Measurement and control of the Hamiltonian of quantum computers

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If the Hamiltonian of a quantum computer can be engineered into another one, measuring the state of the qubits will give information on the Hamiltonian of interest. Being on a lattice, qubits can be used to mimic condensed matter problems [1]. The two requirements for analogue quantum simulation are the Hamiltonian learning problem, ie the Hamiltonian Tomography, and the control of the interactions between the qubits [2]. The Hamiltonian learning problem is also relevant in quantum computing to characterize the unwanted dynamics that are responsible for the quantum gates errors.

Hamiltonian Tomography

Knowing the interaction graph of a quantum computer, the Hamiltonian can be expanded into only one-body and two-body Pauli operators. Then a relation between observables and the Hamiltonian coefficients can be found [3]

$$\langle O(t)\rangle = \mathrm{Tr}(\rho(t)O)$$
$$i\hbar\frac{\partial\rho}{\partial t} = [H,\rho] := H\rho - \rho H$$
$$i\hbar\frac{\partial\langle O(t)\rangle}{\partial t}\Big|_{t=0} = \mathrm{Tr}\Big([H,\rho]\Big|_{t=0} O\Big)$$

$\leftarrow$

$$H = H_{single} + H_{double}$$
$$H_{single} = \sum_{i=1}^{N}\sum_{\Gamma_i = X,Y,Z} c_{\Gamma_i}\Gamma_i$$
$$H_{double} = \sum_{i=1}^{N-1}\sum_{\Gamma_i,\Gamma_{i+1}=X,Y,Z} d_{\Gamma_i,\Gamma i+1}\Gamma_i\otimes\Gamma_{i+1}$$

For the qubit site (i_0, i_0+1)

$$i\hbar\frac{\partial\langle O^{\sigma_{i_0},\sigma_{i_0+1}}(t)\rangle_{\rho^{\tau_{i_0},\tau_{i_0+1}}}}{\partial t}\Big|_{t=0} = \sum_{j=i_0}^{i_0+1}\sum_{\Gamma_j=X,Y,Z} c_{\Gamma_j}\,\mathrm{Tr}([\Gamma_j,\rho^{\tau_{i_0},\tau_{i_0+1}}]O^{\sigma_{i_0},\sigma_{i_0+1}})$$
$$+\sum_{\Gamma_{i_0}=X,Y,Z}\sum_{\Gamma_{i_0+1}=X,Y,Z} d_{\Gamma_{i_0},\Gamma_{i_0+1}}\,\mathrm{Tr}([\Gamma_{i_0}\otimes\Gamma_{i_0+1},\rho^{\tau_{i_0},\tau_{i_0+1}}]O^{\sigma_{i_0},\sigma_{i_0+1}})$$

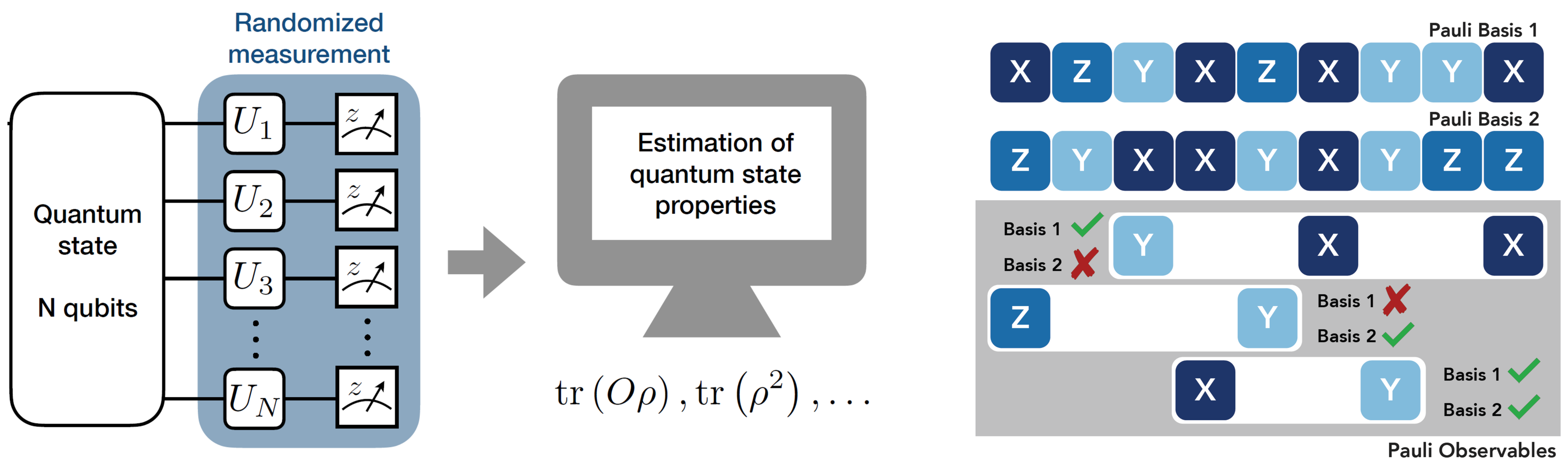
Gathering all the equations for many initial states ρ and all the possibles correlations O, a system can be build, with D a column vector of the derivatives of the correlations, which is something accessible experimentally, M a matrix of algebraic coefficients, that can be easily calculated numerically, and Γ is a column vector of the Hamiltonian coefficients. Solving this system will fully characterize the Hamiltonian of the qubit site (i_0, i_0+1).

$$D = M \times \Gamma$$

Measure | Calculate | Goal: Extract Γ

Efficient correlations measurement

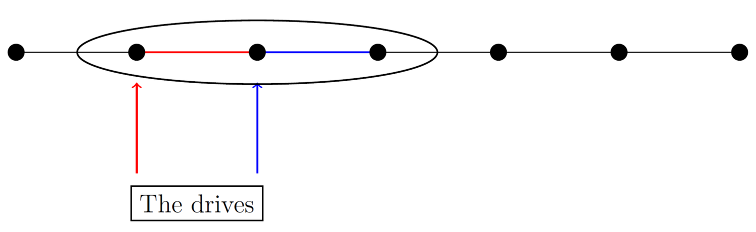
For a N qubit system, by applying N random local unitaries U, measurements can be paralleled to estimate correlations $\langle O\rangle$ efficiently. By instance, for the random sequence $XZYXZXYYX$ on qubit 1 to 9, $\langle Y_3X_6X_9\rangle$ can be estimated, as well as $\langle X_1Z_2\rangle$.



The randomised measurements toolbox: "Measure first, ask questions later" [6]

$$\langle\hat{O}^{(\text{estimated})}\rangle = \frac{\sum_r\sum_{\mathbf{s}}\delta_{U_A^{(r)},V_A}\hat{P}\left(\mathbf{s}\mid U^{(r)}\right)Z_A(\mathbf{s})}{\sum_r\delta_{U_A^{(r)},V_A}} \text{ if } \sum_r\delta_{U_A^{(r)},V_A}\geq 1$$
$$= 0 \text{ otherwise}$$

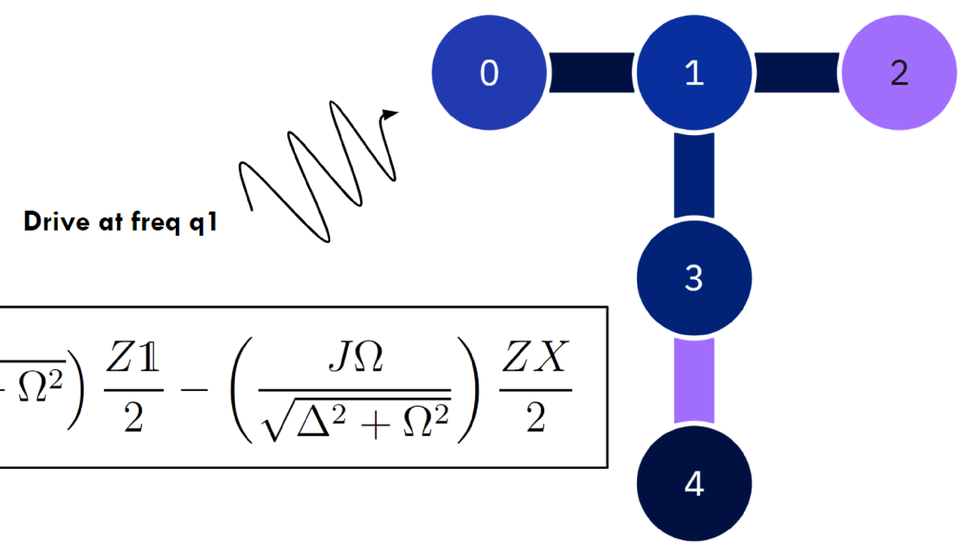
Conclusions and outlook



1. N-qubit Hamiltonians with two-body interactions of any quantum system can be characterised if the derivatives at $t = 0$ of the correlations are accessible experimentally.
2. Perform analogue quantum simulation and study the entanglement growth by measuring Rényi entropies [6].
3. The method can be extended to open quantum systems by deriving the Lindblad master equation [3].

Generating a cross-resonance (CR) interaction

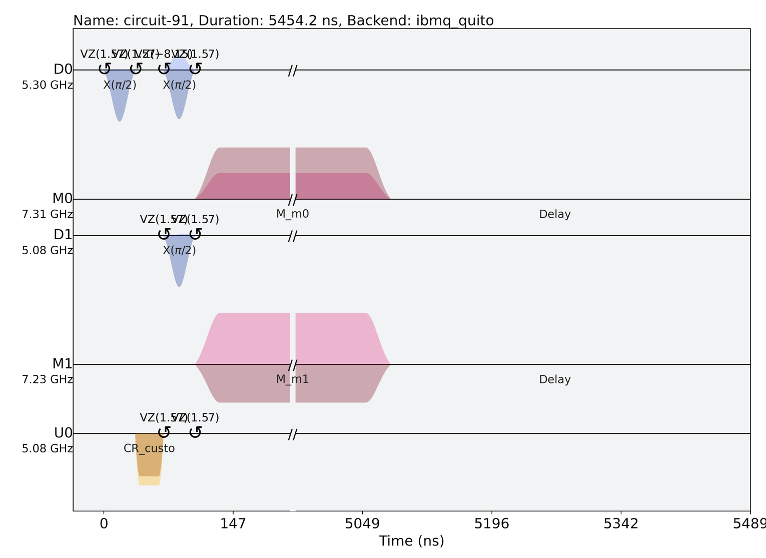
In transmon qubits, the CR technique is used to create entanglement between qubits [4]. By sending a microwave at frequency f_1 on q_0, q_0 will not absorb the excitation because of the detunning, rather, the latter will be transferred to the neighbor q_1, which is at f_1.

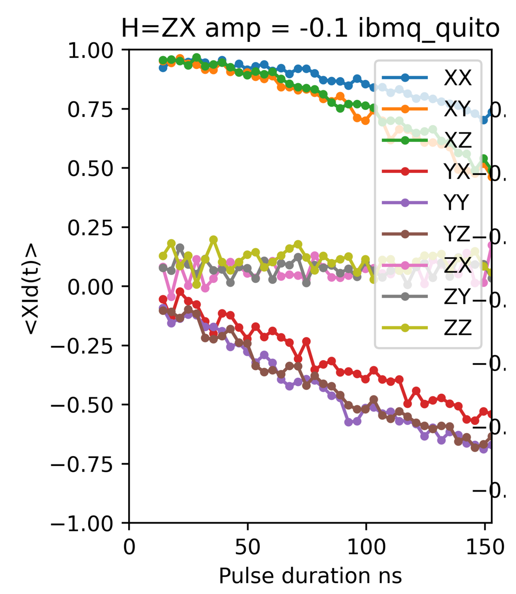


$$H_{\mathrm{CR}} = \left(\Delta - \sqrt{\Delta^2+\Omega^2}\right)\frac{Z\mathbb{1}}{2} - \left(\frac{J\Omega}{\sqrt{\Delta^2+\Omega^2}}\right)\frac{ZX}{2}$$

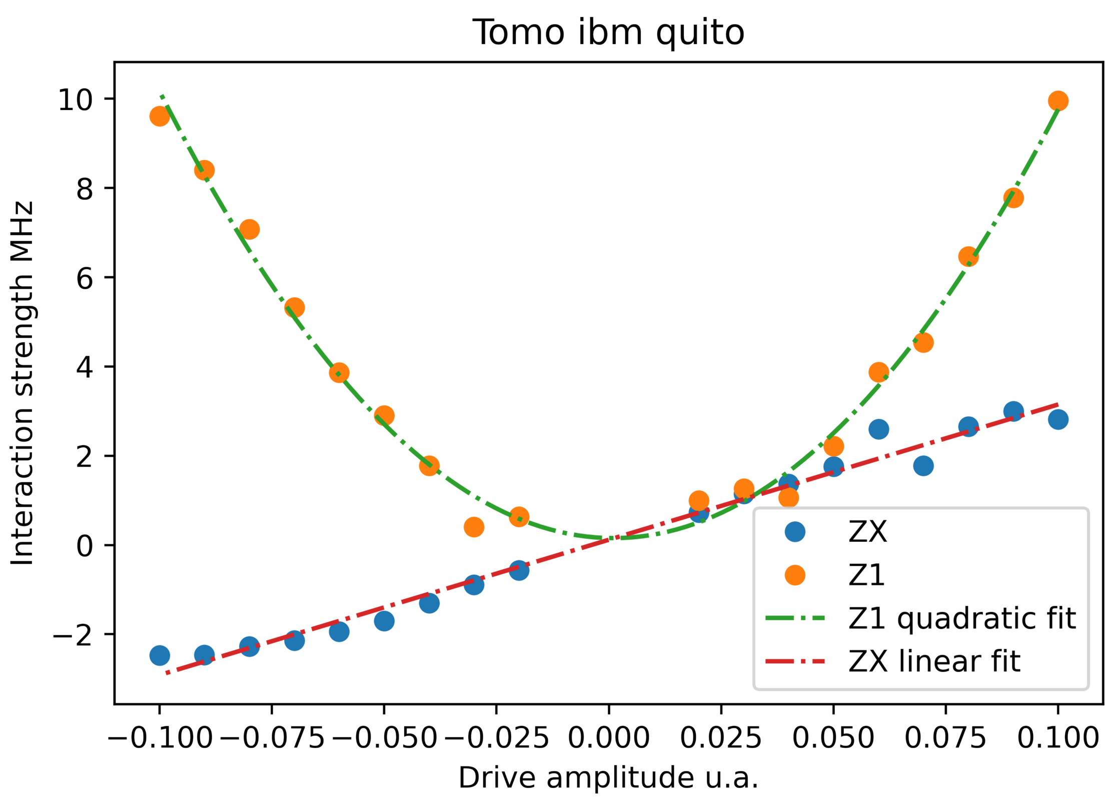
Results from ibmq quito

Via the ibmq quantum computing platform [5] quantum circuits can be run and tested on real devices, such as ibmq quito.





A CR micro-wave was send for many initials states, and all the Pauli operators measured. From the data, derivatives were estimated and the system of equation of the Hamiltonian Tomography technique was solved to determine the Hamiltonian. The Hamiltonian characterisation of the CR micro-wave was done for many amplitudes.



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