# Non-equilibrium cluster-cluster aggregation in the presence of anchoring sites

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Introduction

Synapses are composed of a pre-synaptic part releasing neurotransmitters and a post-synaptic part that ensures their reception. For inhibitory synapses, receptors are localized in front of the pre-synaptic domains underlied by clusters of scaffold proteins.

Some PSDs

5 µm
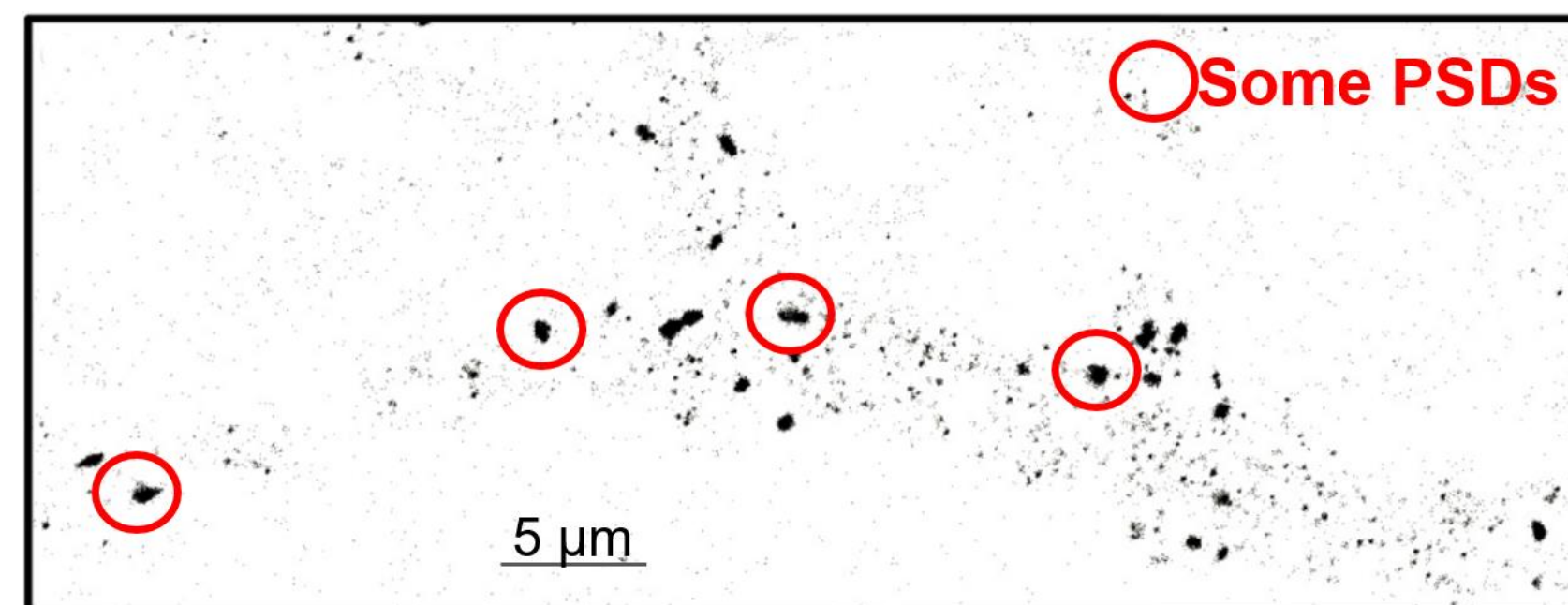
Ranft, Almeida et al Plos CB (2017)

What are the main mechanisms that take place to ensure the formation, maintenance and localization of these clusters ?

Model

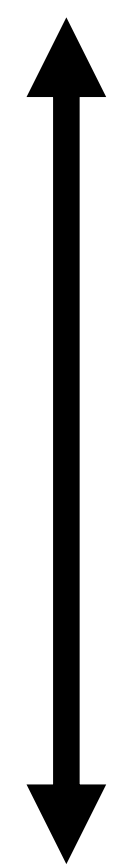Based on biological considerations, three mechanisms have been shown to be of importance :
- Scaffold proteins bind when they enter in contact, forming clusters
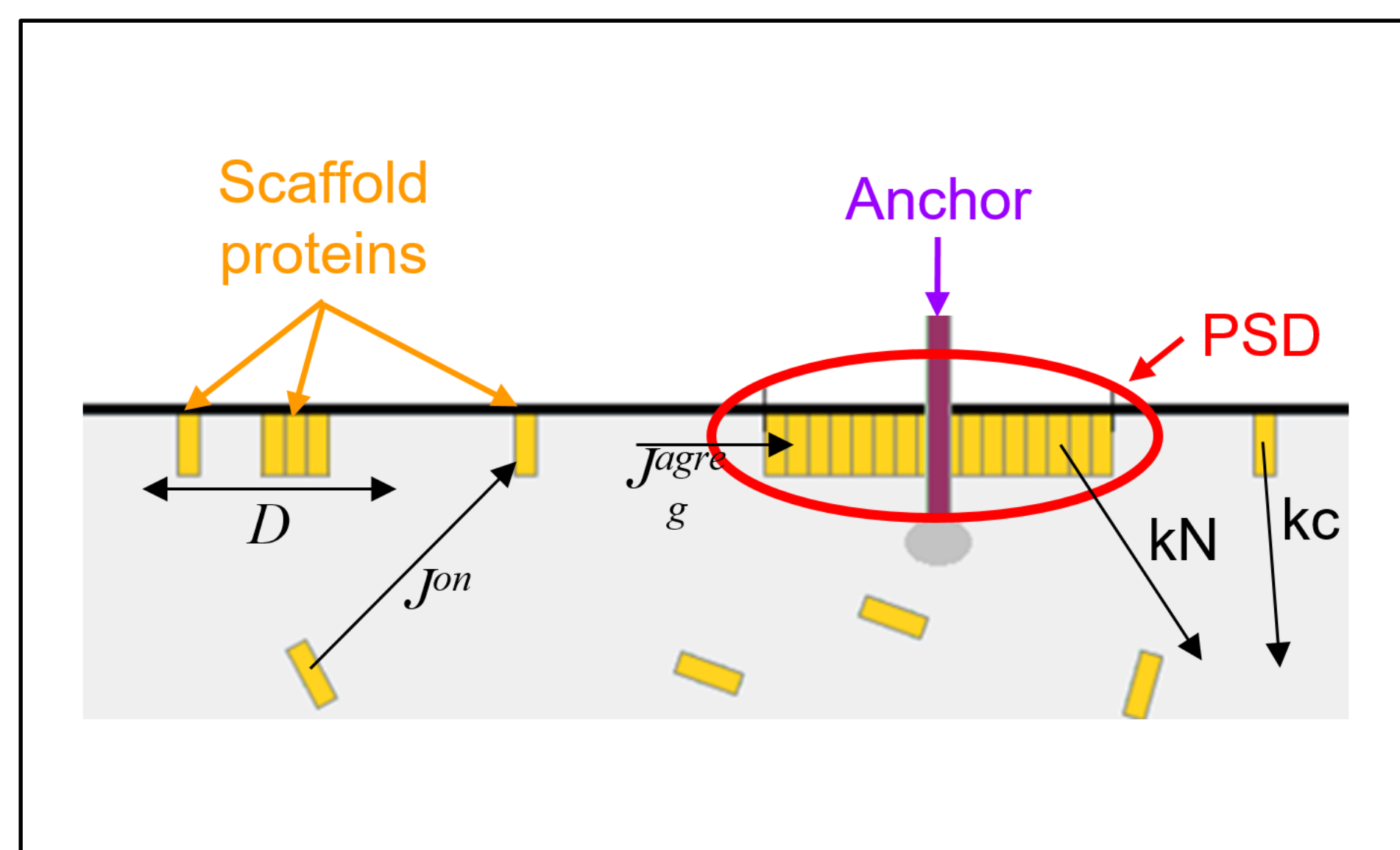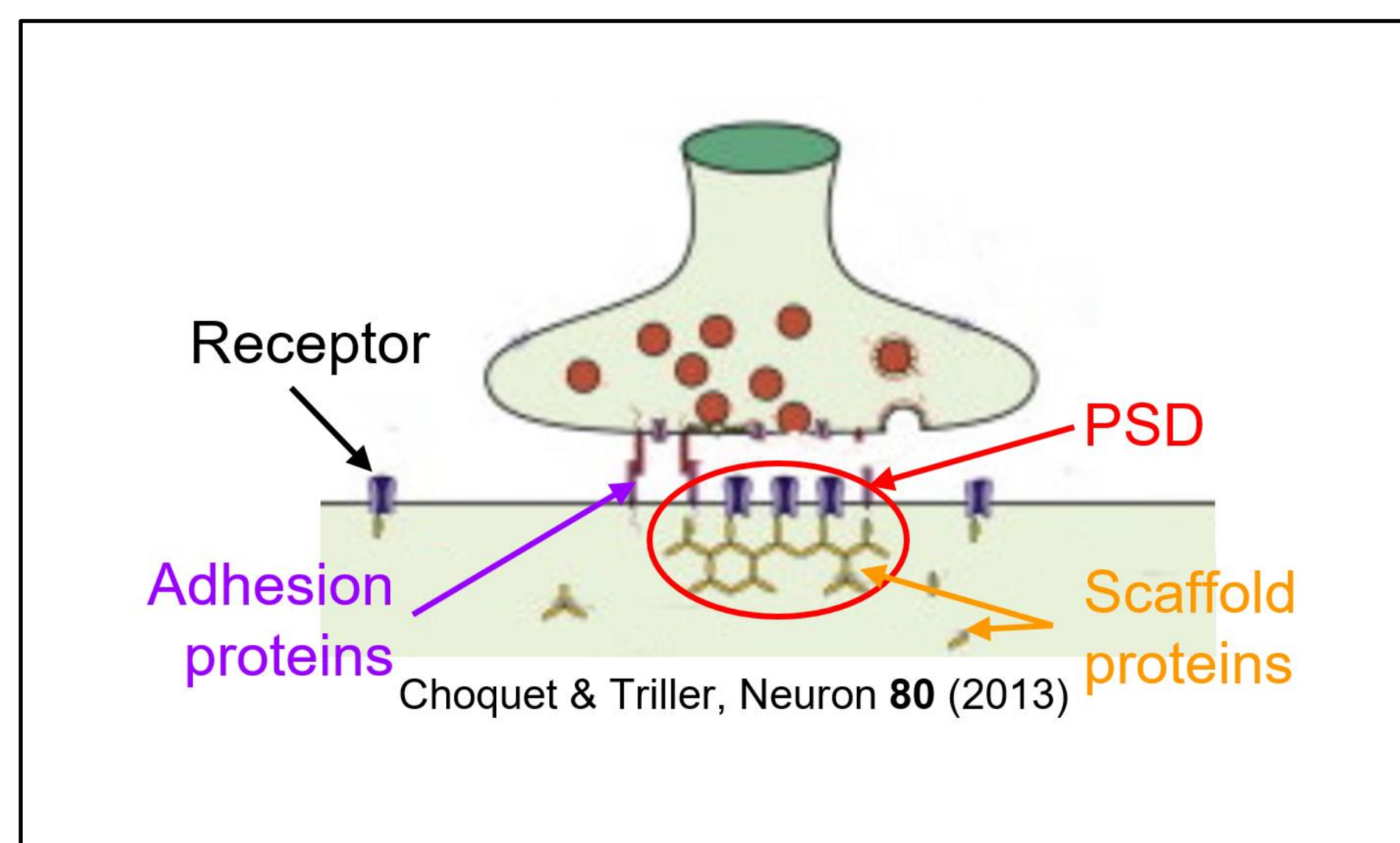- Clusters diffuse on the membrane, depending on their size $D(m)=D_0 \cdot m^{-\sigma}$
- Clusters are renewed : Particles are desorbed/absorbed to/from the cytoplasm at a rate k
We hypothesize that post-synaptic domains take place in front of pre-synaptic part due to fixed adhesion proteins that also bind with clusters.

Biology

Receptor
PSD
Adhesion proteins
Scaffold proteins

Choquet & Triller, Neuron **80** (2013)

Physics

Scaffold proteins
Anchor
PSD
D
$J^{agrég}$
J^{on}
kN
kc

PSD size : $N(t) = \pi R(t)^2 \rho$

$$\frac{dN}{dt} = \underbrace{2\pi RD\nabla c|_R}_{\text{Agrégation}} - \underbrace{kN}_{\text{Recyclage}}$$

Peripheral concentration : $c(r)$

$$\frac{\partial c(r,t)}{\partial t} = 0 \approx \underbrace{D\Delta c}_{\text{Diffusion}} + \underbrace{J^{on}}_{\text{Flux entrant}} - \underbrace{kc}_{\text{Recyclage}}$$

Different levels of coarse-graining

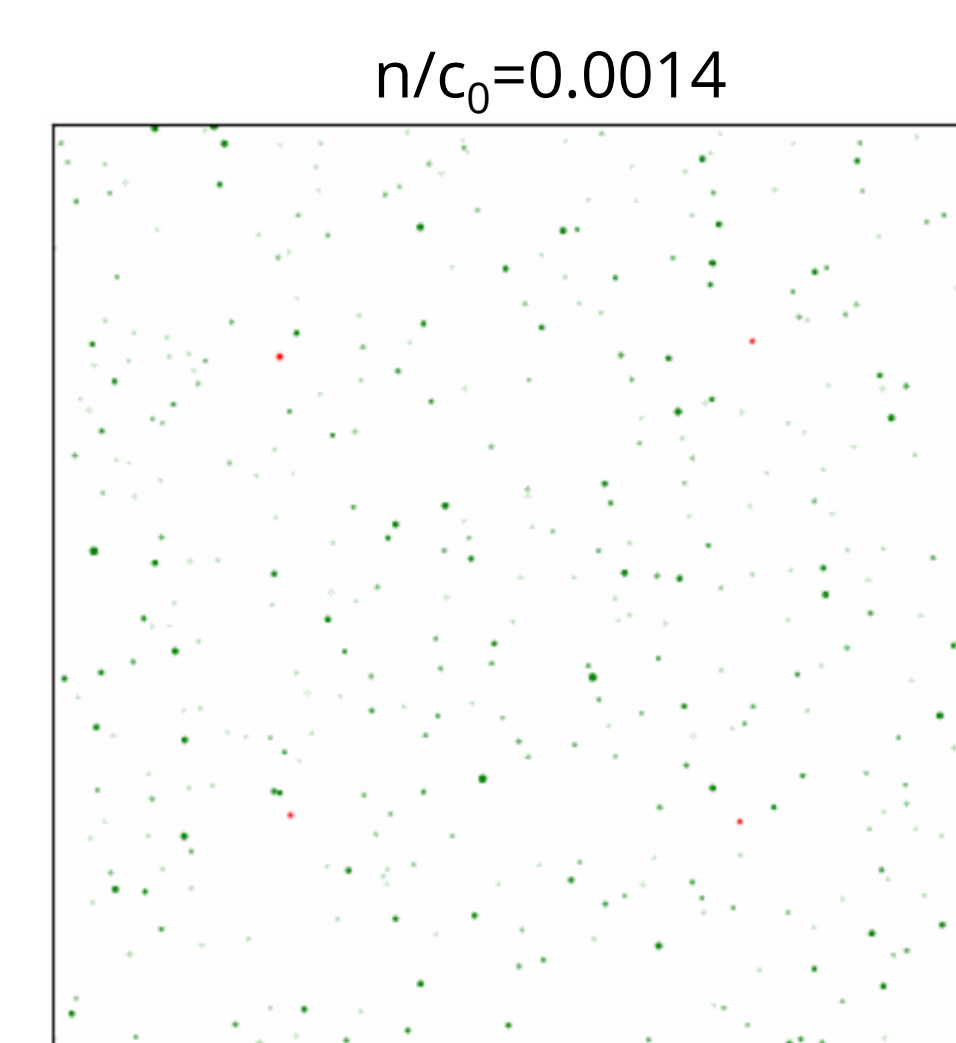Mean-field approximation with **Density of anchors : n**

$$D\Delta c(r) + J^{on} - kc(r) - knN = 0$$

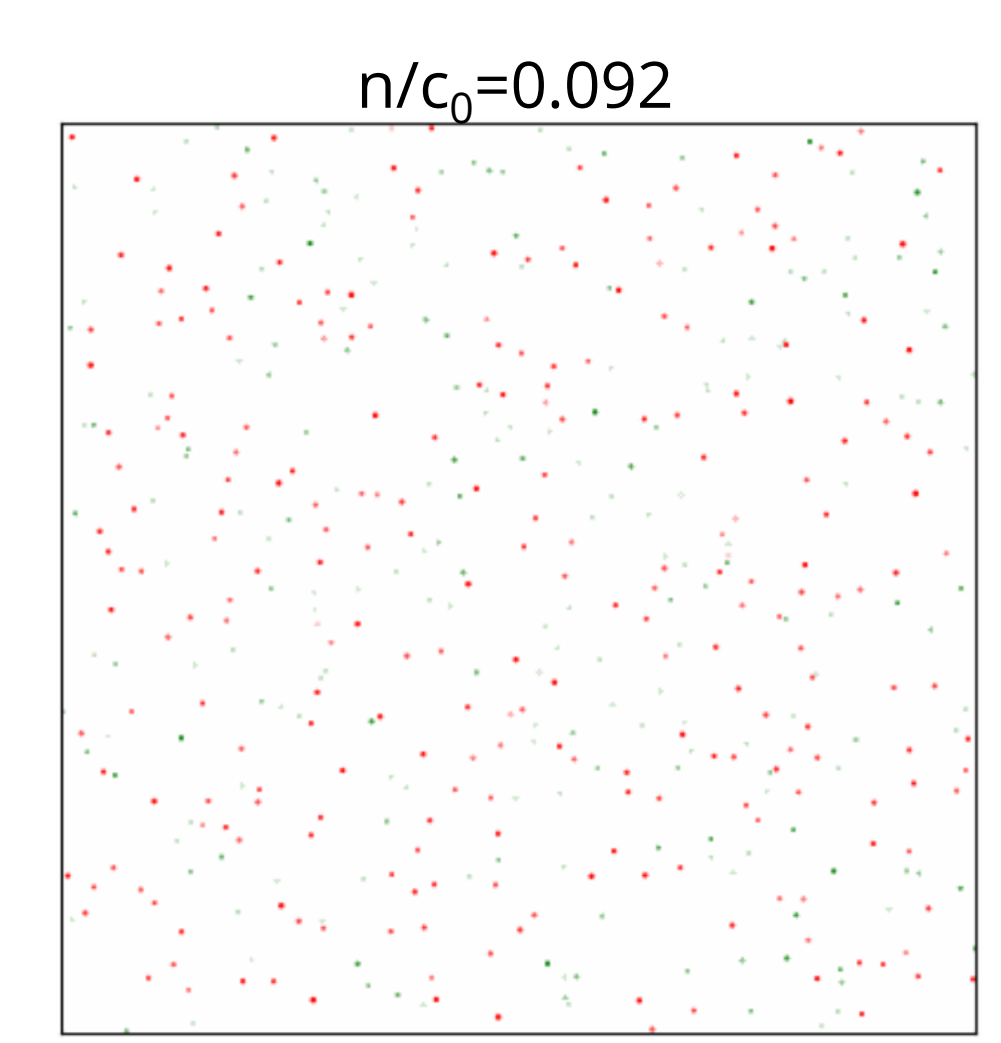Rate equations for **PSD of size l** and **extrasynaptic cluster of size m**

$$\frac{d}{dt}p_l = -klp_l + k(l+1)p_{l+1} - p_l K \sum_j D_j c_j + K \sum_{j<l} D_{l-j}c_{l-j}p_j$$

$$\frac{d}{dt}c_m = -kmc_m + k(m+1)c_{m+1} + J\delta_{m1} - K\sum_j (D_j + D_m)c_jc_m + K\sum_{j<m}(D_j + D_{m-j})c_jc_{m-j} - KD_mc_mn$$

Complete description with numerical simulation :

n/c₀=0.0014

n/c₀=0.092

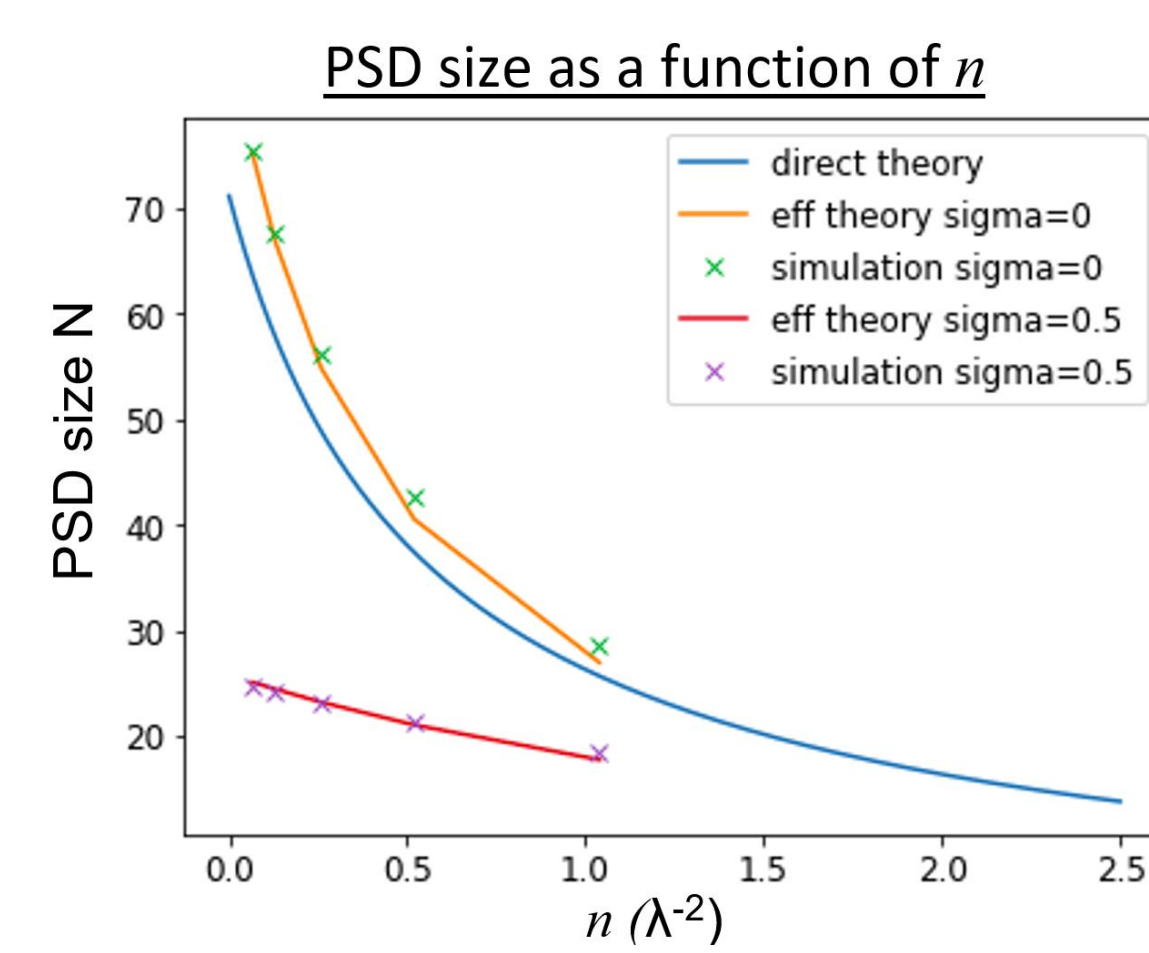Dilute regime
Ordered configuration

Dense regime
Random configuration

Results

PSD size as a function of n

We show that mean-field approximation describes well the case σ=0 where diffusion of the clusters do not depend on their size.

PSD and extrasynaptic cluster size distribution depending on the density of anchors
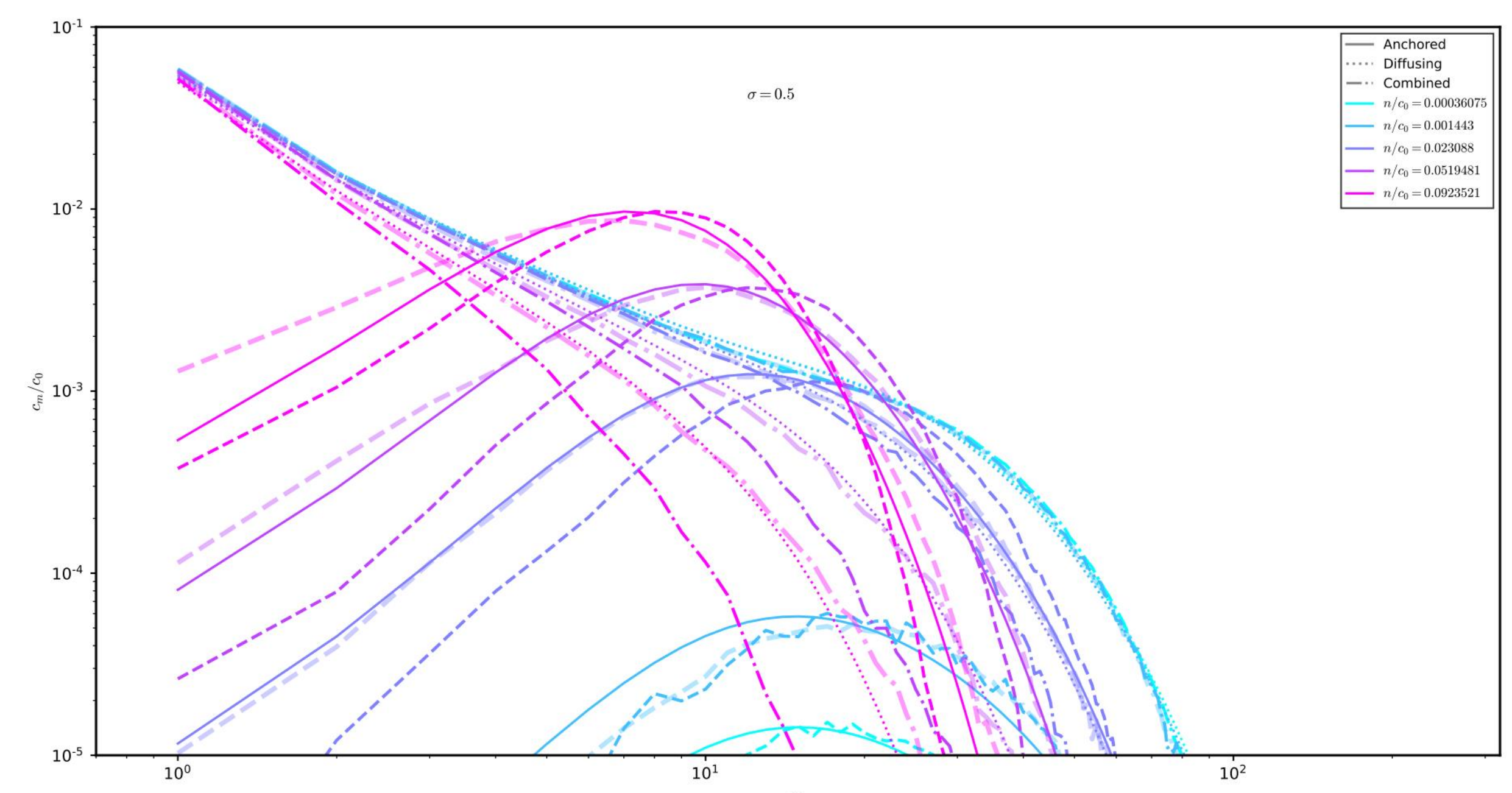
Rate equations reproduce the numerical simulations in the dilute regime in average for random configurations.