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- Introduction : GW theory, GW detectors
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  - GW detector sensitivity, candidates list Ο
- Population properties
  - Fundamental questions, BNS&NSBH, BBH

#### NSBH

GW200115 & GW200105 : detector status, Ο properties, formation channels



# Sub-Solar Mass Search KAGR

• O3a results, implication for PBH

#### Testing GR

Introduction, tests 0

#### • Burst search

All-Sky Search, Candidates, other searches 0









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<sup>\*</sup>Figures and title taken from Tito Dal Canton previous seminar

#### Part 2: Introduction



**GW theory & Ground-based** detector







# **Introduction :**

**CBC** : Compact binary **Coalescence**, systems with neutron stars or black holes (BBH, BNS, NSBH)

**PN**: Post-Newtonian approximation inspiral templates, waveform models of CBC.





• Some useful formula :  $\mathcal{M} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$ .  $(m_1\boldsymbol{\chi_1}+m_2\boldsymbol{\chi_2})\cdot \hat{\boldsymbol{L}}$  $\chi_{\rm eff} =$ 

 $m_1 + m_2$ 

$$q = m_2/m_1 \tag{3}$$

• Virgo: Increased laser power; improved electronics ...



#### Part 3: **GWTC-3**



**GW** detector sensitivity

#### Candidates list



# GWTC-3:



Fig5 : Rate of single-interferometer glitches \*



#### Part 3: GWTC-3

GW detector sensitivity

#### Candidates list





# GWTC-3: Candidates

#### Procedure :

- Search method : Modeled searches (PyCBC GstLal, MBTA ...) & Minimally modeled search (cWB) \*
- Candidates events identification
- Validation by checking for evidence that they were caused by one or more detector noise artifacts following the same procedure as for previous catalogs
- Parameter estimation
- Main list (35 events): candidates with a probability of astrophysical origin (p-astro) > 0.5
- Marginal list\*\* (7 events): p-astro < 0.5 but FAR < 2 per year

#### Likely instrumental artifacts :

Main list: 0 Marginal candidates list : 3

#### Glitch subtraction :

Applied on 8 events before source property analysis

Searches are done on 2 timescales : low-latency & offline re-analysis

\*\*Marginal : Low-significance detections of possible signals



Fig7: Cumulative detection from O1 to O3 (from <a href="https://www.ligo.org/science/Publication-O3bTGR/">https://www.ligo.org/science/Publication-O3bTGR/</a>)

#### Significance estimation :

- False Alarm Rate (FAR) : how often do we expect noise to produce a trigger with the same ranking statistic as a candidate in question. No astrophysical information.
- Probability of astrophysical origin (p-astro) : foreground/background ranking statistics distributions comparison





#### Part 3: GWTC-3



GW detector sensitivity

#### **Candidates list**





# GWTC-3:



Fig8: GW merger detections in O3b observing run (Figure made by Becca Ewing)



### GWTC-3: Candidates properties

Part 3:

**GWTC-3** 

**GW** detector

**Candidates list** 

sensitivity



Chirp mass  $\mathcal{M}\left[M_{\odot}
ight]$ Fig11: Credible-region contours in the plane of chirp mass M and effective inspiral spin xeff for O3b candidates with p-astro > 0.5 plus GW200105-162426 \*

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Fig10: Marginal posterior distributions for the source properties for O3b \*

\* Figures from <a href="https://arxiv.org/pdf/2111.03606.pdf">https://arxiv.org/pdf/2111.03606.pdf</a>



# GWTC-3:

### Summary

- Data releases : <u>https://www.gw-</u> openscience.org/GWTC-3/
- O3 : detector greatest performance to date.
- 35 O3b candidates with p-astro > 0.5, diverse range of masses and spins
- 1st confident NSBH detections









Fig13 : Observing scenarios with targeted sensitivities (from https://arxiv.org/pdf/2105.09247.pdf)

# 5, diverse

#### Introduction

Fundamental questions

BNS & NSBH BBH



# **Astrophysical Populations** Introduction

Population properties of 76 compact binary mergers detected with gravitational waves below a false alarm rate of 1 per year through GWTC-3

- Masses, spins, distances of these events inferred from the GW signal
- Several mass models, 3 spins models, one distance model

#### **Fundamental questions :**

- Which types of mergers are we seeing? In terms of formation channels?
- How many are happening in the Universe ?
- What is the mass distribution of BH and NS ?





Introduction

**Fundamental** questions

**BNS & NSBH** 

BBH



# **Astrophysical population**

### **Fundamental questions**

How many are happening in the Universe?



Study of the merger rate

Multiple models but consistent with the same results :

$$\begin{aligned} \mathcal{R}_{\text{total}} &= 470^{+830}_{-300} \,\, \text{Gpc}^{-3} \,\text{yr}^{-1} \\ \\ \mathcal{R}_{\text{BNS}} &= 250^{+640}_{-200} \,\, \text{Gpc}^{-3} \,\text{yr}^{-1} \\ \\ \\ \mathcal{R}_{\text{NSBH}} &= 170^{+150}_{-89} \,\, \text{Gpc}^{-3} \,\text{yr}^{-1} \\ \\ \\ \mathcal{R}_{\text{BBH}} &= 22^{+9}_{-6} \,\, \text{Gpc}^{-3} \,\text{yr}^{-1} \end{aligned}$$



Fig14: Differential merger rate as a function of component mass for the PDB, MS, and BGP model (from <u>https://arxiv.org/pdf/2111.03634.pdf</u>)





![](_page_11_Picture_2.jpeg)

# **Astrophysical population BNS & NSBH Properties**

![](_page_11_Figure_4.jpeg)

Fig15: Masses for events with at least one candidate neutron \*

Maximum mass observed in the NS population :  $m_{max} = 2.0^{+0.3}_{-0.2} M_{\odot}$ 

Consistent with the mass found with the equation of state & Galactic pulsars

Minimum NS mass in the gravitational wave population inferred to be  $m_{min} = 1.2^{+0.1}_{-0.2} M_{\odot}$  in both the Power and Peak models.

![](_page_11_Figure_10.jpeg)

![](_page_12_Figure_1.jpeg)

Fundamental questions

![](_page_12_Picture_3.jpeg)

![](_page_12_Picture_4.jpeg)

# Astrophysical population **BBH Properties : Mass**

![](_page_12_Figure_6.jpeg)

Fig17: Posterior distribution on the minimum mass truncation hyper-parameter  $m_{min}$ 

\* Figures from <a href="https://arxiv.org/pdf/2111.03634.pdf">https://arxiv.org/pdf/2111.03634.pdf</a>

#### PP-model used for all plots here

![](_page_12_Figure_10.jpeg)

Fig18: The astrophysical BBH primary mass (top) and mass ratio (bottom) distributions \*

Results consistent between GWTC-2 & GWTC-3

![](_page_12_Figure_15.jpeg)

# Introduction Fundamental questions **BNS & NSBH** BBH

# **Astrophysical population BBH Properties : Spins**

![](_page_13_Figure_3.jpeg)

Fig19: The distributions of component spin magnitudes  $\chi$  (left) and spin-orbit misalignment angles  $\theta$  (right) among binary black hole mergers (from https://arxiv.org/pdf/2111.03634.pdf)

![](_page_13_Figure_5.jpeg)

GWTC-3 : a broad or isotropic distribution of spin tilts. GWTC-2 : consistent with tilts concentrated preferentially around  $\cos \theta = 1$ 

Spin magnitude : small but non-zero (concentrate below  $\chi_i < 0.4$ )

![](_page_13_Figure_9.jpeg)

![](_page_14_Figure_1.jpeg)

![](_page_14_Picture_2.jpeg)

![](_page_14_Picture_3.jpeg)

# **Astrophysical population BBH Properties : Merger rates & Redshift**

![](_page_14_Figure_5.jpeg)

https://arxiv.org/pdf/1403.0007.pdf)

- Merger rate density increases with redshift

• In most plausible formation scenarios : we do not expect R(z) to continue growing with arbitrarily high z. Instead, we anticipate that R(z) will reach a maximum beyond which it turns over and falls to zero. -> not observed yet, maybe with Einstein Telescope ?

### Part 5: NSBH

#### Introduction

#### GW200115 & GW200105

![](_page_15_Picture_3.jpeg)

![](_page_15_Picture_4.jpeg)

![](_page_15_Picture_5.jpeg)

# Gravitational waves from NSBH coalescences Introduction

January 2020 : First confident observations of NSBH !

![](_page_15_Figure_8.jpeg)

Fig22 : Graphic of masses of GW announced detections from GWTC-2 catalog + NSBH

# **Gravitational waves from NSBH** coalescences GW200115 & GW200105 detector status

![](_page_16_Figure_1.jpeg)

# NSBH

Part 5:

#### Introduction

#### GW200115 & **GW200105**

![](_page_16_Picture_5.jpeg)

![](_page_16_Picture_6.jpeg)

![](_page_16_Picture_7.jpeg)

### Part 5: NSBH

![](_page_17_Picture_1.jpeg)

#### GW200115 & GW200105

![](_page_17_Picture_3.jpeg)

![](_page_17_Picture_4.jpeg)

![](_page_17_Picture_5.jpeg)

# Gravitational waves from NSBH coalescences GW200115 & GW200105 significance status

![](_page_17_Figure_7.jpeg)

Fig24: Colored shading shows the joint S/N– $\xi^2$  noise probability density function for LIGO Hanford (LHO), LIGO Livingston (LLO), and Virgo. For comparison, the marginal GW190426\_152155 is also shown (from https://iopscience.iop.org/article/10.3847/2041-8213/ac082e/ pdf)

 $= -5 \quad (3) = -10 \quad (3) = -20 \quad (3) = -25 \quad (3) = -25$ 

• Spectrograms do not always show the track of the signal (see previous slide).

• To detect a CBC we use matched-filtering methods but the SNR is not always enough to estimate the significance of a trigger so we also compute the  $\chi^2$  or auto correlation  $\chi^2$ (y-axis of the plot)

#### GW200115 :

- Coincident event
- H1 & L1 do not stand individually

#### GW200105 :

- Single detector
- L1 SNR : 13.6
- In L1, Distinctly separate

# **Gravitational waves from NSBH coalescences** GW200115 & GW200105 skymaps

Part 5:

NSBH

Introduction

GW200115 &

GW200105

![](_page_18_Figure_1.jpeg)

Fig25: Sky localizations for GW200105 (top) and GW200115 (bottom) (from https://iopscience.iop.org/article/10.3847/2041-8213/ac082e/pdf)

Sky localisation :  $600 deg^2$ 

![](_page_18_Figure_4.jpeg)

Fig26: For comparison skymaps from GW170817

![](_page_18_Picture_6.jpeg)

Sky localisation : 7200  $deg^2$ (large sky area arises due to the absence of data from LIGO Hanford)

![](_page_18_Figure_9.jpeg)

# **Gravitational waves from NSBH coalescences** GW200115 & GW200105 source properties

![](_page_19_Figure_1.jpeg)

Fig27: Component masses of GW200105 (red) and GW200115 (blue), represented by their twoand one-dimensional posterior distributions \*

	m1	m2	r
GW200105	$8.9^{+1.2}_{-1.5}M_{\odot}$	$1.9^{+0.3}_{-0.2}M_{\odot}$	r
GW200115	$5.7^{+1.8}_{-2.1}M_{\odot}$	$1.5^{+0.7}_{-0.3}M_{\odot}$	(   

m2: Consistent with maximum NS mass

m1 : BH identified

GW200115 m1: 30% probability of falling in the mass gap 20

Introduction

Part 5:

NSBH

#### GW200115 & **GW200105**

![](_page_19_Picture_9.jpeg)

![](_page_19_Picture_10.jpeg)

![](_page_19_Picture_11.jpeg)

GW200105 GW200115 0.8 -0.8 -0.6 -0.6 -98, 8, 0.40.2 -0.2 -°06 -00 0.0 -°06 0.0 -°Cz °C; .081 J80° \_08I

Fig28: Two-dimensional posterior probability for the spin-tilt angle and spin magnitude for the primary objects (left hemispheres) and secondary objects (right hemispheres) for both events \*

> GW200105 : Primary spin:  $|\chi_1^2| < 0.23$ Secondary spin: unconstrained GW200115 : Primary spin:  $-0.19^{+0.24}_{-0.50}$  $P(\chi_{1,z} < 0) = 88\%$ Secondary spin: unconstrained

\* Figures from <a href="https://iopscience.iop.org/article/">https://iopscience.iop.org/article/</a> 10.3847/2041-8213/ac082e/pdf

# **Gravitational waves from NSBH coalescences** GW200115 & GW200105: Nature of the secondary components

Investigations to establish the nature of the secondary objects :

- Tidal Deformability and Tidal Disruption
- Consistency of Component Masses with the NS Maximum Mass

![](_page_20_Figure_4.jpeg)

Fig30: Estimation of the second object mass in comparison with the maximum mass

ie.PBH).

#### Introduction

Part 5:

NSBH

#### GW200115 & GW200105

![](_page_20_Picture_9.jpeg)

![](_page_20_Picture_10.jpeg)

![](_page_20_Picture_11.jpeg)

Tidal deformability of NSs imprinted in the GW signal. In contrast, BHs have zero tidal deformability.

Tidal deformability inference : Waveform models that include tides.

This measurement cannot establish the presence of NSs (Expected)

No EM counterpart detected for both events (Expected)

![](_page_20_Figure_16.jpeg)

Fig29: Tidal deformation for NS compared to BH

#### Consistency with the maximum NS mass.

BUT does not exclude the possibility that the secondaries could be BHs or exotic compact objects (if such objects also exist within the NS mass range

## Gravitational waves from NSBH coalescences Conclusion

![](_page_21_Figure_1.jpeg)

Fig31: *Inferred probability densities for the NSBH merger rate* (from <u>https://iopscience.iop.org/article/10.3847/2041-8213/ac082e/pdf</u>)</u>

	BBH	BNS	NSBH
Merger rate Gpc <sup>-3</sup> yr <sup>-1</sup>	15-38	80-810	12-242

First robust detections of a black hole merging with a neutron star.

GW200105 ~1.9 and 9  $M_{\odot}$  (two detectors) GW200115 ~ 1.5 and 6  $M_{\odot}$  (three detectors)

Predicted rates of NSBH mergers in the local universe vary by orders of magnitude across the various formation channels.

![](_page_21_Figure_8.jpeg)

Other NSBH detection: GW191219 but only found in offline (that's why it's not the « first » ..)

![](_page_22_Picture_0.jpeg)

![](_page_22_Picture_1.jpeg)

![](_page_22_Picture_2.jpeg)

![](_page_22_Picture_3.jpeg)

![](_page_22_Picture_4.jpeg)

![](_page_22_Picture_5.jpeg)

# **Sub-Solar Mass Search with O3a**

**PBH**: Primordial Black Hole. A theoretical type of BH formed in the early Universe from the collapse of over denses regions of space

![](_page_22_Figure_8.jpeg)

Fig32: The regions searched by recent gravitational-wave analyses of the LIGO and Virgo data as a function of detectorframe primary and secondary mass. \*

GPS time	IFAR (yr)	$m_1/M_{\odot}$	$m_2/M_{\odot}$
1245411568.354	0.084	0.69	0.21
1242817372.434	0.079	0.86	0.11
1246418221.718	0.075	0.13	0.13
1252963276.322	0.062	1.05	0.52
1240000657.632	0.057	3.04	0.10

Table1: The top 5 candidates in our search with the highest inverse false alarm rates (IFAR). \*

![](_page_22_Figure_12.jpeg)

$$R_{90} = \frac{2.3}{VT}$$

where V is the estimated sensitive volume of the analysis assessed at the false alarm rate of the most significant observed candidate and T is the duration of the observation period.

Chirp Mass  $\mathcal{M}$  [ $M_{\odot}$ ] Fig33: Upper limit on the rate of mergers at 90% confidence for the SSM search (purple)

![](_page_22_Figure_16.jpeg)

![](_page_22_Figure_17.jpeg)

![](_page_22_Figure_18.jpeg)

Fig34: The upper limits on the effective fraction of the primordial black hole contribution to dark matter. \*

![](_page_22_Figure_22.jpeg)

![](_page_22_Figure_23.jpeg)

![](_page_22_Picture_24.jpeg)

#### Part 7: Test of GR

#### Introduction

Tests

![](_page_23_Picture_2.jpeg)

![](_page_23_Picture_3.jpeg)

![](_page_23_Picture_4.jpeg)

# **Test of General relativity** Introduction

- The model waveform is constructed using the predictions of General Relativity.
- Gravitational-wave sources offer us unique testbeds for probing strongfield, dynamical and nonlinear aspects of gravity
- Tests predictions of General Relativity by introducing small modifications to our currently available waveform models and compare the data with these "distorted" waveforms
- Three theory-agnostic tests (parameterized tests, inspiral-mergerringdown consistency tests, and gravitational-wave propagation tests)

#### Part 7: Test of GR

#### Introduction

Tests

![](_page_24_Picture_3.jpeg)

![](_page_24_Picture_4.jpeg)

![](_page_24_Picture_5.jpeg)

# Tes

sts : Examples		Event selection : 15 events with IFAR > 1000 yr from O3b Combined with events from GWTC-2, wheneve possible		
Tests	Question to	answer	Description	Results
Residual Test	Are the residual consistent with detector noise?		Subtracts the best-fit GR waveform from the data and asks whether there is any statistically significant residual power.	1.0       
<section-header></section-header>	Is the inspir consistent w $\varphi_{PN}(f) = 2\pi f t_c - \varphi_c$	al phase with GR ? $a^{-} \frac{\pi}{4} + \frac{3}{128\eta} (\pi)$	Inspiral can be treated perturbatively within the post- Newtonian framework. PN coefficients : measurable parameters of the waveform —> sensible consistency test of GR $\tilde{f}$ ) <sup>-5/3</sup> $\sum_{i=0}^{r} \left[ \varphi_i + \varphi_{il} \log(\pi \tilde{f}) \right] (\pi \tilde{f})^{i/3}$	$\delta \varphi_{i}$ $\delta \varphi_{0}$ $\delta \varphi_{0}$ $\delta \varphi_{0}$ $\delta \varphi_{0}$ $\delta \varphi_{0}$ $\delta \varphi_{1}$ $\delta \varphi_{2}$ $\delta \varphi_{0}$ $\delta \varphi_{1}$ $\delta \varphi_{2}$ $\delta \varphi_{2}$ $\delta \varphi_{1}$ $\delta \varphi_{2}$ $\delta \varphi_{1}$ $\delta \varphi_{2}$ $\delta \varphi_{2}$ $\delta \varphi_{2}$ $\delta \varphi_{1}$ $\delta \varphi_{2}$ $\delta \varphi_$

![](_page_24_Figure_10.jpeg)

![](_page_25_Picture_0.jpeg)

Tests

![](_page_25_Picture_2.jpeg)

![](_page_25_Picture_3.jpeg)

![](_page_25_Picture_4.jpeg)

# **Tests : Examples**

Tests	Question to answer	Description	Results
<section-header></section-header>	Modified theory predict dispersion of GW	Affect the morphology of the signal -> effective dephasing of the GW signal can be measured. $E^2 = p^2 c^2 + A_{\alpha} p^{\alpha} c^{\alpha}$ Different choices of $\alpha$ -> leads to a deviation in the GR phasing formula. Mass of the graviton : $m_g = \sqrt{A_0}/c^2$	$\int_{0}^{1} \int_{0}^{1} \int_{0$
<section-header></section-header>	If the merger remnant is not a classical BH but an exotic compact object without an event horizon but a reflective surface	Search for post-merger echoes in a morphology independent way.	1.0 .0.8 0.6 0.6 0.6 0.0 0.0 0.0 0.0 0.0

Fig37-38: from https://arxiv.org/pdf/2112.06861.pdf

![](_page_25_Picture_9.jpeg)

# **Testing GR : Summary**

Many more tests of General Relativity have been done :

- Spin-induced quadrupole moment test
- GW polarizations test
- BH remnant test
- Ringdown test
- ...
- Found no statistically significant evidences for any deviation from GR
- Update bounds on deformation parameters in the case of parametrized tests
- Testing GR is very hard, even if a deformation is found:
  - Is it really GR that is deformed?
  - A problem in the data qualify models ?
  - Waveform not enough precise ?

#### Part 8: **GW Bursts**

#### **All-Sky Search**

#### Candidates

![](_page_27_Picture_3.jpeg)

**Other searches** 

![](_page_27_Picture_4.jpeg)

![](_page_27_Picture_5.jpeg)

# **All-Sky Search for short GW Bursts**

### **Introduction : All-Sky Search**

- There are several plausible sources of short-duration GW transients (GW bursts) that have not yet been observed, such as core-collapse supernovae, neutron star excitations, non-linear memory effects, or cosmic string cusps and kinks
- All-sky search looks for signals arriving at any time from any sky direction : short-duration GW transients, up to a few seconds duration, and longer GW transients, up to  $\sim 10^3$  s duration
- 2 independently developed search algorithms deployed: coherent WaveBurst (cWB) and BayesWave (BW).

### Candidates

Three loudest candidates : statistical significance insufficient to exclude an instrumental origin.

Null result of this search : - Allows setting of rate density upper limits at an inverse false alarm rate threshold of 100 years

- Estimate sensitivity to certain classes of GW signals: CCSNe and isolated NS excitations.

#### Other searches ...

Searches have also been done for many other objects such as continuous GWs and stochastic background —> Currently nothing found !

![](_page_27_Picture_18.jpeg)

![](_page_27_Figure_22.jpeg)

![](_page_27_Figure_24.jpeg)

![](_page_27_Figure_25.jpeg)

# What happened since the last O3 seminar? Summary

- O3a : 44 new events (in GWTC-2.1) -> O3b 35 new events
- First robust NSBH detections
- No evidence for deviation of GR (same as previously)
- O3a sub-solar mass search : no SSM object found

- Forthcoming publications :
- O3b sub-solar mass search

• No exotic source found (Burst, continuous GW ...) only CBC sources (same as previously)

![](_page_28_Picture_13.jpeg)

![](_page_29_Picture_0.jpeg)

![](_page_29_Picture_1.jpeg)

# Thank you !

![](_page_29_Figure_3.jpeg)

![](_page_29_Picture_4.jpeg)

Laboratoire de Physique des 2 Infinis

![](_page_29_Picture_6.jpeg)

![](_page_30_Picture_0.jpeg)

![](_page_30_Picture_1.jpeg)

# Backup

![](_page_30_Figure_3.jpeg)

![](_page_30_Picture_4.jpeg)

Laboratoire de Physique des 2 Infinis

![](_page_30_Picture_6.jpeg)

# **Detector spectrum noise**

![](_page_31_Figure_1.jpeg)

Fig5 : Representative amplitude spectral density of the three interferometers' strain sensitivity

# **Astrophysical population** To go further: Implication for Binary Black hole & Neutron stars formation

BH: The statistical distribution of BH source properties such as their mass, spin and redshift can be used to probe the astrophysics of BH binary formation and evolution:

#### **PREDICTIONS:**

Formation in globular clusters : the resulting BH mass distribution is generally predicted to peak at > 10  $M_{\odot}$  . Large spin–orbit misalignment. Redshifts :  $\kappa \leq 2$ 

Dynamical formation in young clusters : disfavored to explain the whole BH population at m ~ 10  $M_{\odot}$ . large spin–orbit misalignment. Redshifts :  $\kappa \leq 2$ 

Galactic nuclei : a BBH population with a much wider mass spectrum than both young and globular clusters. Large spin-orbit misalignment. Redshifts :  $\kappa \sim 1$ .

Near an AGN disk : a significant population of BBH mergers with a wide mass spectrum. Spin depends on several factors **Isolated binary** evolution models : a peak near m ~ 10  $M_{\odot}$ . Preferentially aligned spins

NS: One result from gravitational wave (GW) observations is tension with the strong preference for 1.35  $M_{\odot}$  mass objects which has been recovered in galactic BNS.

# **GW Burst : CCSN**

![](_page_33_Figure_1.jpeg)

FIG. 3.— Waveform (a) and spectrogram (b) of the characteristic gravitational wave signal for the *fiducial model* at D = 100 kpc. We overplot estimates for the frequency evolution of g-modes at the surface of the PNS (solid-green line), g-modes in the cold inner core (solid-red line), quasi-radial mode (dashed-red line) and f-mode (dotted-blue line). Capital letters point to features described in the main text.

Morozova et al, Astrophys.J. 861 (2018) no.1, 10

Figure 1. Spectrogram (top) and the corresponding waveform (bottom) of the GW signal from the model M10\_SFHo.

![](_page_33_Picture_5.jpeg)

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# **GW200105-15**

![](_page_34_Picture_1.jpeg)

![](_page_34_Picture_2.jpeg)

![](_page_34_Picture_3.jpeg)

![](_page_34_Picture_4.jpeg)

![](_page_34_Picture_6.jpeg)