Quantum sensors exploring SU(2) and SU(3) symmetry

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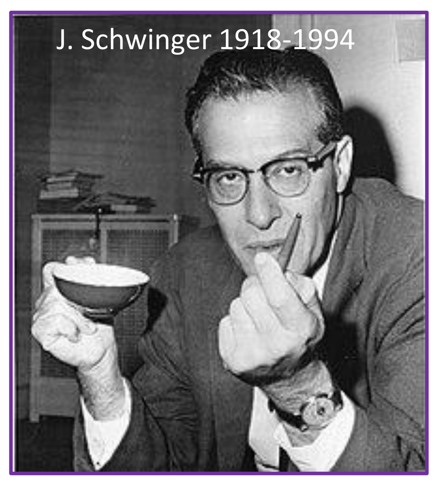
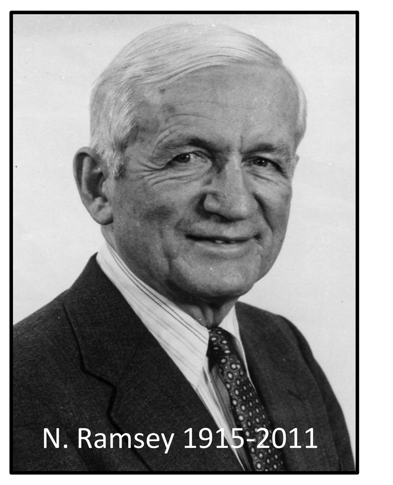
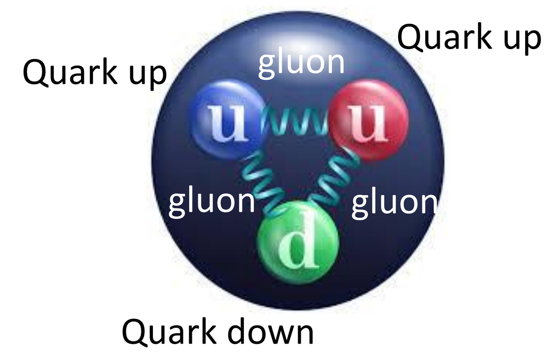




I. SU(d=3) Ramsey qudit rotation dynamics and symmetry breaking:



M. Gell-Mann, Symmetries of baryons and mesons, Phys. Rev. **125**, 1067 (1962).
U. Fano, A.R.P. Rau, *Symmetries in Quantum Physics*, Academic: New York, NY, USA, (1996).

Our proposal is to test a new coherent control technique of a multilevel quantum system interacting simultaneously with multiple color laser fields. The project will explore for the first time the laser SU(3) dynamics which is directly inspired by the same group symmetry driving the quark physics from the Standard model (SM)

IL NUOVO CIMENTO Vol. 106 B, N. 11 Novembre 1991

Matrix Representation of the Evolution Operator for the SU(3) Dynamics.

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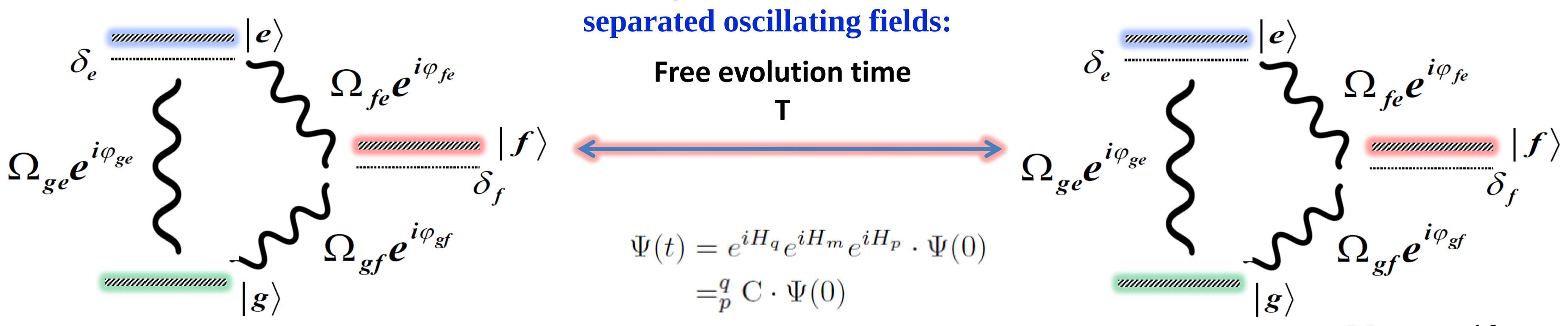
(ricevuto il 2 Ottobre 1990; approvato il 14 Gennaio 1991)

Summary. — The evolution of quantum states ruled by Hamiltonian operators linear in the SU(3) generators is studied within the framework of a method, which combines the Wei-Norman ordering technique with the geometrical representation of the SU(3) generators. The Rabi matrix specifying the motion of the SU(3) Bloch vector is obtained even for a time-dependent Hamiltonian and its usefulness is discussed.

PACS 42.50.Bs – Photon statistics and photon counting.

$$\Psi(t)=C_g(t)|g\rangle+C_f(t)|f\rangle+C_e(t)|e\rangle$$

→Developping SU(3) Ramsey's method of separated oscillating fields:



$$\Psi(t)=e^{iH_q}e^{iH_m}e^{iH_p}\cdot\Psi(0)$$
$$={}^q_p\mathrm{C}\cdot\Psi(0)$$

N. F. Ramsey, A molecular beam resonance method with separated oscillating fields, Phys. Rev. **78**, 695 (1950).

$${}^q_p\mathrm{C}=C_pC_q\left({}^q_p\mathrm{A}-i{}^q_p\mathrm{B}\right)\equiv\begin{pmatrix}{}^q_pC_{gg} & {}^q_pC_{gf} & {}^q_pC_{ge}\\ {}^q_pC_{fg} & {}^q_pC_{ff} & {}^q_pC_{fe}\\ {}^q_pC_{eg} & {}^q_pC_{ef} & {}^q_pC_{ee}\end{pmatrix}$$

$$\begin{aligned}{}^q_p\mathrm{A}=&C_m\left(\lambda_0-\frac{2}{d}\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_p\mathbf{S}_q\}_k}{C_pC_q}\lambda_0-\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_p\odot\mathbf{S}_q\}_k}{C_pC_q}\vec{\lambda}_k\right)-\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_q\times(\mathbf{S}_p\odot\mathbf{S}_m)\}_k}{C_pC_q}\vec{\lambda}_k\\ &-\sum_{k=1}^{d^2-1}\left(\frac{\{\mathbf{S}_q\odot\mathbf{S}_m\}_k}{C_pC_q}+\frac{\{\mathbf{S}_p\odot\mathbf{S}_m\}_k}{C_pC_q}\right)\vec{\lambda}_k+\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_q\odot(\mathbf{S}_p\times\mathbf{S}_m)\}_k}{C_pC_q}\vec{\lambda}_k\\ &-i\left(\sum_{k=1}^{d^2-1}\{\mathbf{S}_m\}_k\vec{\lambda}_k-\sum_{k=1}^{d^2-1}\left(\frac{\{\mathbf{S}_p\times\mathbf{S}_m\}_k}{C_p}-\frac{\{\mathbf{S}_q\times\mathbf{S}_m\}_k}{C_q}\right)\vec{\lambda}_k-\frac{2}{d}\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_q\mathbf{S}_m\mathbf{S}_p\}_k}{C_pC_q}\vec{\lambda}_k+\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_q\times(\mathbf{S}_m\times\mathbf{S}_p)\}_k}{C_pC_q}\vec{\lambda}_k\right)\end{aligned}$$

$$\begin{aligned}{}^q_p\mathrm{B}=&\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_p\}_k}{C_p}\left(C_m\vec{\lambda}_k+\frac{2}{id}\mathbf{S}_m\lambda_0\right)+\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_q\}_k}{C_q}\left(C_m\vec{\lambda}_k+\frac{2}{id}\mathbf{S}_m\lambda_0\right)\\ &-\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_p\times\mathbf{S}_q\}_k}{C_pC_q}\left(C_m\vec{\lambda}_k-\frac{2}{id}\mathbf{S}_m\lambda_0\right)-\frac{2}{d}\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_q\}_k\{S_p\odot\mathbf{S}_m\}_k}{C_pC_q}\lambda_0-\sum_{k=1}^{d^2-1}\frac{\{\mathbf{S}_q\odot(\mathbf{S}_p\odot\mathbf{S}_m)\}_k}{C_pC_q}\vec{\lambda}_k\end{aligned}$$

→Symmetry breaking into irreducible SU(2) rotation when quantum states are equally spaced in energy like a ladder configuration with degenerate fields in amplitude and phase (→ The Wigner-Majorana-Schwinger formula 1977)

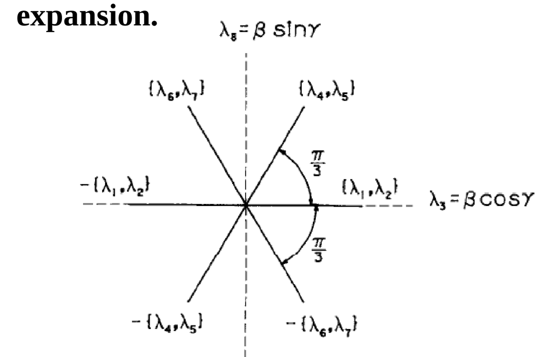
The sets of Gell-Mann matrices as generators of SU(3) are given by:

$$\lambda_1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\ \lambda_2=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix},\ \lambda_3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},$$
$$\lambda_4=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix},\ \lambda_5=\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix},$$
$$\lambda_6=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix},\ \lambda_7=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix},\ \lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

We start with a traceless hermitian 3x3 hamiltonien H where λ_k are Gell-Mann matrices: $k=\{p,q,m\}$

$$H=\frac{1}{2}\sum_{k=1}^{d^2-1}\hat{n}_k\vec{\lambda}_k$$
$$C_2=Tr\left\{H^2\right\}=\frac{1}{2}\left(\sum_{k=1}^{d^2-1}\hat{n}_k^2\right)$$
$$C_3=Tr\left\{H^3\right\}$$

Quadratic and cubic Casimir operators are used to elaborate analytics for SU(d=3) matrix expansion.



Using Kusnesov parametrization, we get the matrix exponentation:

$$e^{iH}=\frac{1}{d}K(\hat{n}_k)-i\sum_k\nabla_{\hat{n}_k}K(\hat{n}_k)\cdot\vec{\lambda}_k$$
$$\equiv C(\hat{n}_k)-i\sum_k S(\hat{n}_k)\cdot\vec{\lambda}_k$$

with SU(3) characteristic functions :

$$K(\hat{n}_k)=e^{i\kappa_1}+e^{i\kappa_2}+e^{i\kappa_3}$$
$$\nabla_{\hat{n}_k}K(\hat{n}_k)=K_1\left[\frac{\cos(2\gamma)}{\beta\cos(3\gamma)}\nabla_{\hat{n}_k}C_2-\frac{4\sin(2\gamma)}{\sqrt{3}\beta^2\cos(3\gamma)}\nabla_{\hat{n}_k}C_3\right]+K_2\left[\frac{\sin(2\gamma)}{\beta\cos(3\gamma)}\nabla_{\hat{n}_k}C_2+\frac{4\cos(2\gamma)}{\sqrt{3}\beta^2\cos(3\gamma)}\nabla_{\hat{n}_k}C_3\right]$$
$$K_1=-\frac{1}{2i}\left(e^{i\kappa_1}-e^{i\kappa_2}\right)$$
$$K_2=-\frac{1}{2i\sqrt{3}}\left(e^{i\kappa_1}+e^{i\kappa_2}-2e^{i\kappa_3}\right)$$

where eigenvalues are:

$$\kappa_1=\frac{\beta}{\sqrt{3}}\sin(\gamma+\pi/3)\qquad \beta=\sqrt{2C_2}$$
$$\kappa_2=\frac{\beta}{\sqrt{3}}\sin(\gamma-\pi/3)\qquad \gamma=\frac{1}{3}\arcsin\left(\frac{\sqrt{6}C_3}{C_2^{3/2}}\right)$$
$$\kappa_3=-\frac{\beta}{\sqrt{3}}\sin(\gamma)$$

We have used the constant generators of the SU(d) Lie algebra and anti-commutation relation:

$$f_{jkl}=-i\frac{1}{4}Tr\left(\lambda_j[\lambda_k,\lambda_l]\right)$$
$$d_{jkl}=\frac{1}{4}Tr\left(\lambda_j\{\lambda_k,\lambda_l\}\right)$$
$$\{\mathbf{S}_p\mathbf{S}_q\}_j=\{S_p\}_j\{S_q\}_j$$
$$\{\mathbf{S}_p\times\mathbf{S}_q\}_j=f_{jkl}\{S_p\}_k\{S_q\}_l=-\{\mathbf{S}_q\times\mathbf{S}_p\}_j$$
$$\{\mathbf{S}_p\odot\mathbf{S}_q\}_j=d_{jkl}\{S_p\}_k\{S_q\}_l=+\{\mathbf{S}_q\odot\mathbf{S}_p\}_j$$

scalar product term

$$\left(\mathbf{S}_p\cdot\vec{\lambda}_k\right)\left(\mathbf{S}_q\cdot\vec{\lambda}_k\right)=\frac{2}{N}\mathbf{S}_p\mathbf{S}_q\cdot\lambda_0+\left(\mathbf{S}_p\odot\mathbf{S}_q+i\mathbf{S}_p\times\mathbf{S}_q\right)\cdot\vec{\lambda}_k$$

dot product term **cross product term**

D. Kusnesov, *Exact matrix expansions for group elements of SU(N)*, J. Math. Phys. **36**, 898 (1995).
S. Weigert, *Baker-Campbell-Hausdorff relation for special unitary groups SU(N)*, J. Phys. A: Math. Gen. **30**, 8739 (1997).
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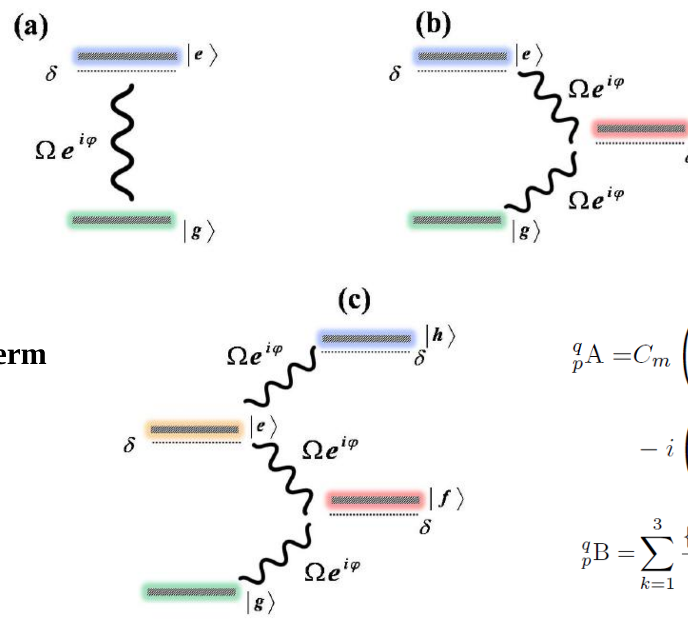


$${}^q_pC_{m,m'}=\frac{1}{(2j)!}\sqrt{\frac{(j+m)!\,(j+m')!}{(j-m)!\,(j-m')!}}\left(\frac{\partial}{\partial\alpha}\right)^{j-m}\left(\frac{\partial}{\partial\beta}\right)^{j-m'}\times\left[{}^q_pC_{gg}+\alpha{}^q_pC_{eg}+\beta{}^q_pC_{ge}+\alpha\beta{}^q_pC_{ee}\right]^{2j}$$

$${}^q_p\mathrm{C}=C_pC_q\left({}^q_p\mathrm{A}-i{}^q_p\mathrm{B}\right)\equiv\begin{pmatrix}{}^q_pC_{gg} & {}^q_pC_{ge}\\ {}^q_pC_{eg} & {}^q_pC_{ee}\end{pmatrix}$$

$$\odot\equiv 0$$
$$f_{123}=1=-f_{132}$$
$$f_{213}=1=-f_{231}$$
$$f_{312}=1=-f_{321}$$

$$\sigma_1=\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix}$$
$$\sigma_2=\sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}$$
$$\sigma_3=\sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

where Pauli matrices are replacing Gell-Mann matrices and functions reduce to :

$$\kappa_1=\beta/2=\sqrt{2C_2}/2$$
$$\kappa_2=-\beta/2=-\sqrt{2C_2}/2$$

trigonometric functions (Bloch sphere)

$$C_k=\cos(\theta_k)$$
$$S_k=-n_k\sin(\theta_k)$$

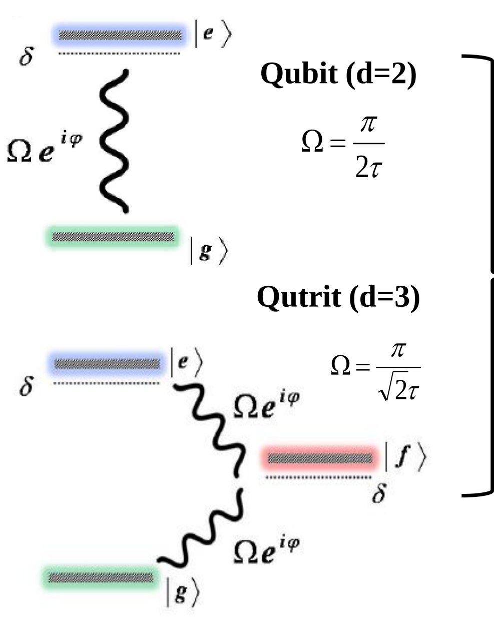
$$\begin{aligned}{}^q_p\mathrm{A}=&C_m\left(\sigma_0-\frac{2}{2}\sum_{k=1}^{3}\frac{\{\mathbf{S}_p\mathbf{S}_q\}_k}{C_pC_q}\sigma_0\right)\\ &-i\left(\sum_{k=1}^{3}\{\mathbf{S}_m\}_k\vec{\sigma}_k-\sum_{k=1}^{3}\left(\frac{\{\mathbf{S}_p\times\mathbf{S}_m\}_k}{C_p}-\frac{\{\mathbf{S}_q\times\mathbf{S}_m\}_k}{C_q}\right)\vec{\sigma}_k-\frac{2}{2}\sum_{k=1}^{3}\frac{\{\mathbf{S}_q\mathbf{S}_m\mathbf{S}_p\}_k}{C_pC_q}\vec{\sigma}_k+\sum_{k=1}^{3}\frac{\{\mathbf{S}_q\times(\mathbf{S}_m\times\mathbf{S}_p)\}_k}{C_pC_q}\vec{\sigma}_k\right)\end{aligned}$$

$${}^q_p\mathrm{B}=\sum_{k=1}^{3}\frac{\{\mathbf{S}_p\}_k}{C_p}\left(C_m\vec{\sigma}_k+\frac{2}{2i}\mathbf{S}_m\sigma_0\right)+\sum_{k=1}^{3}\frac{\{\mathbf{S}_q\}_k}{C_q}\left(C_m\vec{\sigma}_k+\frac{2}{2i}\mathbf{S}_m\sigma_0\right)-\sum_{k=1}^{3}\frac{\{\mathbf{S}_p\times\mathbf{S}_q\}_k}{C_pC_q}\left(C_m\vec{\sigma}_k-\frac{2}{2i}\mathbf{S}_m\sigma_0\right)$$

F. T. Hioe and J. H. Eberly, N-Level Coherence Vector and Higher Conservation Laws in Quantum Optics and Quantum Mechanics, Phys. Rev. Lett. **47**, 838 (1981).
F.T. Hioe, *Dynamic symmetries in quantum electronics*, Phys. Rev. A **28**, 879 (1983).
F.T. Hioe, *N-level quantum systems with SU(2) dynamic symmetry*, J. Opt. Soc. Am. B **4**, 1327 (1987).

II. SU(2) rotation flavour of qudit hyper-clocks ≡ composite pulses with ERG algorithm: PRR, vol 4, 023117 (2022)

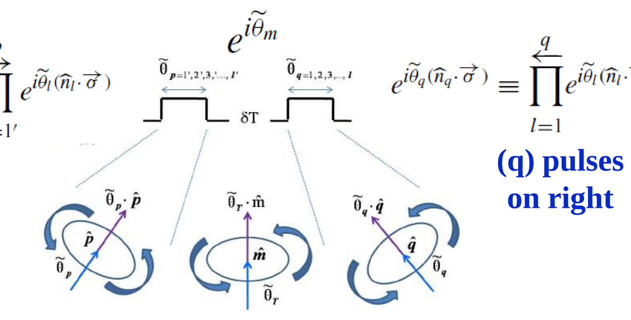
$$\left|{}^q_pC_{gg}\right|^2+\left|{}^q_pC_{ge}\right|^2=1,$$
$${}^q_pC^*_{gg}={}^q_pC_{ee};\quad {}^q_pC_{eg}=-{}^q_pC^*_{ge}$$

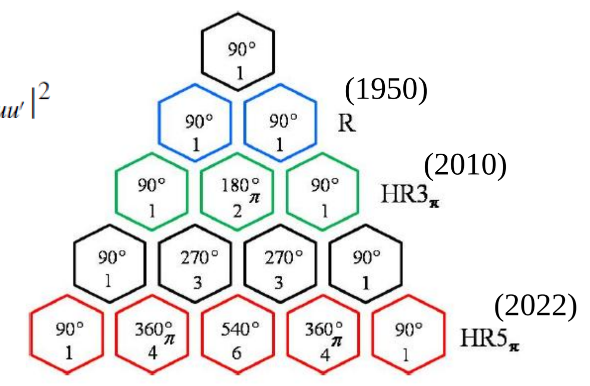


$${}^q_pP_{uu'}=\left|{}^q_pC_{uu'}\right|^2$$

$$e^{i\tilde{\theta}_p(\hat{n}_p\cdot\vec{\sigma})}\equiv\prod_{l=1'}^{\overrightarrow{p}}e^{i\tilde{\theta}_l(\hat{n}_l\cdot\vec{\sigma})}\qquad e^{i\tilde{\theta}_q(\hat{n}_q\cdot\vec{\sigma})}\equiv\prod_{l=1}^{\overleftarrow{q}}e^{i\tilde{\theta}_l(\hat{n}_l\cdot\vec{\sigma})}$$

(p) pulses on left **(q) pulses on right**





Pascal's triangle classification of SU(2) composite pulse protocols

$$\mathrm{C}=\begin{pmatrix}{}^q_pC_{1,1} & {}^q_pC_{1,0} & {}^q_pC_{1,-1}\\ {}^q_pC_{0,1} & {}^q_pC_{0,0} & {}^q_pC_{0,-1}\\ {}^q_pC_{-1,1} & {}^q_pC_{-1,0} & {}^q_pC_{-1,-1}\end{pmatrix}=\begin{pmatrix}{}^q_pC^2_{gg} & \sqrt{2}\cdot{}^q_pC_{gg}\cdot{}^q_pC_{ge} & {}^q_pC^2_{ge}\\ -\sqrt{2}\cdot{}^q_pC_{gg}\cdot{}^q_pC^*_{ge} & \left|{}^q_pC_{gg}\right|^2-\left|{}^q_pC_{ge}\right|^2 & \sqrt{2}\cdot{}^q_pC^*_{gg}\cdot{}^q_pC_{ge}\\ {}^q_pC^{2*}_{ge} & -\sqrt{2}\cdot{}^q_pC^*_{gg}\cdot{}^q_pC^*_{ge} & {}^q_pC^{2*}_{gg}\end{pmatrix}$$

$$\Psi(t)=e^{i\tilde{\theta}_q(\hat{n}_q\cdot\vec{\sigma})}e^{i\theta_m(\hat{m}\cdot\vec{\sigma})}e^{i\tilde{\theta}_p(\hat{n}_p\cdot\vec{\sigma})}\Psi(0)$$
$$={}^q_pC\Psi(0).$$

Matrix components for the two-level system: $${}^q_pC=\begin{pmatrix}{}^q_pC_{gg} & {}^q_pC_{ge}\\ {}^q_pC_{eg} & {}^q_pC_{ee}\end{pmatrix}$$

We have used here the constant generators of the SU(2) Pauli algebra

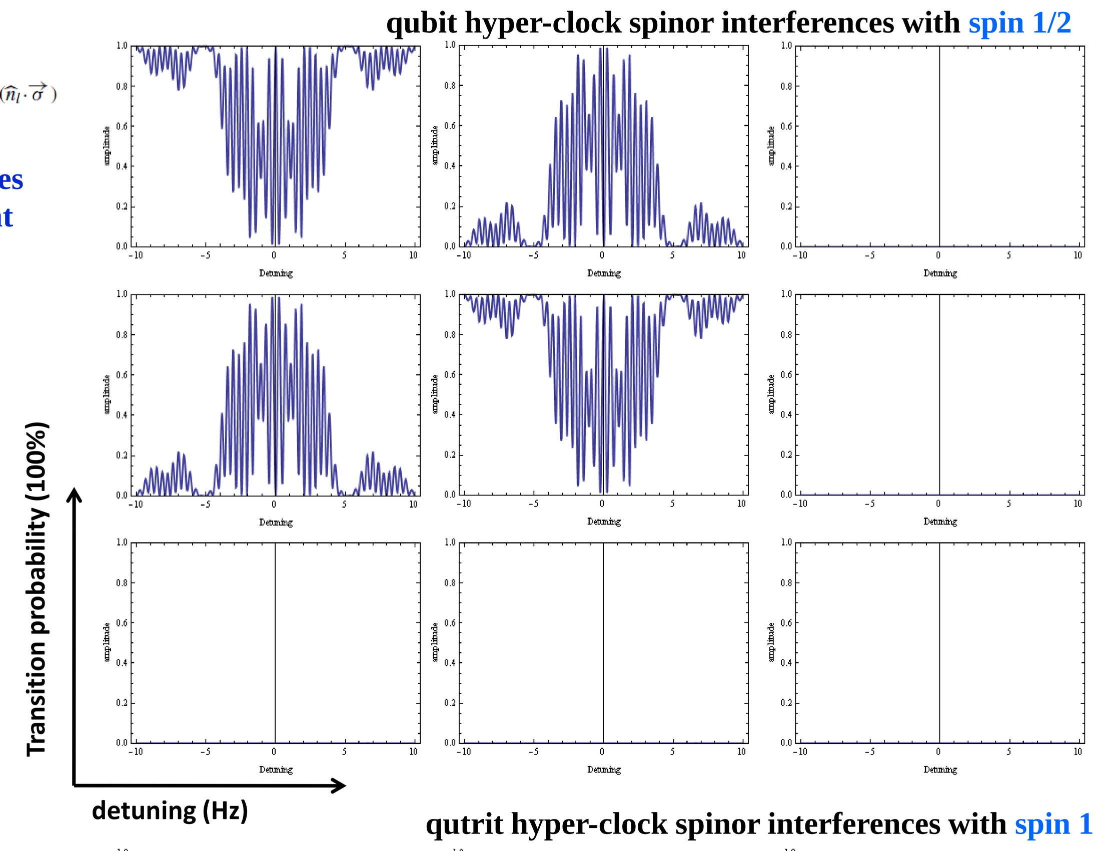
$$e^{i\tilde{\vartheta}_l(\hat{n}_l\cdot\vec{\sigma})}=\sigma_0\cos\tilde{\vartheta}_l+i(\hat{n}_l\cdot\vec{\sigma})\sin\tilde{\vartheta}_l$$
$$(\hat{n}_p\cdot\vec{\sigma})\cdot(\hat{n}_q\cdot\vec{\sigma})=(\hat{n}_p\cdot\hat{n}_q)\sigma_0+i(\hat{n}_p\times\hat{n}_q)\cdot\vec{\sigma}$$

Probability amplitude and phase-shift of the wave-function :

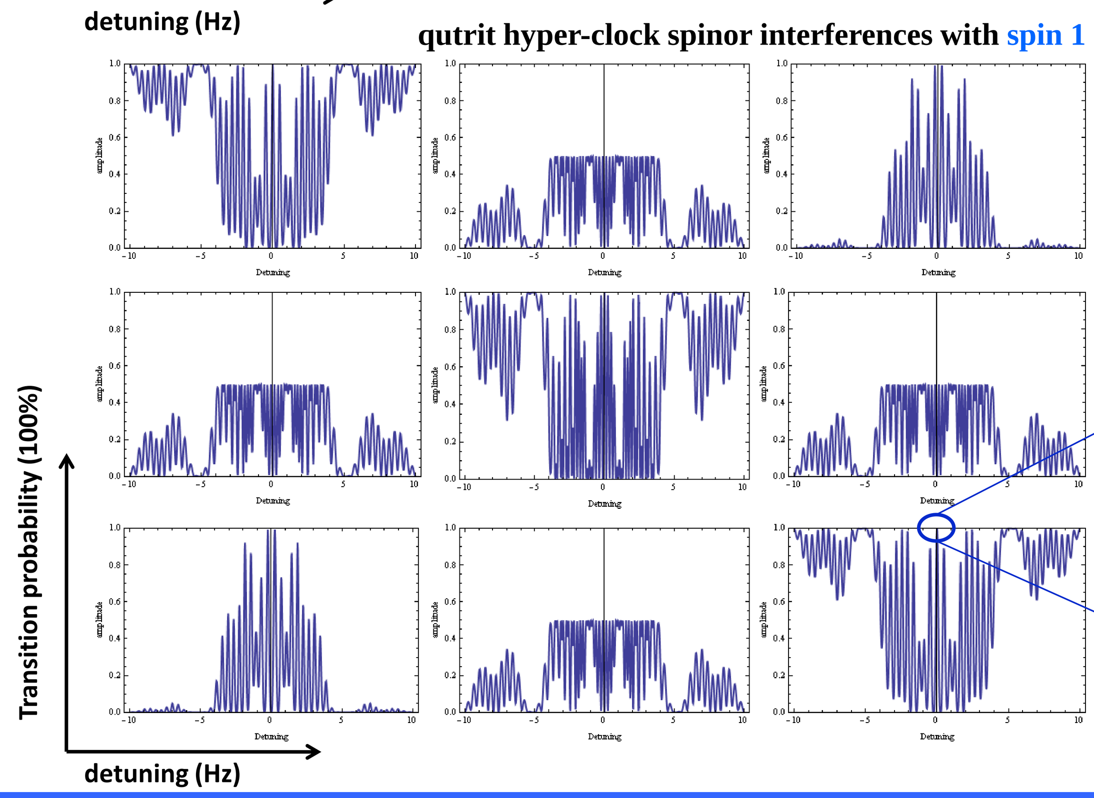
$${}^q_pC_{uu'}={}^q_p\tilde{C}^+_{uu'}e^{i{}^q_p\tilde{\Phi}^+_{uu'}}e^{i\tilde{\theta}_m}+{}^q_p\tilde{C}^-_{uu'}e^{i{}^q_p\tilde{\Phi}^-_{uu'}}e^{-i\tilde{\theta}_m}$$
$${}^q_p\tilde{\Phi}_{uu'}={}^q_p\tilde{\Phi}^+_{uu'}-{}^q_p\tilde{\Phi}^-_{uu'}$$

W. Pauli Jr., On the quantum mechanics of magnetic electrons, Z. Phys. **43**, 601 (1927).
F. Bloch and I.I. Rabi, *Atoms in Variable Magnetic Fields*, Rev. Mod. Phys. **17**, 237 (1945).
I. I. Rabi, N. F. Ramsey, and J. Schwinger, Use of rotating coordinates in magnetic resonance problems, Rev. Mod. Phys. **26**, 167 (1954).

qubit hyper-clock spinor interferences with spin 1/2



qutrit hyper-clock spinor interferences with spin 1



Euler-Rodrigues-Gibbs (ERG) recursive algorithm to evaluate the phase-shift for arbitrary number of pulses (p,q)

$$\tan{}^q_p\tilde{\Phi}^{\pm}_{uu'}\equiv\frac{{}^q_p\hat{N}_+\cdot[\vec{\sigma}\pm\hat{m}\sigma_0]+{}^q_p\hat{N}_\times\cdot[\vec{\sigma}\mp\hat{m}\sigma_0]}{\sigma_0\pm\hat{m}\cdot\vec{\sigma}\pm\left[{}^q_p\hat{N}_-\times\hat{m}\right]\cdot\vec{\sigma}-{}^q_p\hat{N}^{\hat{m}}_\bullet}$$

$${}^q_p\hat{N}_+\equiv\hat{n}_p\tan\tilde{\theta}_p+\hat{n}_q\tan\tilde{\theta}_q,$$
$${}^q_p\hat{N}_-\equiv\hat{n}_p\tan\tilde{\theta}_p-\hat{n}_q\tan\tilde{\theta}_q,$$
$${}^q_p\hat{N}_\times\equiv\hat{n}_p\tan\tilde{\theta}_p\times\hat{n}_q\tan\tilde{\theta}_q,$$
$${}^q_p\hat{N}^{\hat{m}}_\bullet\equiv(\hat{n}_p\cdot\hat{n}_q)_{\hat{m},\vec{\sigma}}\tan\tilde{\theta}_p\tan\tilde{\theta}_q,$$

with a reduced variable,

$$(\hat{n}_p\cdot\hat{n}_q)_{\hat{m},\vec{\sigma}}=(\sigma_0\mp\hat{m}\cdot\vec{\sigma})(\hat{n}_p\cdot\hat{n}_q)\pm[(\hat{m}\cdot\hat{n}_p)\hat{n}_q+(\hat{m}\cdot\hat{n}_q)\hat{n}_p]\cdot\vec{\sigma}.$$

Pauli matrices are used as « pointers » to reconstruct the phase-shift for the 4 matrix components

$$\hat{n}_l\tan\tilde{\theta}_l\mapsto\frac{{}^{l+1}_{\ l}\hat{N}_+\pm{}^{l+1}_{\ l}\hat{N}_\times}{1-{}^{l+1}_{\ l}\hat{N}^0_\bullet}$$

($-$ for p and $+$ for q)

ERG recursive algorithm is used when(p,q) pulses are applied

$$\hat{n}_{l_x}\equiv\frac{\Omega_l}{\omega_l}\cos\varphi_l\qquad \hat{n}_{l_y}\equiv\frac{\Omega_l}{\omega_l}\sin\varphi_l\qquad \hat{n}_{l_z}\equiv\frac{\delta_l}{\omega_l}\qquad \omega_l=\sqrt{\delta_l^2+\Omega_l^2}$$

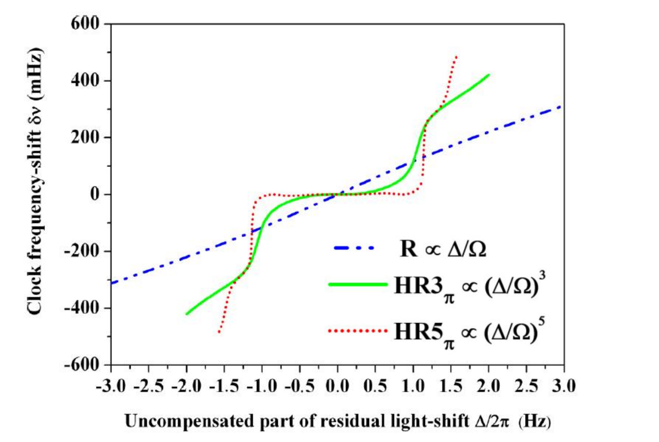
$${}^q_p\tilde{\Phi}^+_{gg}\mp{}^q_p\tilde{\Phi}^-_{gg}=\arctan\left[\frac{\tan{}^q_p\tilde{\Phi}^+_{gg}\mp\tan{}^q_p\tilde{\Phi}^-_{gg}}{1\pm\tan{}^q_p\tilde{\Phi}^+_{gg}\tan{}^q_p\tilde{\Phi}^-_{gg}}\right]$$

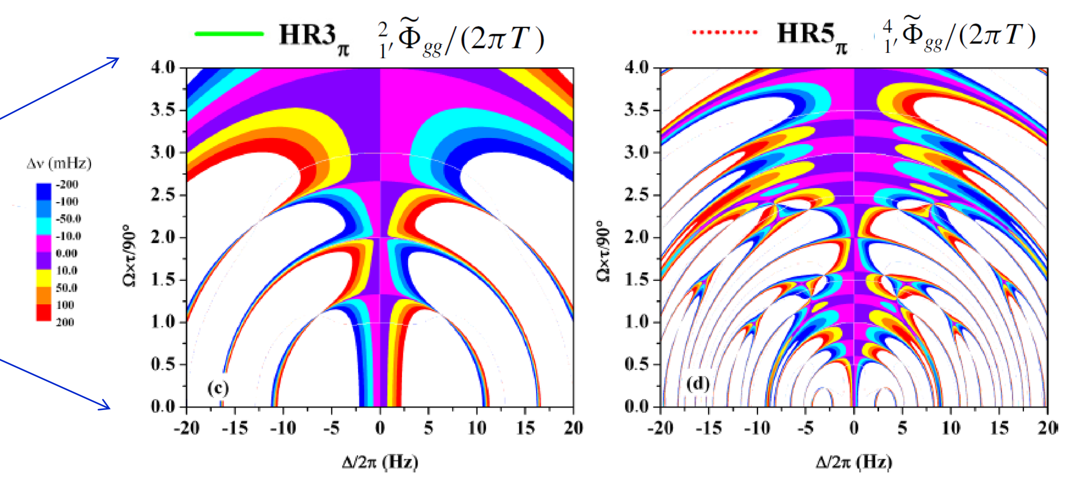
Two contributions from the Oz and Ox, Oy axis

$${}^q_p\tilde{\Phi}^+_{gg}=\arctan\left[\frac{()^p_z+()^q_z}{1-()^p_z()^q_z}\right],$$
$${}^q_p\tilde{\Phi}^-_{gg}=\arctan\left[\frac{()^p_y()^q_x-()^p_x()^q_y}{()^p_x()^q_x+()^p_y()^q_y}\right].$$

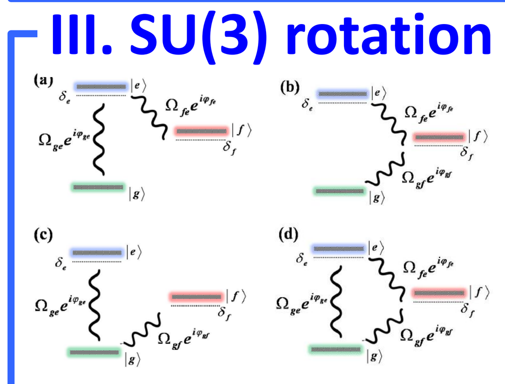
reduced to the Ramsey's phase-shift (1950):

$${}^1_{1'}\tilde{\Phi}_{gg}=\varphi_1-\varphi_{1'}+\frac{\delta_{1'}}{\omega_{1'}}\tan\tilde{\theta}_{1'}+\frac{\delta_1}{\omega_1}\tan\tilde{\theta}_1$$
$$=\varphi_1-\varphi_{1'}+\phi_{1'}+\phi_1,$$



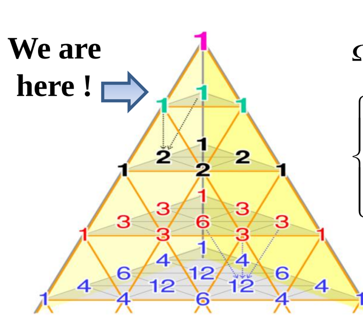


III. SU(3) rotation flavour of a qutrit hyper-clock: chirality of spinor interferences:



Different interaction modes

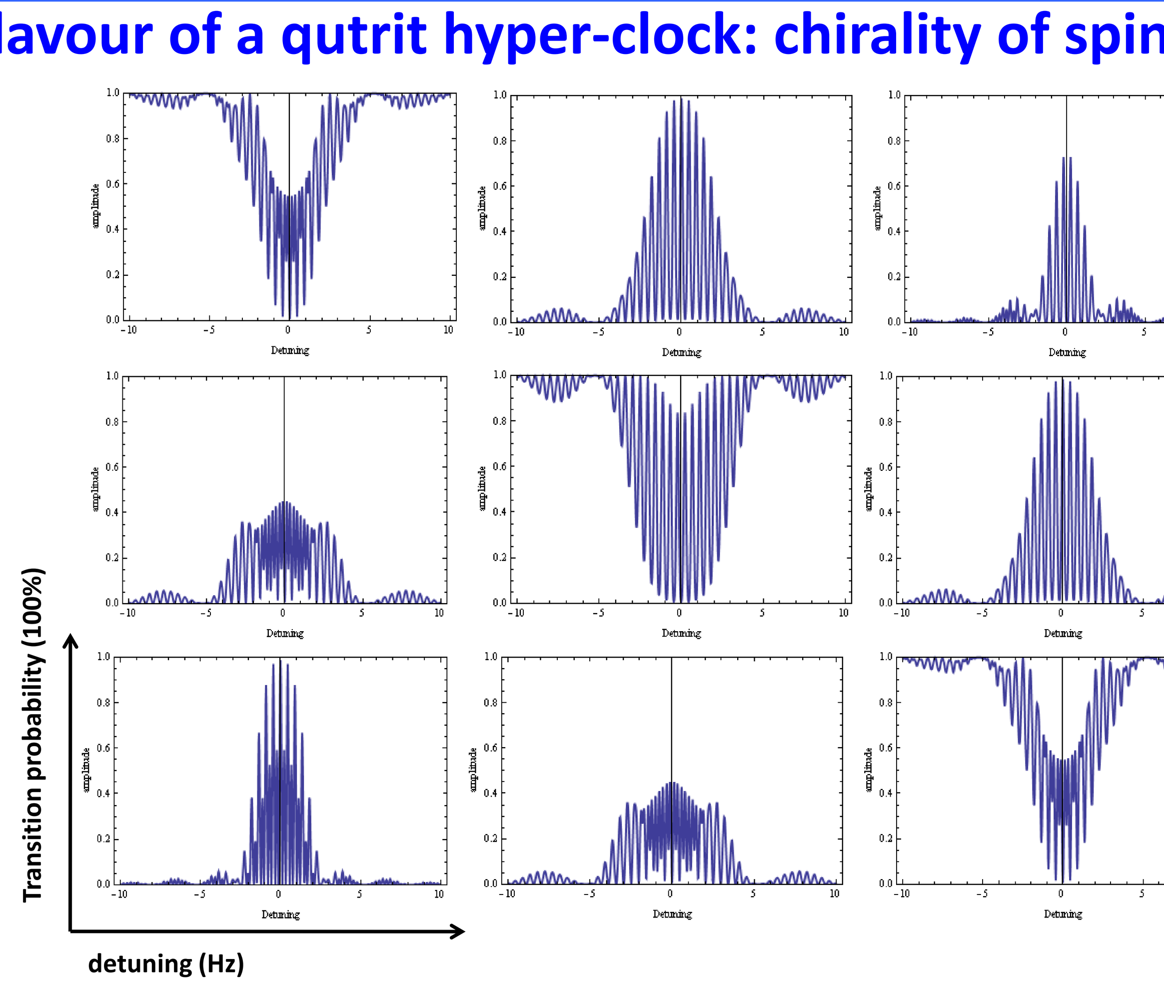
We are here !

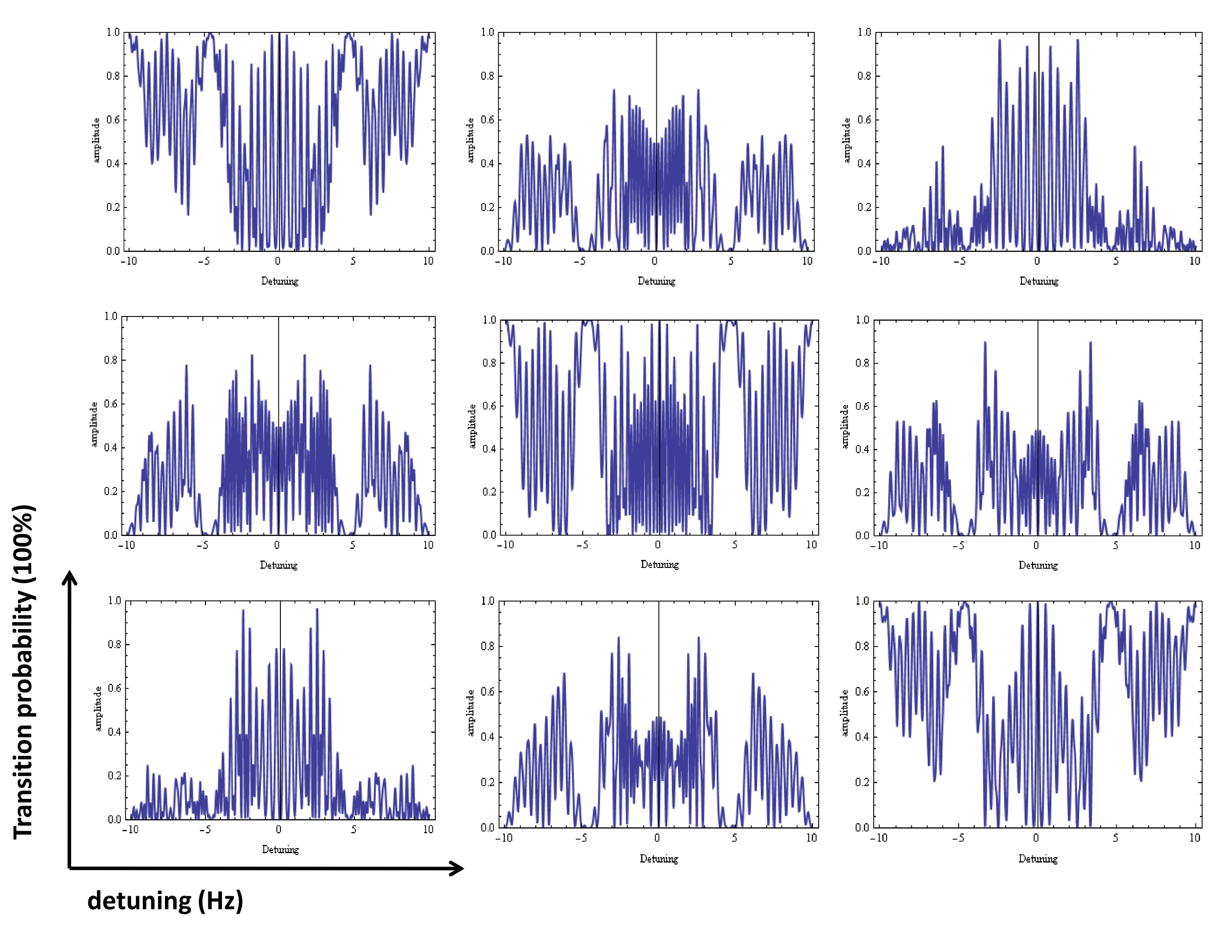


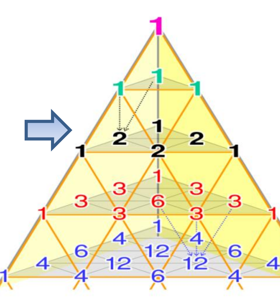
$$\Omega=\frac{5\pi}{8\sqrt{3}\tau}$$
$$\begin{Bmatrix}1_0 & 1_0\\ 1_{\frac{\pi}{2}} & 1_{\frac{\pi}{2}}\\ 1_0 & 1_0\end{Bmatrix}$$

$$\Omega_{ge}=\Omega_{fe}=\Omega_{gf}=\Omega$$
$$\Phi=\Phi_{gf}+\Phi_{fe}-\Phi_{ge}$$

(3-simplex) tetrahedron classification of SU(3) composite pulse protocols







$$\Omega=\frac{2\pi}{\sqrt{3}\tau}$$
$$\begin{Bmatrix}1_0 & 2_\pi & 1_0\\ 1_{3\frac{\pi}{2}} & 2_{3\frac{\pi}{2}} & 1_{3\frac{\pi}{2}}\\ 1_0 & 2_\pi & 1_0\end{Bmatrix}$$

$$\Omega_{ge}=\Omega_{fe}=\Omega_{gf}=\Omega$$
$$\Phi=\Phi_{gf}+\Phi_{fe}-\Phi_{ge}$$

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Boyan T. Torosov,[1] Michael Drewsen,[2] and Nikolay V. Vitanov[3]