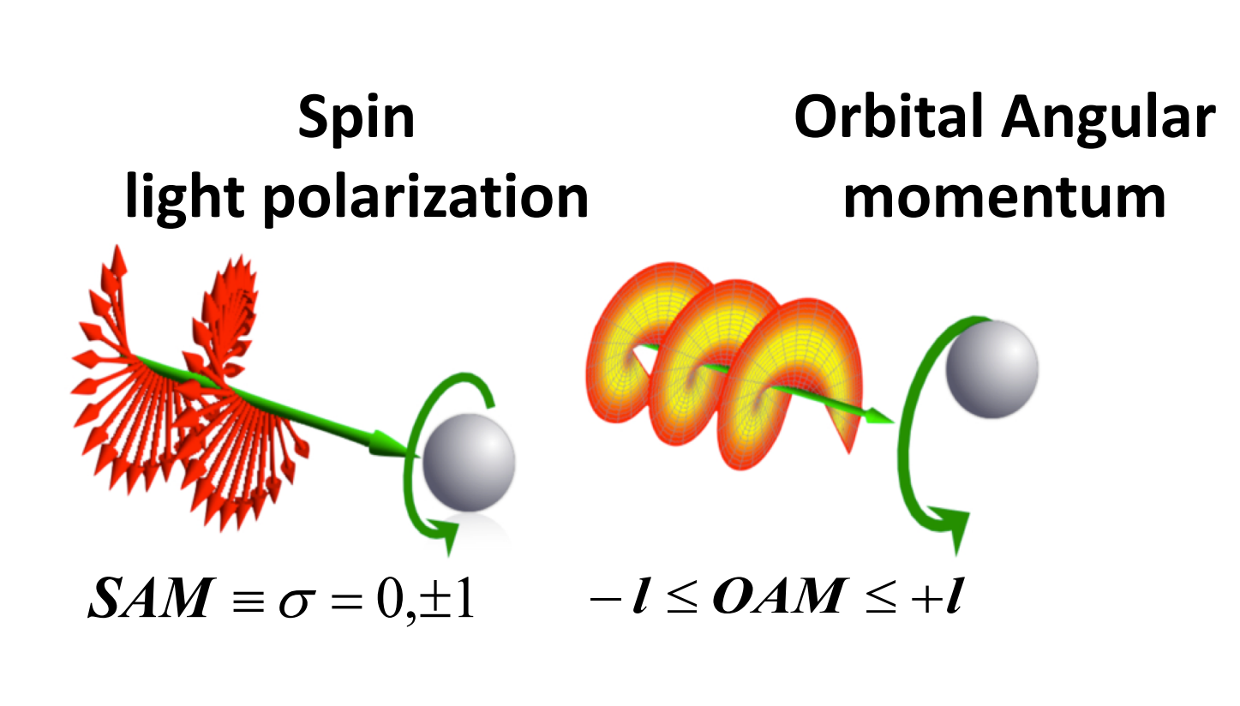

$SAM \equiv \sigma = 0,\pm 1$ $\qquad -l \le OAM \le +l$

Twisted-light field-induced spectroscopy of forbidden optical transitions with application to hyper-clocks

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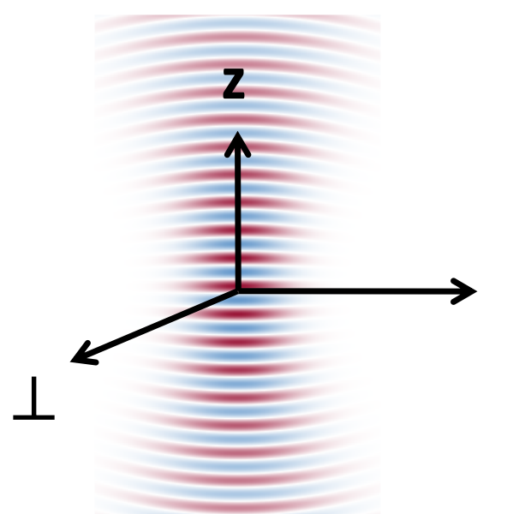
I. Twisted-light (TL) fields: Bessel mode decomposition with Coulomb gauge

G.F. Quinteiro, D.E. Reiter and T. Kuhn, *Formulation of the twisted-light-matter interaction at the phase singularity: Beams with strong magnetic fields*, Phys. Rev. A **95**, 012106 (2017).



$$\left\{\begin{aligned} &\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{A} \text{ and } \mathbf{B} = \nabla \times A \\ &\tilde{\mathbf{A}}(\mathbf{r}) = A_0\left[\mathbf{e}_\sigma J_\ell(q_r r)e^{i\ell\varphi} - i\sigma\mathbf{e}_z\frac{q_r}{q_z}J_{\ell+\sigma}(q_r r)e^{i(\ell+\sigma)\varphi}\right] \\ &B_0 = q_z E_0/c,\ r = \sqrt{x^2+y^2} \text{ and } \varphi = \arctan[y/x]. \end{aligned}\right.$$

$\mathbf{e}_\sigma = (\mathbf{e}_x + i\sigma\mathbf{e}_y)$

$$q_r^2 + q_z^2 = (n\omega/c)^2$$

$$q_r = \frac{2\pi}{w}$$

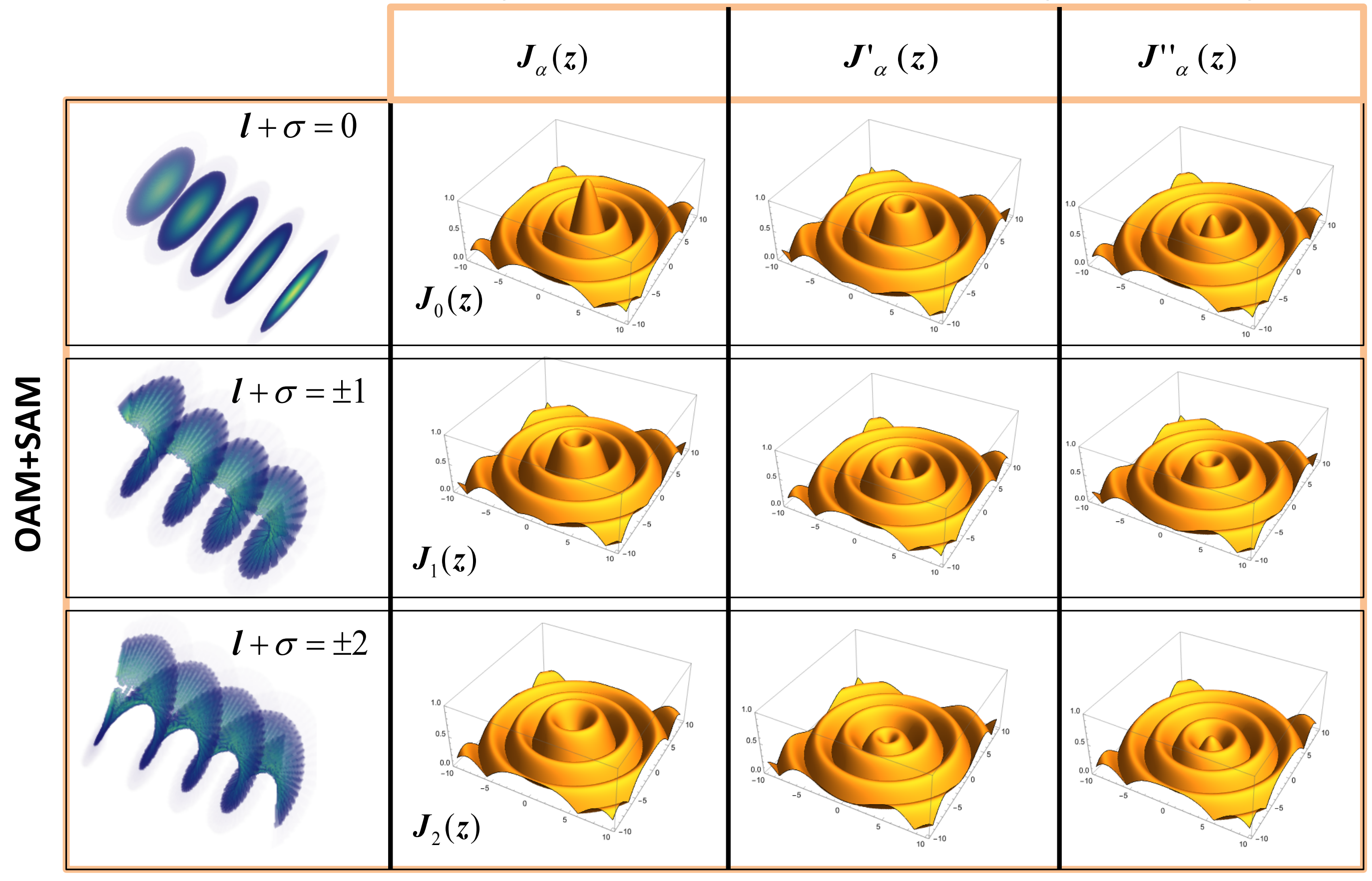
Recurent relations of 1st order Bessel functions

$$J_{\alpha-1}(z) - J_{\alpha+1}(z) = \frac{2\alpha}{z}J_\alpha(z)$$

$$2J'_\alpha(z) = J_{\alpha-1}(z) - J_{\alpha+1}(z)$$

$$J_{\alpha-1}(z) - \frac{\alpha}{z}J_\alpha(z) = -J_{\alpha+1}(z) + \frac{\alpha}{z}J_\alpha(z)$$

Spatial structure of TL beam in the plane (Ox, Oy)



II. Application in atomic spectroscopy: electric octupole E3 transition in ^{171}Yb$^+$

PHYSICAL REVIEW LETTERS **129**, 253901 (2022)

Excitation of an Electric Octupole Transition by Twisted Light

R. Lange,[1] N. Huntemann,[1,*] A. A. Peshkov,[1,2] A. Surzhykov,[1,2,3] and E. Peik[1]
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[2]*Institut für Mathematische Physik, Technische Universität Braunschweig, Mendelssohnstraße 3, 38106 Braunschweig, Germany*
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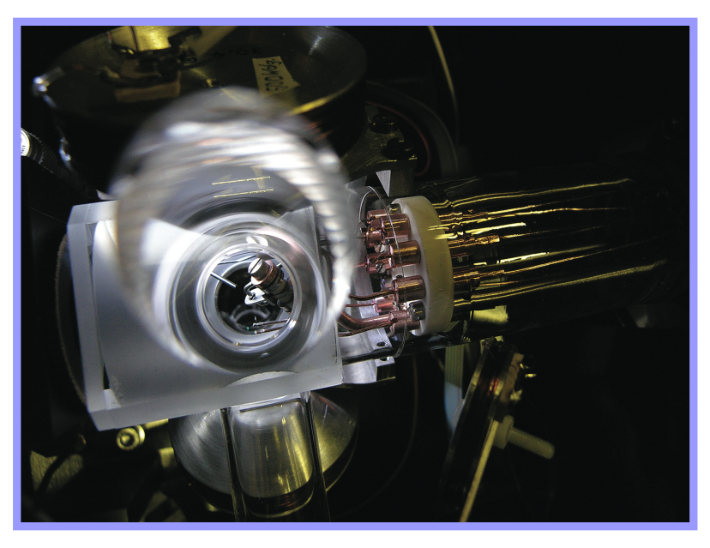
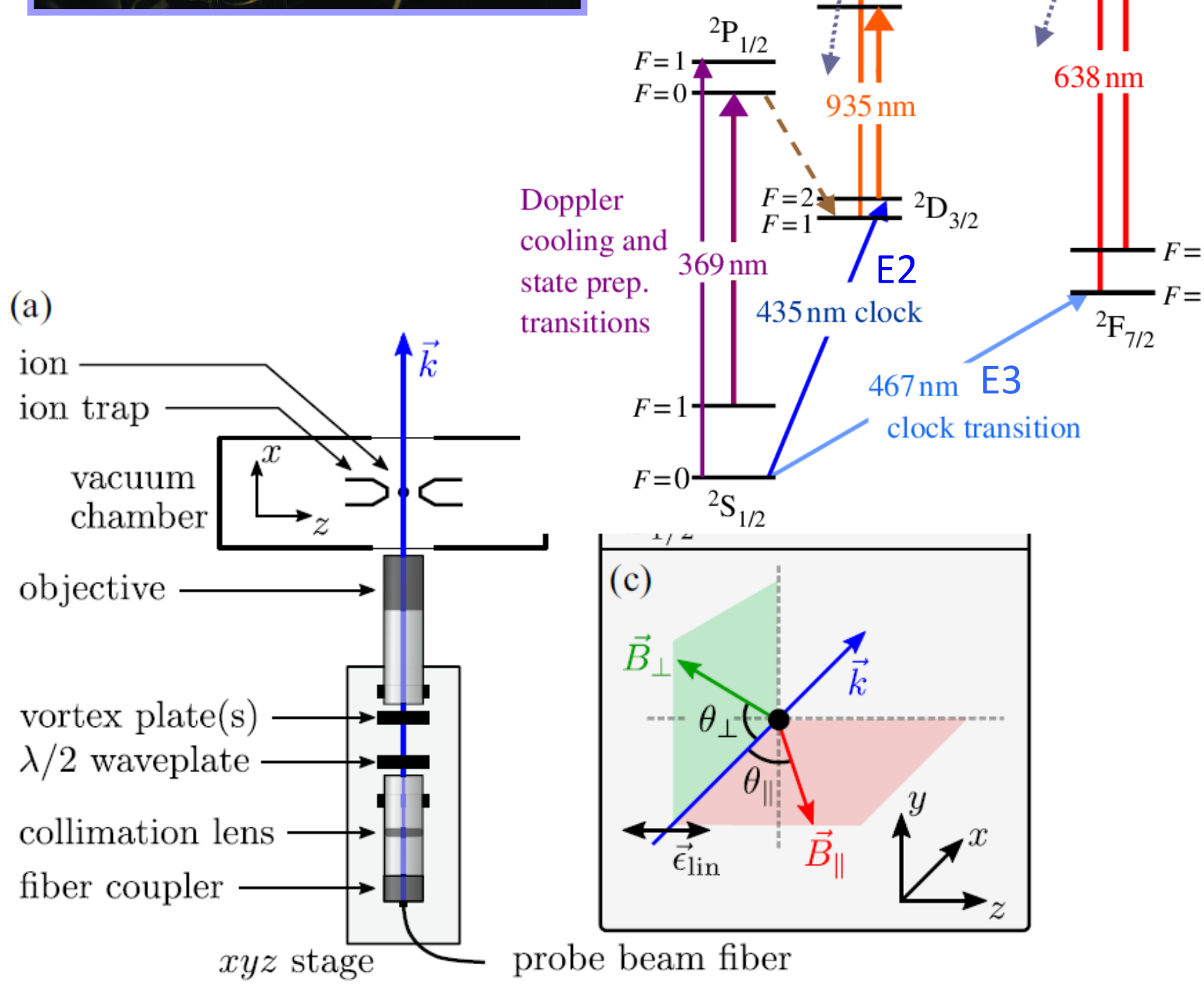
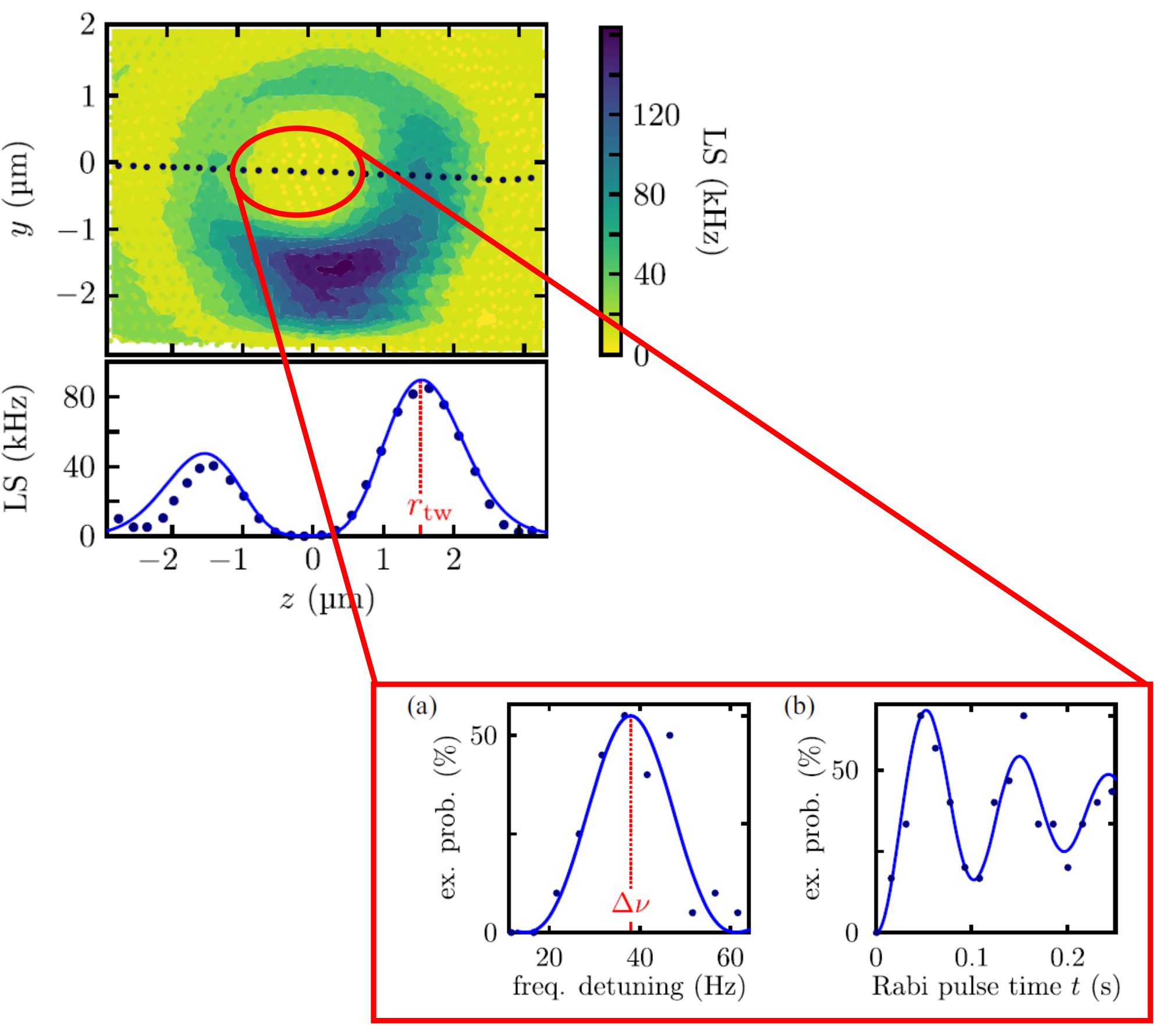
(Received 20 June 2022; accepted 14 November 2022; published 12 December 2022)





LG_0^2

$$\Omega_\perp^{E3} \propto \langle E3\rangle \cdot \nabla_r(\nabla_r E(r))$$



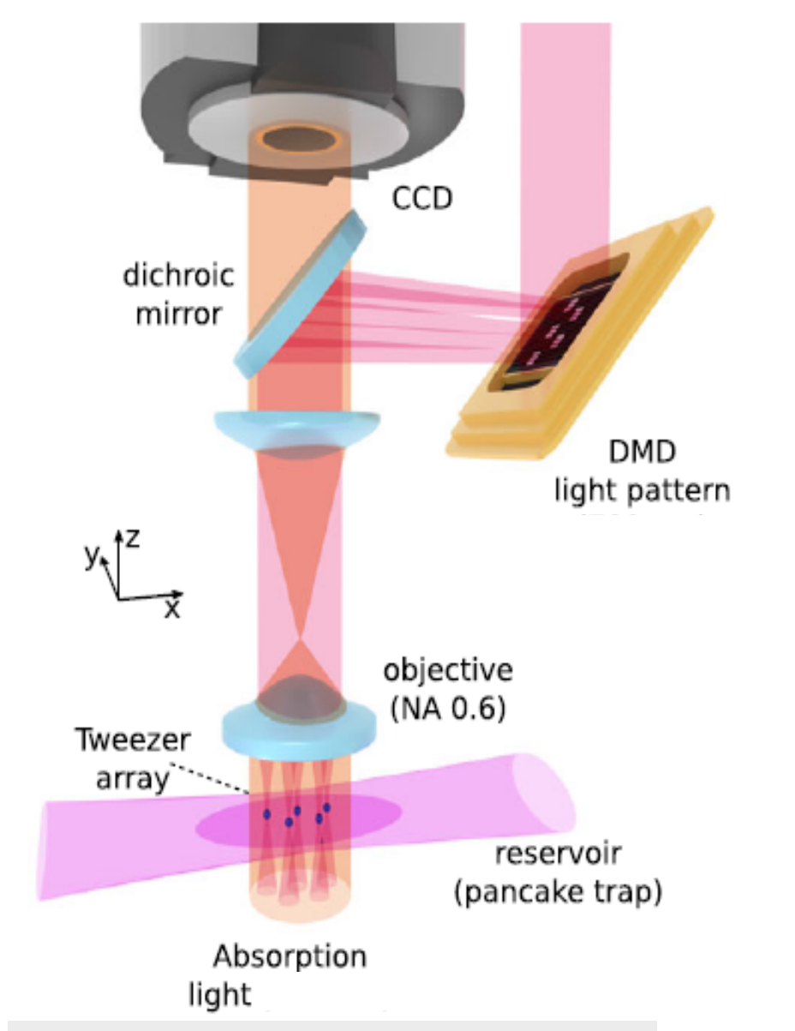
$$E(r) = E_0 \cdot J_2(q_r r)$$

$$2J'_\alpha = J_{\alpha-1} - J_{\alpha+1}$$

$$J'_2 = \frac{1}{2}\left(J_1 - J_3\right) \Rightarrow (J'_2)' = \frac{1}{2}\left(J'_1 - J'_3\right)$$

$$(J'_2)' = \frac{1}{4}(J_0 - J_2) - \frac{1}{4}(J_2 - J_4) \approx \frac{1}{4}J_0 \ne 0$$

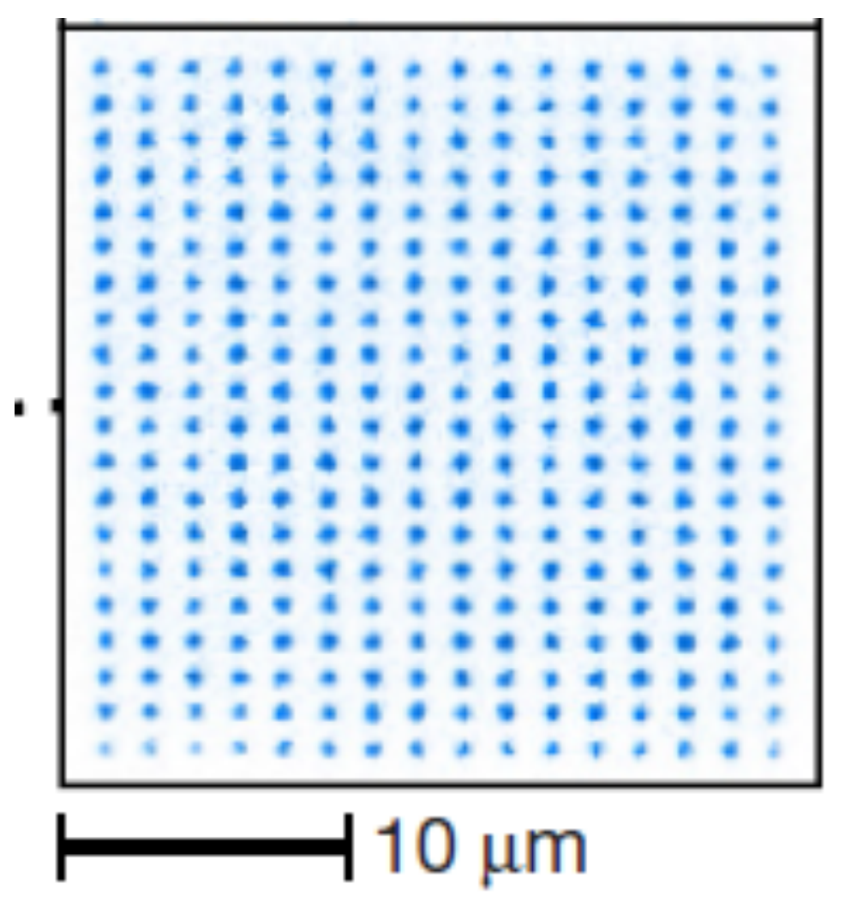
III. Our proposal: TL induced spectroscopy of two clock transitions in ^{88}Sr, ^{172}Yb, ^{200}Hg, ^{40}Ca, ^{24}Mg, ^{112}Cd neutral atoms trapped into optical tweezers



npj Quantum Information (2020)6:54

Preparation of hundreds of microscopic atomic ensembles in optical tweezer arrays

Yibo Wang[1], Sayali Shevate[1], Tobias Martin Wintermantel[1,2], Manuel Morgado[1], Graham Lochead[1] and Shannon Whitlock[1]



| Nature | Vol 588 | 17 December 2020

Half-minute-scale atomic coherence and high relative stability in a tweezer clock

Aaron W. Young[1,2], William J. Eckner[1,2], William R. Milner[1,2], Dhruv Kedar[1,2], Matthew A. Norcia[1,2], Eric Oelker[1,2], Nathan Schine[1,2], Jun Ye[1,2] & Adam M. Kaufman[1,2]

Reduced matrix elements, Einstein's coeffcients and 2nd order Zeeman shifts

	$^1S_0\ ^1P_1$ $\langle E1\rangle/ea_0$	$^1P_1\ ^3P_0$ $\langle M1\rangle/\mu_B$	$^1S_0\ ^3P_1$ $\langle E1\rangle/ea_0$	$^3P_0\ ^3P_1$ $\langle M1\rangle/\mu_B$	λ_ω^{E1M1} nm	A_{M2} mHz	$^1S_0\ ^3P_2$ $\langle M2\rangle/\mu_B a_0$	$\Delta m=0$ MHz/T^2	$\Delta m=\pm1$ MHz/T^2	$\Delta m=\pm2$ Hz/T^2	λ_ω^{M2} nm
^{88}Sr	5.28 [57]	0.023 [33]	0.15 [36]	0.816 [29]	1397	0.13 [61]	11	5.6	4.1		671
^{172}Yb	4.40 [56]	0.103	0.54 [36]	0.815	1157	0.25 [25]	7.5	1.2 [25]	0.92 [25]	-47 [25]	507
^{200}Hg	2.80 [58]	0.140	0.46 [58]	0.804	531	3.6 [60]	3.8	0.45	0.34		227
^{24}Mg	4.03 [57]	0.063	0.0057 [36]	0.814	916	0.44 [61]	7.6	52.5	39.3		456
^{40}Ca	4.91 [57]	0.017	0.036 [36]	0.816	1319	0.13 [61]	10	20.6	15.4		655
^{112}Cd	3.36 [59]	0.036	0.15 [59]	0.815	664	0.96 [60]	10	1.8	1.4		314

Residual decay channels

	A_{M2} $\times10^{-3}s^{-1}$	$^3P_2\to{}^1S_0$ $\langle M2\rangle/\mu_B a_0$	λ_ω^{M2} nm	A_{E2} $\times10^{-6}s^{-1}$	$^3P_2\to{}^3P_0$ $\langle E2\rangle/ea_0^2$	λ_ω^{E2} μm	A_{E2} $\times10^{-6}s^{-1}$	$^3P_2\to{}^3P_1$ $\langle E2\rangle/ea_0^2$	A_{M1} $\times10^{-3}s^{-1}$	$^3P_2\to{}^3P_1$ $\langle M1\rangle/\mu_B$	$\lambda_\omega^{E2,M1}$ μm
^{88}Sr	0.13 [10]	11	671	1 [10]	16.7	17.3	0.3 [10]	24.4	0.83 [10]	∼ 0.7	25.7
^{172}Yb	0.25 [18]	7.5	507			4.1			67 [18]	∼ 0.7	5.8
^{200}Hg	3.6 [19]	3.8	227			1.56					2.15
^{24}Mg	0.44 [10]	7.6	456	3×10^{-6} [10]	7.9	163	1×10^{-6} [10]	12.2	9.12×10^{-4} [10]	∼ 0.7	242
^{40}Ca	0.13 [10]	10	655	10^{-3} [10]	13.4	63.3	3×10^{-4} [10]	20	1.6×10^{-2} [10]	∼ 0.7	94.4
^{112}Cd	0.96 [19]	10	314			5.8					8.5

$$\Omega_{\perp,z}^{E1M1}(\mathbf{r}) = \sum_k\sum_{i\ne j}\frac{\langle{}^1S_0|H_{E1}^i|k\rangle_{\perp,z}\langle k|H_{M1}^j|{}^3P_0\rangle_{\perp,z}}{\hbar\omega_i - E(k)}$$

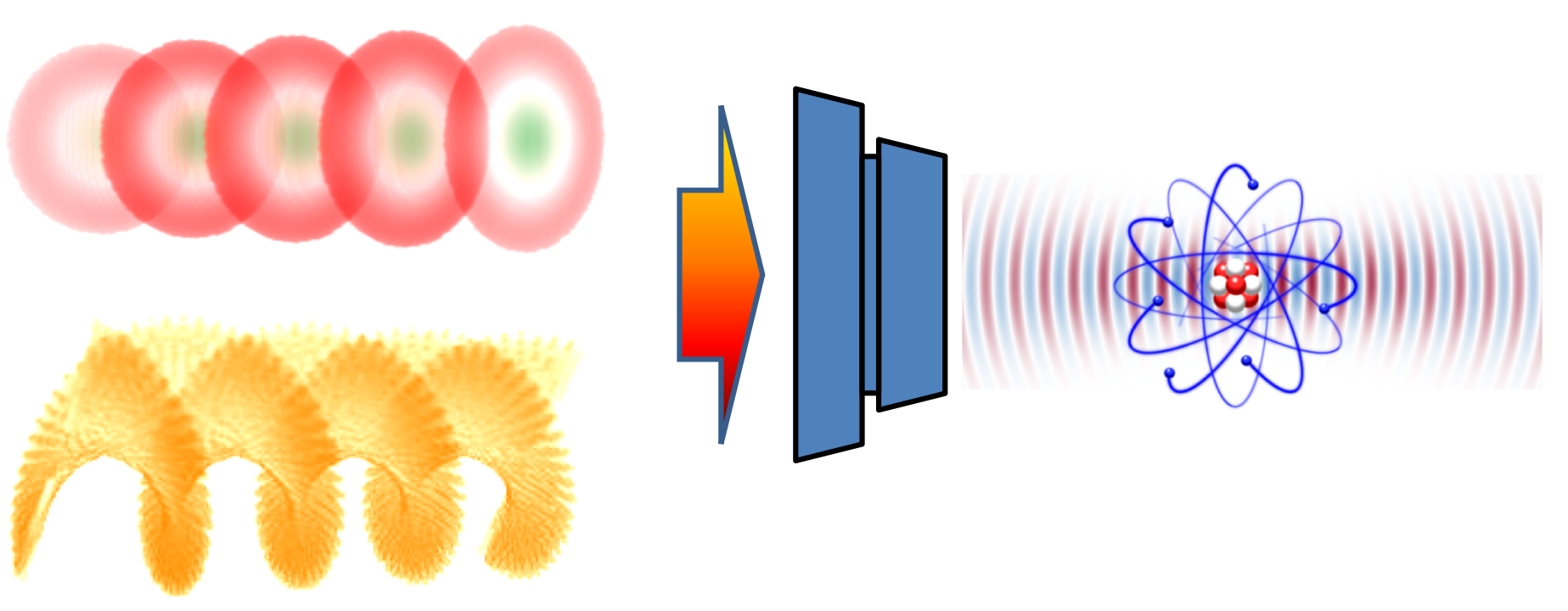
$$\Omega_{\perp,z}^{M2}(\mathbf{r}) = -\frac{1}{6}\langle M2\rangle\cdot\nabla_{\mathbf{r}}\int_0^1 u\widetilde{B}_{\perp,z}(u\mathbf{r})du$$

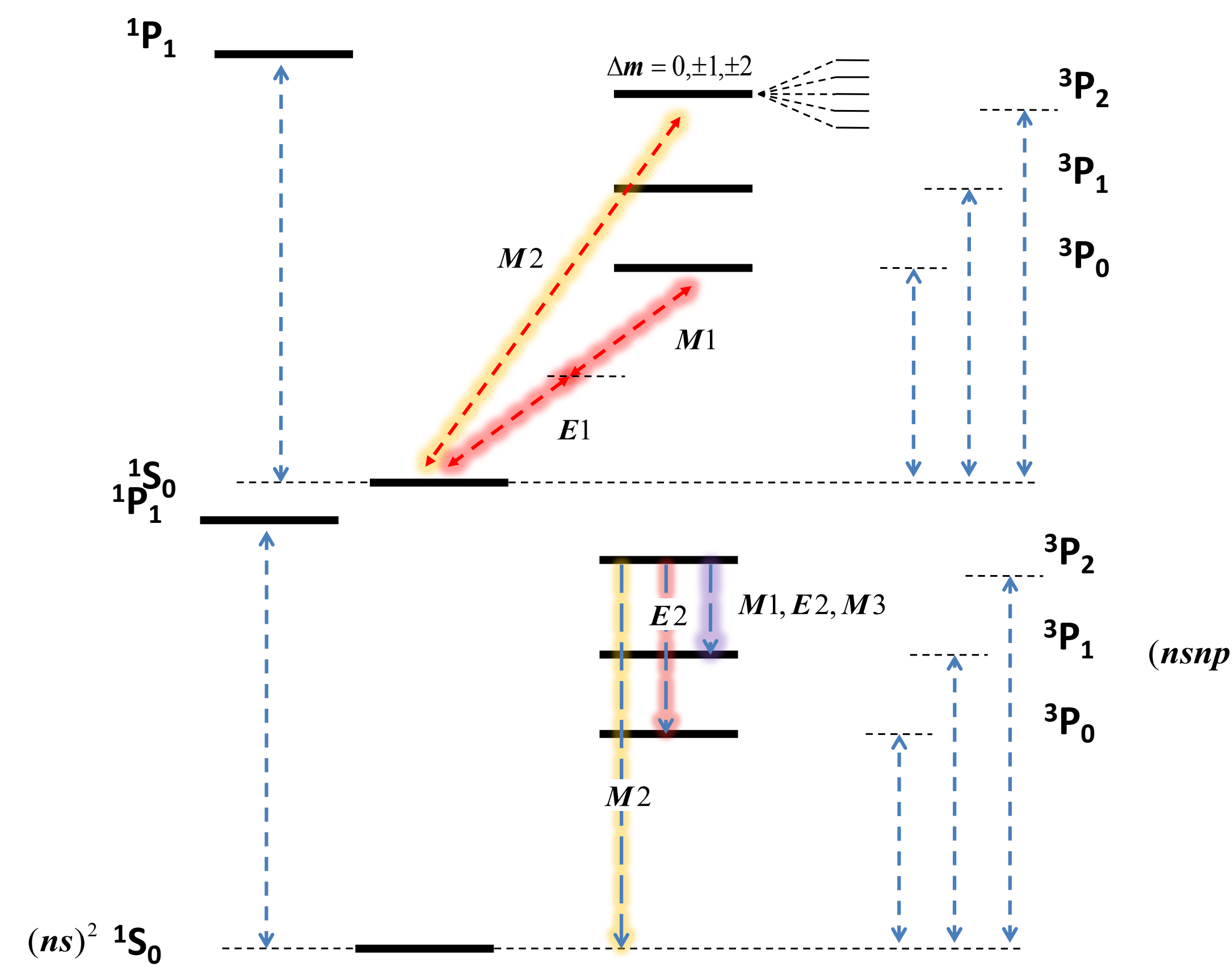
$l=\pm1, \sigma=\mp1$

$l=\pm1, \sigma\equiv0$

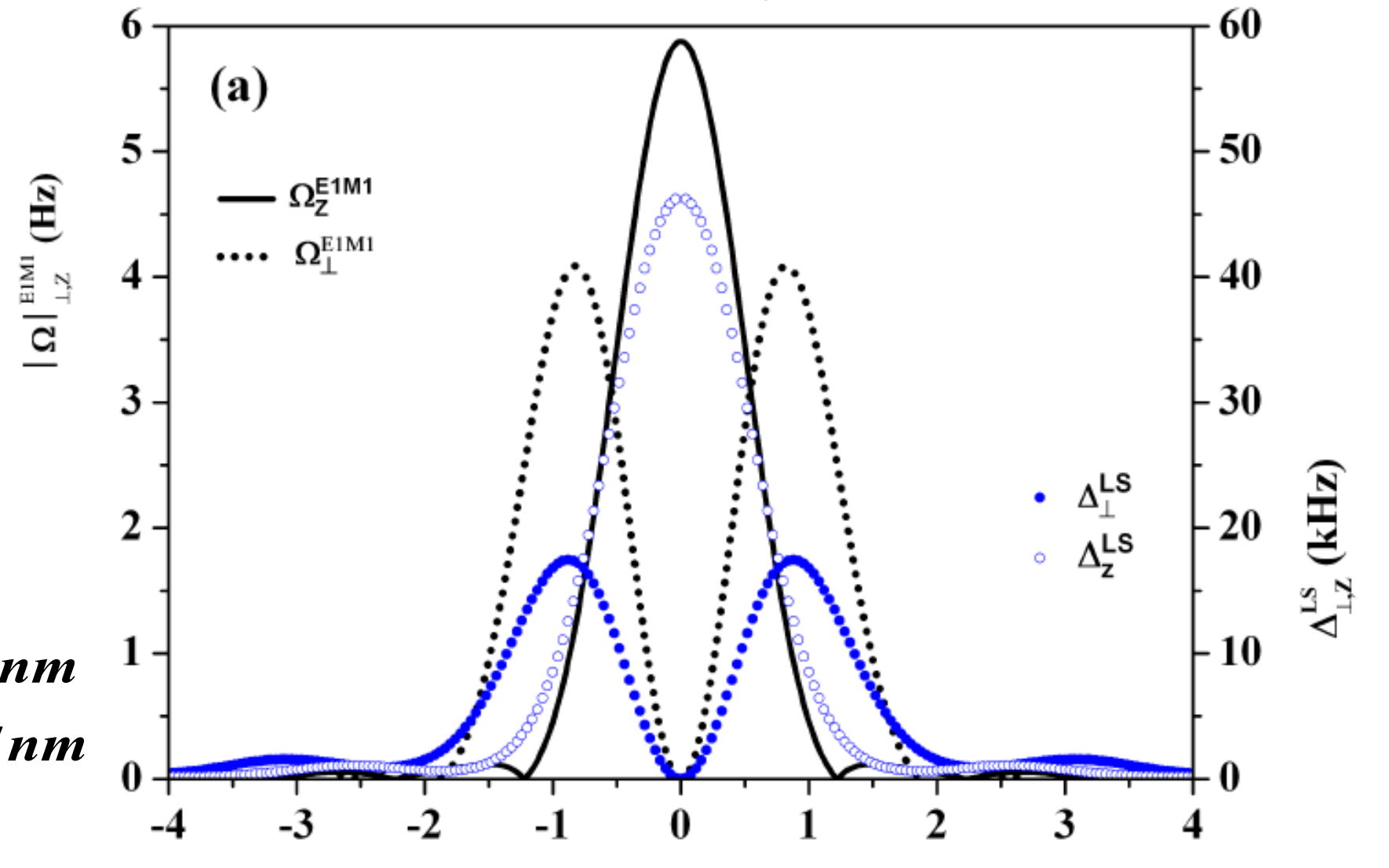
$l=\pm2, \sigma=\mp1$

$l=\pm1, \sigma=\pm1$



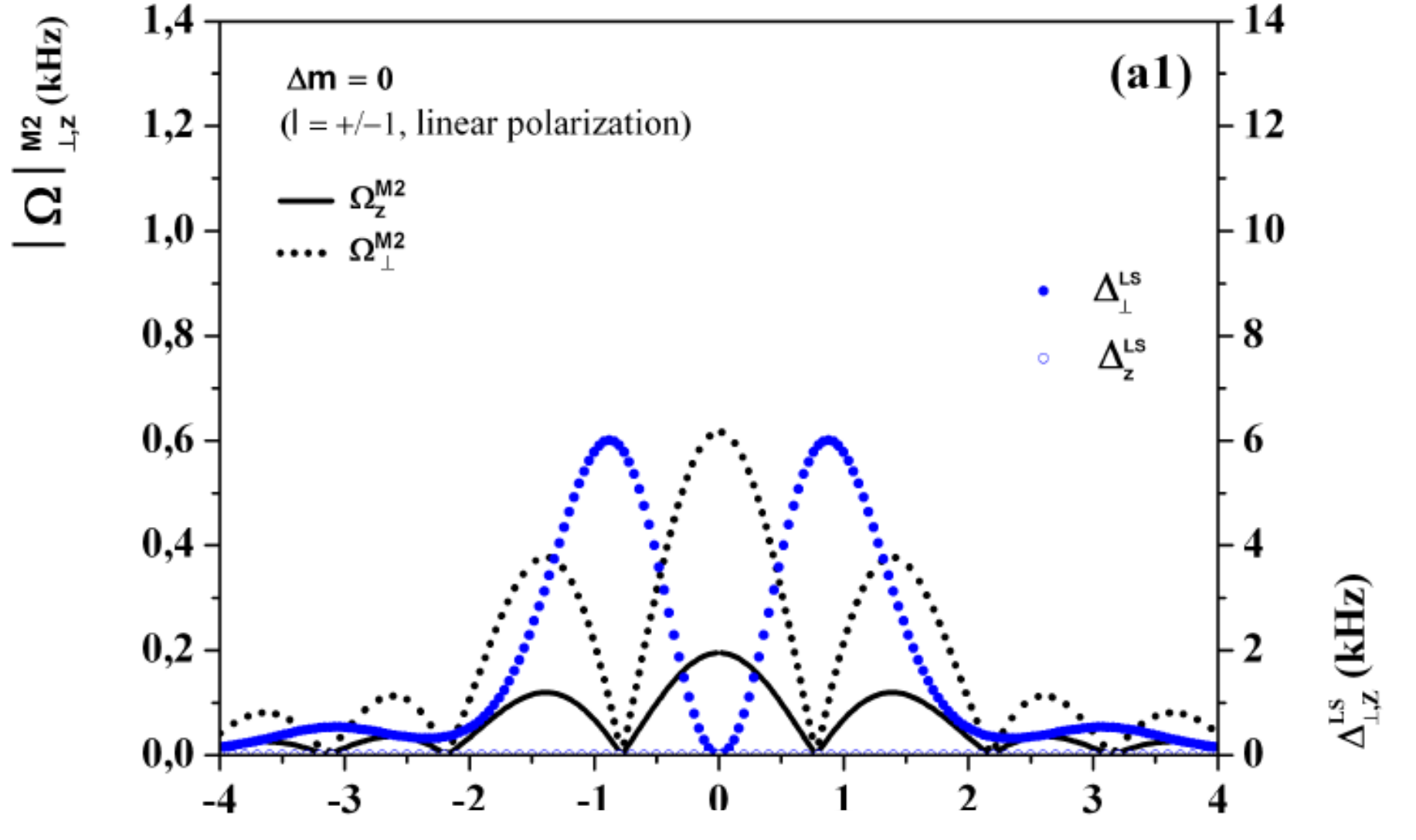


monochromatic two-photon E1M1

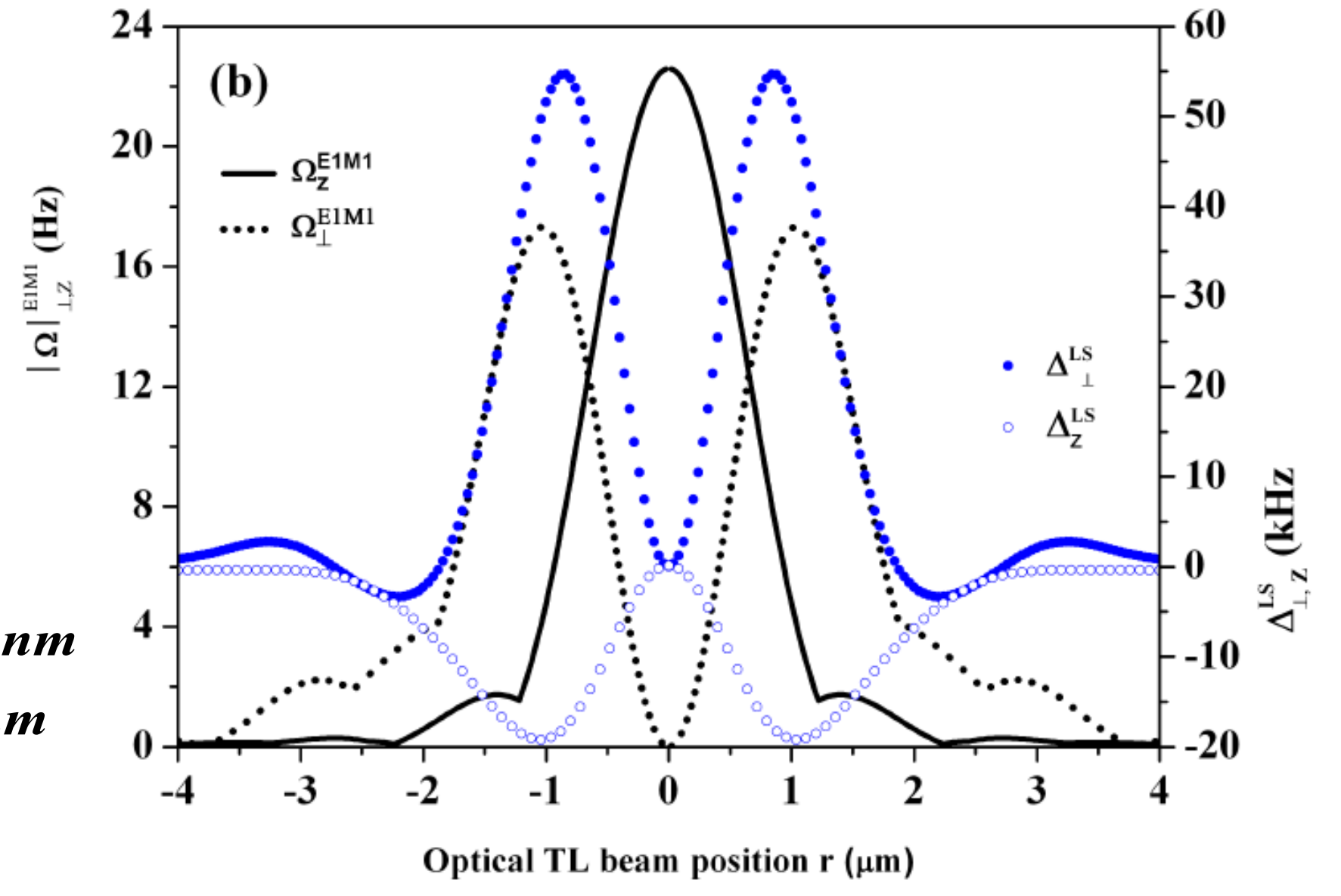


$\lambda_{E1} = 1397nm$
$\lambda_{M1} = 1397nm$

Magnetic quadrupole M2

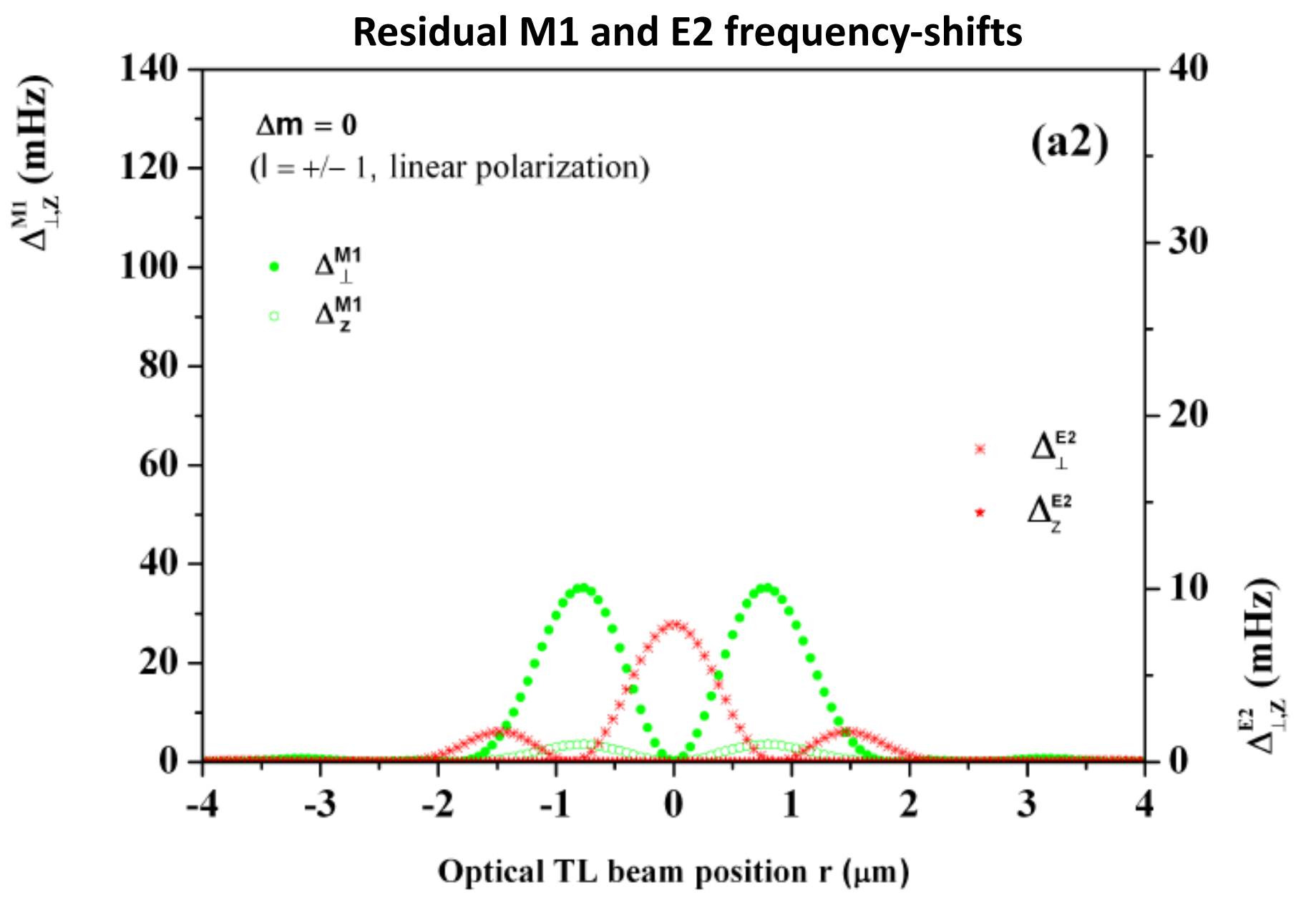


Bichromatic two-photon E1M1



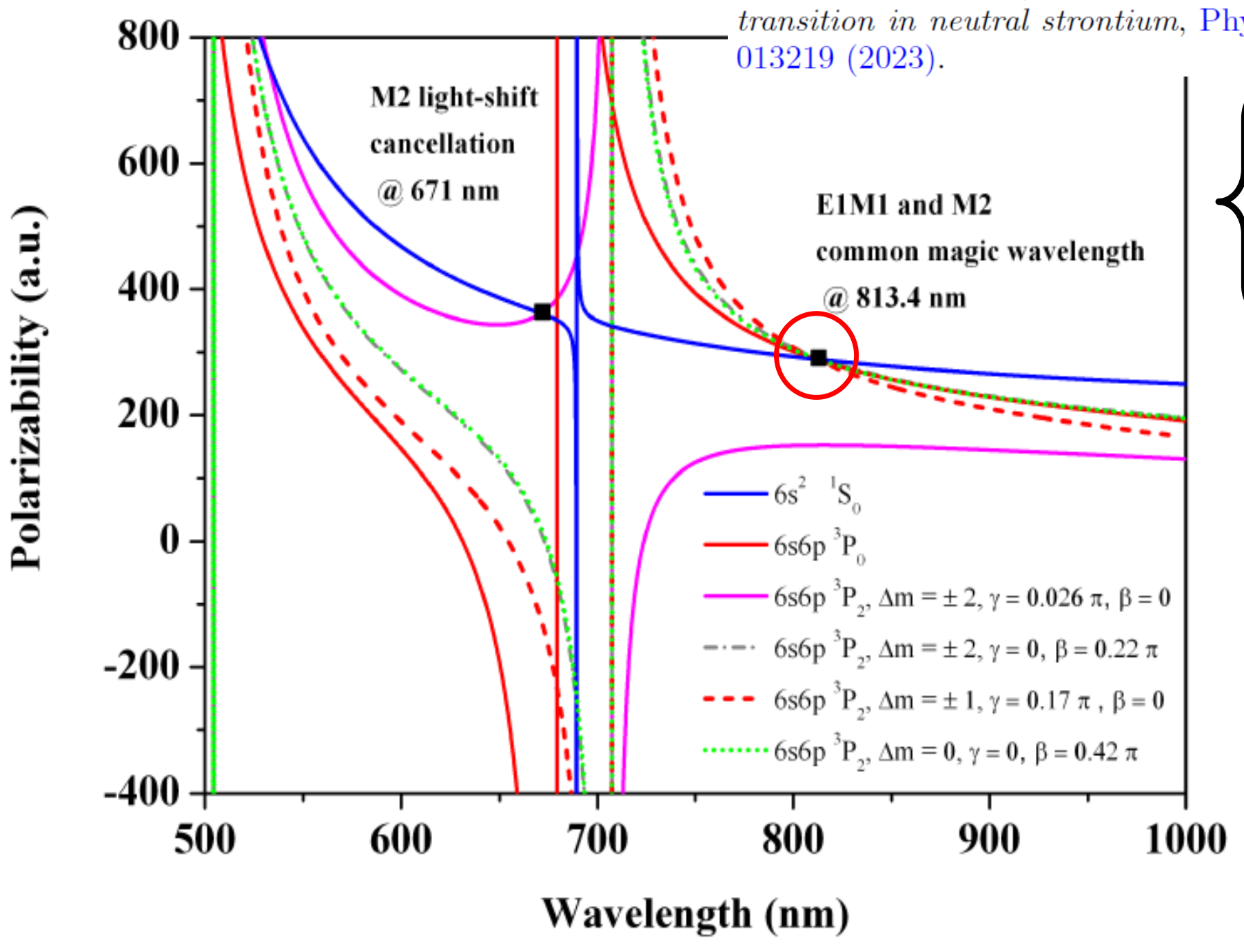
$\lambda_{E1,M1} = 3236nm$
$\lambda_{M1,E1} = 890nm$

Residual M1 and E2 frequency-shifts



IV. Realizing a ^{88}Sr two-color TL hyper-clock @ 813.4 nm robust against light-shift and residual multipole shifts

J. Trautmann, D. Yankelev, V. Klüsener, A. J. Park, I. Bloch and S. Blatt, *The* $^1S_0$-$^3P_2$ *magnetic quadrupole transition in neutral strontium*, Phys. Rev. Research. **5**, 013219 (2023).

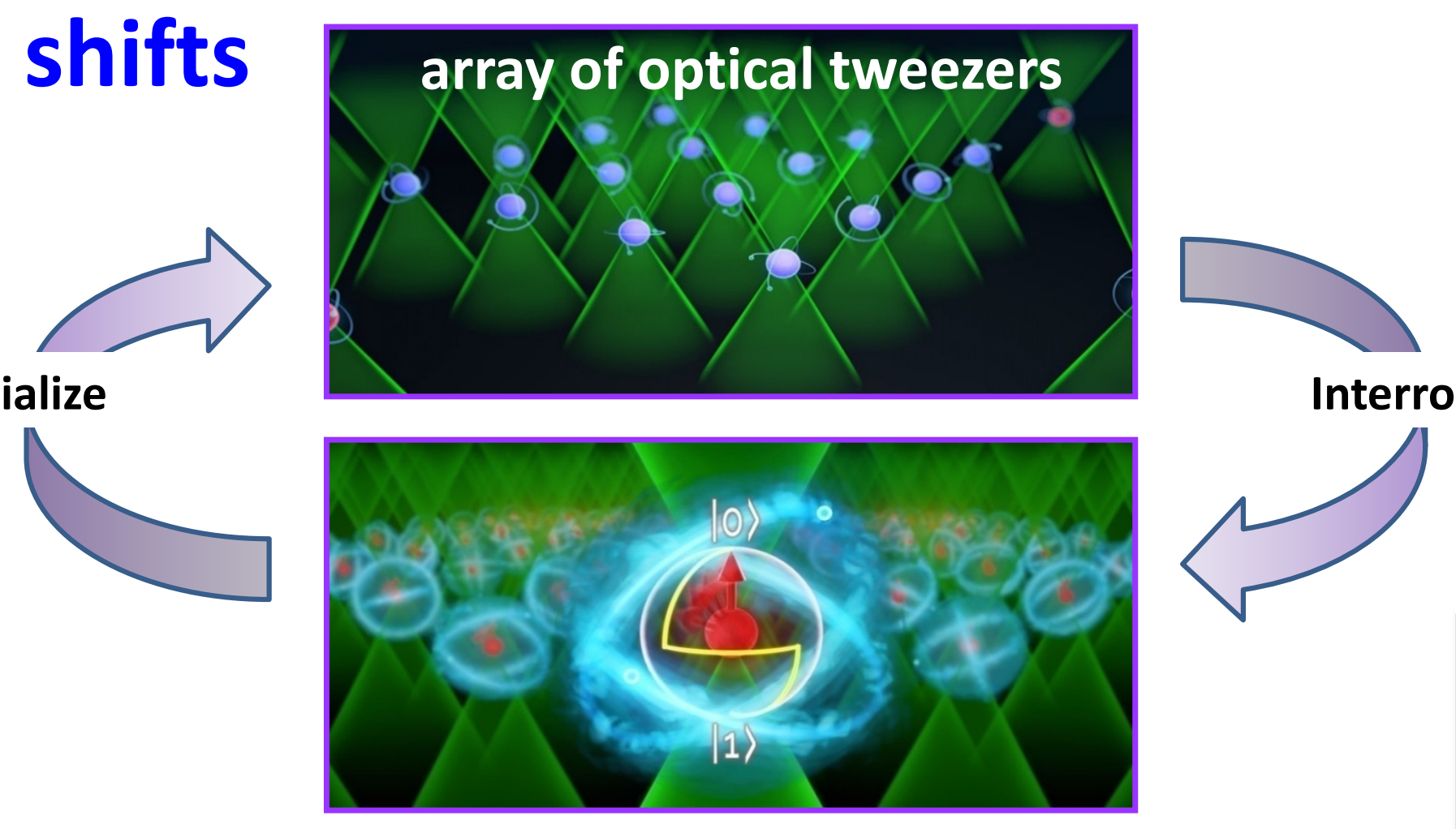


$$\left\{\begin{aligned} &\alpha_s : scalar \\ &\alpha_v : vector \\ &\alpha_t : tensor \end{aligned}\right.$$

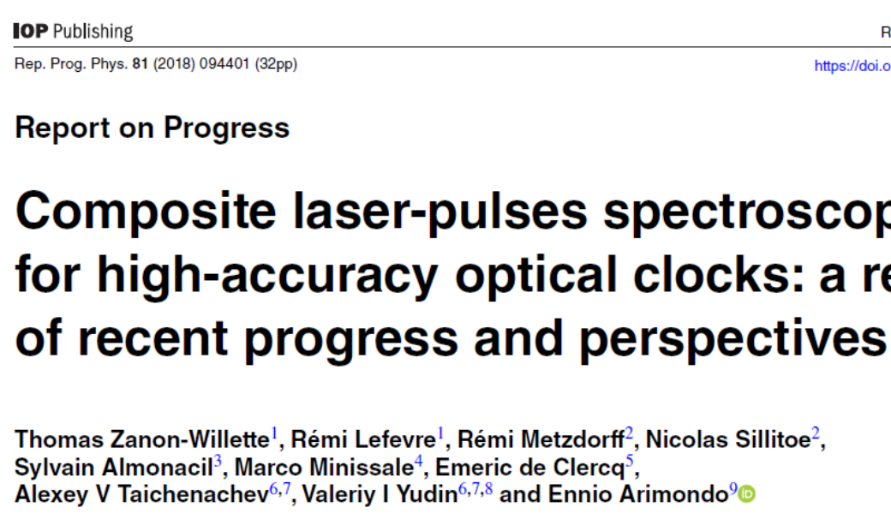
$$\Delta_{\perp,z}^{E1}(\mathbf{r}) = -\frac{1}{4}\Delta\alpha\widetilde{E}_{\perp,z}^2(\mathbf{r})$$

$$\alpha_i = \alpha_i^s + \alpha_i^v\sin(2\gamma)\frac{m_{J_i}}{2J_i} + \alpha_i^t\frac{3\cos^2(\beta)-1}{2}\frac{3m_{J_i}^2 - J_i(J_i+1)}{J_i(2J_i-1)}$$

Tunable parameters of trapping light:
γ → polarization ellipticity
β → angle between polarization and quantization axis defined by an external weak static B field



Hyper-clock ≡ clock using composite pulses



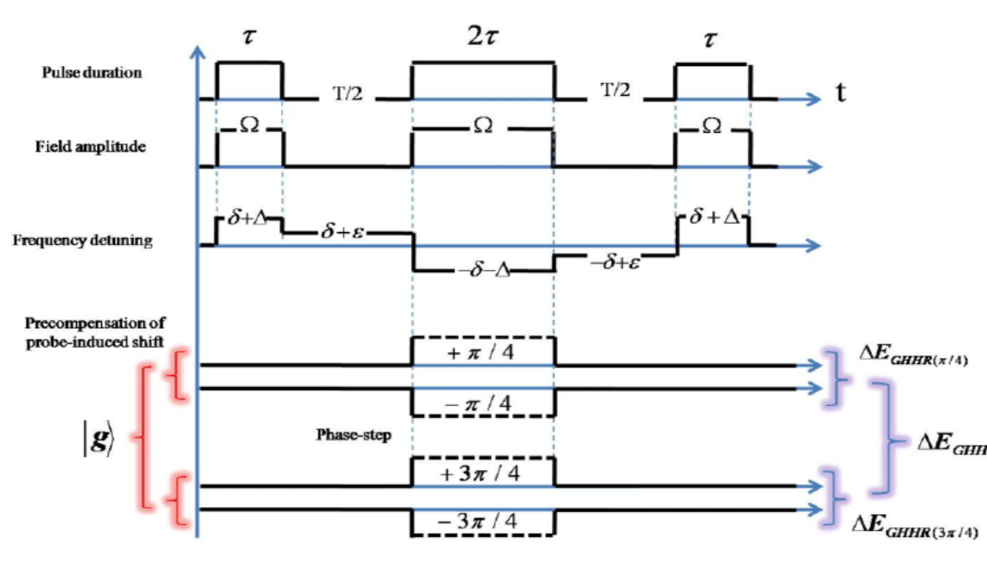
IOP Publishing Reports on Progress in Physics
Rep. Prog. Phys. 81 (2018) 094401 (32pp) https://doi.org/10.1088/1361-6633/aac9e9

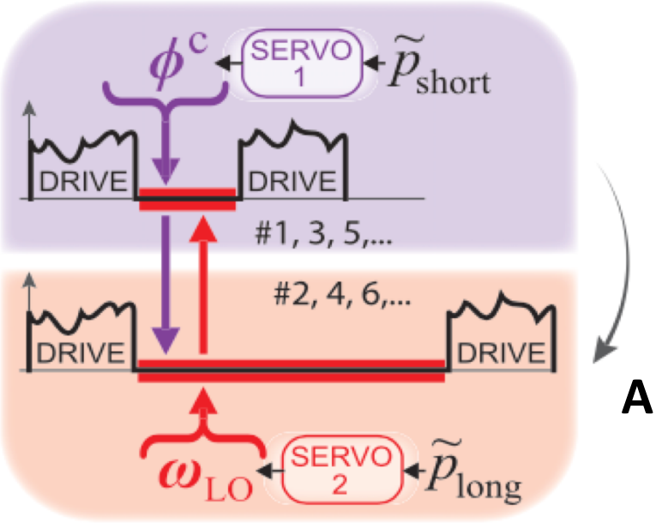
Report on Progress

Composite laser-pulses spectroscopy for high-accuracy optical clocks: a review of recent progress and perspectives

Thomas Zanon-Willette[1], Rémi Lefevre[1], Rémi Metzdorff[2], Nicolas Sillitoe[2], Sylvain Almonacil[3], Marco Minissale[4], Emeric de Clercq[5], Alexey V Taichenachev[6,7], Valeriy I Yudin[6,7,8] and Ennio Arimondo[9]

- GHHR(π/4,3π/4) in 2022





Auto-balanced Ramsey spectroscopy (2018)