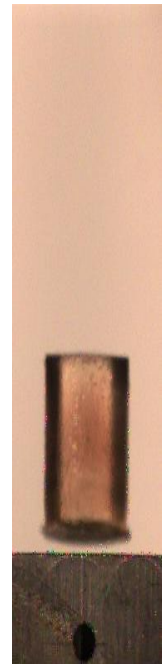
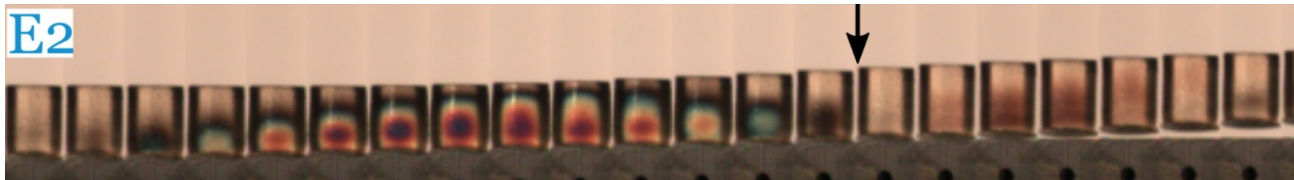
Young modulus 12 kPa

Deformation wave speed  $c = 3.4$  m/s

Typical length  $L$ : 3 - 30 mm

Eigenfrequency  $f_0 = c/(2L)$ : 60 - 600 Hz

## □ Typical time sequence



$A \sim 1$  mm  
 $f \sim 50$  Hz



# Solution: soft elastic projectiles

## □ Results

Effect of the size for a given frequency  $f$



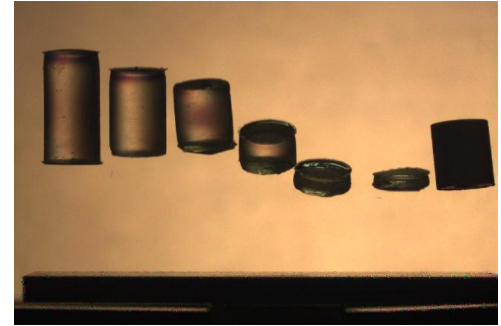
$f \sim 50$  Hz

↑  
rigid

# Solution: soft elastic projectiles

## □ Results

Effect of the size for a given frequency  $f$



# Solution: soft elastic projectiles

## □ Results

Effect of the size for a given frequency  $f$

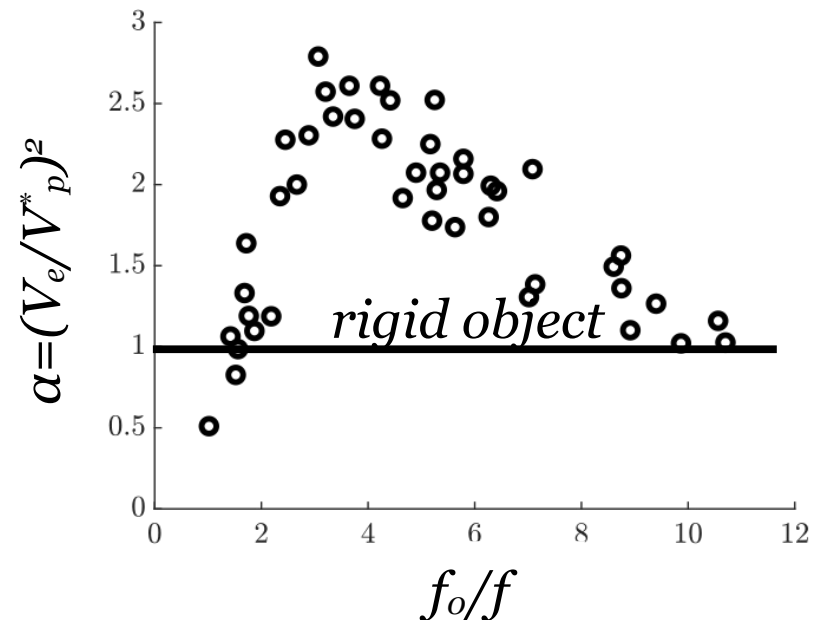
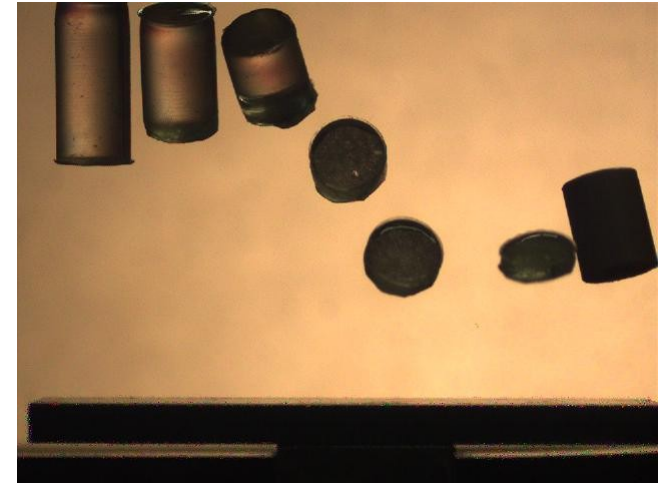
## □ Energy transfer factor

$$\alpha = (V_e/V_p^*)^2$$

Effect of the dimensionless frequency  $f_o/f$

Optimal ratio  $f_o/f \approx 3-4$  gives  $\alpha \approx 2.5$

Specific resonance effect



# Solution: soft elastic projectiles

## □ Results

Effect of the size for a given frequency  $f$

## □ Energy transfer factor

$$\alpha = (V_e/V_p^*)^2$$

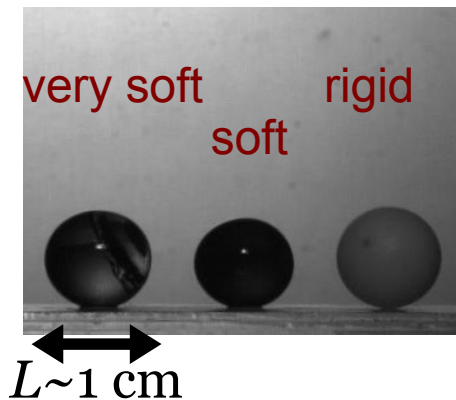
Effect of the dimensionless frequency  $f_o/f$

Optimal ratio  $f_o/f \approx 3-4$  gives  $\alpha \approx 2.5$

Specific resonance effect

## □ Other material/geometry

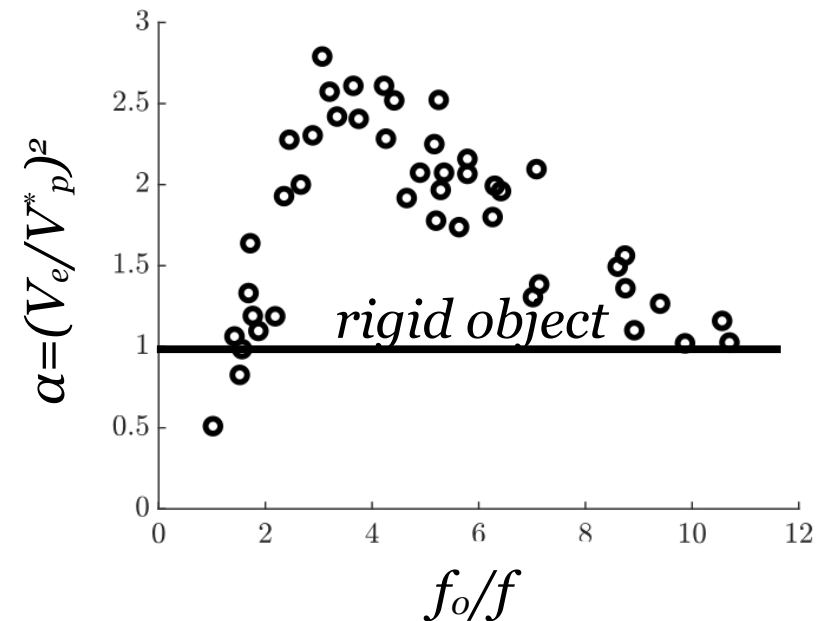
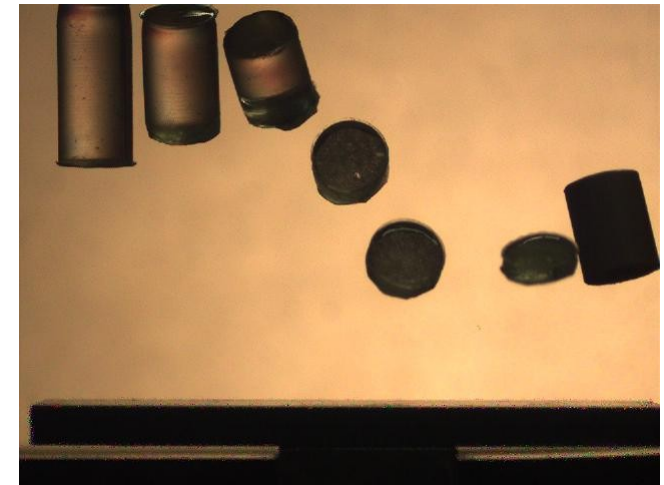
Polyacrylamide beads



$$E \simeq 1 - 10 \text{ kPa}$$

$$f_o \sim \frac{1}{L} \sqrt{\frac{E}{\rho}}$$

$$50 - 200 \text{ Hz}$$



# Solution: soft elastic projectiles

## □ Results

Effect of the size for a given frequency  $f$

## □ Energy transfer factor

$$\alpha = (V_e/V_p^*)^2$$

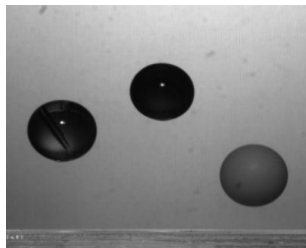
Effect of the dimensionless frequency  $f_o/f$

Optimal ratio  $f_o/f \approx 3-4$  gives  $\alpha \approx 2.5$

Specific resonance effect

## □ Other material/geometry

Polyacrylamide beads

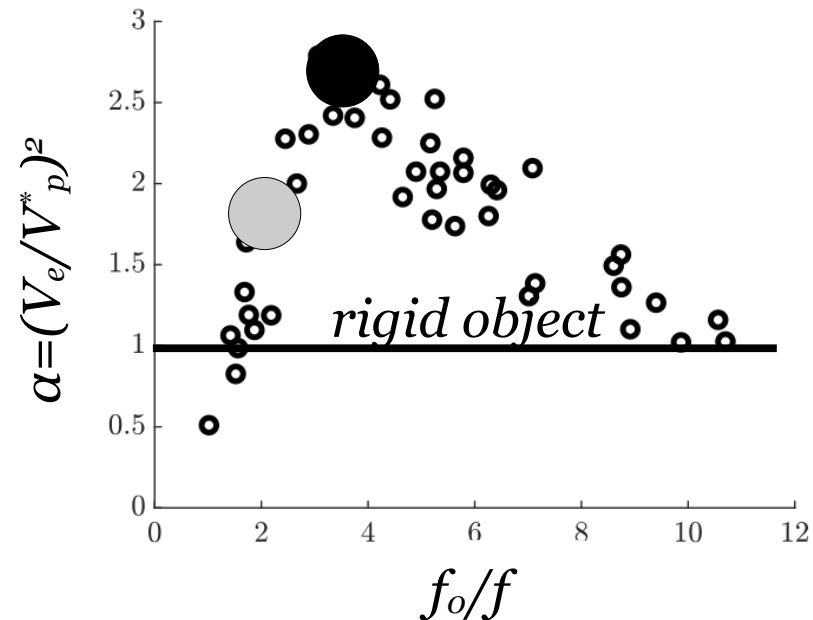
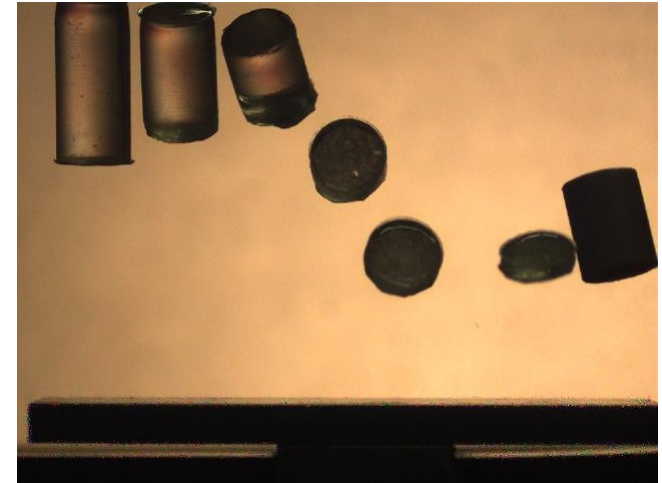


$f \sim 67$  Hz

$$E \simeq 1 - 10 \text{ kPa}$$

$$f_o \sim \frac{1}{L} \sqrt{\frac{E}{\rho}}$$

50 - 200 Hz



# Solution: soft elastic projectiles

## □ Results

Effect of the size for a given frequency  $f$

## □ Energy transfer factor

$$\alpha = (V_e/V_p^*)^2$$

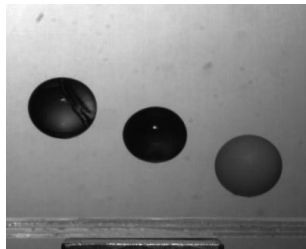
Effect of the dimensionless frequency  $f_o/f$

Optimal ratio  $f_o/f \approx 3-4$  gives  $\alpha \approx 2.5$

Specific resonance effect

## □ Other material/geometry

Polyacrylamide beads

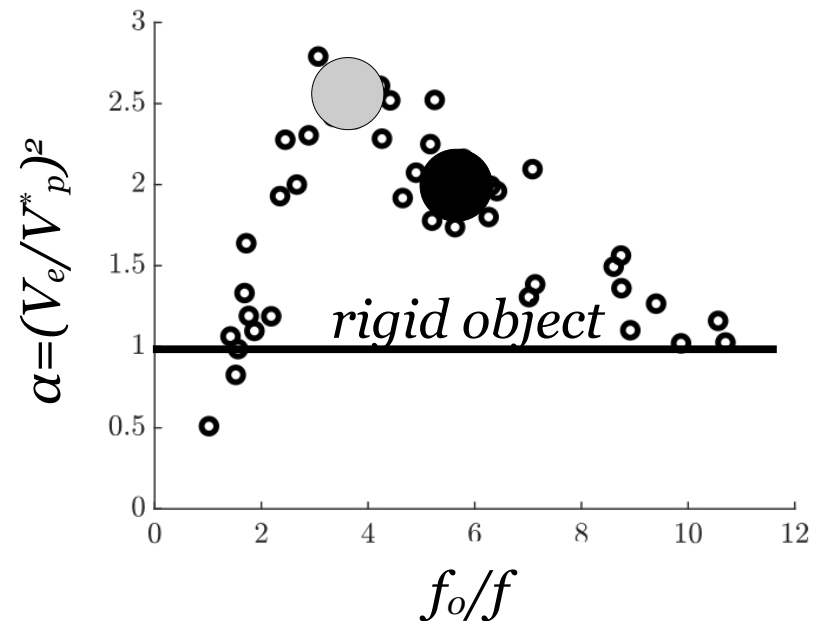
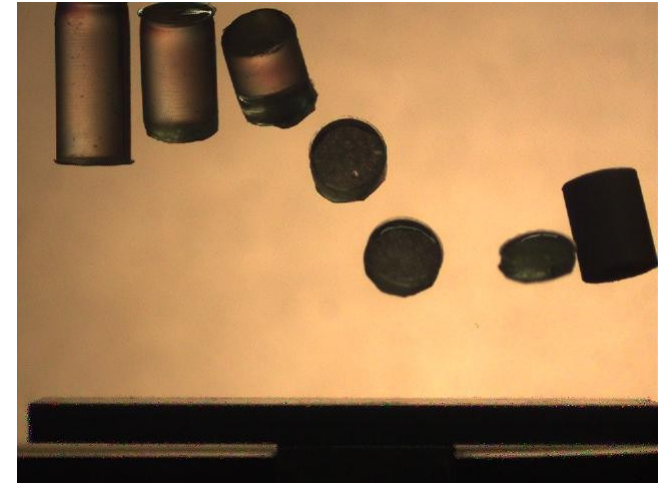


$f \sim 26$  Hz

$$E \simeq 1 - 10 \text{ kPa}$$

$$f_o \sim \frac{1}{L} \sqrt{\frac{E}{\rho}}$$

50 - 200 Hz



# Solution: soft elastic projectiles

## □ General mechanism: superpropulsion

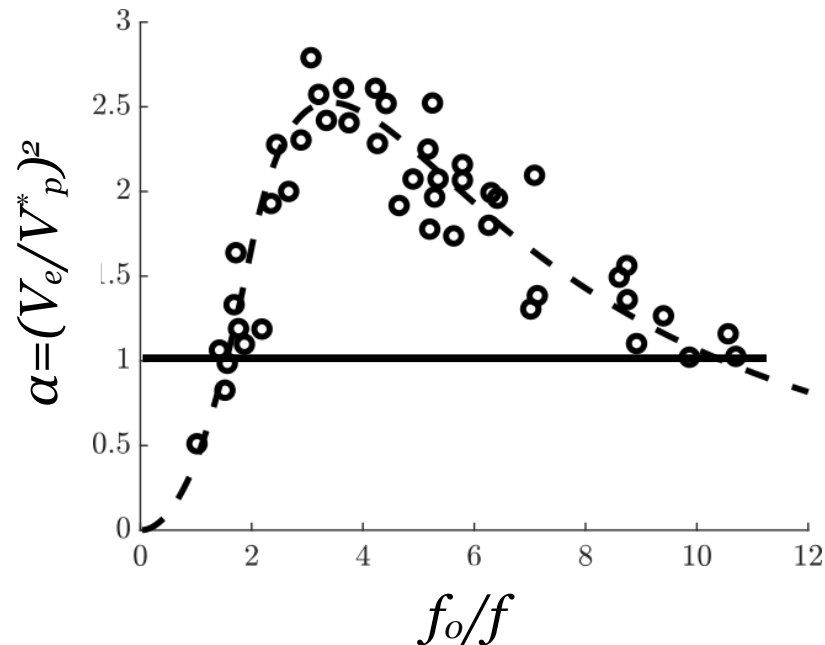
Matching deformation/throw dynamics

Optimal value of the parameter  $f_o/f \approx 3-4$

Gain in kinetic energy  $\alpha \approx 2.4-2.7$

## □ Perfect agreement with models

$f_o/f = 3.4$  and  $\alpha = 2.5$



# Application 1: droplet ejection

## □ Droplet dynamics

Deformation associated with surface tension

Eigenfrequency

$$f_0 \sim \sqrt{\frac{\gamma}{\rho R^3}}$$



# Application 1: droplet ejection

## □ Droplet dynamics

Deformation associated with surface tension

Eigenfrequency

$$f_0 \sim \sqrt{\frac{\gamma}{\rho R^3}}$$

1mm

## □ Parameters

drop radius  $R \sim 1\text{mm}$ ,  $f_0 \sim 300\text{Hz}$

catapult amplitude  $A \sim 1\text{-}10\text{mm}$

catapult frequency  $f \sim 20\text{-}100\text{Hz}$

catapult acceleration  $\sim 10g$

ejection velocity  $V_e \sim 1\text{m/s}$



adhesionless substrates  
collaboration with chemists  
 $f=37\text{Hz}$   $A=1.4\text{mm}$

# Application 1: droplet ejection

## □ Droplet dynamics

Deformation associated with surface tension

Eigenfrequency

$$f_0 \sim \sqrt{\frac{\gamma}{\rho R^3}}$$

## □ Parameters

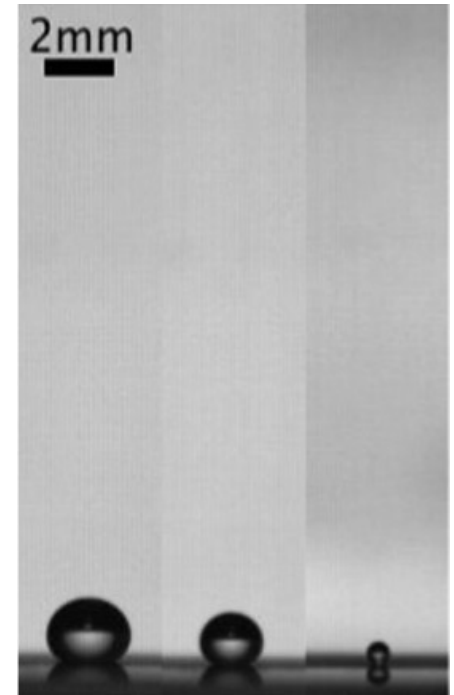
drop radius  $R \sim 1\text{mm}$ ,  $f_0 \sim 300\text{Hz}$

catapult amplitude  $A \sim 1\text{-}10\text{mm}$

catapult frequency  $f \sim 20\text{-}100\text{Hz}$

catapult acceleration  $\sim 10g$

ejection velocity  $V_e \sim 1\text{m/s}$



adhesionless substrates  
collaboration with chemists  
 $f=37\text{Hz}$   $A=1.4\text{mm}$

# Application 1: droplet ejection

## □ Droplet dynamics

Deformation associated with surface tension

Eigenfrequency

$$f_0 \sim \sqrt{\frac{\gamma}{\rho R^3}}$$

## □ Parameters

drop radius  $R \sim 1\text{mm}$ ,  $f_0 \sim 300\text{Hz}$

catapult amplitude  $A \sim 1\text{-}10\text{mm}$

catapult frequency  $f \sim 20\text{-}100\text{Hz}$

catapult acceleration  $\sim 10g$

ejection velocity  $V_e \sim 1\text{m/s}$



adhesionless substrates  
collaboration with chemists  
 $f=37\text{Hz}$   $A=1.4\text{mm}$

# Application 1: droplet ejection

## □ Droplet dynamics

Deformation associated with surface tension

Eigenfrequency

$$f_0 \sim \sqrt{\frac{\gamma}{\rho R^3}}$$

## □ Parameters

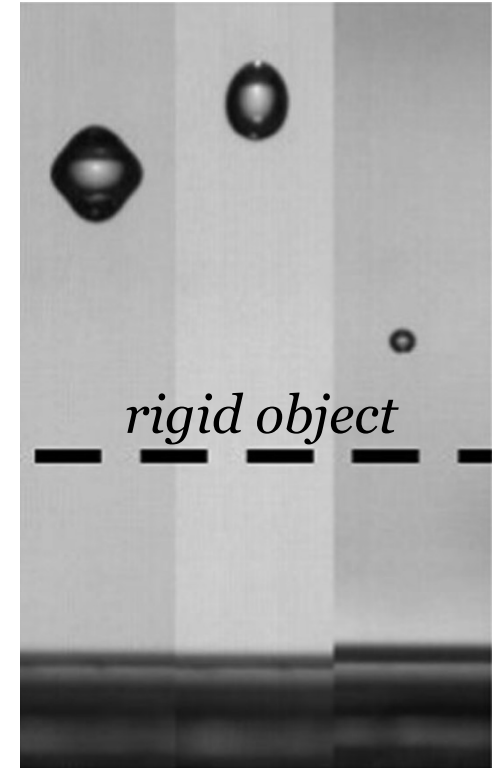
drop radius  $R \sim 1\text{mm}$ ,  $f_0 \sim 300\text{Hz}$

catapult amplitude  $A \sim 1\text{-}10\text{mm}$

catapult frequency  $f \sim 20\text{-}100\text{Hz}$

catapult acceleration  $\sim 10g$

ejection velocity  $V_e \sim 1\text{m/s}$



adhesionless substrates  
collaboration with chemists  
 $f=37\text{Hz}$   $A=1.4\text{mm}$

# Application 1: droplet ejection

## □ Droplet dynamics

Deformation associated with surface tension

Eigenfrequency

$$f_0 \sim \sqrt{\frac{\gamma}{\rho R^3}}$$

## □ Parameters

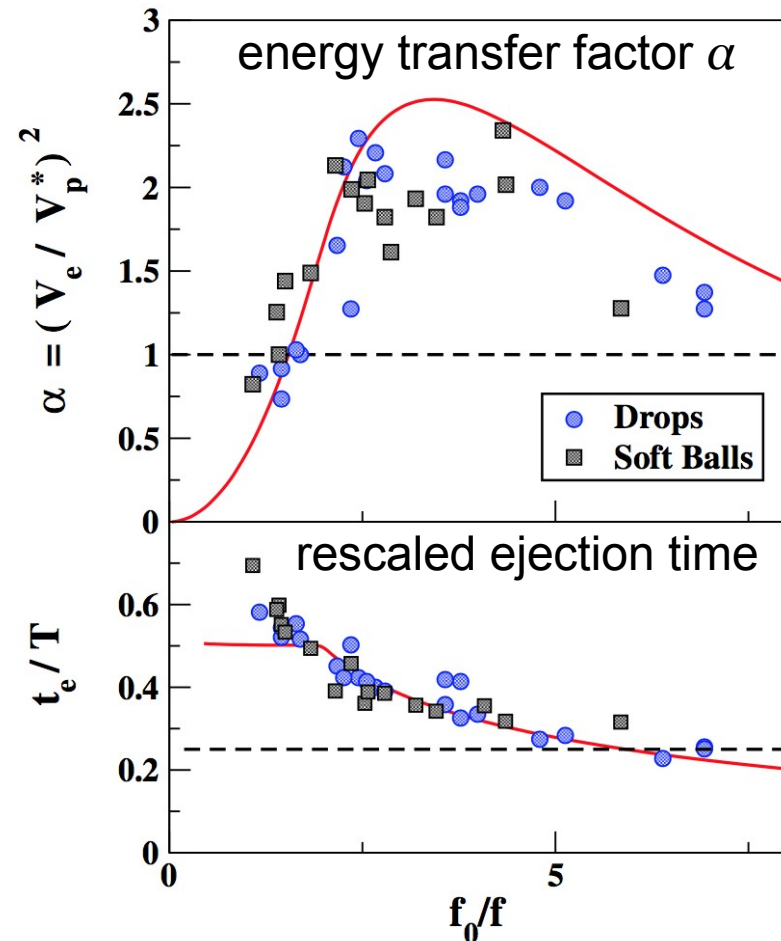
drop radius  $R \sim 1\text{mm}$ ,  $f_0 \sim 300\text{Hz}$

catapult amplitude  $A \sim 1\text{-}10\text{mm}$

catapult frequency  $f \sim 20\text{-}100\text{Hz}$

catapult acceleration  $\sim 10g$

ejection velocity  $V_e \sim 1\text{m/s}$



# Application 1: droplet ejection

## Applications

Droplet actuation and sorting

Energy saving

Already present in nature !

# Application 1: droplet ejection

## □ Applications

Droplet actuation and sorting

Energy saving

Already present in nature !



Droplet superpropulsion in an energetically constrained insect  
Sharpshooters need to evacuate 300x their mass in urine everyday !  
Challita et al., Nature Com. 2023

# Application 1: droplet ejection

## □ Applications

Droplet actuation and sorting

Energy saving

Already present in nature !



Droplet superpropulsion in an energetically constrained insect  
Sharpshooters need to evacuate 300x their mass in urine everyday !  
Challita et al., Nature Com. 2023

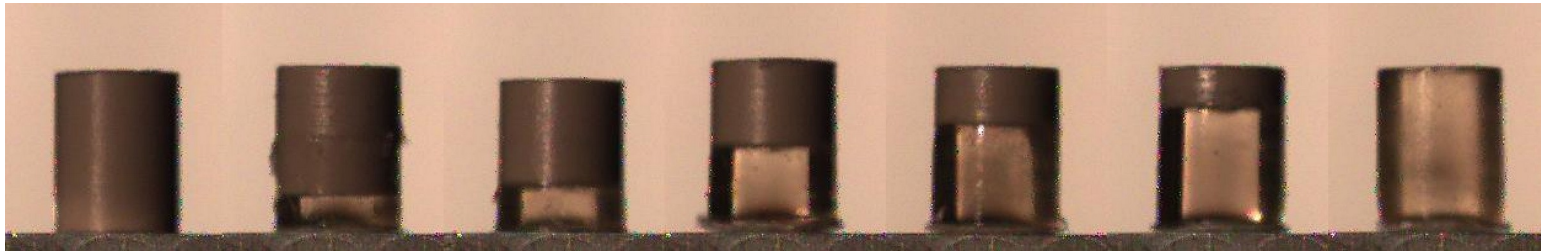


## Application 2: boosting rigid projectiles

# Application 2: boosting rigid projectiles

## □ Idea

Add a layer of soft elastic material at the bottom of rigid objects



$x=0$

rigid case

$x=1$

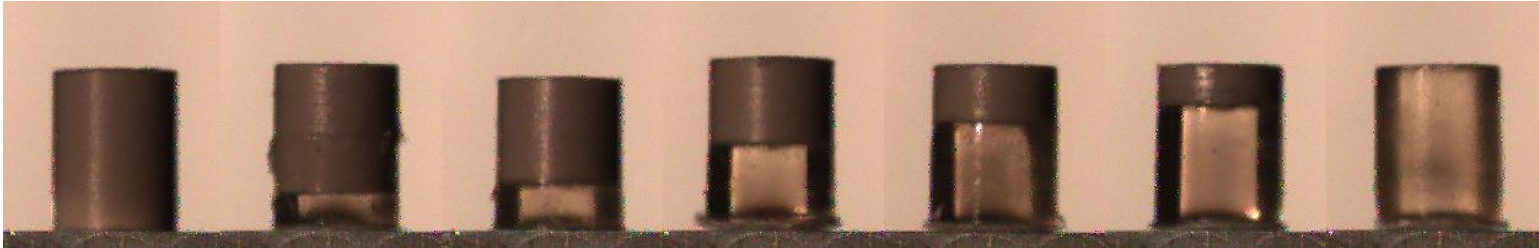
soft case

□ Relevant parameters  $f_o/f \rightarrow c_s/Lf$  and  $x$

# Application 2: boosting rigid projectiles

□ Idea

Add a layer of soft elastic material at the bottom of rigid objects



$x=0$

rigid case

$x=1$

soft case

□ Relevant parameters  $f_o/f \rightarrow c_s/Lf$  and  $x$

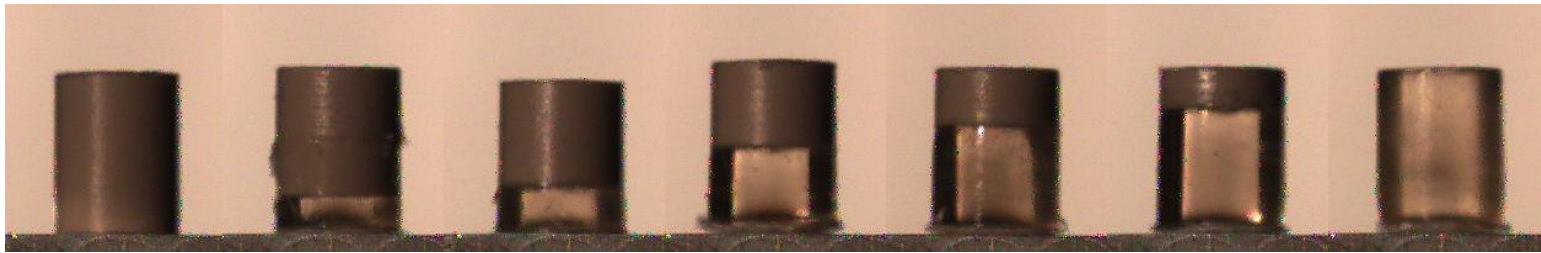
$c_s/Lf=6.5$



# Application 2: boosting rigid projectiles

## □ Idea

Add a layer of soft elastic material at the bottom of rigid objects



$x=0$

rigid case

$x=1$

soft case

□ Relevant parameters  $f_0/f \rightarrow c_s/Lf$  and  $x$

$c_s/Lf=2.9$

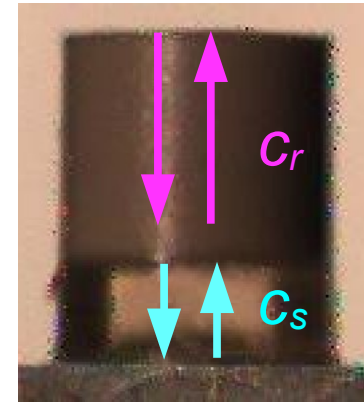


# Application 2: boosting rigid projectiles

## □ Numerical approach

1D wave equation in both layers ( $c_r \gg c_s$ )

right boundary conditions

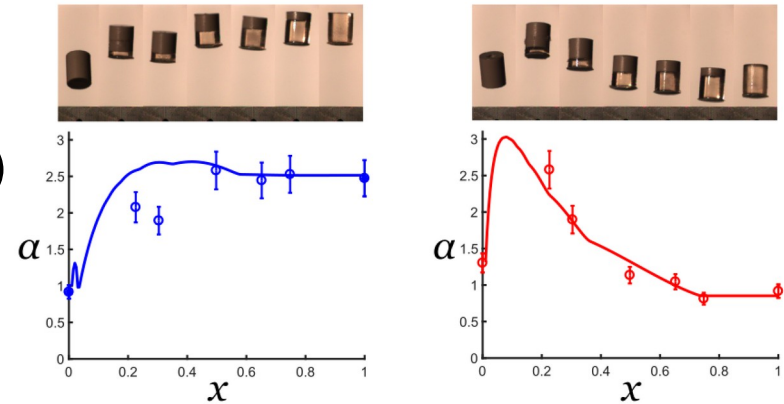


# Application 2: boosting rigid projectiles

## □ Numerical approach

1D wave equation in both layers ( $c_r \gg c_s$ )

right boundary conditions



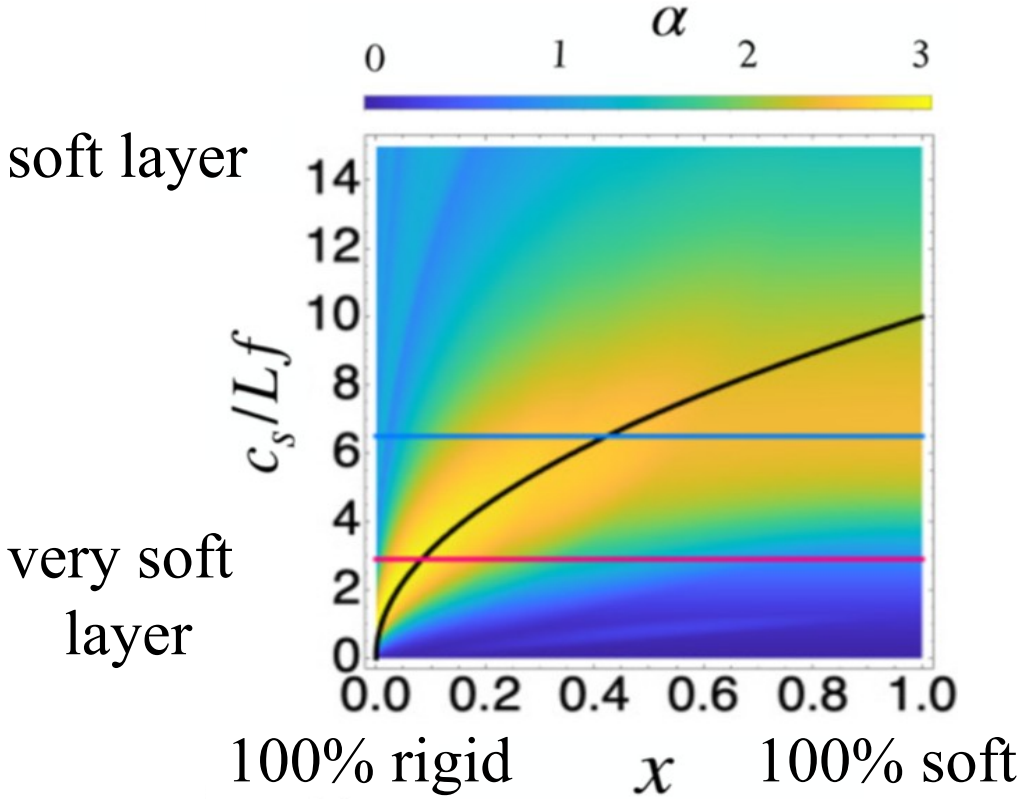
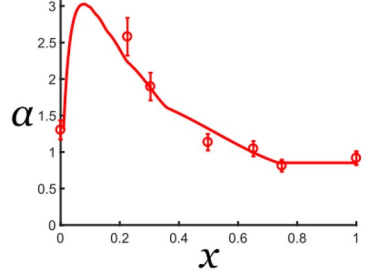
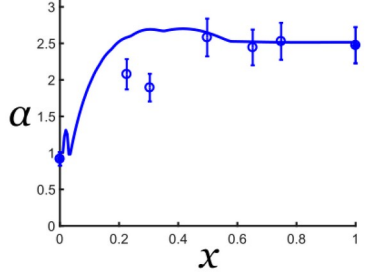
# Application 2: boosting rigid projectiles

## □ Numerical approach

1D wave equation in both layers ( $c_r \gg c_s$ )  
right boundary conditions

## □ Results

Superpropulsion whatever  $x$   
Optimal crest



# Application 2: boosting rigid projectiles

## □ Numerical approach

1D wave equation in both layers ( $c_r \gg c_s$ )  
right boundary conditions

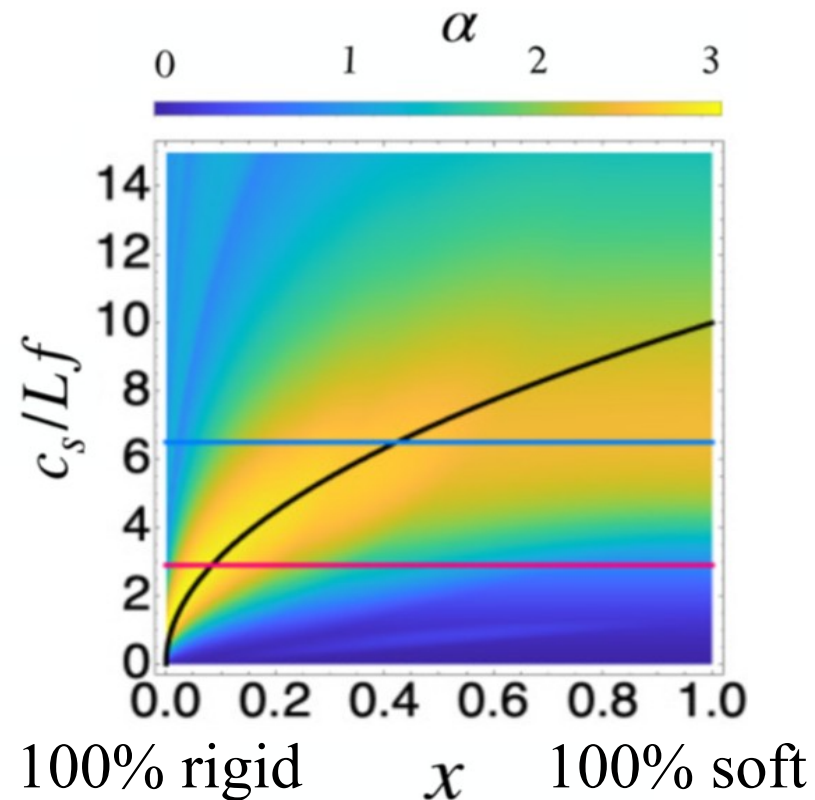
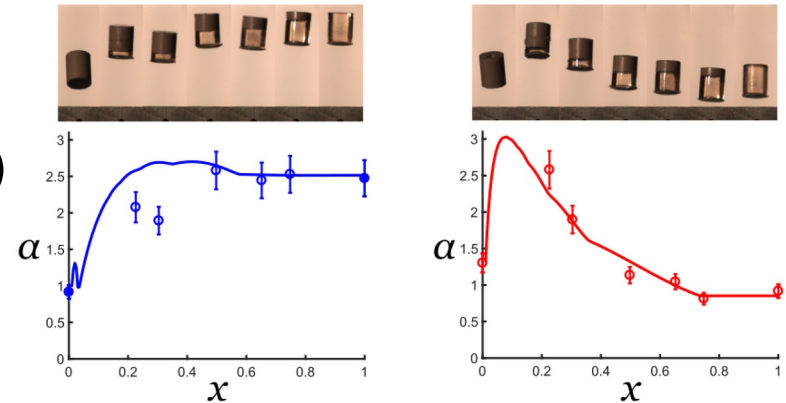
## □ Results

Superpropulsion whatever  $x$   
Optimal crest

## □ Two limits

$x \rightarrow 1$ ,  $\alpha_{max} = 2.5$   
wave dynamics inside the soft layer  
 $f_o/f = 3.4$  for the optimal case

$x \rightarrow 0$ ,  $\alpha_{max} = 3$   
mass-spring system  
 $f_o/f = 1.6$  for the optimal case



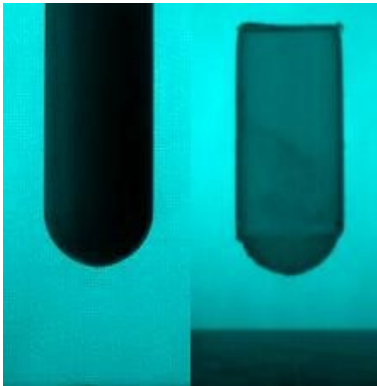


# Conclusion

- **General mechanism: superpropulsion**
  - Matching deformation/throw dynamics
  - Specific resonance (physics and model dependent)
    - optimal value of the parameter  $f_o/f$
  - Different systems – same effect:
    - waves, mass-spring system, surface tension, ...
  
- **Input from soft matter and materials physics**
  - Tunable properties, low elastic moduli
  - Typical acceleration time around 10-100 ms ... can be extended !
  
- **Applications in throws**
  - 250-300% gain in kinetic energy for light objects

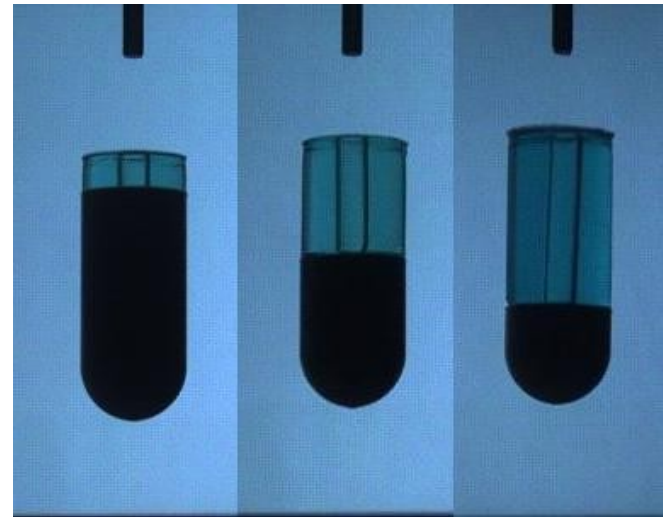
# Related works

- Impact of bilayered projectiles



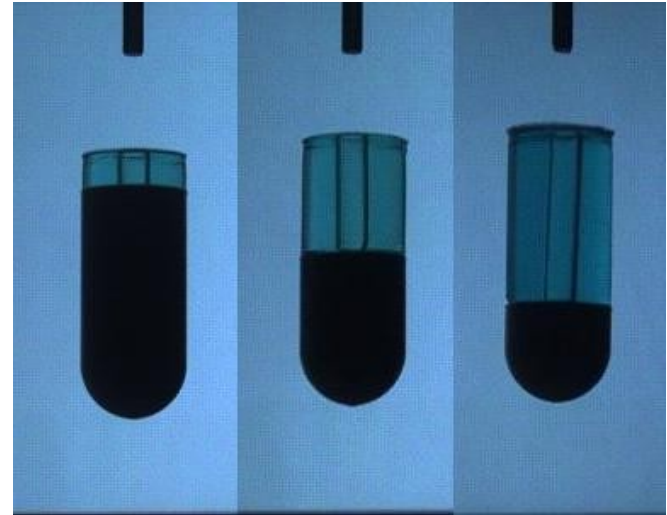
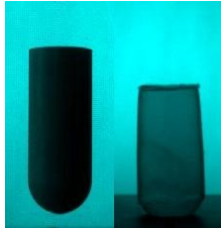
100% rigid  
hard plastic

100% soft  
gelatin hydrogel



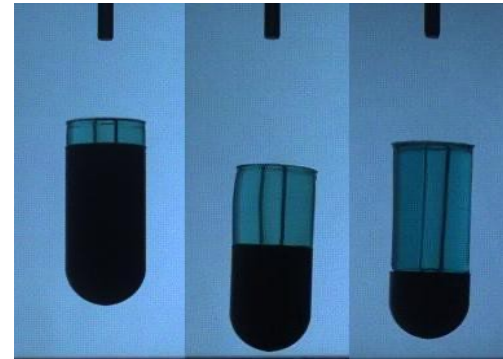
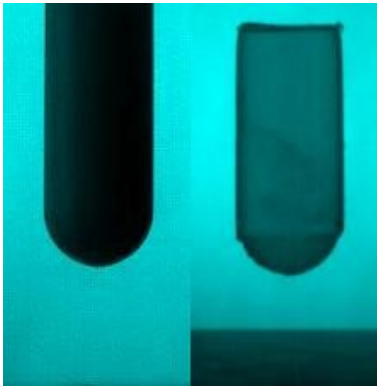
# Related works

- Impact of bilayered projectiles



# Related works

- Impact of bilayered projectiles



# Acknowledgments

## Investigators at the physics institute of Nice



Franck  
Celestini



Christophe  
d'Angelo



Guillaume  
Giombini



Médéric  
Argentina



Cyrille  
Claudet

## Collaborators and sponsors



Joachim Mathiesen  
Niels Bohr Institute  
Copenhagen



Laurence Viennot  
MSC  
Paris



institut  
universitaire  
de France



AGENCE  
INNOVATION  
DÉFENSE