Optimal control of Bose-Einstein Condensates in an optical lattice

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B. Peaudecerf[1], N. Dupont[1], F. Arrouas[1], G. Chatelain[1], L. Gabardos[1], N. Ombredane[1], M. Arnal[1], J. Billy[1], D. Sugny[2] and D. Guéry-Odelin[1]

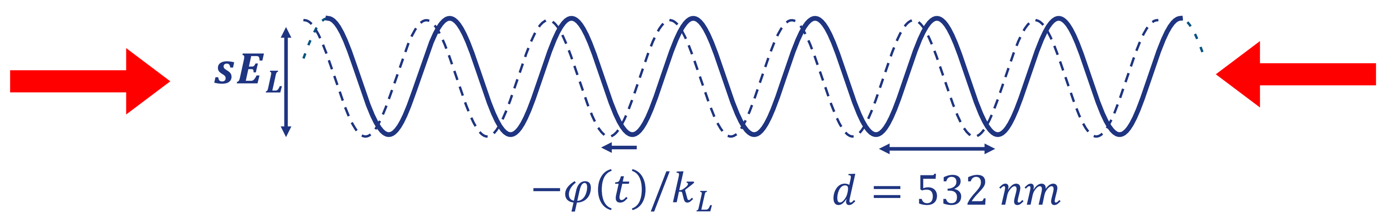
[1]Laboratoire Collisions Agrégats Réactivité, UMR5589, Université Toulouse III/CNRS, Toulouse, France
[2]Laboratoire Interdisciplinaire Carnot de Bourgogne, UMR 6303, Université de Bourgogne/CNRS, Dijon, France

Quantum optimal control is a framework under which the controlled dynamics of a quantum system can be made to satisfy a number of constraints. It is a promising direction for quantum simulation – for state and dynamics engineering, or for characterization or sensing of quantum systems [1].
We perform optimal control of the quantum state of a Bose-Einstein condensate of ^{87}Rb atoms in a one-dimensional optical lattice. We prepare a broad range of quantum state that we fully characterize. This enables further use of optimal control for quantum simulation and sensing with cold atoms on a lattice.

Experimental system

Bose-Einstein Condensate in a modulated 1D optical lattice

$$\mathcal{H}(x,p,t) = \frac{p^2}{2m} + V(x,t) = \frac{p^2}{2m} - \frac{s\,E_L}{2}\cos(k_L x + \varphi(t)) \qquad k_L = \frac{2\pi}{d},\ E_L = \frac{\hbar^2 k_L^2}{2m}$$

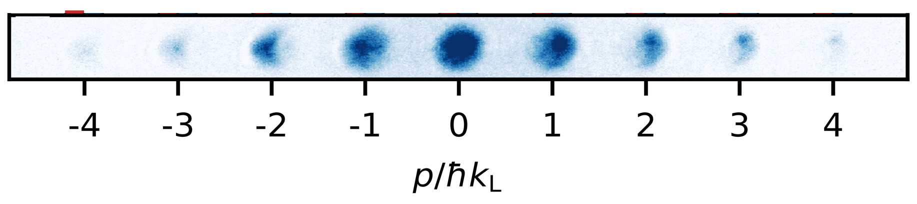


Spatial periodicity of the wavefunction ↔ discrete distribution in momentum space

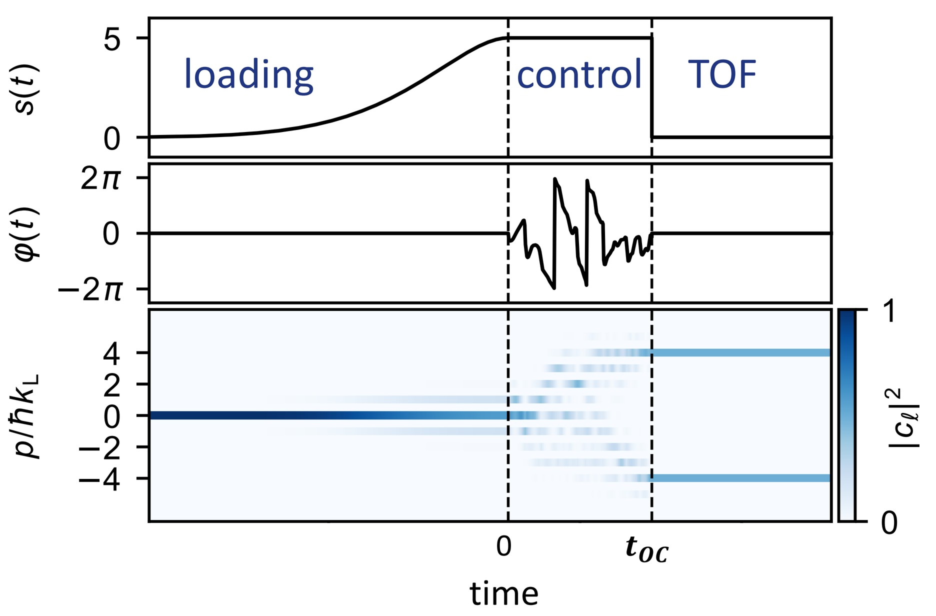
$$|\Psi\rangle = \sum_\ell c_\ell |\ell\rangle$$

$|\ell\rangle$: plane waves $\propto e^{i\ell k_L x}$
$c_\ell \in \mathbb{C}$ and $\sum_\ell |c_\ell|^2 = 1$

Measurement :
Momentum distribution $|c_\ell|^2$ after time-of-flight (TOF)

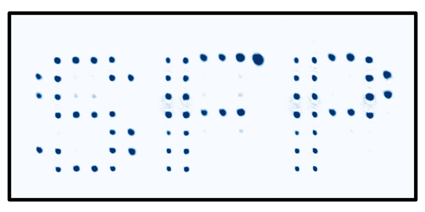


Protocol and first experiments [2]



- Control field : phase $\varphi(t)$
- Precise calibration of depth s

Control of momentum distribution



e.g. create all 128 equal-weights superpositions of 7 central orders

Full quantum state reconstruction [3]

Maximum likelihood algorithm

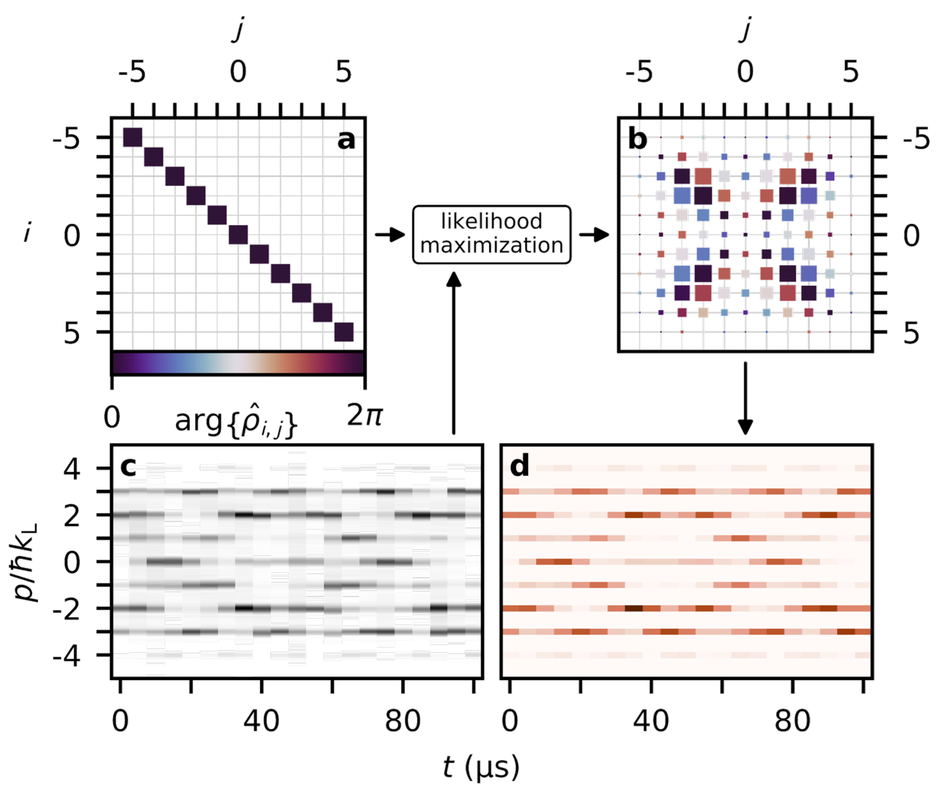
Iteratively find density matrix $\hat{\rho}_{\mathrm{ML}}$ maximizing likelihood

$$\mathcal{L}[\hat{\rho}] = \prod_j \pi_j^{f_j}$$

f_j = measured *frequencies*

π_j = theoretical *probabilities*

Use measurements of free evolution in static lattice after preparation



Quantum optimal control (QOC)

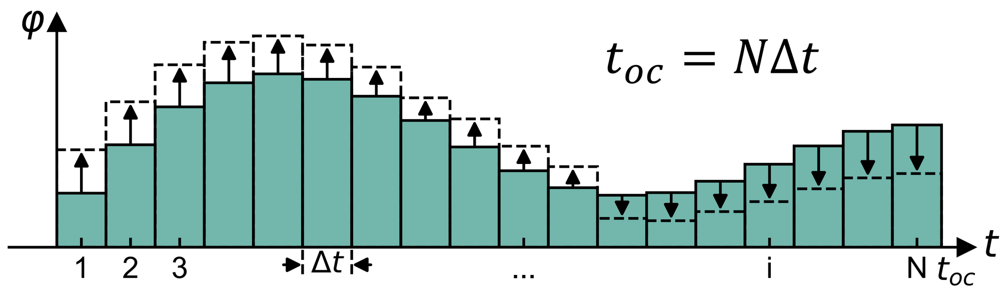
Goal : find the control field $\varphi(t)$ maximising the fidelity F to a target $|\Psi_T\rangle$

$$F = |\langle\Psi(t_{oc})|\Psi_T\rangle|^2$$

Schrödinger equation : $i\hbar|\dot{\Psi}\rangle = \mathcal{H}(\varphi(t))|\Psi\rangle$

Discretization of the field :

φ : piecewise constant φ_i over N time steps Δt



Pontryagin's Maximum Principle (PMP) :

Optimal field **maximizes** Pontryagin's Hamiltonian : $H_P(t) = Re[\langle p(t)|\dot{\Psi}(t)\rangle]$

$|p\rangle$ = adjoint state : - obeys the Schrödinger equation
- final condition $|p(t_{oc})\rangle = \frac{\partial F}{\partial\langle\Psi(t_{oc})|}$

Gradient Ascent algorithm to maximise H_P :

For 1 iteration, step i

$$\delta\varphi_i = \epsilon\frac{\partial H_P(i)}{\partial\varphi_i}$$

Correction factor > 0

(1) Initialize all $\{\varphi_i\}$
(2) Using $\mathcal{H}(\{\varphi_i\})$, evolve :
$|\Psi(0)\rangle \rightarrow |\Psi(t_{oc})\rangle$
$|p(t_{oc})\rangle \rightarrow |p(0)\rangle$
(3) Calculate corrections $\{\delta\varphi_i\}$ for all i.
(4) Calculate new fidelity F.
Loop until F is maximized

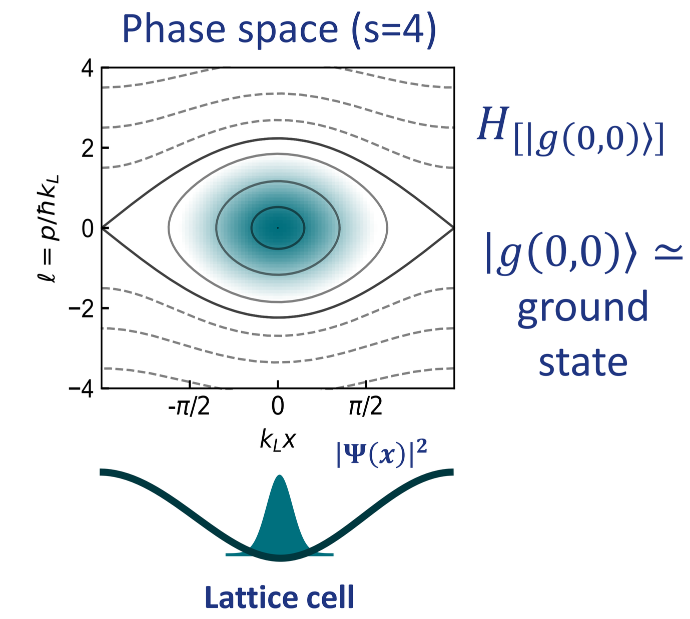
Control in phase space [3]

Gaussian states $|g(x_0,p_0)\rangle$ centered in (x_0,p_0) :

$$|g(x_0,p_0)\rangle = \mathcal{N}\sum_\ell e^{-i\ell x_0}\, e^{-(\ell-p_0)^2/\sqrt{s}}\, |\ell\rangle$$

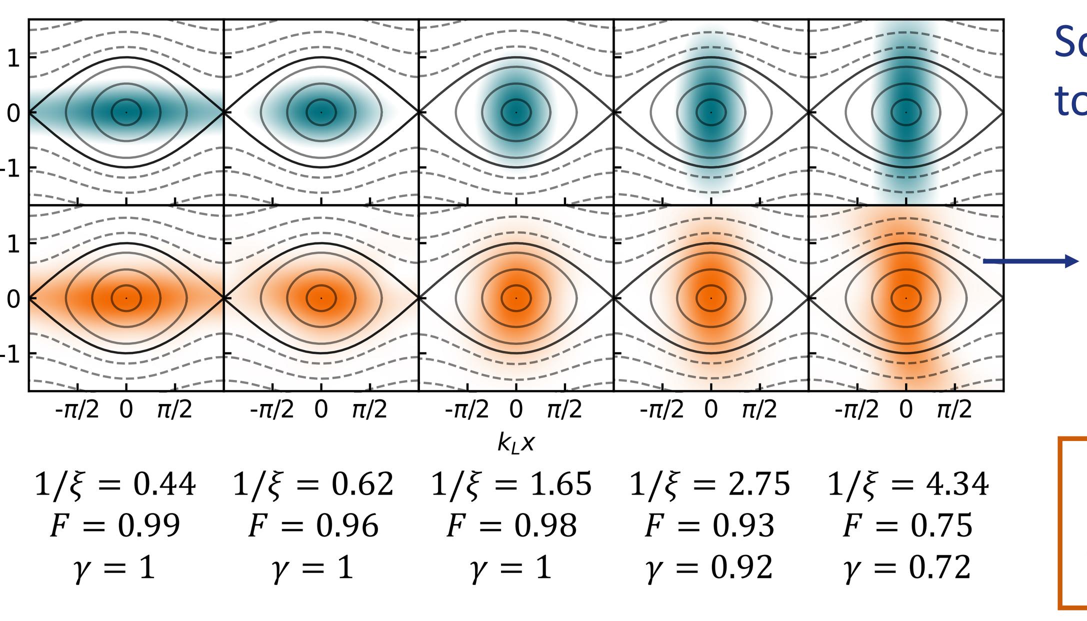
Husimi quasi-distribution :

$$H_{[\hat{\rho}]}(x_0,p_0) = \frac{1}{2\pi}\langle g(x_0,p_0)|\hat{\rho}|g(x_0,p_0)\rangle$$



Prepare and reconstruct squeezed states

Gaussian states with $\Delta x = \xi\Delta x_{\mathrm{ground}}$



$1/\xi = 0.44$	$1/\xi = 0.62$	$1/\xi = 1.65$	$1/\xi = 2.75$	$1/\xi = 4.34$
$F = 0.99$	$F = 0.96$	$F = 0.98$	$F = 0.93$	$F = 0.75$
$\gamma = 1$	$\gamma = 1$	$\gamma = 1$	$\gamma = 0.92$	$\gamma = 0.72$

Squeezing ξ at depth s equivalent to ground state at depth $s_{\mathrm{eff}} = s/\xi^4$

$s = 5.62 \pm 0.25$
$s_{eff} = 2000$

Reach non trivial states faster than adiabatic protocol

Fidelity $F = \langle\Psi_T|\hat{\rho}_{ML}|\Psi_T\rangle$ *Purity* $\gamma = \mathrm{tr}(\hat{\rho}_{ML}^2)$

$t_{OC} < 2T_0$, $T_0 \sim$ trap period

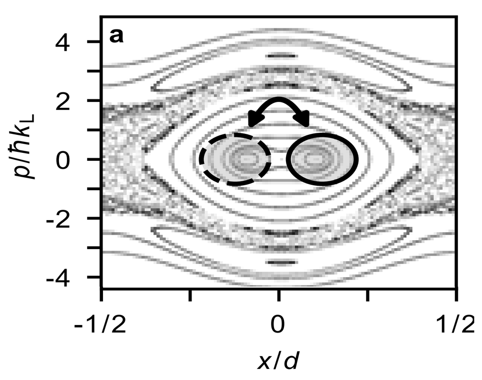
Application to quantum simulation [3]

Enhancement of dynamical tunneling signal

Amplitude-modulated lattice

$$V(x,t) = -\frac{s\,E_L}{2}(1+\varepsilon\cos(2\pi t/T))\cos(k_L x + \varphi(t))$$

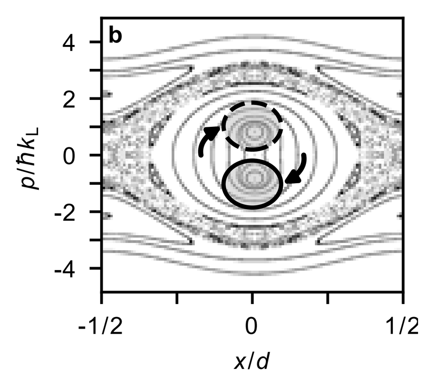
Stroboscopic phase space



Classical trajectories : ***chaotic*** or ***regular***
A particle observed every $2T$ stays on a regular island

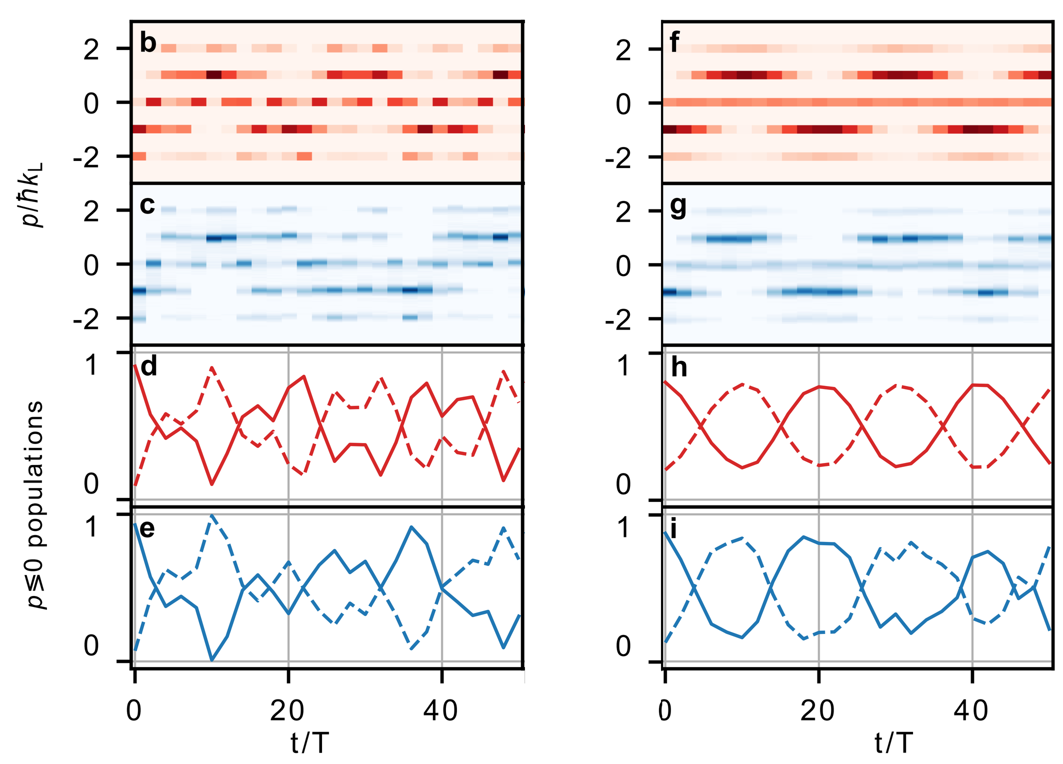
Quantum particles undergo dynamical tunneling **[4]**

Measurement



Rotation in phase-space exchange $x \leftrightarrow p$

"loading the island" is key (initial state)



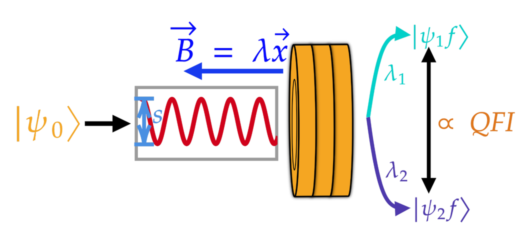
Initial state = shifted ground state

Initial state = optimal Floquet superposition

Outlook

- Extension to the optimization of a unitary transform/Floquet operator
- Quantum sensing :

Maximize separation of states for close values of a parameter *e.g.* magnetic field gradient λ



One approach : optimize Fisher information, quantum (QFI) or classical (CFI)

[1] Koch et al., *EPJ Quantum Technol.* 9, 19 (2022)
[2] Dupont et al., *PRX Quantum* 2, 040303 (2021)
[3] Dupont et al., *New J. Phys.* 25, 013012 (2023)
[4] Arnal et al. *Sci. Adv. 6 : eabc4886 (2020)*

LCAR Laboratoire Collisions Agrégats Réactivité
icb Laboratoire Interdisciplinaire Carnot de Bourgogne
FERMI cnrs Université Toulouse III Paul Sabatier Université de Toulouse anr nano-X