From black hole's atmosphere to thermal quenches

… analog curved spacetime in quantum materials

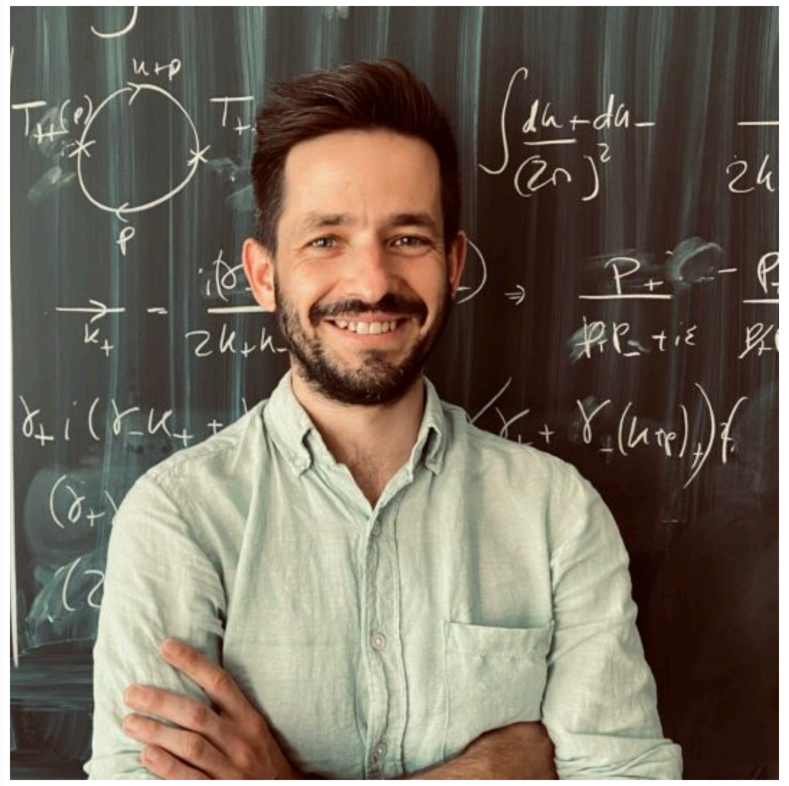
David Carpentier

(Ecole Normale Supérieure de Lyon)

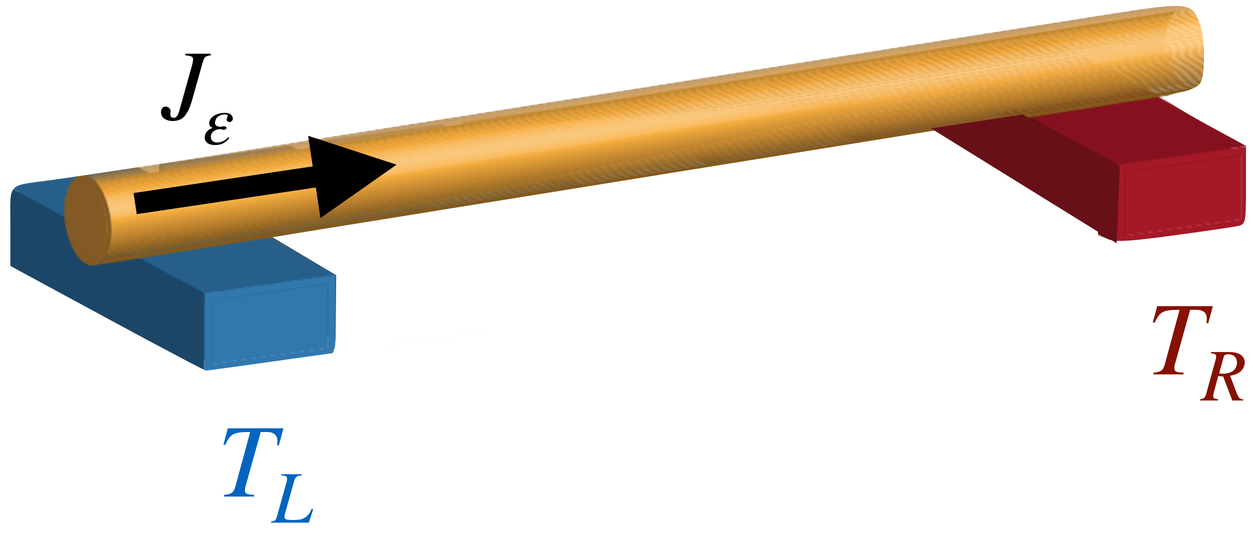
ENS DE LYON

Baptiste Bermond

Maxim Chernodub

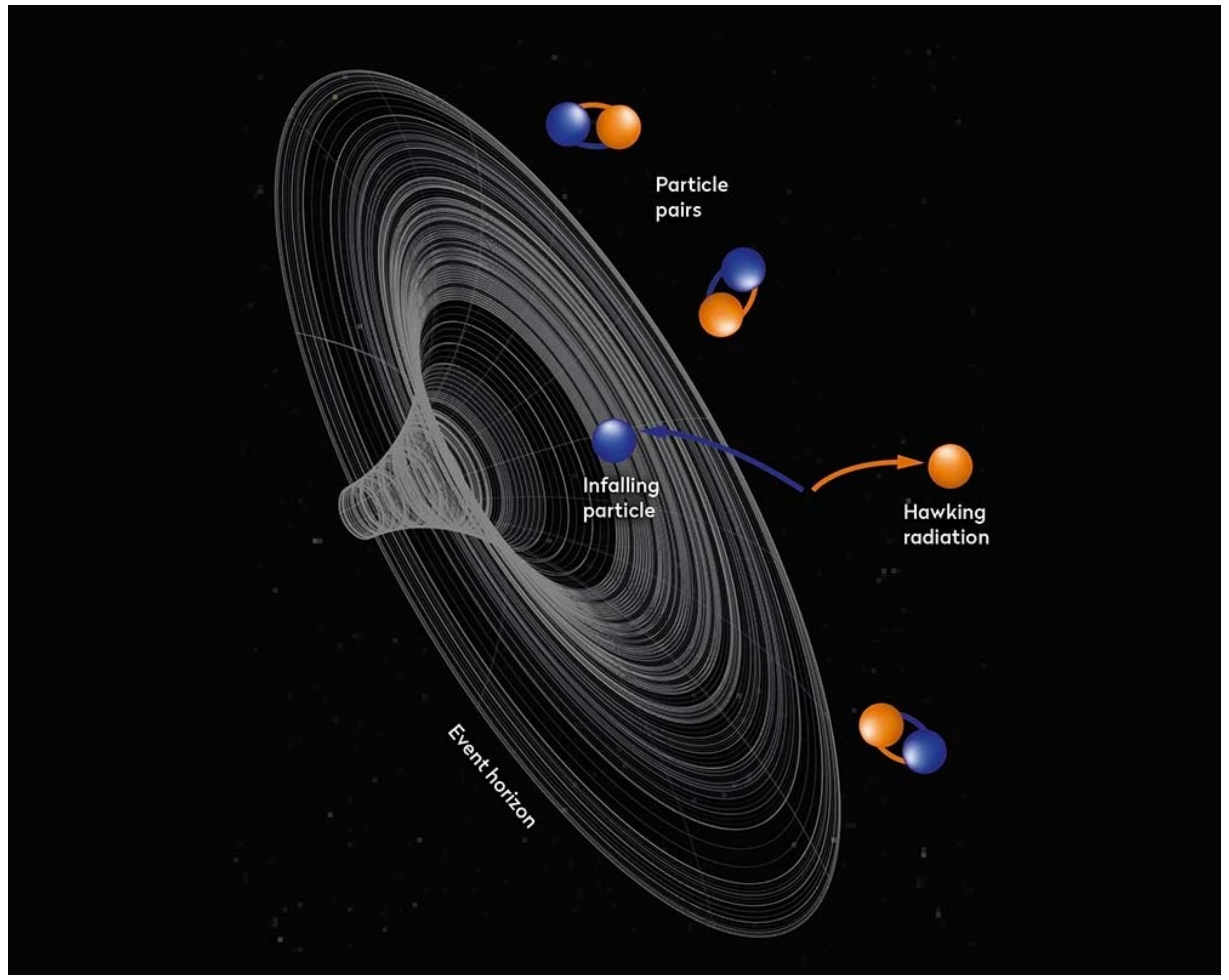


Adolfo Grushin

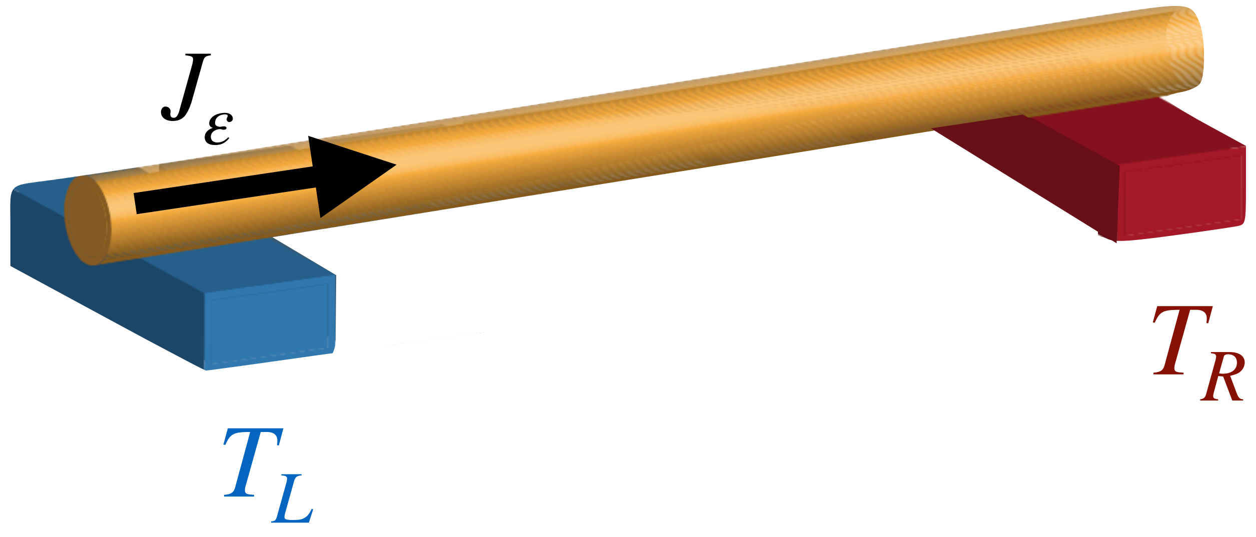


Thermal transport in a laboratory

? $\leftrightarrow$



Black body radiation close to a blackhole



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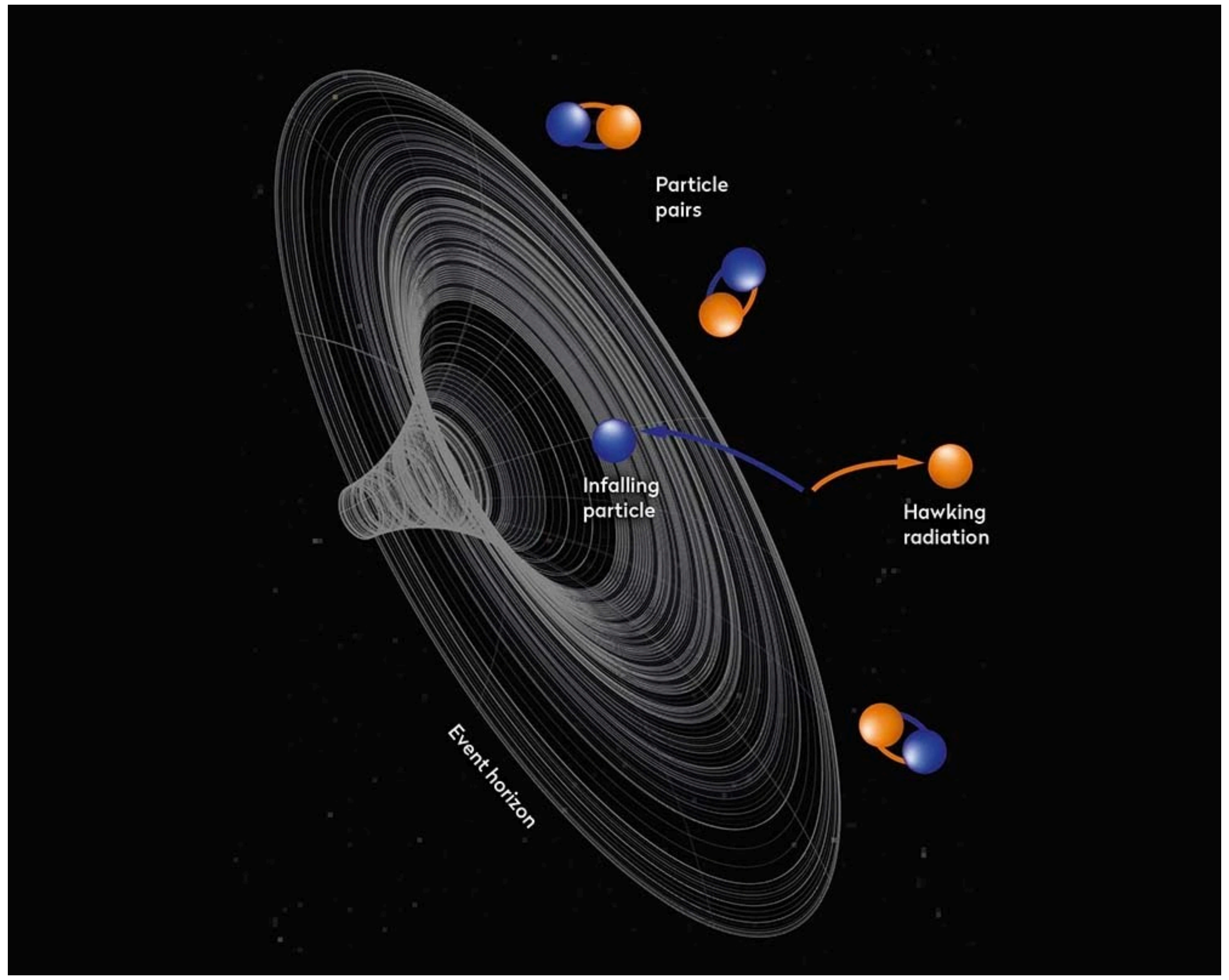
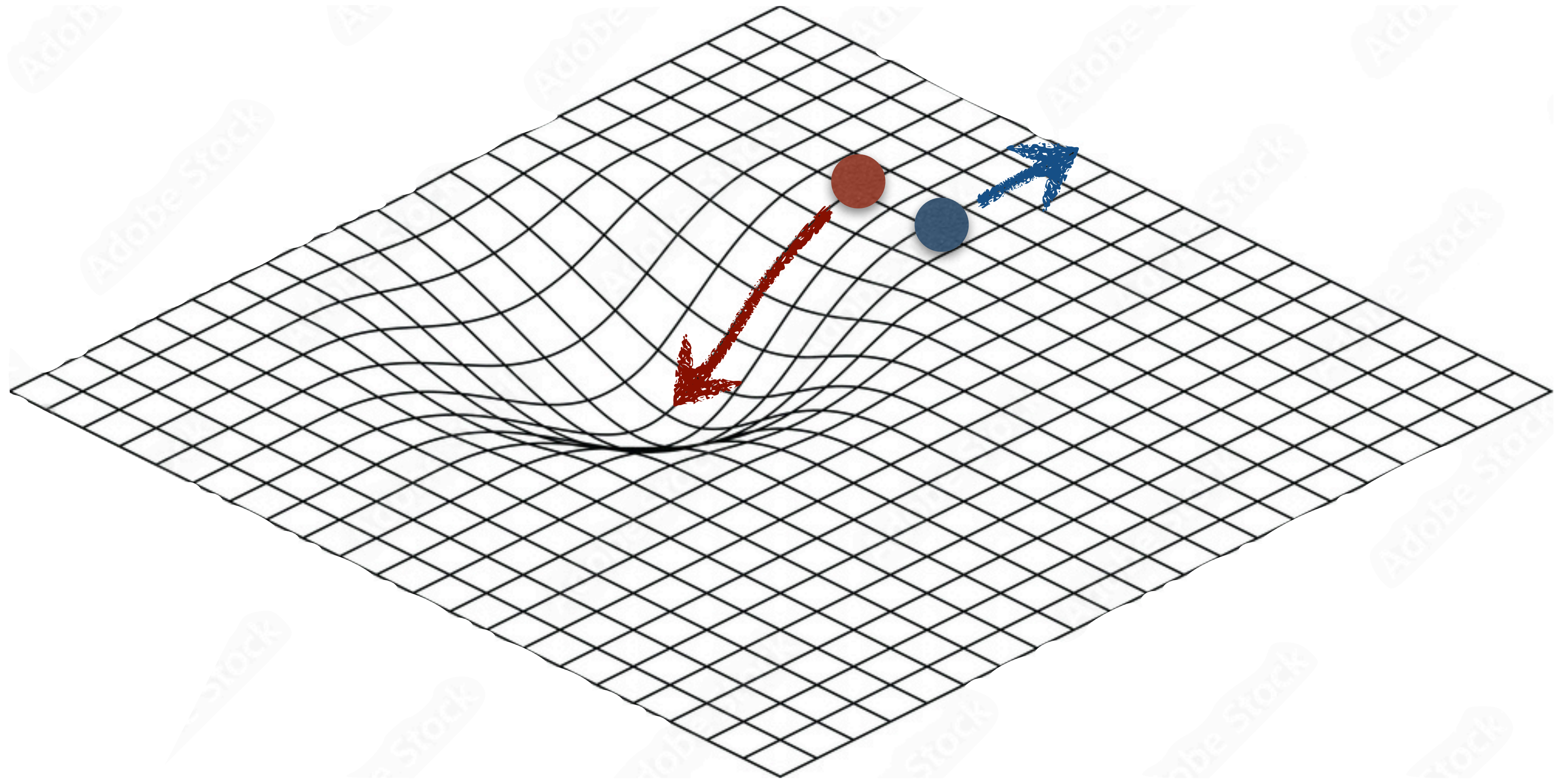


Black body radiation close to a blackhole

Anomalous vacuum fluctuations induced by spacetime curvature

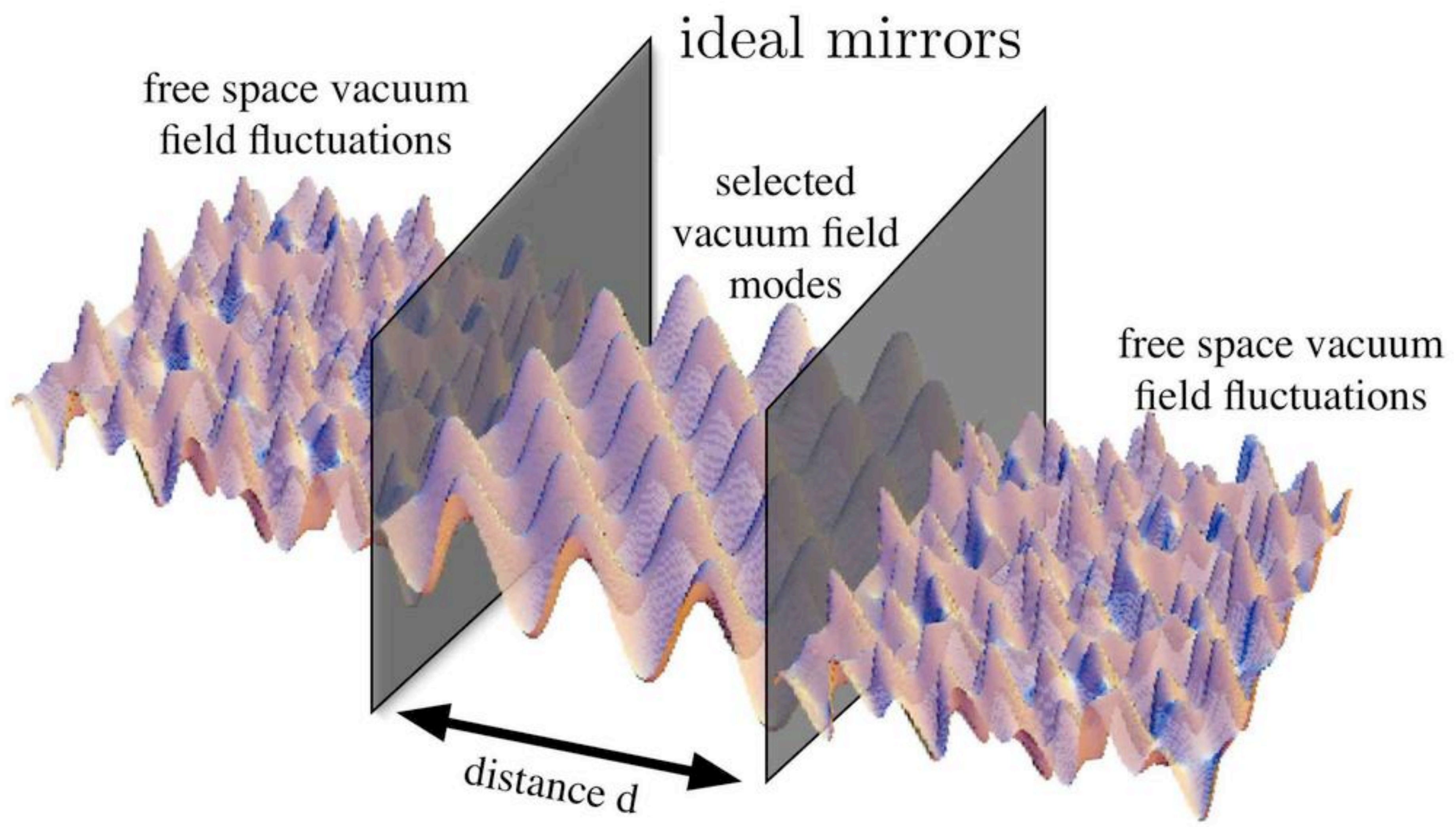


When geometry affects quantum fluctuations

Anomalous vacuum fluctuations
induced by **geometrical confinement**

- **Casimir effect**

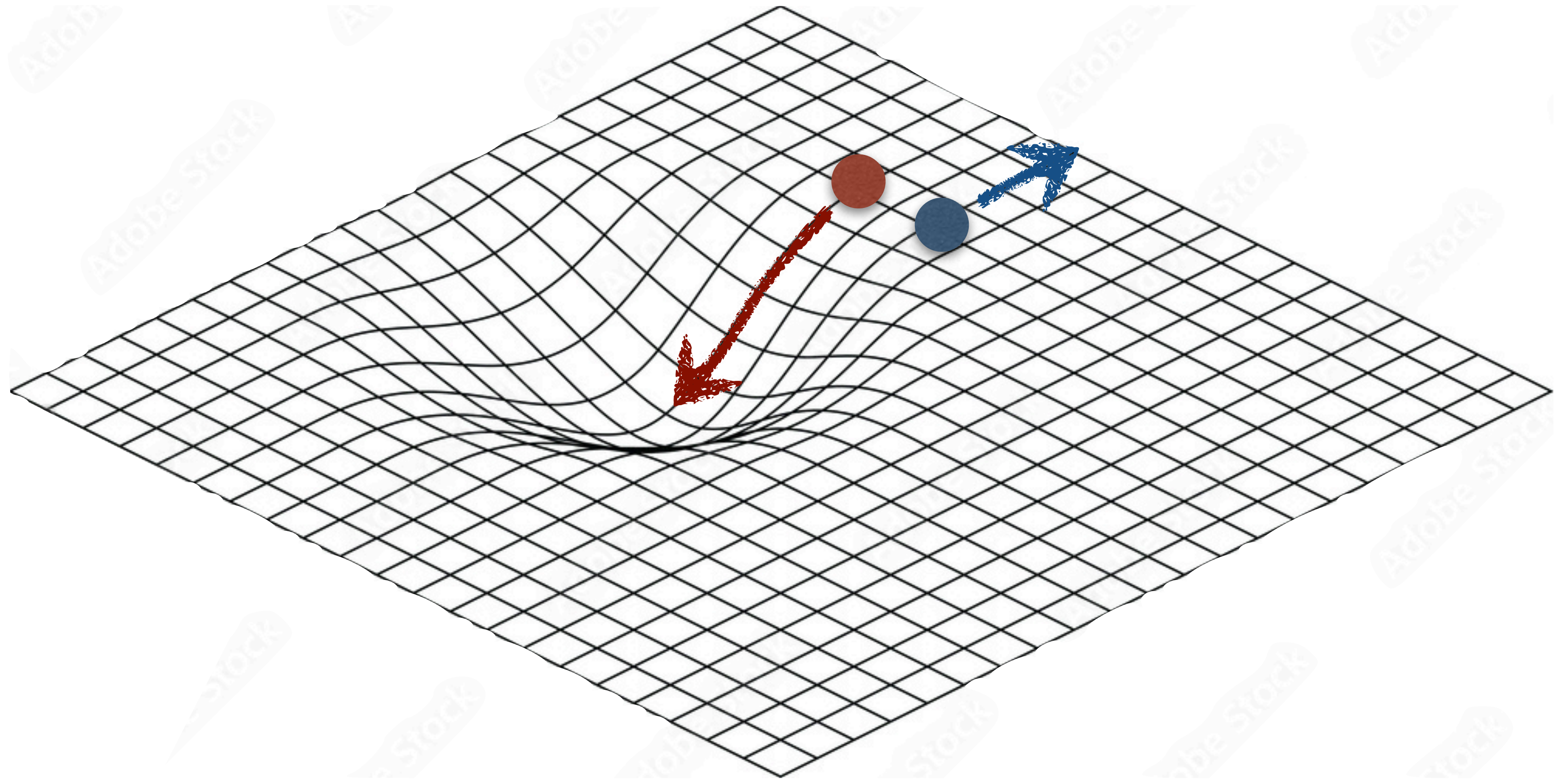
Casimir (1948)
Ruser (2007)



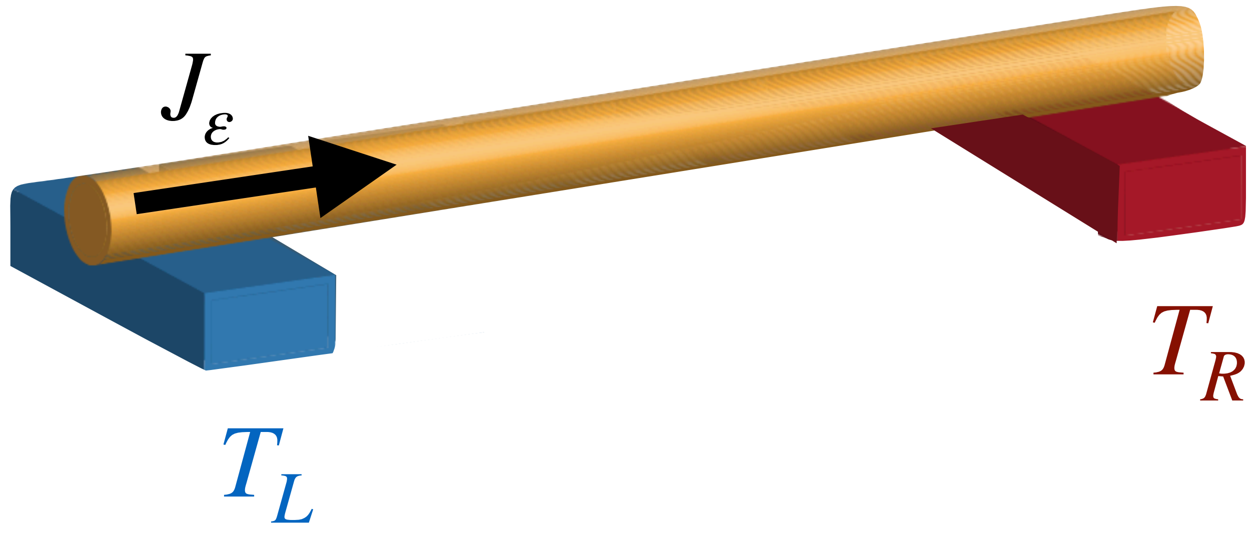
Anomalous vacuum fluctuations
induced by **spacetime curvature**

- **Gravitational anomalies** of **relativistic** field theories

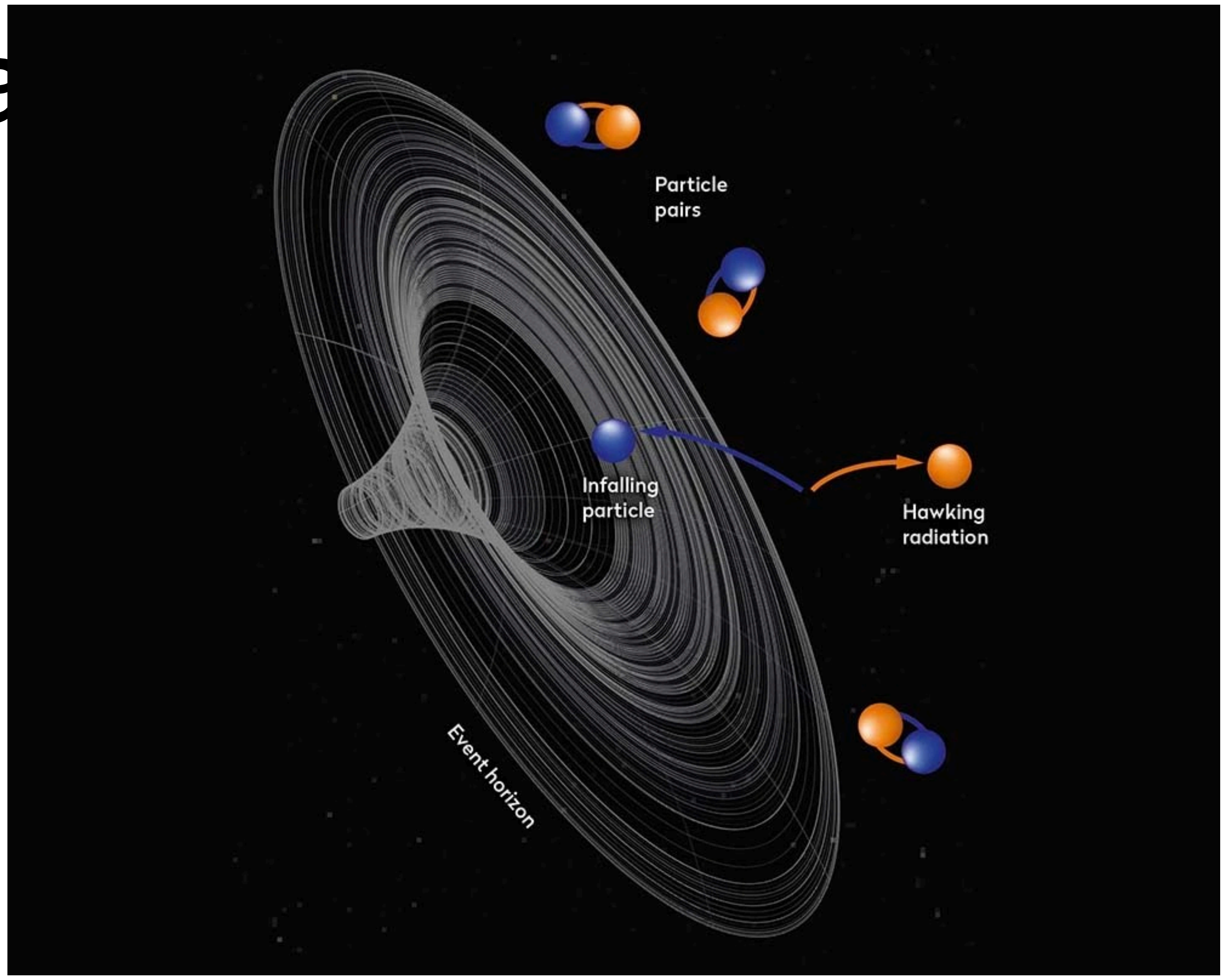
Bertlmann (2001)



When geometry affects quantum fluctua



Thermal transport in **a laboratory**



Black body radiation close to **a blackhole**

Anomalous vacuum fluctuations

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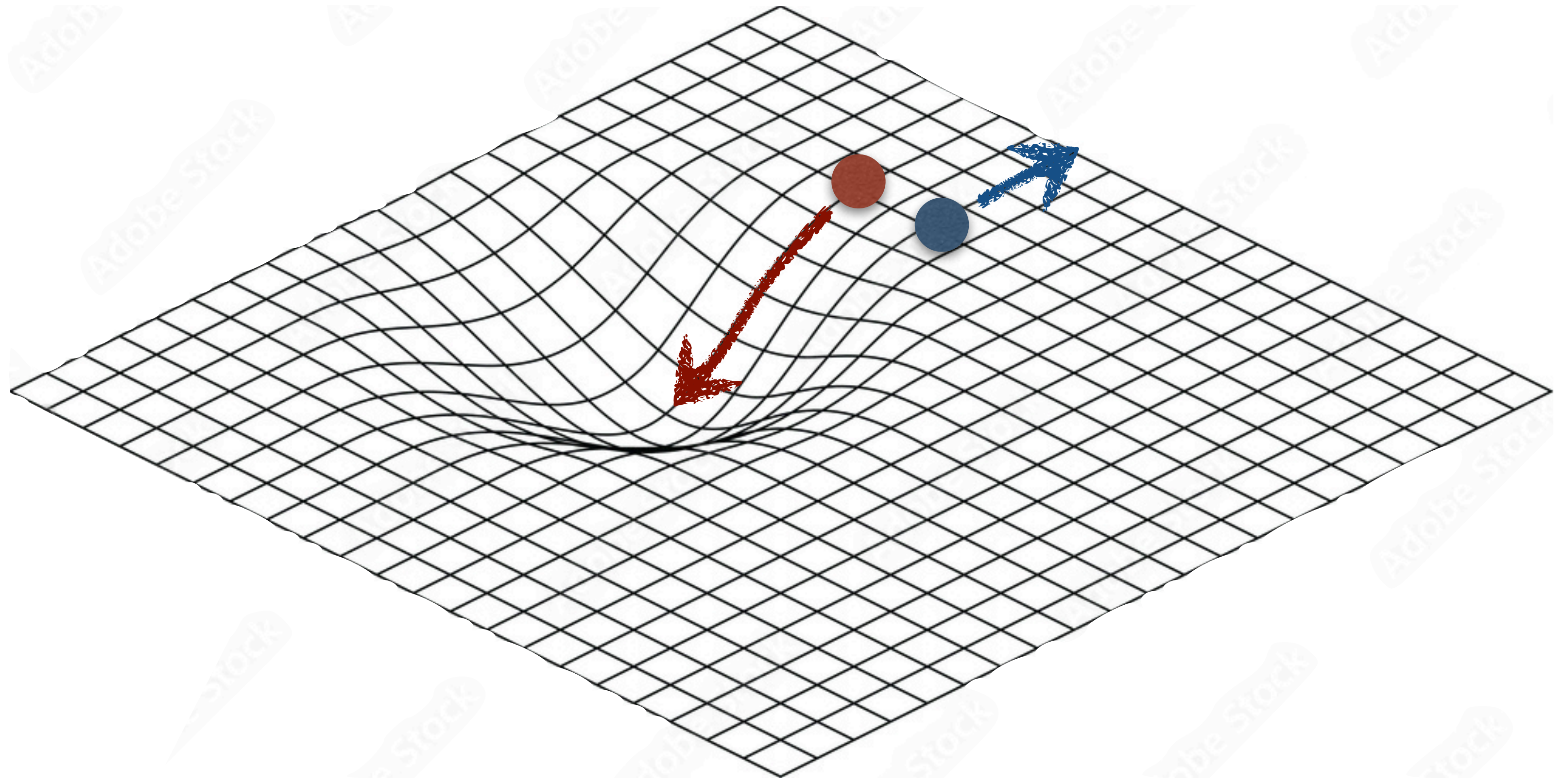
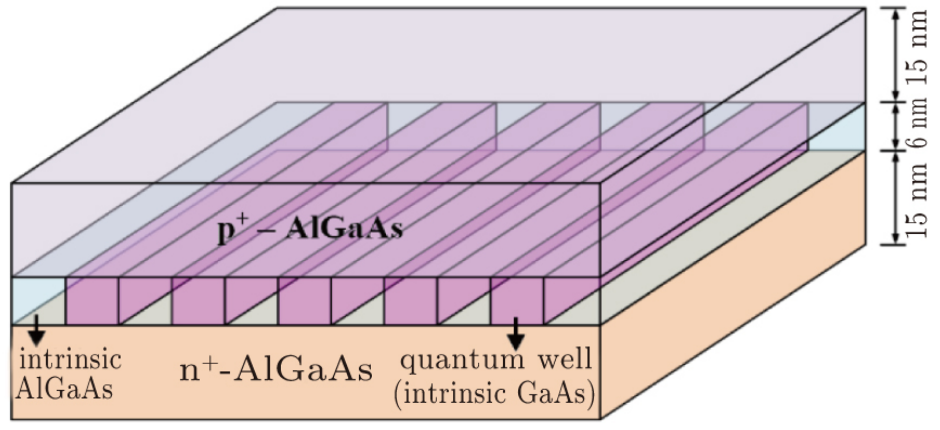


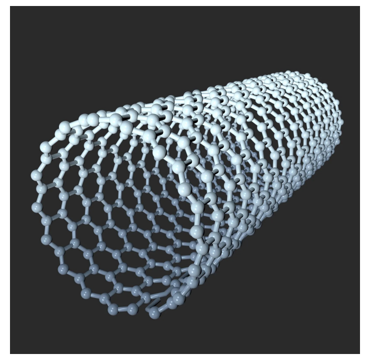
Massless relativistic particles in materials

vacuum fluctuations ↔ massless relativistic particles

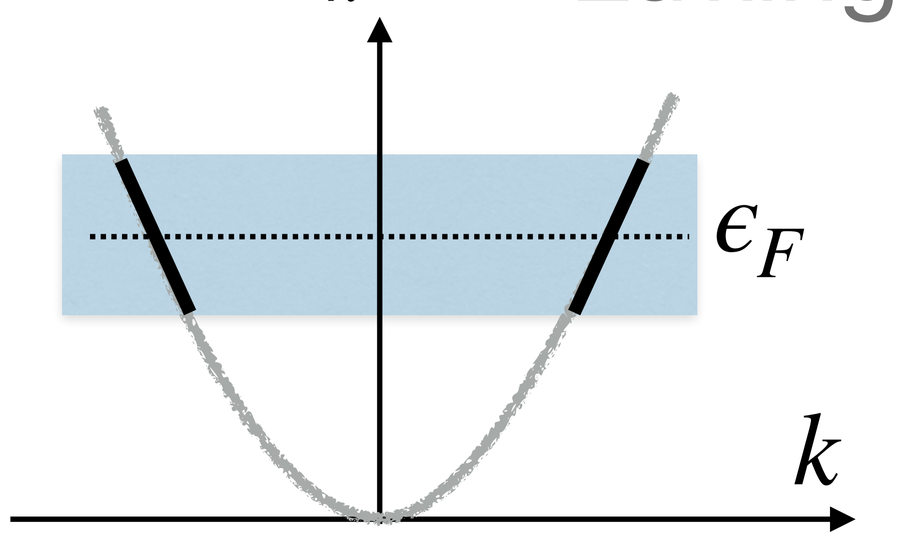
D=1+1

Quantum wires Nanotubes





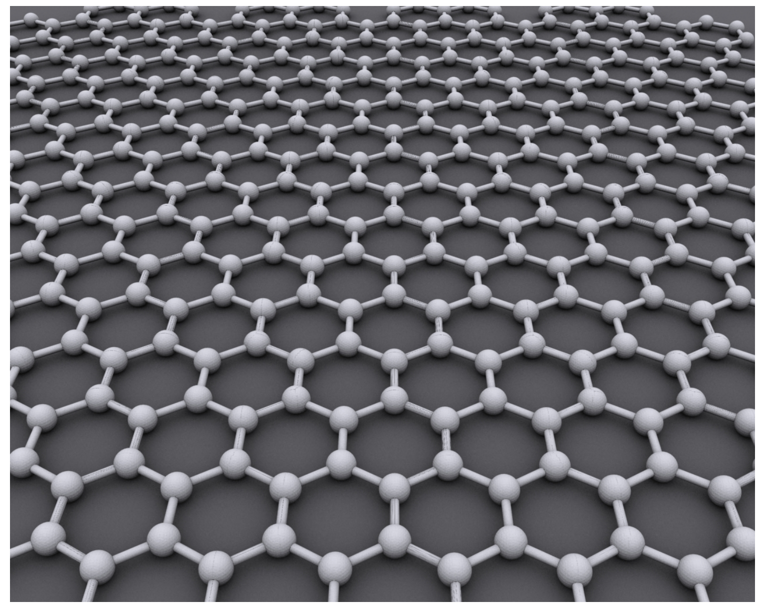
Luttinger liquid

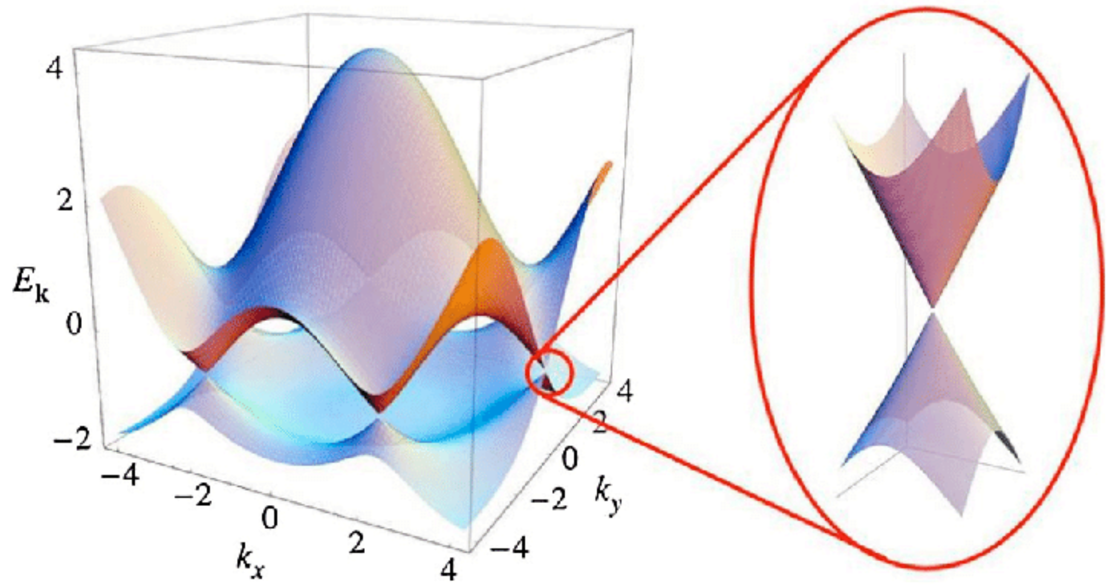


$$H(k) = \sigma_z\, k$$

D=2+1

Graphene

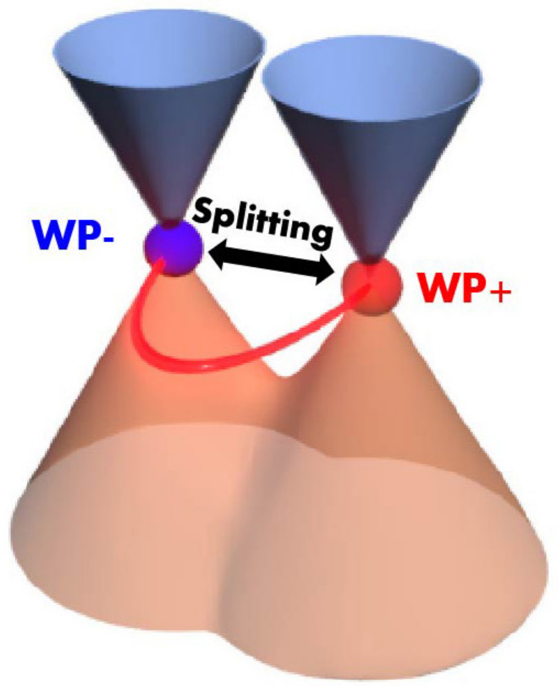




$$H(k) = \sigma_x\, k_x + \sigma_y\, k_y$$

D=3+1

Dirac and Weyl semimetals



$$H(k) = \sigma_x\, k_x + \sigma_y\, k_y + \sigma_z\, k_z$$

Luttinger Trick (1964)

J.M.Luttinger

PHYSICAL REVIEW VOLUME 135, NUMBER 6A 14 SEPTEMBER 1964

Theory of Thermal Transport Coefficients*

J. M. LUTTINGER
Department of Physics, Columbia University, New York, New York

(Received 20 April 1964)

Just as the space- and time-varying external electric potential produced electric currents and density variations, so a varying gravitational field will produce, in principle,[7] energy flows and temperature fluctuations. The reason for this is that an energy density $h(\mathbf{r})$ behaves as if it had a mass density $h(\mathbf{r})/c^2$, as far as its interaction with a gravitation field goes. Calling the gravitational potential $-c^2\psi(\mathbf{r},t)$, we have an interaction term in the Hamiltonian of the form

$$\int h(\mathbf{r})\psi(\mathbf{r},t)d\mathbf{r}\,,$$

where $h(\mathbf{r})$ is the Hamiltonian density of the unperturbed system. Clearly a varying ψ will give rise to a varying energy density, which, in turn, will correspond to a varying temperature. We shall see this in more

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[7] These effects are actually extremely small, far too small to be observed in any ordinary experiment. They were first considered by A. Einstein, Ann. Physik **38**, 443 (1912). See also R. C. Tolman, Phys. Rev. **35**, 904 (1930) and R. C. Tolman and P. Ehrenfest, *ibid*. **36**, 1791 (1930). (I am indebted to Professor G. Uhlenbeck for calling these interesting references to my attention.) Although the effect is very small, in practice we are only interested in questions of principle, and an arbitrarily small effect is just as good as a large one. In fact, if the gravitational field didn't exist, one could invent one for the purposes of this paper.

… based on the Tolman-Ehrenfest theorem

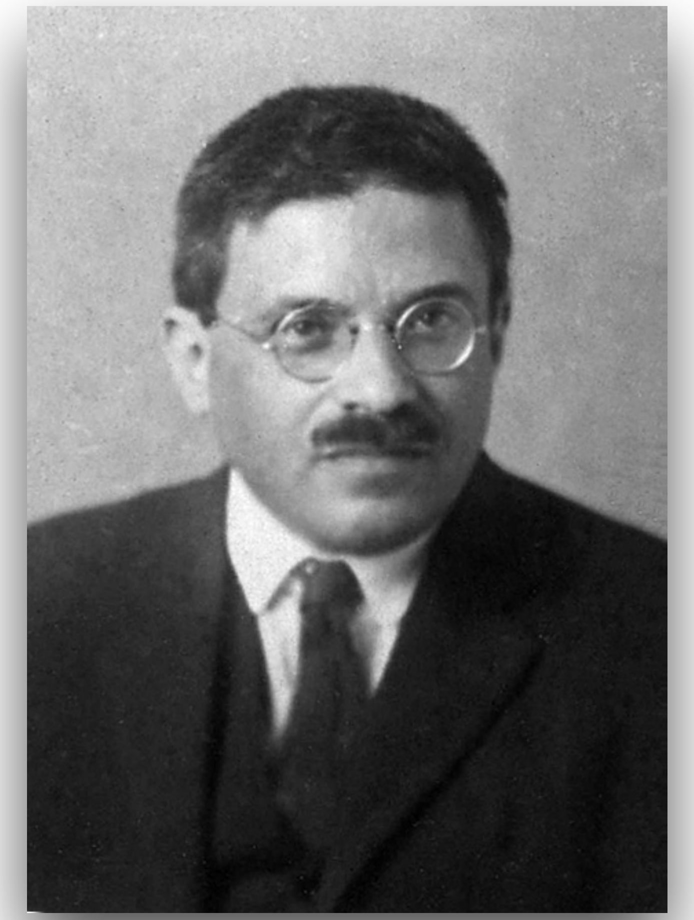

R.C. Tolman

DECEMBER 15, 1930 PHYSICAL REVIEW VOLUME 36

TEMPERATURE EQUILIBRIUM IN A STATIC GRAVITATIONAL FIELD

By Richard C. Tolman and Paul Ehrenfest
Norman Bridge Laboratory of Physics, Pasadena, California

(Received October 27, 1930)


P. Ehrenfest

Thermal equilibrium in a curved spacetime (here in D=1+1)

Curved spacetime

$$ds^2 = g_{00}(x) v_F^2 \, dt^2 - dx^2$$

⇨

Inhomogeneous equilibrium temperature

$$T(x) = T_0 \sqrt{\frac{g_{00}(x_0)}{g_{00}(x)}} \,, \quad T_0 = T\left(x_0\right)$$

… based on the Tolman-Ehrenfest theorem

R.C Tolman and P. Ehrenfest (1930)

Curved Spacetime

$$ds^2 = \left(\frac{T_0}{T(x)}\right)^2 v_F^2\, dt^2 - dx^2$$

Inhomogeneous $T(x)$

Equilibrium, $J_\varepsilon = 0$

=

Flat Spacetime

$$ds^2 = v_F^2\, dt^2 - dx^2$$

Inhomogeneous $T(x)$

Out of Equilibrium, $J_\varepsilon \neq 0$

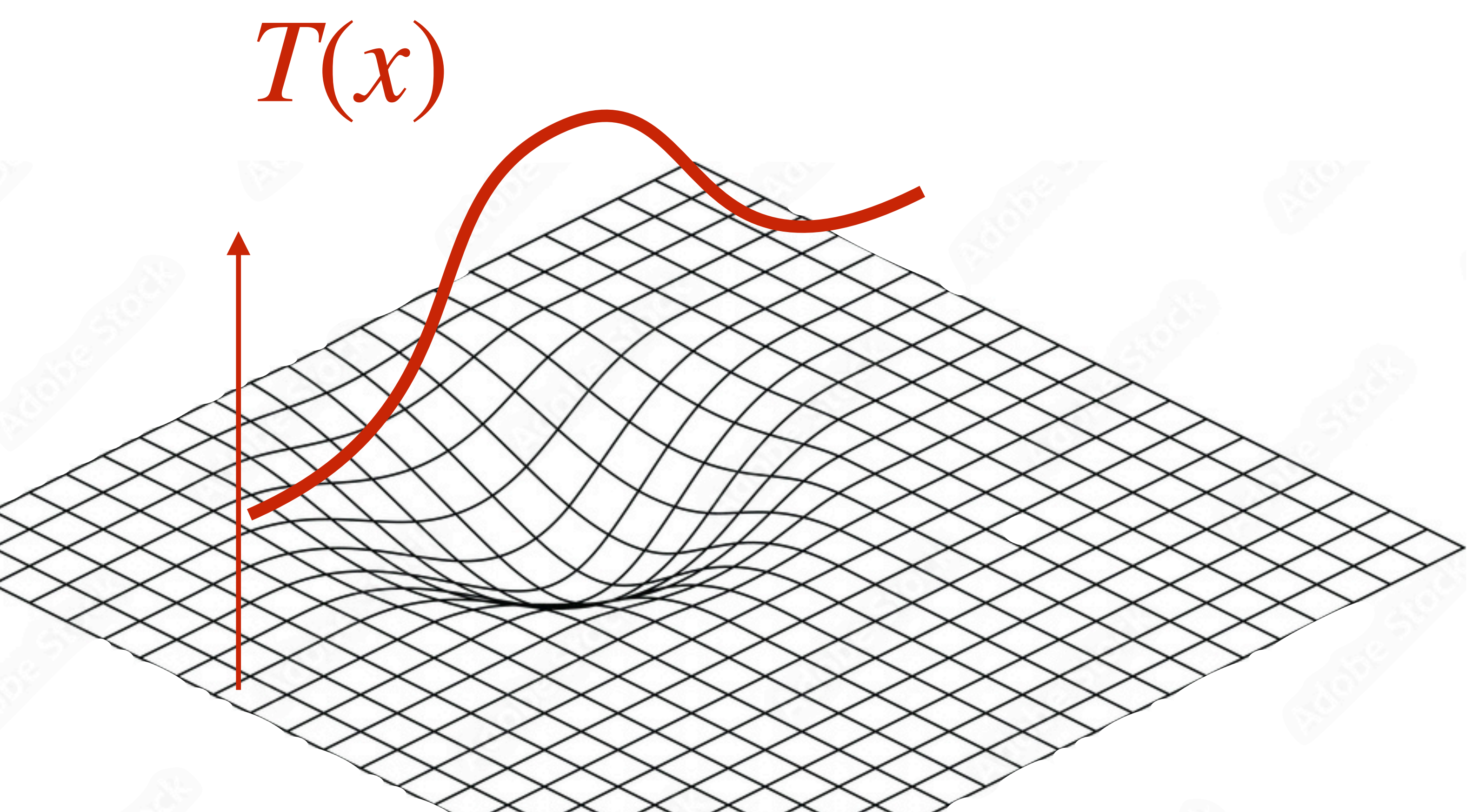
+

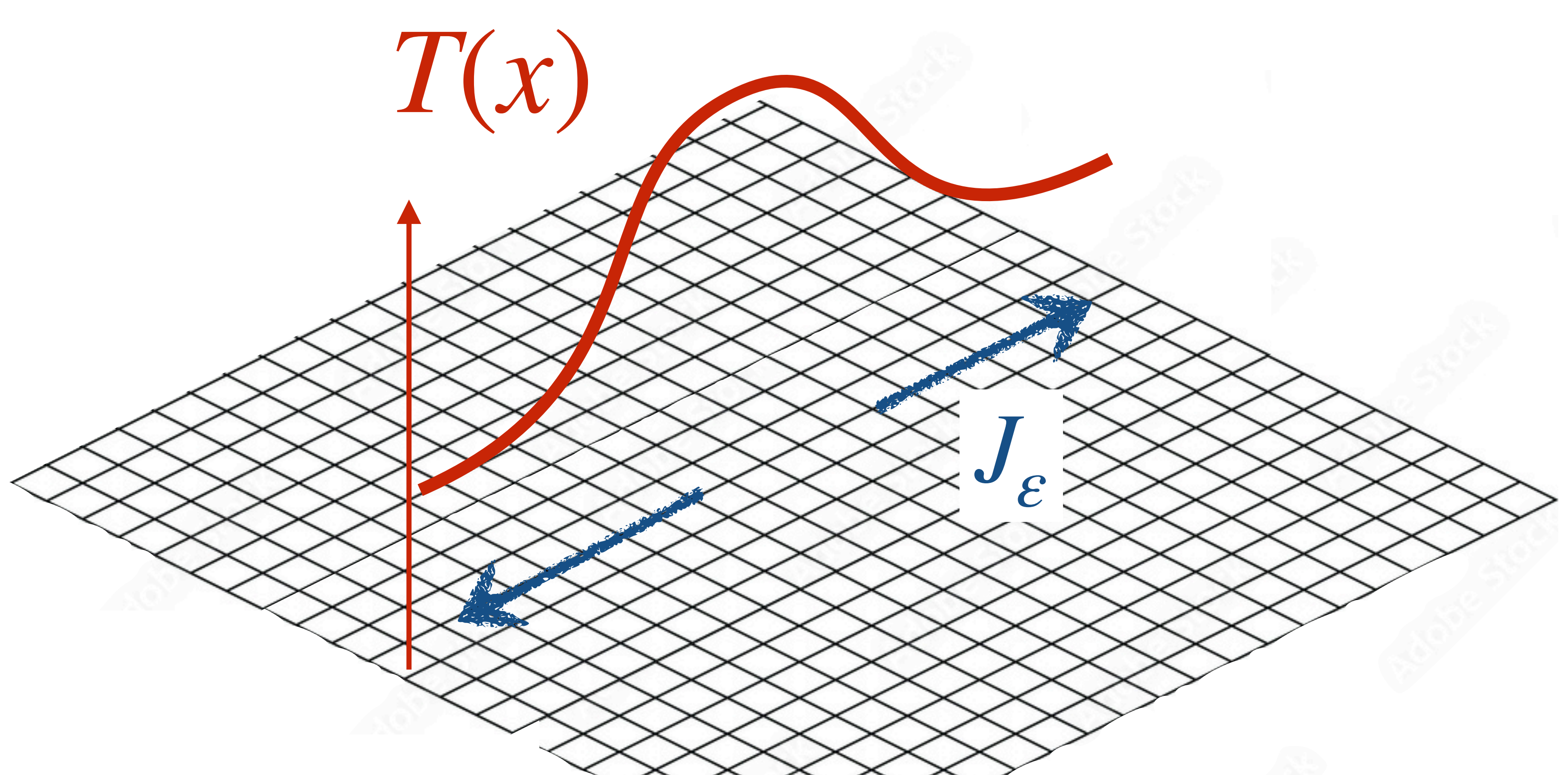
Curved spacetime

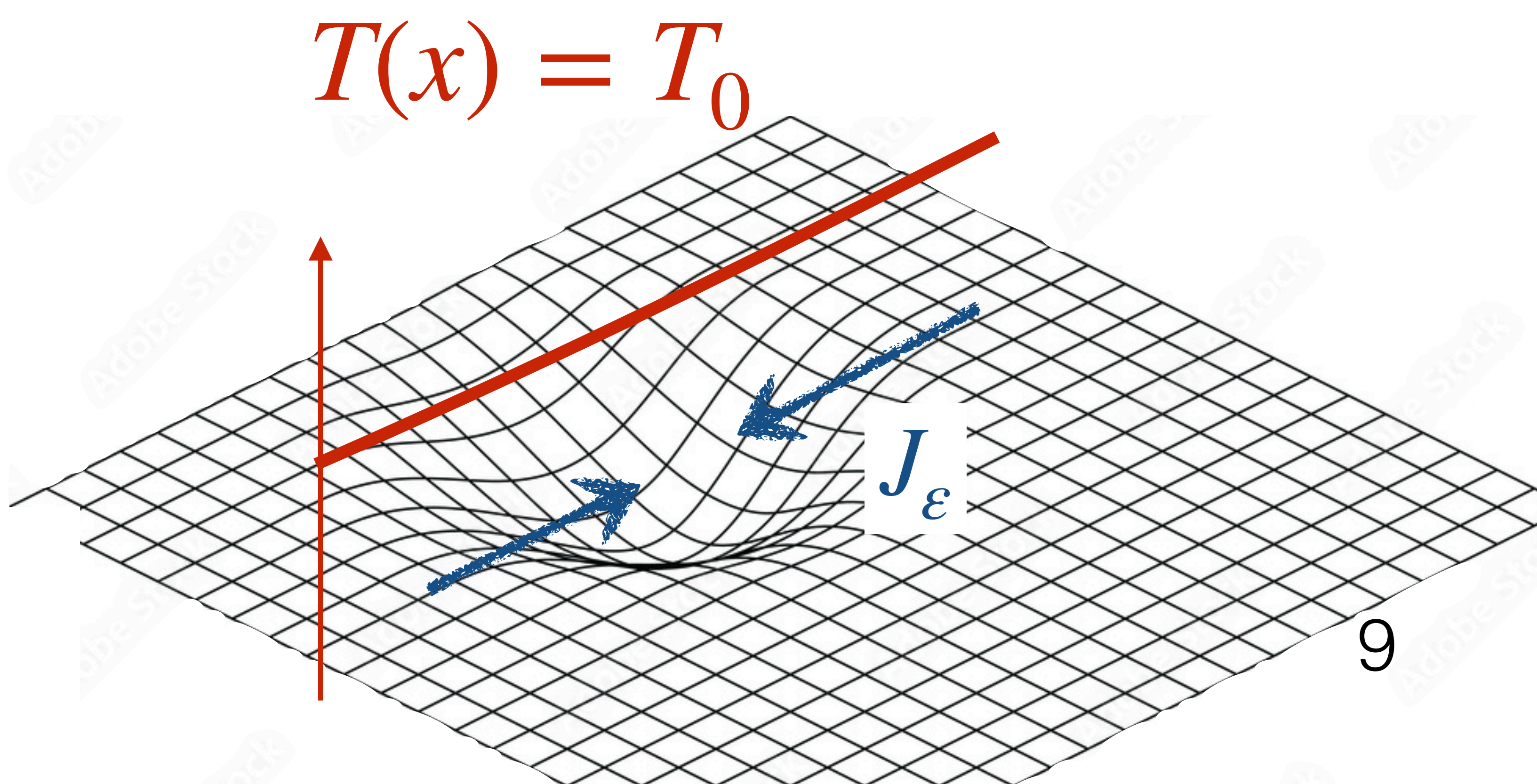
$$ds^2 = \left(\frac{T_0}{T(x)}\right)^2 v_F^2\, dt^2 - dx^2$$

Homogeneous T_0

Out of Equilibrium, $J_\varepsilon \neq 0$







… based on the Tolman-Ehrenfest theorem

J. Luttinger, (1964)

Luttinger equivalence

Luttinger metric $g_{\mu\nu} = \begin{pmatrix} e^{2\phi(x)} & 0 \\ 0 & 1 \end{pmatrix}$

$$ds^2 = e^{2\phi(x)} v_F^2\, dt^2 - dx^2$$

with gravitational potential $\phi(x)$

$$e^{2\phi(x)} = \left(\frac{T_0}{T(x)}\right)^2$$

$$\partial_x \phi(x) = -\frac{\partial_x T(x)}{T(x)}$$

Flat Spacetime

$$ds^2 = v_F^2\, dt^2 - dx^2$$

Inhomogeneous $T(x)$

Out of Equilibrium, $J_\varepsilon \neq 0$

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Curved spacetime

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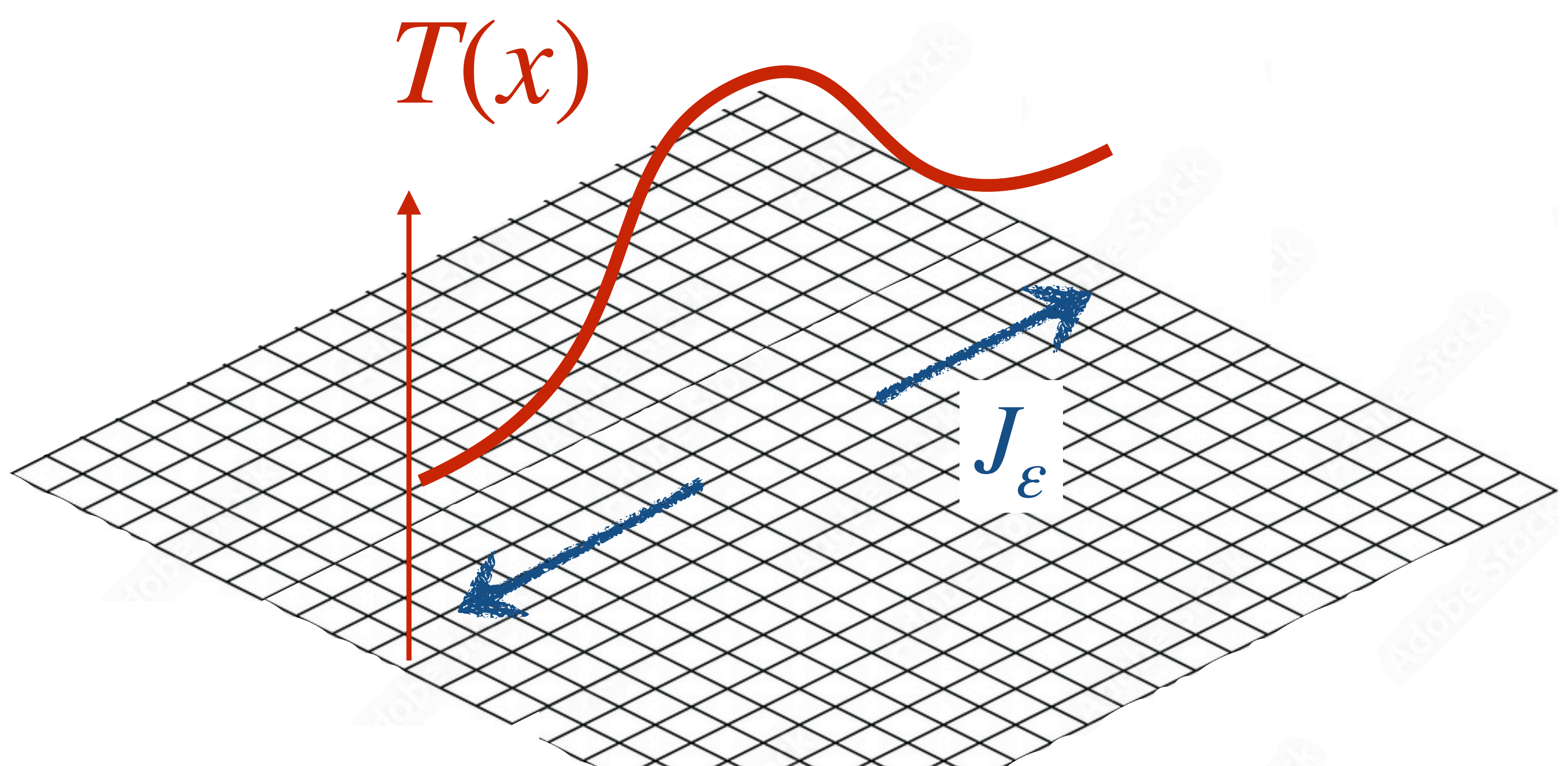
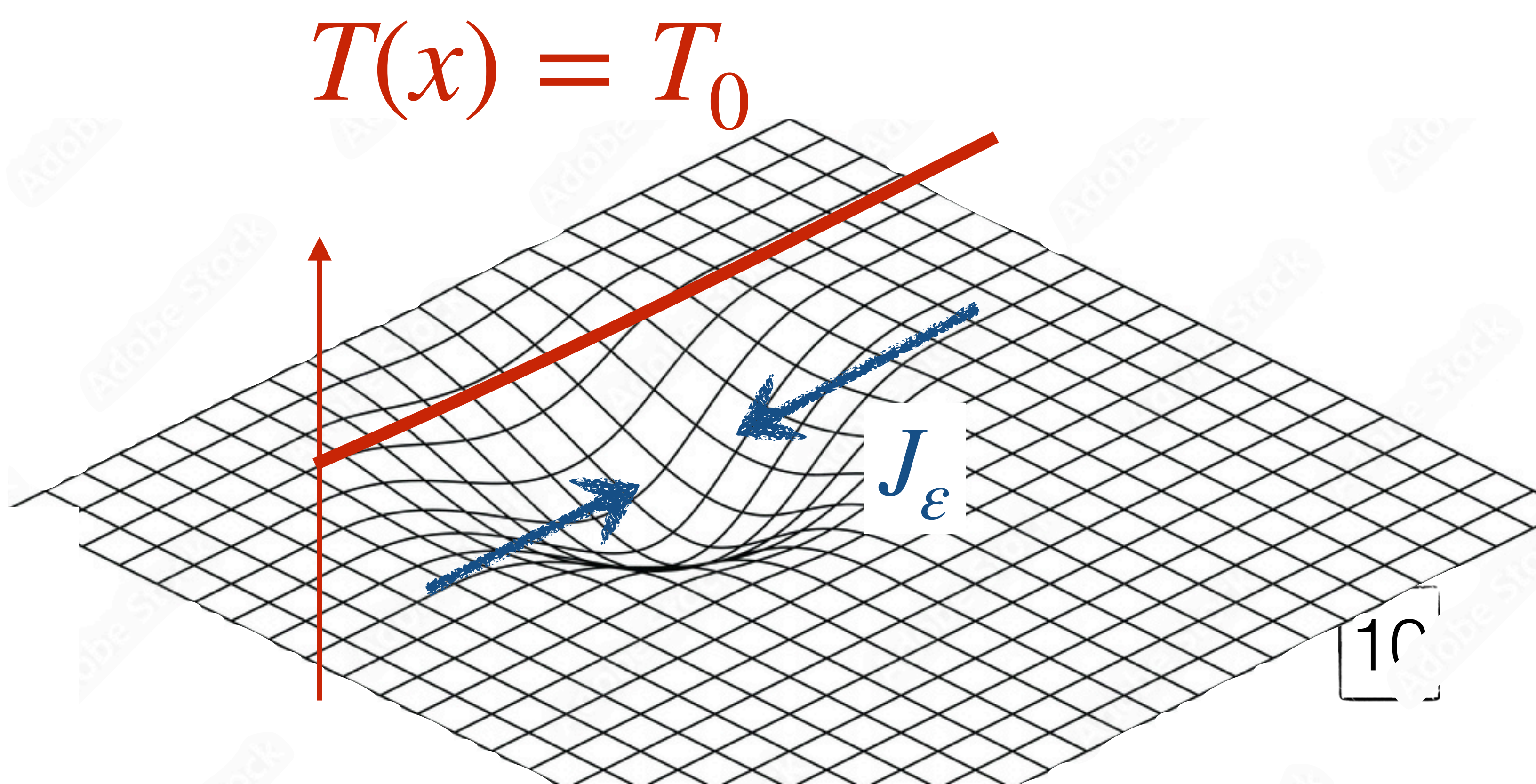




Anomaly and vacuum fluctuations

- Conformal/Weyl symmetry
- Lorentz invariance
- Diffeomorphism invariance

} Symmetries of the Hamiltonian / action

Anomaly: in a quantum theory is a symmetry of the action, but not of the measure

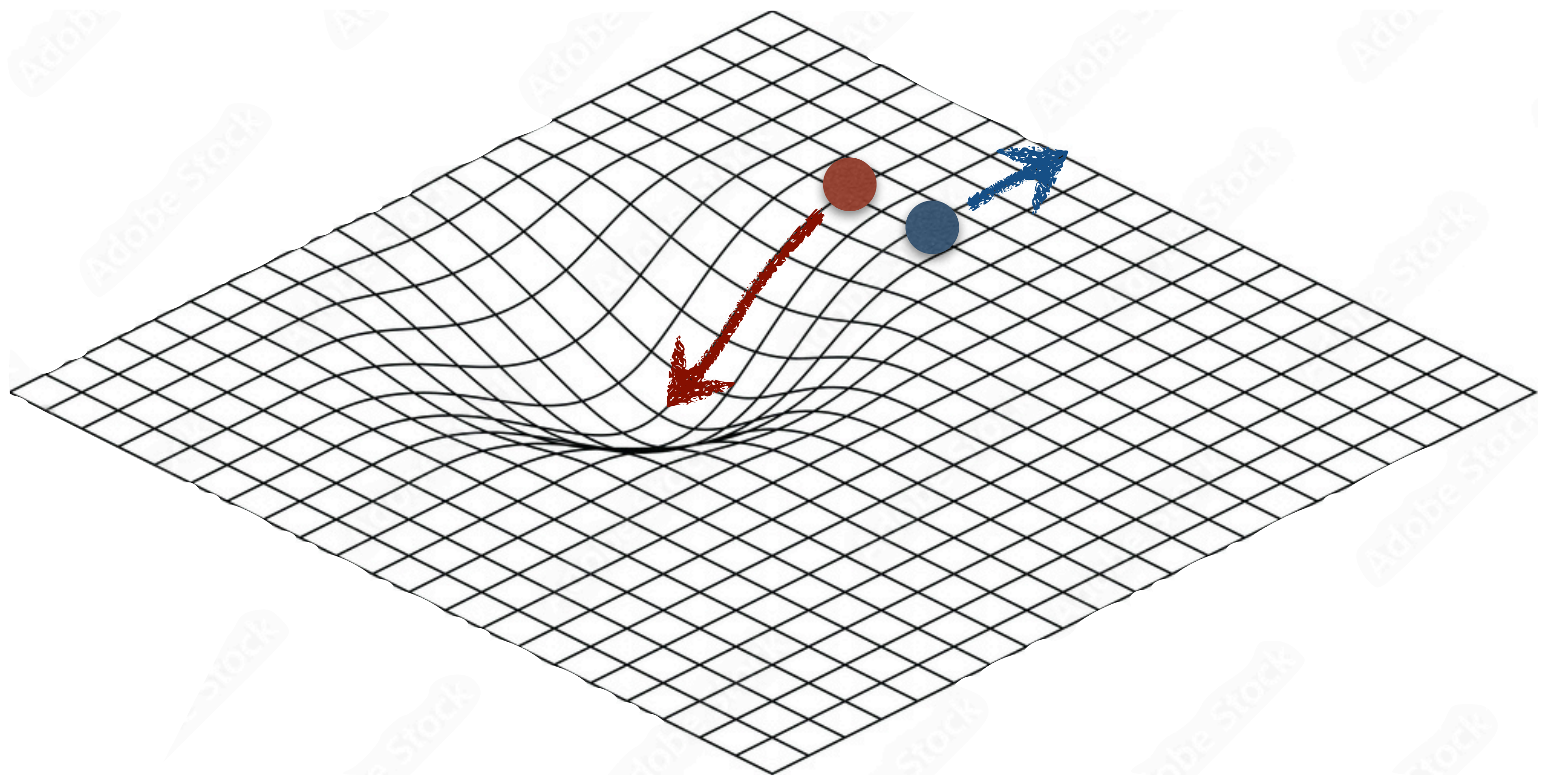
➡ signals anomalous quantum fluctuations

➡ Conservation law spoiled by quantum fluctuations

Bertlmann, Anomalies in Quantum Field Theory (2001)

Relativistic quantum theory in a curved spacetime :

gravitational anomalies: anomalous vacuum fluctuations



Anomalous equilibrium temperature

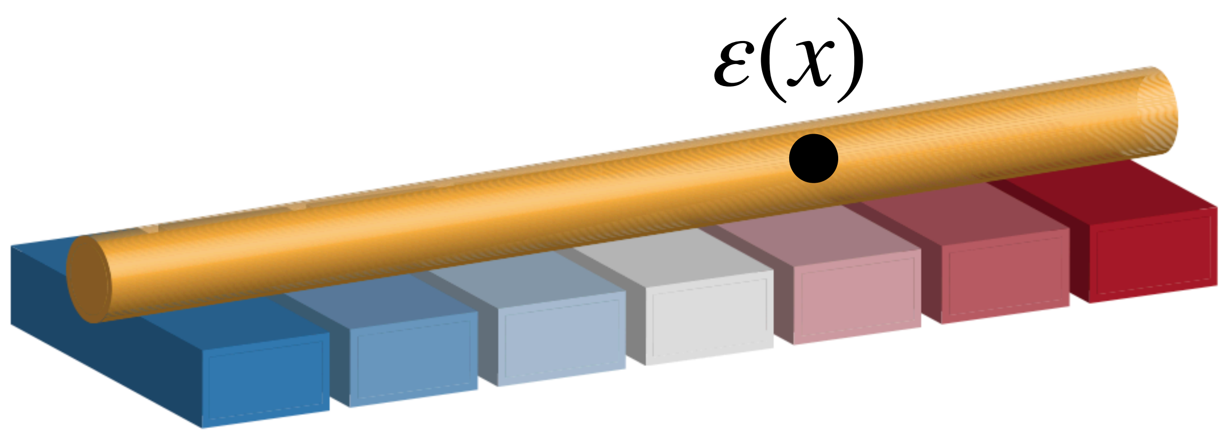
Curvature of spacetime $\mathscr{R}$ sets new energy scales: $\epsilon_q^{(1)} = \frac{\hbar v_F}{48\pi}\mathscr{R}$ $\quad \epsilon_q^{(2)} = \epsilon_q^{(1)} - \frac{1}{f_1(x)}\int_0^x \epsilon_q^{(1)}\partial_x f_1$

- via thermodynamics (Stefan-Boltzman law):

  energy density ε and pressure P

Conformal anomaly

$$\varepsilon - P = C_w\, \epsilon_q^{(1)} \qquad \varepsilon + P = C_w \gamma_{1D} T_m^2$$

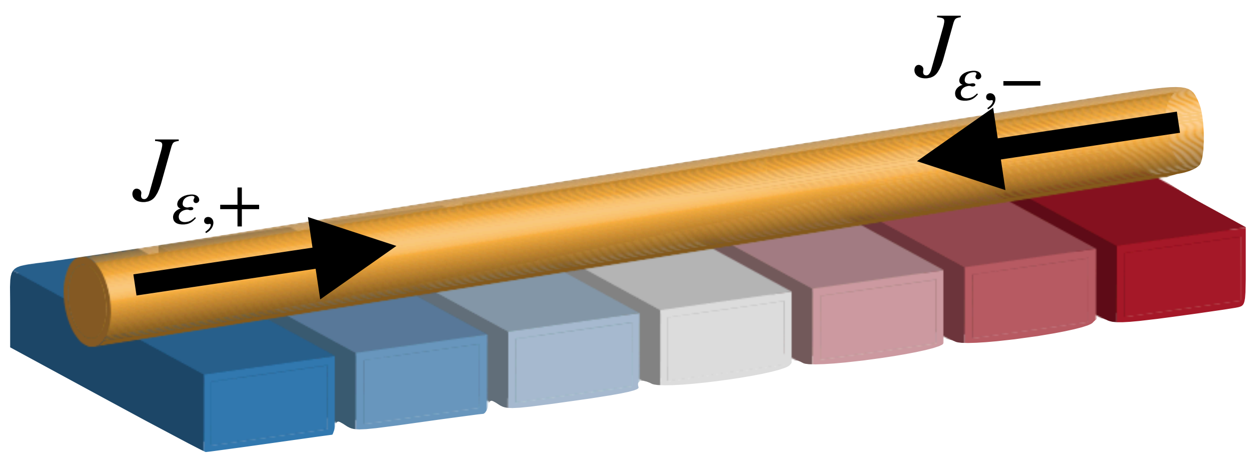


Anomalous eq. Temperature

$$\gamma_{1D} T_m^2(x) = \gamma_{1D}\, T_{TE}^2(x) + \epsilon_q^{(2)}(x)$$

- via transport (current):

  energy current J_ε and momentum Π



Einstein anomaly

$$v_F^{-1} J_\varepsilon = v_F \Pi = C_g\, \gamma_{1D}\, T_m^2$$

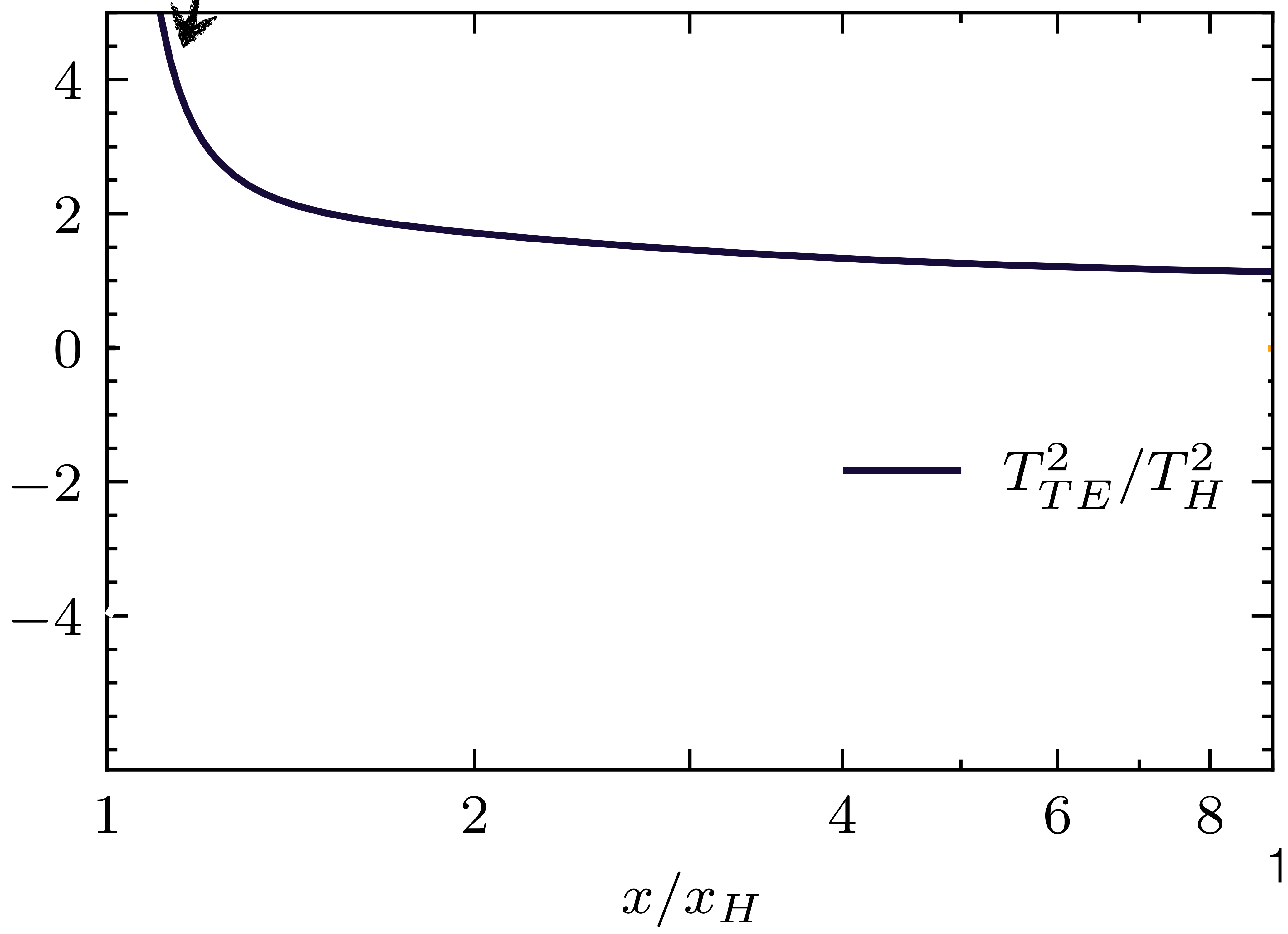
Hawking radiation and Black-hole atmosphere

$$g_{\mu\nu} = \begin{pmatrix} f(x) & 0 \\ 0 & -1/f(x) \end{pmatrix}$$

$$f(x) = 1 - \frac{x_H}{x} \qquad f'(x_H^+) = 2\kappa c^{-2}$$

Christensen et al. (1977)
Robinson et al. (2005)
M.Eune et al. (2017)

- Classical $T_{TE}^2 = \frac{T_H^2}{f}$ diverges at the horizon



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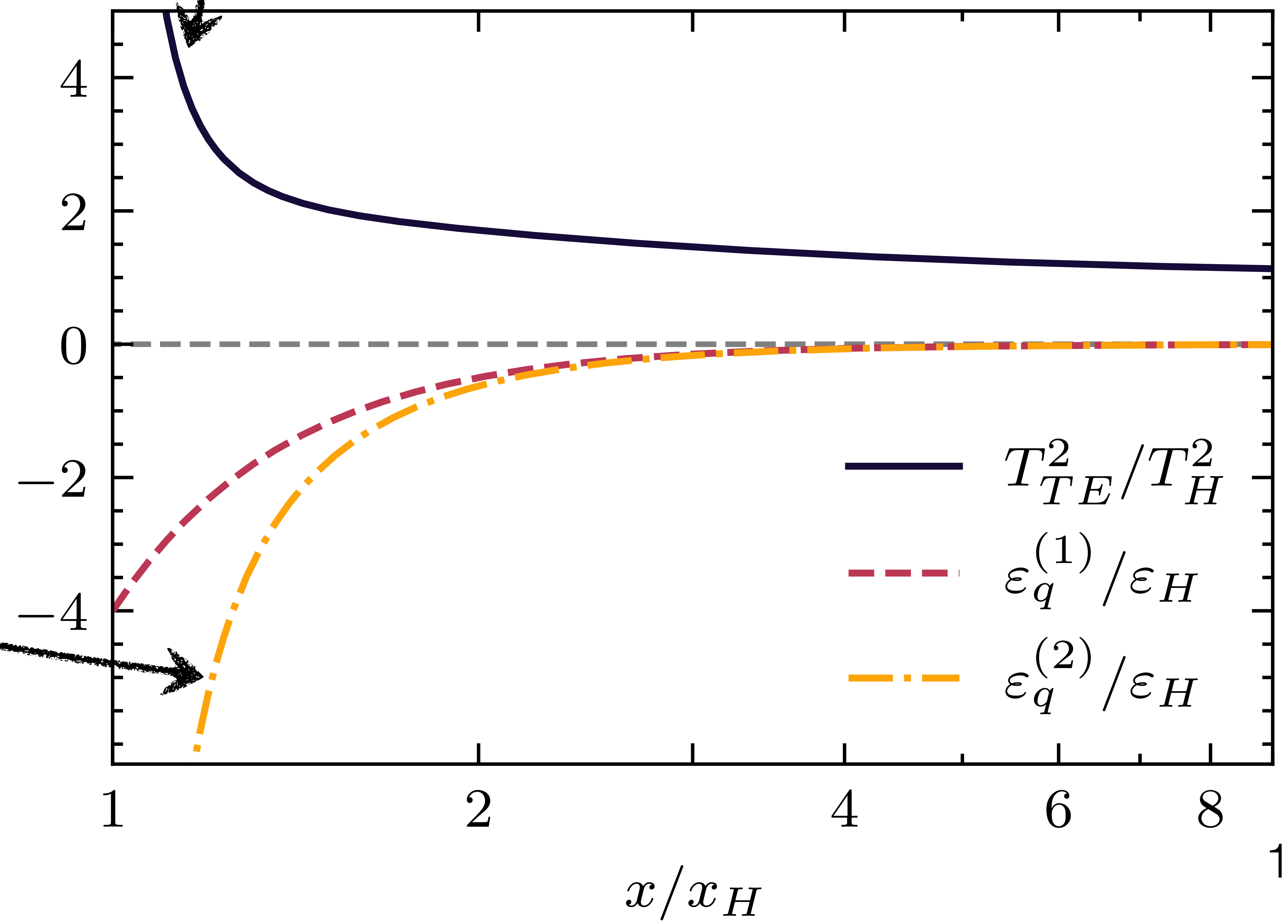
- Classical $T_{TE}^2 = \frac{T_H^2}{f}$ diverges at the horizon
- Anomalous eq. Temperature
  $\gamma_{1D} T_m^2(x) = \gamma_{1D} T_{TE}^2(x) + \epsilon_q^{(2)}$
  with new energy scale:
  $\epsilon_q^{(2)} = \frac{\hbar c}{48\pi} \left[f'' - \frac{(f')^2}{2f} \right]$ diverges at the horizon
- both divergences cancel
  provided $k_B T_H = \frac{\hbar}{2\pi c}\kappa$



Hawking radiation and Black-hole atmosphere

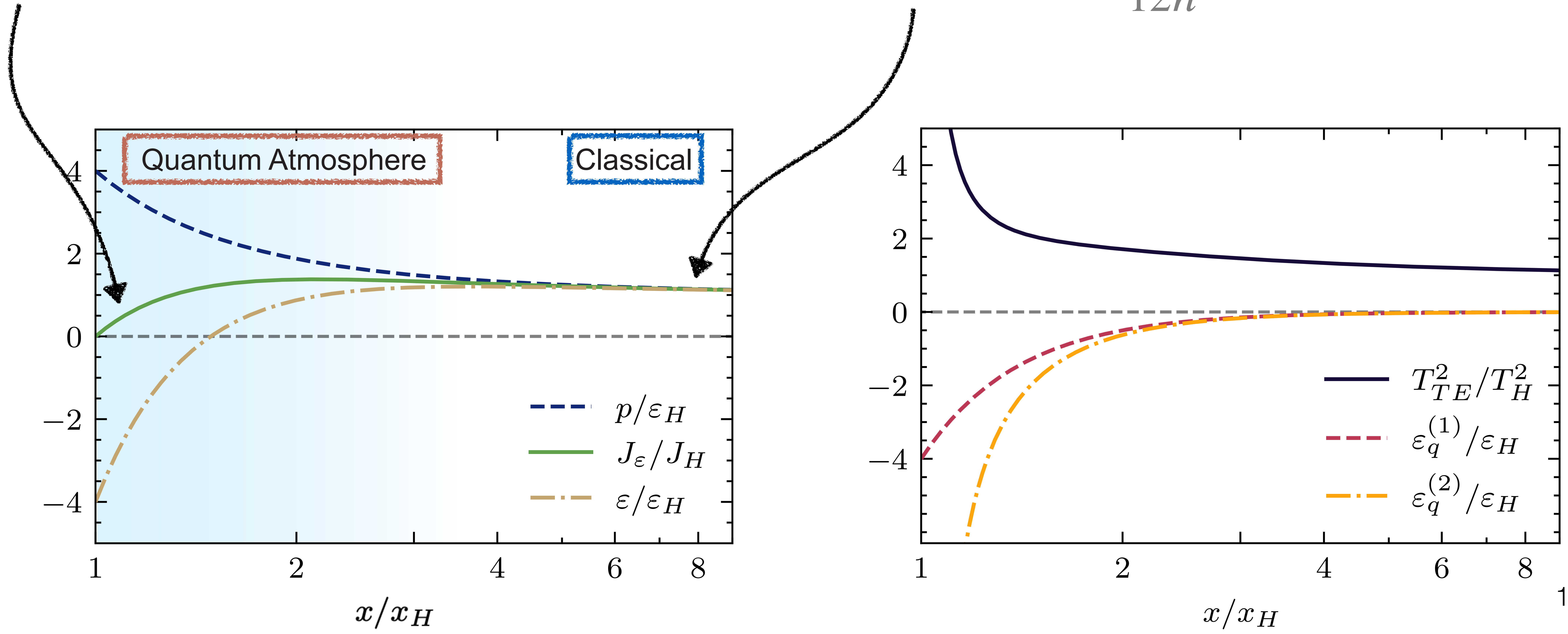
Giddings (2016)

Outgoing energy current / Temperature

$J_\varepsilon\left(x \to x_H\right) = 0$: **Nothing exits the black-hole**

Asymptotic outgoing energy current:

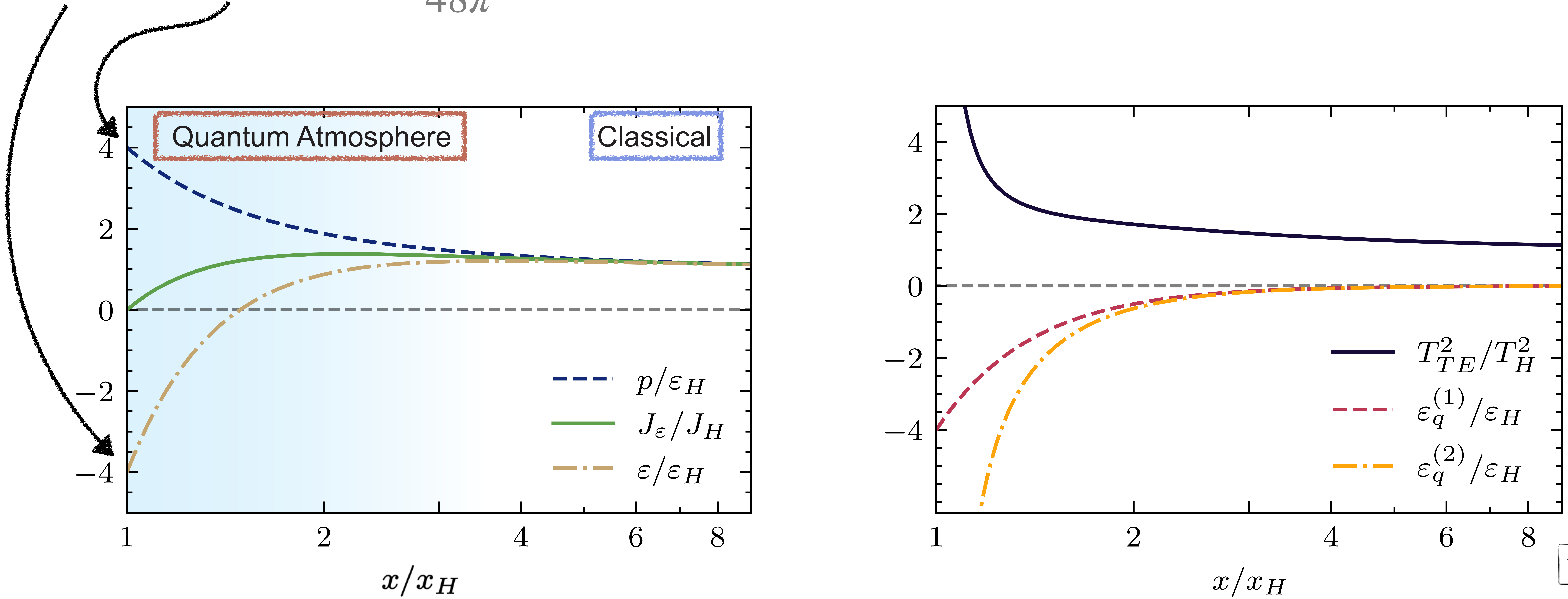
$J_{\varepsilon,+}(x \to \infty) = \frac{\pi}{12\hbar} T_H^2$: **Hawking Radiation**



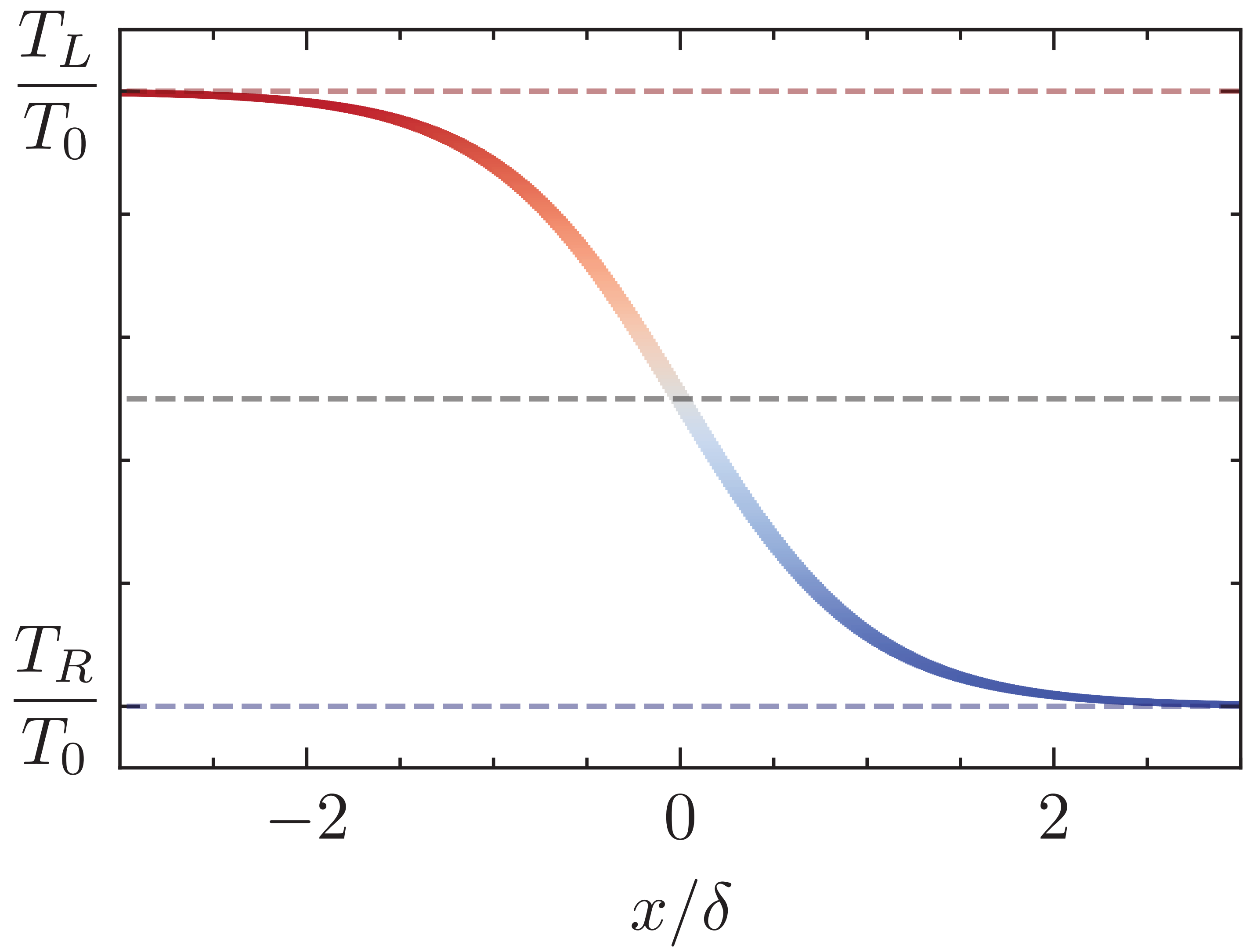
Hawking radiation and Black-hole atmosphere

In the close atmosphere (quantum troposphere)

$$\varepsilon\left(x_H\right) = -P\left(x_H\right) = \frac{\hbar c}{48\pi} f''(x_H)$$: **Analog of the Casimir effect**



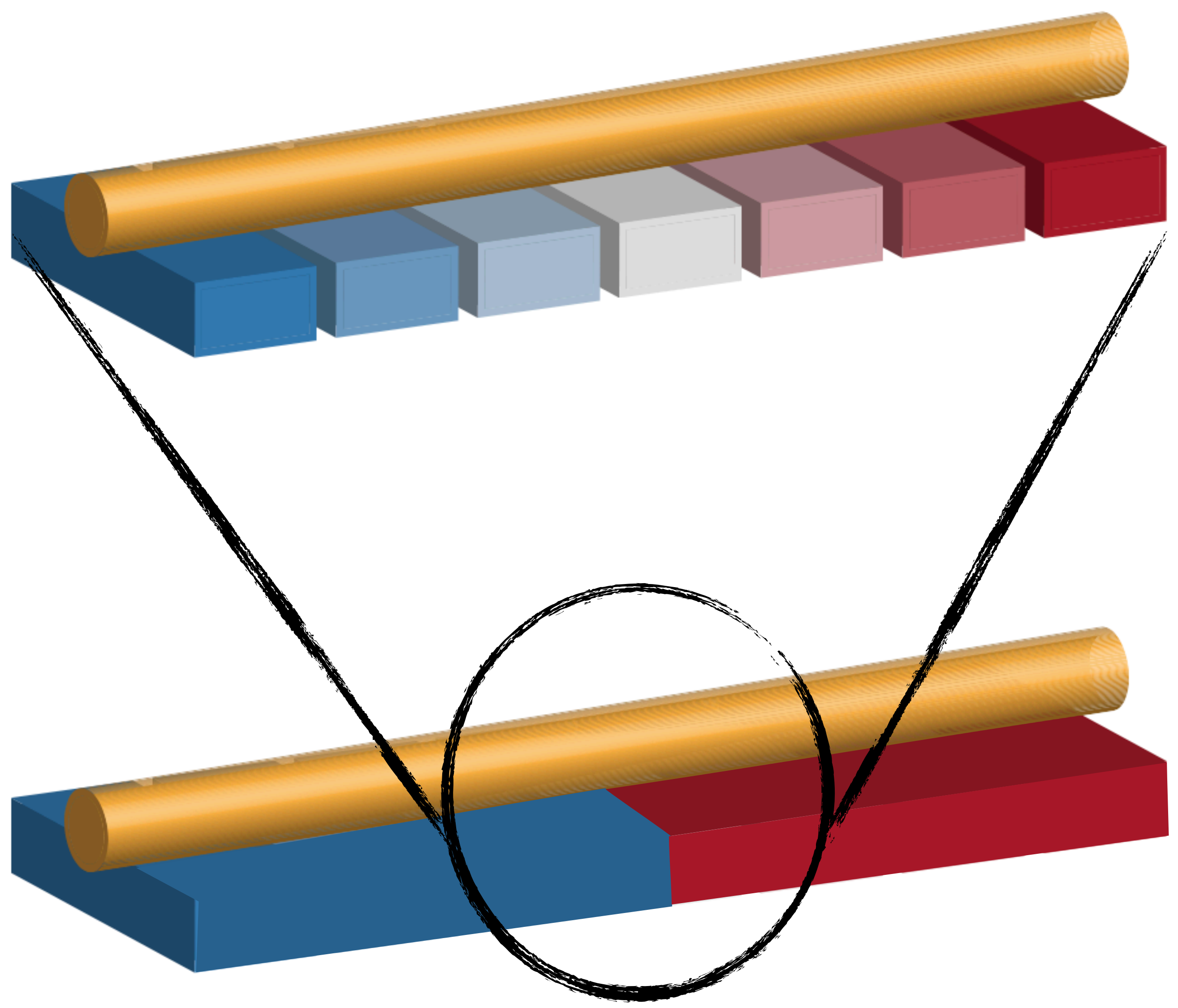
Out-of-equilibrium states by an inhomogeneous T



Imposing externally a temperature profile $T(x)$:
Modified Gibbs measure

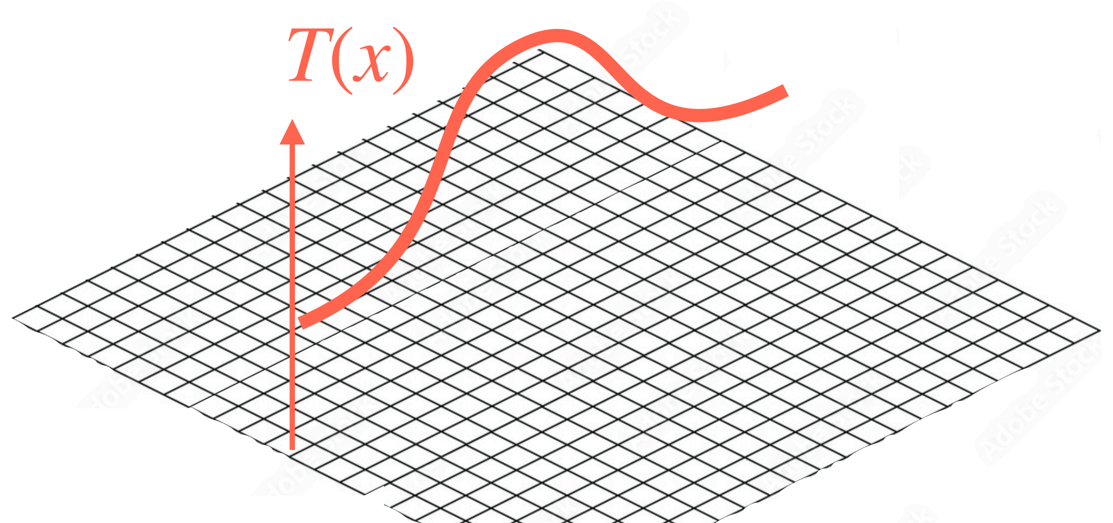
$$\rho = \frac{e^{-\beta H}}{Tr(e^{-\beta H})} \quad \text{with } \beta H = \int dx\ \beta(x) h(x)$$

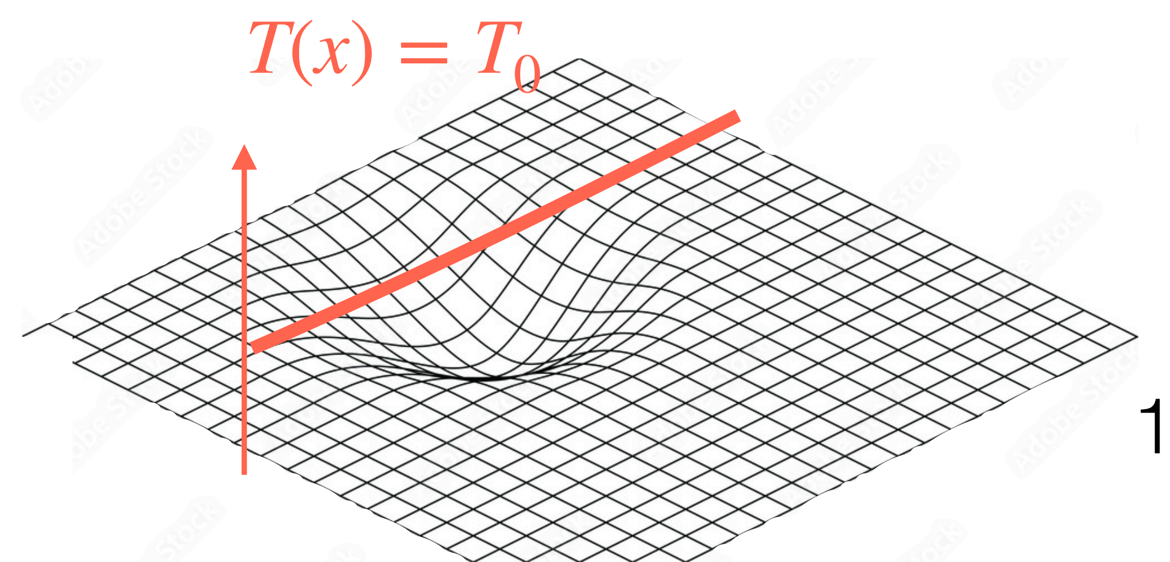
Local temperature $k_B T(x) \neq \beta^{-1}(x)$



Inhomogeneous $T(x)$ **Flat** Space ⇔ **Homogeneous** T_0 **Curved** space $g_{\mu\nu}$

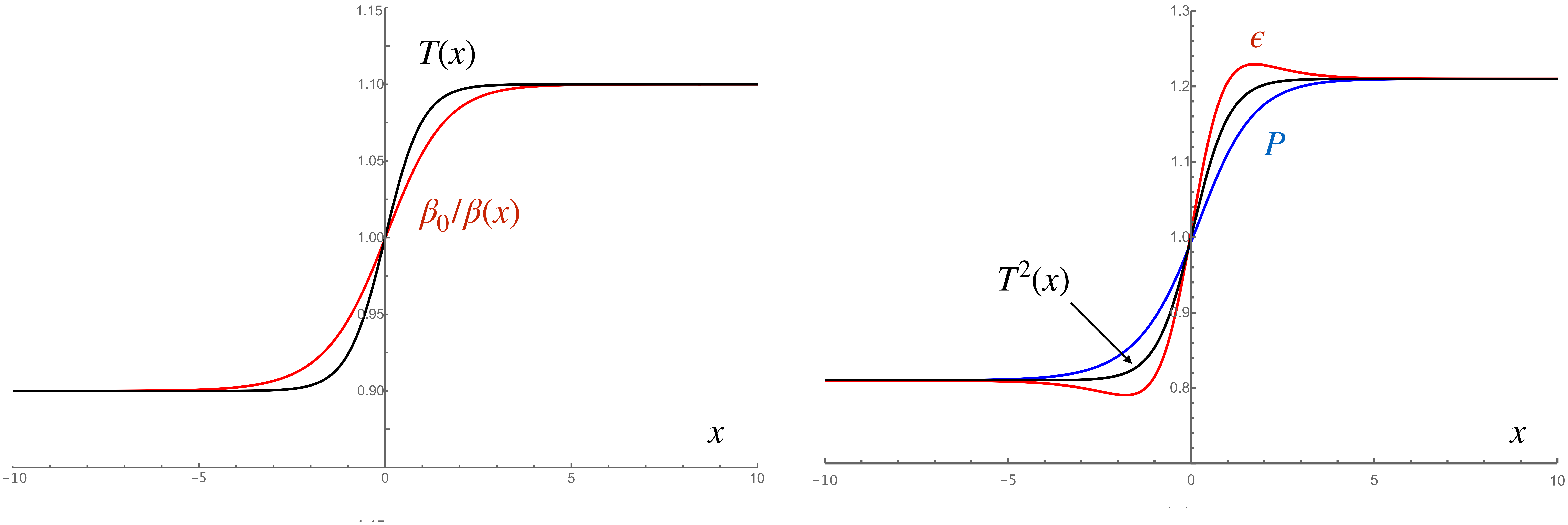
Extended Luttinger equivalence





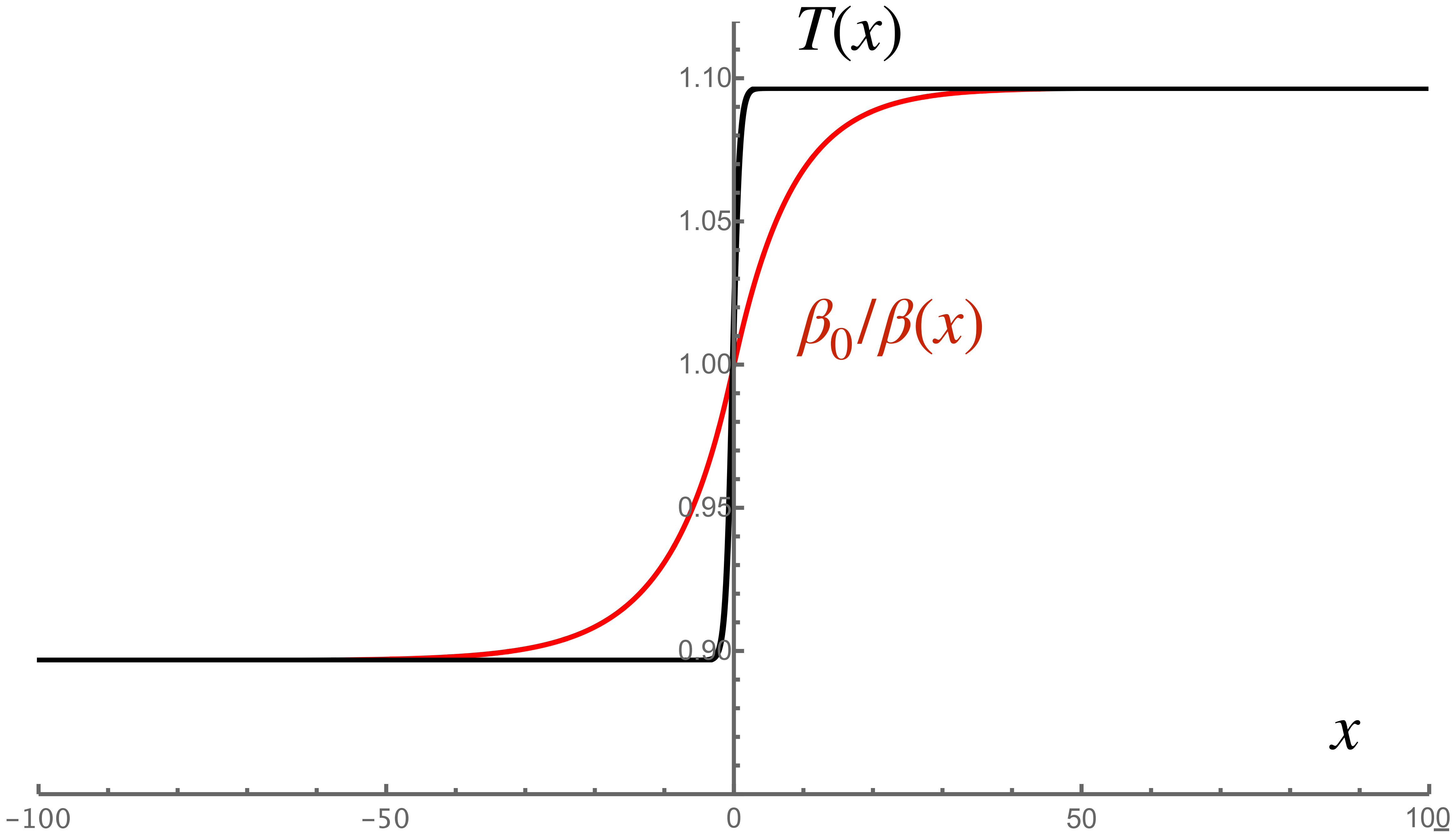
Out-of-equilibrium states by an inhomogeneous T

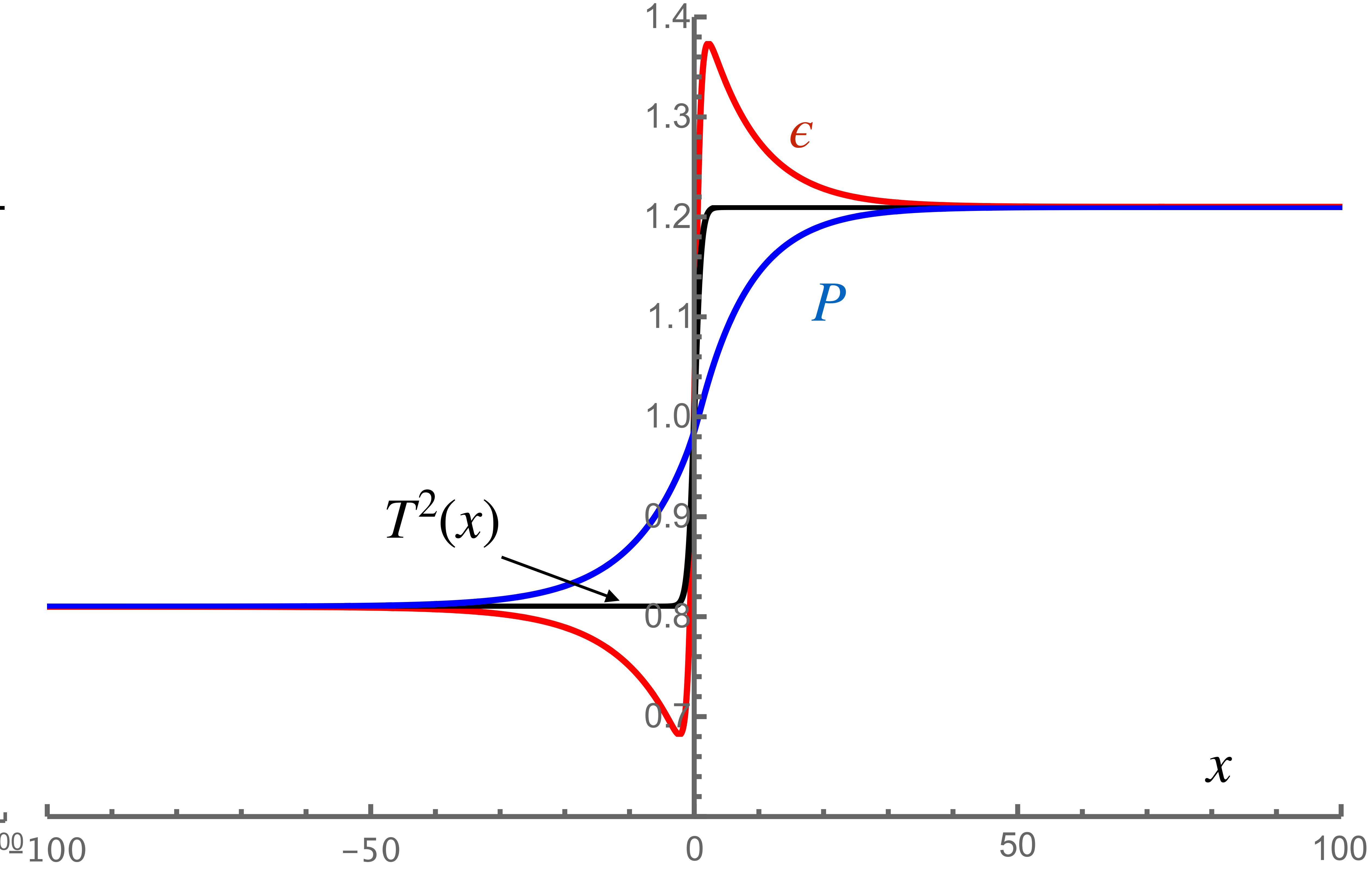
$\delta = 10\mu m, \Delta T = 20mK, T_0 = 100mK, v_F = 10^6$



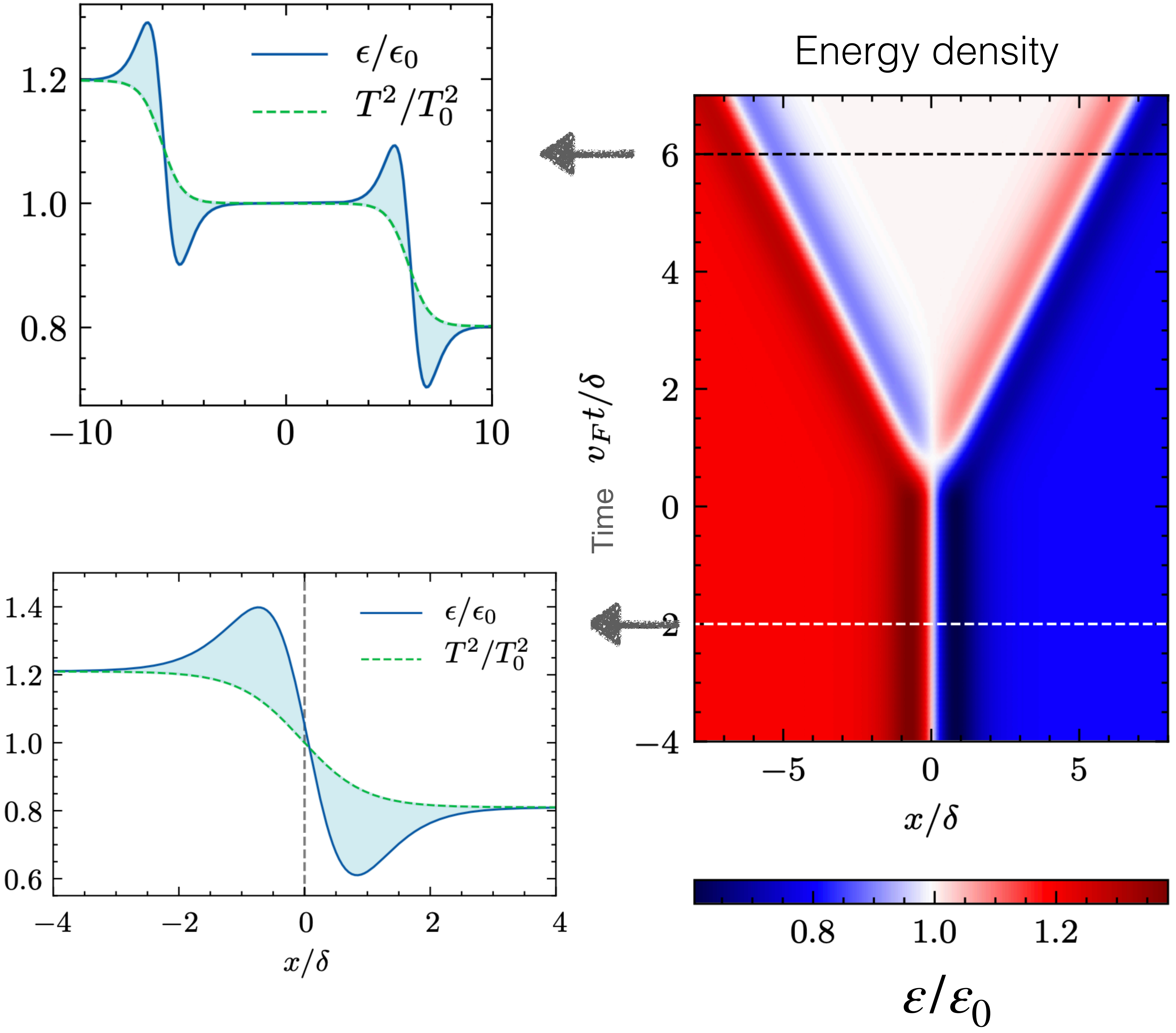
Out-of-equilibrium states by an inhomogeneous T

$\delta = 1\mu m, \Delta T = 20mK, T_0 = 100mK, v_F = 10^6$





Quench dynamics



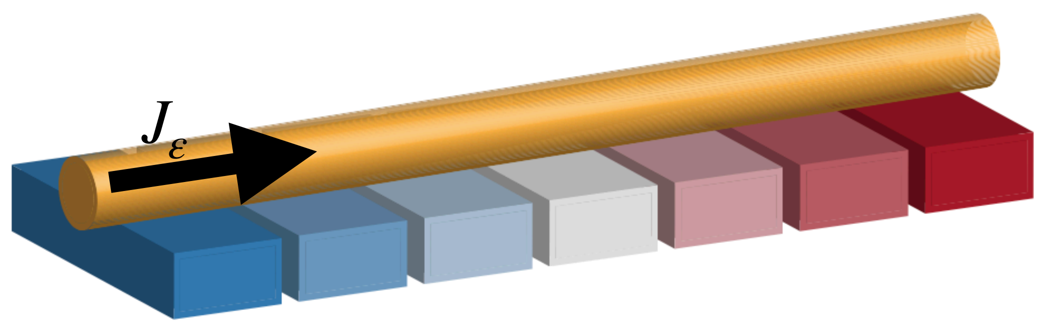
2 « heat waves » of **energy** propagating at a velocity $\pm v_F$

Profile : signature of anomalous fluctuations

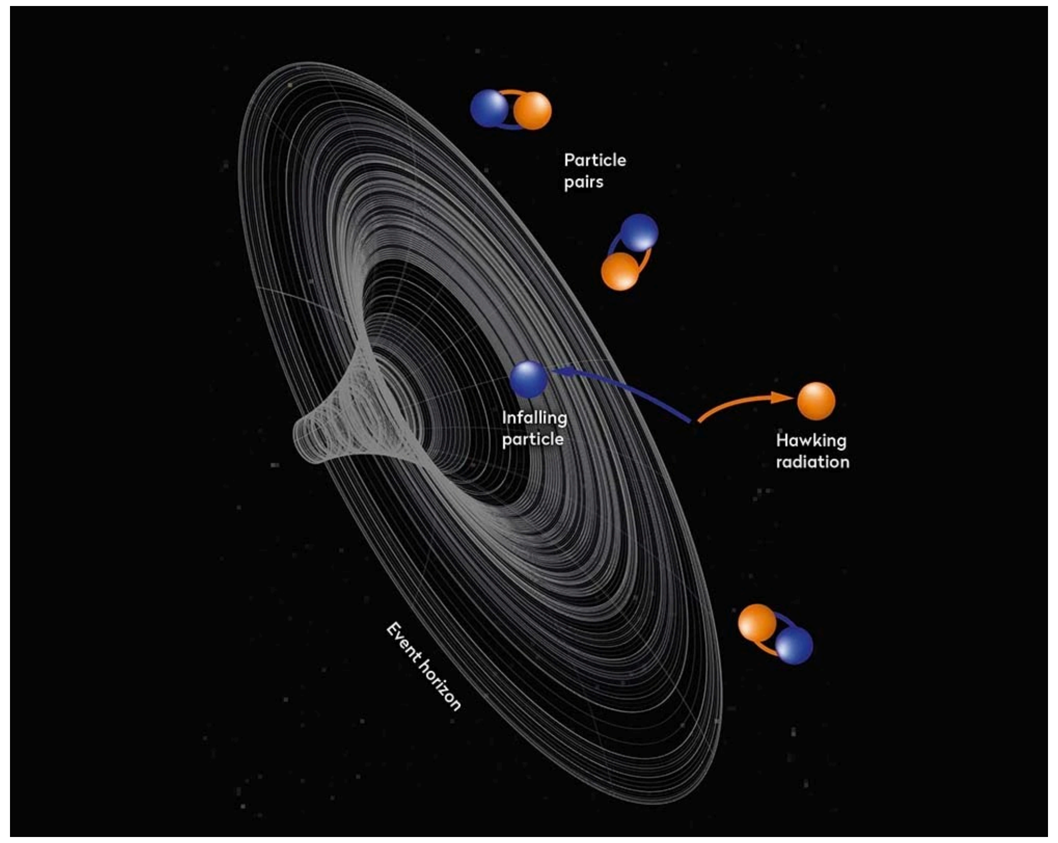
$v_F = 10^6 m/s,$

$T_0 = 100mK, \Delta T = 20mK, \delta = 1\mu m$

Conclusion



Anomalous vacuum fluctuations
induced by **spacetime curvature**
(Gravitational anomalies)



Thermal transport in **a laboratory**

Black body radiation
close to **a blackhole**

- Experimental signatures ?
- Induced by acceleration instead of curvature (Unruh effect) ?
- Extension to D=3+1 ?

Anomalous Luttinger relation for energy transport: From black hole's atmosphere to thermal quenches
B. Bermond, M. Chernodub, A. Grushin, D. Carpentier, arXiv:2206.08784