Cosmological structure formation with negative mass

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Antimatter and gravitation

• Why we see no antimatter in today’s universe?
  – Antimatter created in the primordial universe, but…
  – Observations exclude large amounts of antimatter in visible universe
    ➢ No signature of gamma rays produced during annihilations
  – CP violation cannot explain observed asymmetry

• Open questions on gravitation
  – Acceleration of the expansion of the universe (1998) – Dark Energy
  – Matter content of the universe – Dark Matter
  – Primordial inflation (~1980) – Inflaton field

• Gravitational behavior of antimatter
  – Current experiments at CERN: GBAR, ALPHA-g, AEGIS
Cosmological models

- **The standard model: ΛCDM**
  - $\Lambda \rightarrow$ dark energy (70%) $\rightarrow$ accelerated expansion ($\approx$ repulsive gravity!)
  - **CDM** (Cold Dark Matter): 25%
  - Ordinary matter (baryonic): 5%
  - Scale factor:
    - $a(t) \sim t^{2/3}$ (matter-dominated age)
    - $a(t) \sim e^{\Lambda t}$ ($\Lambda$-dominated age)

- **Dirac-Milne universe** (see: A. Benoit-Lévy and G. Chardin, A&A 537, A78 (2012))
  - Matter-antimatter symmetric universe
    - Repulsion between matter and antimatter (negative mass)
    - Antimatter spreads almost uniformly across the universe
  - Total matter content = 0 ($\Omega_M = 0$)
  - No cosmological constant ($\Omega_\Lambda = 0$); No need for inflationary phase
  - Scale factor: $a(t) \sim t$
Mass in Newtonian mechanics

- Active gravitational mass $m_a$:  \[ \Delta \phi = 4\pi G \rho = 4\pi G m_a n \]
- Passive gravitational mass $m_p$:  \[ F = -m_p \nabla \phi \]
- Inertial mass $m_i$:  \[ \mathbf{p} = m_i \mathbf{\dot{r}} \]
- Equation of motion:  \[ \ddot{\mathbf{r}} = -\left( \frac{m_p}{m_i} \right) \nabla \phi. \]

<table>
<thead>
<tr>
<th>matter</th>
<th>Active grav. mass</th>
<th>Passive grav. mass</th>
<th>Inertial mass</th>
</tr>
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<tbody>
<tr>
<td>matter</td>
<td>A (standard)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
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<td>B (antiplasma)</td>
<td>-</td>
<td>-</td>
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<tr>
<td>antimatter</td>
<td>C (Bondi)</td>
<td>-</td>
<td>+</td>
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<tr>
<td>antimatter</td>
<td>D (antiinertia)</td>
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**EP:** $m_p = m_i$
Mass in Newtonian mechanics

- Active gravitational mass $m_a$: $\Delta \phi = 4\pi G \rho = 4\pi G m_a n$
- Passive gravitational mass $m_p$: $F = -m_p \nabla \phi$
- Inertial mass $m_i$: $p = m_i \dot{r}$
- Equation of motion: $\ddot{r} = -(m_p/m_i) \nabla \phi$.

$$\text{EP: } m_p = m_i$$

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Bondi: runaway acceleration
However, the above scenarios are not suited to model the Dirac-Milne universe.

**Antiplasma:**
- Does not respect the EP
- Allows formation of negative mass structures

**Bondi:**
- Requires negative inertial mass to ensure energy conservation
- Unlikely features such as runaway acceleration

We need a generalization of Newtonian gravity for two particle species:

<table>
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<th>Type of matter</th>
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<th>Interaction</th>
</tr>
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<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>Attraction</td>
</tr>
<tr>
<td>−</td>
<td>−</td>
<td>Repulsion</td>
</tr>
<tr>
<td>−</td>
<td>+</td>
<td>Repulsion</td>
</tr>
<tr>
<td>+</td>
<td>−</td>
<td>Repulsion</td>
</tr>
</tbody>
</table>

- Antimatter spreads uniformly
- Matter coalesces into structures

Cannot be realized with a single Poisson’s equation:

\[
\Delta \phi_+ = 4\pi G m (n_+ - n_-), \\
\Delta \phi_- = 4\pi G m (n_+ - n_-)
\]
General matrix formalism

\[ \Phi = \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}, \quad n = \begin{pmatrix} n_+ \\ n_- \end{pmatrix}, \quad \hat{M} = \begin{pmatrix} M_{++} & M_{+-} \\ M_{-+} & M_{--} \end{pmatrix}, \quad M_{ij} = \pm 1 \]

Since \( M_{++} = 1 \), there are \( 2^3 = 8 \) possible cases (one trivial, with all elements = +1)

**Antiplasma**\hspace{2cm}**Bondi**\hspace{2cm}**Anti-inertia**\hspace{2cm}**Dirac-Milne**

\[ \hat{M}_{\text{ap}} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad \hat{M}_{\text{Bondi}} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}, \quad \hat{M}_{\text{ai}} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}, \quad \hat{M}_{\text{DM}} = \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \]

**Open question**: how to incorporate this approach into General Relativity

- Bimetric theory?
Expanding universe – Comoving coordinates

Equation of motion

\[ \frac{d^2 r}{dt^2} = E_r(r, t), \]

Scale factor

\[ r = a(t) \hat{r}, \]

\[ a(t) \propto e^{\Lambda t} \quad \Lambda CDM \text{ late times} \]

\[ a(t) \sim t^{2/3} \quad \text{Einstein–de Sitter (} \Lambda \text{CDM early times)} \]

\[ a(t) \sim t \quad \text{Dirac – Milne} \]
One-dimensional geometry
$\Lambda$CDM cosmology

### Density

- $z=116$.  
- $z=24$.  
- $z=4$.  
- $z=0$.  
- $z=-0.95$.

### Phase space

Now
Dirac-Milne cosmology

Density

Phase space

Density

Phase space

Now
Matter-density power spectrum

Dirac-Milne

$P(k) \ (h^{-3} \text{Mpc}^3)$

$z=0.9$
$z=0.0$
$z=0.9$
$z=9.9$
$z=34.1$
$z=108.0$

$K \ (h \text{Mpc}^{-1})$

$10^{-3}$
$10^{-2}$
$10^{-1}$
$1$
$10$

$10^{-6}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$

$\Lambda$CDM

$P(k) \ (h^{-3} \text{Mpc}^3)$

$z=-0.95$
$z=0.0$
$z=4.4$
$z=24.0$
$z=116.0$
$z=1080.0$

$K \ (h \text{Mpc}^{-1})$

$10^{-3}$
$10^{-2}$
$10^{-1}$
$1$
$10$

$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$

$P_{\text{peak}} \ (h^{-3} \text{Mpc}^3)$

$K_{\text{peak}} \ (h \text{Mpc}^{-1})$

$a$

$10^{-3}$
$10^{-2}$
$10^{-1}$
$1$
$10$

$10^{-2}$
$10^{-1}$
$1$

$\Lambda$CDM

$\text{Dirac-Milne}$
Matter-density power spectrum

Two-component Dirac-Milne system
3D simulations and dark matter/MOND

RAMSES code simulation

Faber-Jackson relation

\[ m \propto v^\alpha, \alpha \approx 2.6 \]
3D simulations and dark matter/MOND

\[
\begin{align*}
\nabla^2 \phi_+ &= 4\pi G (\rho_+ - \rho_-), \\
\nabla^2 \phi_- &= 4\pi G (-\rho_+ - \rho_-).
\end{align*}
\]

\[\rho_-(r) = \rho_0 \exp \left( \frac{-m\phi_- + \mu}{k_B T} \right),\]

![Graph showing density profile](image)

![Graph showing potential profile](image)
Conclusions

• **Newtonian gravity with negative mass**
  – Standard cases with various choices of $m_i$, $m_a$, $m_p$ (Bondi, antiplasma, …)
  – Alternative “bimetric” theories → Dirac-Milne

• **Cosmological structure formation with negative mass**
  – Comparison between $\Lambda$CDM and Dirac-Milne
  – In Dirac-Milne universe, structure formation begins at an earlier epoch and freezes before $\approx 10^{10}$ Gy
  – Present power spectra are qualitatively similar to $\Lambda$CDM

• **Local MOND-like behavior**
  – 3D simulations show depletion zone and antimatter halos
  – Compatible with Faber-Jackson relation with exponent 2.6
  – Flattening of rotation curves

