Throwing objects with the superpropulsion effect

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A fascinating ability

- Throwing
  an action which consists in accelerating a projectile and then releasing it so that it follows a ballistic trajectory. (From Wikipedia)

- Throw and records
  >12000 occurrences in Guinness World Records
distance, speed, precision, frequency

  “Longest throw of an object with no tail” (427.2 m)
  “Fastest Jai-Alai (Pelota) throw” (305.77 km/h)
  “Most basketball free throws in three minutes” (201)
  “Furthest distance to throw and catch an egg” (98.51 m)
  “Farthest throw of a washing machine” (4.45 m)
  “Most tea bags thrown into mugs in 30 seconds” (30)
Evolution of throwing in humans

- **Humans**
  - slow, weak, lack natural weapons
  - unique abilities among primates
  - hunting 2 Myr ago

- **Anatomical features**
  - rotation of the shoulder
  - elbow flexion

- **Later development of tools/weapons**
  - context: hunting, warfare, sports
  - spear - 0.5 Myr ago
  - bow - 70000 yr ago
  - counterweight trebuchet 900 yr ago

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Hand throwing

- **Biomechanical aspects**
  - sequential activation of many muscles: legs, hips, torso, shoulder, elbow, wrist
  - role of tendons
  - elastic energy storage and release
  - accumulation and transmission of kinetic energy

- **Available energy in shot putters**
  - muscle power ~ 100 W/kg
  - muscle weight ~ 25 kg (20% of body mass)
  - activation time ~ 200 ms
  - available energy ~ 500 J

- **Kinetic energy of the shot**
  - \( v_{\text{shot}} \sim 10 \text{ m/s}, m_{\text{shot}} \sim 10 \text{ kg} \)
  - \( KE_{\text{shot}} = \frac{1}{2} m_{\text{shot}} v^2_{\text{shot}} \sim 500 \text{ J} \)
Always efficient? A simple experiment

- Effect of the projectile mass
  example in overarm throw
  not efficient with light projectiles

- Simple model
  kinetic energy of the projectile
  \[ E_0 = KE = \frac{1}{2}mv^2 \]
  or \[ V = \sqrt{\frac{2E_0}{m}} \]
  available energy in muscles

Always efficient? A simple experiment

- Effect of the projectile mass
  *example in overarm throw*
  *not efficient with light projectiles*

- Simple model
  
  \[ E_0 = K \cdot E = \frac{1}{2} m V^2 \quad \text{or} \quad V = \sqrt{2 E_0 / m} \]

  
  available energy in muscles

Always efficient? A simple experiment

- **Effect of the projectile mass**
  example in overarm throw
  not efficient with light projectiles

- **Simple model**
  kinetic energy (KE)
  of the projectile
  \[ E_0 = \frac{1}{2} m V^2 + \frac{1}{2} M V^2 \]
  available energy in muscles
  kinetic energy remaining in the body through the motion of a virtual mass \( M \)

- **Implications**
  \( M \approx 2 \) kg corresponds to hand and forearm
  Difficulties to throw at large distances
  Higher risk of injuries with light objects

\[ KE = \frac{E_0}{1 + M/m} \]

Light projectiles: need for a tool

- **Efficiency KE/E₀**

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☐ Other strategies with tools
- hitting (golf, tennis, ...)
- spinning (sling, hammer throw ...)
- loading (bow, slingshot ...)

Scientific questions

☐ How to increase the throw efficiency of light objects?

☐ Can we find other strategies than the use of a tool?
  • Mimic the action of tendons

☐ What input from soft matter and materials physics?
  • Find the good materials and geometries to reach relevant time scales
Main idea

- Basic geometry

Perfect throwing engine ($M \gg m$)
Harmonic motion
Amplitude $A$, frequency $f$

$$z_p(t) = A[1 - \cos(2\pi ft)]$$
$$V_p^* = 2\pi fA$$ maximum speed
Main idea

- **Basic geometry**
  - Perfect throwing engine ($M \gg m$)
  - Harmonic motion
  - Amplitude $A$, frequency $f$
  - Equation: $z_p(t) = A[1 - \cos(2\pi ft)]$
  - Maximum speed: $V_p^* = 2\pi fA$

- **Case of a rigid object**
  - Ejection speed: $V_e = V_p^*$
  - Transferred energy: $\int_0^{t_e} F(t)\dot{z}_p(t)\,dt$
Main idea

- Basic geometry
- Case of a rigid object

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\[ V_e = V_p^* \]

Transferred energy:
\[ \int_0^{t_e} F(t) \dot{z}_p(t) \, dt \]

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Amplitude \( A \), frequency \( f \)

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Solution: soft elastic projectiles

**Requirements**
- Delayed response and tunable time scale
- Good elastic restitution

**Examples of quasi-1D gelatin hydrogels**
- Young modulus 12 kPa
- Deformation wave speed \( c = 3.4 \text{ m/s} \)
- Typical length \( L: 3 - 30 \text{ mm} \)
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- **Typical time sequence**
  - $A \sim 1 \text{ mm}$
  - $f \sim 50 \text{ Hz}$
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☐ Results
Effect of the size for a given frequency $f$

$1cm$

$f \approx 50 \text{ Hz}$
rigid
Solution: soft elastic projectiles

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□ Results
  Effect of the size for a given frequency $f$

□ Energy transfer factor
  $\alpha = (V_e/V_p^*)^2$
  Effect of the dimensionless frequency $f_0/f$
  Optimal ratio $f_0/f \approx 3-4$ gives $\alpha \approx 2.5$
  Specific resonance effect
Solution: soft elastic projectiles

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- **Other material/geometry**
  - Polyacrylamide beads
    - \( E \approx 1 - 10 \text{ kPa} \)
    - \( f_0 \approx \frac{1}{L} \sqrt{\frac{E}{\rho}} \)
    - \( 50 - 200 \text{ Hz} \)
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Solution: soft elastic projectiles

- General mechanism: superpropulsion
  - Matching deformation/throw dynamics
  - Optimal value of the parameter $f_0/f \approx 3-4$
  - Gain in kinetic energy $\alpha \approx 2.4-2.7$

- Perfect agreement with models
  - $f_0/f = 3.4$ and $\alpha = 2.5$
Application 1: droplet ejection

- Droplet dynamics
  - Deformation associated with surface tension
  - Eigenfrequency

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- Parameters
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catapult amplitude \( A \sim 1-10\text{mm} \)
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adhesionless substrates
collaboration with chemists
\( f=37\text{Hz} \quad A=1.4\text{mm} \)
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\[ \alpha = \left(\frac{V_e}{V_p}\right)^2 \]

Energy transfer factor \( \alpha \)

Rescaled ejection time \( t_e/T \) vs. \( f_0/f \)
Application 1: droplet ejection

- Applications
  - Droplet actuation and sorting
  - Energy saving
  - Already present in nature!
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Droplet superpropulsion in an energetically constrained insect
Sharpshooters need to evacuate 300x their mass in urine everyday!
Challita et al., Nature Com. 2023
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Application 2: boosting rigid projectiles
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Idea
Add a layer of soft elastic material at the bottom of rigid objects

Relevant parameters $f_0/f \rightarrow c_s/Lf$ and $x$
Application 2: boosting rigid projectiles

- **Idea**
  Add a layer of soft elastic material at the bottom of rigid objects

- **Relevant parameters**
  \[ \frac{f_0}{f} \rightarrow \frac{c_s}{Lf} \] and \( x \)

- \( \frac{c_s}{Lf} = 6.5 \)
Application 2: boosting rigid projectiles

- **Idea**
  Add a layer of soft elastic material at the bottom of rigid objects

- **Relevant parameters**
  \[ \frac{f_0}{f} \rightarrow \frac{c_s}{Lf} \text{ and } x \]

- \( c_s/Lf = 2.9 \)
Application 2: boosting rigid projectiles

- Numerical approach
  1D wave equation in both layers ($c_r >> c_s$)
  right boundary conditions
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- **Results**
  Superpropulsion whatever $x$
  Optimal crest
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- **Results**
  Superpropulsion whatever $x$
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- **Two limits**
  $x \rightarrow 1$, $a_{max} = 2.5$
  wave dynamics inside the soft layer
  $f_0/f = 3.4$ for the optimal case

  $x \rightarrow 0$, $a_{max} = 3$
  mass-spring system
  $f_0/f = 1.6$ for the optimal case
Conclusion

- **General mechanism: superpropulsion**
  - Matching deformation/throw dynamics
  - Specific resonance (physics and model dependent)
    - optimal value of the parameter $f_0/f$
  - Different systems – same effect:
    - waves, mass-spring system, surface tension, ...

- **Input from soft matter and materials physics**
  - Tunable properties, low elastic modulii
  - Typical acceleration time around 10-100 ms … can be extended!

- **Applications in throws**
  - 250-300% gain in kinetic energy for light objects
Related works

- Impact of bilayered projectiles

100% rigid 100% soft
hard plastic gelatin hydrogel
Related works

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