Measurements of Quantum correlations, and characterization of quantum thermalization, in a dipolar interacting spin system

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Chromium atoms loaded in a deep 3D lattice: a spin system driven by dipolar interactions

A platform to study an original quantum magnetism

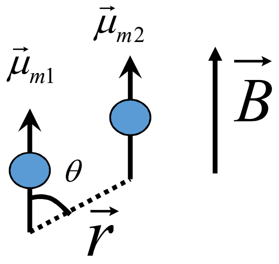
Real spins

Large s=3 spins (6 electrons)

Atoms loaded in a deep
3D optical lattice→ Mott state

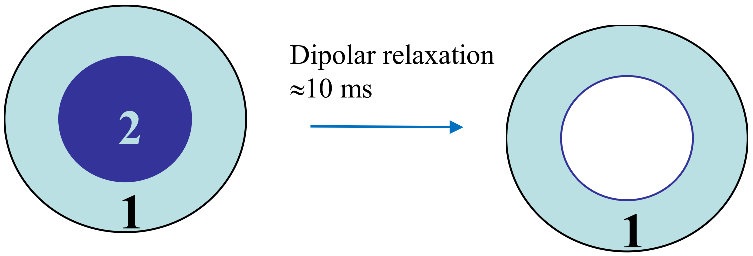
Spins directly coupled by long range interactions

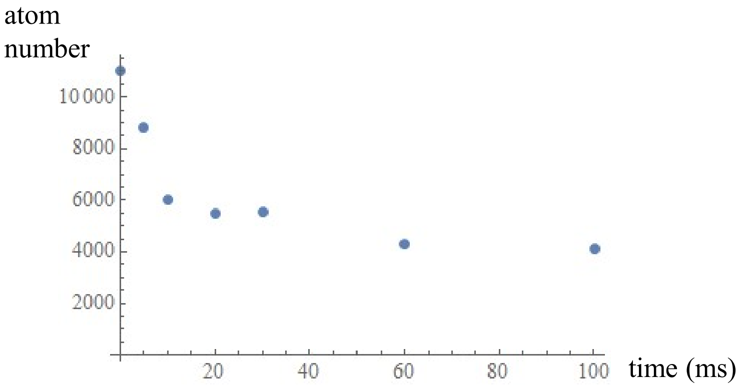
$$\hat{H}_{\text{dd eff}} = \sum_{i>j} \frac{\mu_0}{4\pi}(g\mu_B)^2 \frac{1-3\cos^2\theta_{ij}}{R_{ij}^{\ 3}}\left(\hat{S}_i^z\hat{S}_j^z - \frac{1}{2}\left(\hat{S}_i^x\hat{S}_j^x + \hat{S}_j^y\hat{S}_j^y\right)\right)$$



Not a perfect spin system

Lossy system





Finite size effects

Presence of holes(?)

Principle of or out of equilibrium spin dynamics experiments

Spin =3 for chromium

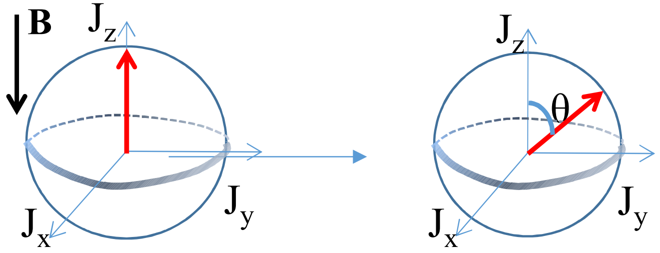
Initial preparation: $\Psi_{initial} = \left|-3_z,-3_z,....,-3_z,-3_z\right\rangle$ BEC in absolute ground state m_s=-3 loading in a 3D lattice

1- Excite the spins

$$\Psi_{(t=0)} = \left|-3_\theta,-3_\theta,....,-3_\theta,-3_\theta\right\rangle$$

Use of Radio Frequency: induce spin rotations

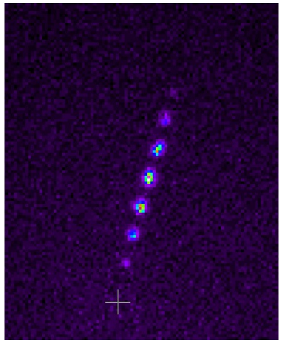
$f_{RF} = f_{Larmor} = g\,\mu_B\,B$



2- Free evolution under the effect of interactions

3- Measurement of Spin populations

Stern Gerlach separation



Fluorescence imaging

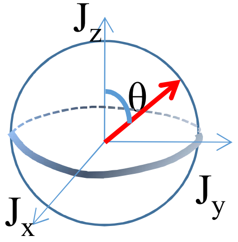
4- Measure collective quantities – Derive something interesting

Alternative: 4- Local measurements (quantum microscope, M. Greiner, W. Bakr)

Out of equilibrium spin dynamics experiments: summary of our previous results

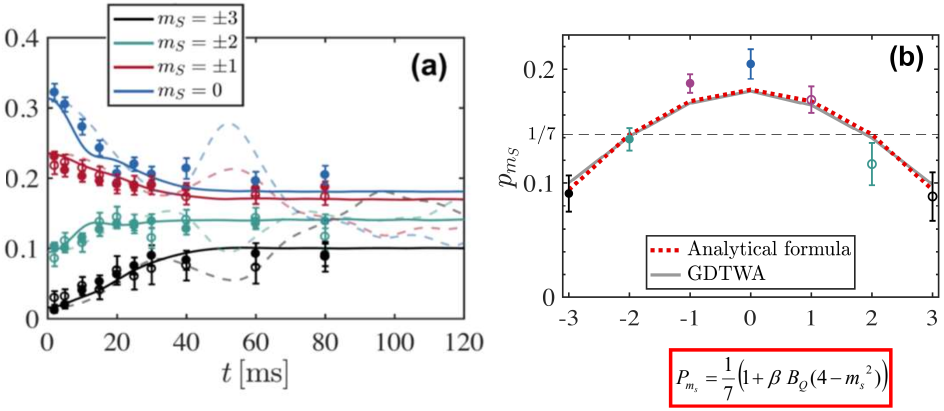
$$\hat{H} = \sum_{i>j} V_{i,j}\left(\hat{S}_i^z \hat{S}_j^z - \frac{1}{2}\left(\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y\right)\right) + B_Q \sum_i \hat{S}_i^{z\,2}$$

$$\Psi_{(t=0)} = \left| -3_\theta, -3_\theta, ..., -3_\theta, -3_\theta \right\rangle$$



Spin populations dynamics well reproduced by quantum simulations (GDTWA) and **not** by mean field simulations

Asymptotic populations in agreement with a scenario of Quantum Thermalization

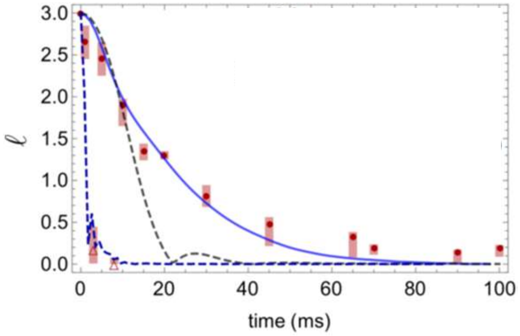


$$\boxed{P_{m_s} = \frac{1}{7}\left(1 + \beta\, B_Q\left(4 - m_s^{\,2}\right)\right)}$$

Lepoutre et al, NatCom 2019

Dynamics of the Collective spin length Compatible with a dipolar evolution (but slower)

$$\left|\vec{J}\right| = \left(\left\langle \hat{J}_x \right\rangle^2 + \left\langle \hat{J}_y \right\rangle^2 + \left\langle \hat{J}_z \right\rangle^2\right)^{1/2} \qquad 0 \le \ell = \frac{\left|\vec{J}\right|}{N} \le 3$$



Gabardos et al, PRL 2020

All these experimental features are in agreement with a quantum thermalization scenario: the isolated spin system evolves towards an asymptotic state where local quantities acquire a thermal character while the entire system remains pure and get entangled

New characterization of the thermalization process: measuring directly the growth of correlations

$$\Delta\hat{J}_z^{\,2} = \left\langle \hat{J}_z^{\,2} \right\rangle - \left\langle \hat{J}_z \right\rangle^2 = \sum_i \Delta\hat{s}_z^{i\,2} + C_z$$

$$C_z = \sum_{i \neq j} \left(\left\langle \hat{s}_z^i \hat{s}_z^j \right\rangle - \left\langle \hat{s}_z^i \right\rangle \left\langle \hat{s}_z^j \right\rangle \right)$$

$$\boxed{C_z = \Delta\hat{J}_z^{\,2} - \sum_i \Delta\hat{s}_z^{i\,2}}$$

$\Delta\hat{J}_z^{\,2}$ = Variance of $\hat{J}_z$

⇨ Repeat the experiment= acquire statistics

$\sum_i \Delta\hat{s}_z^{i\,2}$ = Sum of individual variances

⇨ Can be measured from collective measurement assuming homogeneity

homogeneous spin system

$$\sum_i \Delta\hat{s}_z^{i\,2} \overset{\downarrow}{=} N\left(\sum_{m_s} p_{m_s} m_s^{\,2} - \left(\sum_{m_s} p_{m_s} m_s \right)^2 \right)$$

$$-3 \leq m_s \leq +3$$

p_{m_s} = fractionnal population, $\sum_{m_s} p_{m_s} = 1$

$$\boxed{C_{z\,\mathrm{norm}} \underset{\theta=\frac{\pi}{2}}{=} \frac{1}{N}\Delta\hat{J}_z^{\,2} - \sum_{m_s} p_{m_s} m_s^{\,2}}$$

$\frac{1}{N}\Delta\hat{J}_z^{\,2} = \frac{3}{2}$ expected constant for a pure dipolar dynamics

$\sum_{m_s} p_{m_s} m_s^{\,2}$ grows as dynamics proceeds

We can measure a correlation witness for large spin systems from collective measurements

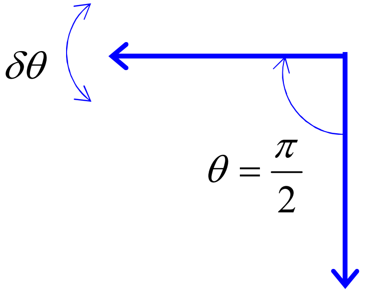
Measuring spin fluctuations at the level of the quantum noise: a difficult task!

We do not assume $\frac{1}{N}\Delta\hat{J}_z^{\,2} = \frac{3}{2}$, we measure it

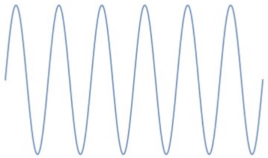
$$\frac{\Delta\hat{J}_z}{N} \underset{\text{theory}}{=} \sqrt{\frac{3}{2N}} = \text{SQN}$$

Our best values for magnetization fluctuations at *t*=0 $\Delta J_{z\,\exp} \approx 2\times\text{SQN}$ $\Delta J_{z\,\exp} > \text{SQN}$ because:

* Spin preparation is not perfect:



π/2 RF pulse

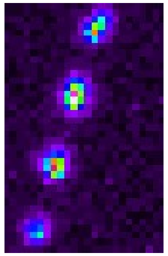


6-8 Larmor periods $f_{\text{Rabi}} \approx 100$ kHz

Large spins
more sensitive
to technical noise!

$\Delta J_{z\ \text{preparation}} = \text{SQN}$ if $\delta\theta = 3\times10^{-3}$ rd

* Atom counting is not perfect:
limited signal to noise ratio→ fit noise



* Detection noise = Shot noise
Poisson statistics of light

$$\Delta S = \sqrt{2G}\sqrt{S}$$

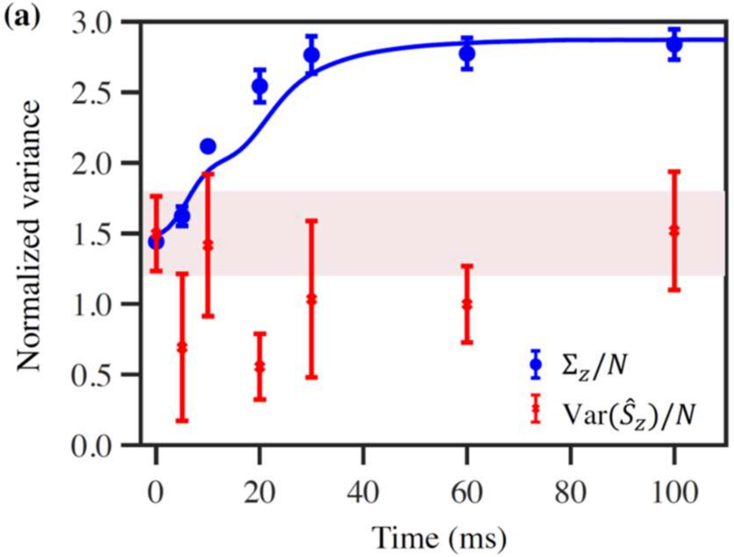
G = gain of EMCCD camera
S = signal

$$\frac{\Delta J_z}{N} \underset{\text{shot noise}}{=} \frac{\sqrt{\sum_{m_s} m_s^{\,2} G S_{m_s}}}{\sqrt{\sum_{m_s} S_{m_s}}}$$

Spin Correlations build up during dynamics: experimental results

$$\boxed{\sum_{m_s} p_{m_s} m_s{}^2}$$

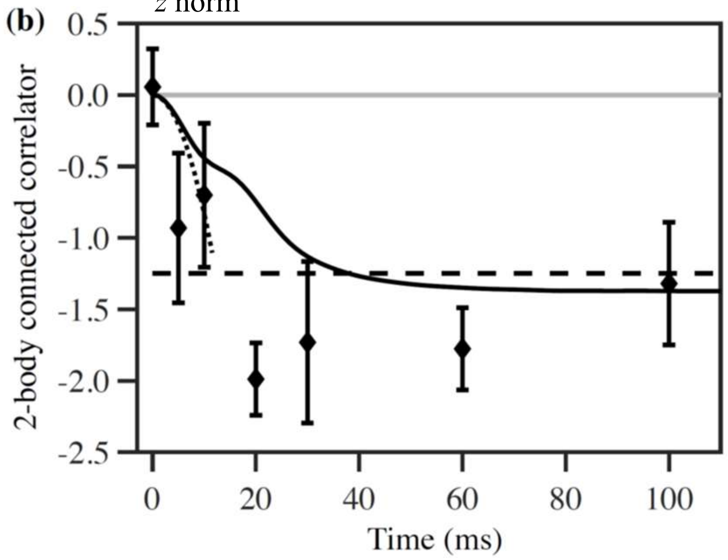
$$\boxed{\frac{1}{N}\Delta\hat{J}_z{}^2}$$



Proof of the growth of correlations is demonstrated

$$\boxed{C_{z\,\mathrm{norm}} \underset{\theta=\frac{\pi}{2}}{=} \frac{1}{N}\Delta\hat{J}_z{}^2 - \sum_{m_s} p_{m_s} m_s{}^2}$$

$C_{z\ \mathrm{norm}}$



The measured correlations are compatible with expected quantum correlations

$$\boxed{C_z = \Delta\hat{J}_z{}^2 - \sum_i \Delta\hat{s}_z^{i\,2} = \Delta\hat{J}_z{}^2 - \Sigma_z}$$

$$G_{z\,i,j} = \left\langle \hat{s}_z^i \hat{s}_z^j \right\rangle - \left\langle \hat{s}_z^i \right\rangle \left\langle \hat{s}_z^j \right\rangle \equiv G_z(r_{i,j})$$

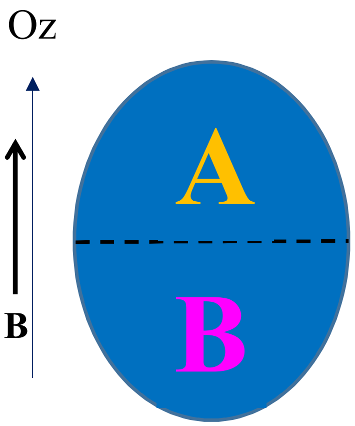
$$G_z(r) \approx -G_z(0)\exp\left(\frac{-r}{\xi}\right) \qquad G_z(0) \approx \frac{\Sigma_z}{N}$$

$$\frac{C_z}{N} = \sum_{r\neq 0} G_z(r_{i,j}) \approx \frac{-\Sigma_z}{N}\sum_{r\neq 0}\exp\left(\frac{-r}{\xi}\right)$$

$$\Rightarrow \xi \approx 0.3 \ \text{lattice units}$$

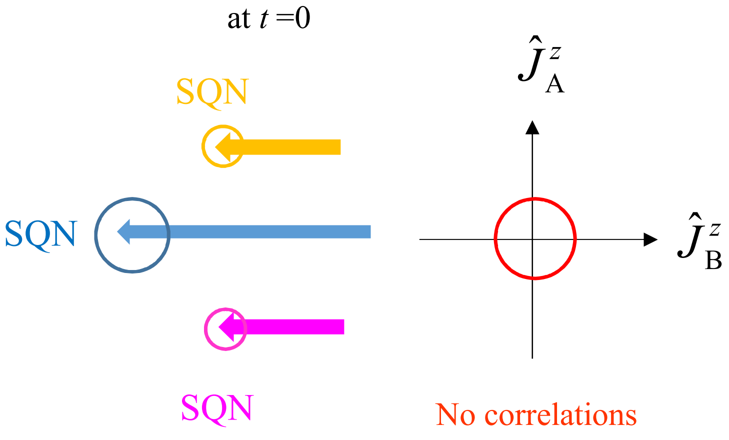
characteristic correlation length associated with the dynamical onset of correlations in the system

Demonstrating spin correlations from bipartition measurements

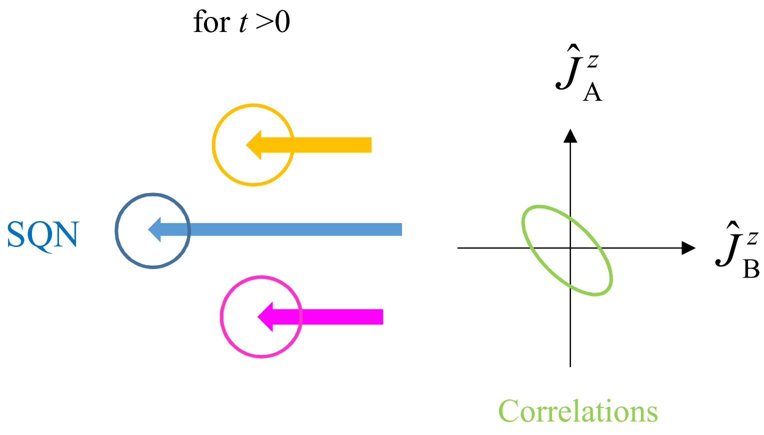


$$\langle \hat{J}^z_{\text{tot}} \rangle = 0 = \langle \hat{J}^z_{\text{A}} \rangle = \langle \hat{J}^z_{\text{B}} \rangle$$

$$\underset{\underset{\text{constant}}{\uparrow}}{\langle \hat{J}^{z\,2}_{\text{tot}} \rangle} = \underbrace{\langle \hat{J}^{z\,2}_{\text{A}} \rangle + \langle \hat{J}^{z\,2}_{\text{B}} \rangle + 2\langle \hat{J}^z_{\text{A}} \hat{J}^z_{\text{B}} \rangle}_{\text{Can change as spin dynamics proceed}}$$



$$\frac{3}{2}N = \frac{3}{4}N + \frac{3}{4}N + 0$$



$$\frac{3}{2}N = x\frac{3}{4}N + x\frac{3}{4}N + (1-x)\frac{3}{2}N,\ x > 1$$

Spin fluctuations of sub ensembles
get larger because they get anticorrelated

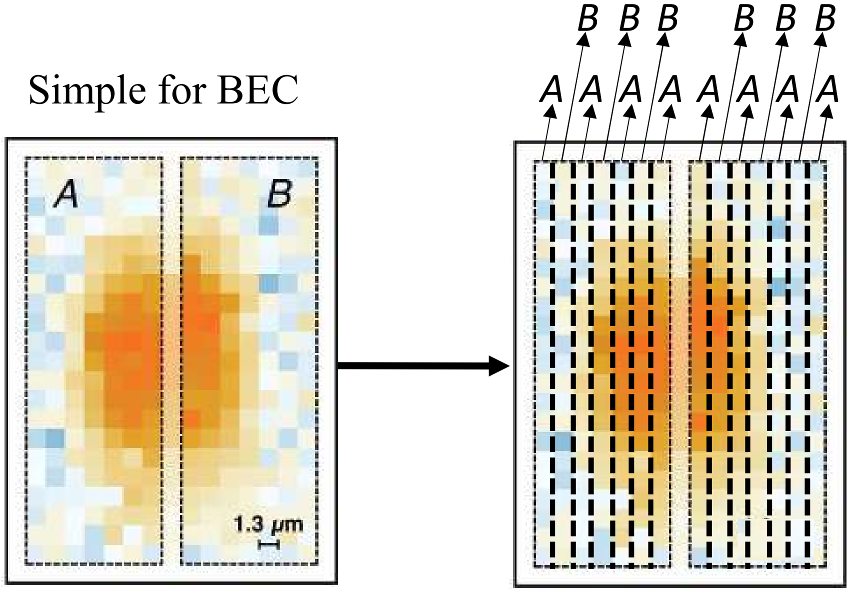
1- Realize a bipartition

2- Measure fluctuations

$$\langle \hat{J}^{z\,2}_{\text{tot}} \rangle, \langle \hat{J}^{z\,2}_{\text{A}} \rangle, \langle \hat{J}^{z\,2}_{\text{B}} \rangle, \langle \hat{J}^z_{\text{A}} \hat{J}^z_{\text{B}} \rangle$$

Realization of Bipartition : our strategy

Simple for BEC

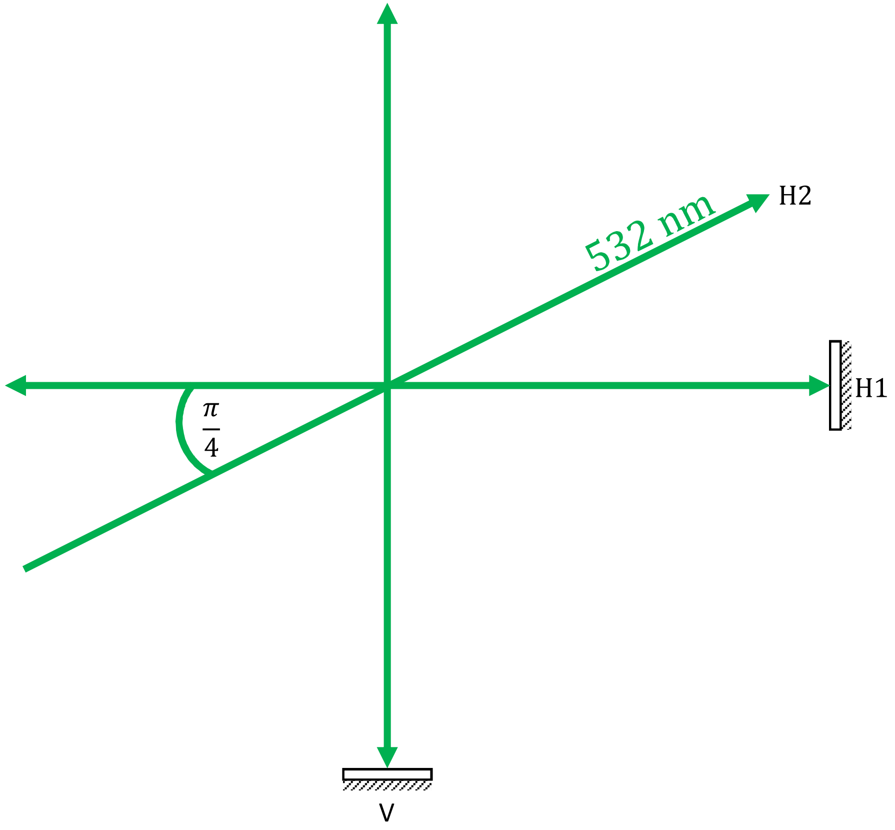


More adapted for lattice systems

Idea: create a
Bichromatic lattice after spin dynamics

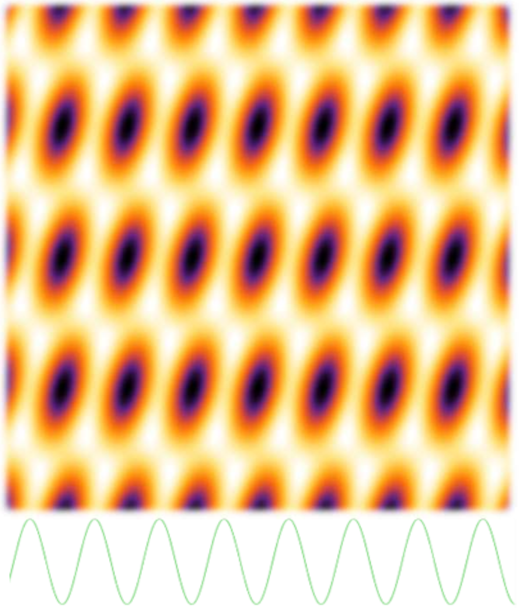
- Une bipartition adaptée à l’anisotropie des interactions
- Une bipartition adaptée à la courte portée des interactions
- Une bipartition créant de larges zones de contact entre les deux sous ensembles A et B (area law)

Bipartition : experimental realization

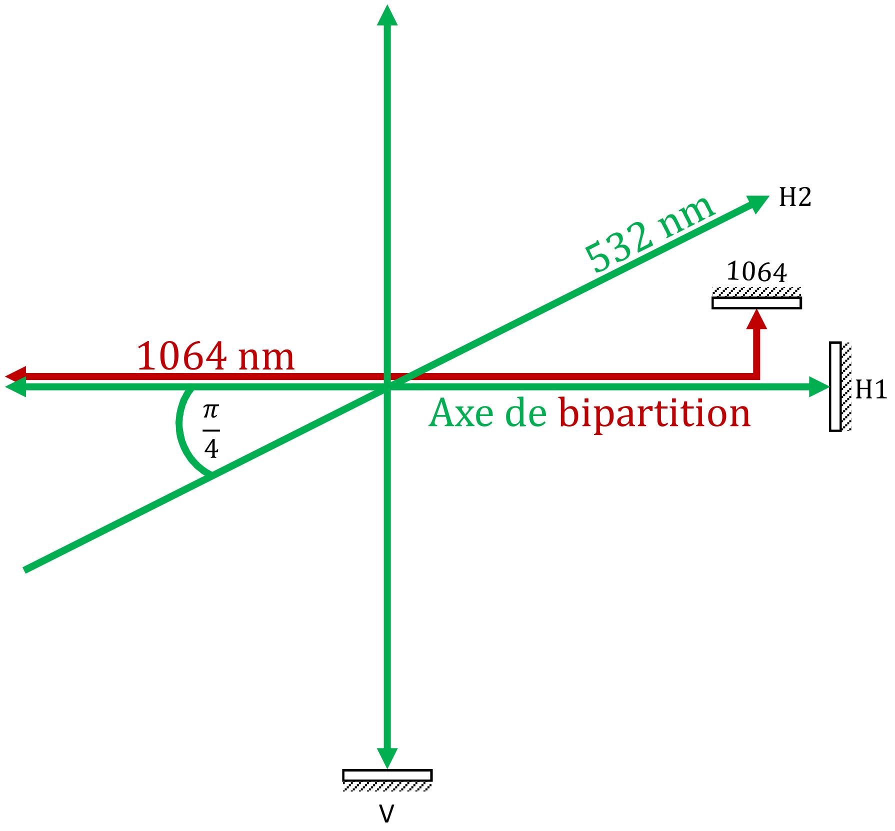


Cinq faisceaux à 532 nm nous permettent de créer
le réseau 3D primitif de l'expérience

Potentiel dipolaire horizontal
532 nm

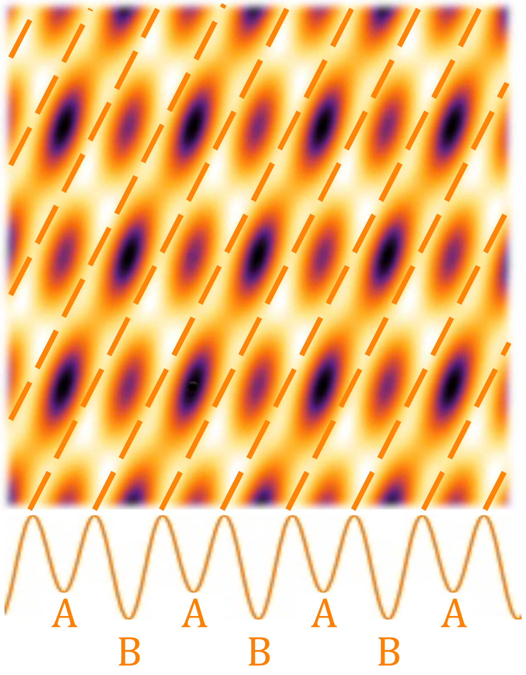


Bipartition : experimental realization



The 1064 nm laser creates a superlattice

Potentiel dipolaire horizontal bichromatique



Bipartition scheme

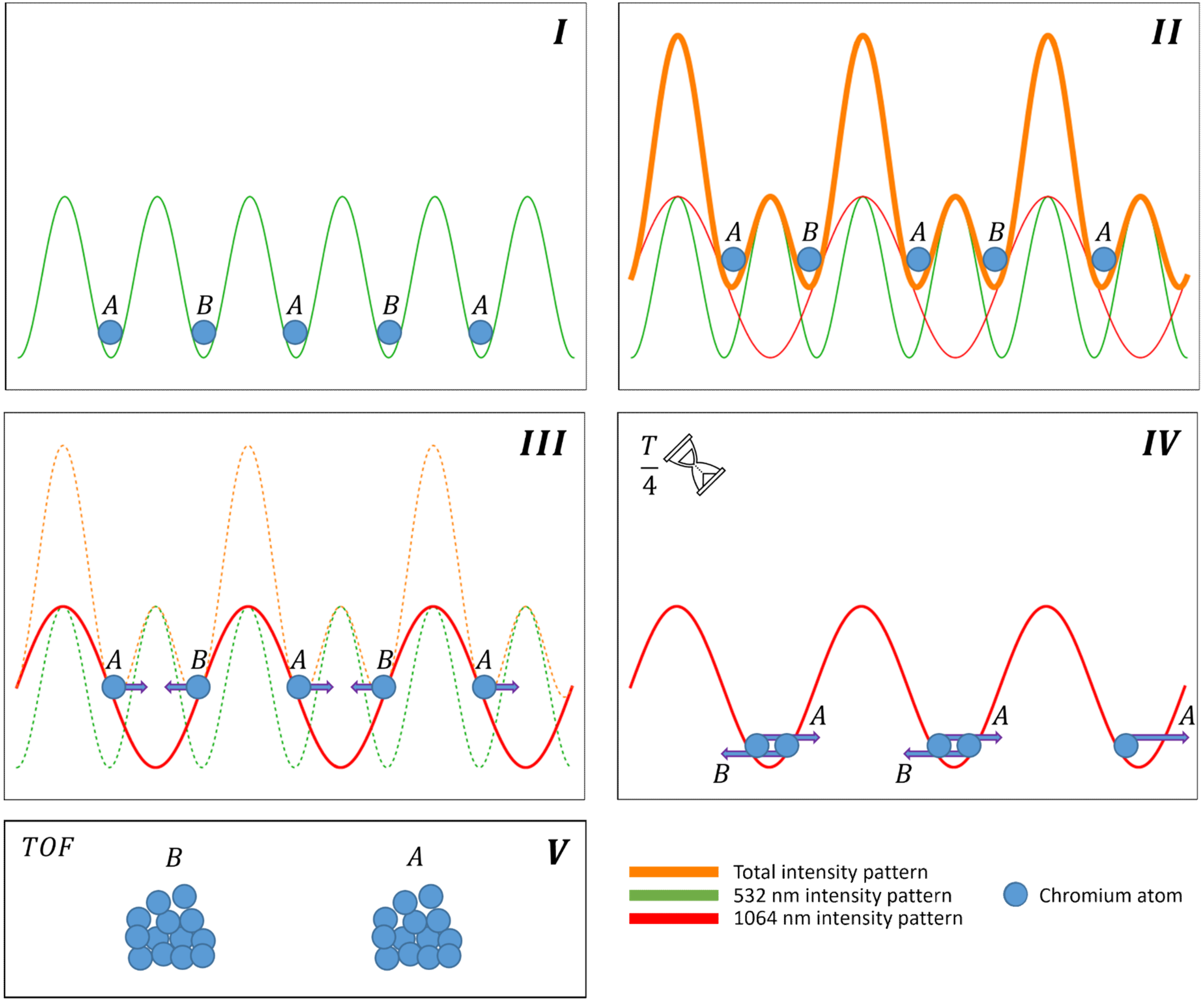
I Spin dynamics

II 1604 nm laser added

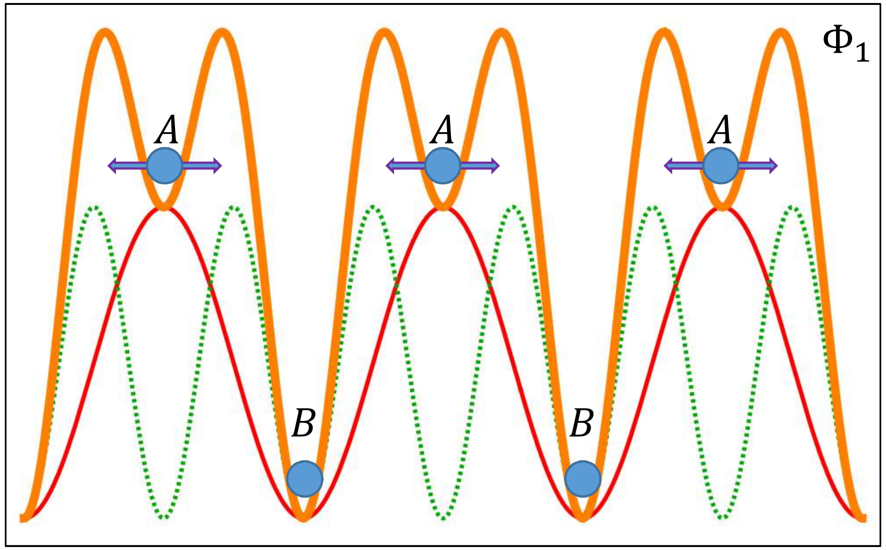
III 532 nm laser off

IV *T*/4 evolution in the 1064 nm lattice

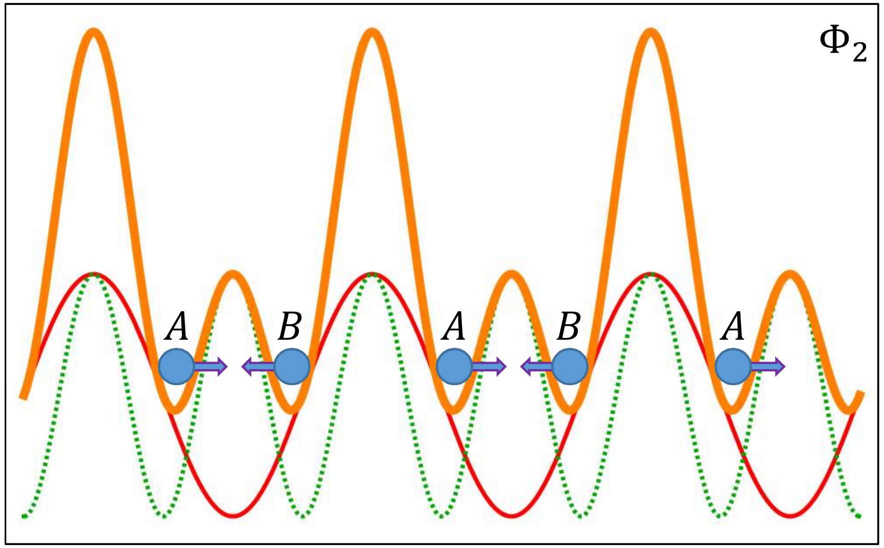
V Separation after time of flight



On the importance of the intensity pattern during the bipartition process



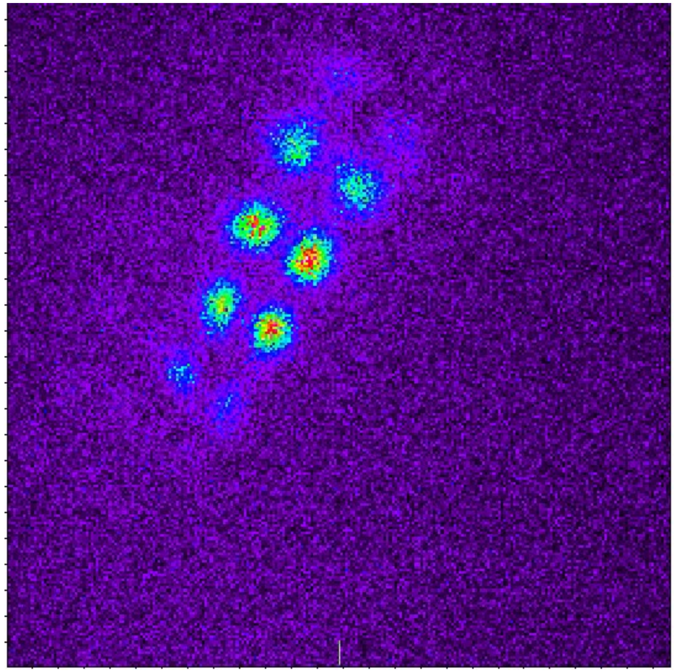
bad



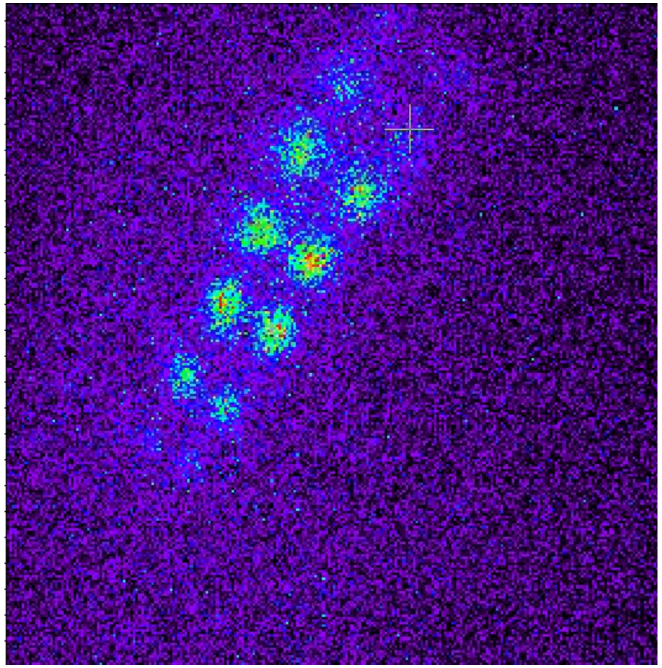
good

Bipartite measurements: experimental realization

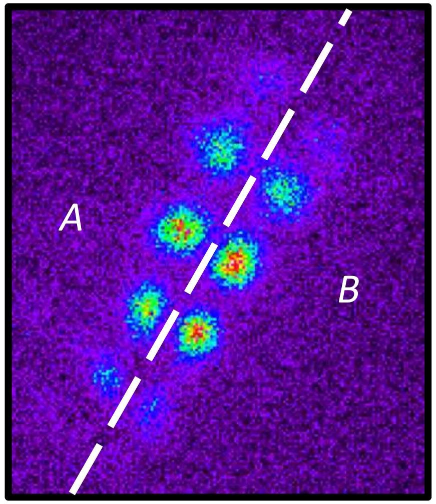
0 ms



15 ms



Bipartite correlation witnesses



$$C_z^A = \sum_{i\neq j}^{A} \langle \hat{s}_i^z \hat{s}_j^z \rangle - \langle \hat{s}_i^z \rangle \langle \hat{s}_j^z \rangle = \mathrm{Var}\left(\hat{S}_z^A\right) - \Sigma_z^A$$

Corrélations intra-famille A

$$C_z^B = \sum_{i\neq j}^{B} \langle \hat{s}_i^z \hat{s}_j^z \rangle - \langle \hat{s}_i^z \rangle \langle \hat{s}_j^z \rangle = \mathrm{Var}\left(\hat{S}_z^B\right) - \Sigma_z^B$$

Corrélations intra-famille B

$$C_z^{AB} = \sum_{\substack{i\in A \\ j\in B}} \langle \hat{s}_i^z \hat{s}_j^z \rangle - \langle \hat{s}_i^z \rangle \langle \hat{s}_j^z \rangle = \frac{C_z - C_z^A - C_z^B}{2}$$

Corrélations inter-famille AB

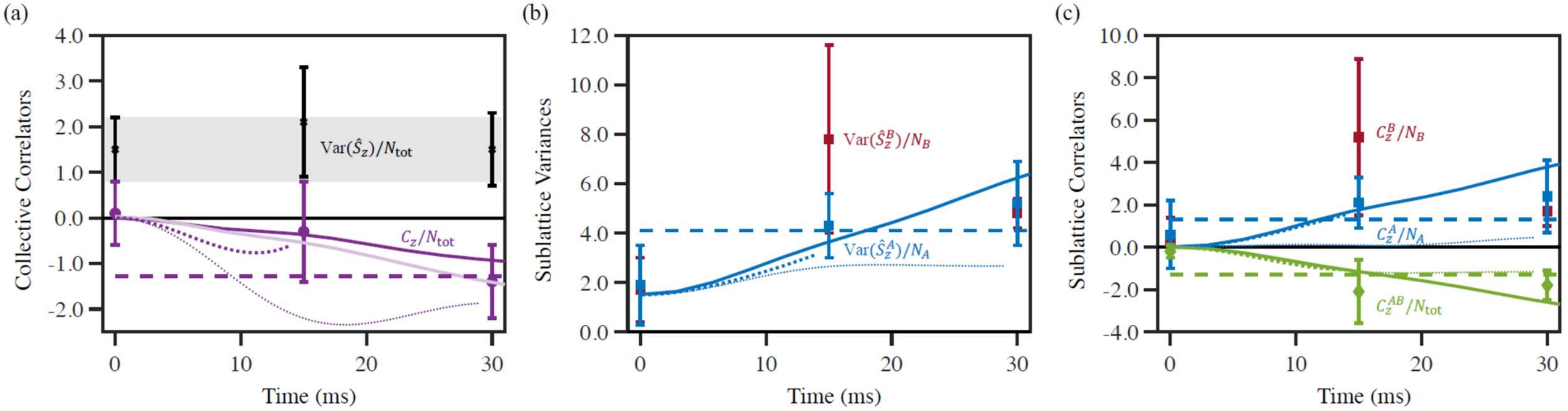
$$C_z^{AB} = \mathrm{cov}(\hat{S}_z^A, \hat{S}_z^B)$$

Bipartite measurements: results

Entire system

Subpartite fluctuations

Subpartite correlations



- - - - - - - Expected thermalized value

——— Numerical simulations

$\mathrm{Var}S_z^{A,B}$ ↗

C_z^{AB}<0

C_z^A, C_z^B>0

Conclusion

We study experimentally and theoretically the growth of spin correlations in a dipolar spin dynamics which leads to quantum thermalization

From collective measurement we measure spin correlations

$$C_{z\,\text{norm}} \underset{\theta=\frac{\pi}{2}}{=} \frac{1}{N}\Delta\hat{J}_z^{\,2} - \sum_{m_s} p_{m_s} m_s^{\,2}$$

Negative correlations develop

$$C_z < 0$$

From bipartition measurement we measure fluctuations and correlations of subsystems

Subsystem fluctuations grow

$\mathrm{Var}S_z^{A,B}$ ↗

Anisotropy of the system is revealed by subpartite correlations

C_z^{AB}<0 C_z^{A}, C_z^{B}>0

Isotropic system
$C_z^A = C_z^B = C_z^{AB}$

Development of new theoretical tools necessary

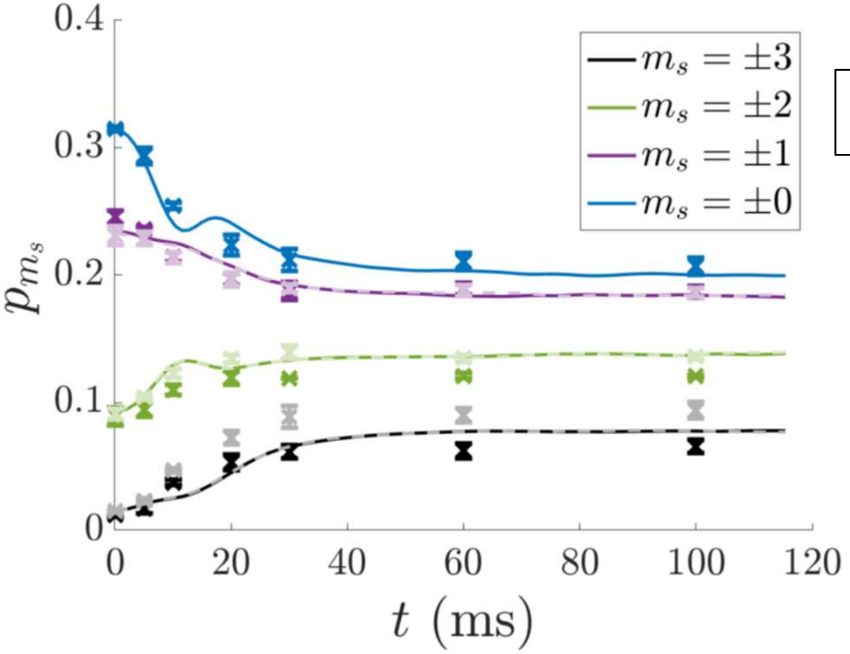
Two point correlators are difficult to evaluate

Spin population dynamics: experimental results and numerical simulations

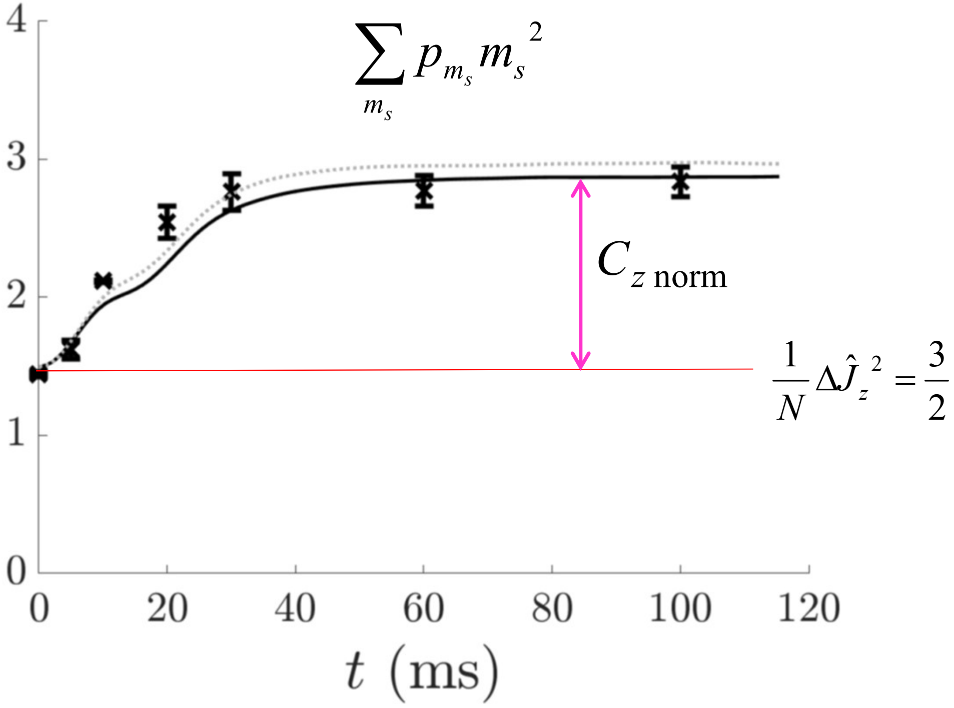
$$C_{z\,\text{norm}} \underset{\theta=\frac{\pi}{2}}{=} \frac{1}{N}\Delta\hat{J}_z^{\,2} - \sum_{m_s} p_{m_s} m_s^{\,2}$$

As soon as spin dynamics proceeds, correlations grow

numerical simulations = semi classical = GDTWA



Lines: numerical simulations
Symbols: experimental results



p_{m_s} = fractionnal population, $\sum_{m_s} p_{m_s} = 1$

Dipolar spin dynamics gives rise to measureable correlations for large spins *j*>1/2

Now: what about the real system ?

$\Delta\hat{J}_z^{\,2}$ = Variance of $\hat{J}_z$ = ???

Measuring spin fluctuations at the level of the quantum noise: our approach

Independent noises:

$$\Delta J_{z\,\mathrm{exp}} = \sqrt{\underbrace{\Delta J_{z\,\text{spin noise}}^{2}}_{\text{Our target}} + \underbrace{\Delta J_{z\,\text{fit}}^{2} + \Delta J_{z\,\text{shot noise}}^{2} + \Delta J_{z\,\text{preparation}}^{2}}_{\text{Parasite noises}}}$$

$$\frac{\Delta \hat{J}_z}{N} \underset{\text{coherent}}{=} \sqrt{\frac{3}{2N}} = \text{SQN}$$

We use the data at $t=0$ to infer the preparation noise; then we obtain the spin noise for $t>0$

→ the spin noise = atomic contribution is equal to the SQN for $t=0$

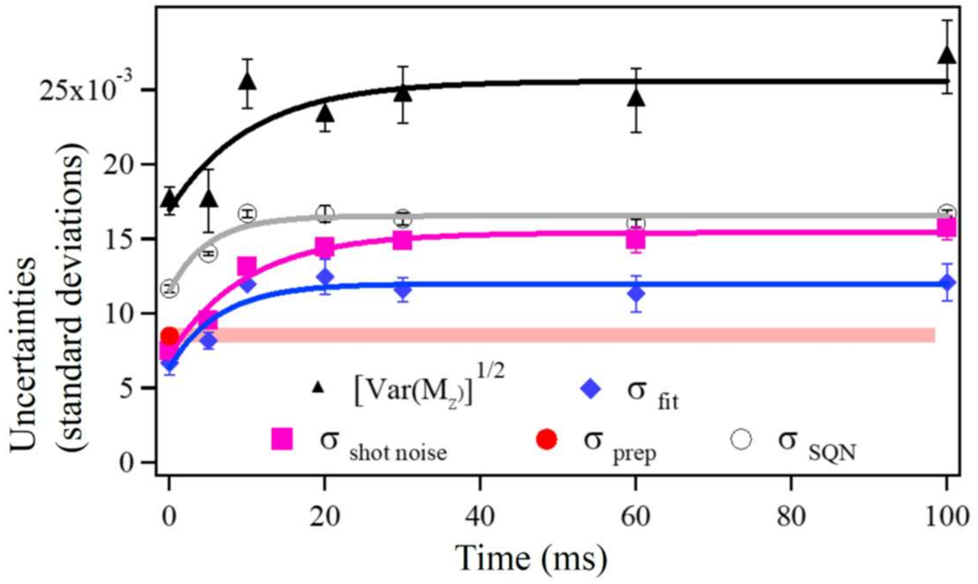
$$\Delta J_{z\,\mathrm{exp}} = \sqrt{\text{SQN}^{2} + \Delta J_{z\,\text{fit}}^{2} + \Delta J_{z\,\text{shot noise}}^{2} + \Delta J_{z\,\text{preparation}}^{2}} \Rightarrow \Delta J_{z\,\text{preparation}}$$

measured	measured (N)	given by data analysis	derived from first principles	inferred

$$\Delta J_{z\,\mathrm{exp}} = \sqrt{\Delta J_{z\,\text{spin noise}}^{2} + \Delta J_{z\,\text{fit}}^{2} + \Delta J_{z\,\text{shot noise}}^{2} + \Delta J_{z\,\text{preparation}}^{2}} \Rightarrow \Delta J_{z\,\text{spin noise}}(t)$$

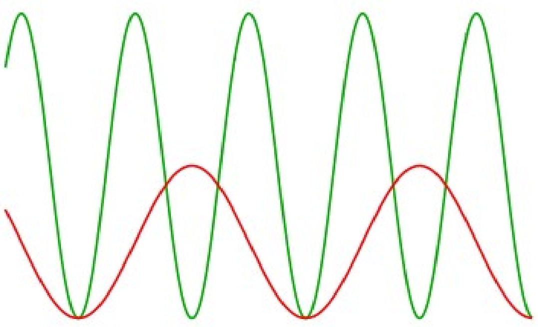
$$\Delta\left(\frac{J_{z\,\text{preparation}}}{N}\right) = \text{constant}$$

Normalized standard deviation $\sigma = \Delta\left(\frac{J_z}{N}\right)$



Bipartite measurements: experimental realization

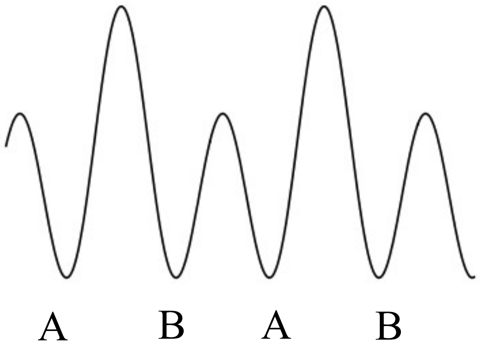
Use of a bicolor lattice to isolate two families of spins



$\lambda_{IR}=2\ \lambda_{Green}$

=

Double-well lattice



→ very favorable to detect
Local correlations
Dipolar interactions $\propto \frac{1}{r^3}$

Requires achieving a good spatial separation between the two spin families

A Frequency difference between the two lasers create this intensity profile

$$\Delta\phi = \frac{\pi}{4} = 2\pi\frac{\Delta f\ L}{c}$$

$$\Delta f = f_{IR} - f_{Green}/2$$

We can compensate for frequency drift by use of an AOM

L = distance atoms - back reflection mirrors ≈ 50 cm galery