Absorbing states in granular matter

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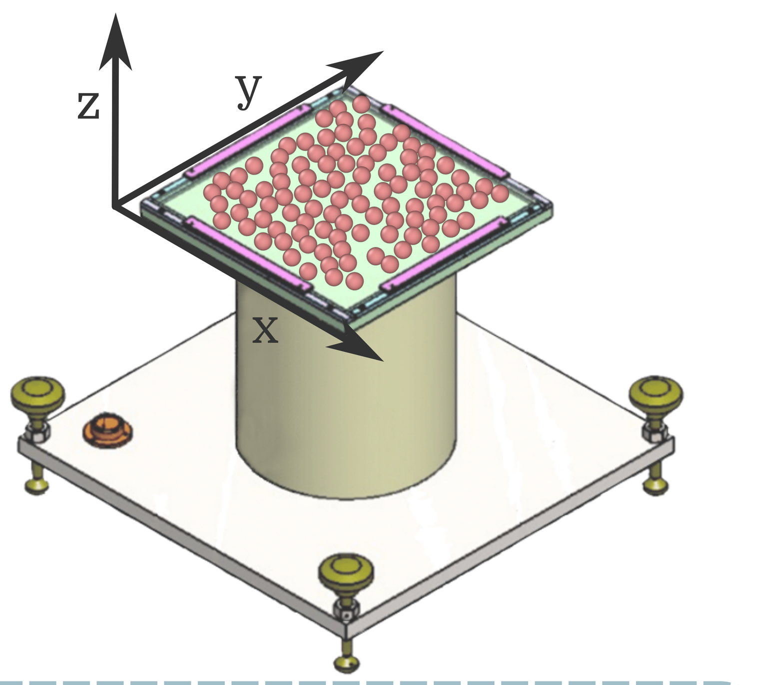
ABSTRACT

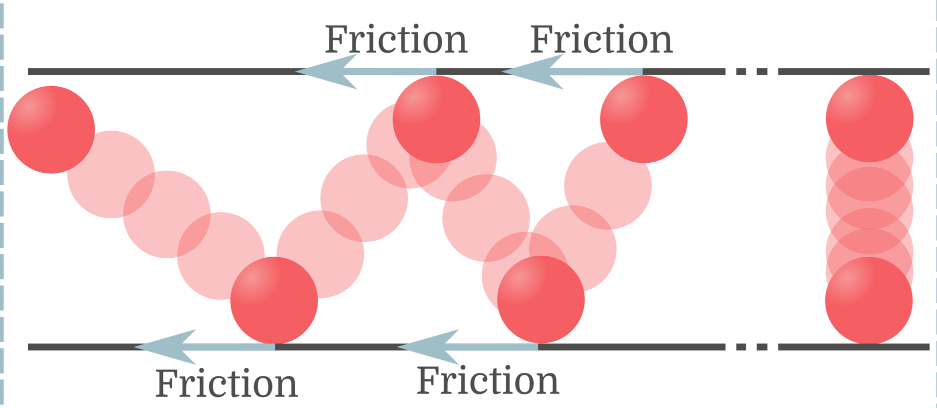
Absorbing phase transitions, where a system goes from a dynamic "alive" to a static "dead" state from which there is no return are an archetypal exemple of out of equilibrium phase transitions. In this study, we investigate such a process using computer simulations of a vertically shaken ensemble of granular beads confined in a quasi-2D setup [1]. We find that the nature of the transition is deeply linked to a dynamics of synchronization which in turn alters the dissipative nature of the system. To gain deeper insights, we develop a coarse-grained two-dimensional hard-disk model as a representation of the initial quasi-2D system and employ it to elucidate the observed transition. The simulations and the kinetic theory both predict the transitions observed in the realistic model.

Dynamics of beads in a quasi-2D setup

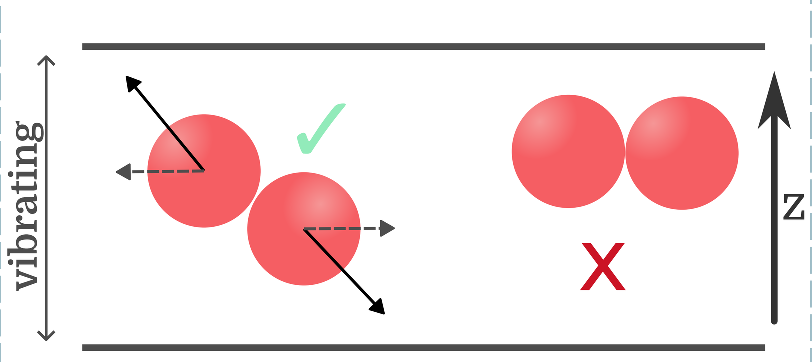
Grains placed on a **shaker** and confined in the z direction gain energy by vertical shaking. **Three main effects** will drive the phase transition:



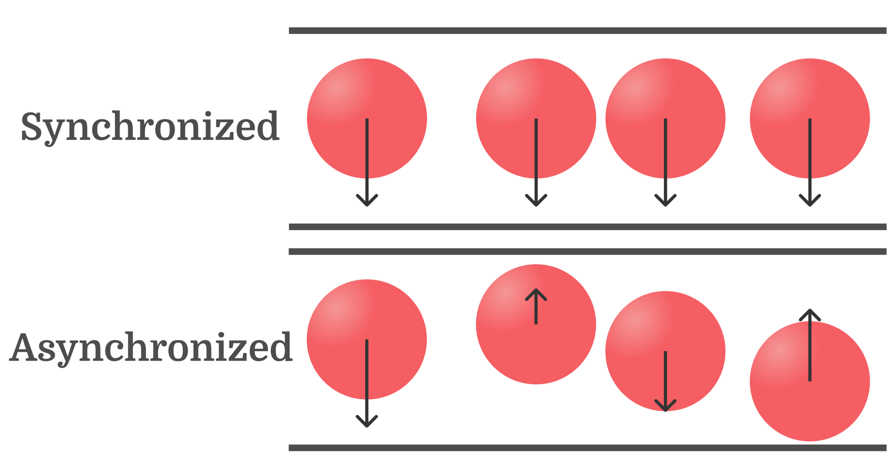
2. Collisions with the plates will **act as friction** and tend to reduce the xy momentum of the beads.



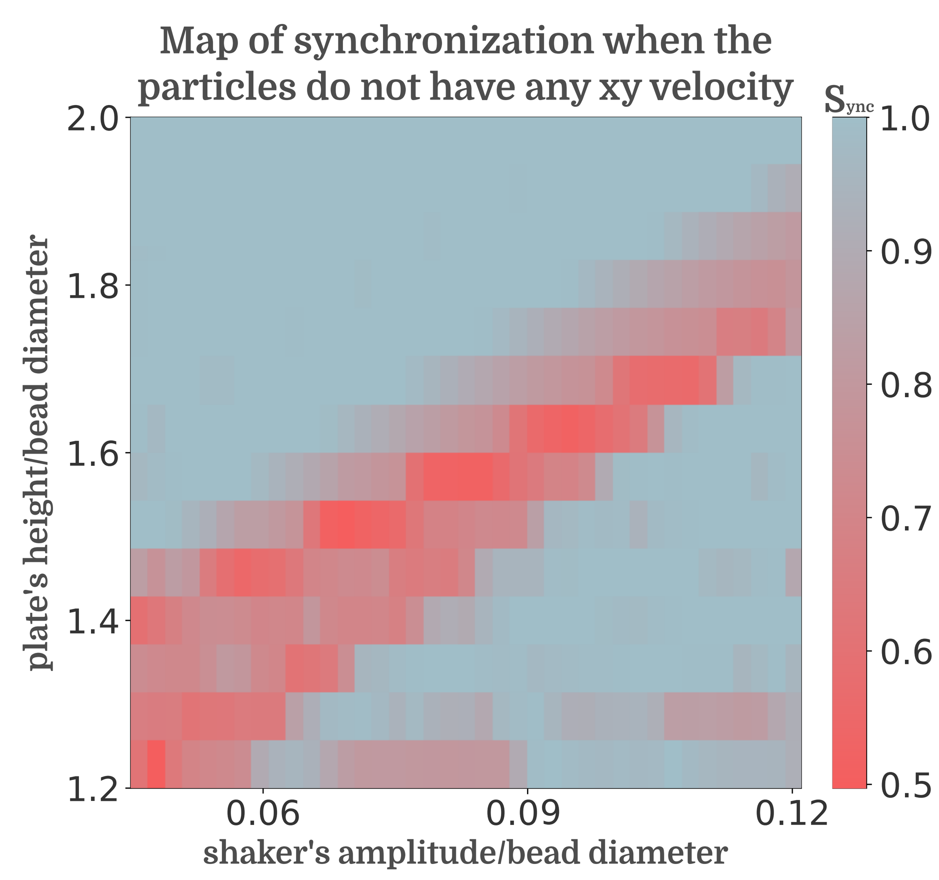
1. Collisions between beads will **transfer the vertical energy** gained from vibration only if they collide off-center.



3. Depending on the amplitude, frequency,... **particles may synchronize** if they do not collide.

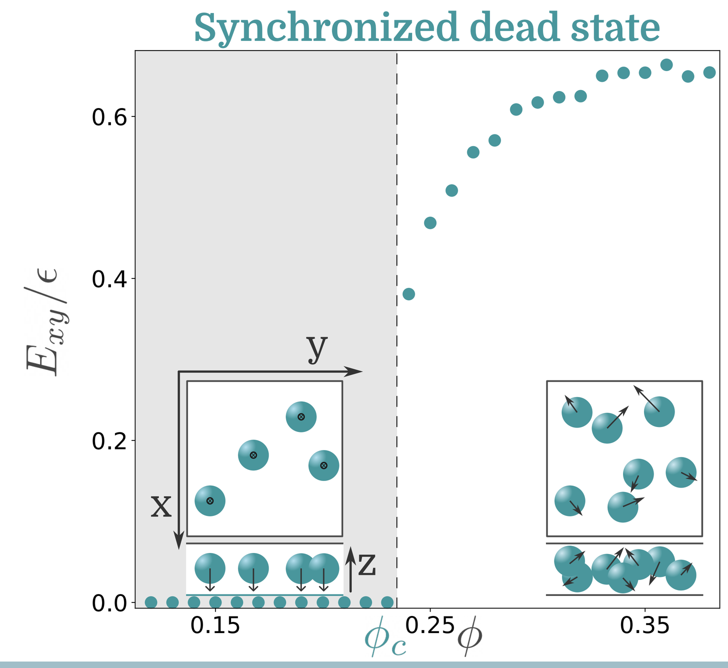


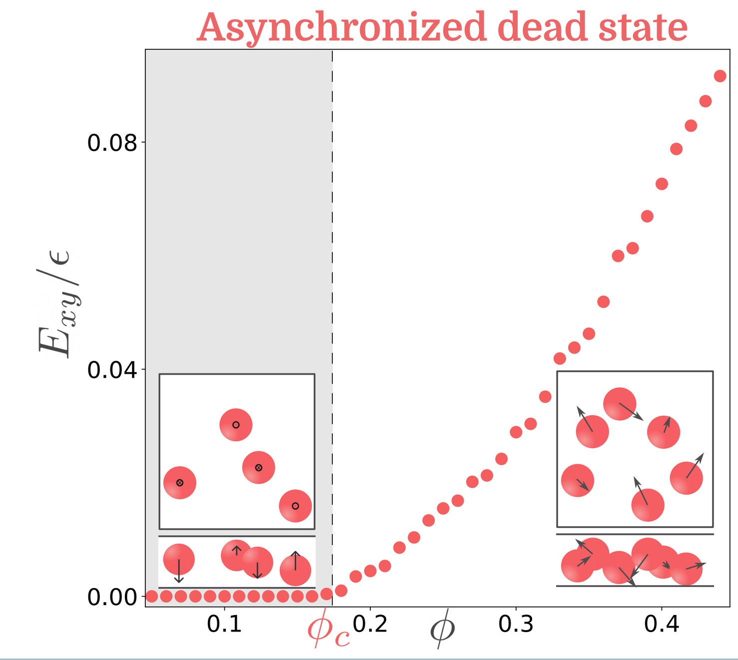
Absorbing phase transition behavior



- Starting from particles **without any velocity in the xy plane** (only jumping vertically) we can create a map of synchronization.

- Starting from particles **with a non-zero velocity in the xy plane** we can study the final steady state the system will reach (especially its xy velocity). **At high densities** we expect the energy injection mechanism **1.** to be dominant due to the large number of collisions while **at low densities** we expect the energy drain mechanism **2.** to be dominant.





Building a simpler model: integrating the z component of the complex model

In order to investigate the dynamics of the phase transition on a larger scale and develop a theoretical understanding of it we build a computationally simpler model by **coarse-graining out the z component of the quasi-2D system** to be left with a purely 2D model.

Simple 2D model

Energy injection mechanism 1.

Particles are modeled as dissipative hard disks gaining at each collision an additional velocity Δ simulating the off-center collisions of the quasi-2D model [2].

$$\mathbf{v}_i' = \mathbf{v}_i + \frac{1+\alpha}{2}(\mathbf{v}_{ij}\cdot\hat{\boldsymbol{\sigma}}_{ij})\hat{\boldsymbol{\sigma}}_{ij} + \Delta\hat{\boldsymbol{\sigma}}_{ij}$$

$$\mathbf{v}_j' = \mathbf{v}_j - \frac{1+\alpha}{2}(\mathbf{v}_{ij}\cdot\hat{\boldsymbol{\sigma}}_{ij})\hat{\boldsymbol{\sigma}}_{ij} - \Delta\hat{\boldsymbol{\sigma}}_{ij}$$

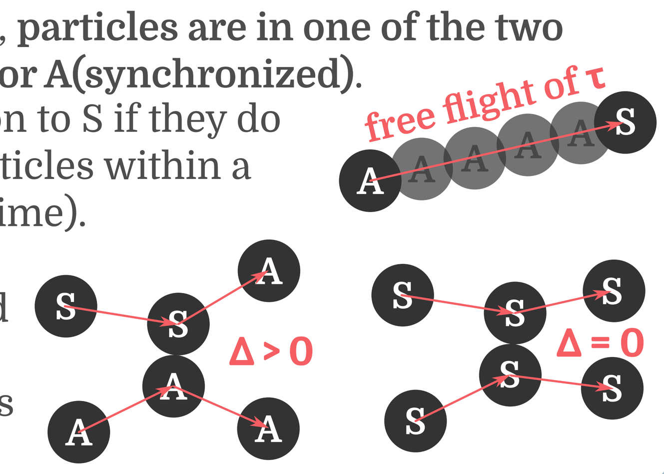
Energy damping mechanism 2.

During their free flight, particles, in our simplified model, are damped to simulate collisions with the plate

$$v_{\text{free flight}} = v_0 e^{-\gamma t}$$

Synchronization mechanism 3.

In our simplified model, **particles are in one of the two states: S(ynchronized) or A(synchronized).** Particles A will transition to S if they do not collide with any particles within a time τ (this is a synch. time). Colision between S particles should not add energy into the system. Moreover, A-S collisions make **S** transition to **A**.



Theory and simulation of the simple model

To describe our system we will use two coupled equations. A **hydrodynamic** one capturing the mean field evolution of the global temperature of the system and a **master equation** dealing with the population dynamics between A and S particles.

$$\frac{dP_A}{dt} = \omega(T)P_AP_S - \frac{e^{\omega(T)\tau}}{\tau}P_A.$$

A particles become **S** after a time τ if they don't collide (a Poissonian process is assumed).

S particles transition to **A** when they collide with an **A** particle.

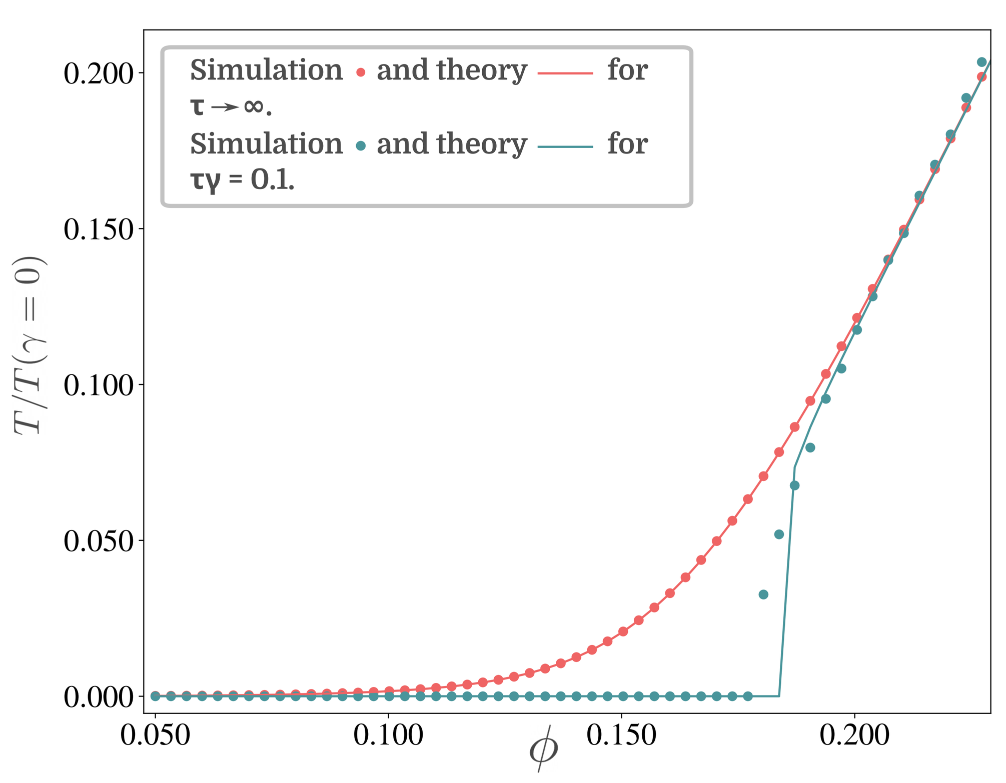
$P_A = N_A/N$: the fraction of A particles

$\omega(T)$: the corrected collision frequency assuming an equilibrium system at temperature T.

$$\frac{dT}{dt} = (P_A^2 + 2P_AP_S)\frac{\omega(T)}{2}\langle E'-E\rangle_{\text{coll},\Delta>0} + P_S^2\frac{\omega(T)}{2}\langle E'-E\rangle_{\text{coll},\Delta=0} - 2\gamma T$$

Energy change due to A-S and A-A collisions

Energy change due to S-S collisions



The simplified model successfully captures both types of phase transitions and the predictions of the kinetic theory align with the observations.

In the case of the continuous transition, the population dynamics is straightforward, involving only A particles. In this scenario, the theory works well, except for some deviations at the transition point that arise from **the gaussian/equilibrium approximation in the collision frequency**.

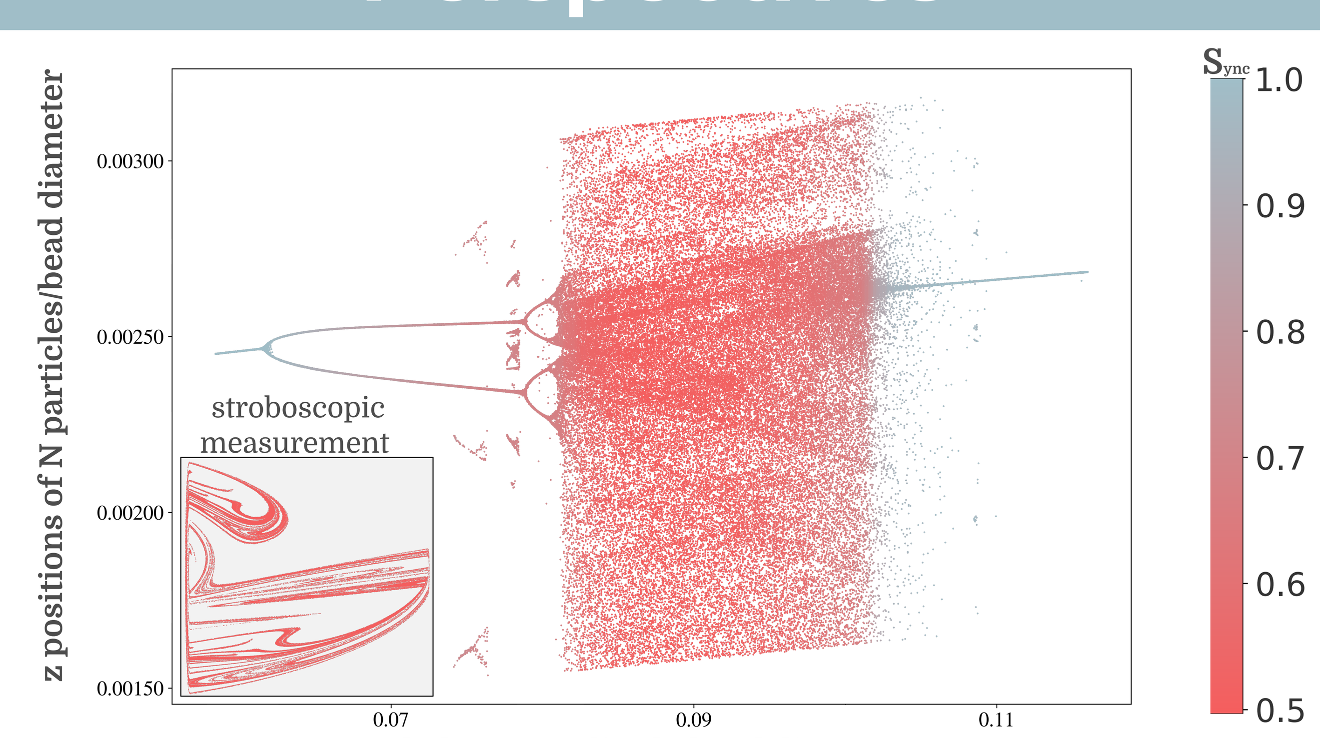
However, for the discontinuous transition, the mean field approach encounters challenges because **spatial correlations emerge as a result of the local nature of the population dynamics**.

Conclusion

- In a quasi-2D shaken granular gas, a phase transition occurs as a result of the **interplay between energy injection and energy dissipation processes**.

- The specific nature of the phase transition is determined by a synchronization behavior emerging in the absorbing state of the transition. This synchronization leads to **purely dissipative collisions** within the system, enabling it to **reach a steady state as the cooling state of a granular gas.**

Perspectives



[1] Néel, Baptiste, et al. "Dynamics of a first-order transition to an absorbing state." Physical Review E 89.4 (2014).

[2] Brito, Ricardo, Dino Risso, and Rodrigo Soto. "Hydrodynamic modes in a confined granular fluid." Physical Review E 87.2 (2013).