Experimental Signatures of Higher Order Topology

Mesoscopic Physics group (LPS, Université Paris-Saclay)

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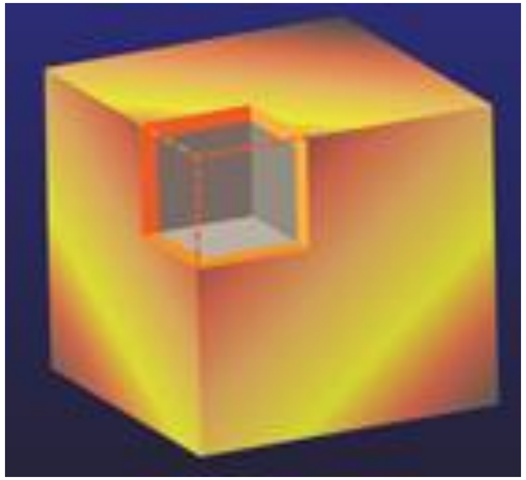
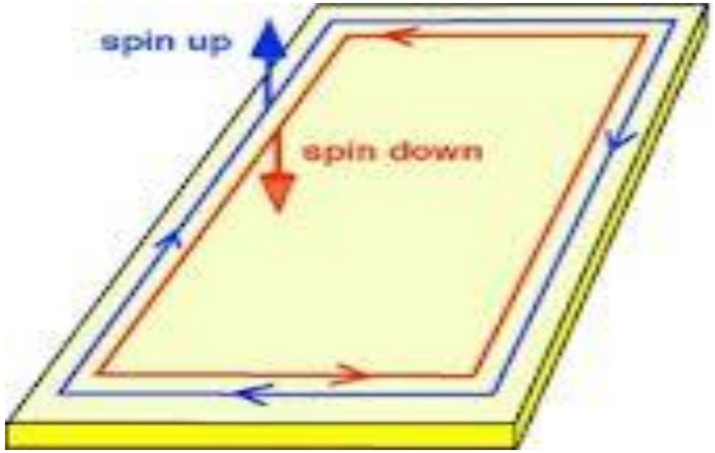
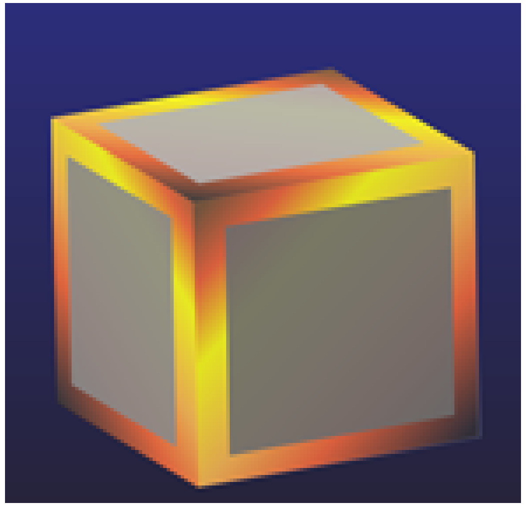
SFP 2023 - Paris

Higher order Topological Insulators







3D topological insulator

3D insulating bulk

2D Conducting surfaces

2D topological insulator

2D insulating bulk

1D conducting edges

Second Order Topological Insulator

3D insulating bulk

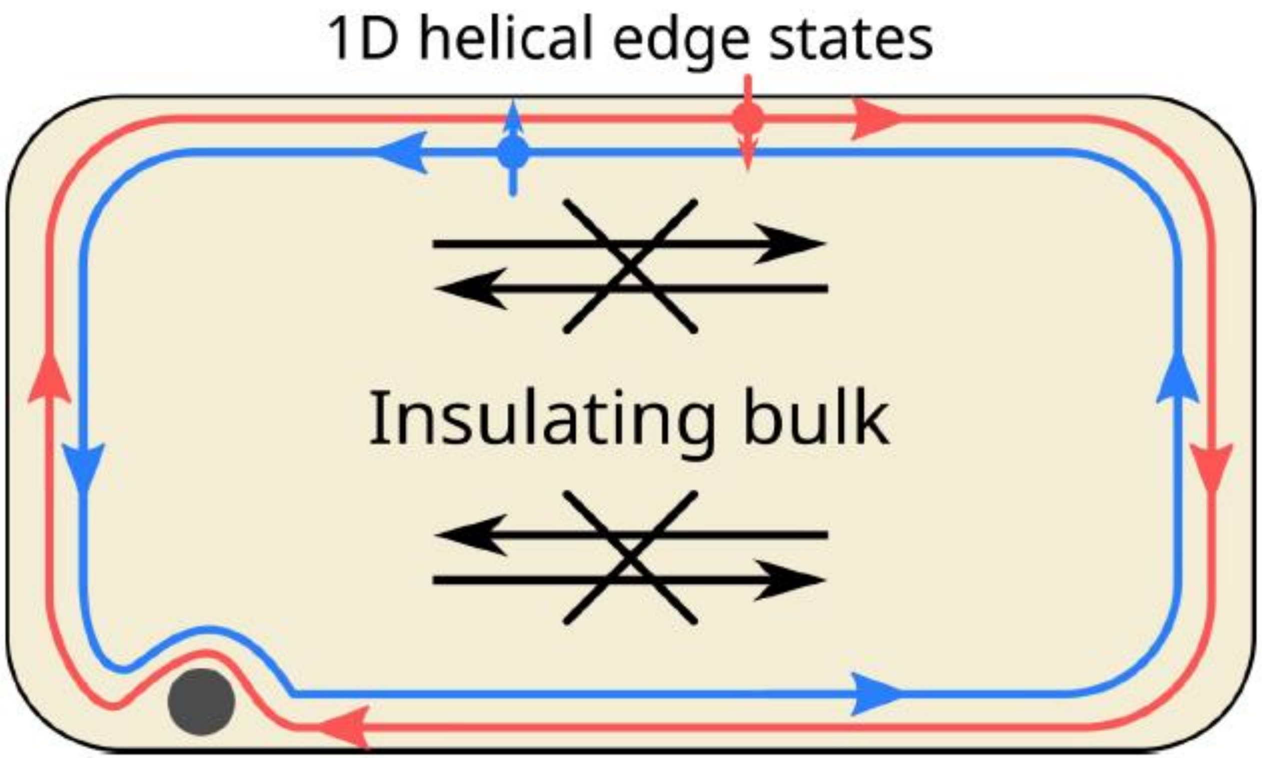
2D insulating surfaces

1D conducting helical « hinges »

Gapless surface state protected by bulk band topology

Schindler *et al*, Sci. Adv. (2018)
Geier *et al*, Phys. Rev. B (2018)
Xie *et al*, Nat. Rev. Phys. (2021)

Topological protection of 1D helical edge states



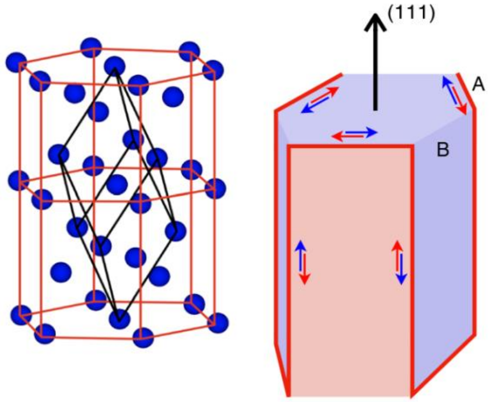
Spin-momentum locking
Correlation of spin and momentum

Topological protection
Ballistic transport, topological superconductivity

Experimental evidence of higher order topology in condensed matter

Bismuth : STM : Drozdov *et al,* Nature Phys (2014)

and Josephson junctions (this talk)



WTe_2 : - monolayer : quantum spin hall

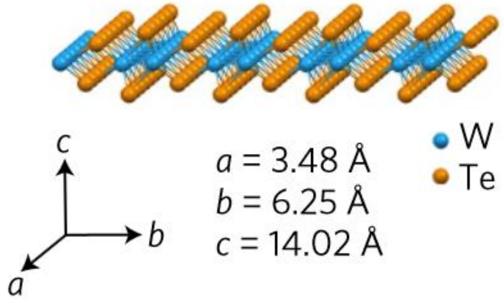
Wu *et al,* Science (2018), Fei *et al,* Nature Phys (2017)

- multilayer : SOTI, ballistic hinge states

Josephson interferometry (Choi *et al,* Nat. Mater. (2020))

Current phase relation measurement (M. Endres *et al,* arXiv:2211.10273)

L. Bugaud's poster (today 18h30)

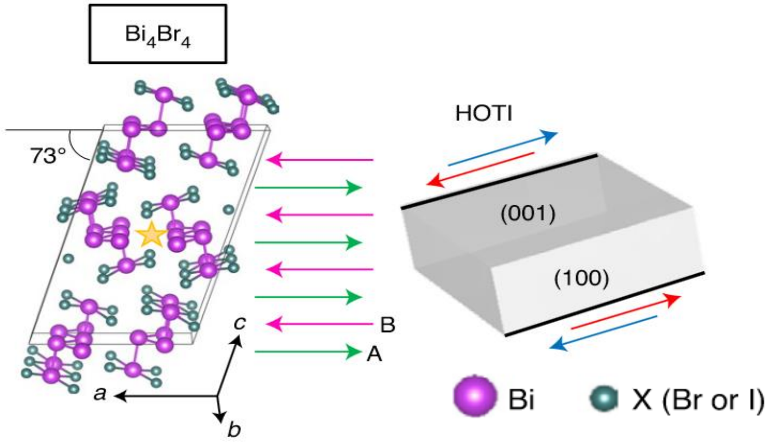


Bi_4Br_4 : SOTI with a high band gap (200meV)

ARPES : R. Noguchi *et al,* Nature Materials (2021)

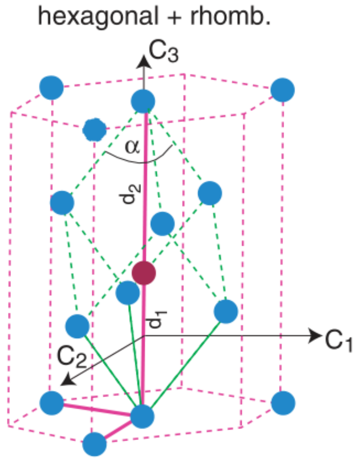
STM : N. Shumiya *et al,* Nat. Mater. (2022)

J. Lefeuvre's Talk (this morning)



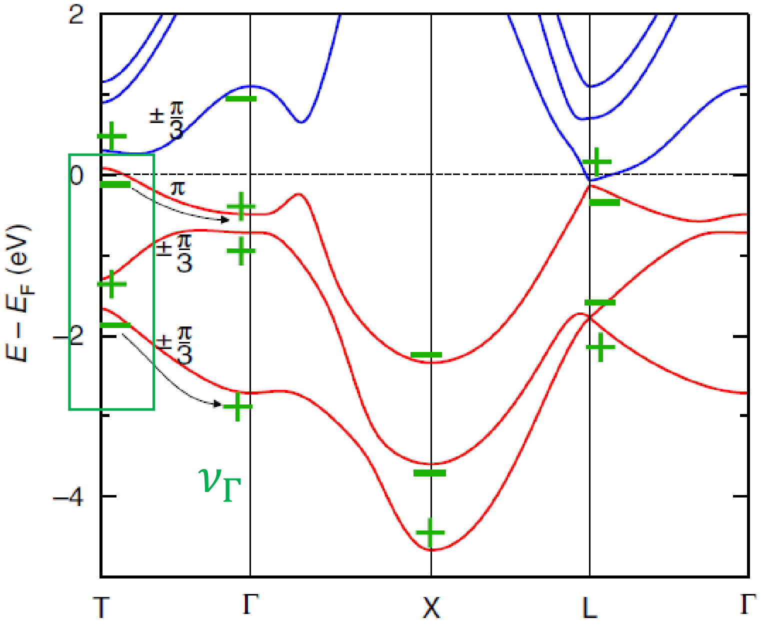
Bismuth, a Second Order Topological Insulator ?

Crystal structure

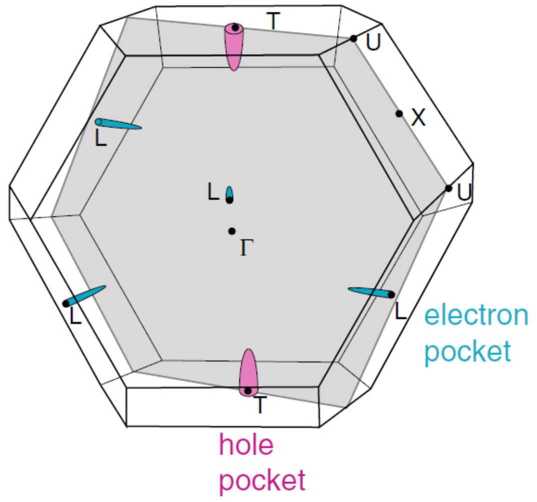


Bi has inversion (I) and
3-fold rotation symmetries (C_3)
+TRS

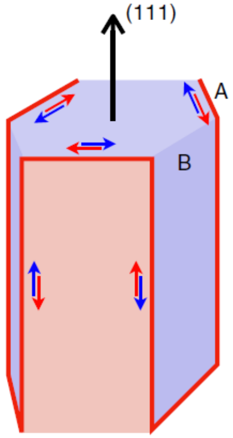
Band structure



Brillouin zone and Fermi surfaces



Bulk single-crystal Bi is not an insulator
but a semi-metal



topologically-protected
helical hinge states

diffusive semi-metallic
bulk states ($\lambda_{F,B} \sim 50\ nm$)

diffusive metallic
surface states ($\lambda_{F,S} \sim 5\ nm$)

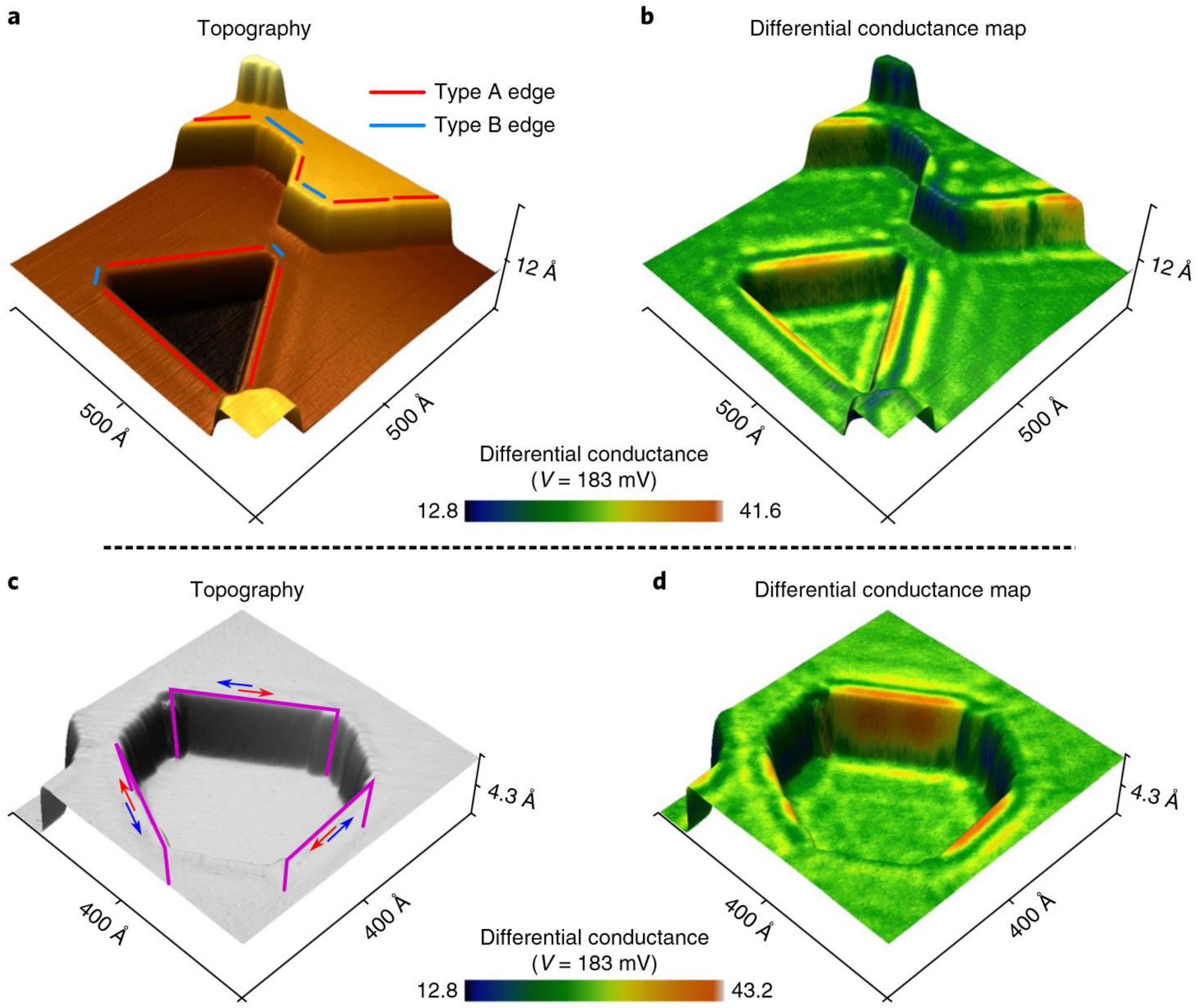
Even number of band inversions: $\Longrightarrow$ not a topological insulator… to first order

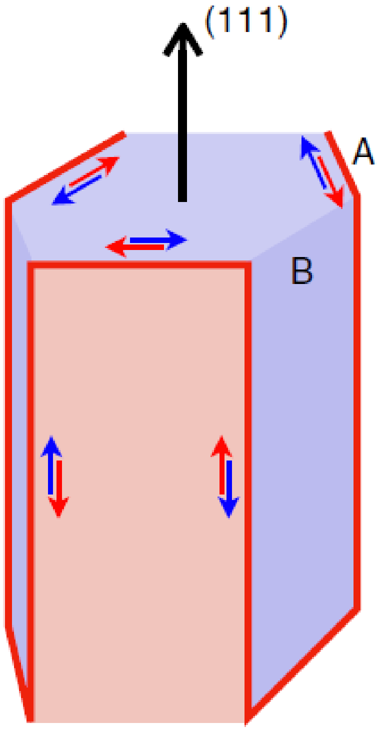
Odd number of band inversions in each C_3-subspace $\Longrightarrow$ HOTI!

Schindler *et al*, Nat. Phys. (2018)

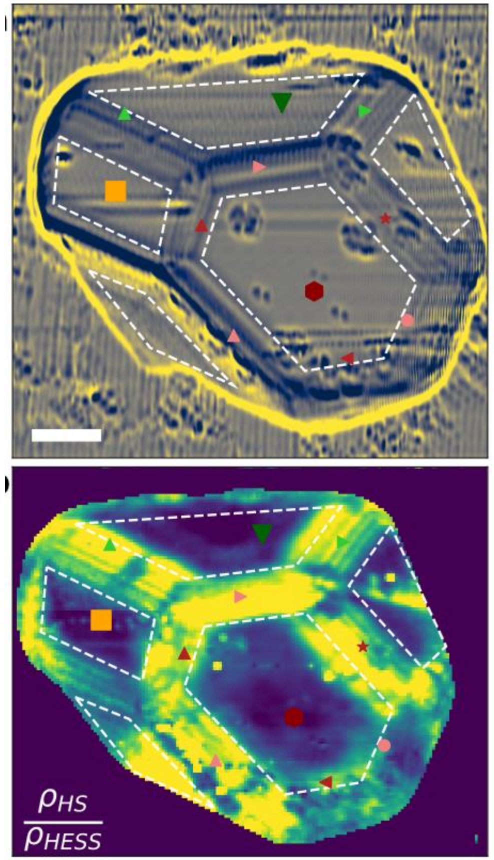
STM probe of edge states in Bismuth

Drozdov *et al,* Nature Phys (2014)





Zhang *et al*, arXiv:2303.06722

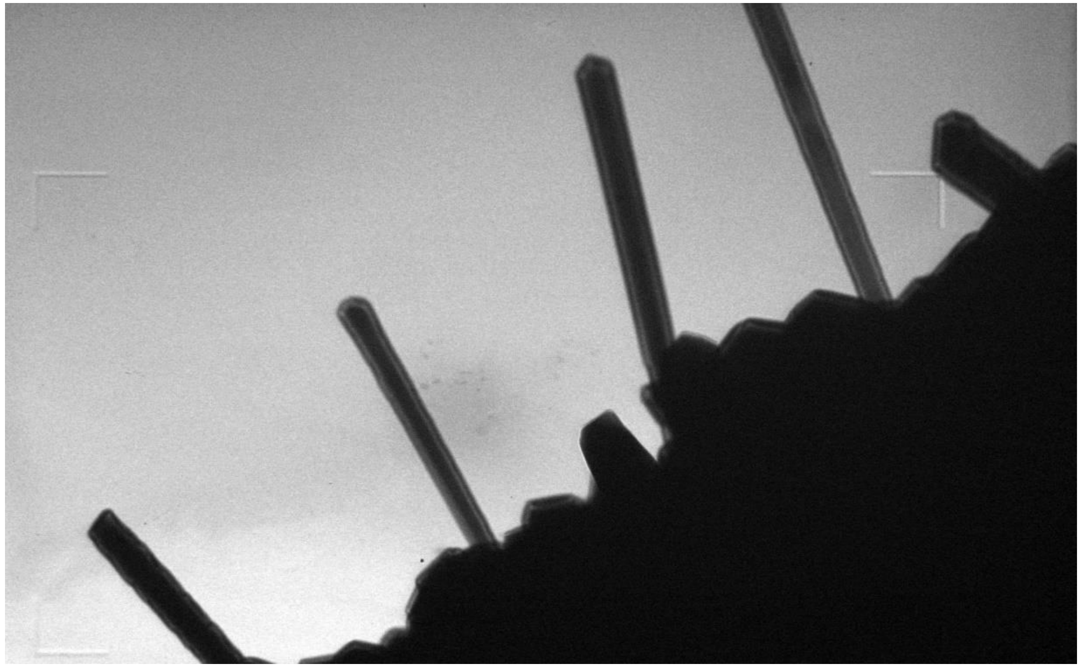


What about transport experiment ?

Monocrystalline Bismuth nanowires

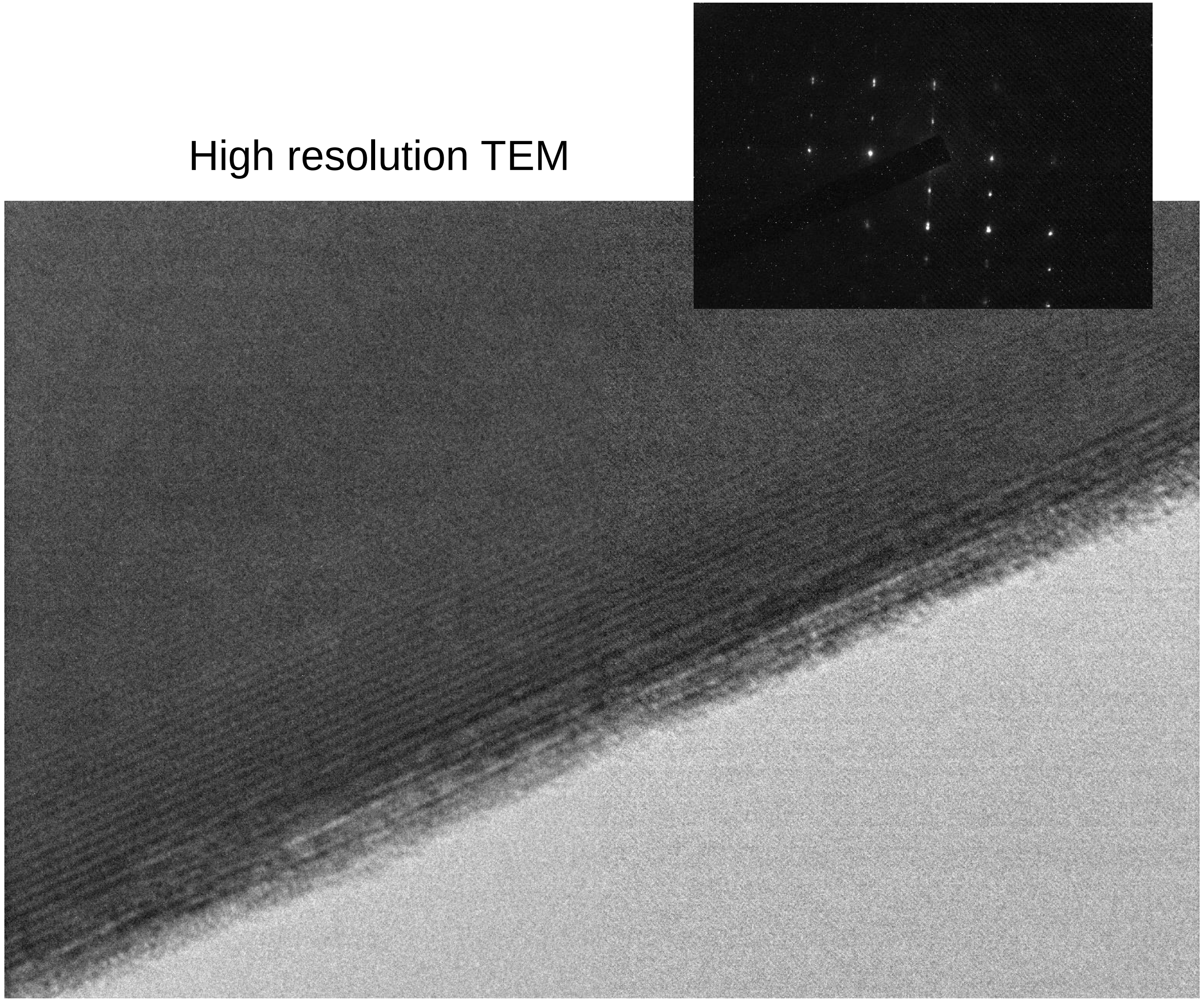
High quality single crystals
Sputtering, buffer layer of Fe or V
(A. Kasumov)

Diameter ~ 100-200 nm



Low magnification,
Transmission Electron Microscope

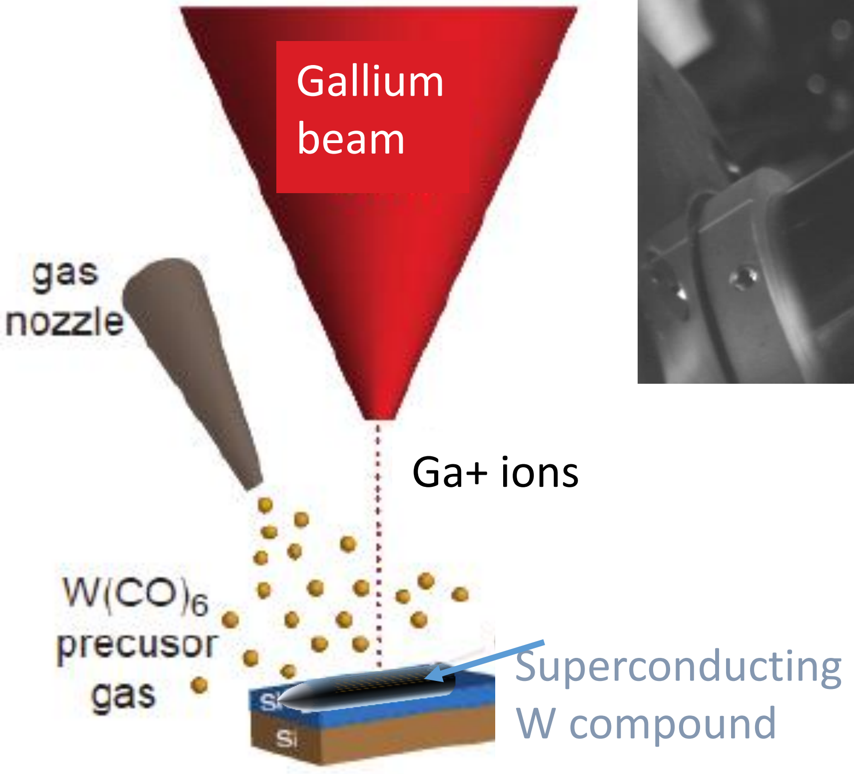
High resolution TEM

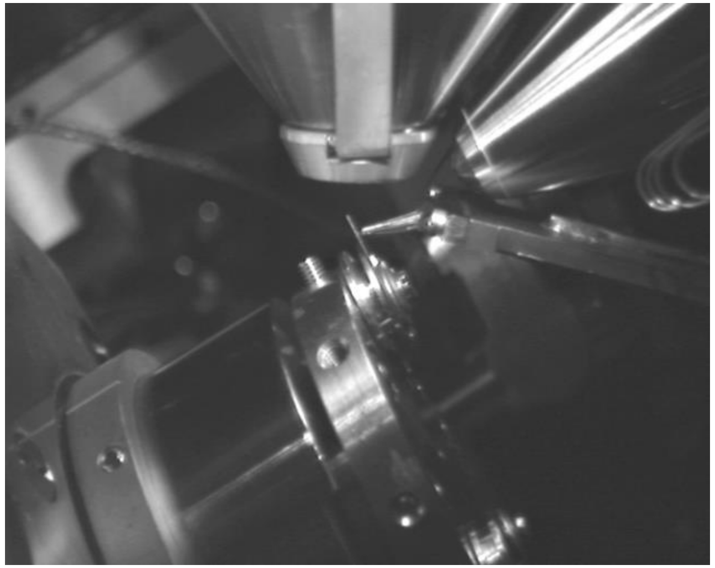


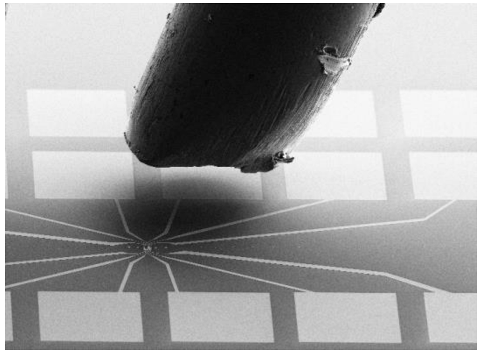
Nanowires with superconducting FIB contacts

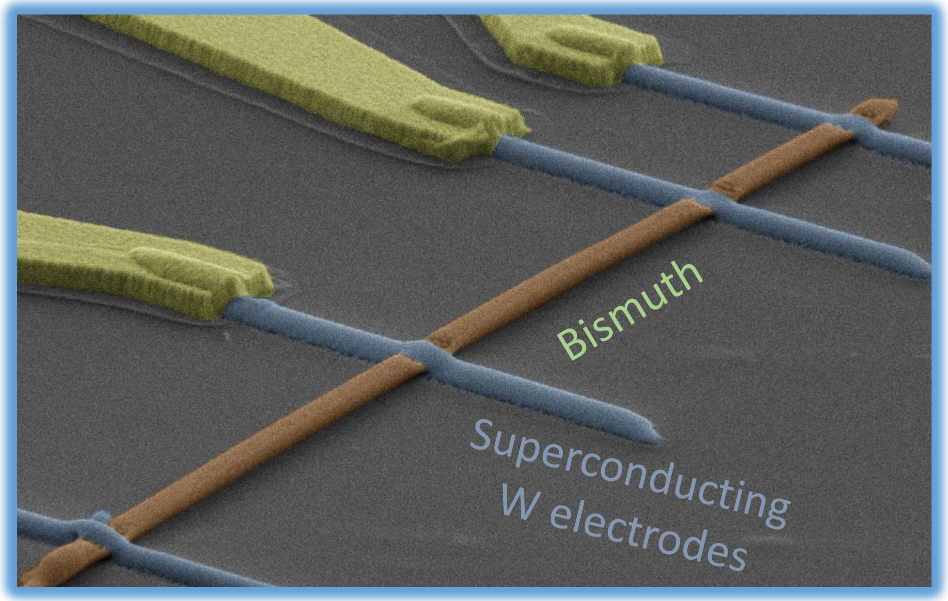
Kasumov 2005

focused-ion-beam (FIB) assisted deposition







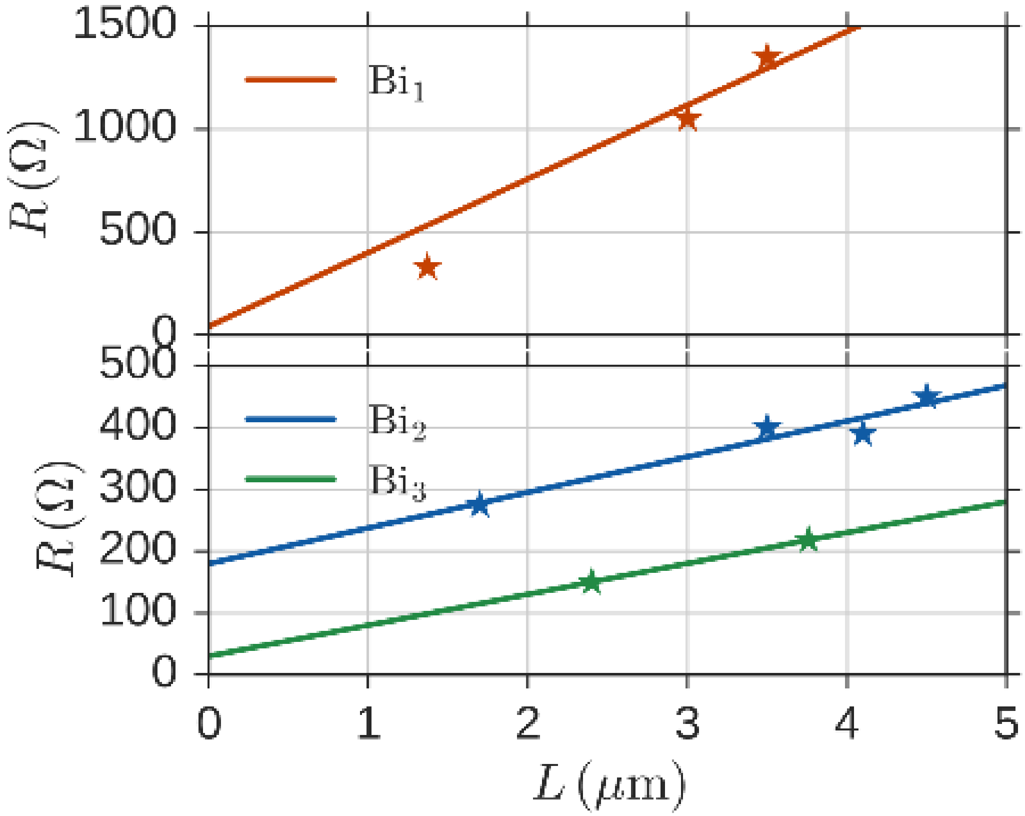


Superconducting electrodes: C and Ga-doped amorphous tungsten 200 nm thick and wide

Great superconducting properties: T_c~4 K, Δ~0.8 meV, H_c~12 Tesla!

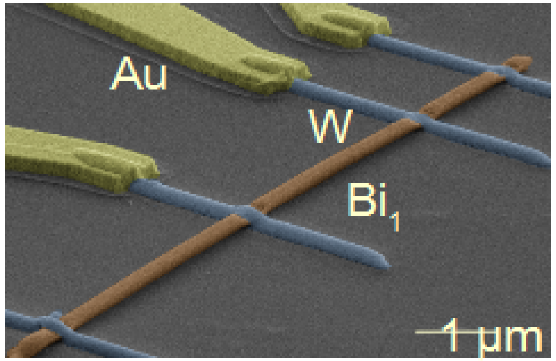
Transport in normal state : dominated by surface states

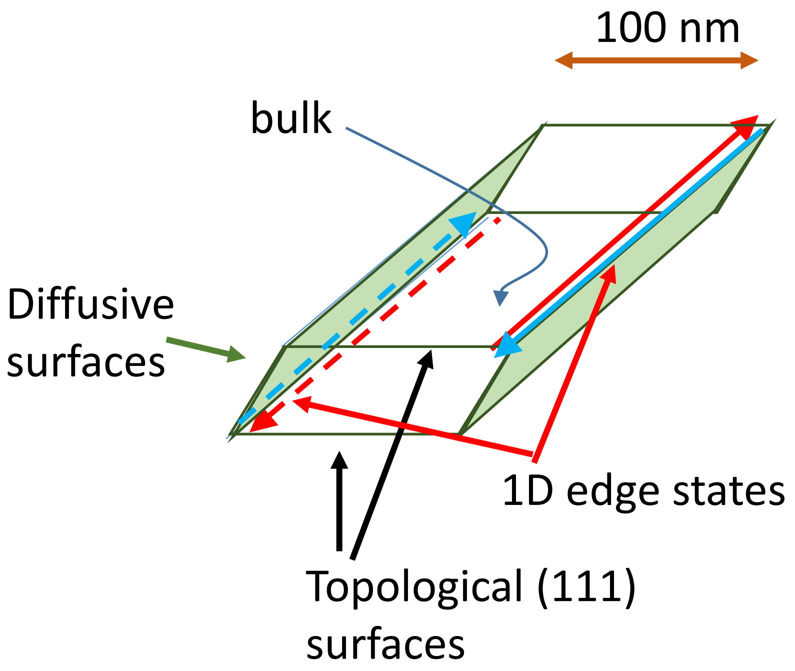
Bulk $\lambda_F \simeq 50\,\mathrm{nm}$
Surface $\lambda_F \simeq 5\,\mathrm{nm}$
} Roughly 50 times more surface states than bulk states



$$R(L) = R_c + \frac{R_Q}{M}\frac{L}{l_e}$$

Thus $l_e \lesssim 200\,\mathrm{nm}$

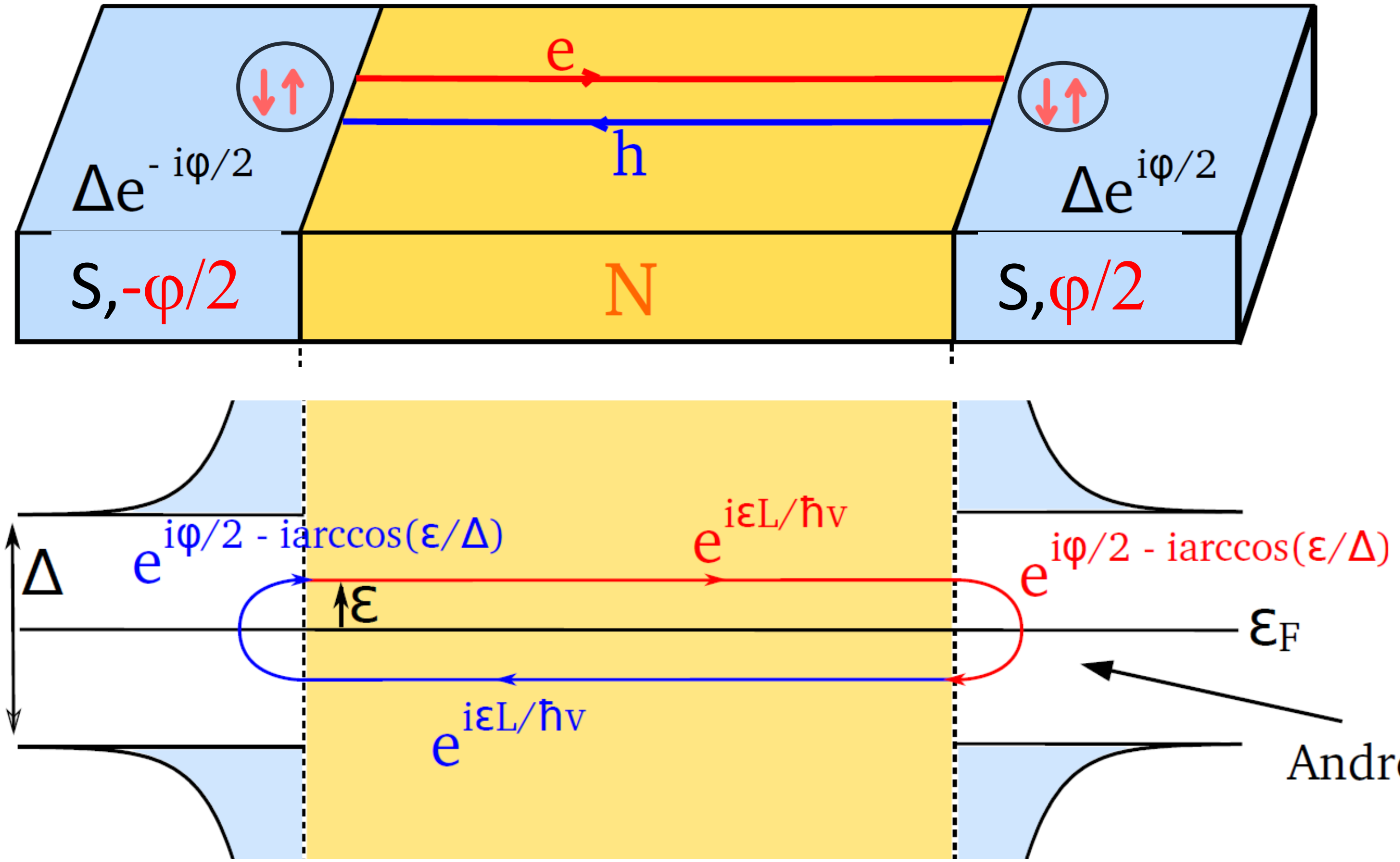




Diffusive surfaces states carry the normal current

Probe supercurrent to enhance visibility of ballistic/topological states

Andreev Bound States in a phase-biased SNS junction



Resonance condition on accumulated phase : Andreev Bound States with eigenenergies ε_n.

$$\frac{2\epsilon L_N}{\hbar v_F} - 2\,arccos\frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

Propagation Through N

Interface reflection

Superconducting phase difference

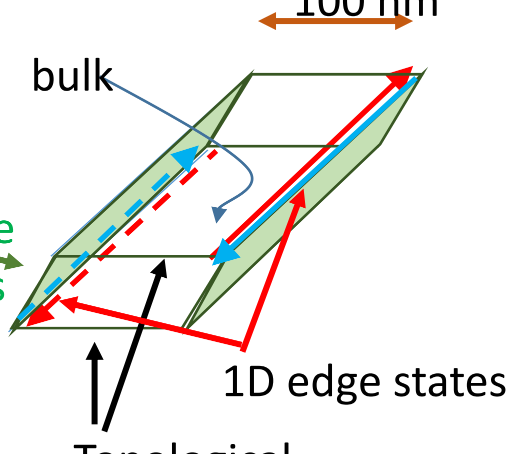
Andreev bound states with phase dependent energy levels

SUPERCURRENT $I = \sum_{-\infty}^{0} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$

Induced superconductivity enhances contribution of helical states

- Critical current carried by diffusive states is much smaller than critical current carried by ballistic/helical states

~ 6 ballistic edge channels, ~ 100 diffusive surface channels, elastic mean free path l_e

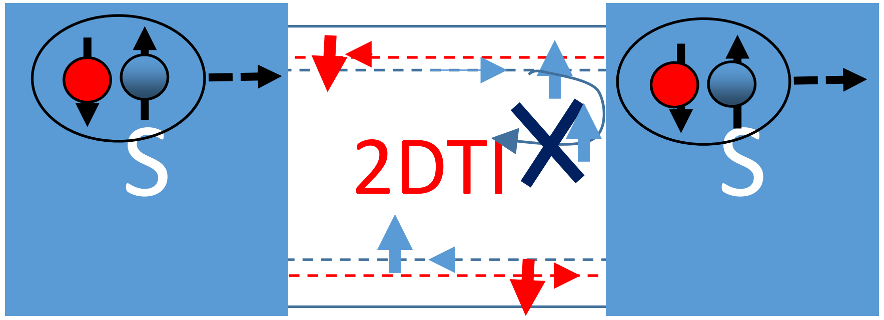


$$I_{c\ 1channel,\ ballistic} \sim \min\left(\Delta, \frac{hv_F}{L}\right)\frac{h}{e^2}$$

$$I_{c\ 1channel,\ diffusive} \sim \min\left(\Delta, \frac{hv_F}{L}\right)\frac{h}{e^2}\frac{l_e^{\,2}}{L^2}$$

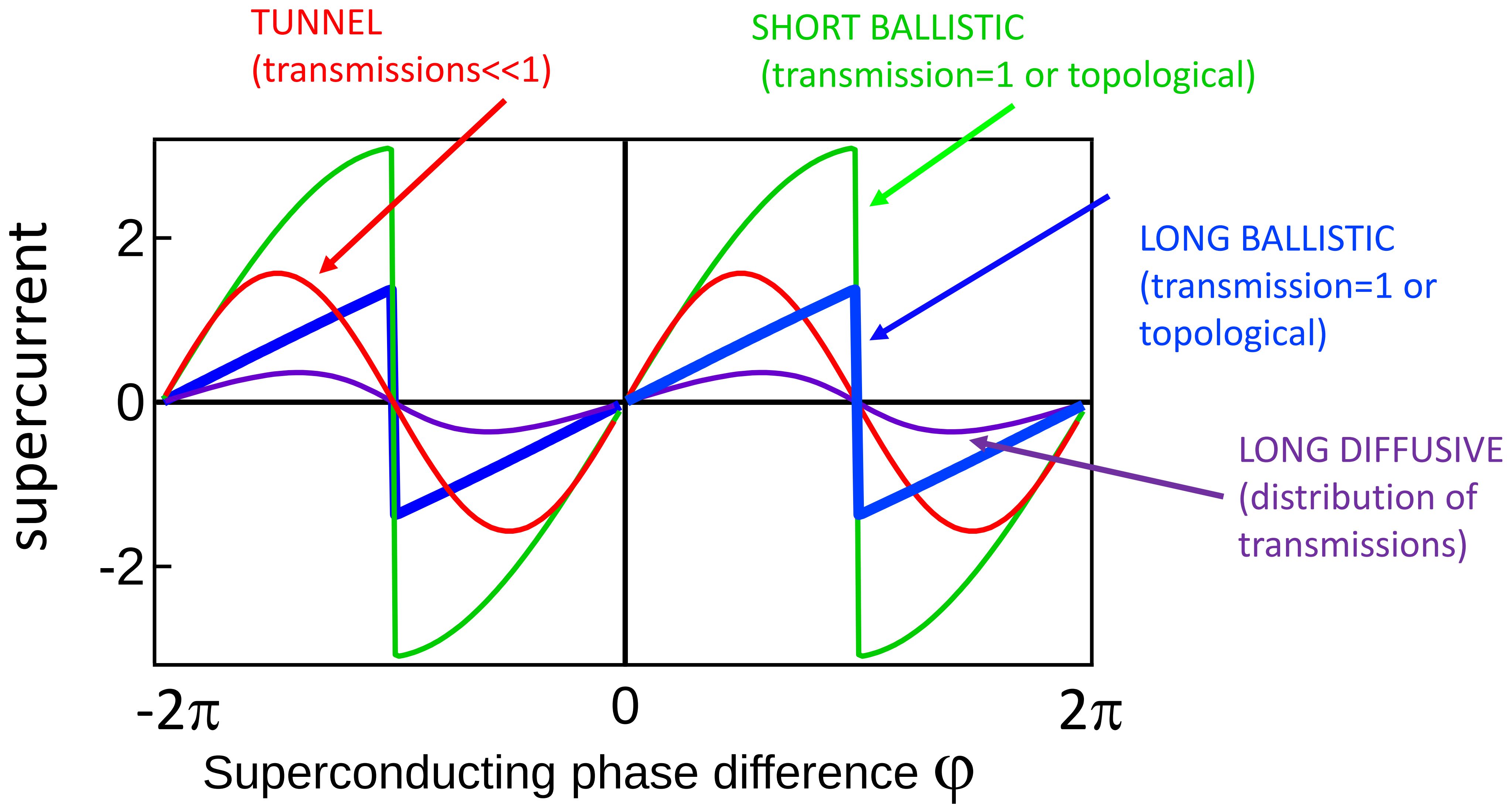
100 to 1000 times smaller than ballisitic

- In addition, helical channels should have perfect transmission into S (not true of diffusive channels)



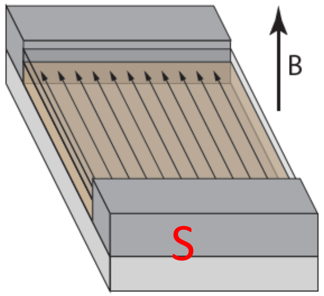
Supercurrent mainly determined by the helical edge states

Supercurrent to probe the nature of the normal part



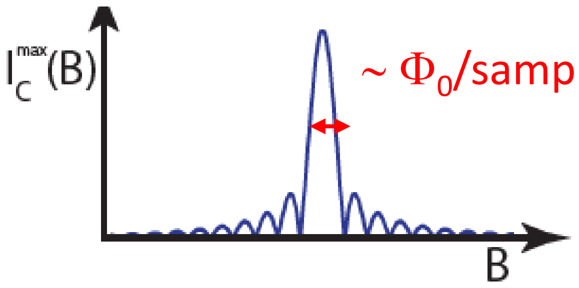
- Amplitude of the current phase relation : critical current
- Measure the current-phase relation?

Josephson interferometry : supercurrent distribution



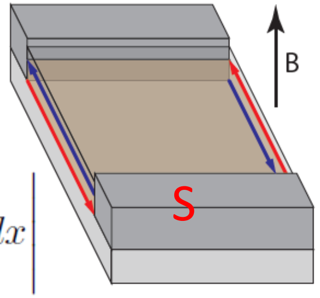
Many paths WIDE
(ballistic or diffusive)

$$I_c(B) = \left| \int_{-W/2}^{W/2} J(x) \cdot e^{2\pi i L B x/\Phi_0} dx \right|$$

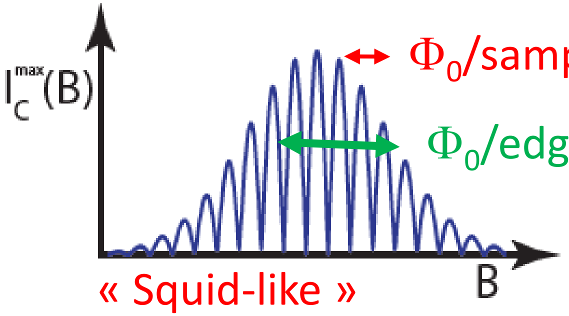


~ Φ_0/sample area

« Fraunhoffer pattern »



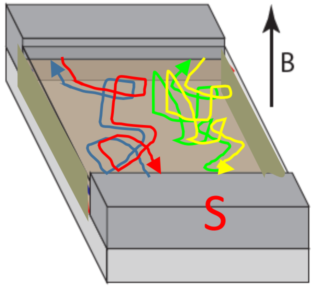
Only 2 paths (edges)
ballistic



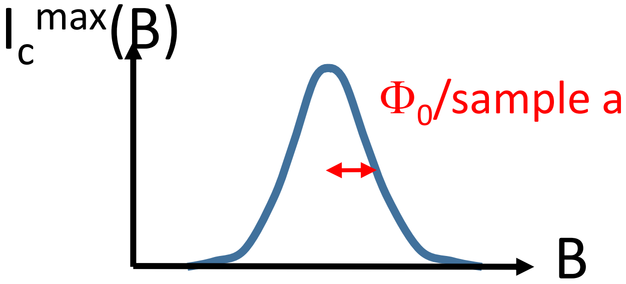
Φ_0/sample area

Φ_0/edge state area

« Squid-like »



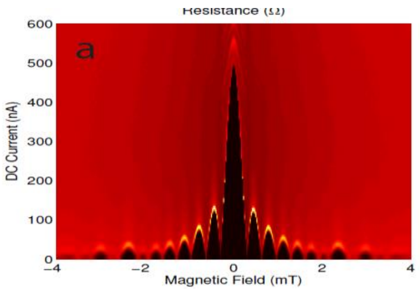
Many paths
NARROW,
diffusive



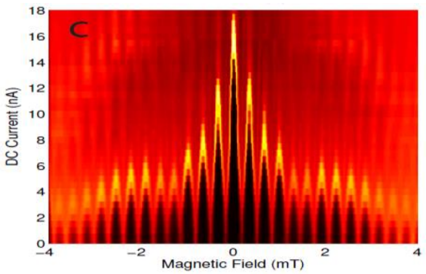
Φ_0/sample area

Many diffusive paths
Gaussian decay

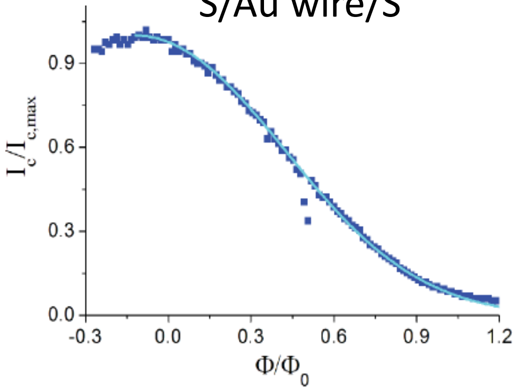
S/Non topo HgTe QW/S,
Hart *et al*, Nat. Phys. (2014)



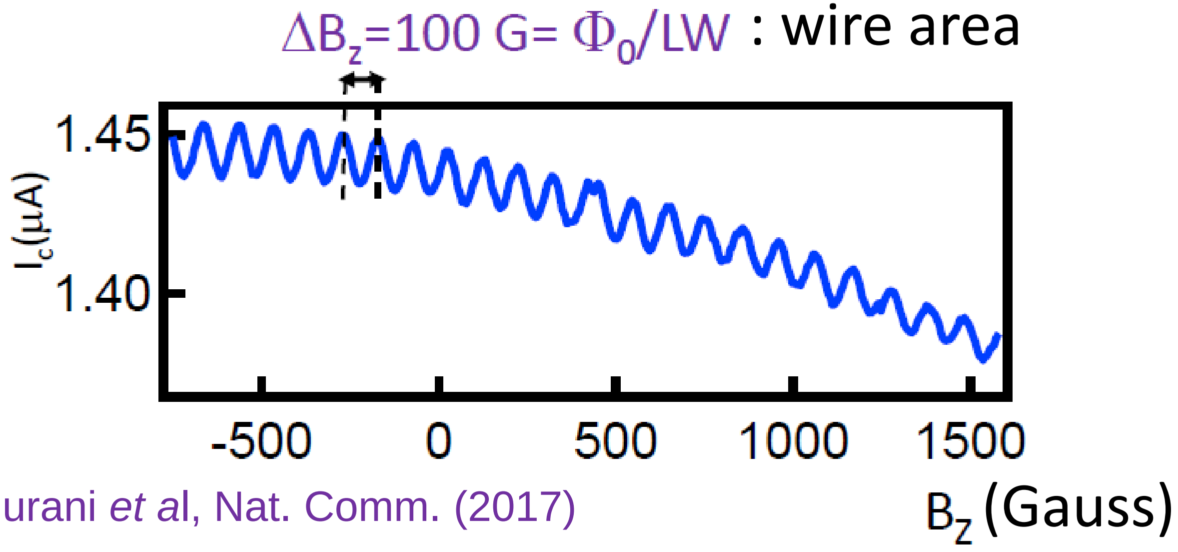
S/Topo/S HgTe QW,
Quantum Spin Hall
Hart *et al*, Nat. Phys. (2014)



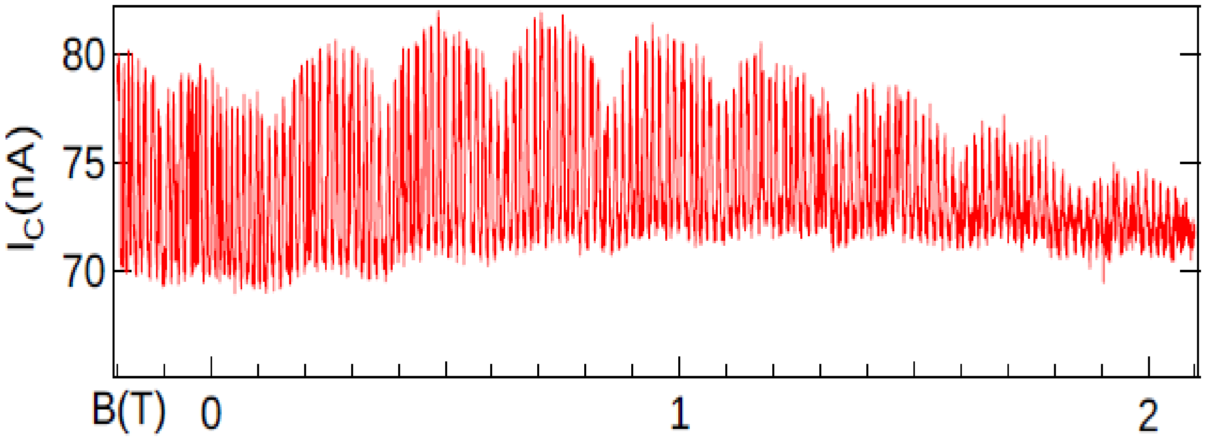
Chiodi *et al*, PRB (2012)
S/Au wire/S



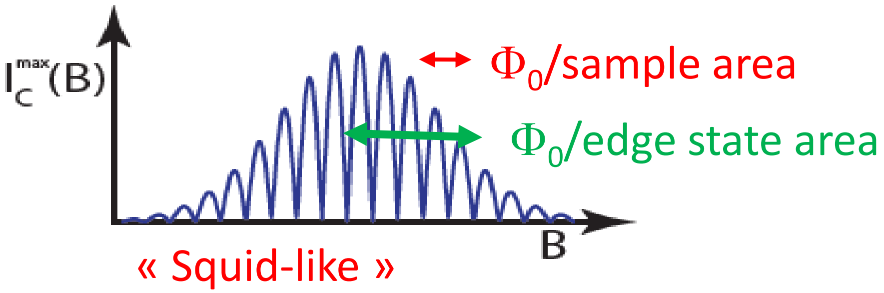
SQUID like behaviour in bismuth nanowires

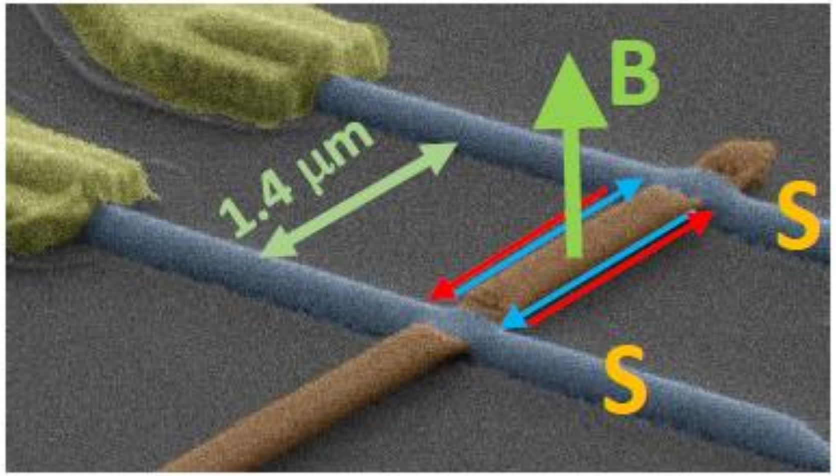


Murani *et al*, Nat. Comm. (2017)



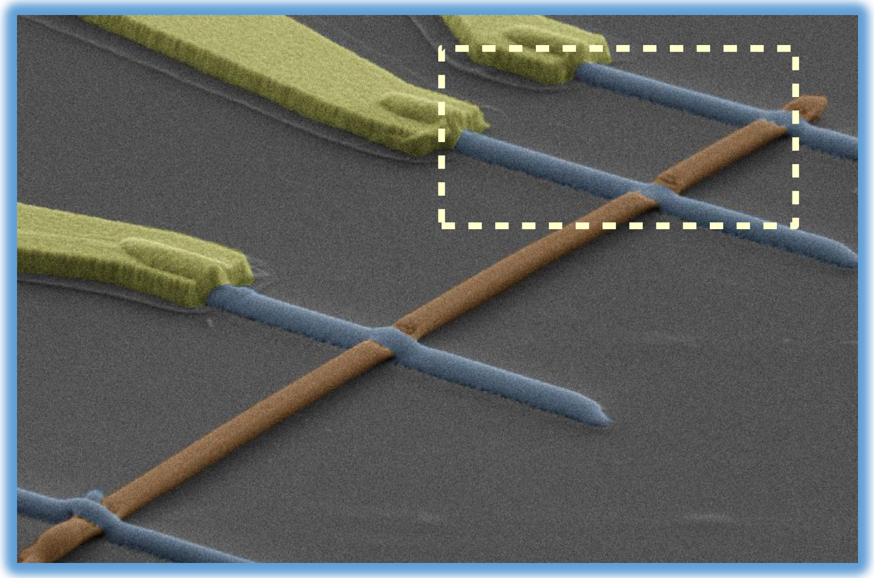
Li *et al*, PRB (2014)



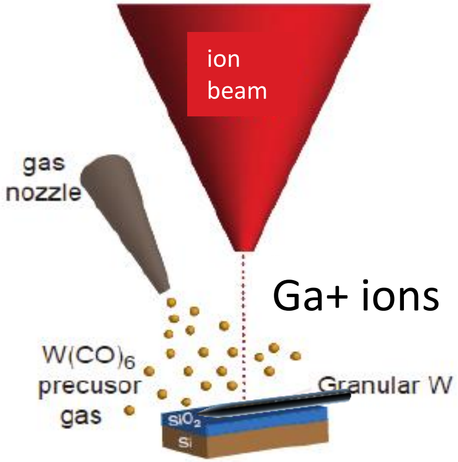


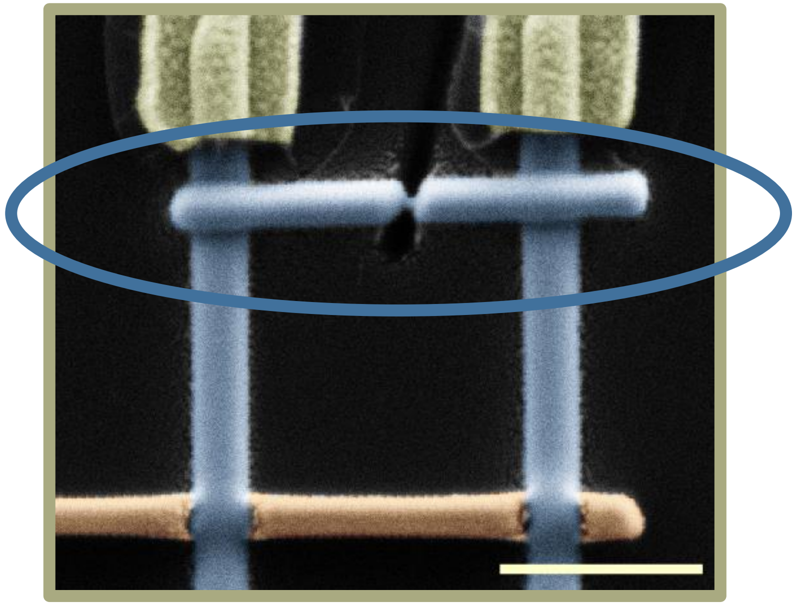
- Oscillations: supercurrent travels at the two acute wire edges
- High field (Tesla) decay scale : narrow channels (nm!)
- High critical current : well transmitted channels

Current-phase relation on very same sample



Add superconducting constriction in parallel





1 μm

Build an asymmetric SQUID to measure the I(φ) relation

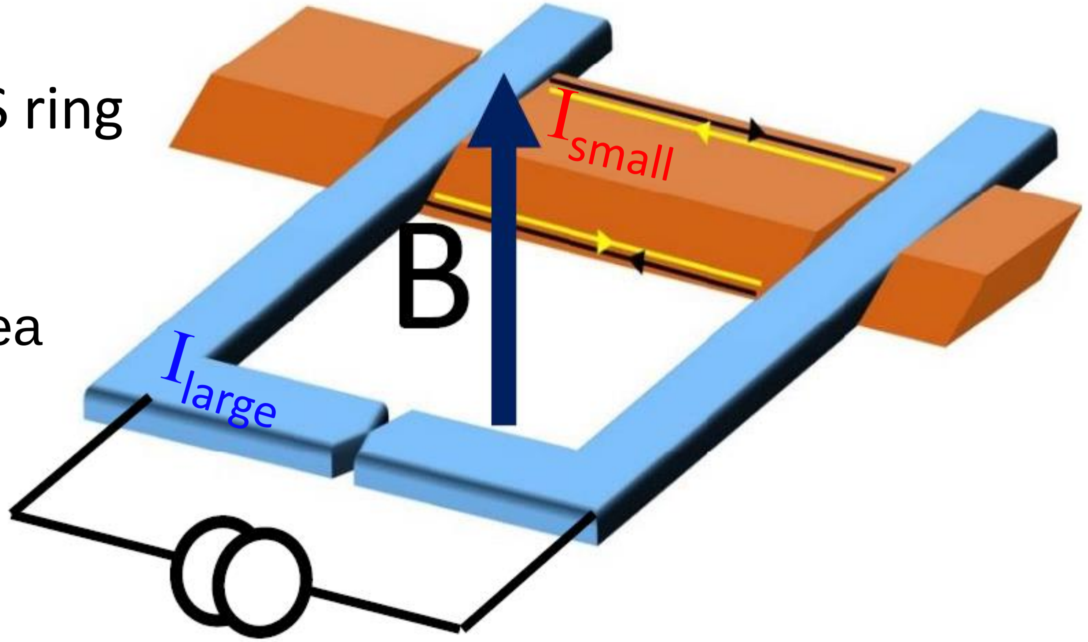
Probing the current-Phase relation with an asymmetric SQUID

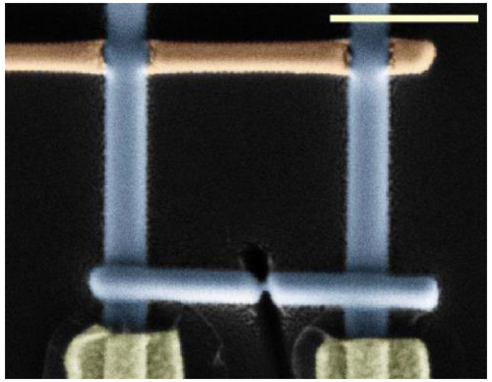
Principle of CPR measurement:

phase-biased SNS ring

$\varphi = -2\pi\, \Phi/\Phi_0$

Φ =B S, S loop area

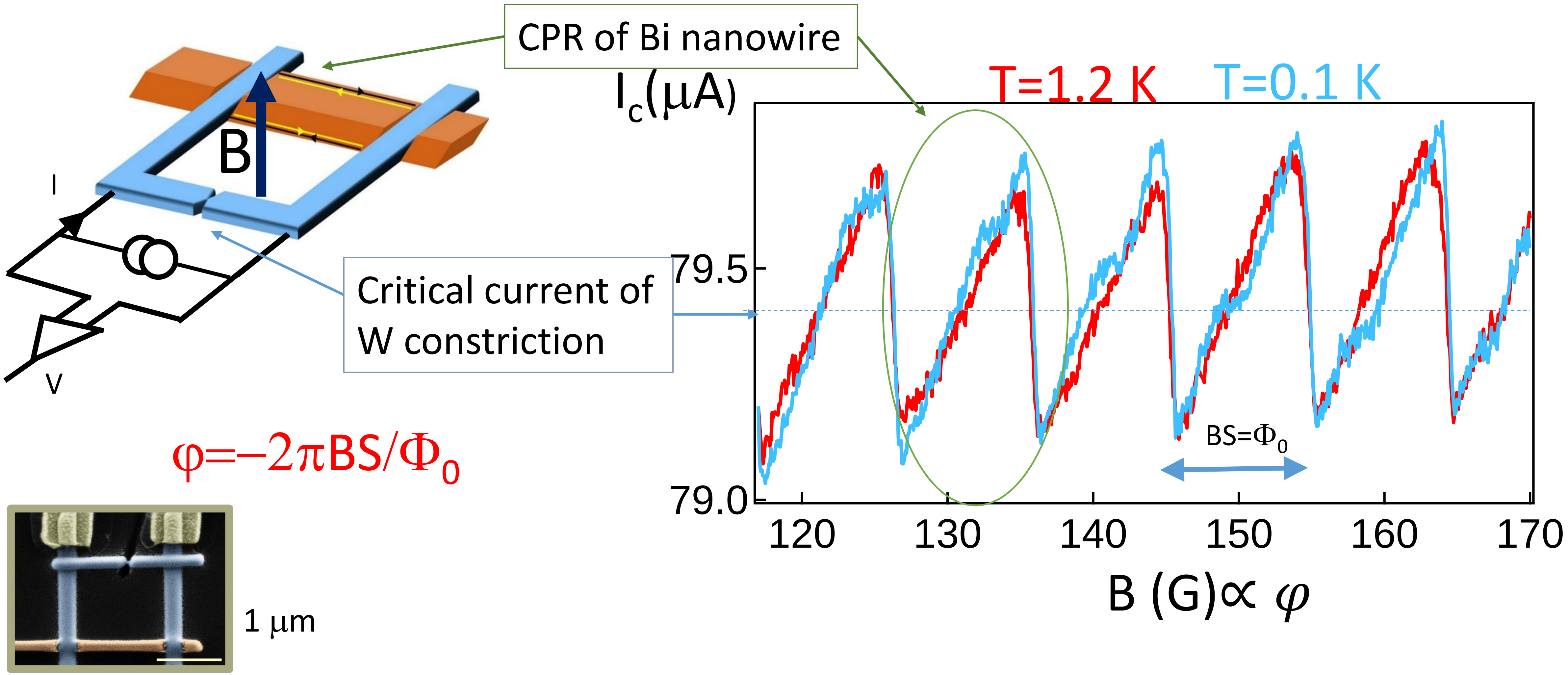




$$I_c(\text{SQUID}) \simeq I_{large}(\varphi_{max}) + I_{small}\left(\varphi_{max} + 2\pi \frac{\Phi}{\Phi_0}\right)$$

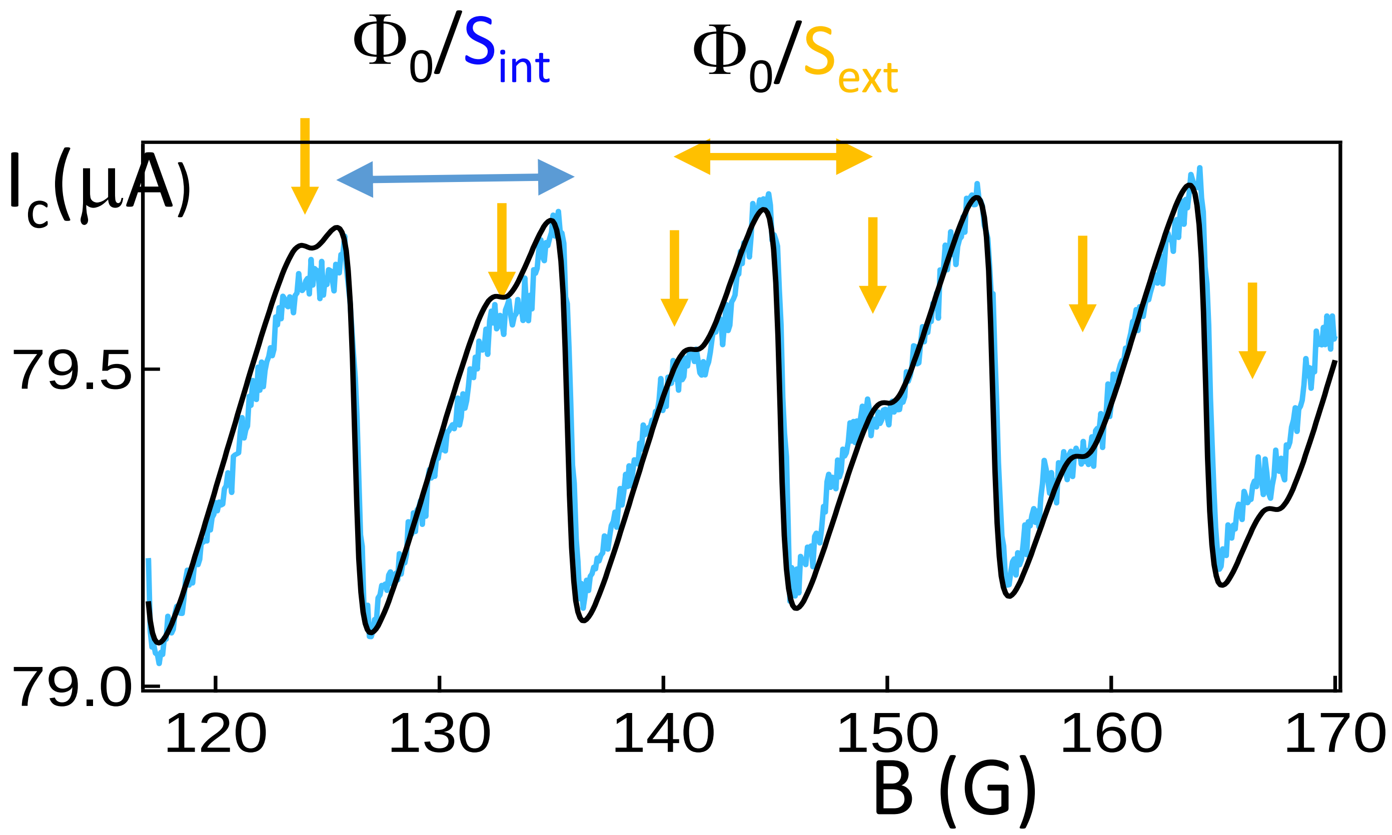
Current-Phase relation of I_{small}

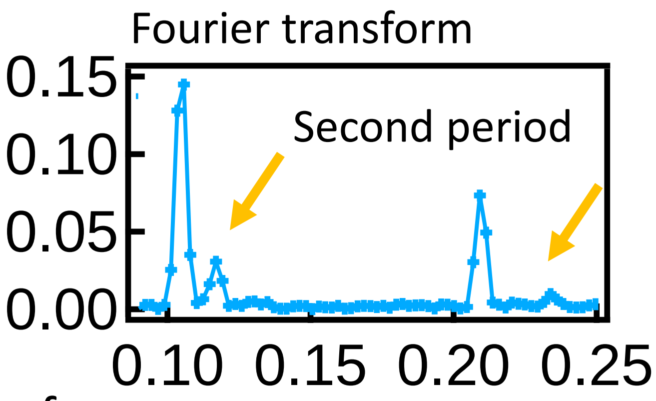
Supercurrent-versus Phase relation of Bi Josephson junction

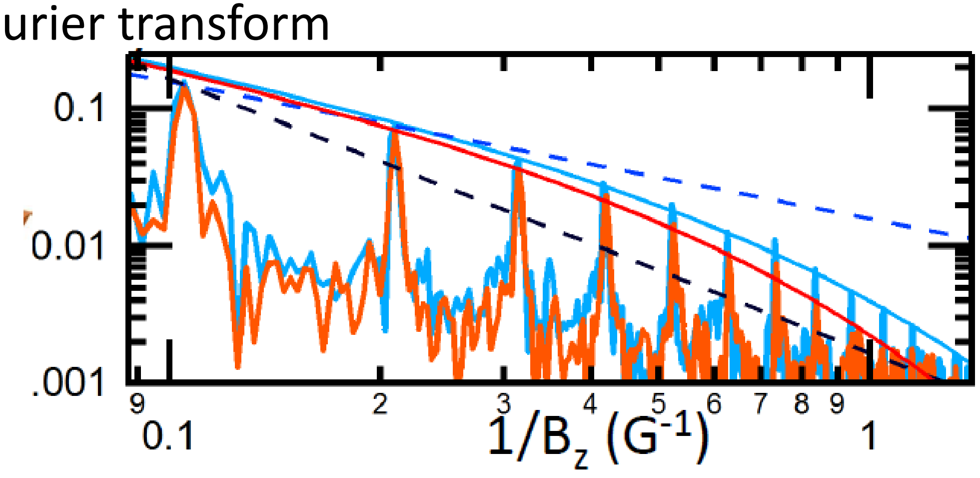


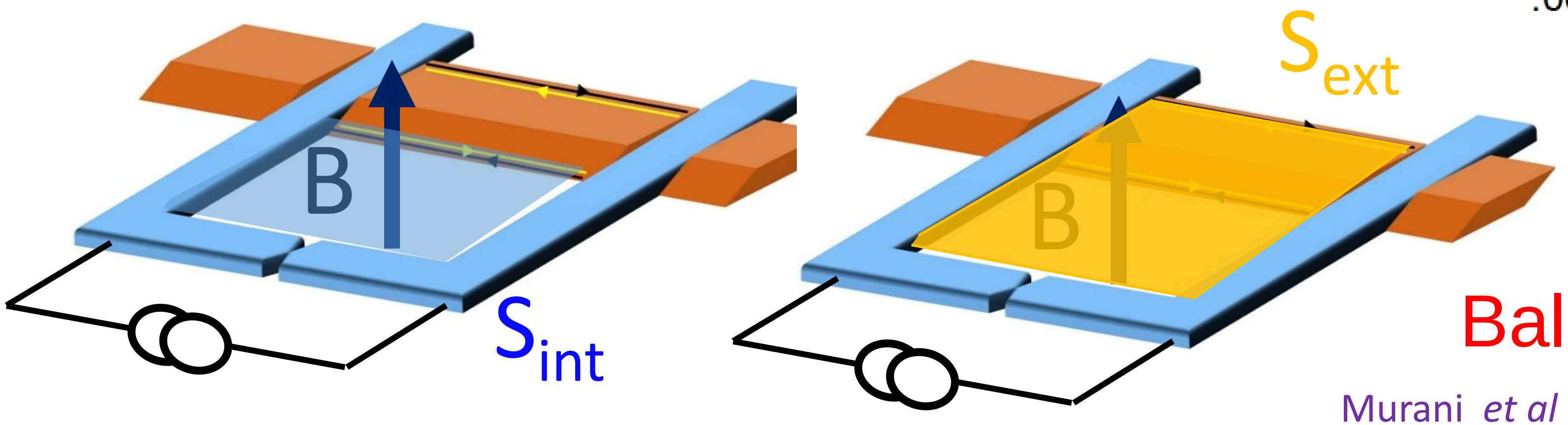
Sawtooth-shaped current phase relation: long ballistic!

Two sawtooths ?





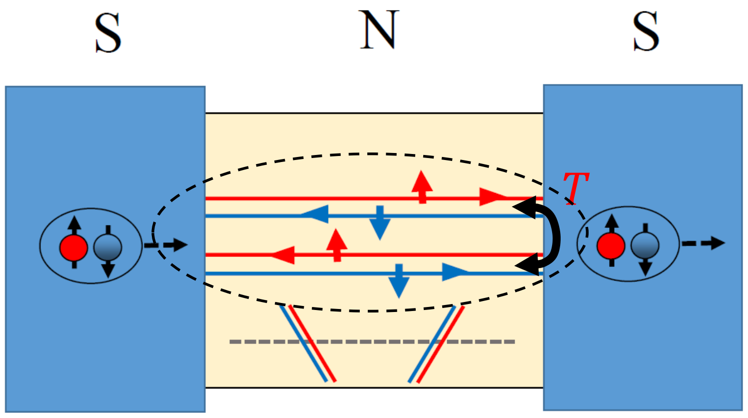




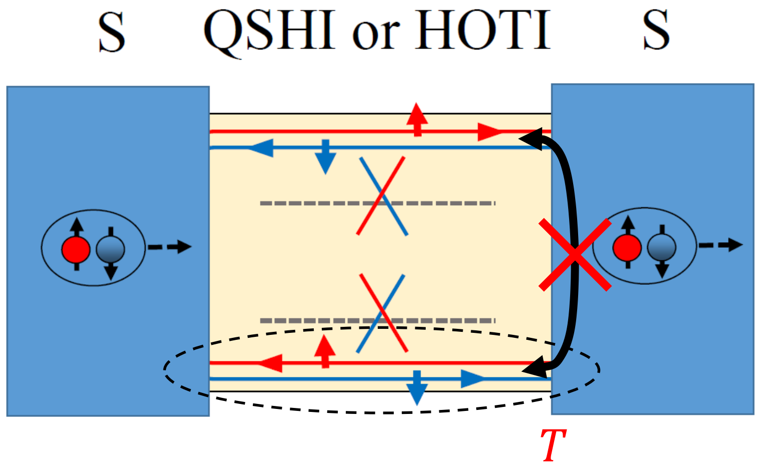
Ballistic states at two edges

Murani *et al* Nat. Comm. (2017) , Schindler *et al* Nat. Phys. (2018)

Conventional versus topological junction



Introducing imperfect transmission T



For short junctions

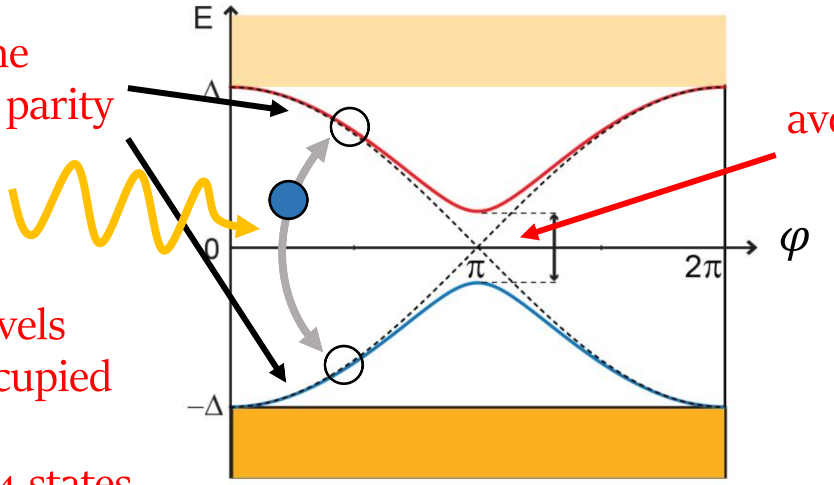
Two helical ABS

same fermion parity

both levels can be occupied

describes 4 states

avoided crossing



$$\varepsilon(\varphi) = \pm\Delta\sqrt{1 - T\sin^2(\phi/2)}$$

Spinless ground state

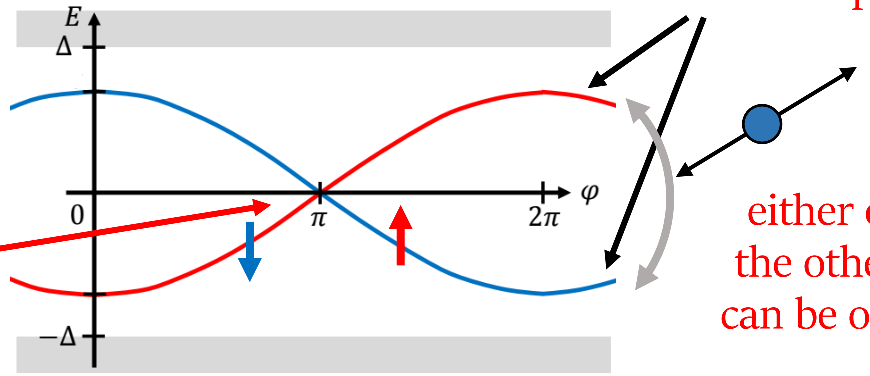
One helical ABS

different fermion parities

crossing protected by parity

either one or the other level can be occupied

describes 2 states

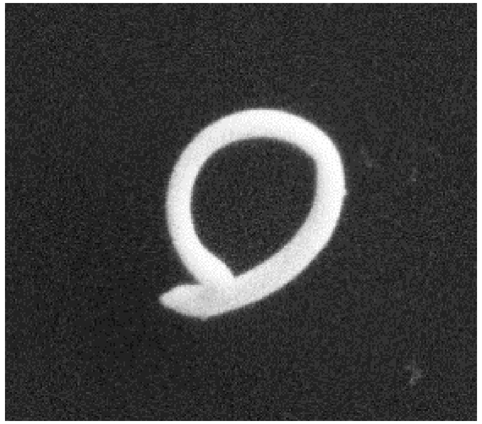


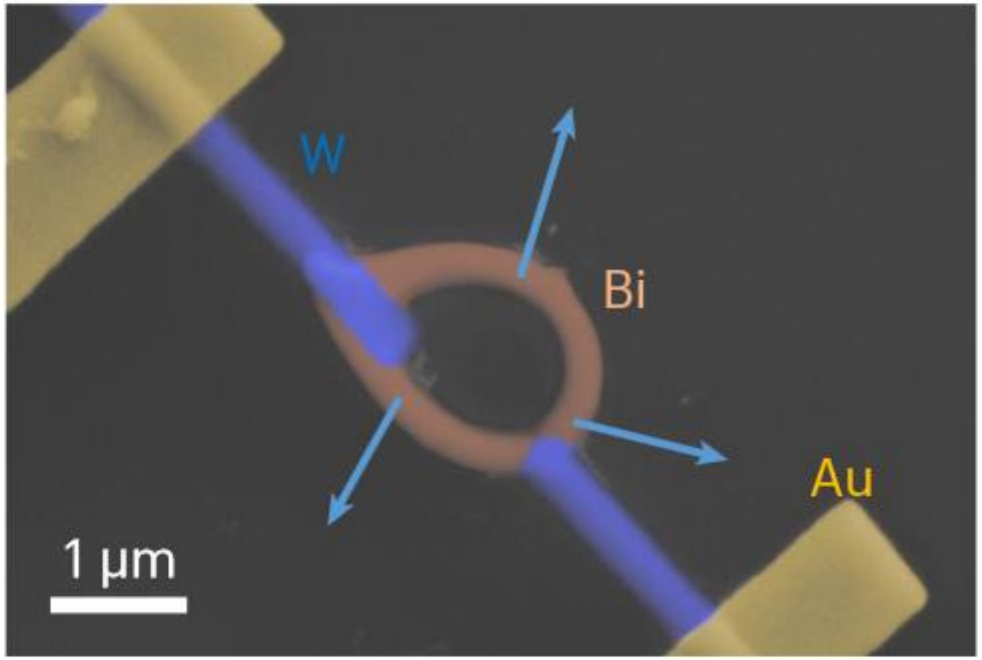
$$\varepsilon(\varphi) = \pm\Delta\sqrt{T}\cos(\phi/2)$$

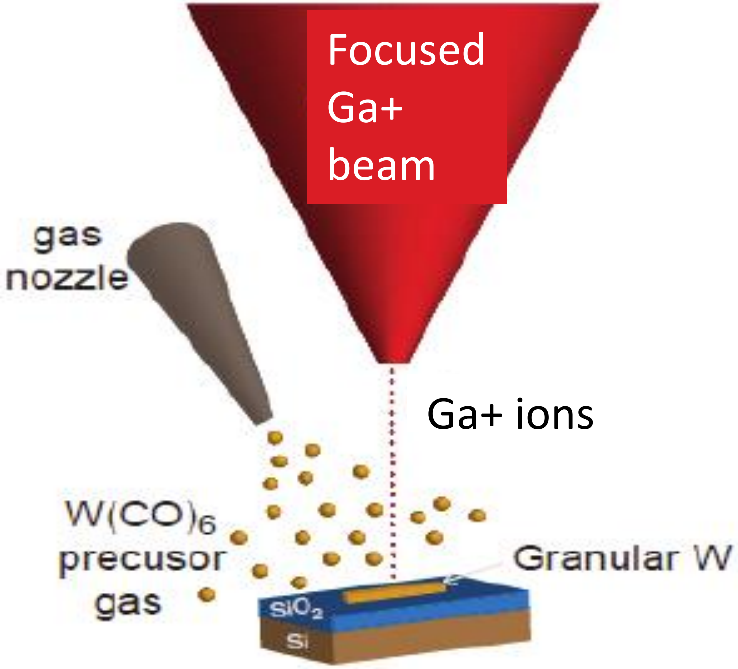
Spinfull ground states

intrinsic dc squid with a bismuth nanoring

Monocrystalline bismuth nanoring
(A. Kasumov, released with laser shock wave)



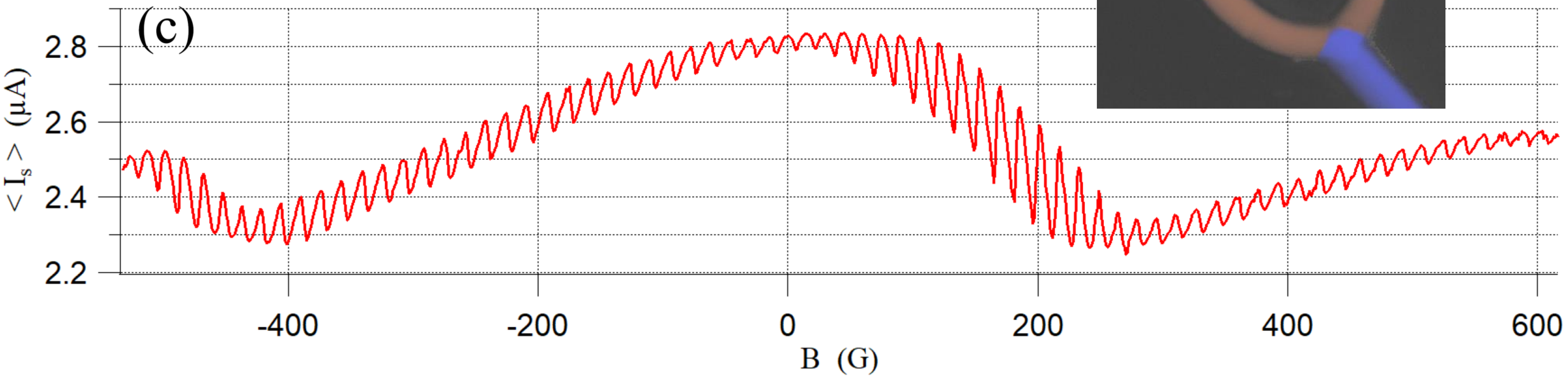


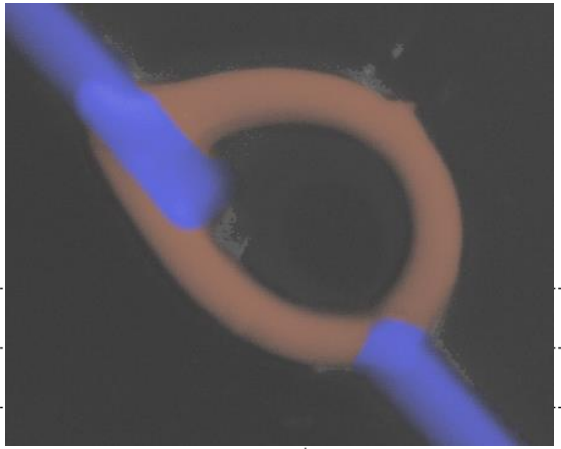


Kasumov 2005
T_c ~4 K
H_c >10 T

Bi-SQUID

Average over 250 switching events





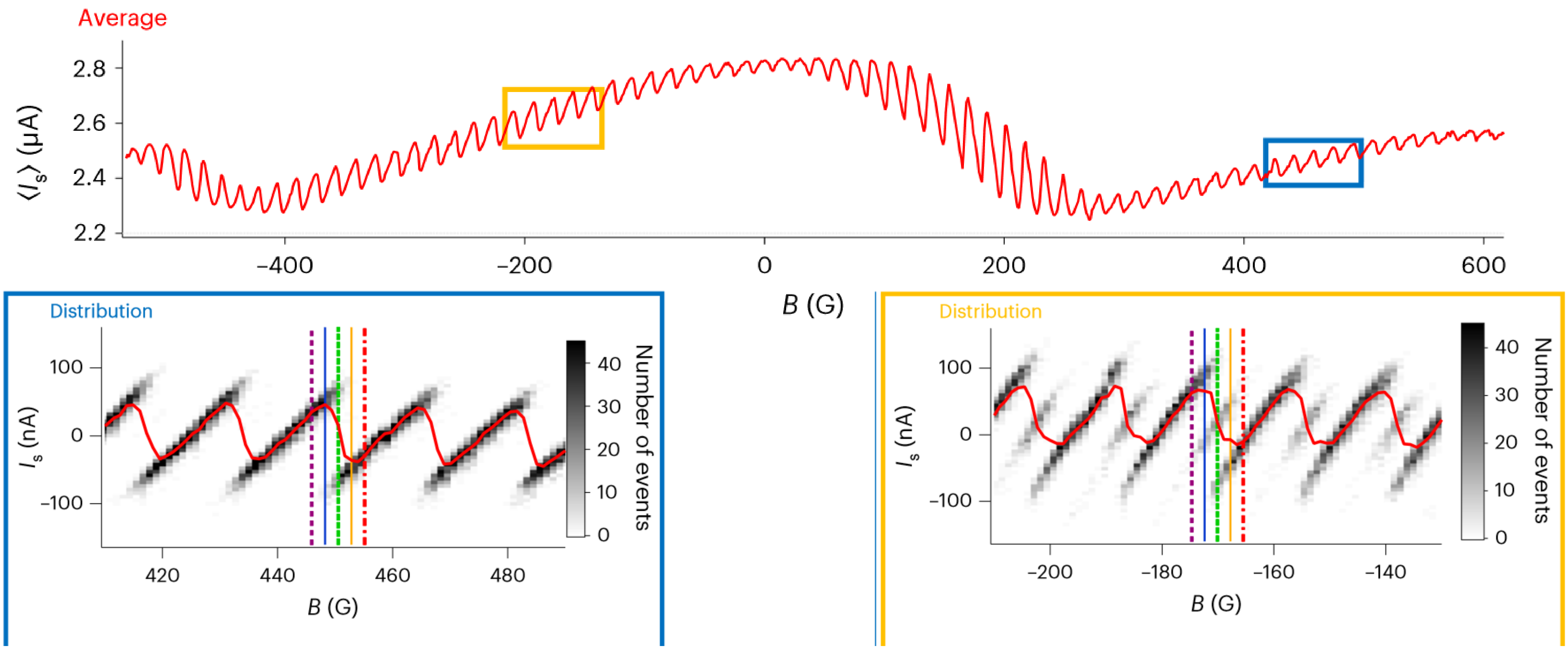
- Period=16 G, OK with flux through ring
- Modulation over background: ~ asymetric SQUID
- Suggests sawtooth CPR

$$I_c^{tot} = I(\varphi^{max}) + i(\varphi^{max} + 2\pi\frac{B.S}{\phi_0})$$

agrees with Murani *et al* Nat. Comm. (2017) , Schindler *et al* Nat. Phys. (2018)

⇒ Ballistic (or topologically protected) transport. Can we say more?

Switching current histograms



- Baseline removed
- Sawtooth shape (long ballistic junction)
- For a given phase, one or two possible switching current values

- Baseline removed
- Sawtooth shape (long ballistic junction)
- For a given phase, one or three possible switching current values

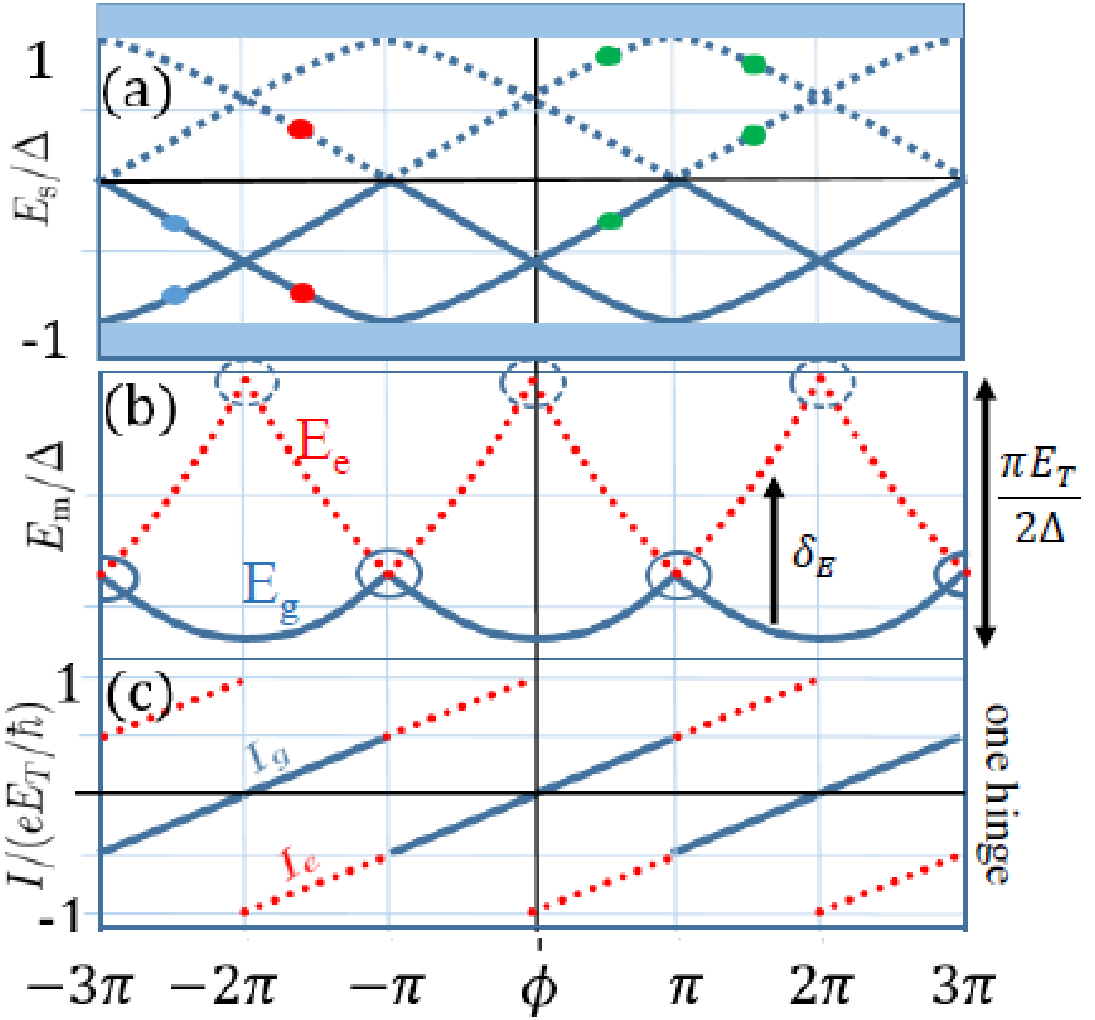
Spectrum and CPR of one long helical junction

ground state lowest excited state higher excited states

Andreev spectrum
(single particle picture)

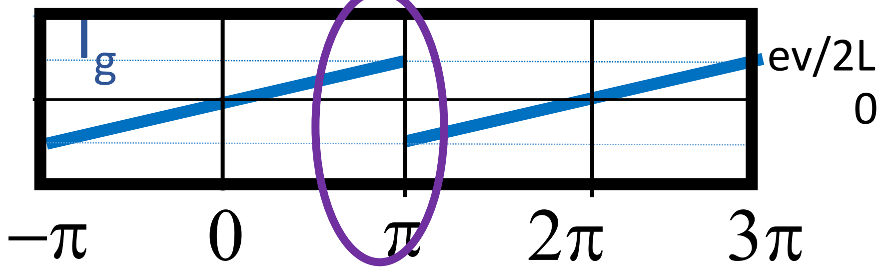
Andreev spectrum
(many-body picture)

CPR of one helical channel



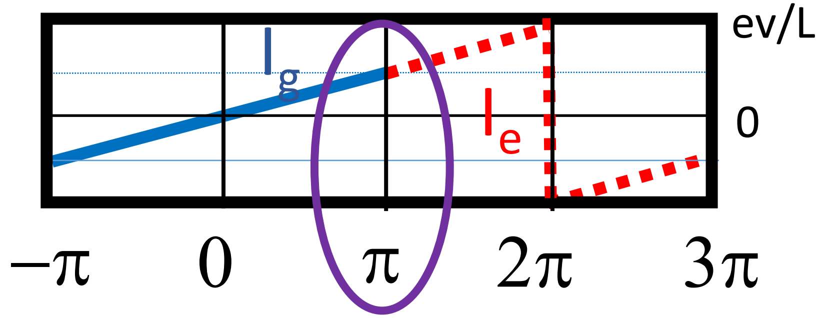
If junction always in the ground state

2π-periodic supercurrent



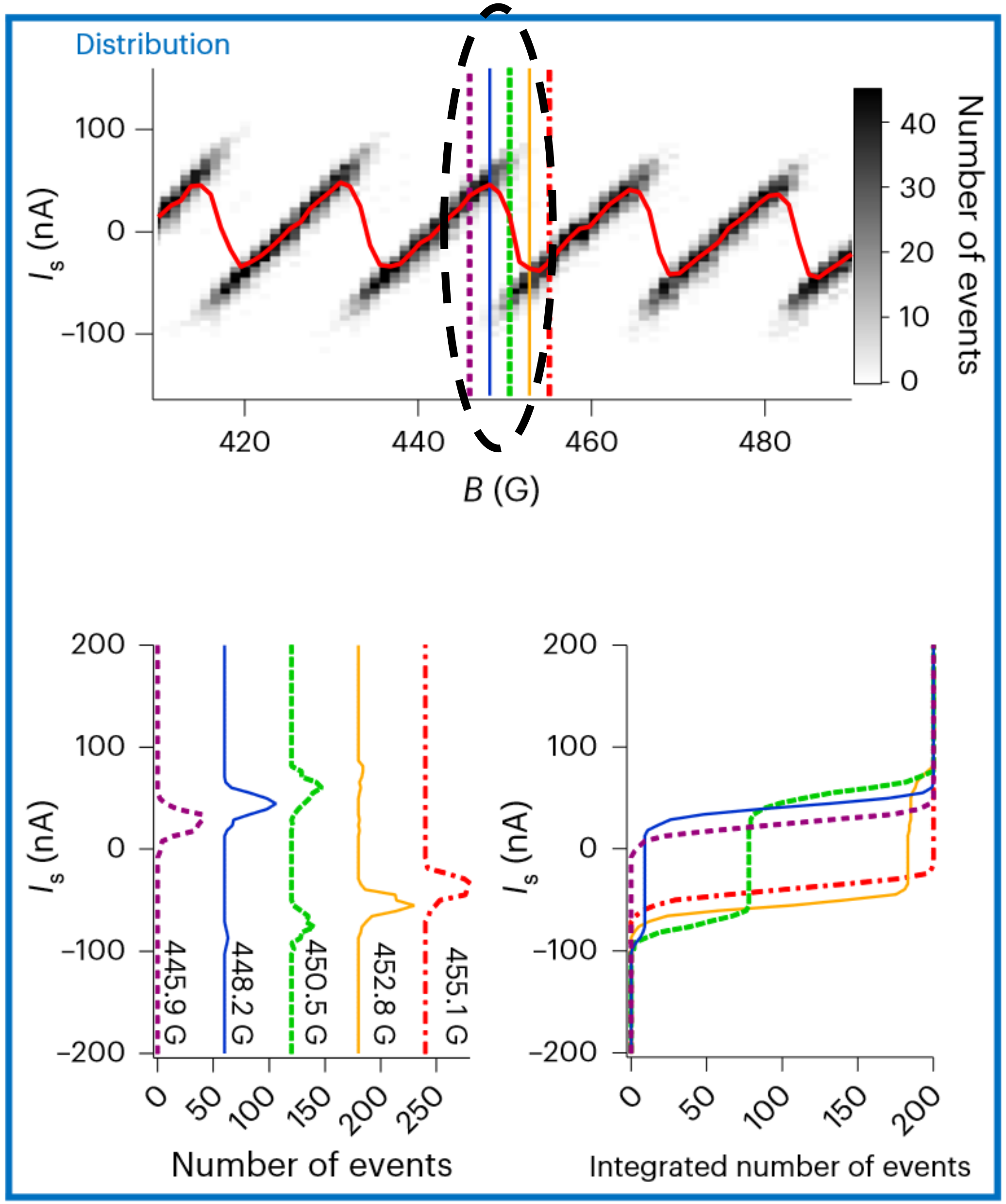
If parity conservation

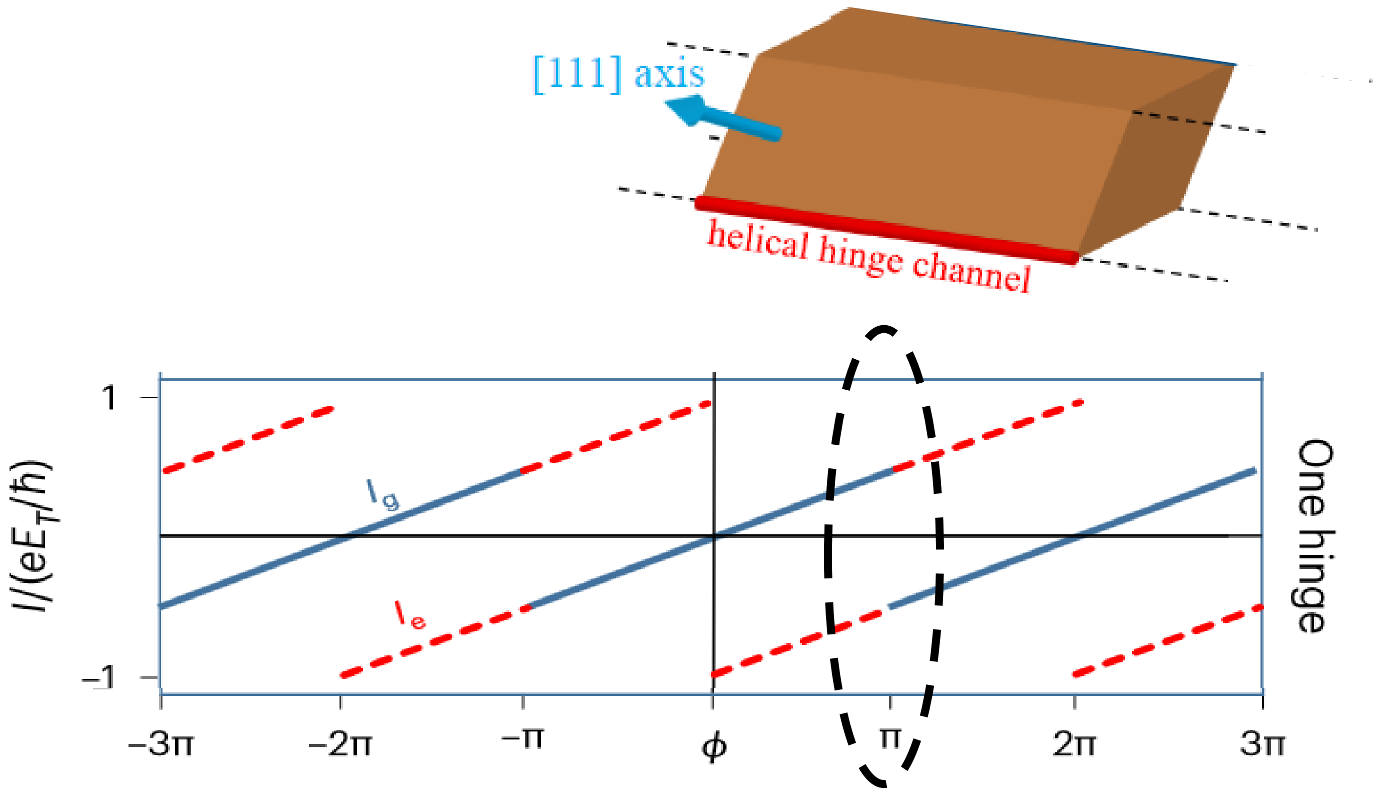
4π-periodic supercurrent



Difference between non-topological and topological junctions at π

Parity conservation and switching current histogram



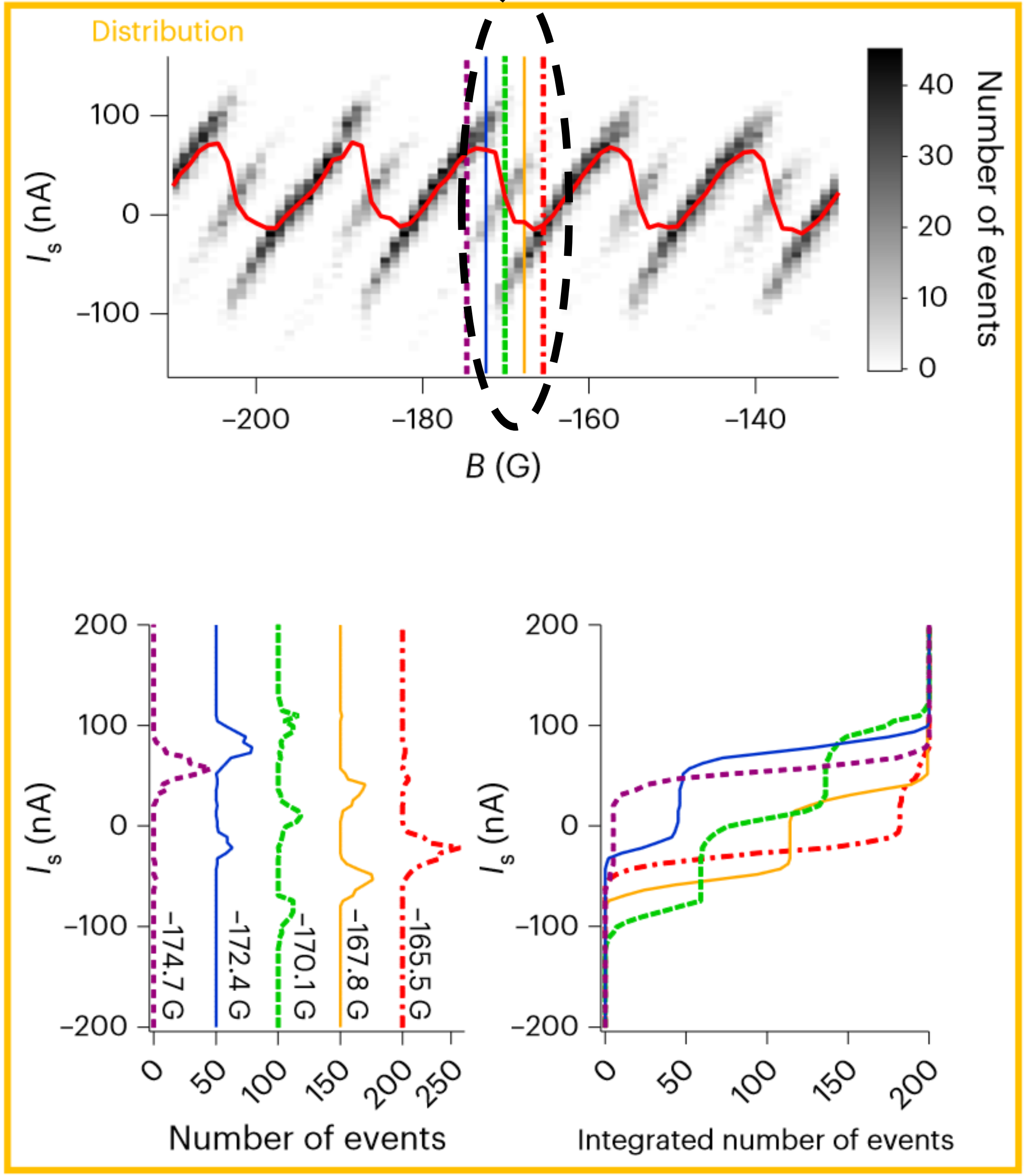


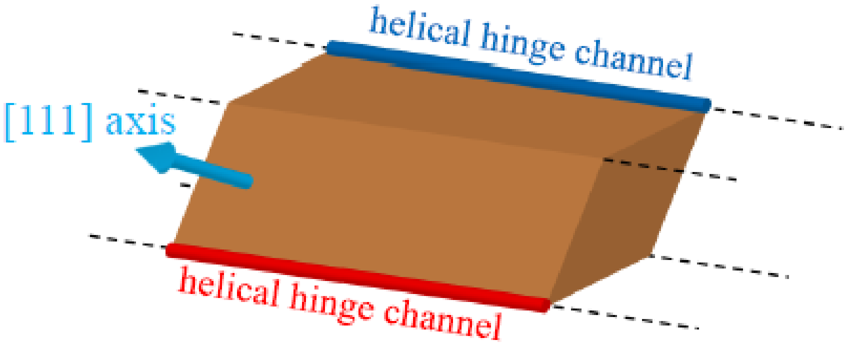
For phase around π [2π] : signature of two states

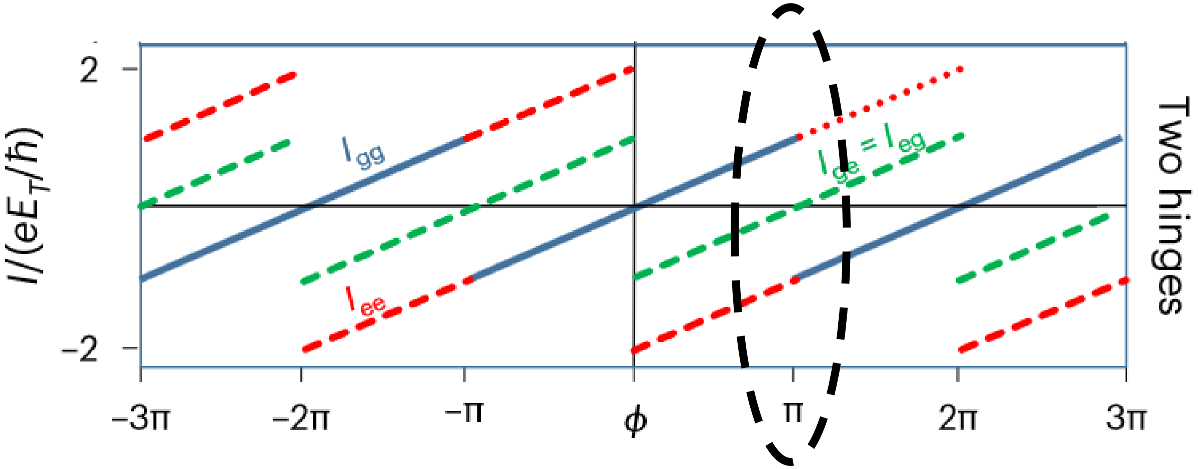
- ground state (I_g)
- excited state (I_e)

Evidence of partial parity conservation ?

Two helical hinge states and switching current histograms







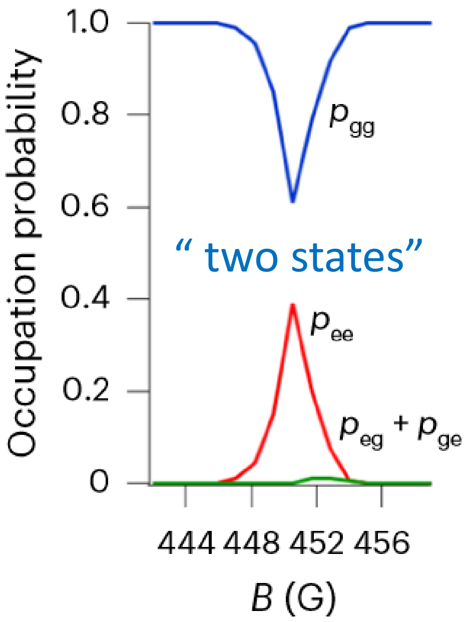
Two hinges + ground and excited states : 4 states

- 2 hinges in the ground state : I_{gg}
- 2 hinges in the excited state : I_{ee}
- 1 hinge in the ground state, the other in the excited state : I_{eg}, Ige

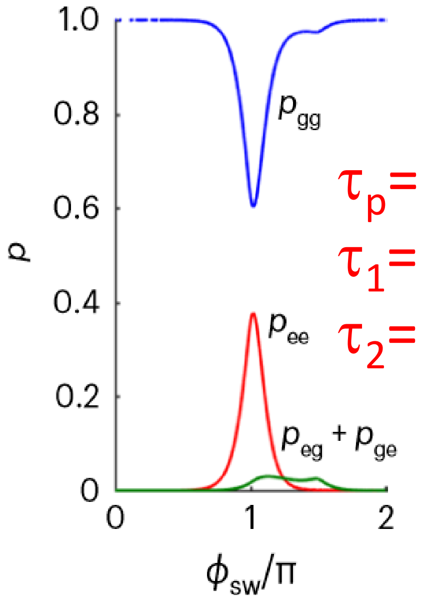
Occupation probability and relaxation times

EXPERIMENT

THEORY : two hinges + master equation + pair and quasiparticle relaxation times



" two states"



τ_p= 1.8 ms

τ_1= 25 ms

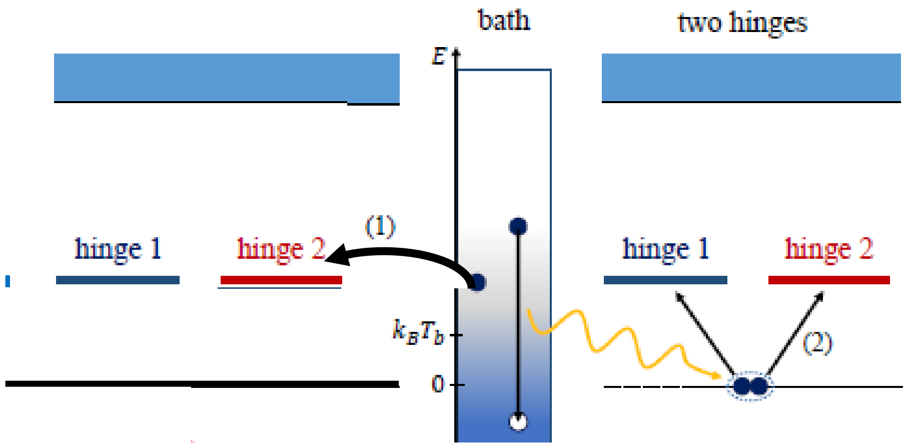
τ_2=10 τ_1

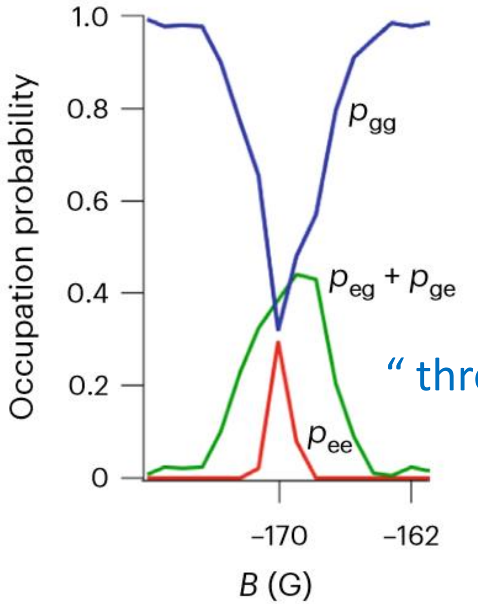
1 quasiparticle from bath

gg -> ge/eg : poisoning

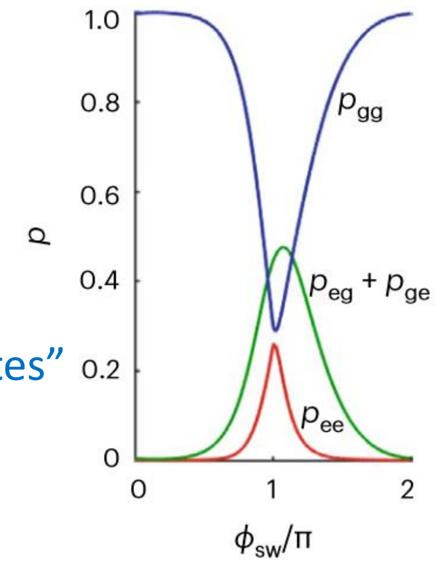
energy from bath, split Cooper pair

gg -> ee : pair excitation or relaxation



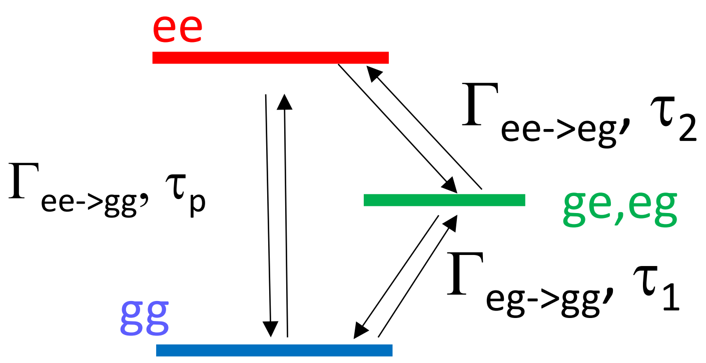


" three states"



τ_p= 2 ms

τ_{qp}= 10 ms

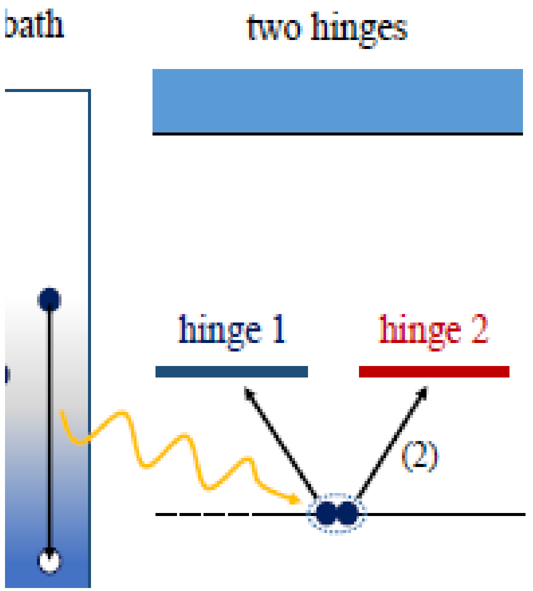


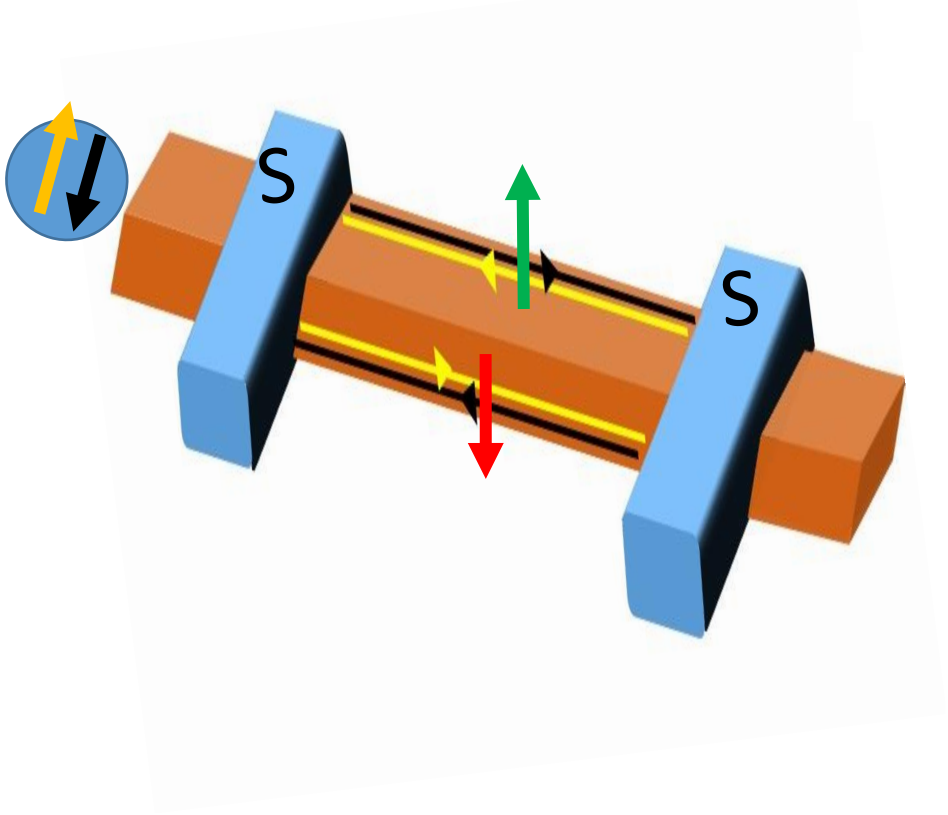
Coll. Y. Peng, Y. Oreg, F. von Hoppen

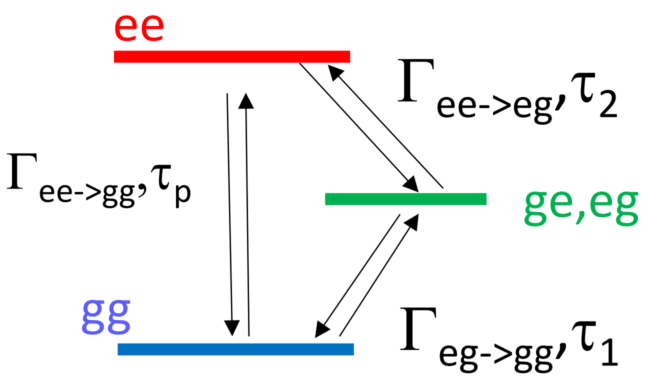
Slow pair relaxation rate : topological hinge modes?

Slow pair relaxation : τ_p~ ms (instead of μs in non engineered e.m. environment)

To go from ee->gg or gg-> ee, need to split or form a Cooper pair with one quasiparticle from each hinge
difficult for hinges further apart than ξ_s
-> slow pair excitation or relaxation (would be much easier if spin-degenerate edge)

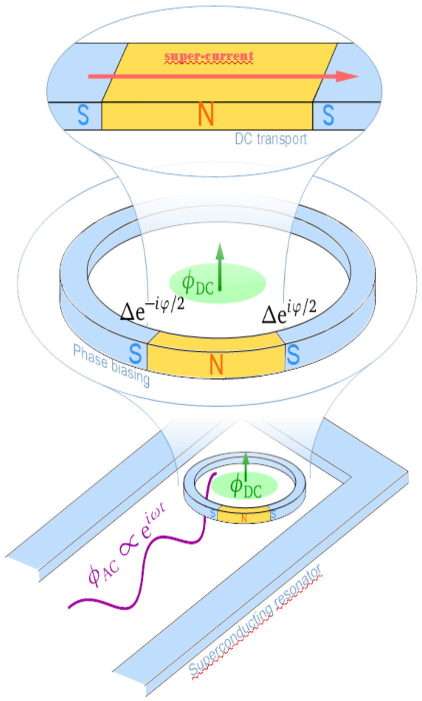






A. Bernard *et al*, Nat. Phys. (2023)

Conclusion and outlooks



Used induced superconductivity to probe possible SOTI character

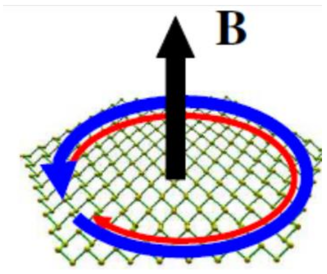
- Interference in the critical current ✔

- Supercurrent versus phase relation in a ring geometry ✔

- Switching histograms : long pair relaxation time

- ac susceptibility $\chi = dI/d\varphi$ to demonstrate protected crossing at phase π (not presented today, Murani et al, Phys.Rev. Lett. (2019)) ✔

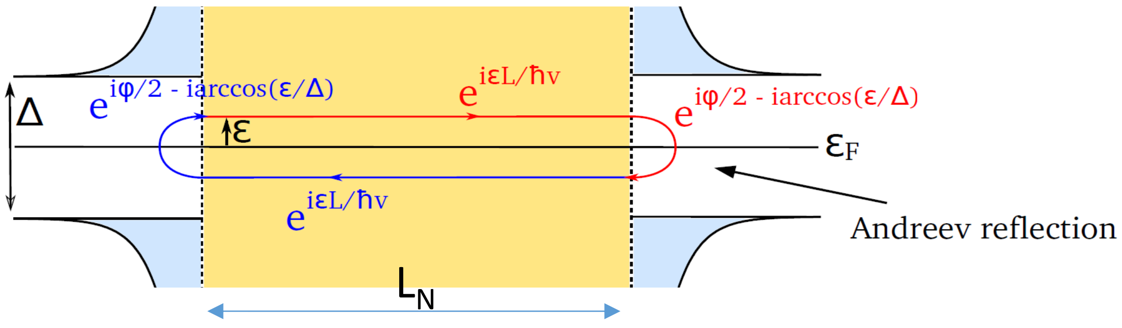
- rf spectroscopy of helical Andreev states? PhD Lucas Bugaud



- Without superconductivity: -Detect Edge current in an isolated flake? (new detector of orbital currents) : PhD Matthieu Bard

-Other materials ? WTe_2 (PhD L. Bugaud and X. Ballu), Bi_4Br_4 (PhD J. Lefeuvre)

Andreev spectrum and supercurrent in short ballistic junction

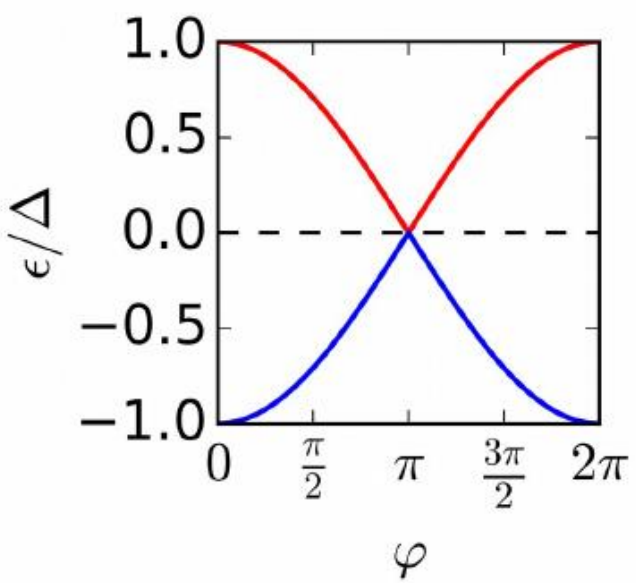


$$\cancel{\frac{2\epsilon L_N}{\hbar v_F}} - 2\,arccos\frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

propagation

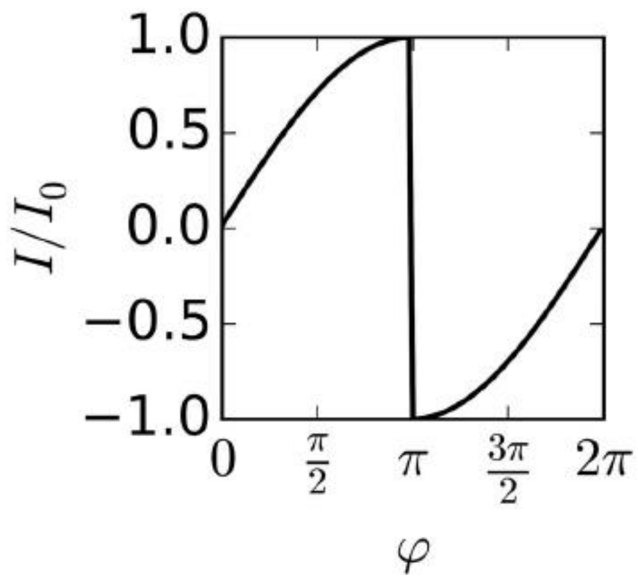
spectrum: branches of **cos(φ/2)**

few states in gap



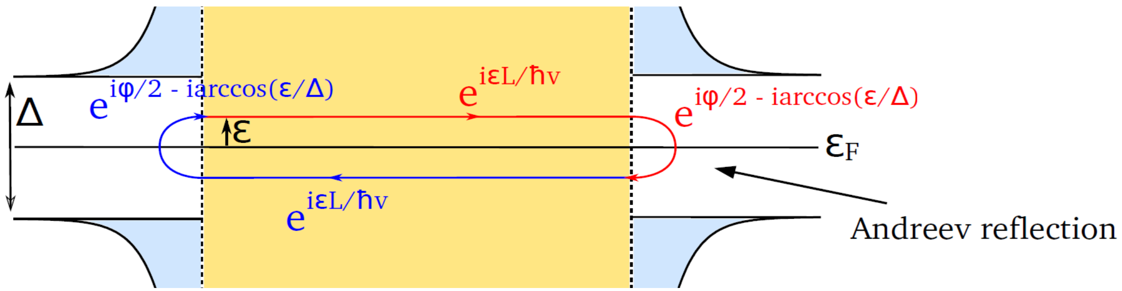
$$I = \sum_{-\infty}^{0} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$

supercurrent



I(φ)~branches of sin(φ/2) **with jump at π**

Example: Andreev spectrum and supercurrent in long ballistic SNS junction: L>> $\xi_s = \frac{hV_F}{\Delta}$

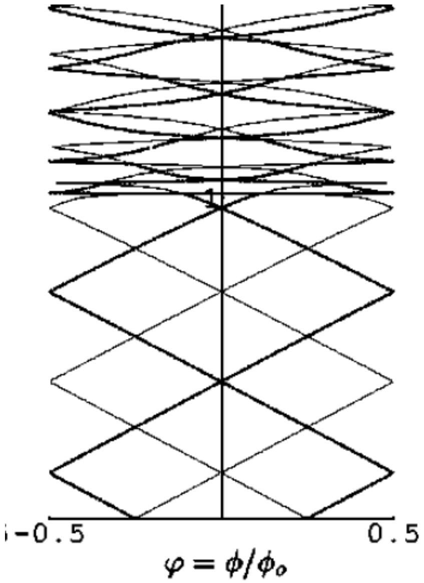


$$\frac{2\epsilon L_N}{\hbar v_F} - 2\,\arccos\frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

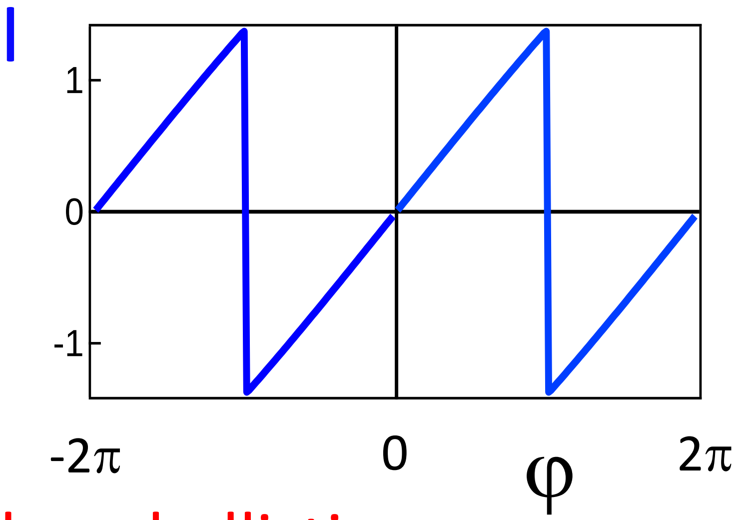
propagation in N

spectrum: more states in gap,
quasi linear

I(φ) ~ linear segments with jumps at π

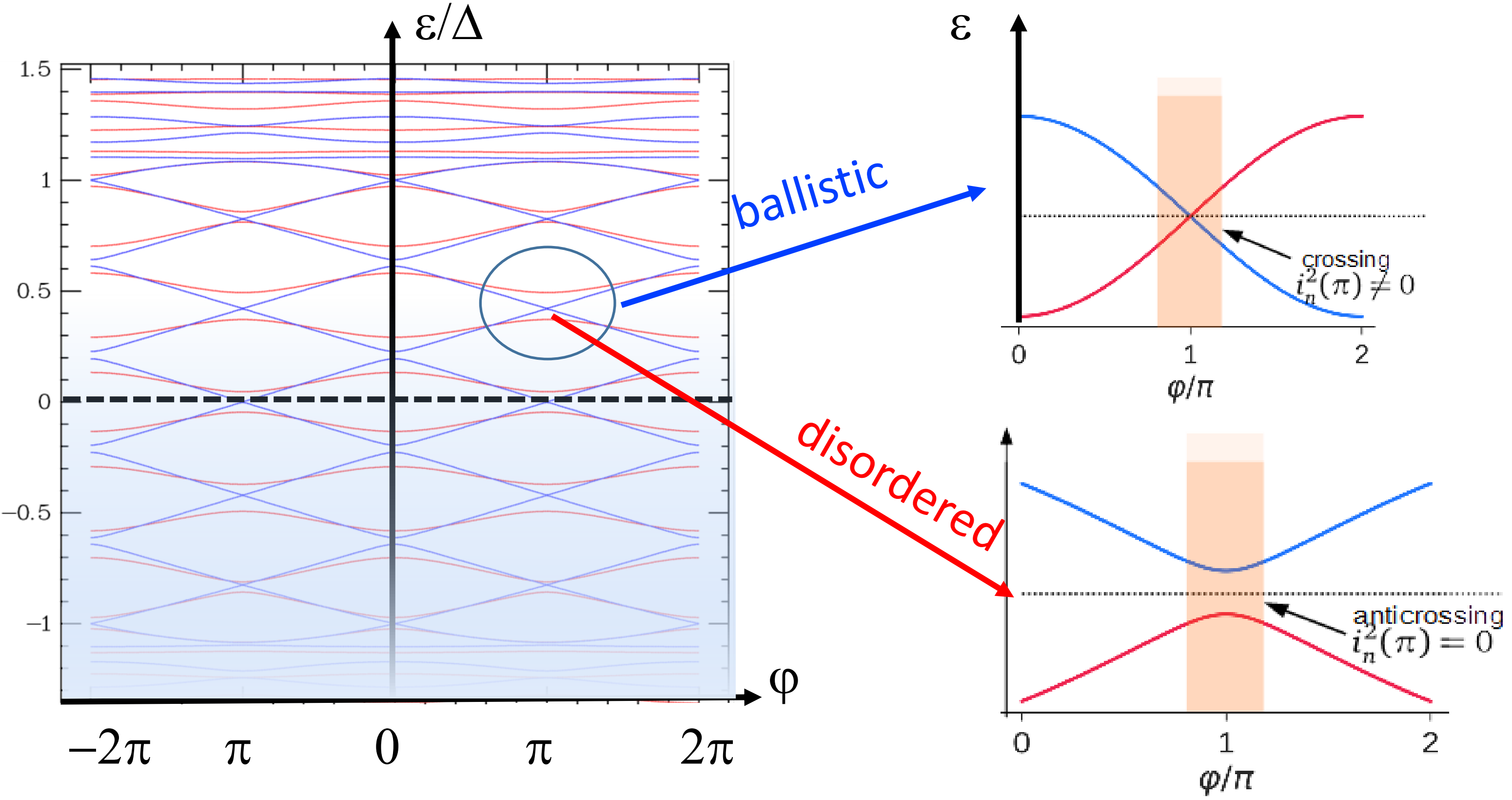


$$I = \sum_{-\infty}^{\infty} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$



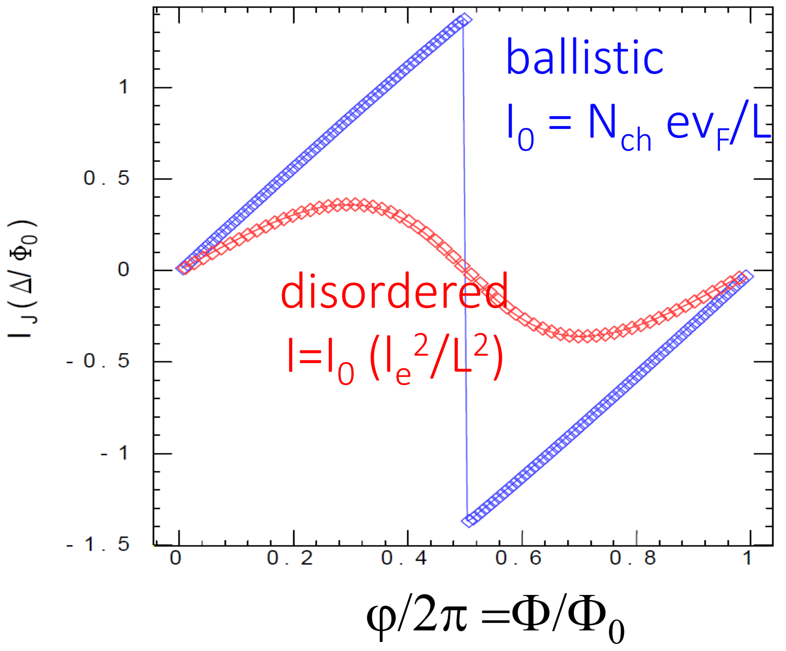
Sawtooth I(φ) characteristic of long ballistic

Influence of disorder on Andreev spectrum and supercurrent



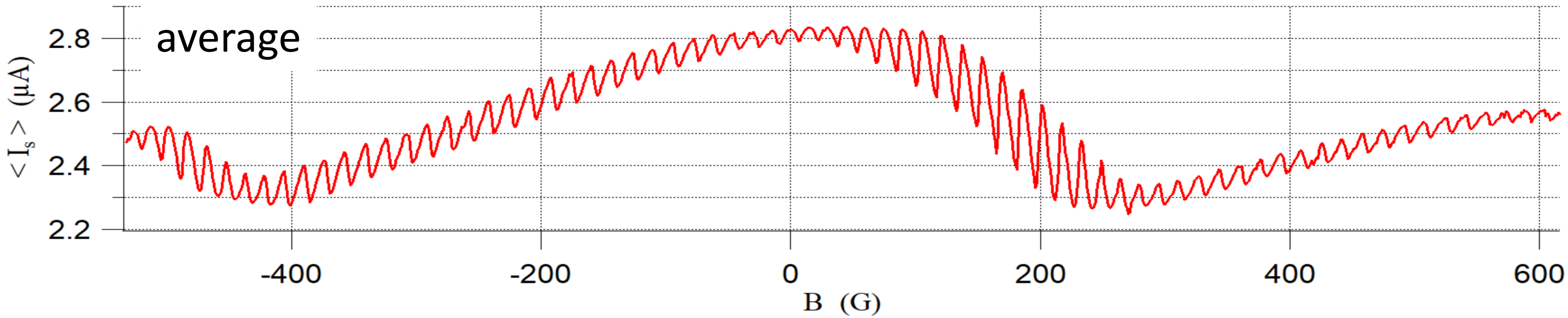
Dc supercurrent versus phase

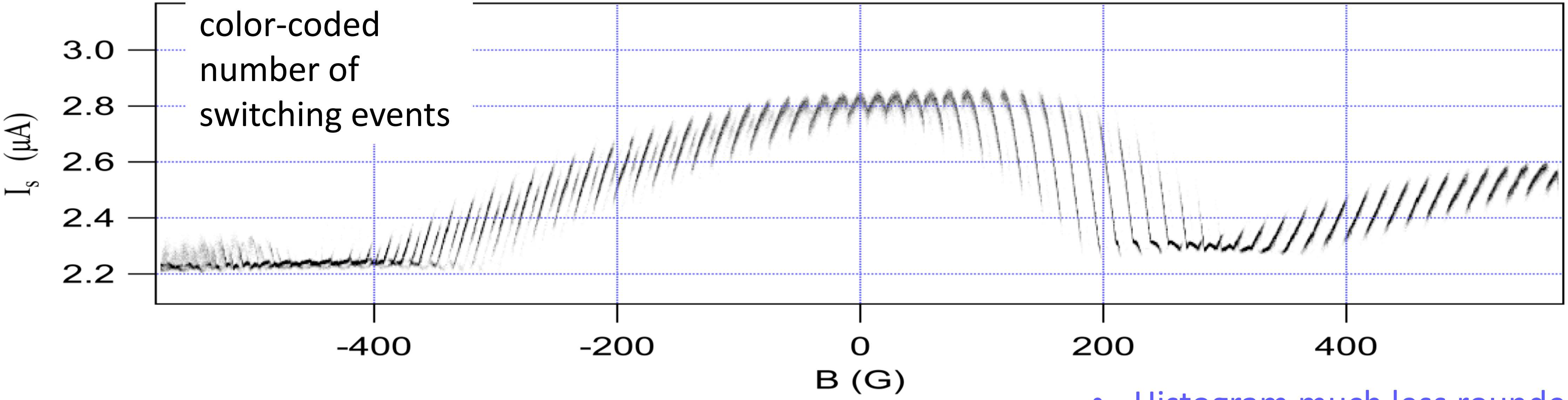
$$I = \sum_{-\infty}^{\infty} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$



Disorder lifts Andreev level degeneracy at π and rounds I(φ)

Beyond average switching current: full statistics





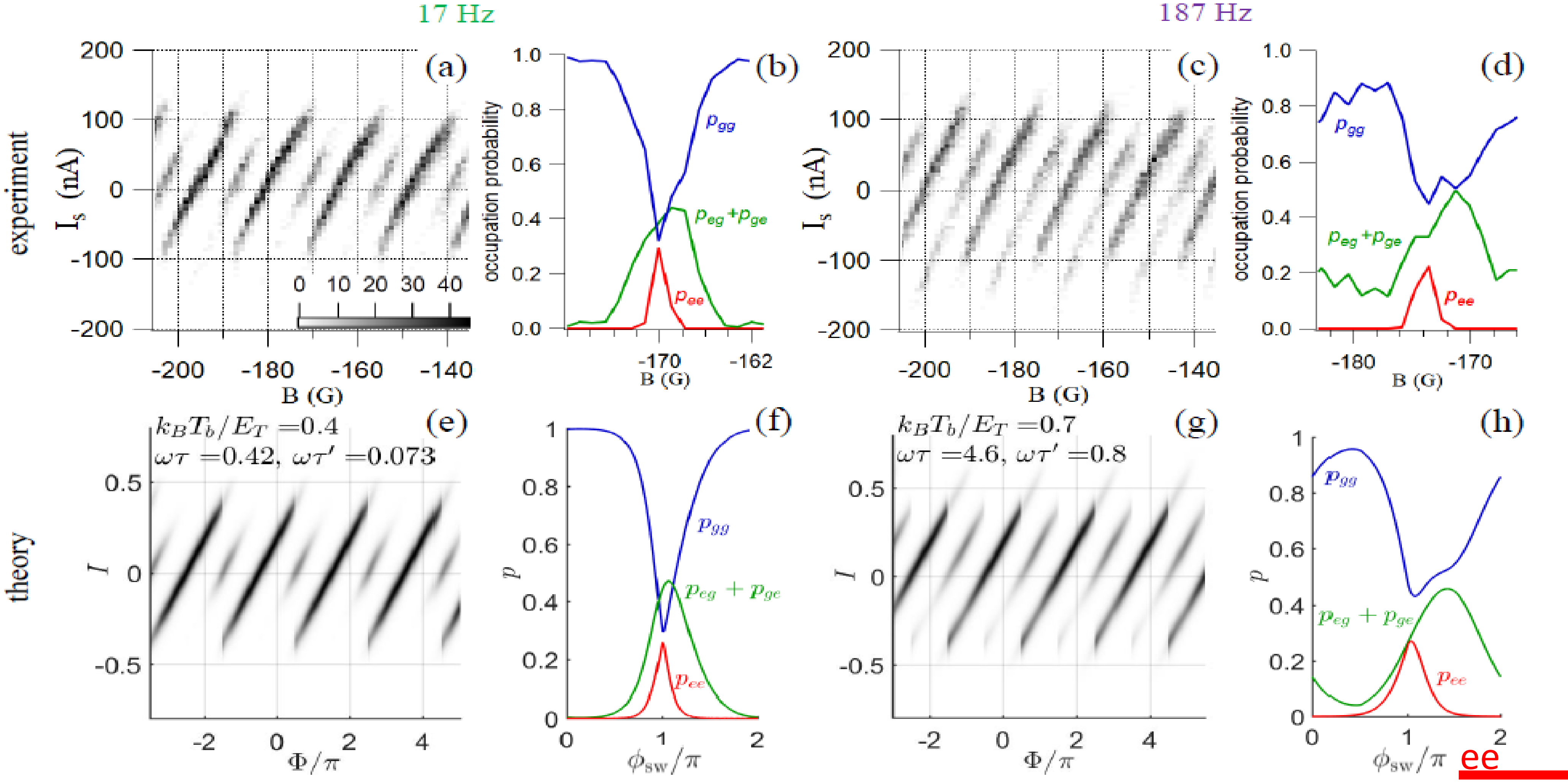
- No rounding of CPR:

⇒ ballistic over more than 1 µm

⇒ suggests topologically protected level crossing

- Histogram much less rounded than average!
- Clear sawtooth behaviour, clear jumps

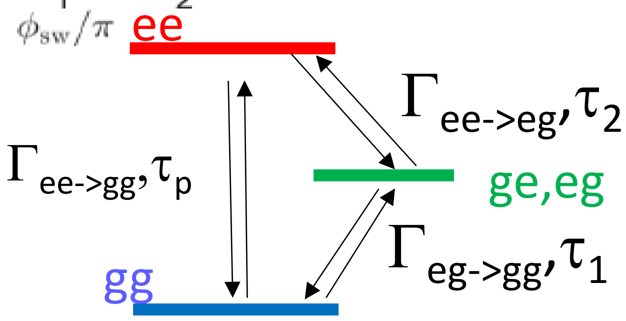
Comparison experiment /theory (region with poisoning)



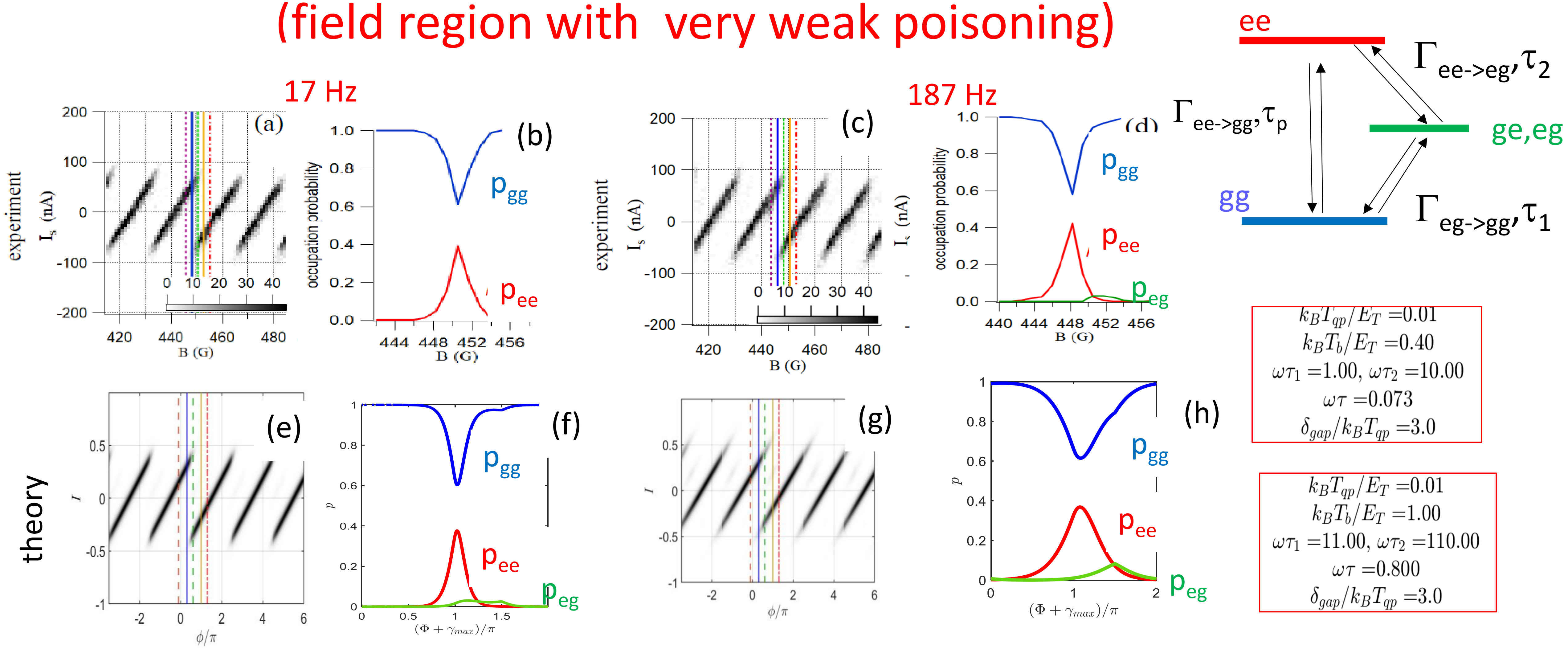
pair relaxation τ_p= 2 ms (exceptionally long, other systems: μs)

quasiparticle relaxation (poisoning) = τ_{qp}= 10 ms

T_{qp} = T_{pairs}=0.6-1 K depending on sweep rate



Comparison experiment /theory
(field region with very weak poisoning)



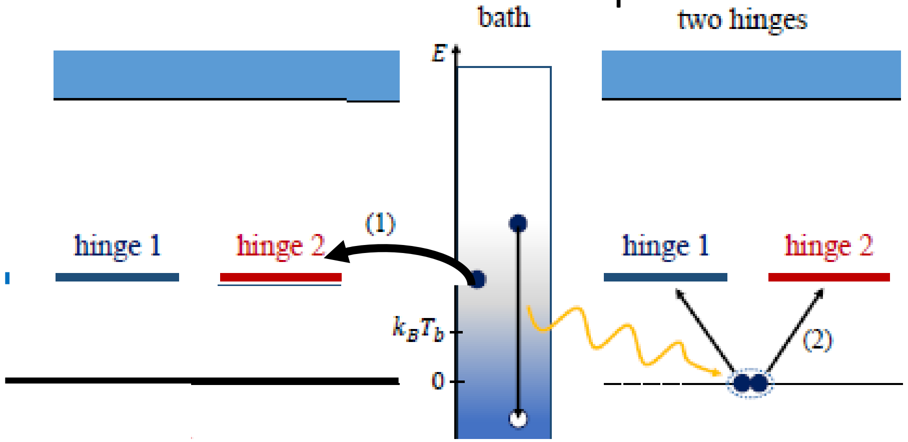
τ_{pair}=1.8 ms, $\tau_{qp,1}$=25 ms, $\tau_{qp,2}$=250 ms , T_{qp} =15 mK, T_{pairs}=0.6-1.5 K

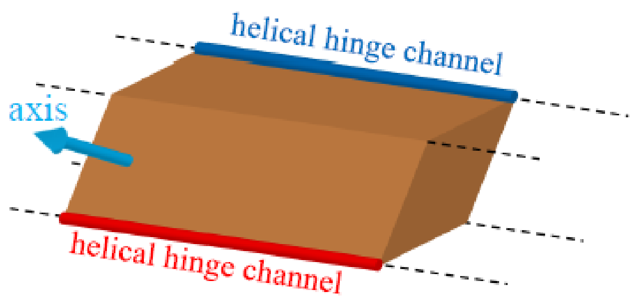
Here had to introduce a small gap in the spectrum, and $T_{qp} << T_{pairs}$

Phenomenological model

1 quasiparticle from bath
gg->ge/eg : poisoning

energy from bath,
split Cooper pair
gg->ee
pair excitation or relaxation

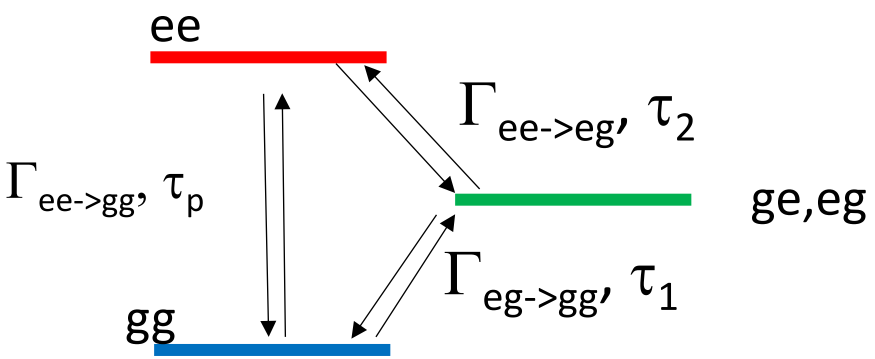




Solve rate equations

$$\frac{dp_{gg}}{dt} = -2\Gamma_{eg\leftarrow gg}p_{gg} + 2\Gamma_{gg\leftarrow eg}p_{eg} - \Gamma_{ee\leftarrow gg}p_{gg} + \Gamma_{gg\leftarrow ee}p_{ee}$$

$$\frac{dp_{eg}}{dt} = -\Gamma_{gg\leftarrow eg}p_{eg} + \Gamma_{eg\leftarrow gg}p_{gg} - \Gamma_{ee\leftarrow eg}p_{eg} + \Gamma_{eg\leftarrow ee}p_{ee}),$$

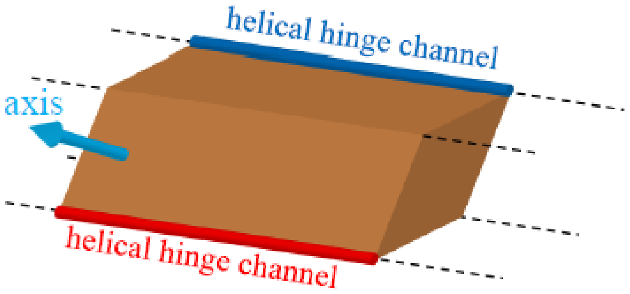


poisoning
$$\Gamma_{ee\text{<->}eg} = f(\pm\delta_E(\phi)/k_BT_{qp})/\tau_2$$
$$\Gamma_{eg\text{<->}gg} = f(\pm\delta_E(\phi)/k_BT_{qp})/\tau_1$$

pair relaxation
$$\Gamma_{ee\text{<->}gg} = \frac{2\delta_E(\phi)}{E_T\tau_p}n_B\left(\frac{2\delta_E(\phi)}{k_BT_b}\right) / \left[1 + n_B\left(\frac{2\delta_E(\phi)}{k_BT_b}\right)\right]$$

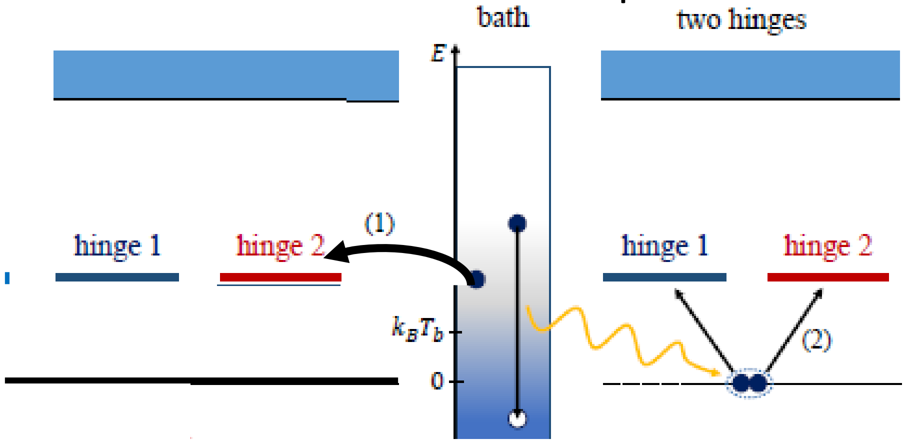
Yields Andreev level occupation probabilites p_{gg}, p_{eg}, p_{ee}

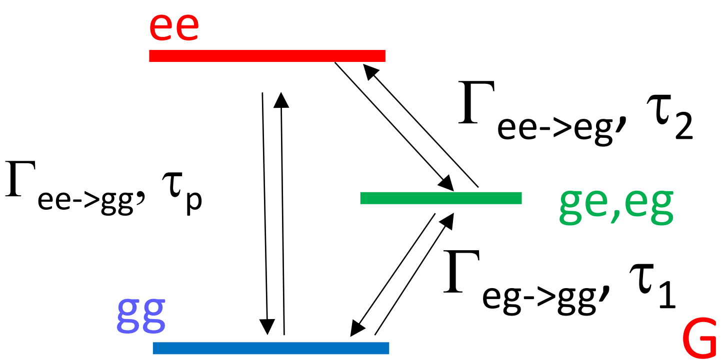
Phenomenological model



1 quasiparticle from bath
gg->ge/eg : poisoning

energy from bath,
split Cooper pair
gg->ee
pair excitation or relaxation





1- Solve rate equations

Andreev level occupation probabilites p_{gg}, p_{eg}, p_{ee}

2- Switching probability as sum of state-dependent probabiity

$$P(I,\phi) = \sum_{l,l' \in \{e,g\}} p_{ll'}(\phi) P_{\mathrm{sw}}^{ll'}(I,\phi)$$

smooth step function centered around I(ϕ) (switching is stocchastic)

3- Generate histogram and switching current distribution

Get

-Andreev level occupation probabilites p_{gg}, p_{eg}, p_{ee}

-pair and quasiparticle relaxation times

-bath temperature(s)

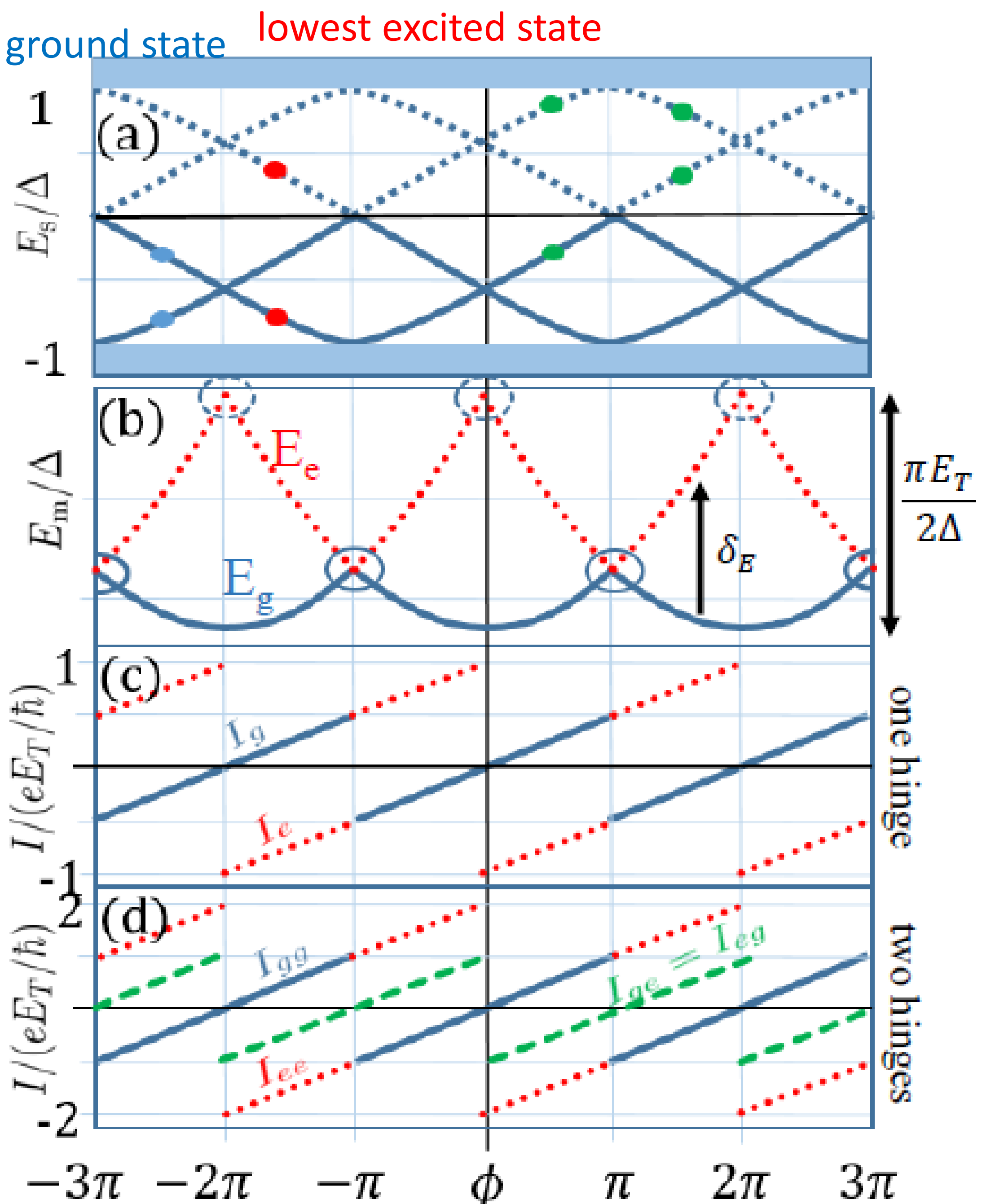
Spectrum and CPR of long helical (topological, QSH) junction

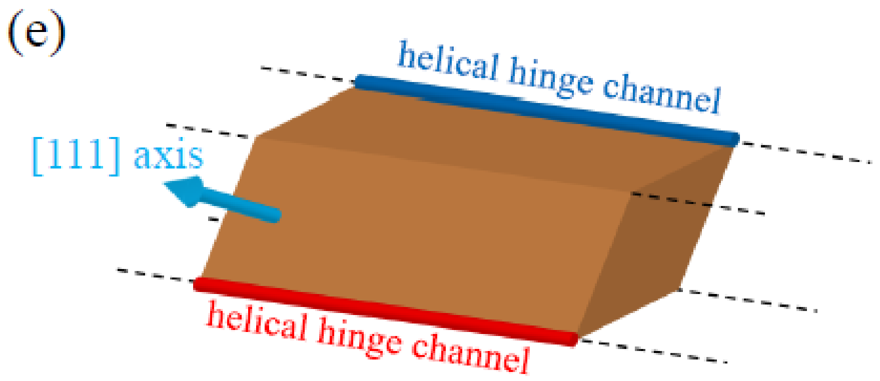
Andreev (single particle excitation) spectrum

Andreev (many-body) spectrum

CPR one helical channel

CPR two helical channels

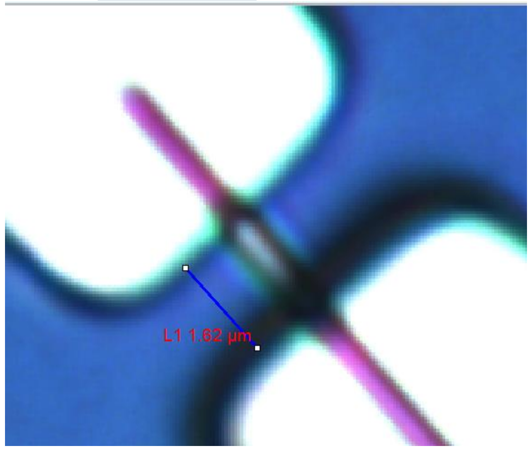




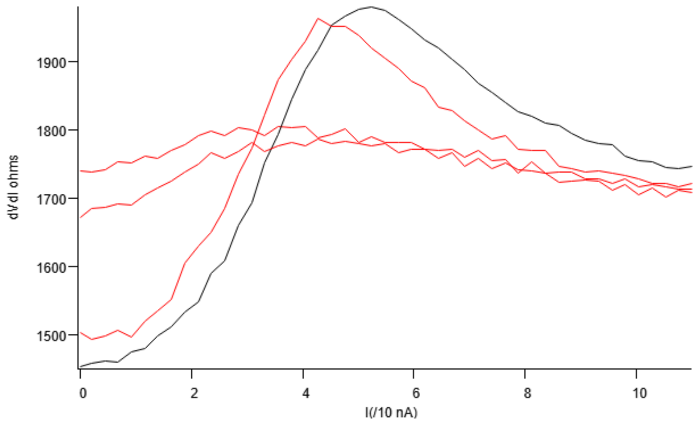
gg,ee,ge:

With two hinges, poisoning in the form of shifted CPR

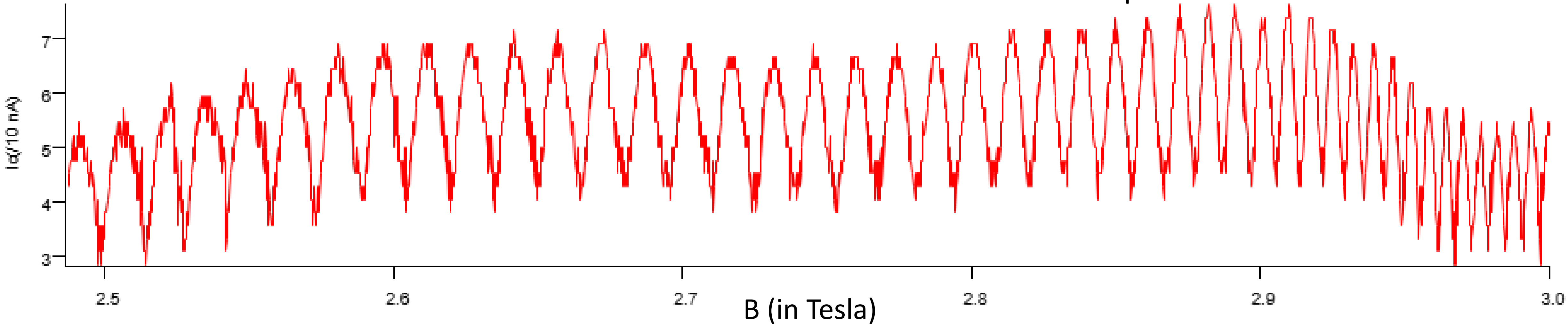
Bismuth with more conventional contacts
(NbN, no Pd, no FIB-assisted W deposition)



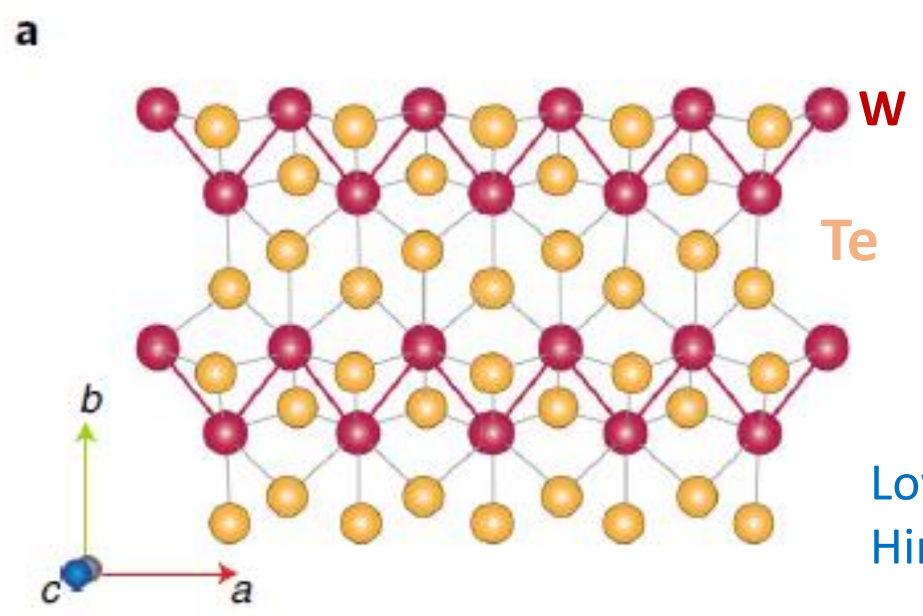
Meydi Ferrier, sabbatical U. Sherbrooke,
Institut Quantique. May 2023



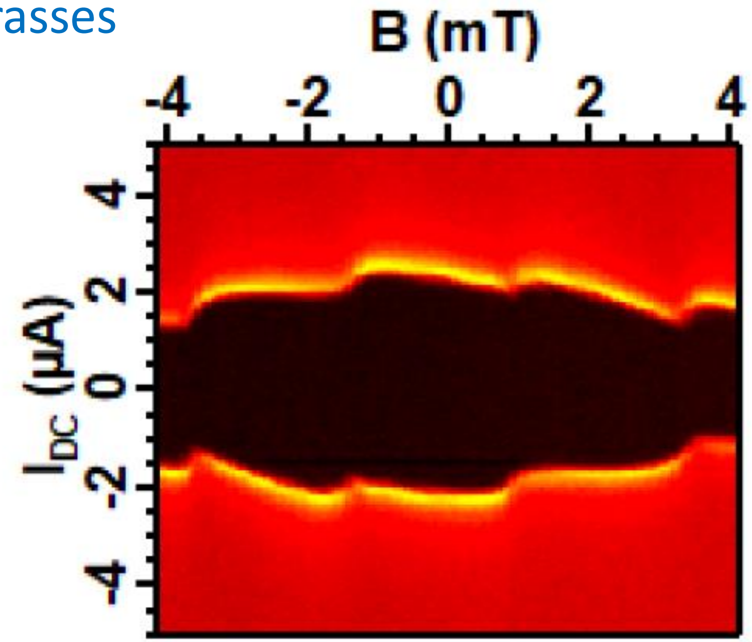
Phase and amplitude modulation: 4 channels?



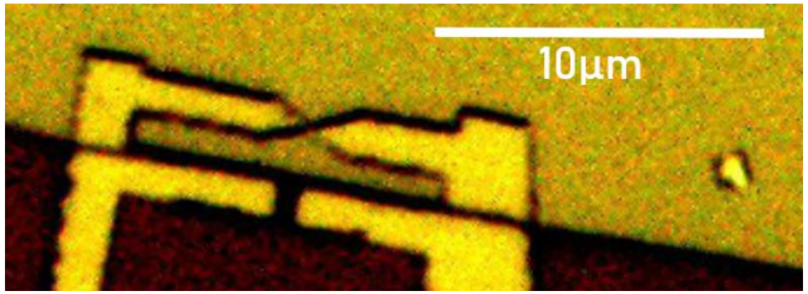
Other systems beyond Bismuth: WTe2 multilayers

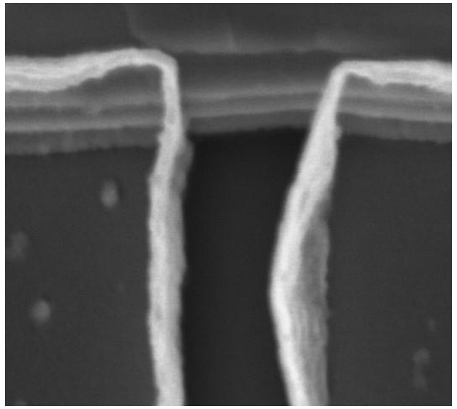


Lower junction: Several terrasses
Hinge states in thin flakes



Asymmetric SQUID (bulk and edge)





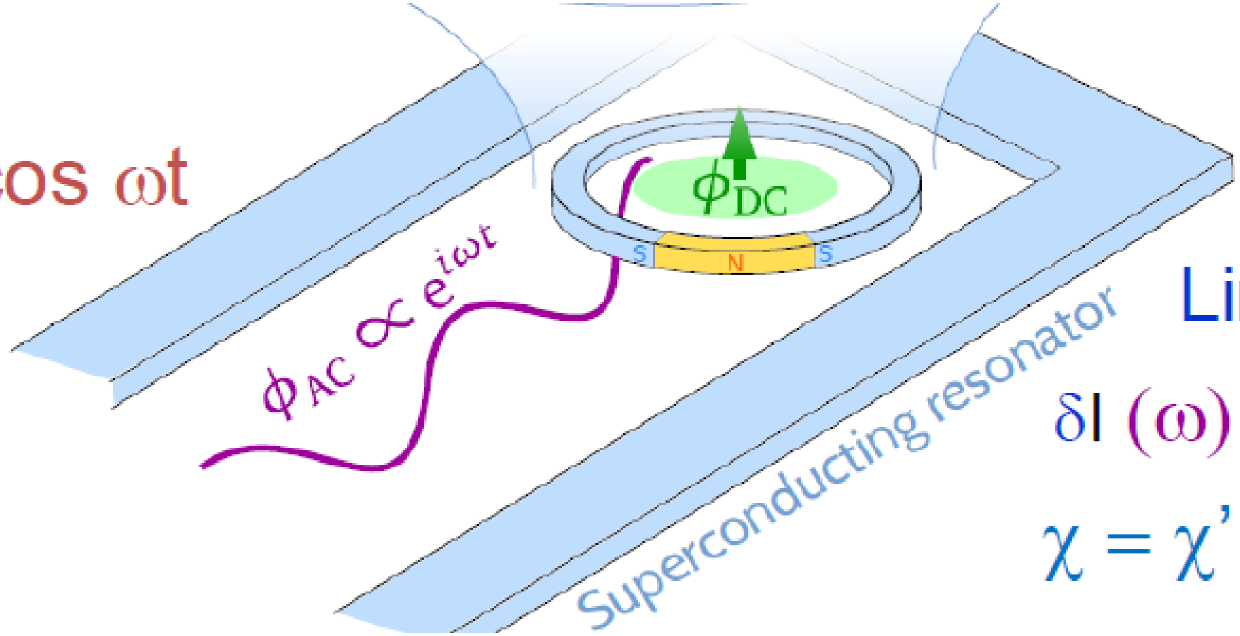
Pd/Nb contacts

Sawtooth current phase relation

Ballu et al., 2022 coll with B. Cava and L. Shoop Princeton

(dc+) ac phase-driven proximity effect

$\varphi(t)=\varphi_{dc}+ \varphi_{ac} \cos \omega t$



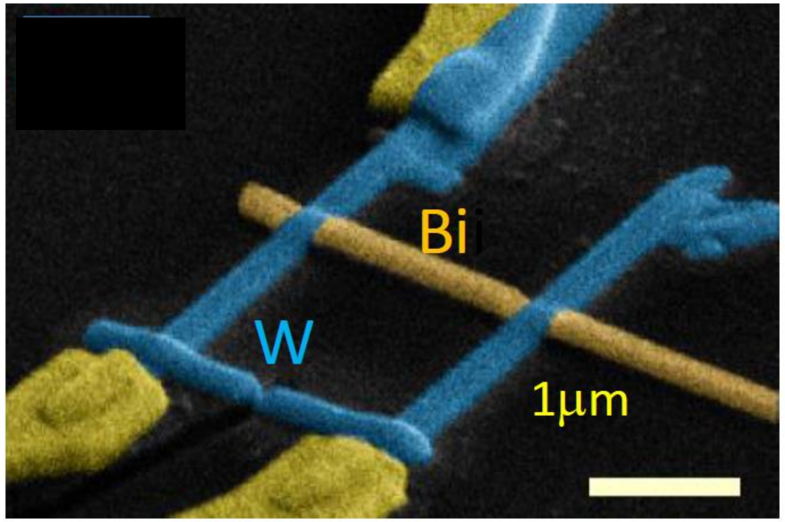
Linear response

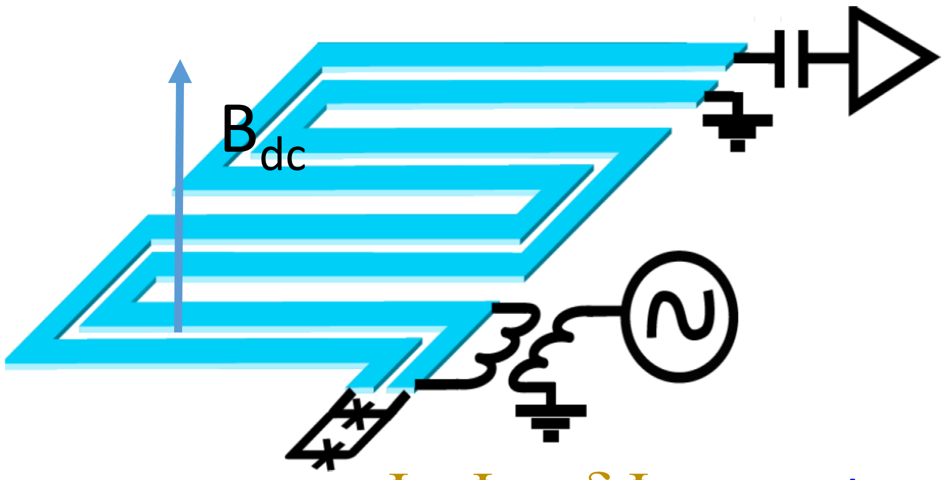
$\delta I (\omega) = \chi(\omega) (\varphi_{ac} \exp -i\omega t$

$\chi = \chi' + i\chi''$

$\chi(\varphi,\omega)$ probes spectrum and dynamics close to equilibrium

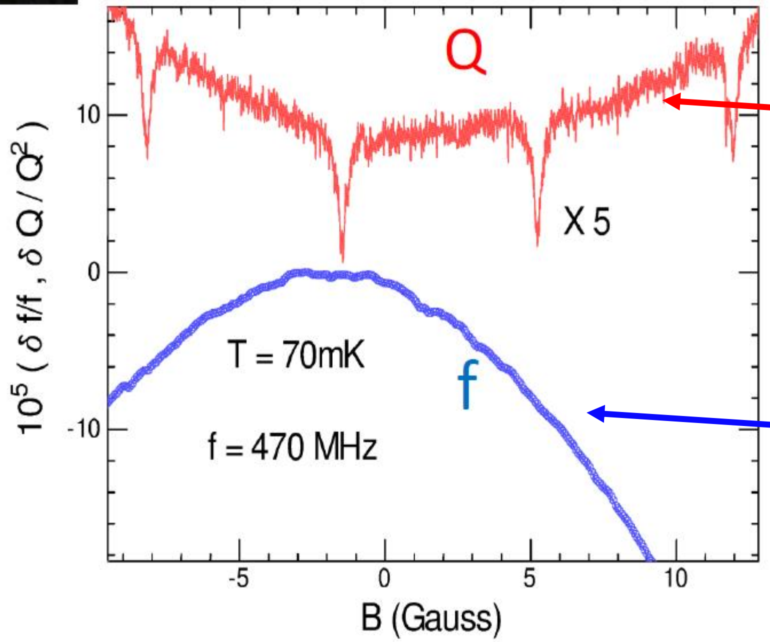
In practice : multimode resonator coupled to S/Bi/S asymmetric SQUID





$\Phi=\Phi_{dc}+\delta\Phi_{ac}\cos\omega t$

Measure δQ(B) and δf(B) at each resonator eigenfrequency



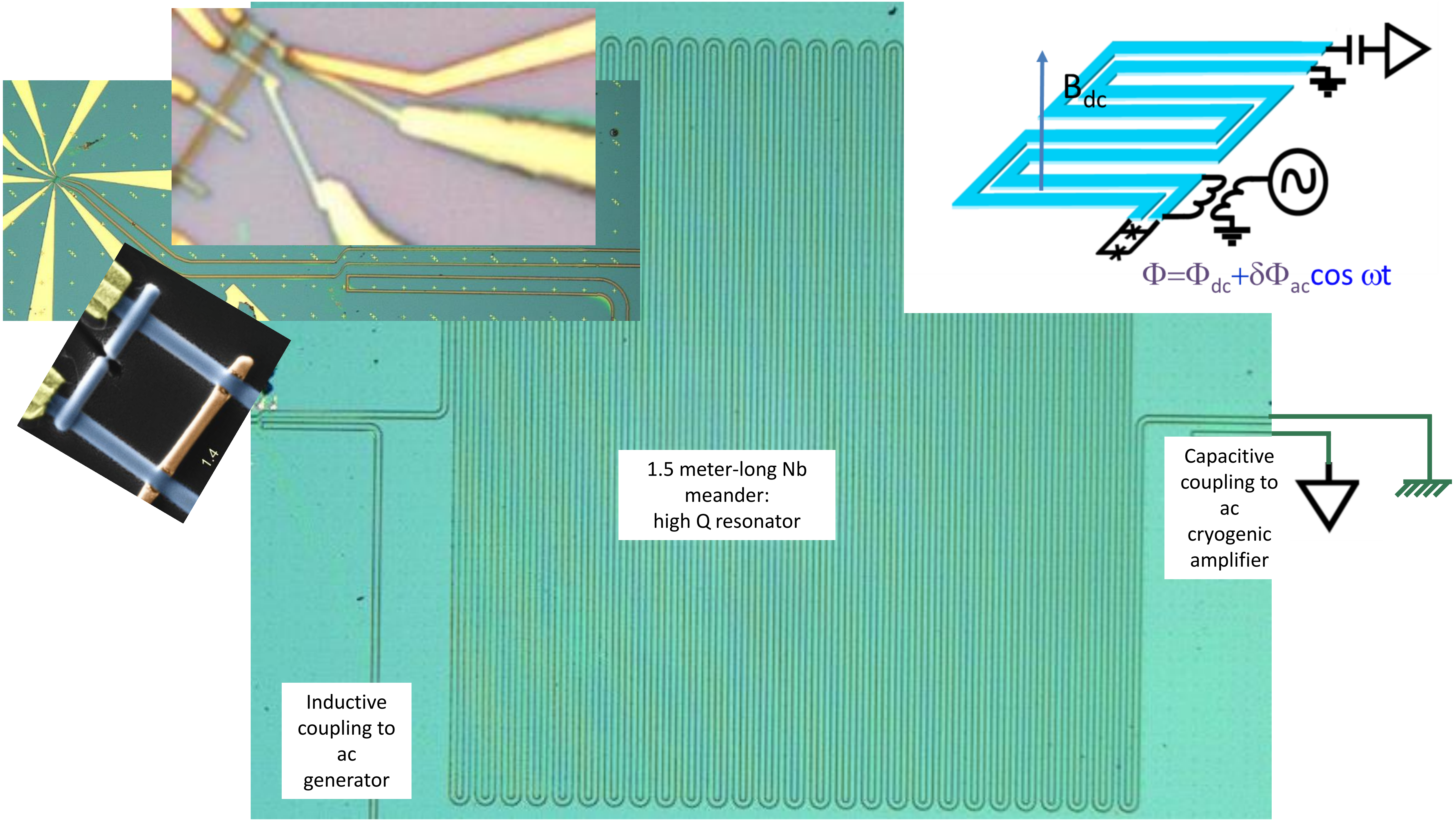
Dissipative response

$$\chi''(\varphi) = \frac{L_R}{L_W^2}\delta\left[\frac{1}{Q_n}\right](\Phi)$$

Non-dissipative response

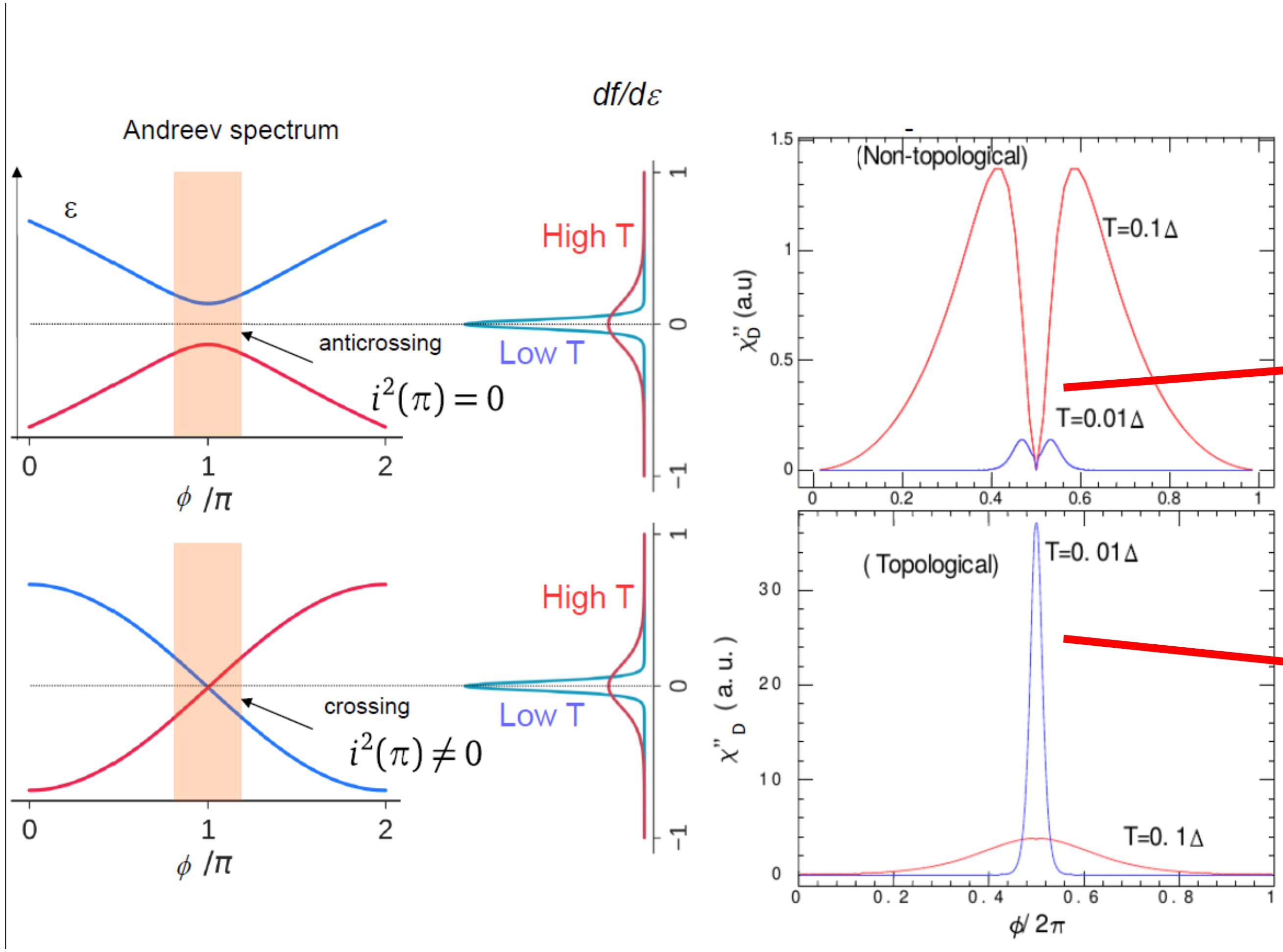
$$\chi'(\varphi) = -\frac{L_R}{L_W^2}\frac{\delta f_n(\Phi)}{2f_n}$$

Absorption peaks at π !



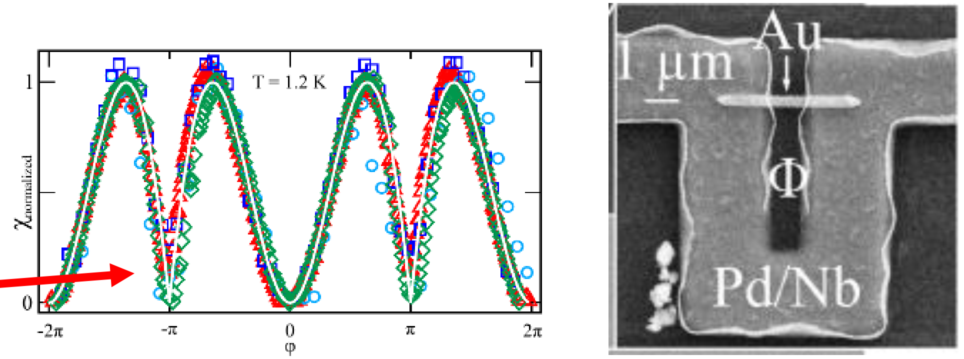
B_{dc}
$\Phi=\Phi_{dc}+\delta\Phi_{ac}\cos\omega t$
1.4
1.5 meter-long Nb meander: high Q resonator
Capacitive coupling to ac cryogenic amplifier
Inductive coupling to ac generator

Comparison of ac susceptibility of S/Bi/S and S/diffusive Au/S (albeit different temperature ranges)



In SNS: zero absorption at π!

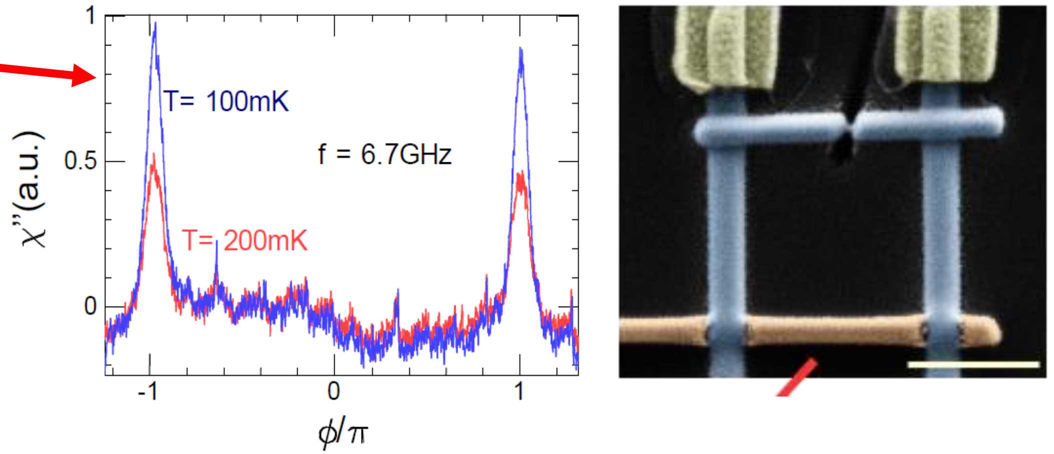
S/diffusive Au/S



Dassonneville2013

In S/Bi/S:max absorption at π!

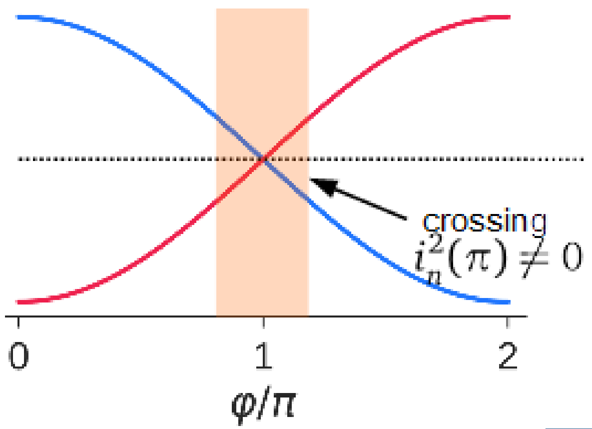
S/Bi/S



Murani2019

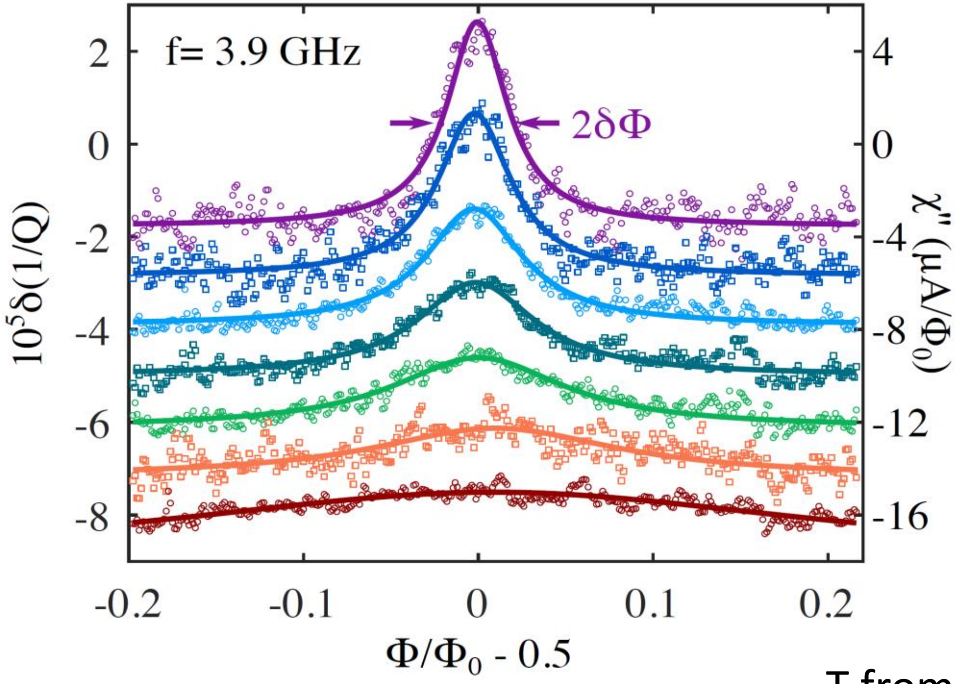
T dependence of absorption peaks at φ=π OK with protected crossing

$\delta\,(1/Q) = L_c^2 / L_R\, \chi''$



$$\underbrace{\sum_n \frac{\omega}{\omega + i\gamma_{nn}} \left(\frac{\partial \epsilon_n}{\partial \Phi}\right)^2 \frac{\partial f(\epsilon_n)}{\partial \epsilon_n}}_{\chi_D}$$

i_n^2



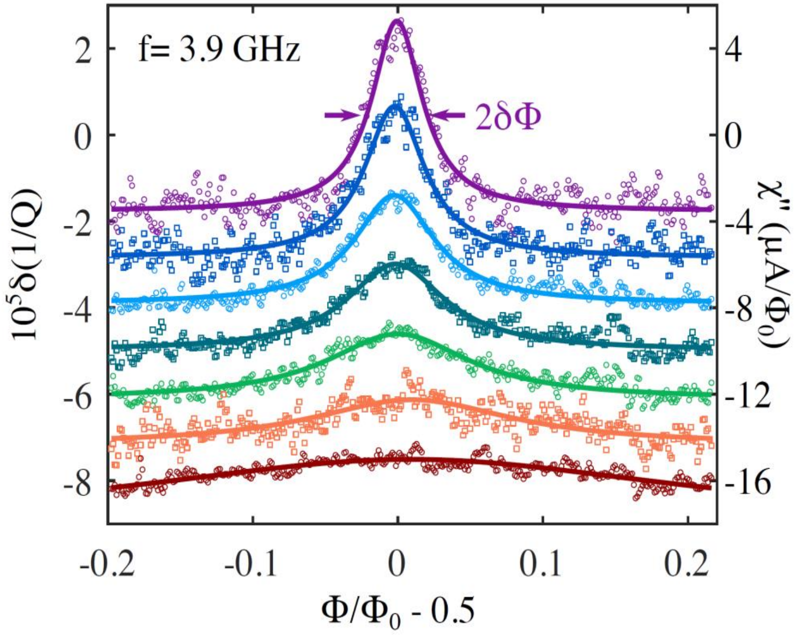
$$\chi''_D = i_0^2 \frac{\omega\gamma}{\omega^2 + \gamma^2} \frac{1}{2T\cosh^2\left(\frac{\epsilon_T}{2T}(\varphi/\pi - 1)\right)}$$

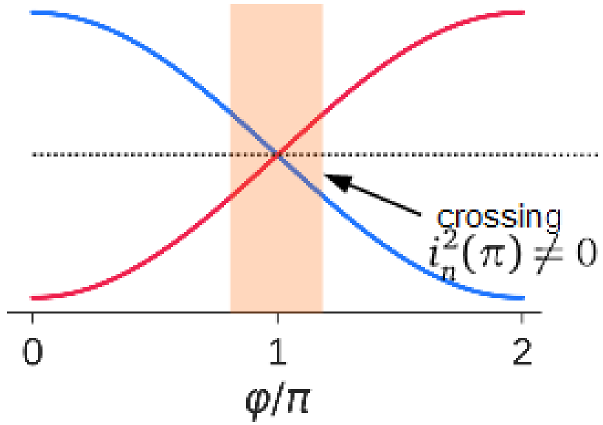
$i_n = i_0 = E_{Th}/\Phi_0$

T from 1K to 70 mK

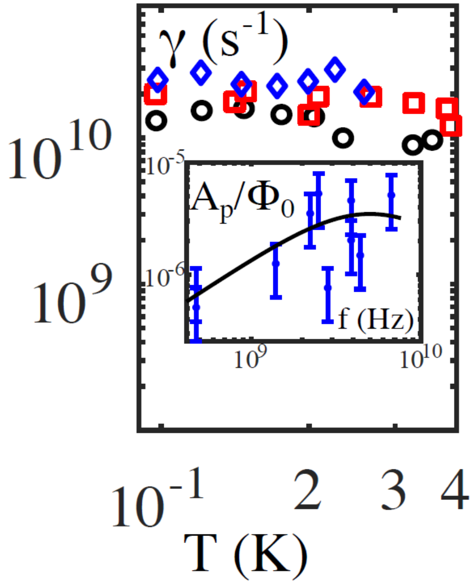
This is the thermal noise of a QSH insulator (Fu Kane)!

But fast relaxation





$$\chi''_{\mathrm{D}} = i_0^2 \frac{\omega\gamma}{\omega^2+\gamma^2} \frac{1}{2T\cosh^2\left(\frac{\epsilon_T}{2T}(\varphi/\pi - 1)\right)}$$

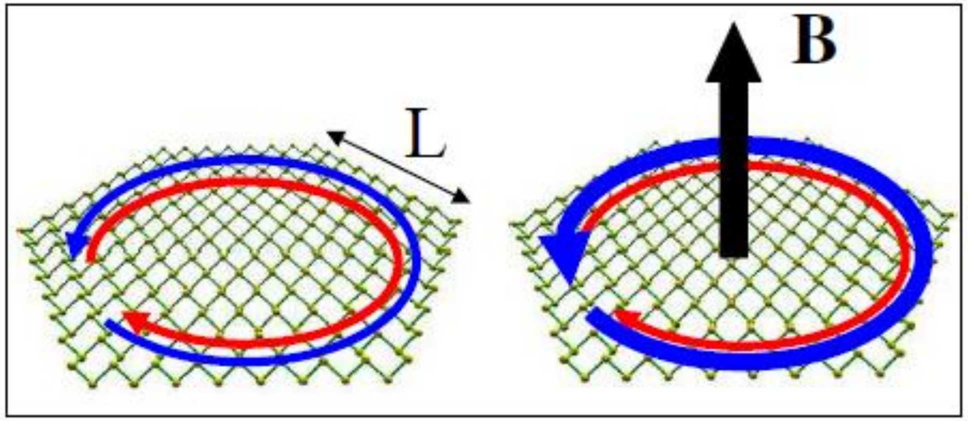


- Frequency dependence gives γ ~ 1ns^{-1} : Fast poisoning!
  Due to soft gap, quasiparticles, broadband environment

Enabled us to see a response, but room for improvement…

- Protected crossing better than 30 mK (experimental resolution)

Persistent (charge) current best discriminator?



- Take a platelet (no need for a ring), no leads

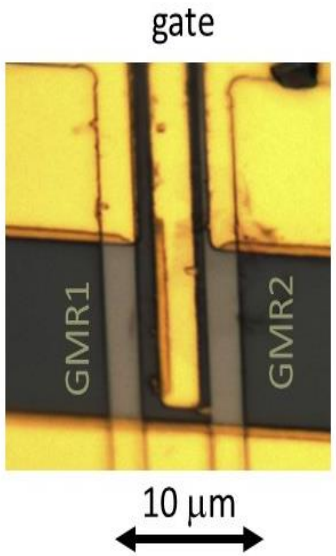
- Diffusive states have tiny persistent current ~ evF/L (le/L)
 (as if only one diffusive channel=)

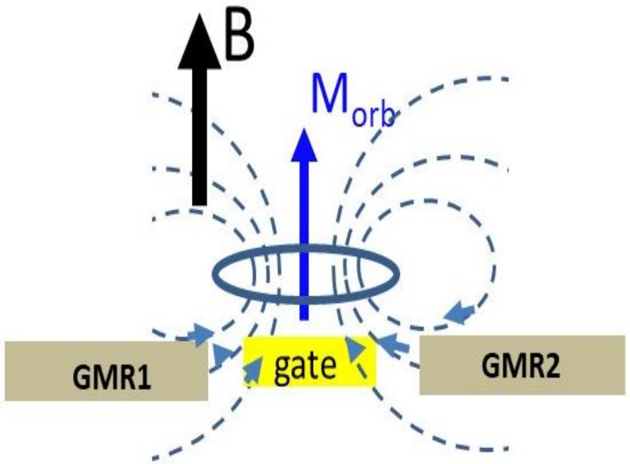
- -1D edge states have evF/L: 100 nA

- Only edge states would have a well-defined period

P. Potasz and J. Fernández-Rossier, Nano Lett. (2015).

We already have the magnetic probe ready: GMR detector





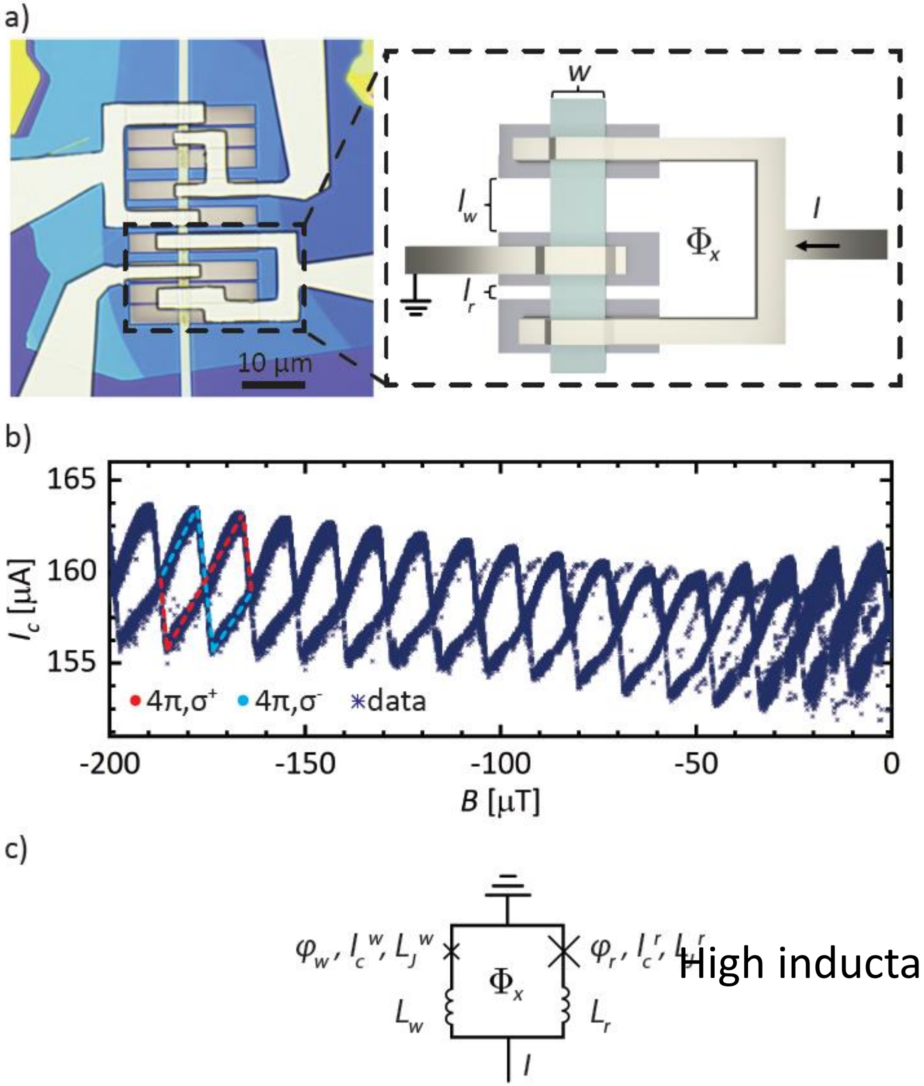
Other scenarios for interpretation of same data?

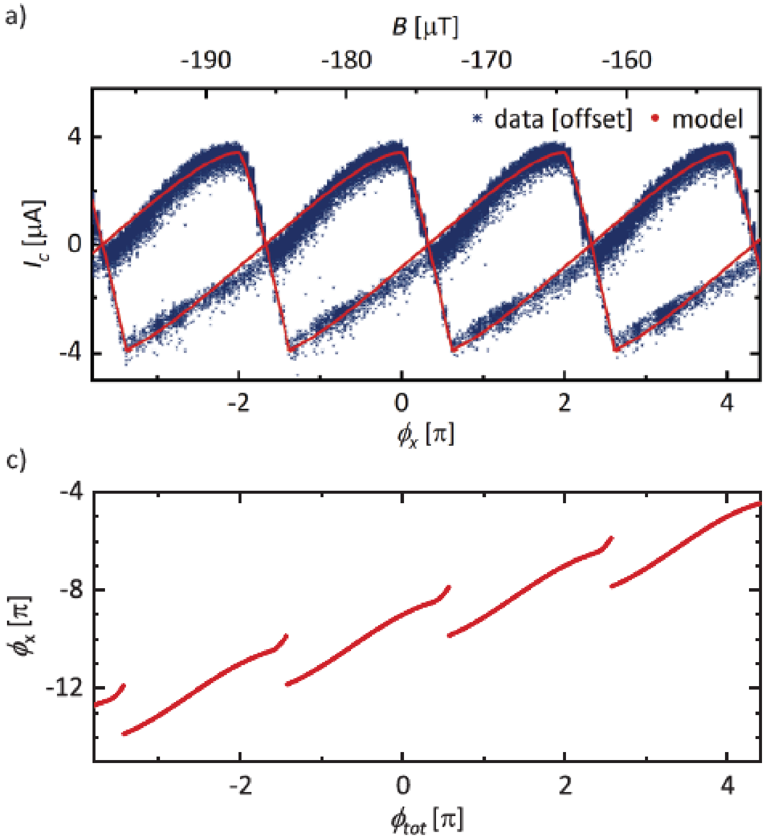
- Inductance in series with weak junction can induce multivalued Ic and distort CPR

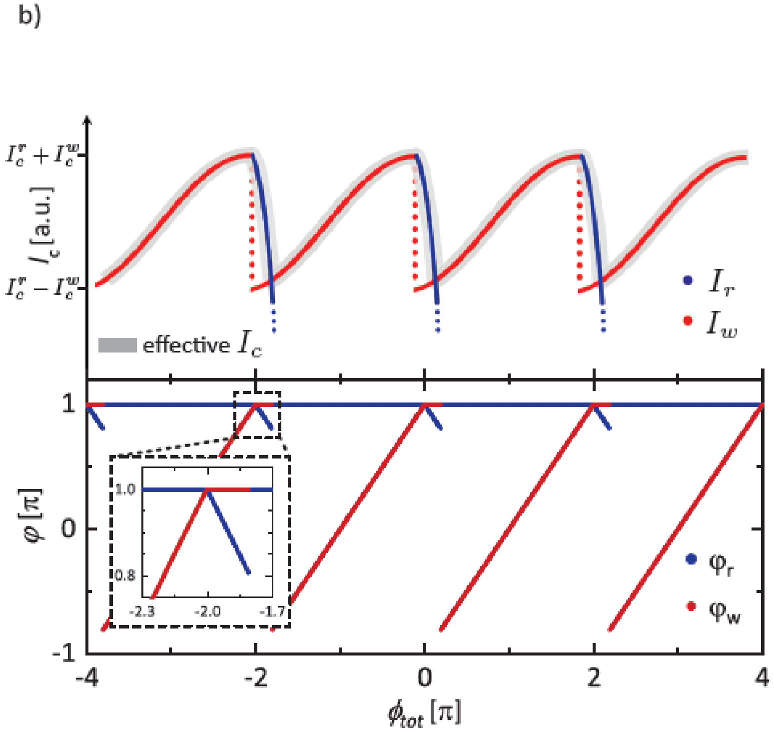
- Asymmetric inductance can mimic asymmetric SQUID (A. Bernard, PhD thesis). But sharpness of CPR is robust.

Current-phase relation of a WTe_2 Josephson junction

Martin Endres,[1,*] Artem Kononov,[1,*] Hasitha Suriya Arachchige,[2] Jiaqiang Yan,[2,3] David Mandrus,[4,2,3] Kenji Watanabe,[5] Takashi Taniguchi,[6] and Christian Schönenberger[1,7,†]

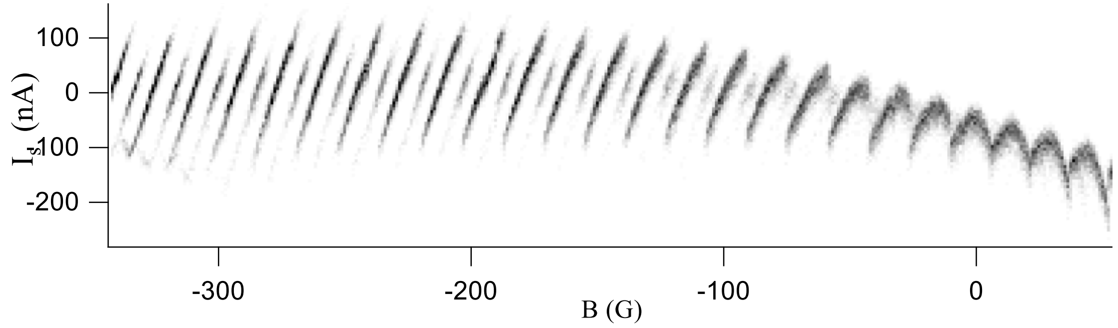


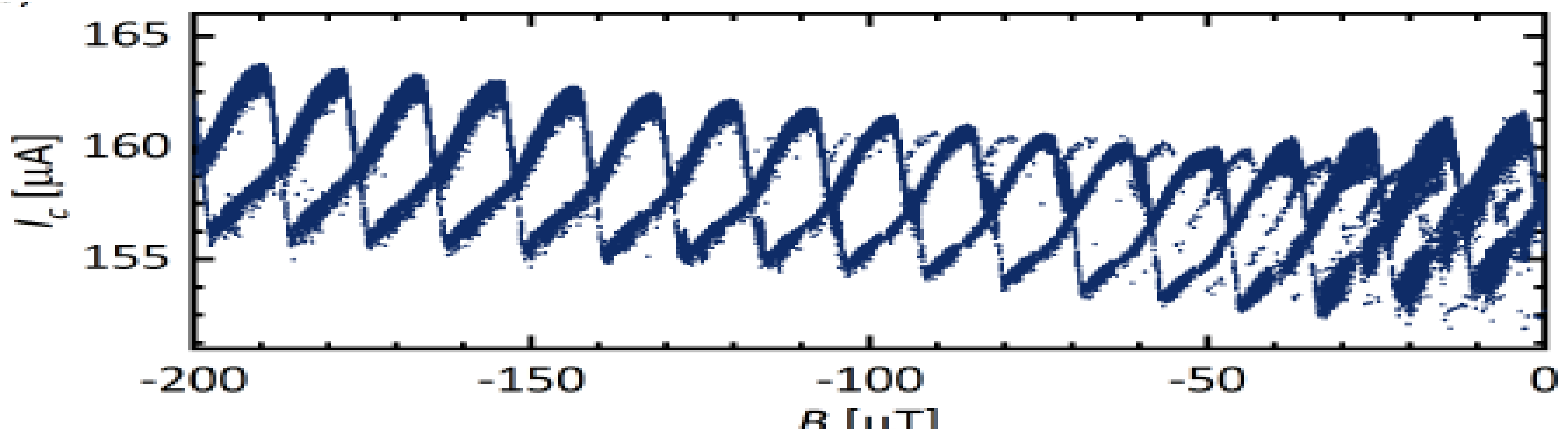




High inductance in weak branch causes multivalued phase, bistability, need to reconstruct CPR

But very different looking data…





$$I(\phi_1, \phi_2) = i_1(\phi_1) + i_2(\phi_2)$$

$$\phi_1 - \phi_2 = \frac{2\pi}{\Phi_0}\Phi_{int} = \frac{2\pi}{\Phi_0}(\Phi_{ext} - L_1 i_1 + L_2 i_2)$$

Asymmetric inductance can mimic asymmet SQUID (A. Berna PhD thesis). But sharpness of CP robust.

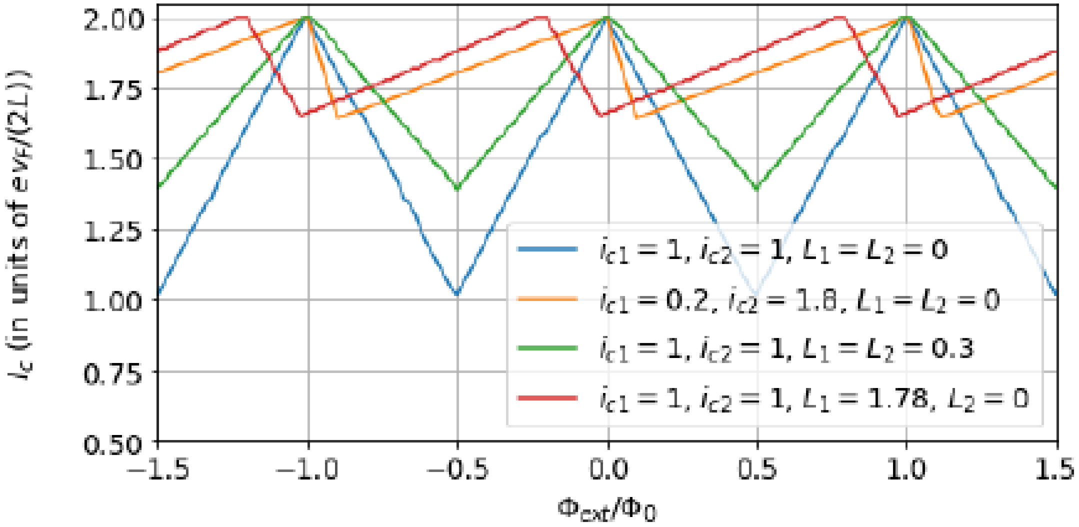
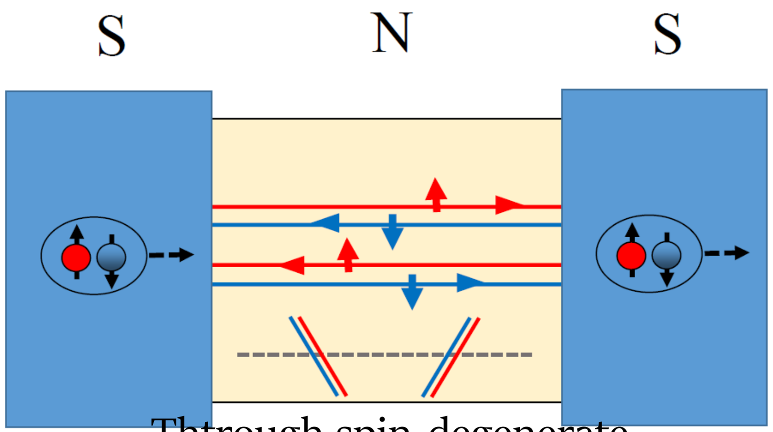
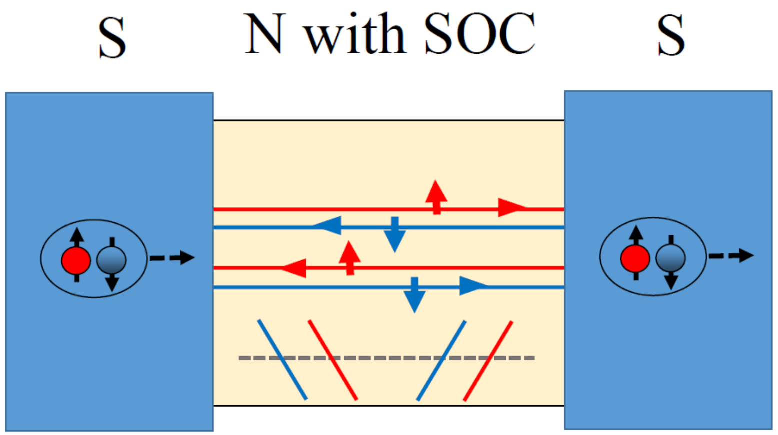


Figure 1.34 – Critical current of a DC SQUID with two long ballistic (or helical) junctions as a function of magnetic flux Φ_{ext} applied through the SQUID surface via an external magnetic field. The junctions are labelled 1 and 2, with critical currents i_{c1} and i_{c2}, and are in series with inductances L_1 and L_2, respectively. $i_{c1,c2}$ are expressed in units of $ev_F/(2L)$, and $L_{1,2}$ are expressed in Φ_0 per unit of current.

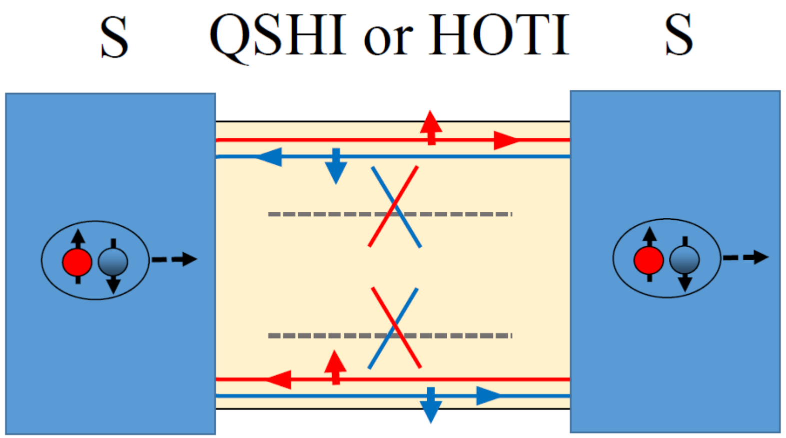
Superconductivity induced in different materials



Thtrough spin-degenerate channels



spin-split Andreev levels



Helical channels

What are the signatures of the topologically-protected helical hinge states of a SOTI junction?