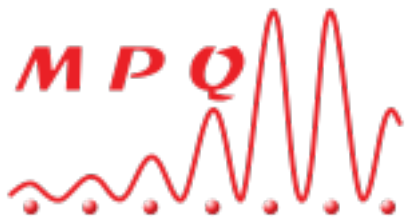


# The wonders of moiré graphene materials

Christophe Mora



Laboratoire Matériaux et Phénomènes Quantiques

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# Outline

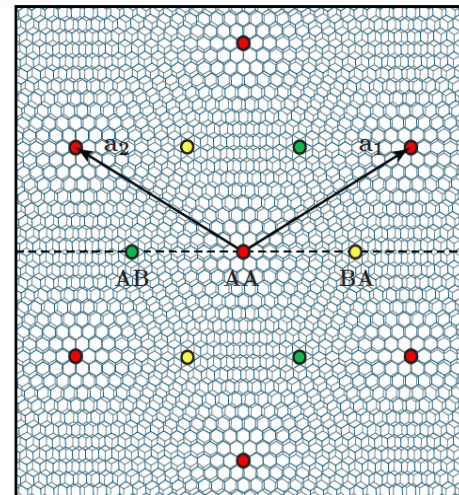
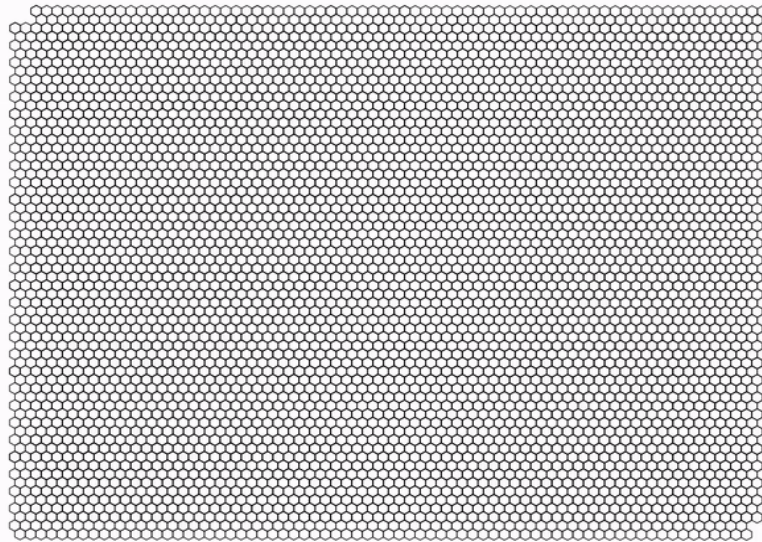
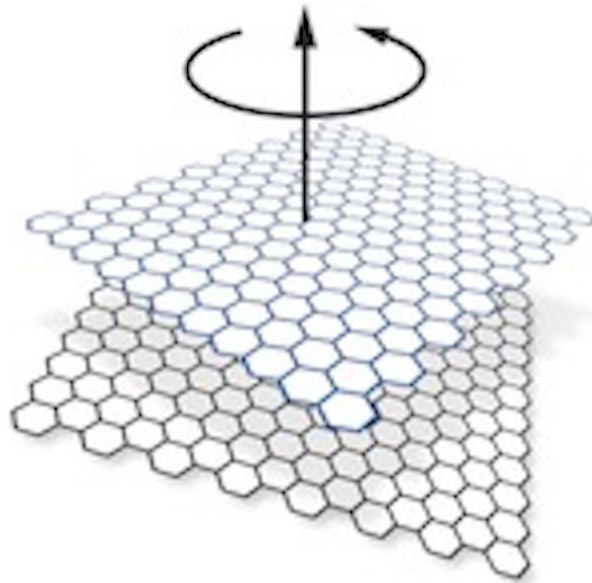
- I. Twisted bilayer graphene: superconductivity, correlations and topology
- II. Chern mosaic in a trilayer geometry

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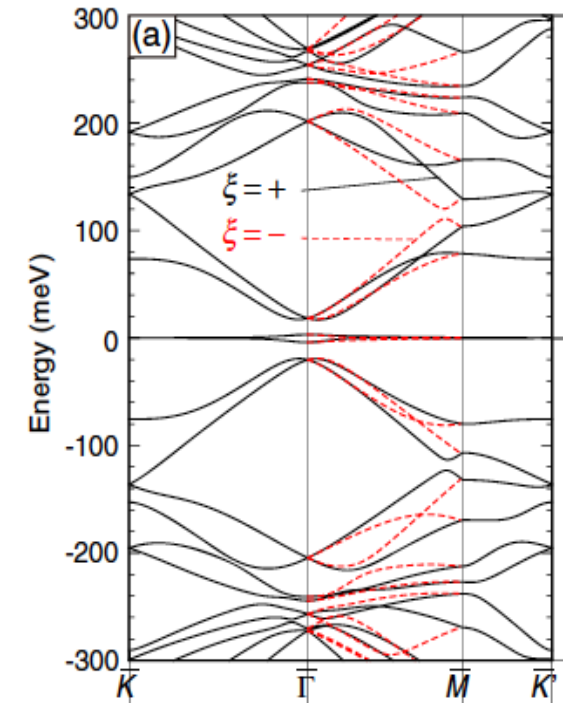
# Twisted bilayer graphene: superconductivity, correlation and topology

# Twisted bilayer graphene

Cao et al., Nature 2018 (MIT)



Moiré pattern



$$\theta = 1.05^\circ$$

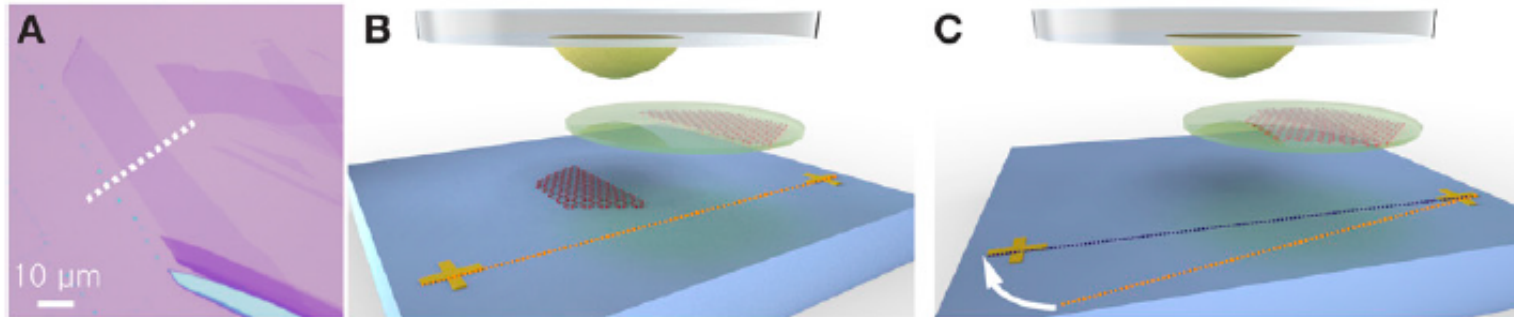
Band folding: electrons are slowed down

Electronic interactions (correlations) prevail

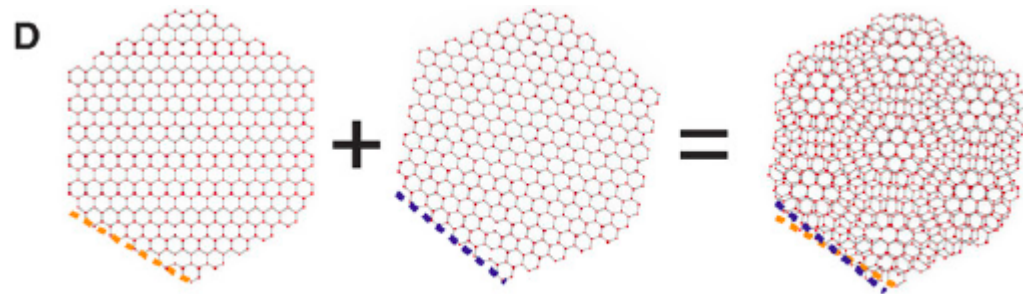
Lu, Efetov, Nature 2019

Yankowitz, Science 2019 (Colombia)

# Fabrication - rotation



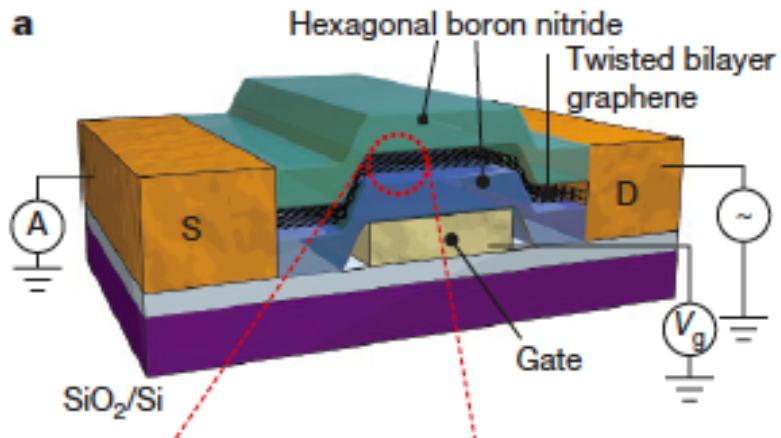
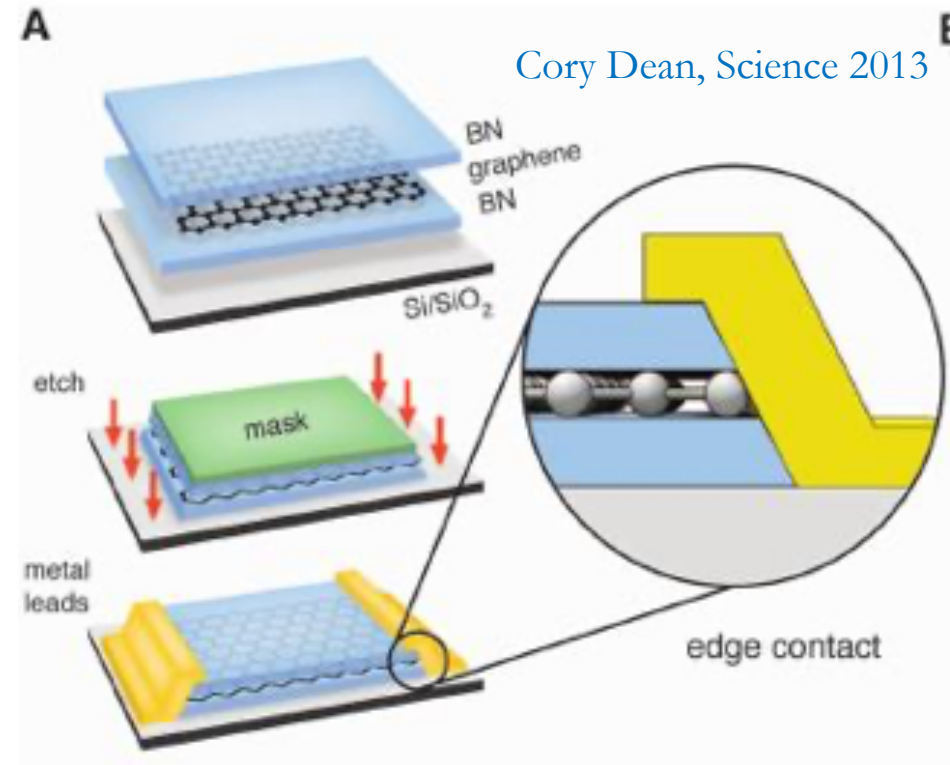
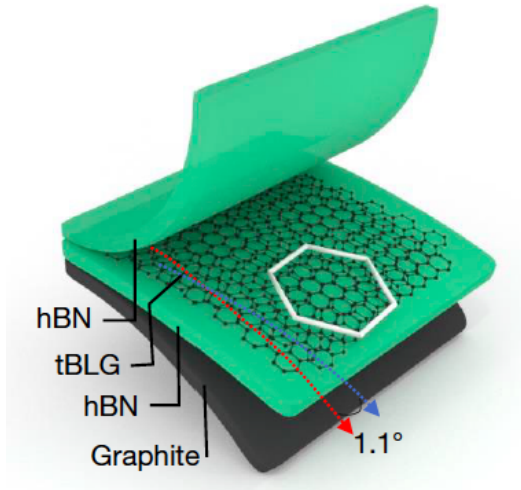
Hemispherical handle (polymer) picks up sequentially hBN substrate and graphene layers



Rotation between two graphene layer pick-up steps

Kim, Tutuc, PNAS 2017, Nano Lett. 2016

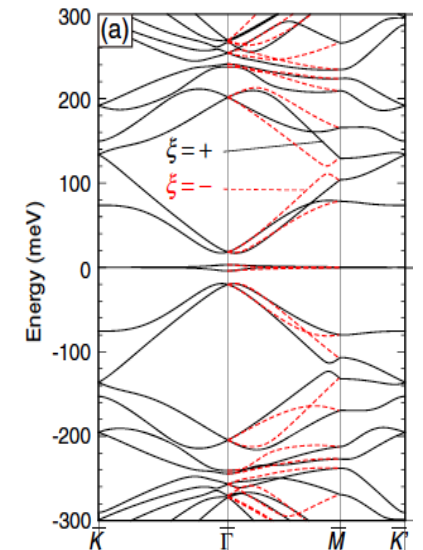
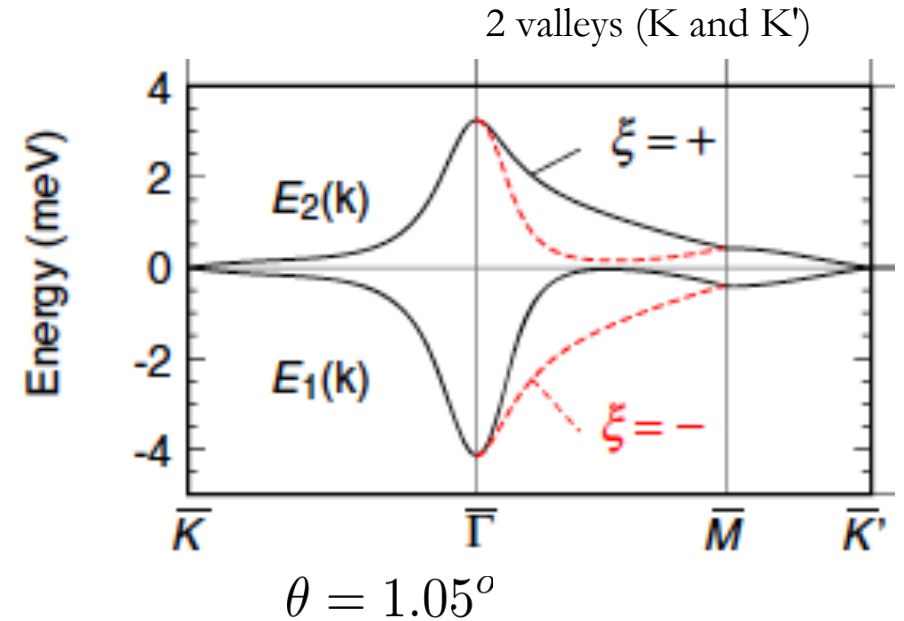
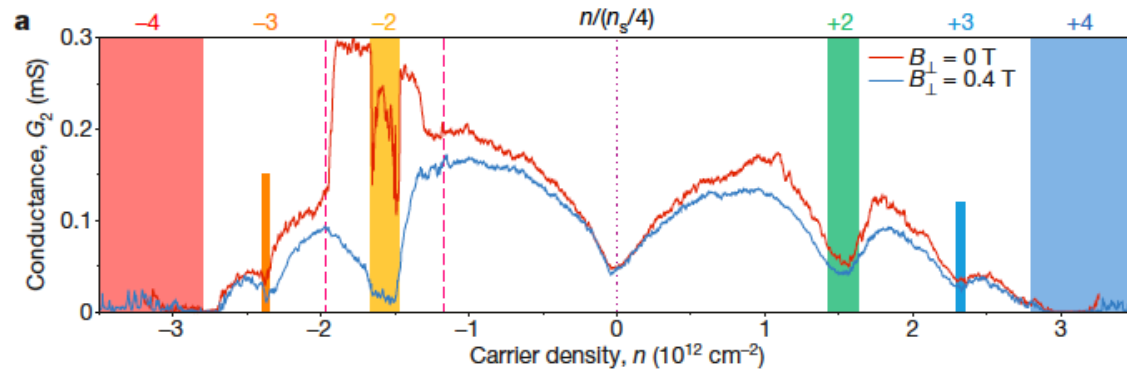
# Fabrication - encapsulation



Cao et al., Nature 2018 (MIT)

# Flat bands

Cao et al., Nature 2018 (MIT)

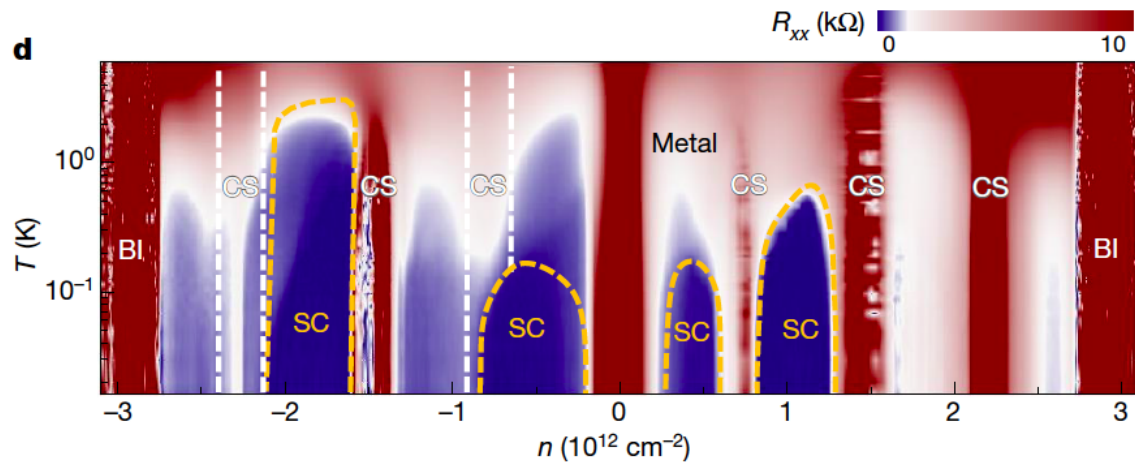
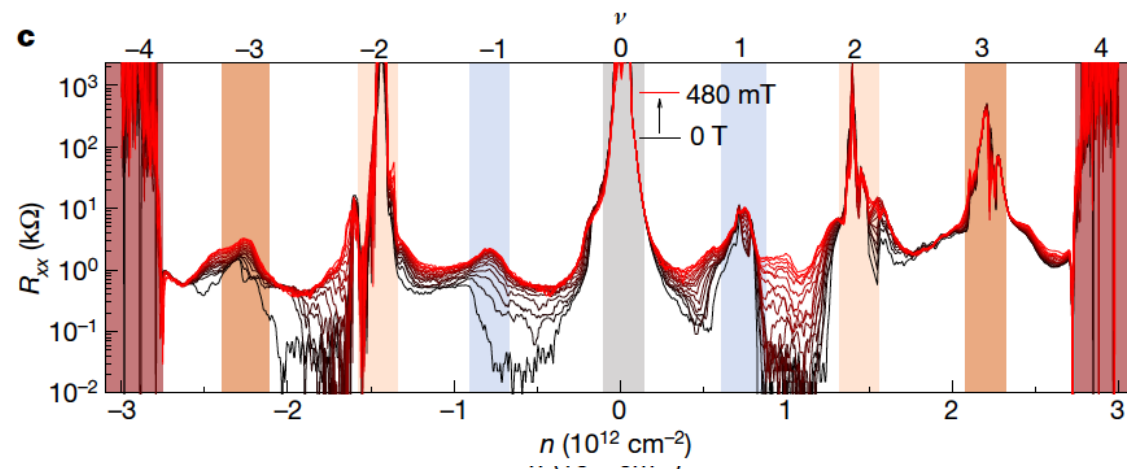


Magic angle where band is relatively non-dispersive

Gate voltage fills the Moiré band with four electrons (spin + valley)

Conductance dip (resistance peak) at each integer filling

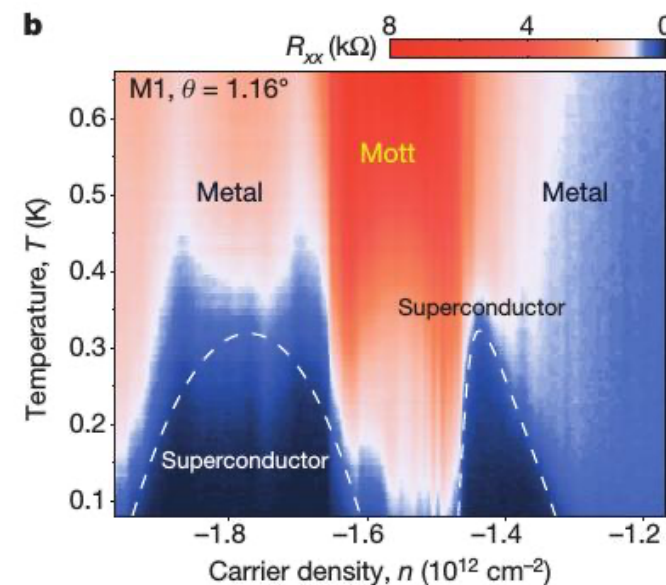
# Phase diagram



Lu, Efetov, Nature 2019

More complete phase diagram, contains

- 1) insulating phases - driven by interactions
- 2) superconducting regions



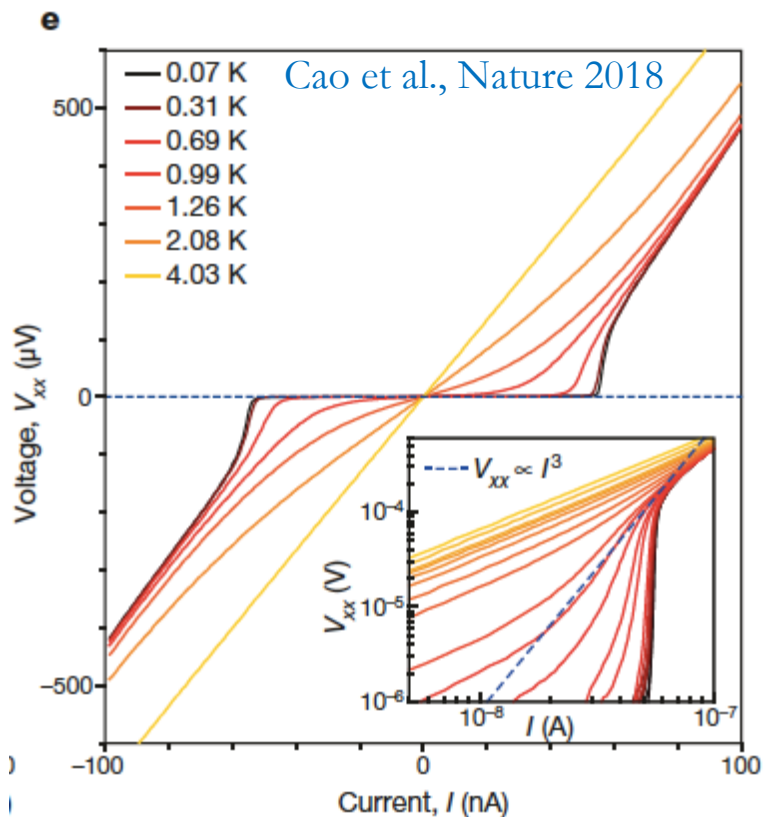
Cao et al., Nature 2018 (MIT)

Yankowitz, Science 2019 (Colombia)

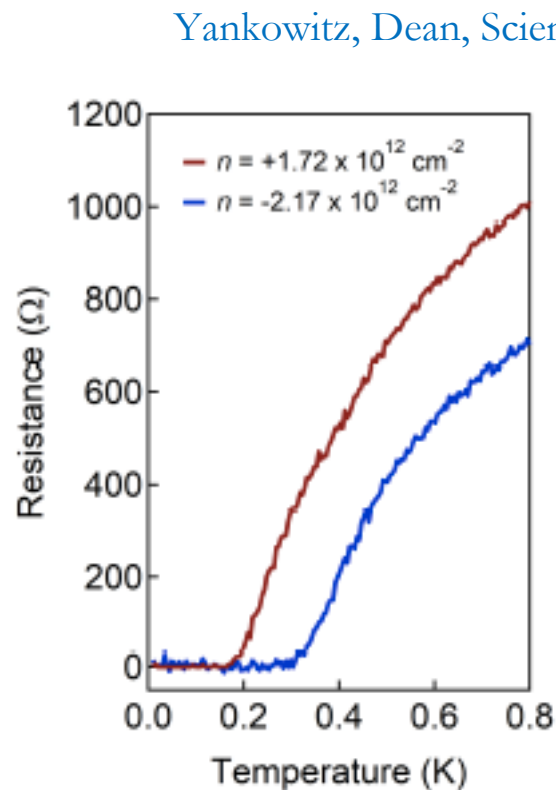


# (unconventional) superconductivity

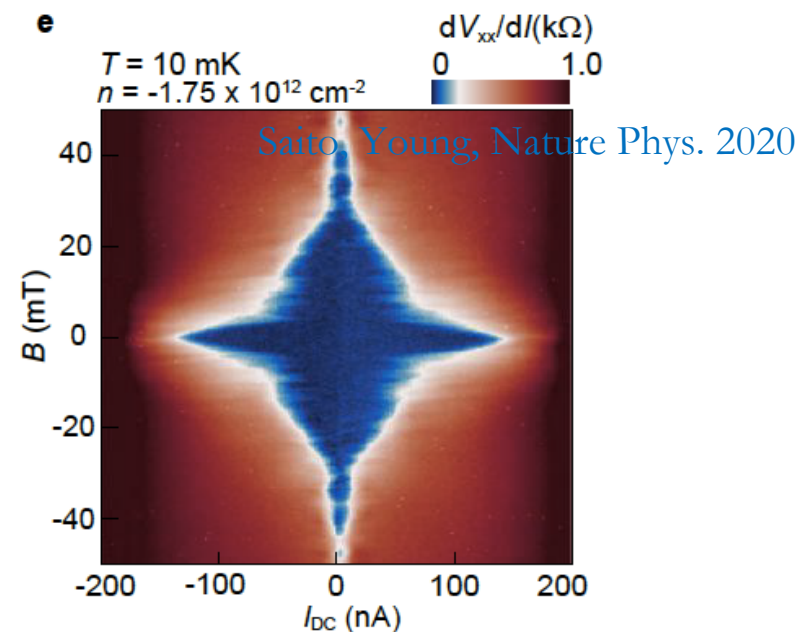
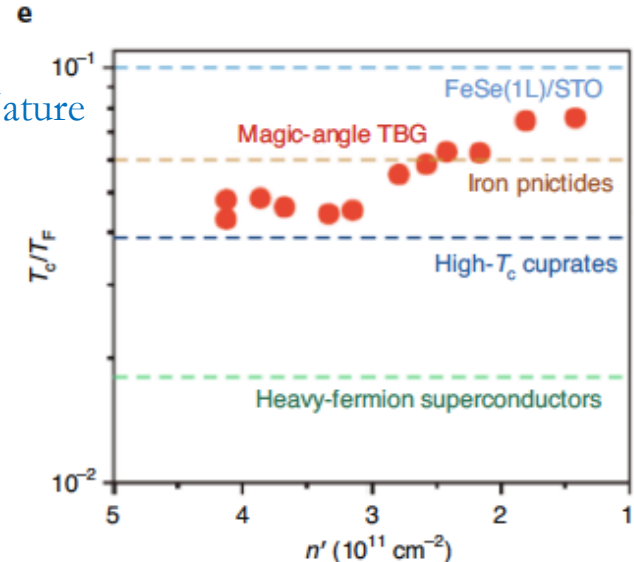
Andrei, MacDonald, Nature Mat. 2020



Non-linear BKT resistance

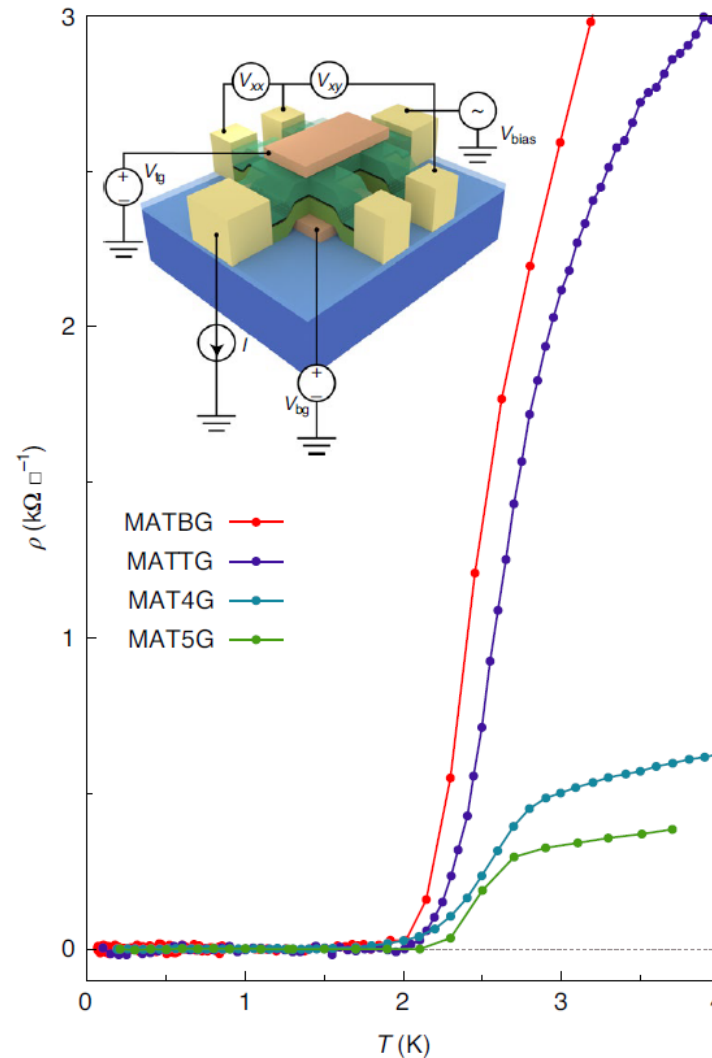
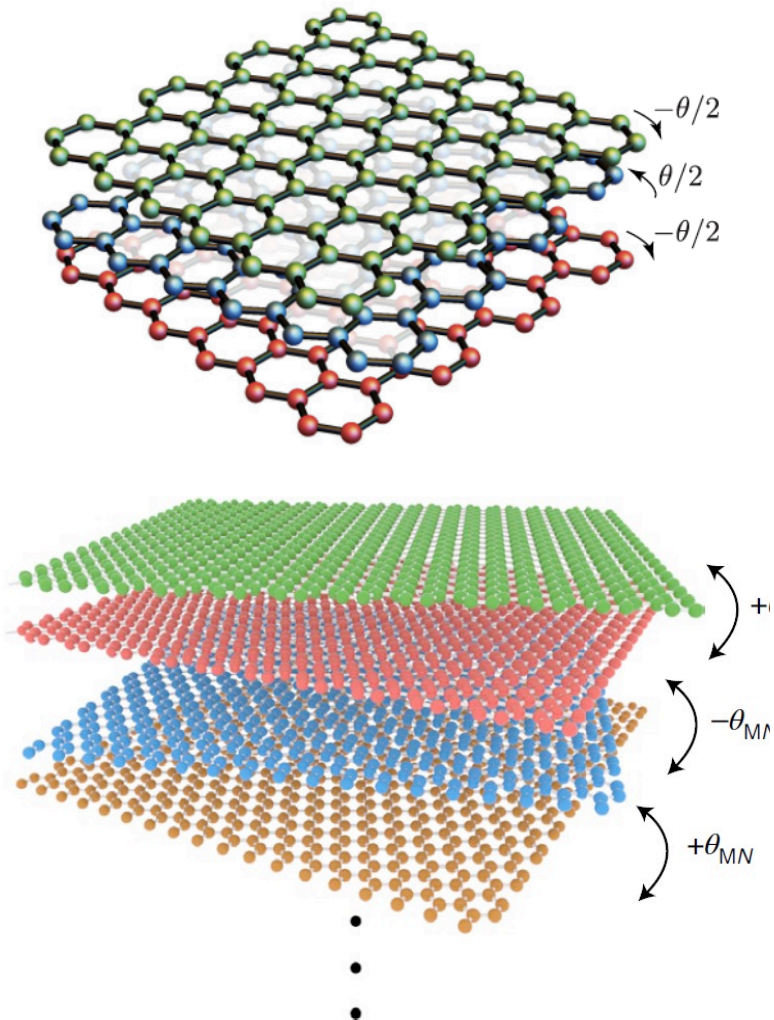


Vanishing resistance



Fraunhofer oscillations

# Multi-layer twisted graphene



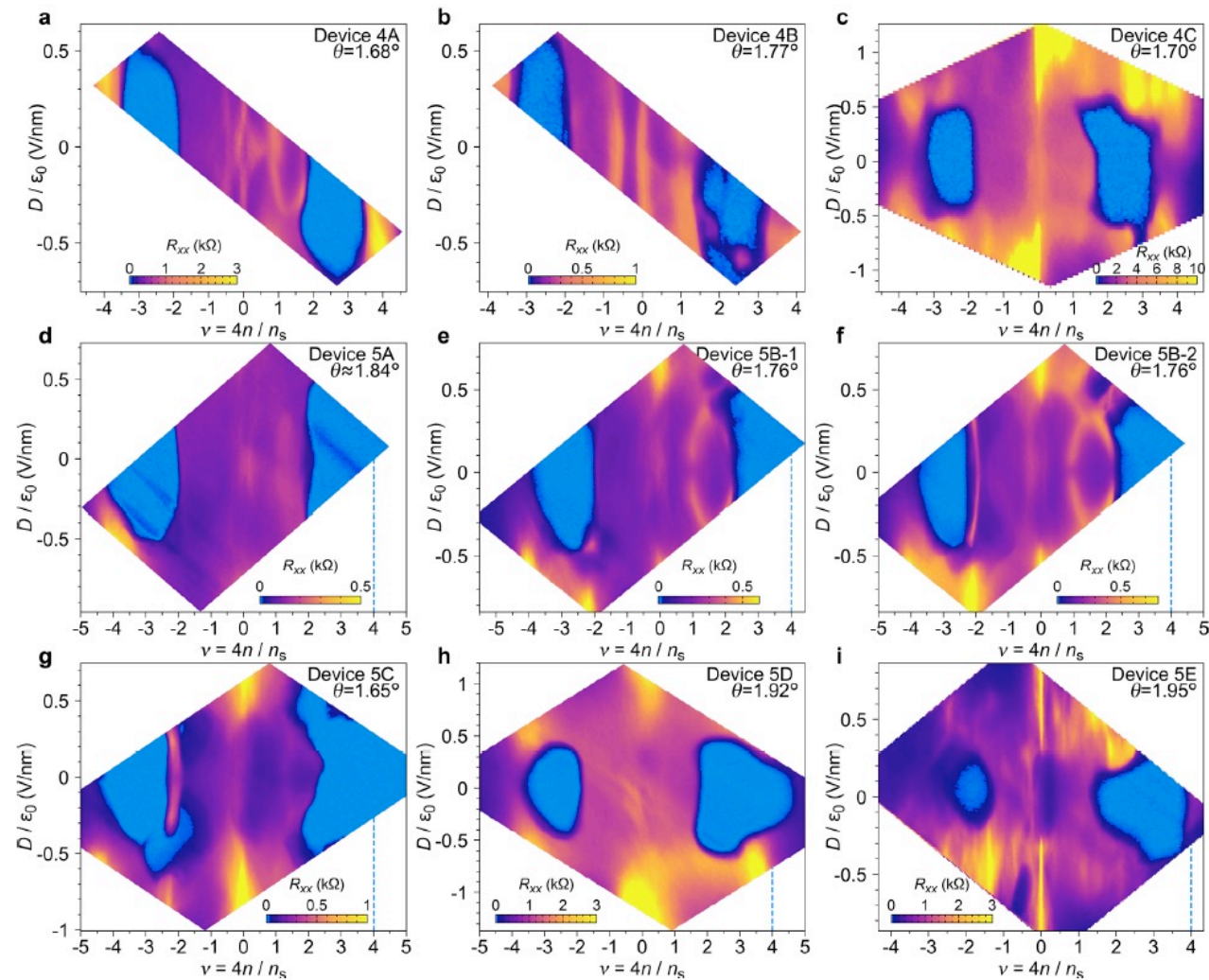
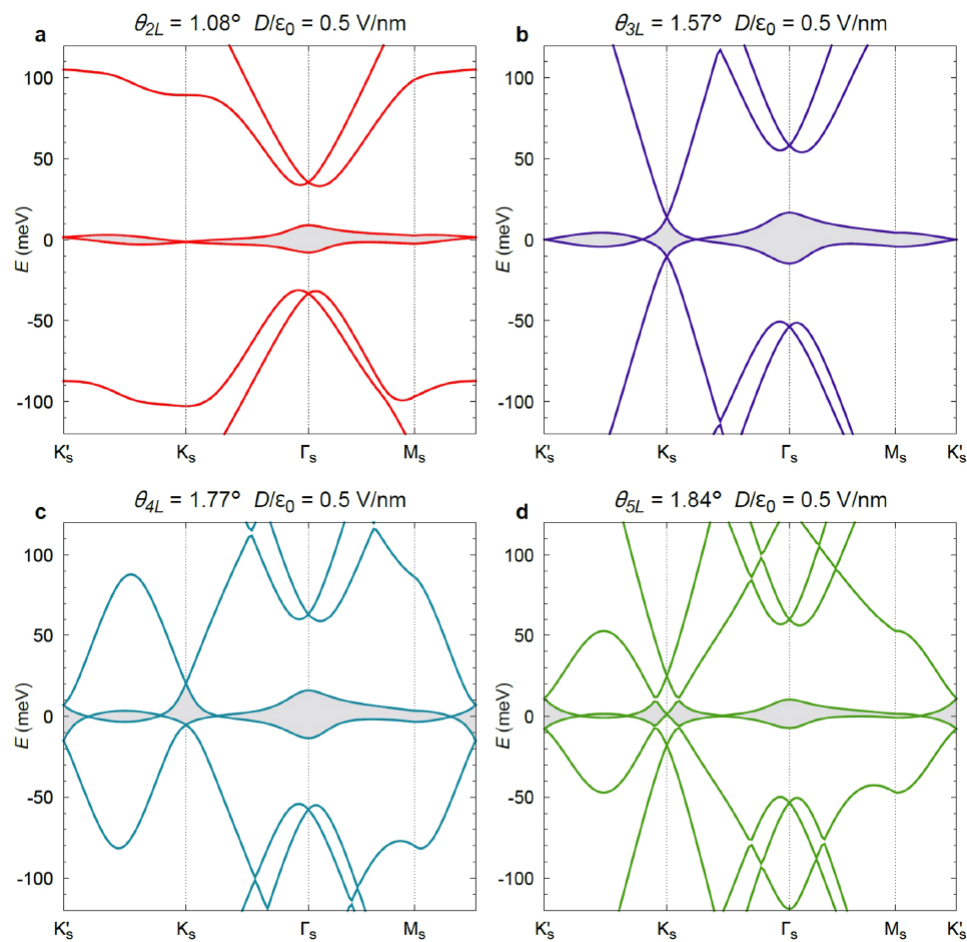
Stacking of 3, 4, 5 rotated graphene monolayers

All exhibit robust signatures of superconductivity

Pauli limited is exceeded for  $N > 2$

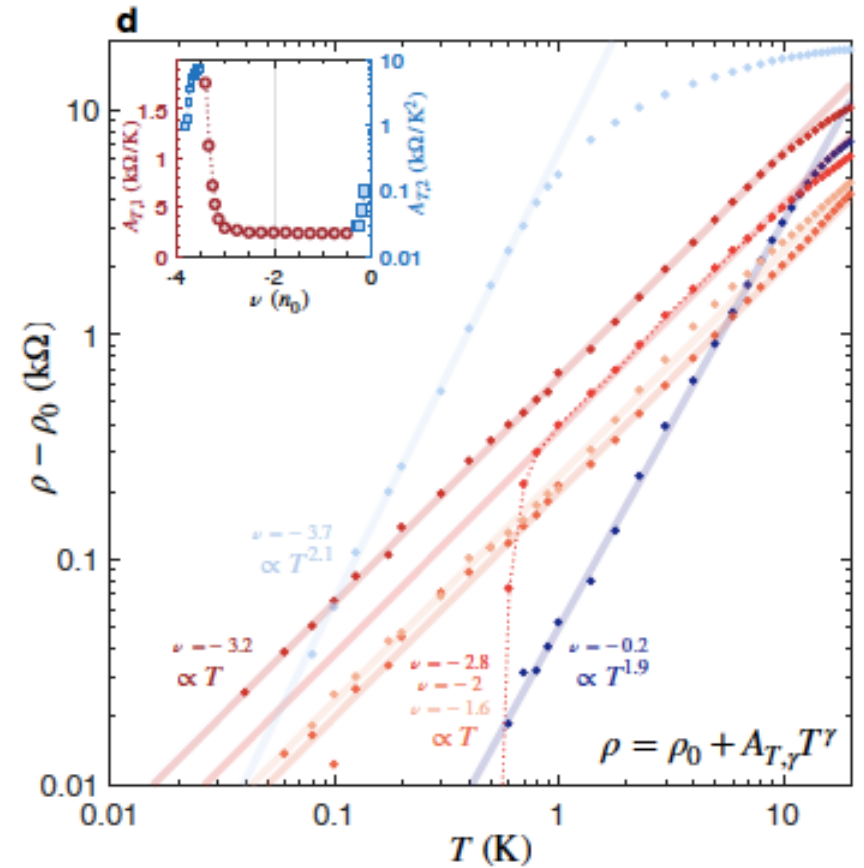
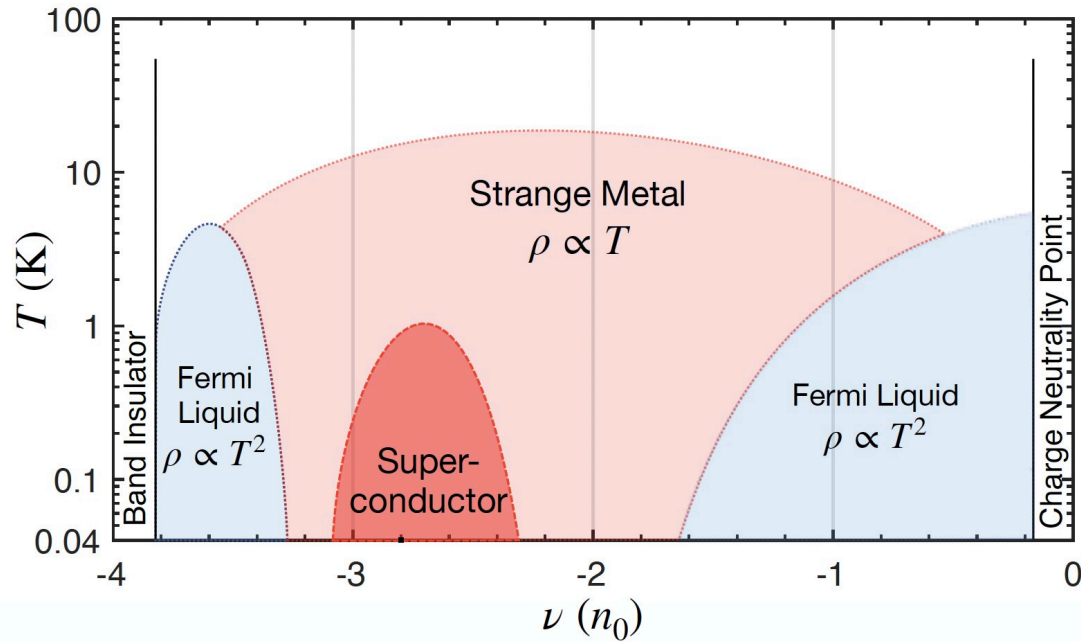
Park, Jarillo-Herrero et al., Nature Mat 2023

# Multi-layer twisted graphene



Park, Jarillo-Herrero et al., Nature Mat 2023

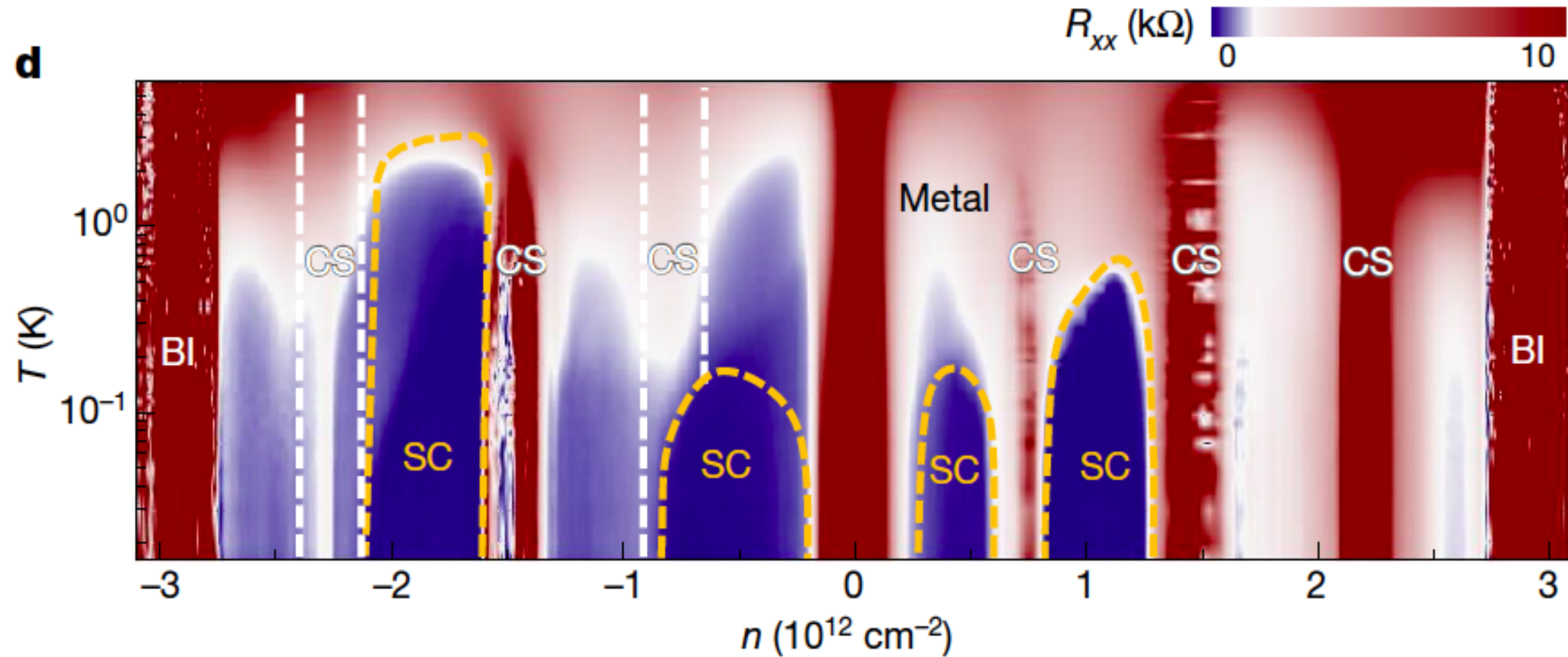
# Strange metal behavior



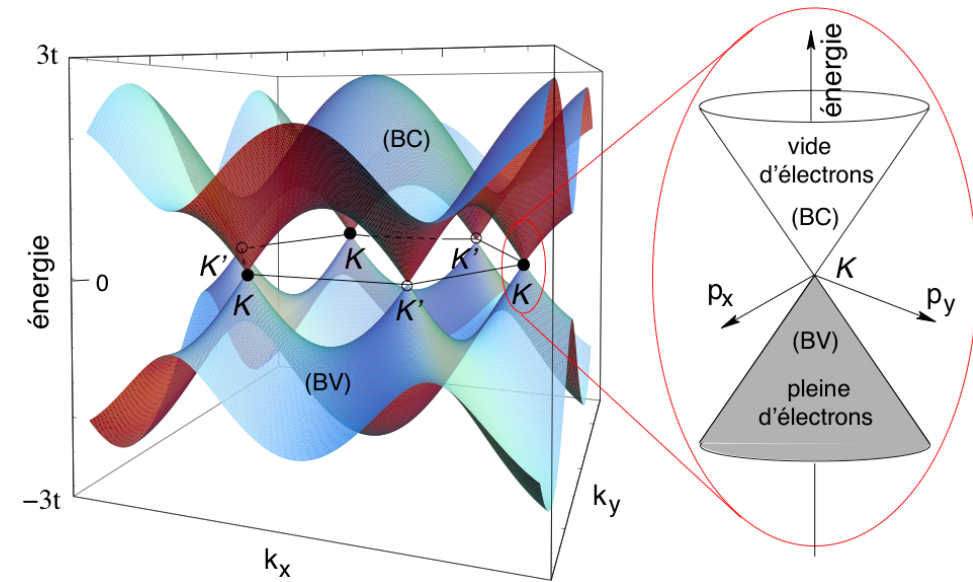
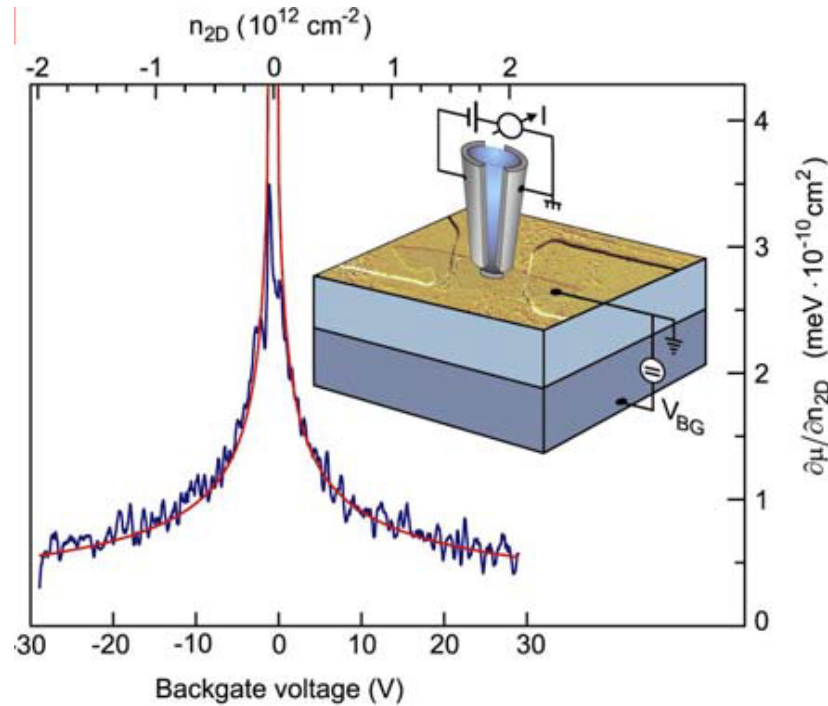
Linear in T resistivity: quantum critical fluctuations (origin is not known)

Similarity with many correlated materials (cuprates, ruthenates, pnictides, heavy fermions)

# Correlated states and symmetry breaking



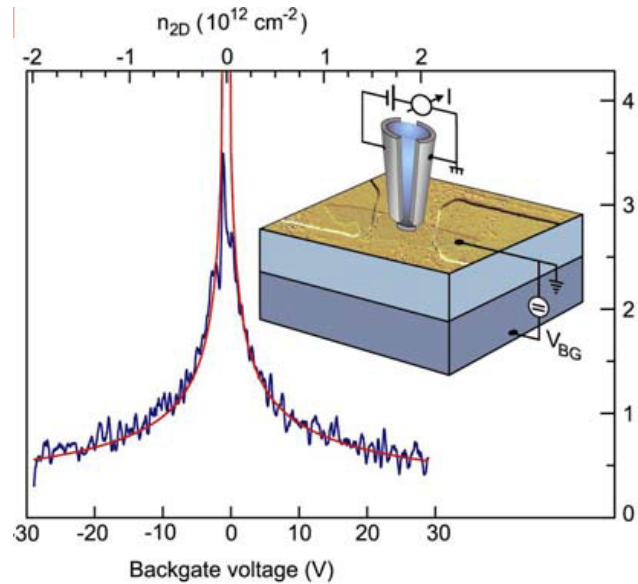
# Compressibility in monolayer graphene



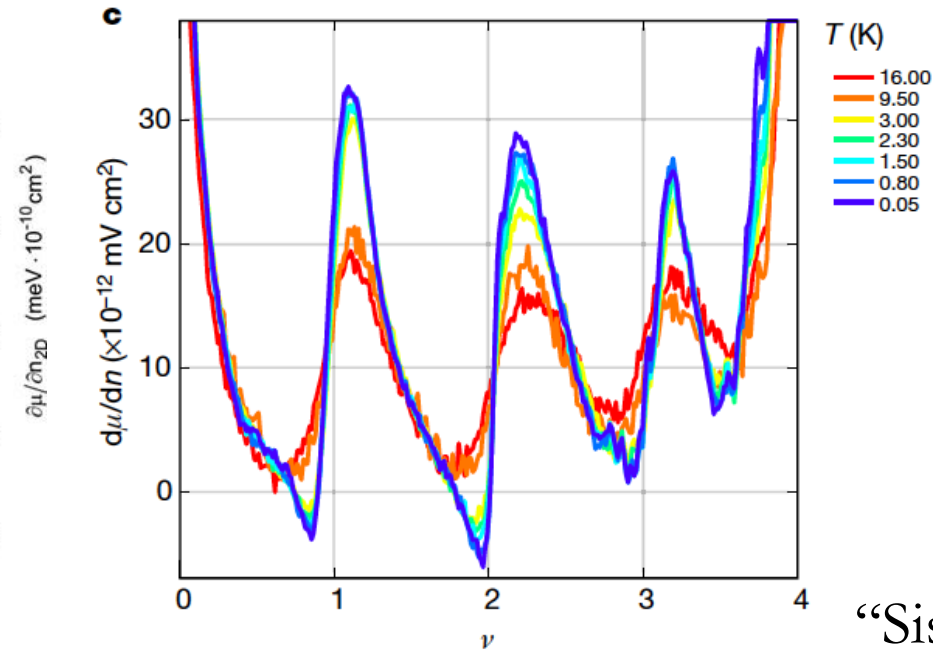
Martin, Yacoby, Nat. Phys. 2008

Compressibility shows a peak at charge neutrality in graphene (vanishing density of states)

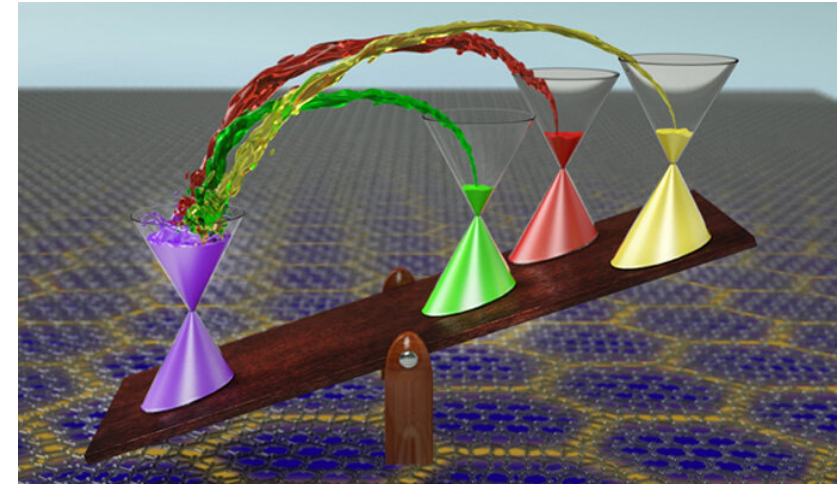
# Symmetry-breaking phases



Monolayer graphene



Twisted bilayer graphene



“Sisyphus” resetting to the Dirac point

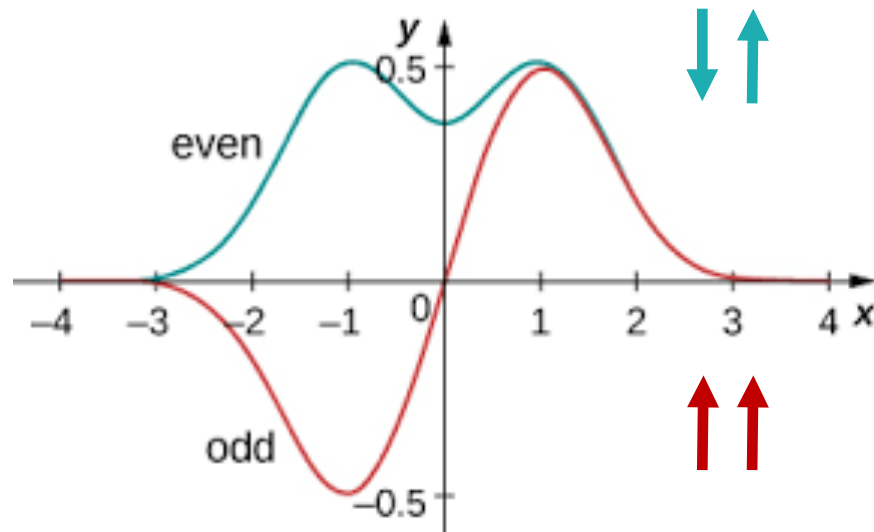
At (relatively) higher temperature/magnetic field

Measure of compressibility reveals a cascade of Stoner-like ferromagnetic transitions

Zondiner, Ilani, Nature 2020 ; Wong, Yazdani, Nature 2020 ; Saito, Young, Nature Physics 2021

# Exchange energy

Gregarious tendency of interacting electrons



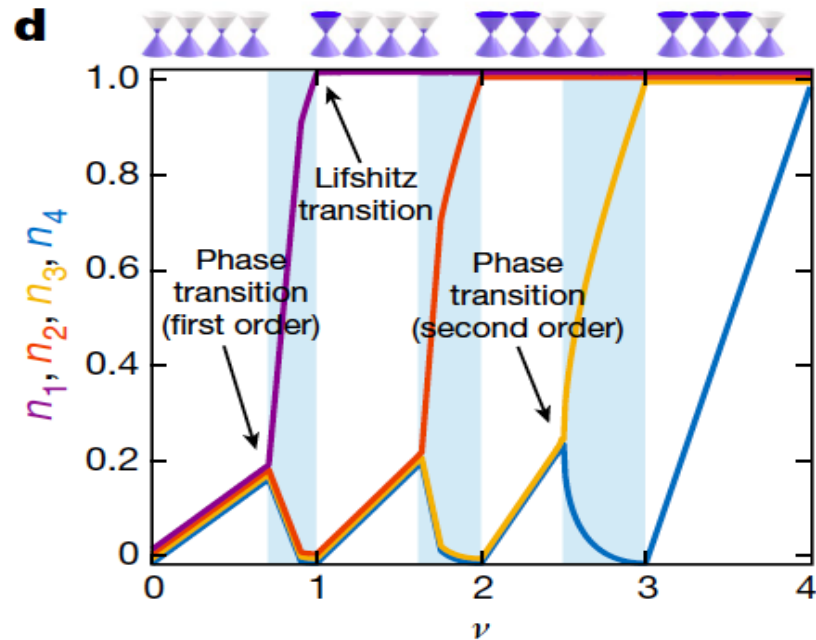
$$\int dr \frac{e^2}{\epsilon_0 r} |\psi_{\text{odd}}(r)|^2 < \int dr \frac{e^2}{\epsilon_0 r} |\psi_{\text{even}}(r)|^2$$

Odd two-electron wavefunction (singlet spin state) is less energetic than even (triplet spin)

$\implies$  Favors symmetric spin configuration (ferromagnetic)



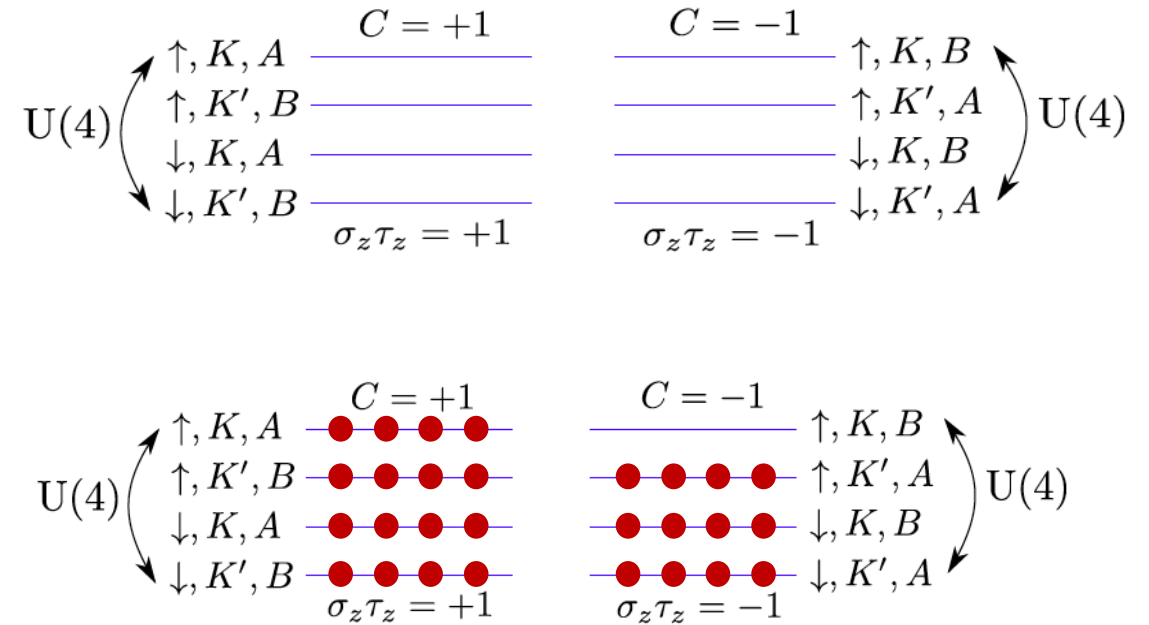
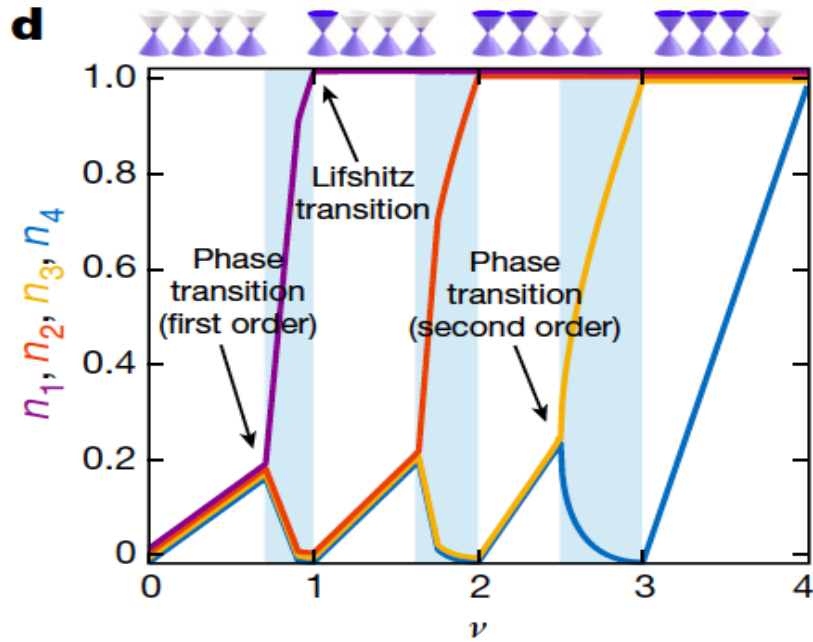
# Stoner mechanism and ferromagnetism



Stoner transition to an isospin (valley+spin)  
magnet to maximize exchange energy

Zondiner, Ilani, Nature 2021

# Stoner mechanism and ferromagnetism

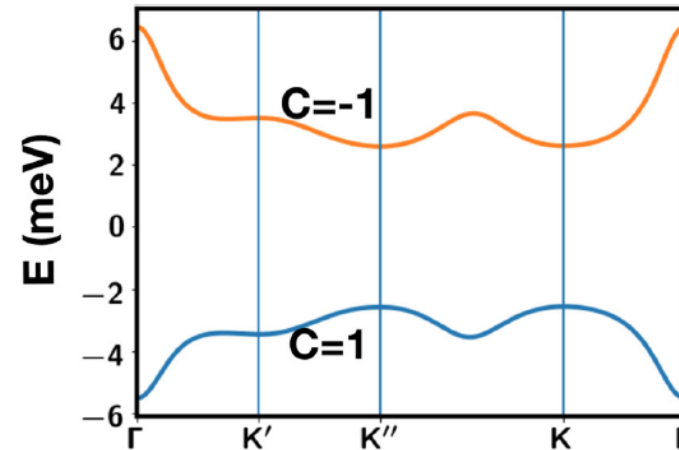
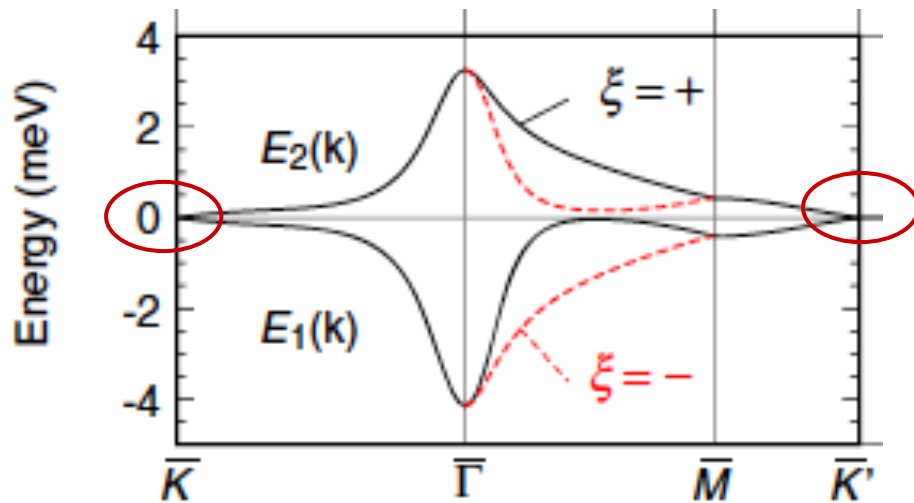


Stoner transition to an isospin (valley+spin) magnet to maximize exchange energy

Zondiner, Ilani, Nature 2021

# Topology and Chern bands

hBN substrate - aligned with TBG - opens a gap at the two Dirac points

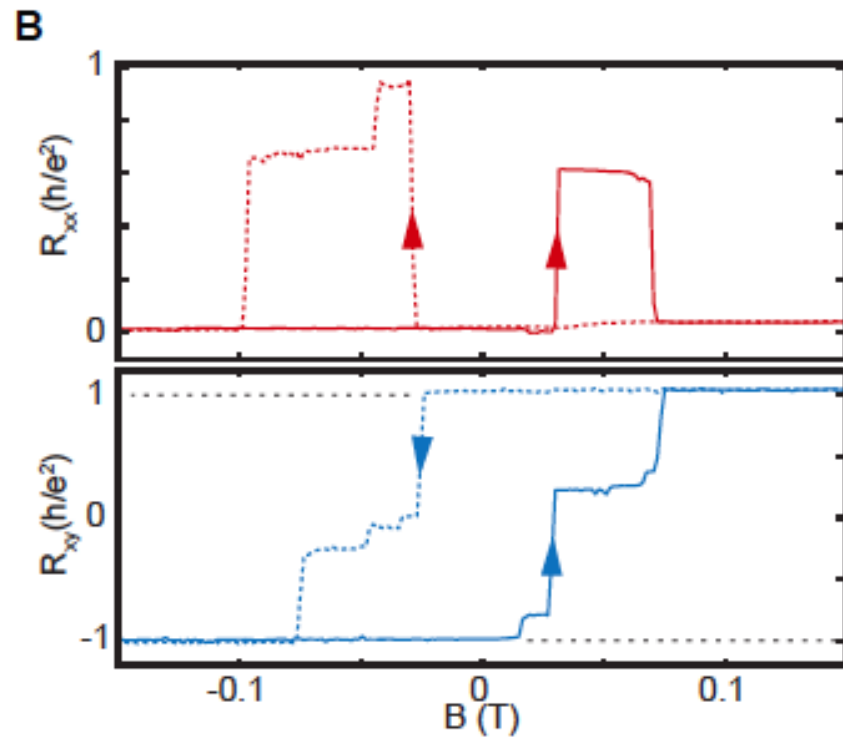


Non-zero Chern numbers

Resulting bands are topological because the two Dirac points have the same helicity (Berry phase)

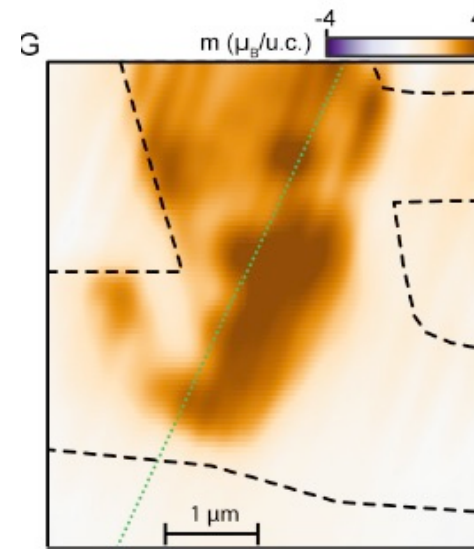
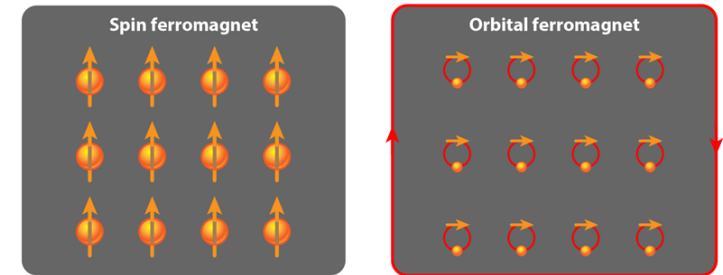
# Anomalous quantum Hall effect

Filling of bands with isospin symmetry breaking



Quantized transverse resistance with no magnetic field !

(anomalous) quantum Hall effect without magnetic field



Measurement of orbital magnetization

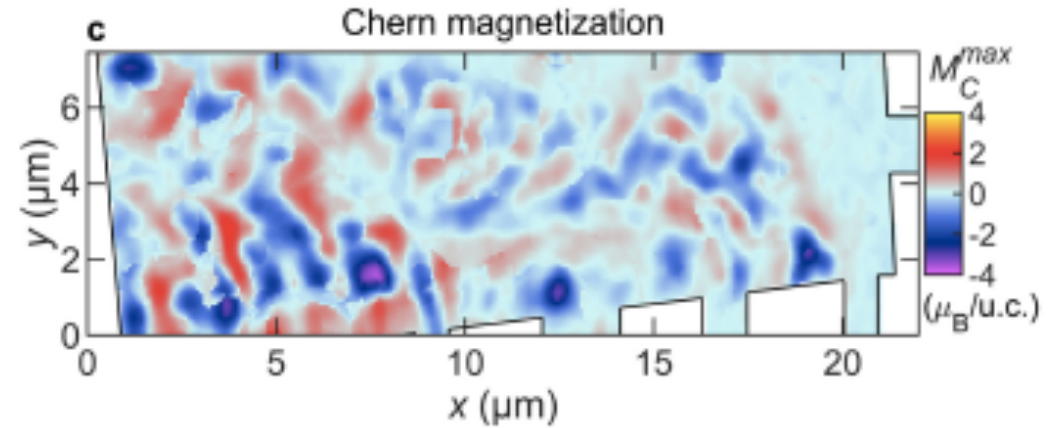
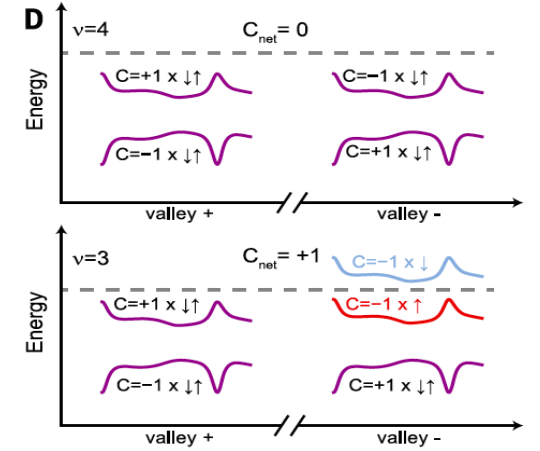
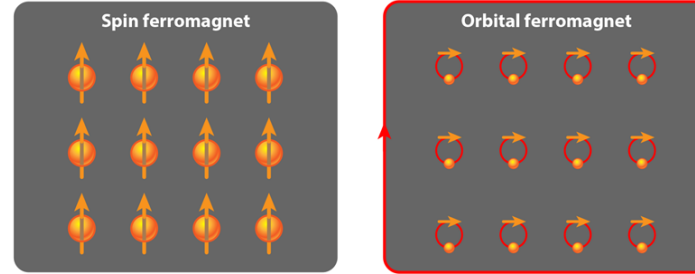
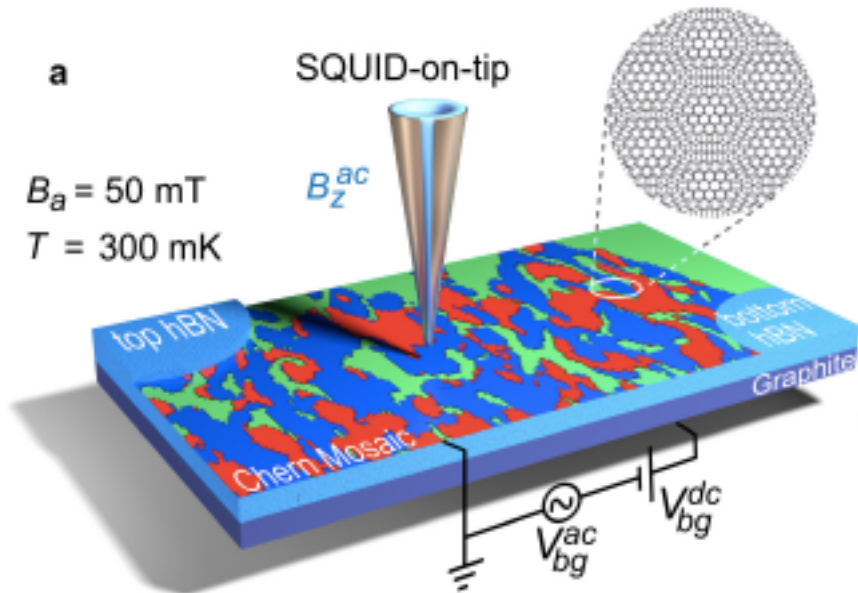
Tschirhart, Young, et al. Science 2021

# Chern mosaic

Grover, Efetov, Zeldov, Nature Phys 2022

Chern bands in the moiré minibands

Spontaneous isospin (valley) symmetry breaking

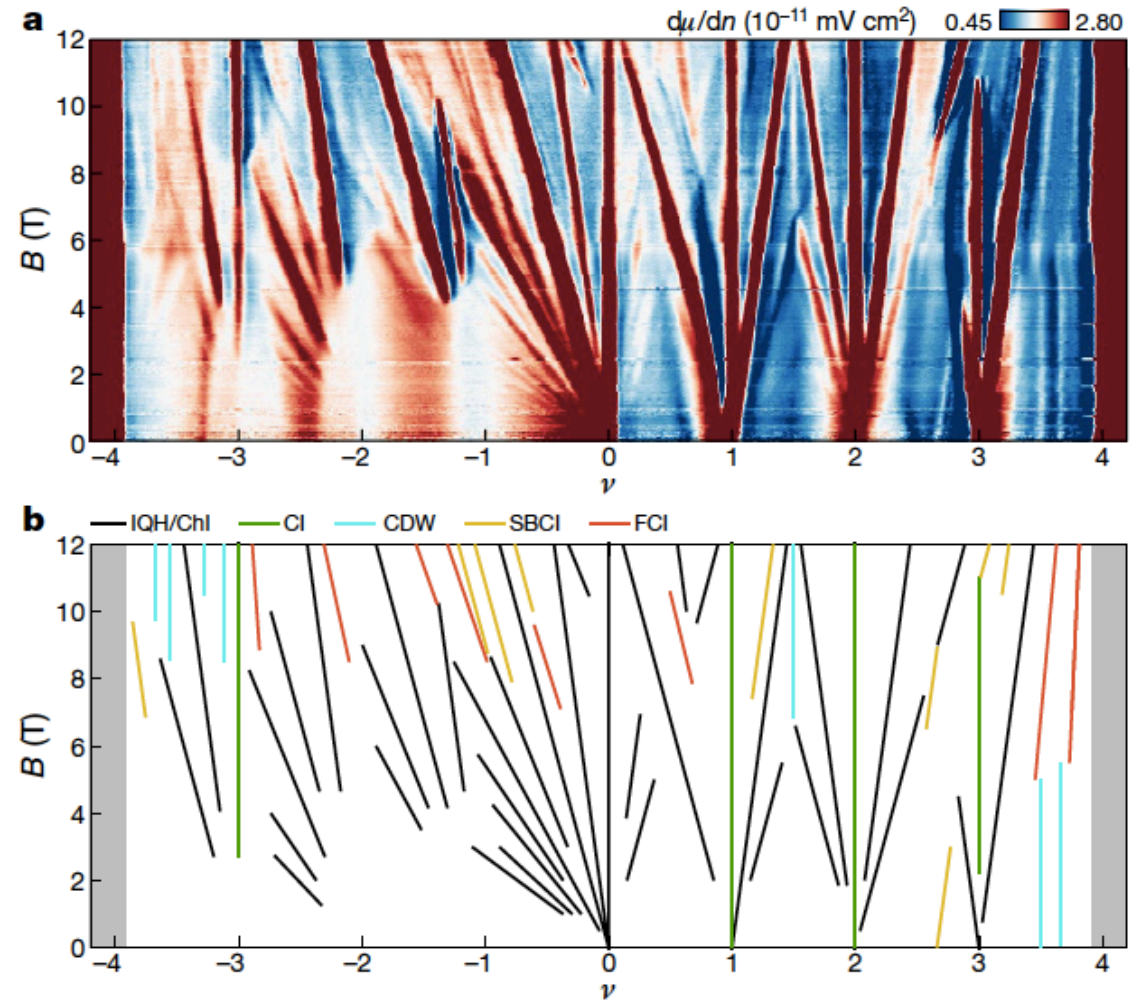


# Fractional Chern insulators ?

Xie, Jarillo-Herrero, Yacoby, Nature 2021

Fractional Chern insulators identified in Fan diagrams

But they appear ( are stabilized ?) above 5T

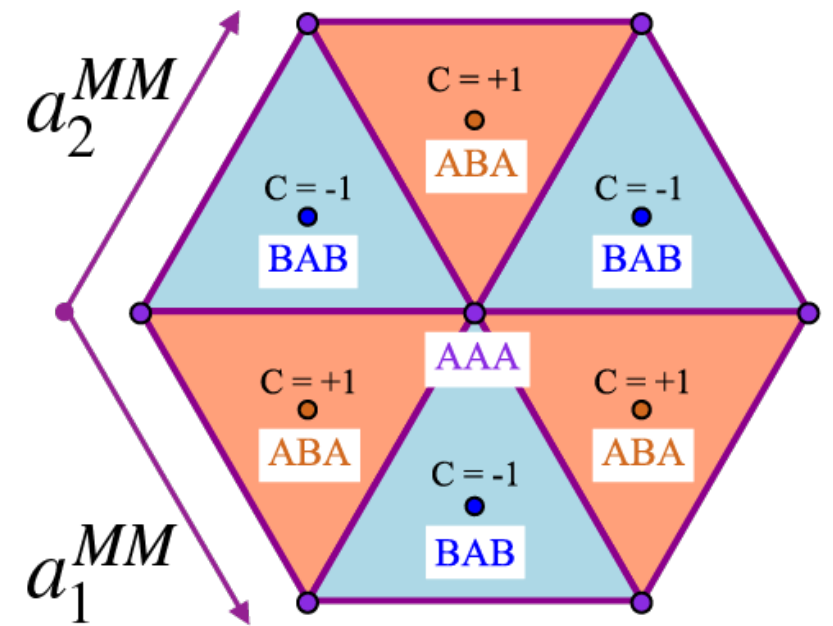
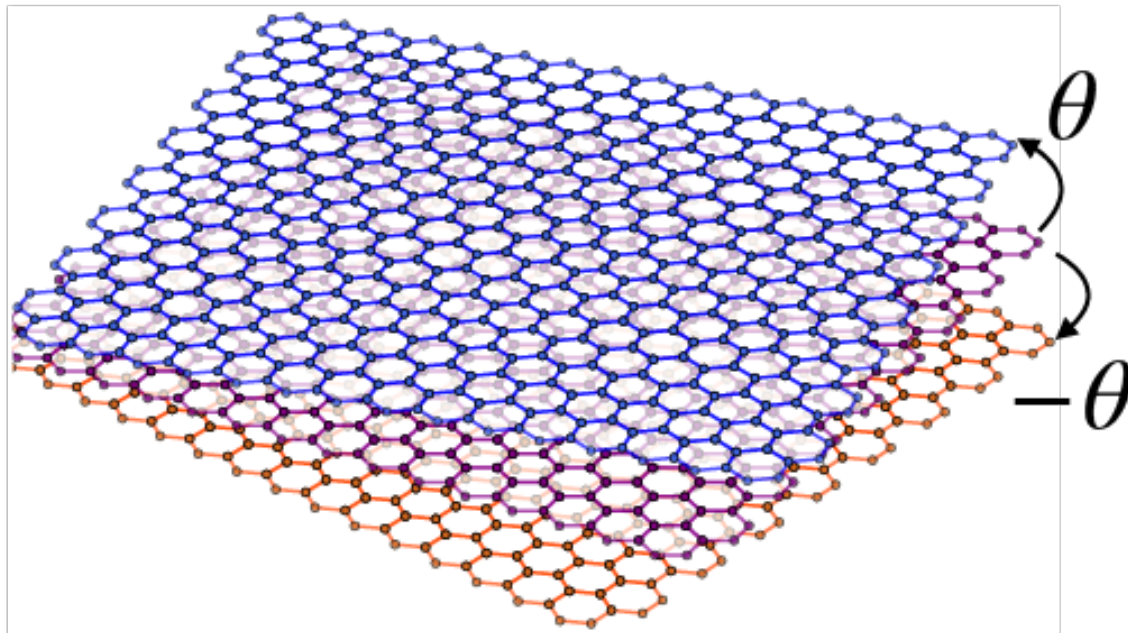


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# Chern mosaic in twisted trilayer graphene

# Helical twisted trilayer graphene

Stacking develops two incommensurate moiré patterns: there is a **supermoiré** modulation on top of the **moiré** pattern



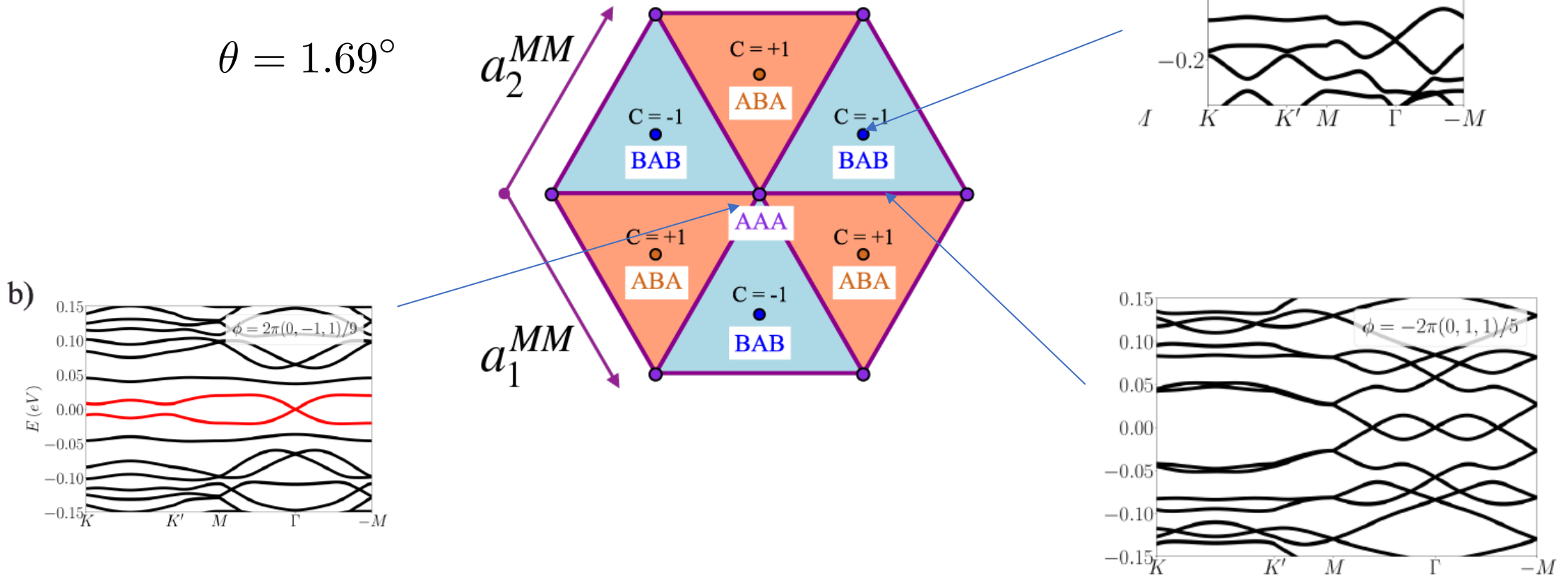
At the supermoiré scale, we find a triangular lattice of topological Chern bands separated by chiral edge channels



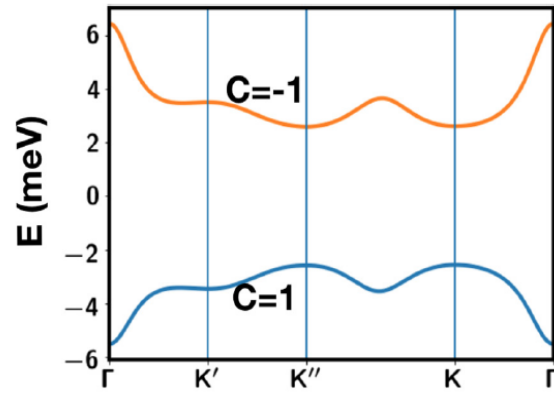
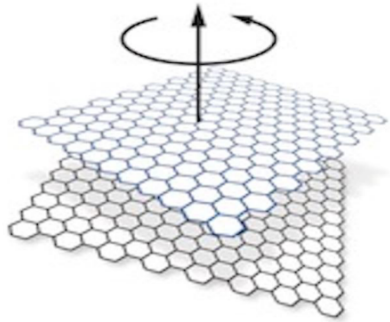
# Chern mosaic of topological bands

The two central bands carry a non-vanishing Chern value

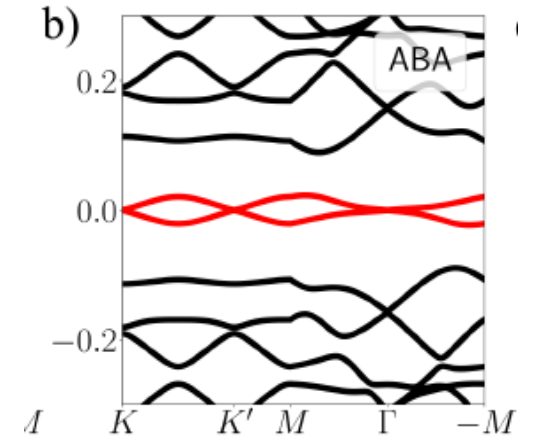
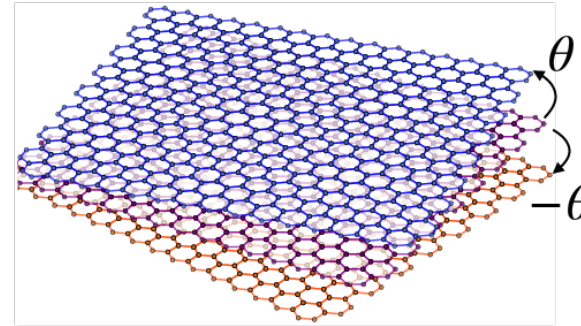
$$\theta = 1.69^\circ$$



# Difference with TBG



The two central bands carry opposite Chern values  
Need of a mechanism to split them (with HBN)  
Also symmetry breaking is required



The two central bands have directly  $C=1$   
The magic twist angle is larger than for TBG

# Why flat bands in TBG ?

$$H_{\text{Dirac}} = (\mathbf{k} - e\mathbf{A}) \cdot \boldsymbol{\sigma} = \mathbf{k} \cdot \boldsymbol{\sigma} - e \begin{pmatrix} 0 & A_x - iA_y \\ A_x + iA_y & 0 \end{pmatrix}$$

Any nonuniform coupling between the two sublattices act as an effective magnetic field

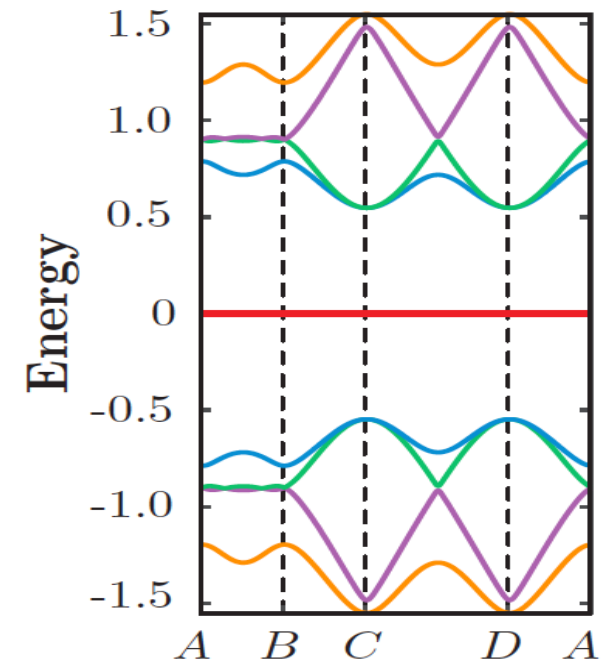
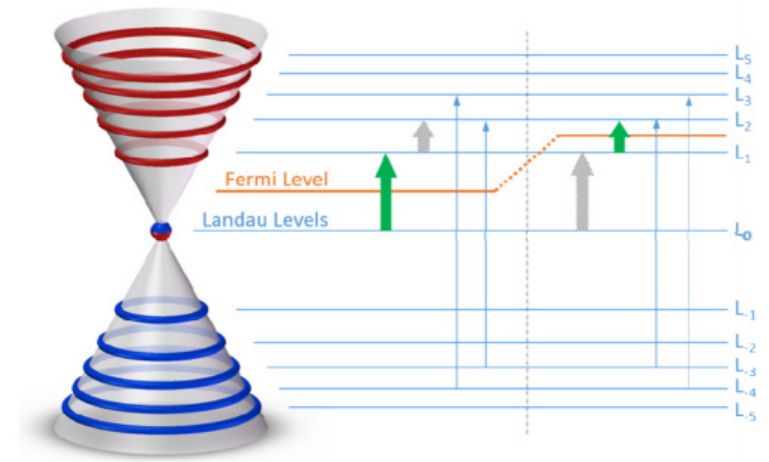
Continuum model for TBG

$$H = \begin{pmatrix} \mathbf{k} \cdot \boldsymbol{\sigma} & T(\mathbf{r}) \\ T^\dagger(\mathbf{r}) & \mathbf{k} \cdot \boldsymbol{\sigma} \end{pmatrix}$$

$$T(\mathbf{r}) = \begin{pmatrix} w(r) & f_1(\mathbf{r}) \\ f_2(\mathbf{r}) & w(r) \end{pmatrix}$$

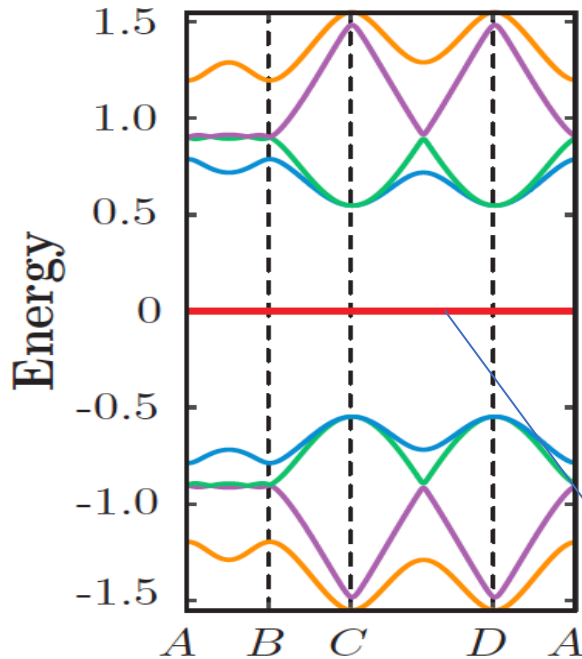
## Chiral limit

$w=0$ : exactly flat band at magic angle !



Tarnopolsky et al, PRL 2019

# Ideal flat bands in the chiral limit (TBG)



The quantum geometric tensor

Involves a covariant derivative with the Berry connection

$$Q_{ab}(k) = \langle D_{k_a} u_k | D_{k_b} u_k \rangle$$

$$D_{k_a} = \partial_{k_a} + i\mathcal{A}_a(k)$$

$$\mathcal{A}_a(k) = i\langle u_k | \partial_a u_k \rangle$$

Quantum metric

$$g_{ab}(k) = \frac{Q_{ab}(k) + Q_{ba}(k)}{2}$$

Berry curvature

$$\Omega(k) = i\varepsilon_{ab}Q_{ab}(k)$$

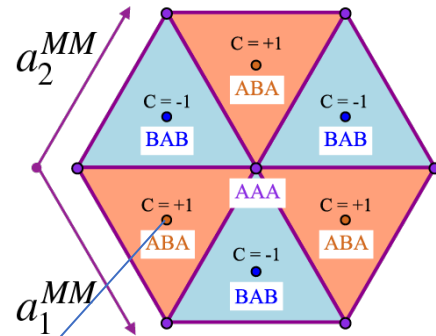
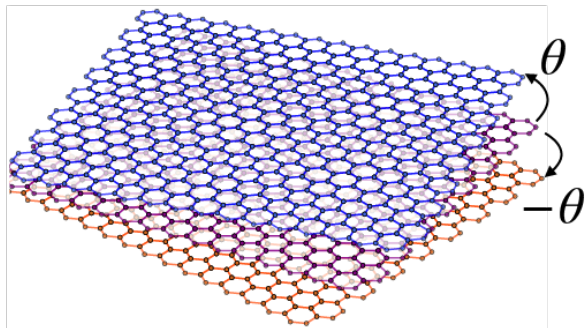
When the trace condition is met  $\text{Tr}g(k) = |\Omega(k)|$



Fractional Hall states emerge as solutions of interacting electrons

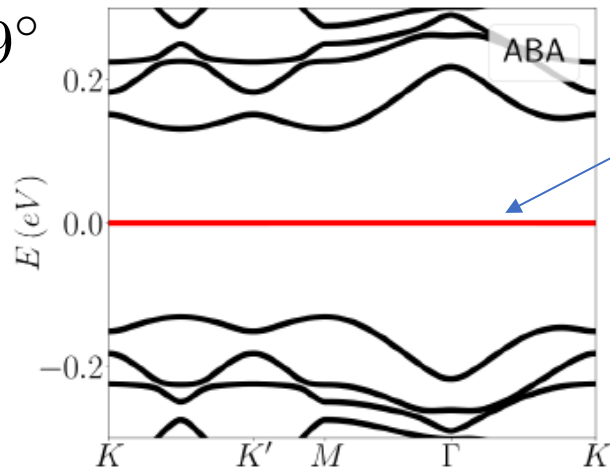
Ledwith, Vishwanath et al, PRR 2020

# Exactly flat bands in the chiral limit of helical trilayer



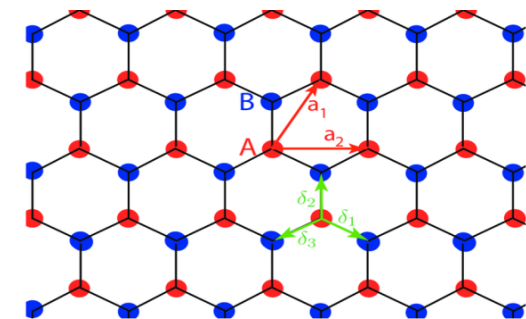
Bands are also **ideal** in the sense that the **trace condition** is satisfied  $\text{Tr}g(k) = |\Omega(k)|$

$\theta = 1.69^\circ$



Two bands with Chern number +2 and -1 (sublattice-polarized)

Chiral limit



# Exactly flat bands in the chiral limit of helical trilayer

We find an analytical expression for the color-entangled exact wavefunction with Chern  $C=+2$

$$a_k = \vartheta_1[(k - K)/b_2, \omega]$$

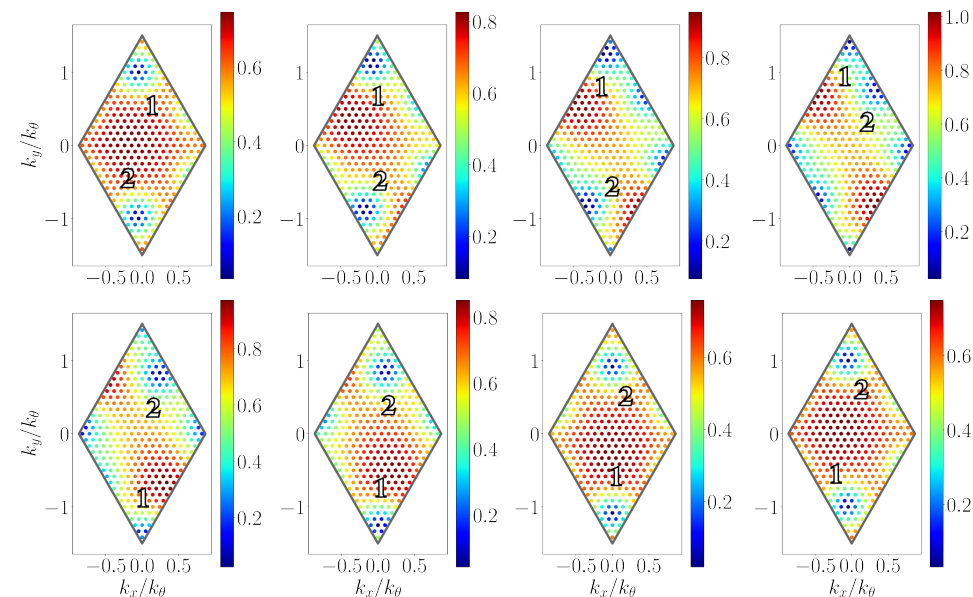
$$\omega = e^{2i\pi/3}$$

$$\psi_k(r) = a_k \eta_{k-K}(z) \psi_K(r) + a_{-k} \eta_{k-K'}(z) \psi_{K'}(r)$$

$$\eta_k(z) = e^{ik_1 z/a_1} \frac{\vartheta_1[z/a_1 - k/b_2, \omega]}{\vartheta_1[z/a_1, \omega]}$$

Lowest landau level wavefunction on the torus

Guerci, Mao and Mora, Arxiv 2023



# Acknowledgements

**MPQ**

**Flatiron Institute**

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Daniele Guerci

**ENS/Princeton**

**Berlin**

**LPS (Uni. Paris-Saclay)**

**Weizmann**

Andrei Bernevig  
Nicolas Regnault

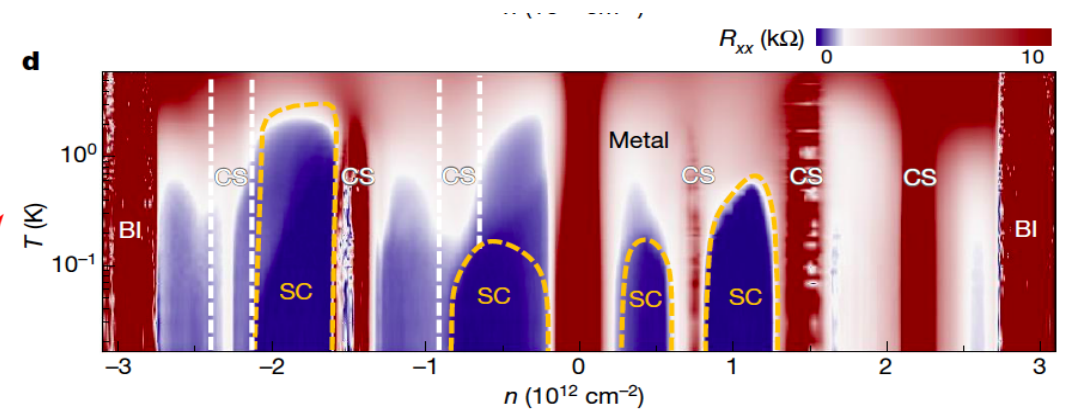
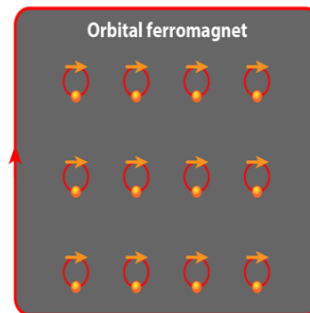
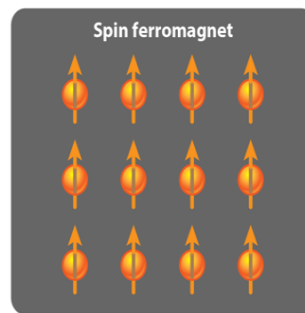
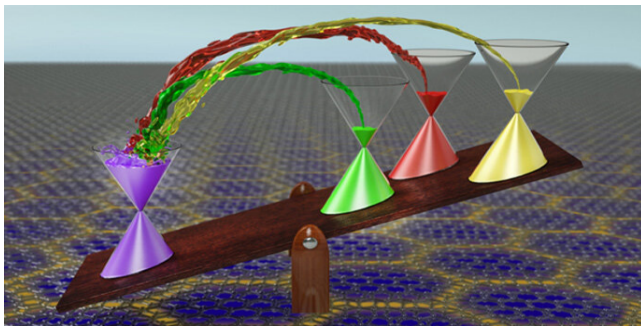
Felix von Oppen  
Krystov Kolar

Pascal Simon

Yuval Oreg  
Gal Shavit

# Conclusion

Twisted bilayer graphene: fantastic platform to explore novel correlated states, topological phases, superconductivity...

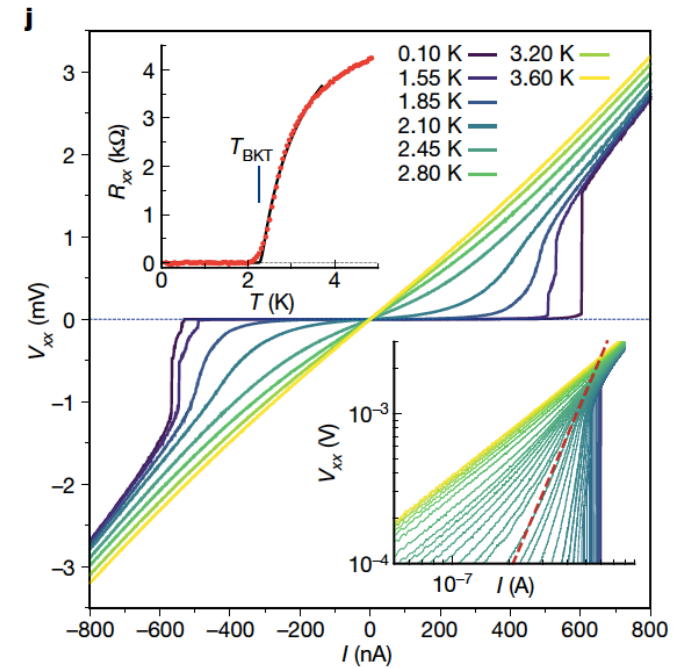
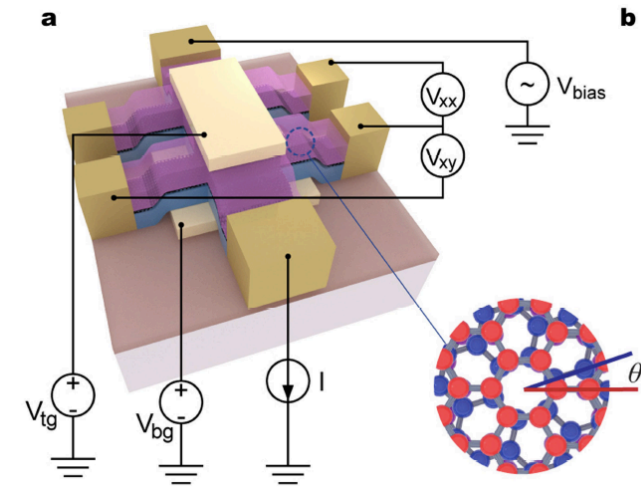
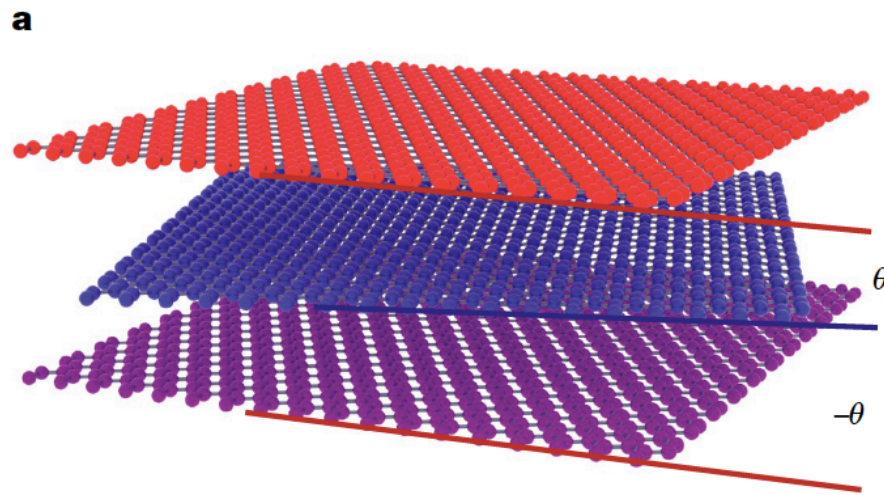
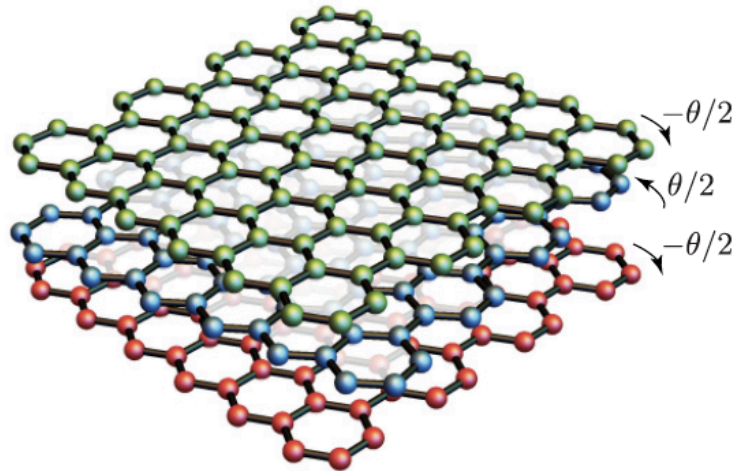


Other interesting effects: nematicity, broken translation invariance, fractional Chern insulators, unconventional superconductivity, anomalous Josephson junctions, strange metal...

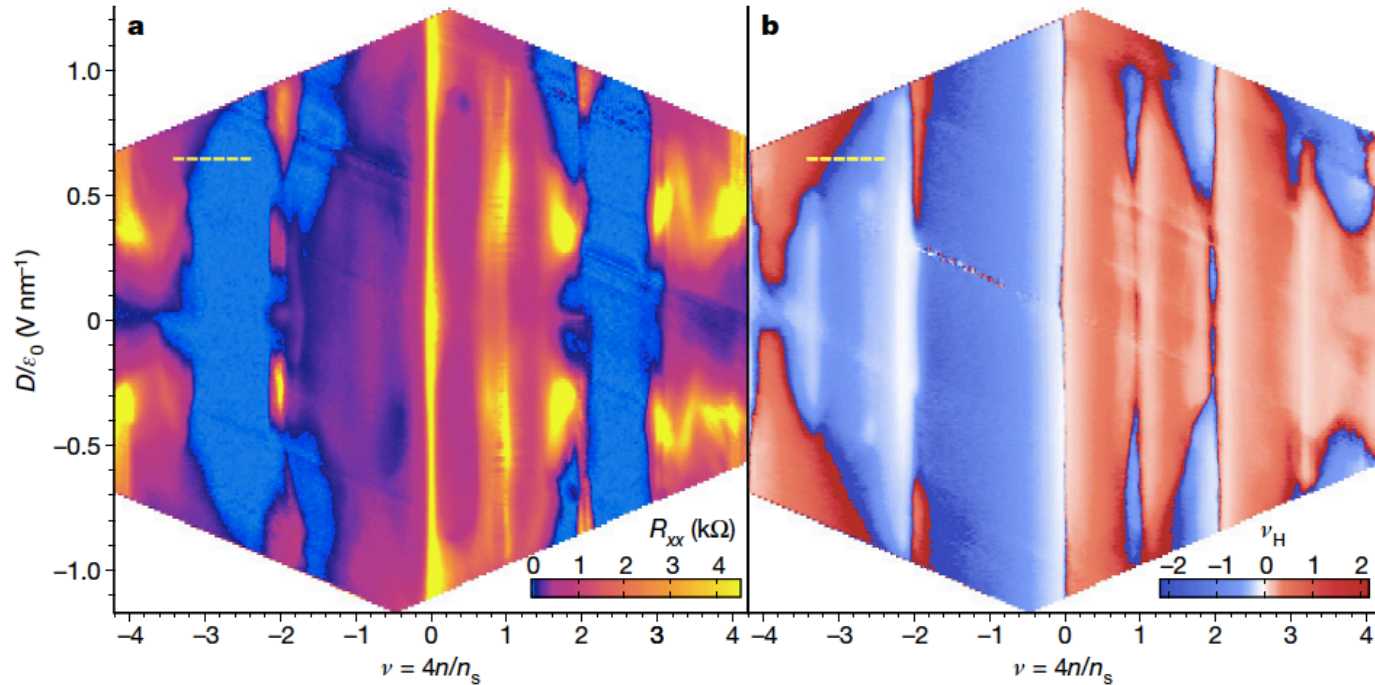
Helical trilayer graphene shows interesting topological features with a Chern mosaic and unconventional  $C=+2$  Chern bands



# Twisted trilayer graphene



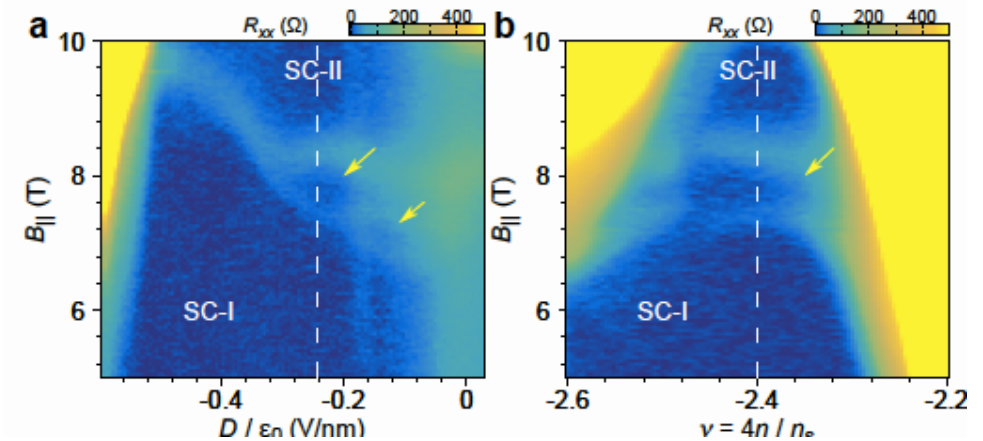
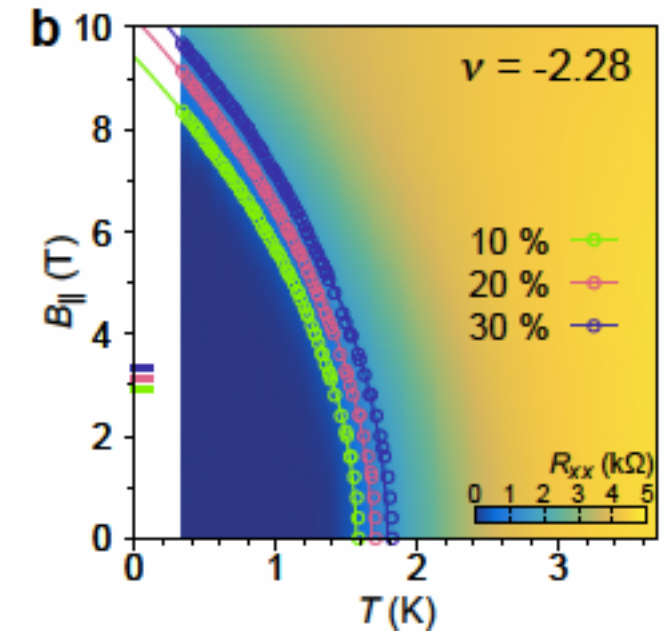
# Twisted trilayer graphene



Insulating ferromagnetic phases

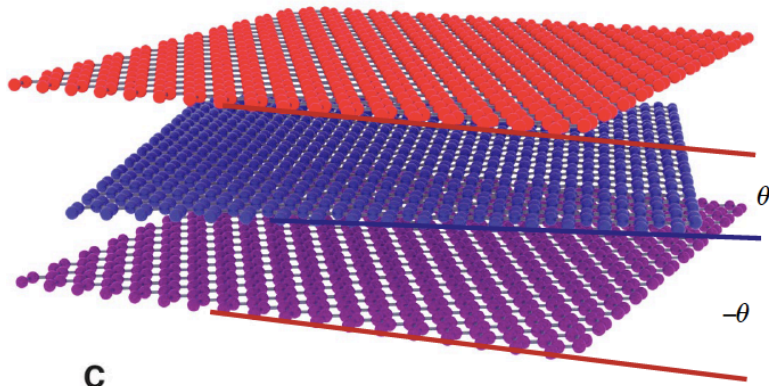
Superconducting states with high critical field (p-wave ?)

Superconducting regions bordered by van Hove singularity

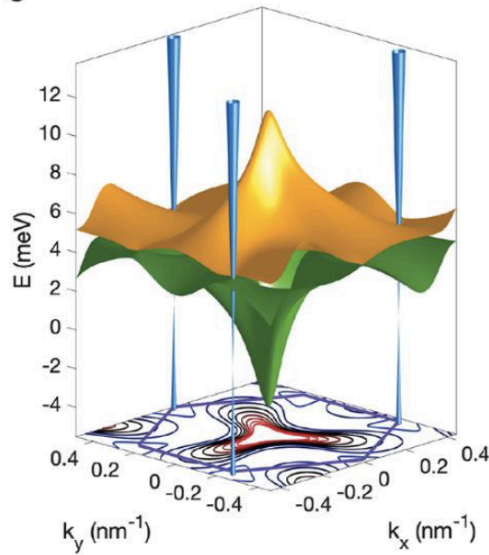


# Twisted trilayer graphene

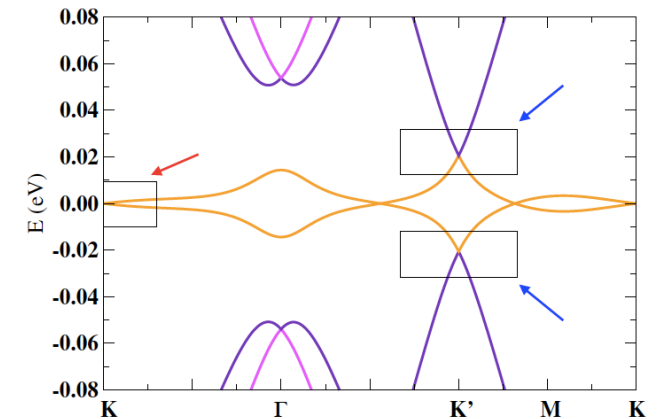
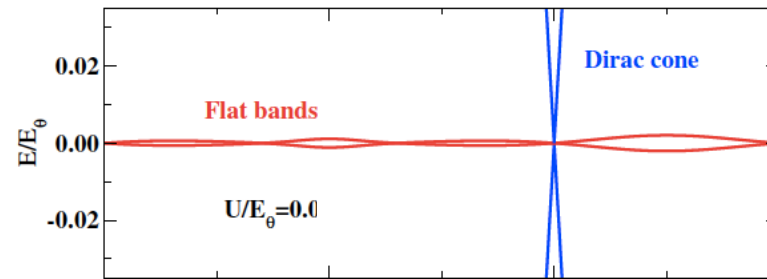
a



c



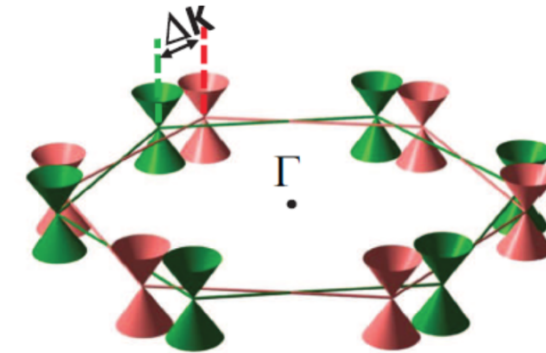
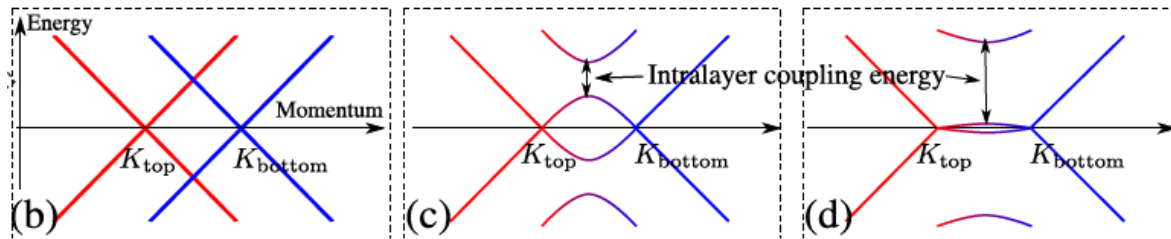
Zero electric field



Finite electric field

# Shifted Dirac cones

T. Heikkilä & T. Hyart 2019

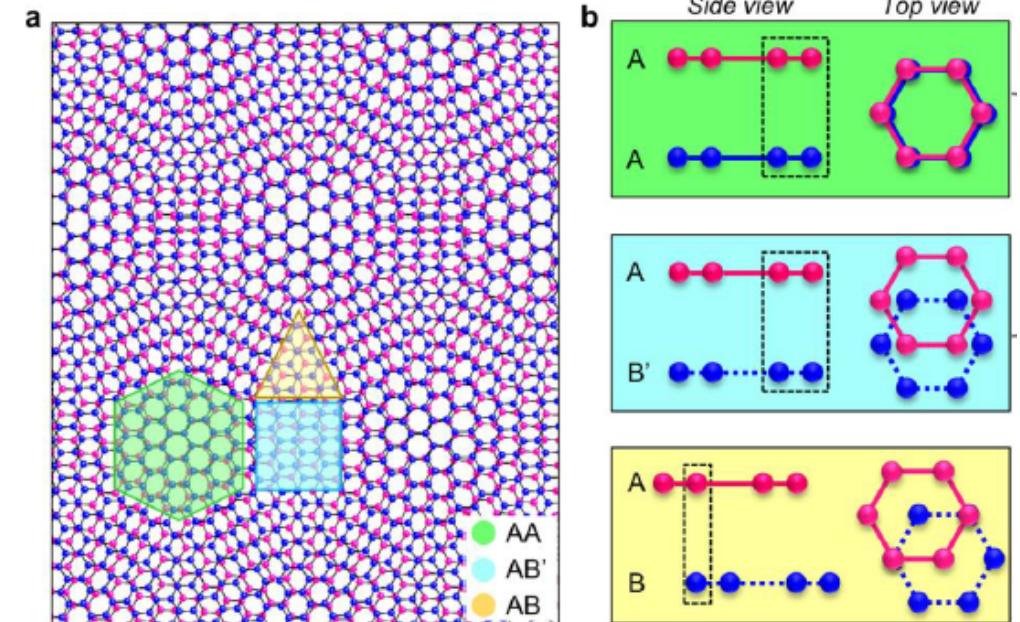


Mini-graphene spectrum with two Dirac cones

Interlayer coupling renormalizes down the Dirac velocity

$$H = \begin{pmatrix} \mathbf{k} \cdot \boldsymbol{\sigma} & T(\mathbf{r}) \\ T^\dagger(\mathbf{r}) & \mathbf{k} \cdot \boldsymbol{\sigma} \end{pmatrix} \quad T(\mathbf{r}) = \begin{pmatrix} w(r) & f_1(\mathbf{r}) \\ f_2(\mathbf{r}) & w(r) \end{pmatrix}$$

Valley decoupling at small angle



# Bilayer with a shifting stacking position

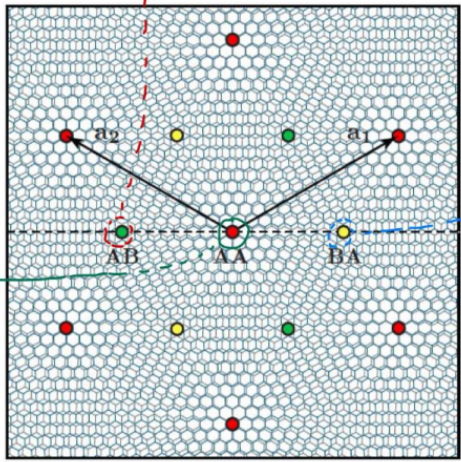
•  $\tau = \tau_{AB}$

$$H = \begin{pmatrix} \hbar v_F \vec{k} \cdot \vec{G} & 0 & 0 \\ 0 & 3t_{\perp} & 0 \\ 0 & 0 & \hbar v_F \vec{k} \cdot \vec{G} \end{pmatrix}$$

AB Stacking

AA Stacking

•  $\tau = 0$

$$H = \begin{pmatrix} \hbar v_F \vec{k} \cdot \vec{G} & 3t_{\perp} & 0 \\ 3t_{\perp} & 0 & 3t_{\perp} \\ 0 & 3t_{\perp} & \hbar v_F \vec{k} \cdot \vec{G} \end{pmatrix}$$


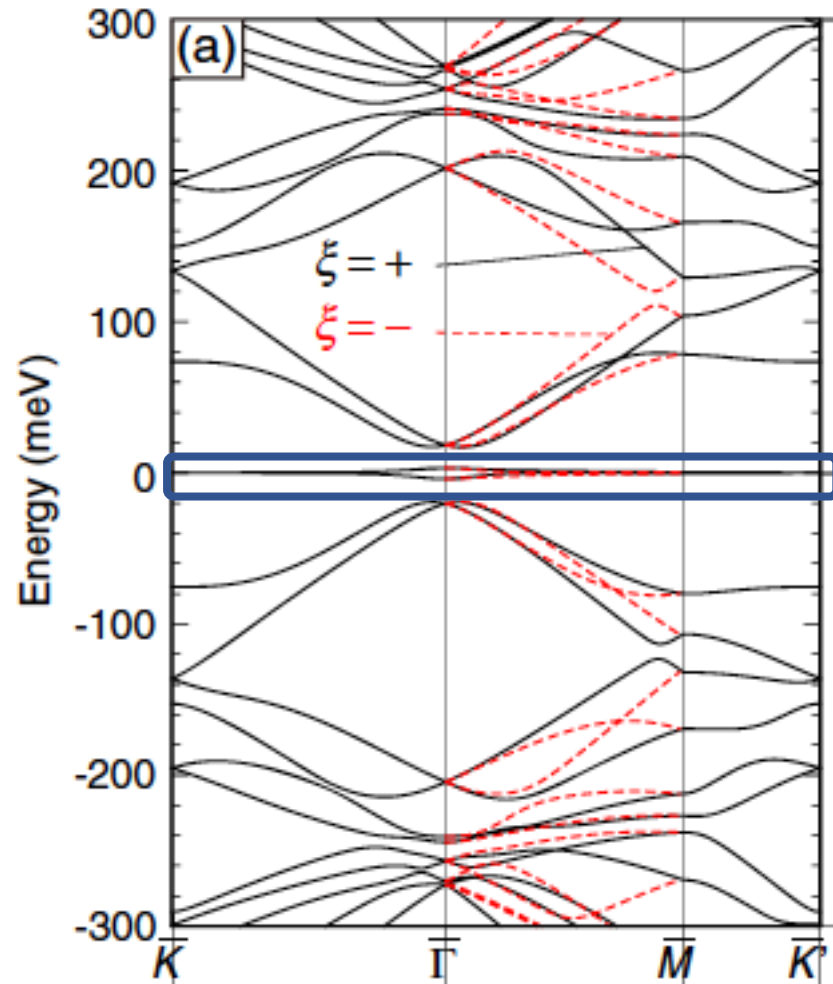
BA Stacking

•  $\tau = \tau_{BA}$

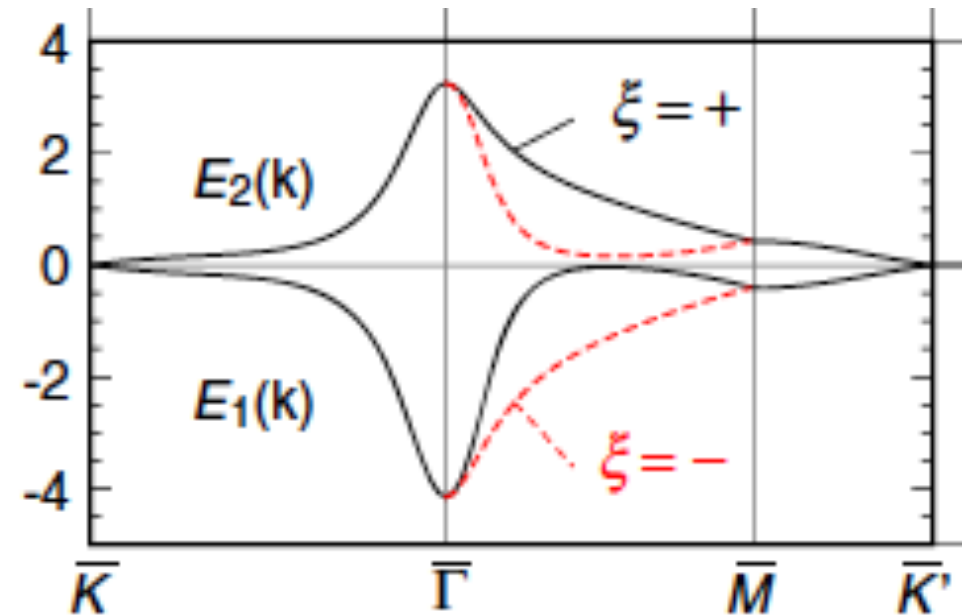
$$H = \begin{pmatrix} \hbar v_F \vec{k} \cdot \vec{G} & 0 & 3t_{\perp} \\ 0 & 0 & 0 \\ 3t_{\perp} & 0 & \hbar v_F \vec{k} \cdot \vec{G} \end{pmatrix}$$

# Two narrow bands close to the Fermi energy

$$\theta = 1.05^\circ$$

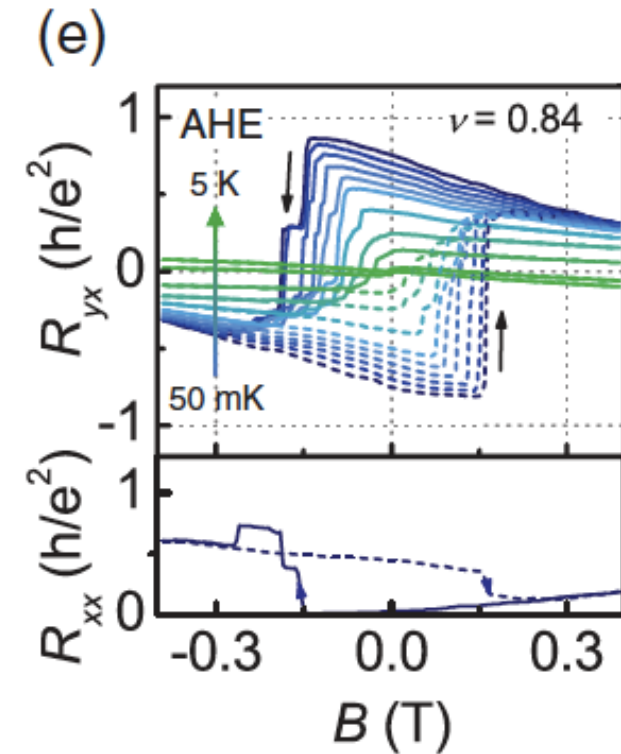
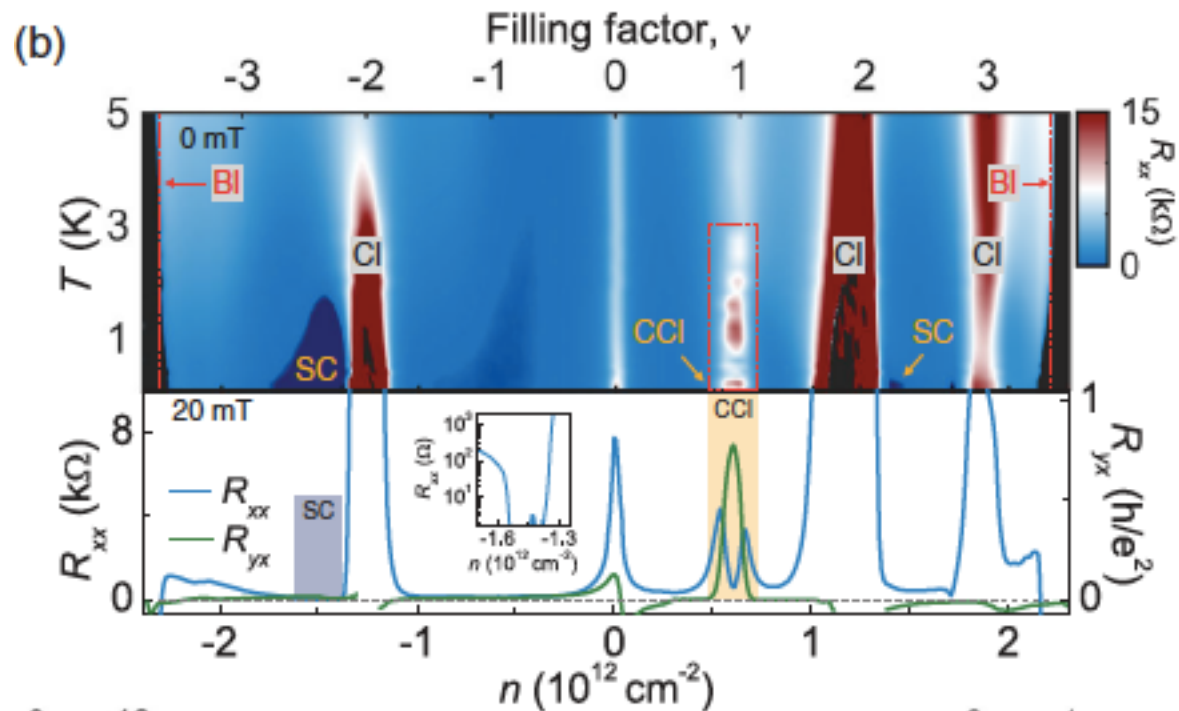


Energy (meV)



Koshino et al, PRX 2018

# Zero-field Chern insulator (anomalous Hall effect)



Intrinsic QAHE: no aligned hBN substrate

Stepanov, Efetov, PRL 2021

# Versatile playground of 2D materials

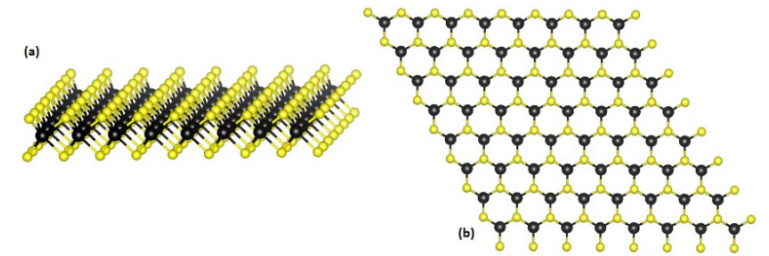
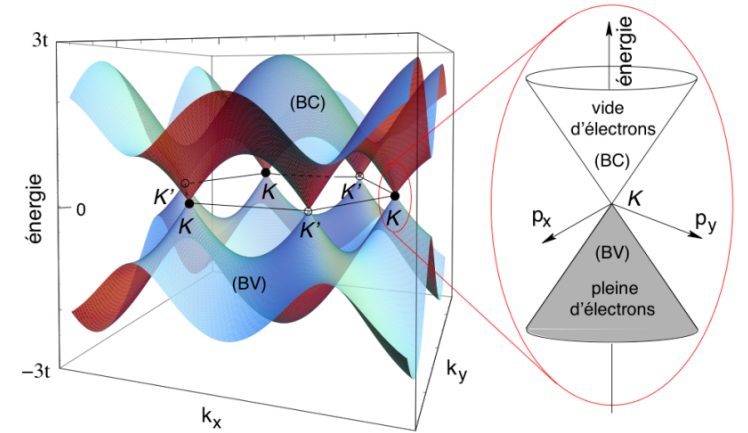
**Flourishing topic** since 2004: 2D materials are accessible on the surface, tunable via gating, low densities. Stacking of layers provides infinite possibilities.

**Graphene:** monolayer and multilayer stacked structures + twistrionics

**Transition metal dichalcogenide** (MoS<sub>2</sub>, TeW<sub>2</sub>, ...)

Strong **spin-orbit**: interplay between valley and spin

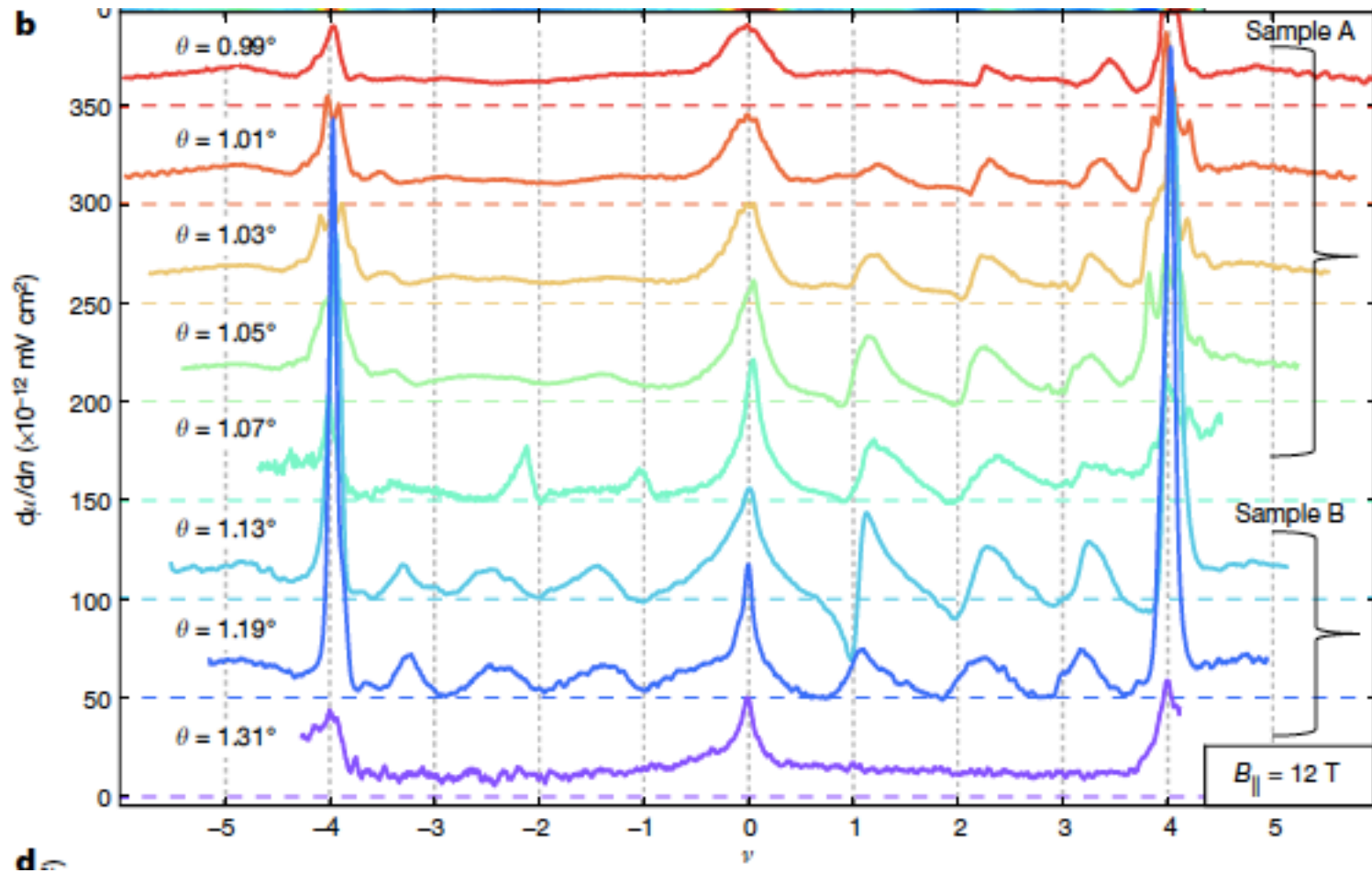
Strong interactions: **Wigner crystal**, **correlated insulators**, **ferromagnetism**, etc.



Graphene family	Graphene	hBN 'white graphene'	BCN	Fluorographene	Graphene oxide
2D chalcogenides	MoS <sub>2</sub> , WS <sub>2</sub> , MoSe <sub>2</sub> , WSe <sub>2</sub>	Semiconducting dichalcogenides: MoTe <sub>2</sub> , WTe <sub>2</sub> , ZrS <sub>2</sub> , ZrSe <sub>2</sub> and so on	Metallic dichalcogenides: NbSe <sub>2</sub> , NbS <sub>2</sub> , TaS <sub>2</sub> , TiS <sub>2</sub> , NiSe <sub>2</sub> and so on		
			Layered semiconductors: GaSe, GaTe, InSe, Bi <sub>2</sub> Se <sub>3</sub> and so on		
2D oxides	Micas, BSCCO	MoO <sub>3</sub> , WO <sub>3</sub>	Perovskite-type: LaNb <sub>2</sub> O <sub>7</sub> , (Ca,Sr) <sub>2</sub> Nb <sub>3</sub> O <sub>10</sub> , Bi <sub>4</sub> Ti <sub>3</sub> O <sub>12</sub> , Ca <sub>2</sub> Ta <sub>2</sub> TiO <sub>10</sub> and so on		Hydroxides: Ni(OH) <sub>2</sub> , Eu(OH) <sub>2</sub> and so on
	Layered Cu oxides	TiO <sub>2</sub> , MnO <sub>2</sub> , V <sub>2</sub> O <sub>5</sub> , TaO <sub>3</sub> , RuO <sub>2</sub> and so on			Others



# Angle dependence of strong interactions



Zondiner, Ilani, Nature 2020