Experimental Signatures of Higher Order Topology

Mesoscopic Physics group (LPS, Université Paris-Saclay)

Higher order Topological Insulators

3D topological insulator
3D insulating bulk
2D Conducting surfaces

2D topological insulator
2D insulating bulk
1D conducting edges

Second Order Topological Insulator
3D insulating bulk
2D insulating surfaces
1D conducting helical « hinges »

Gapless surface state protected by bulk band topology

Topological protection of 1D helical edge states

Spin–momentum locking
Correlation of spin and momentum

Topological protection
Ballistic transport, topological superconductivity
Experimental evidence of higher order topology in condensed matter

and Josephson junctions (*this talk*)

**WTe$_2$**: - monolayer: quantum spin hall
  - multilayer: SOTI, ballistic hinge states
    L. Bugaud’s poster (*today 18h30*)

**Bi$_4$Br$_4$**: SOTI with a high band gap (200meV)
  STM: N. Shumiya *et al*, Nat. Mater. (2022)
  J. Lefeuvre’s Talk (*this morning*)
Bi has inversion ($I$) and 3-fold rotation symmetries ($C_3$) +TRS

Even number of band inversions: $\Rightarrow$ not a topological insulator... to first order

Odd number of band inversions in each $C_3$-subspace $\Rightarrow$ HOTI!


Bulk single-crystal Bi is not an insulator but a semi-metal

topologically-protected helical hinge states
diffusive semi-metallic bulk states ($\lambda_{F,B} \sim 50 \, nm$)
diffusive metallic surface states ($\lambda_{F,S} \sim 5 \, nm$)
STM probe of edge states in Bismuth


What about transport experiment?

Zhang et al, arXiv:2303.06722
Monocrystalline Bismuth nanowires

High quality single crystals
Sputtering, buffer layer of Fe or V
(A. Kasumov)

Diameter ~ 100-200 nm

Low magnification,
Transmission Electron Microscope

High resolution TEM
Superconducting electrodes: C and Ga-doped amorphous tungsten 200 nm thick and wide

Great superconducting properties: $T_c \approx 4$ K, $\Delta \approx 0.8$ meV, $H_c \approx 12$ Tesla!
Transport in normal state: dominated by surface states

Diffusive surfaces states carry the normal current

Probe supercurrent to enhance visibility of ballistic/topological states
Andreev Bound States in a phase-biased SNS junction

Resonance condition on accumulated phase:
Andreev Bound States with eigenenergies $\varepsilon_n$.

$$I = \sum_{\varepsilon_n} \frac{\partial \varepsilon_n}{\partial \phi} f(\varepsilon_n)$$
Induced superconductivity enhances contribution of helical states

- Critical current carried by diffusive states is much smaller than critical current carried by ballistic/helical states
  \[ I_{c\,\text{1channel, ballistic}} \sim \min (\Delta, \frac{h v_F}{L}) \frac{h}{e^2} \]
  \[ I_{c\,\text{1channel, diffusive}} \sim \min (\Delta, \frac{h v_F}{L}) \frac{h}{e^2} \left( \frac{l_e^2}{L^2} \right) \]
  100 to 1000 times smaller than ballistic

- In addition, helical channels should have perfect transmission into S (not true of diffusive channels)

Supercurrent mainly determined by the helical edge states
Supercurrent to probe the nature of the normal part

- Amplitude of the current phase relation: critical current
- Measure the current-phase relation?
Josephson interferometry: supercurrent distribution

Many diffusive paths NARROW, diffusive

Fraunhoffer pattern

S/Topo/S HgTe QW,
Quantum Spin Hall

Chiodi et al, PRB (2012)

S/Au wire/S

~ $\Phi_0$/sample area

$\Phi_0$/edge state area

Many paths WIDE (ballistic or diffusive)

$\Phi_0$/sample area

Only 2 paths (edges) ballistic

WIDE, ballistic

Many paths

~ $\Phi_0$/sample area

B

$B$

$B$

$B$

S/Non topo HgTe QW/S,

S/Non topo HgTe QW/S,

S/Au wire/S

$I_c(B) = \int_{-W/2}^{W/2} J(x) \cdot e^{2\pi i LBx/\Phi_0} dx$

$F_0$/sample area

$F_0$/sample area

$F_0$/edge state area

Ballistic

Squid-like

Topo

$S$/HgTe QW,
Quantum Spin Hall
SQUID like behaviour in bismuth nanowires

- Oscillations: supercurrent travels at the two acute wire edges
- High field (Tesla) decay scale: narrow channels (nm!)
- High critical current: well transmitted channels

$\Delta B_z = 100 \text{ G} = \Phi_0/LW : \text{wire area}$

$L_i(\mu A)$ vs $B_z(\text{Gauss})$

$\Phi_0$/sample area

$\Phi_0$/edge state area

Li et al, PRB (2014)

Current-phase relation on very same sample

Add superconducting constriction in parallel

Build an asymmetric SQUID to measure the I(φ) relation
Probing the current-Phase relation with an asymmetric SQUID

Principle of CPR measurement:

phase-biased SNS ring

\[ \varphi = -2\pi \frac{\Phi}{\Phi_0} \]
\[ \Phi = B S, \text{ S loop area} \]

\[ I_c(\text{SQUID}) \approx I_{\text{large}}(\varphi_{\text{max}}) + I_{\text{small}}(\varphi_{\text{max}} + 2\pi \frac{\Phi}{\Phi_0}) \]

Current-Phase relation of \( I_{\text{small}} \)
Supercurrent-versus Phase relation of Bi Josephson junction

\[ I_c (\mu A) \]

\[ \varphi = -2\pi BS/\Phi_0 \]

CPR of Bi nanowire

Critical current of W constriction

1 \, \mu m

Sawtooth-shaped current phase relation: long ballistic!
Two sawtooths?

Two sawtooths?

**Fourier transform**

- Second period

**Ballistic states at two edges**

Conventional versus topological junction

Introducing imperfect transmission $T$

For short junctions

Two helical ABS
- same fermion parity
- both levels can be occupied
- describes 4 states

One helical ABS
- avoided crossing
- crossing protected by parity

Spinless ground state

Spinfull ground states

$\varepsilon(\varphi) = \pm \Delta \sqrt{1 - T \sin^2(\phi/2)}$

$\varepsilon(\varphi) = \pm \Delta \sqrt{T \cos(\phi/2)}$

different fermion parities
- either one or the other level can be occupied
- describes 2 states
**intrinsic dc squid with a bismuth nanoring**

Monocrystalline bismuth nanoring (A. Kasumov, released with laser shock wave)

Kasumov 2005

\[ T_c \sim 4 \text{ K} \]

\[ H_c > 10 \text{ T} \]
Period=16 G, OK with flux through ring
Modulation over background: ~ asymmetric SQUID
Suggests sawtooth CPR

\[ I_{c}^{tot} = I(\phi_{max}) + i(\phi_{max} + 2\pi \frac{B.S}{\phi_0}) \]


⇒ Ballistic (or topologically protected) transport. Can we say more?
Switching current histograms

- Baseline removed
- Sawtooth shape (long ballistic junction)
- For a given phase, one or two possible switching current values

- Baseline removed
- Sawtooth shape (long ballistic junction)
- For a given phase, one or three possible switching current values
Spectrum and CPR of one long helical junction

Andreev spectrum (single particle picture)

Andreev spectrum (many-body picture)

CPR of one helical channel

Difference between non-topological and topological junctions at $\pi$
For phase around $\pi$ $[2\pi]$ : signature of two states
- ground state ($I_g$)
- excited state ($I_e$)

Evidence of partial parity conservation?
Two helical hinge states and switching current histograms

- Two hinges in the ground state: $I_{gg}$
- Two hinges in the excited state: $I_{ee}$
- One hinge in the ground state, the other in the excited state: $I_{eg}, I_{ge}$
Occupation probability and relaxation times

**EXPERIMENT**

- "two states"
- "three states"

**THEORY**: two hinges + master equation + pair and quasiparticle relaxation times

1 quasiparticle from bath
- $gg \rightarrow ge/eg$: poisoning
- $gg \rightarrow ee$: pair excitation or relaxation

\[
t_p = 1.8 \text{ ms}\]
\[
t_1 = 25 \text{ ms}\]
\[
t_2 = 10 \text{ ms}\]

Coll. Y. Peng, Y. Oreg, F. von Hoppen
Slow pair relaxation rate: topological hinge modes?

Slow pair relaxation: $\tau_p \sim \mu$s (instead of $\mu$s in non engineered e.m. environment)

To go from ee->gg or gg->ee, need to split or form a Cooper pair with one quasiparticle from each hinge difficult for hinges further apart than $\xi_s$

$->$ slow pair excitation or relaxation (would be much easier if spin-degenerate edge)

A. Bernard et al, Nat. Phys. (2023)
Conclusion and outlooks

Used induced superconductivity to probe possible SOTI character
- Interference in the critical current ✔
- Supercurrent versus phase relation in a ring geometry ✔
- Switching histograms: long pair relaxation time
- ac susceptibility $\chi = \frac{dI}{d\phi}$ to demonstrate protected crossing at phase $\pi$ (not presented today, Murani et al, Phys.Rev. Lett. (2019)) ✔
- rf spectroscopy of helical Andreev states? PhD Lucas Bugaud

- Without superconductivity: -Detect Edge current in an isolated flake? (new detector of orbital currents) : PhD Matthieu Bard
- Other materials? WTe$_2$ (PhD L. Bugaud and X. Ballu), Bi$_4$Br$_4$ (PhD J. Lefevre)
Andreev spectrum and supercurrent in short ballistic junction

Spectrum: branches of \( \cos(\varphi/2) \)
Few states in gap

\[
I = \sum_{n} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)
\]

\[
I(\varphi) \sim \text{branches of } \sin(\varphi/2) \text{ with jump at } \pi
\]

\[
-2 \arccos \frac{\epsilon}{\Delta_0} \pm \Delta \varphi = 2\pi m
\]
Example: Andreev spectrum and supercurrent in long ballistic SNS junction: $L >> \xi_s = \frac{hV_F}{\Delta}$

- $2\epsilon L_N \frac{\epsilon}{h\nu_F} - 2 \arccos \frac{\epsilon}{\Delta_0} \pm \Delta \phi = 2\pi m$

propagation in $N$

- $I(\phi) \sim$ linear segments with jumps at $\pi$

Sawtooth $I(\phi)$ characteristic of long ballistic spectrum: more states in gap, quasi linear

$I = \sum_{-\infty}^{\infty} \frac{\partial \epsilon_n}{\partial \phi} f(\epsilon_n)$
Influence of disorder on Andreev spectrum and supercurrent

Dc supercurrent versus phase

\[ I = \sum_{-\infty}^{\infty} \frac{\partial\epsilon_n}{\partial \phi} f(\epsilon_n) \]

Disorder lifts Andreev level degeneracy at \( \pi \) and rounds \( I(\phi) \)

ballistic\n\[ I_0 = N_{ch} e v_f / L \]

disordered\n\[ I = I_0 (I_e^2 / L^2) \]
Beyond average switching current: full statistics

- No rounding of CPR:
  ⇒ ballistic over more than 1 µm
  ⇒ suggests topologically protected level crossing

- Histogram much less rounded than average!
- Clear sawtooth behaviour, clear jumps
pair relaxation $\tau_p = 2$ ms (exceptionally long, other systems: $\mu$s)
quasiparticle relaxation (poisoning) = $\tau_{qp} = 10$ ms
$T_{qp} = T_{pairs} = 0.6-1$ K depending on sweep rate
Comparison experiment / theory (field region with very weak poisoning)

\( \tau_{\text{pair}} = 1.8 \text{ ms}, \ \tau_{\text{qp},1} = 25 \text{ ms}, \ \tau_{\text{qp},2} = 250 \text{ ms}, \ T_{\text{qp}} = 15 \text{ mK}, \ T_{\text{pairs}} = 0.6-1.5 \text{ K} \)

Here had to introduce a small gap in the spectrum, and \( T_{\text{qp}} \ll T_{\text{pairs}} \)
Phenomenological model

1 quasiparticle from bath
\( gg \rightarrow ge/eg \): poisoning

energy from bath, split Cooper pair
\( gg \rightarrow ee \)
pair excitation or relaxation

two hinges

Solve rate equations

\[
\frac{dp_{gg}}{dt} = -2\Gamma_{eg\leftarrow gg}p_{gg} + 2\Gamma_{gg\leftarrow eg}p_{eg} - \Gamma_{ee\leftarrow gg}p_{gg} + \Gamma_{gg\leftarrow ee}p_{ee}
\]

\[
\frac{dp_{eg}}{dt} = -\Gamma_{gg\leftarrow eg}p_{eg} + \Gamma_{eg\leftarrow gg}p_{gg} - \Gamma_{ee\leftarrow eg}p_{eg} + \Gamma_{eg\leftarrow ee}p_{ee},
\]

\[
\begin{align*}
\Gamma_{ee\leftarrow eg} &= f(\pm \delta_E(\phi)/k_BT_{qp})/\tau_2 \\
\Gamma_{eg\leftarrow gg} &= f(\pm \delta_E(\phi)/k_BT_{qp})/\tau_1 \\
\Gamma_{ee\leftarrow gg} &= \frac{2\delta_E(\phi)}{E_T\tau_p} n_B \left( \frac{2\delta_E(\phi)}{k_BT_b} \right) \left[ 1 + n_B \left( \frac{2\delta_E(\phi)}{k_BT_b} \right) \right]
\end{align*}
\]

Yields Andreev level occupation probabilities \( p_{gg}, p_{eg}, p_{ee} \)
Phenomenological model

1 quasiparticle from bath
$gg \rightarrow ge/eg$ : poisoning

energy from bath, split Cooper pair
$gg \rightarrow ee$

pair excitation or relaxation

1- Solve rate equations

Andreev level occupation probabilities $p_{gg}, p_{eg}, p_{ee}$

2- Switching probability as sum of state-dependent probability

$$P(I, \phi) = \sum_{l,l' \in \{e,g\}} p_{ll'}(\phi) P_{sw}(I, \phi)$$

smooth step function centered around $l(\phi)$

switching is stochastic

3- Generate histogram and switching current distribution

Get

-Andreev level occupation probabilities $p_{gg}, p_{eg}, p_{ee}$

-pair and quasiparticle relaxation times

-bath temperature(s)
Spectrum and CPR of long helical (topological, QSH) junction

Andreev (single particle excitation) spectrum

Andreev (many-body) spectrum

CPR one helical channel

CPR two helical channels

gg, ee, ge:
With two hinges, poisoning in the form of shifted CPR
Bismuth with more conventional contacts (NbN, no Pd, no FIB-assisted W deposition)

Meydi Ferrier, sabbatical U. Sherbrooke, Institut Quantique. May 2023

Phase and amplitude modulation: 4 channels?
Other systems beyond Bismuth: WTe2 multilayers

Asymmetric SQUID (bulk and edge)

Lower junction: Several terrasses
Hinge states in thin flakes

Pd/Nb contacts

Sawtooth current phase relation

Ballu et al., 2022 coll with B. Cava and L. Shoop Princeton
(dc+) ac phase-driven proximity effect

\[ \phi(t) = \phi_{dc} + \phi_{ac} \cos \omega t \]

\[ \phi_{AC} \propto e^{i\omega t} \]

Linear response

\[ \delta l(\omega) = \chi(\omega) (\phi_{ac} \exp(-i\omega t)) \]

\[ \chi = \chi' + i\chi'' \]

\( \chi(\varphi,\omega) \) probes spectrum and dynamics close to equilibrium
In practice: multimode resonator coupled to S/Bi/S asymmetric SQUID

\[ \Phi = \Phi_{dc} + \delta \Phi_{ac} \cos \omega t \]

Measure \( \delta Q(B) \) and \( \delta f(B) \) at each resonator eigenfrequency.

Absorption peaks at \( \pi \)!

\[ \chi''(\varphi) = \frac{L_R}{L_W^2} \delta \left[ \frac{1}{Q_n} \right] (\Phi) \]

Non-dissipative response

\[ \chi'(\varphi) = -\frac{L_R}{L_W^2} \frac{\delta f_n(\Phi)}{2 f_n} \]

Dissipative response
Inductive coupling to ac generator

Capacitive coupling to ac cryogenic amplifier

1.5 meter-long Nb meander: high Q resonator

\[ \Phi = \Phi_{dc} + \delta \Phi_{ac} \cos \omega t \]
Comparison of ac susceptibility of S/Bi/S and S/diffusive Au/S (albeit different temperature ranges)

In SNS: zero absorption at $\pi$! S/diffusive Au/S

In S/Bi/S: max absorption at $\pi$! S/Bi/S
T dependence of absorption peaks at $\varphi=\pi$ OK with protected crossing

$$\delta (1/Q) = \frac{L_c^2}{L_R} \chi''$$

This is the thermal noise of a QSH insulator (Fu Kane)!
But fast relaxation

Frequency dependence gives $\gamma \sim 1\text{ns}^{-1}$: Fast poisoning! Due to soft gap, quasiparticles, broadband environment Enabled us to see a response, but room for improvement...

Protected crossing better than 30 mK (experimental resolution)
Persistent (charge) current best discriminator?

- Take a platelet (no need for a ring), no leads

- Diffusive states have tiny persistent current $\sim evF/L (l_e/L)$ (as if only one diffusive channel=)

- 1D edge states have $evF/L$: 100 nA

- Only edge states would have a well-defined period


We already have the magnetic probe ready: GMR detector
Other scenarios for interpretation of same data?

- Inductance in series with weak junction can induce multivalued $I_c$ and distort CPR

- Asymmetric inductance can mimic asymmetric SQUID (A. Bernard, PhD thesis). But sharpness of CPR is robust.
High inductance in weak branch causes multivalued phase, bistability, need to reconstruct CPR
But very different looking data...
Asymmetric inductance can mimic asymmetric SQUID (A. Bernard, PhD thesis). But sharpness of CP robust.

\[ I(\phi_1, \phi_2) = i_1(\phi_1) + i_2(\phi_2) \]

\[ \phi_1 - \phi_2 = \frac{2\pi}{\Phi_0} \Phi_{int} = \frac{2\pi}{\Phi_0} (\Phi_{ext} - L_1 i_1 + L_2 i_2) \]

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![Graph](image)

Figure 1.34 – Critical current of a DC SQUID with two long ballistic (or helical) junctions as a function of magnetic flux \( \Phi_{ext} \) applied through the SQUID surface via an external magnetic field. The junctions are labelled 1 and 2, with critical currents \( i_{c1} \) and \( i_{c2} \), and are in series with inductances \( L_1 \) and \( L_2 \), respectively. \( i_{c1,c2} \) are expressed in units of \( e\nu_F/(2L) \), and \( L_{1,2} \) are expressed in \( \Phi_0 \) per unit of current.
Superconductivity induced in different materials

What are the signatures of the topologically-protected helical hinge states of a SOTI junction?