4D emittance measurement and coupling in transverse phase planes

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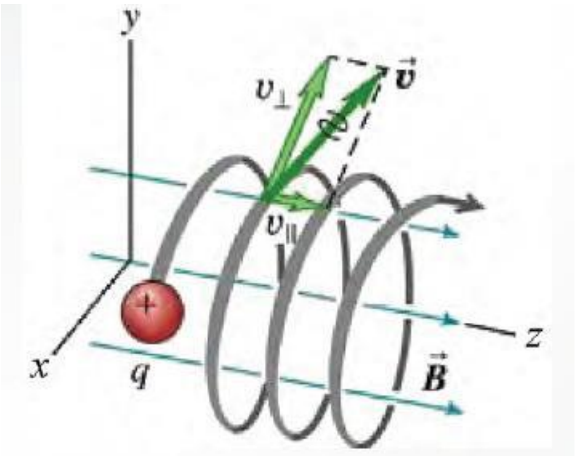
Introduction

- Coupling between 3 phase space planes in LEBT lines are usually considered negligeable with good approximation and 2D emittances are constant. Due to possible beam coupling in the case on non-linear fields and solenoid some inter-plane correlation moments become non-zero (xy’, x’y, x’y’) and 2D emittances are variable
- Objective of the note is investigating beam coupling effects at the ECRIS extraction and beam transport with solenoid and to quantify errors of 2D emittance measurements

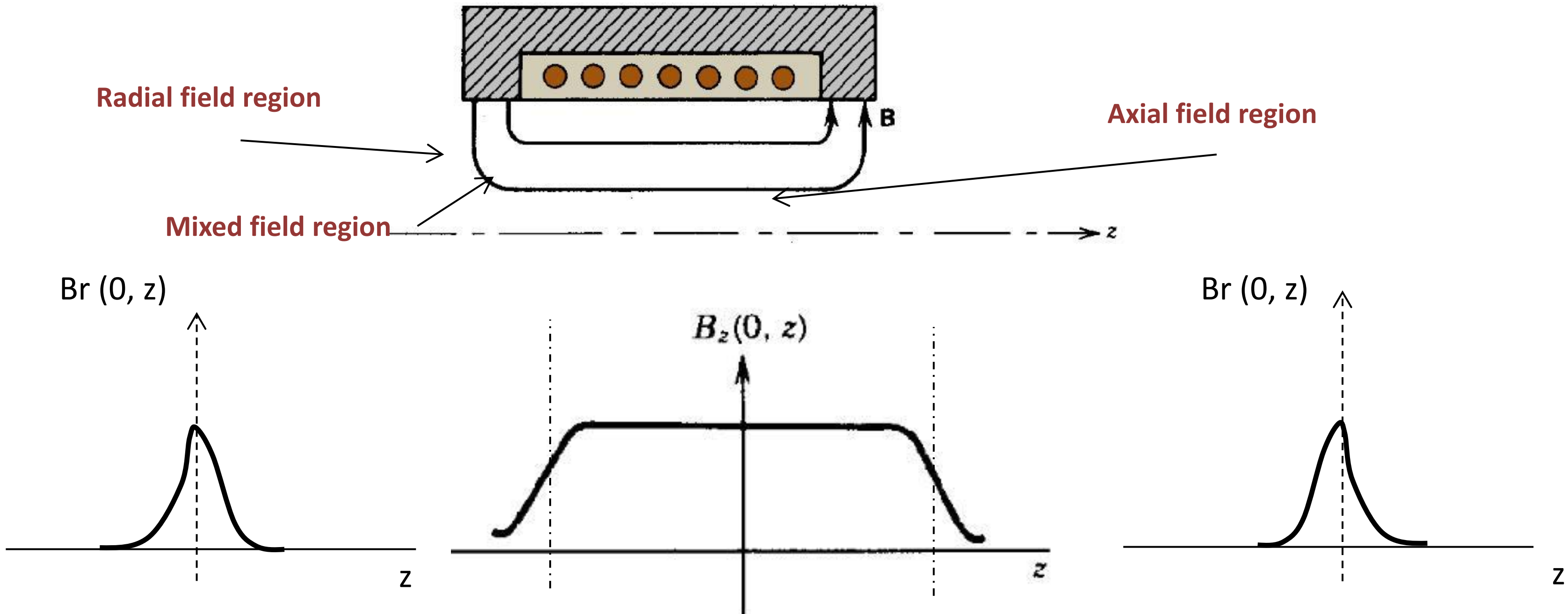
Basic case: single axial field component

- A charged particle with initial momentum is placed in a static axial Bz field. There is a simple gyroscopic trajectory with constant azimuthal velocity. The motion of the charged particle is coupled in the transverse planes
- To notice, in a frame rotating around the z axis by the Larmor frequency: ω = qBz / m, both the transverse planes decouple and the phase space area in each of the planes is conserved

Motion of particle in axial magnetic Bz field and no radial Br component



Solenoid study case

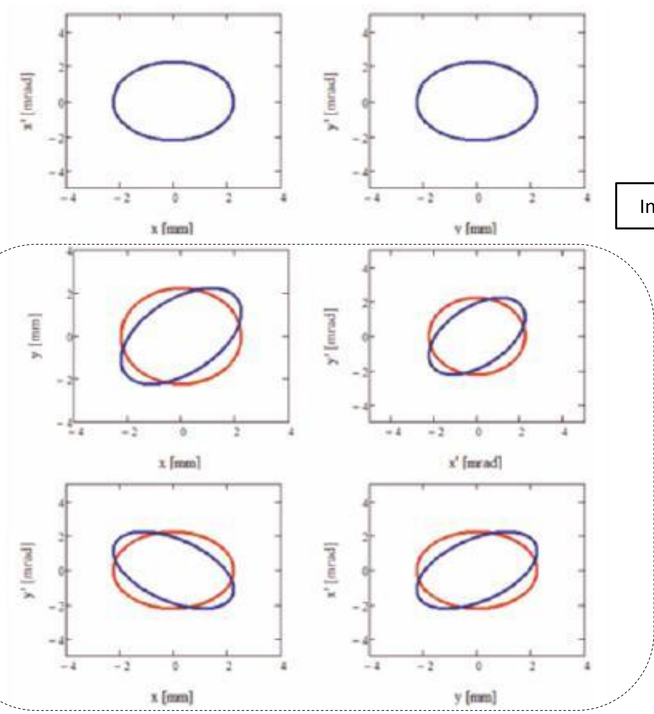


Entrance of solenoid | Center of solenoid | Exit of solenoid

At entrance and exit of solenoid the charged particule is submitted to a radial magnetic field component even higher for larger distance from axis. Lorentz force applied to the particule is given by the relation (qv_z * Br). Azimuthal force created by radial field is complemented by a radial force of axial field. They generate an azymuthal motion with deflection toward the axis. The result is coupling between the coordinates (r, Θ, z), centrifugal and coriolis forces have to be taken into account

Effect of solenoid coupling on RMS emittance

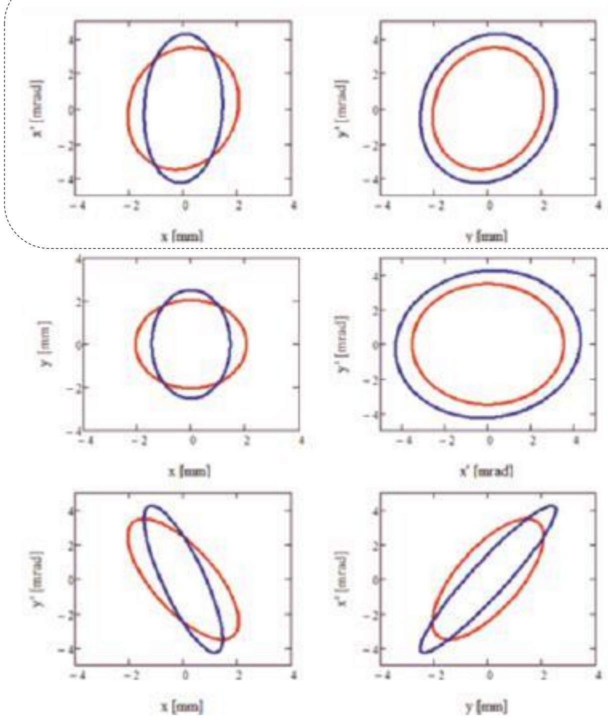
Before solenoid



Initial coupling

After solenoid

Modified x/y emittances



Figures credit [17]

Simulation at GSI performed with 11 MeV/u **uncoupled (red)** and **coupled (blue)** uranium beam [17]. Before solenoid coupling exists on xy, xy', yx' and x'y' planes due to initial conditions applied on beam. After solenoid coupling effect is noticeable on all planes due to existence of interplane correlations. Nevertheless modification of emittance is mostly effective in vertical y plane: 20-30 % increase

Step comment

- Last result with simulation is obtained with ideal conditions. In real world things are a little bit different. IPHC 2D emitm delivers correct measurements before solenoid but with usual intrinsic 10-20 % error (standard value). After solenoid, measurement in horizontal x plane is consistent taking into account system accuracy. Some correction is required in vertical y plane

- If the beam line shows no coupling element as solenoid and skewed quad, emittances remain constant and single 2x2D emittance measurement at extraction is suffisient for the design and the beam matching. In case of coupling element the projected emittances will be modified and thus corrections should be performed to obtain good beam matching. In that case one or two solenoids or skew quads can be used to decorrelate moments of the beam matrix and thus modify projected 2D emittances

ECRIS study case

In case of ECRIS there is an axial magnetic field component due to mirror configuration and radial component due to sextupole magnet both used for plasma confinement. Due to sextupole field map beam cross section shows a triangular feature. Asymmetry will play in favor of more coupling. The rapid decreasing magnetic field at the extraction induces an azimuthal force (rotational motion), resulting orbit is spiral and both transverse planes will be coupled. As transverse motion components present coupling, projected RMS-emittances ε_x and ε_y increase after extraction due to inter-plane correlations. According to Liouville's theorem ε_{4D} is invariant of the motion

Transmission matrix of axial magnetic field at ECRIS extraction may be non-symmetric. Four transverse phase space coupling elements ($\sigma 13$, $\sigma 14$, $\sigma 23$, $\sigma 24$) are non-zero and represent the inter-plane correlations

Step comment

Modification of projected beam emittances under preservation of the full 6D emittance with transfer from one plane to the other is a rather recent issue. It led to the development of flat beam concept with one emittance much smaller than the other one and found application for electron beam in 2001 and ECRIS in 2006 in order to achieve high mass resolution [13]

High transmission efficiency of the beam extracted from ECRIS requires well understanding of transverse coupling. There are a lot of experiments with Allison emittance-meters for which coupling can be neglected or ignored, see for example papers from SNS (work of M.P Stockli et al.) [2-4], FAIR (work of O. Tuske et al.) [5], FRIB (work of C.Y.J Wong et al.) [6], TRIUMF (work of A. Laxdal et al.) [7], GANIL (work of R. Ferdinand and C. Jamet et al.) [8-9], LNL (work of M. Cavenago et al.) [10], ITER (work of C. Poggi et al.) [11], and ARRONAX [12]

That doesn't mean that it is not an issue but that coupling should be relativized in comparison of other sources of error which contribute to the result and may be dominant as:

- Ghost signals (contaminants, neutrals, electrons, etc.)
- Noise and background
- Space charge and dynamic neutralization (compensation)
- Accuracy of back-tracking simulations
- Capacity to measure low charge densities

Fragments of phase space theory

Emittance definition

$$\varepsilon_x = \beta x'^2 + 2\alpha x x' + \gamma x^2$$

(according to Courant-Snyder, equation of an ellipse, same for y plane)

RMS value

$$\varepsilon_x^{RMS} = \sqrt{\overline{x^2 x'^2} - \overline{(xx')^2}}$$

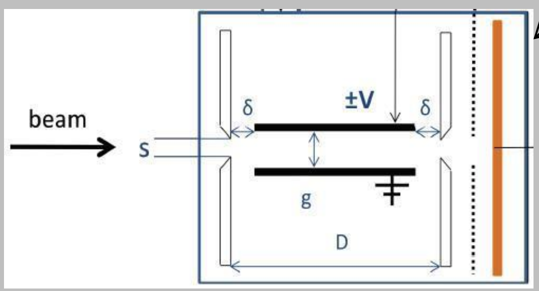
For IPHC system

$$x' = \Delta V\, D \,/\, 4U\, g$$

$$x'_{max} = 2g \,/\, D$$

$$\Delta V_{max} = 8g^2\, U \,/\, D^2$$

Beamlet current is integrated by FC (no resolution)



ΔV voltage between deflecting plates, U acceleration potential

Source of error

Emittance is the 6-dimensional hyper volume $\mathcal{V}_6$ occupied by N particles in the (x,y,z,p_x,p_y,p_z) phase space with

$$\mathcal{V}_6 = \iiiint\!\!\iint dx \cdot dy \cdot dz \cdot dp_x \cdot dp_y \cdot dp_z$$

Conservation law/Liouville theorem gives:

$$\Delta\mathcal{V}_6 = 0$$

with assumption of conservative forces, analogy with incompressible fluid

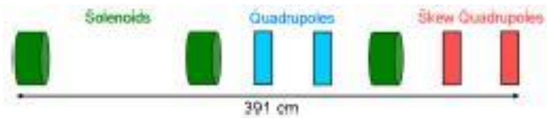
and $$\mathcal{V}_6 = \iiiint dx \cdot dy \cdot dp_x \cdot dp_y \cdot \iint dz \cdot dp_z$$

with assumption of no coupling between transverse and longitudinal motions

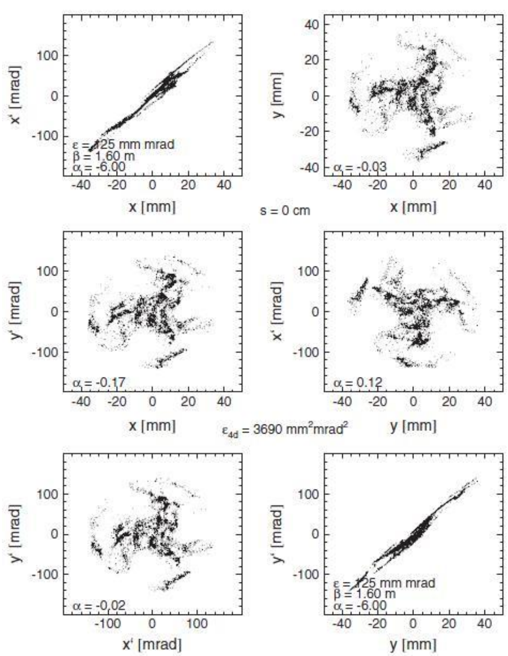
and $$\mathcal{V}_4^T = \iint dx \cdot dp_x \cdot \iint dy \cdot dp_y$$

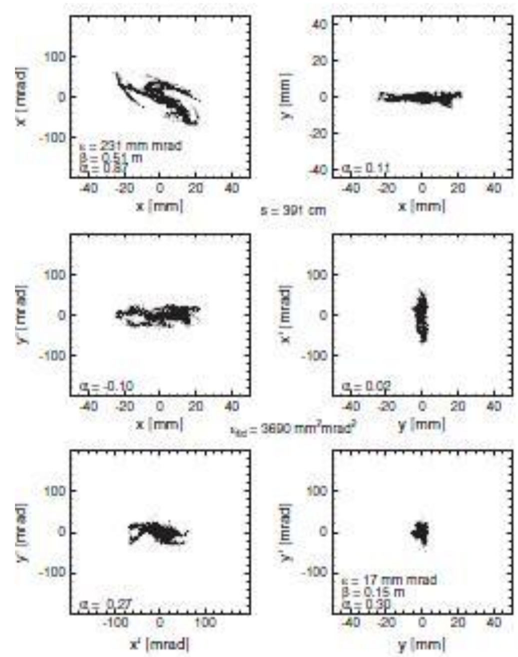
with assumption of no coupling between both tranverse motions, that's not always the case!

Initial experiment at GSI



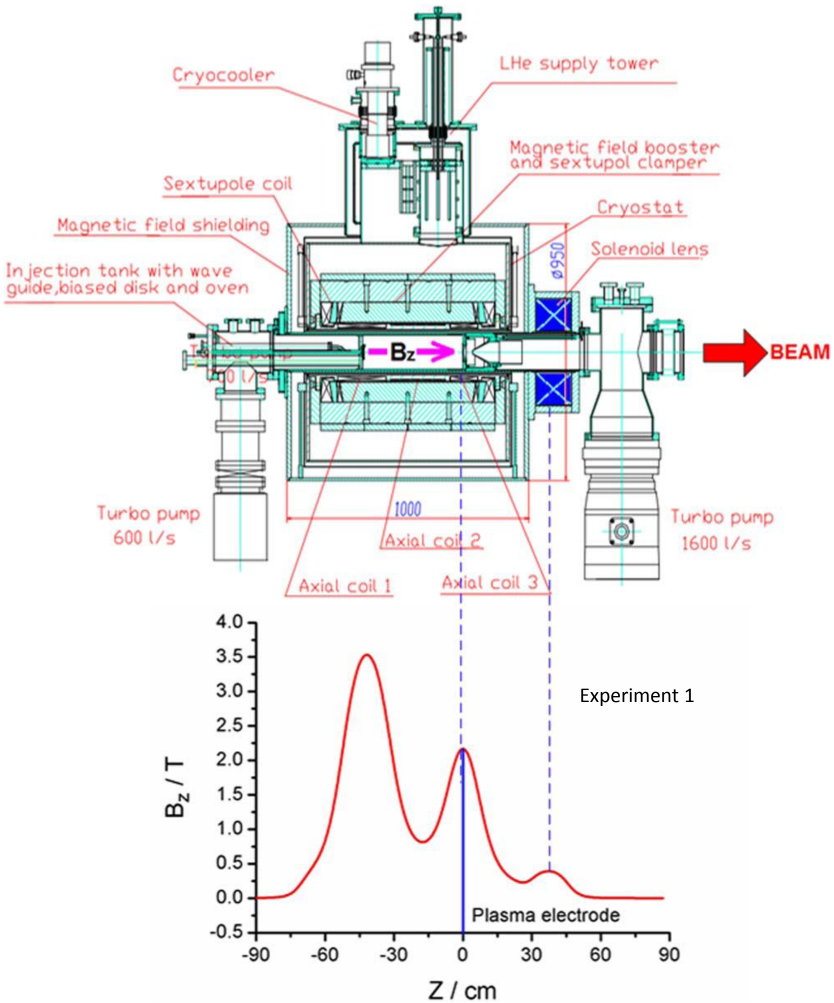
LEBT after ECRIS of the HLI facility. A first solenoid is used to simultaneously focus beam in both transverse planes. Emittance magnitude is controled by additional solenoids and quad lenses. Finally, four interplane correlation moments are minimized by two skew quads [14]



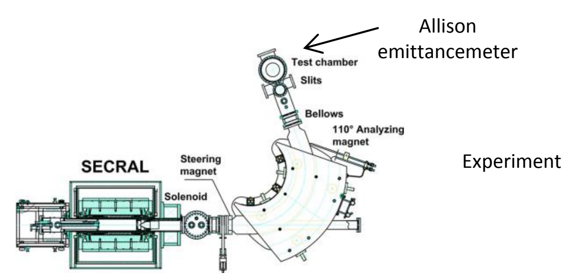


2D emittances and 4 interplane correlation parameters of $^{16}O^{3+}$ beam at 2.5 keV/u on ECRIS extraction , entrance of LEBT on left side and exit at right side. Due to interplane correlations and beam coupling product of projected 2D emittances becomes larger than 4D one behind third solenoid: $\varepsilon_x * \varepsilon_y \approx 12.5\ \varepsilon_{4D}$ but after skew quads product will be minimized to almost unity and interplane correlation moments are almost completely removed [14-17]

Field experiment at IMP



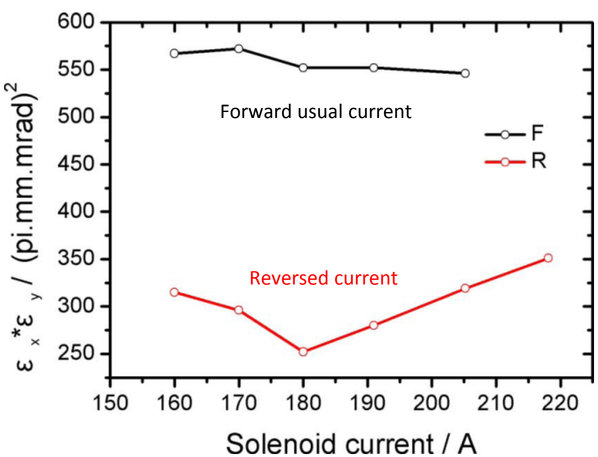
- An experimental campaign has been performed on SECRAL (Superconducting Electron Cyclotron Resonance ion source with Advanced design in Lanzhou) and its platform at Institute of Modern Physics (IMP) [22]. SECRAL is a fully SC ECR ion source operating at 18–28 GHz. The SC magnet confinement configuration of the ion source consists of three axial solenoid coils and six sextupole coils with a cold iron structure as field booster and clamping. For 28 GHz operation, the magnet assembly can produce peak mirror fields on axis of 3.6 T at injection, 2.2 T at extraction, and a radial sextupole field of 2.0 T at plasma chamber wall. As usual, the plasma electrode is located near the center of the extraction solenoid coil which directly mounted on IS exit
- Standard $^{129}Xe^{29+}$ analysed beam current is of 19 eμA, with a total extracted current of 950 eμA from the ion source at 25 kV



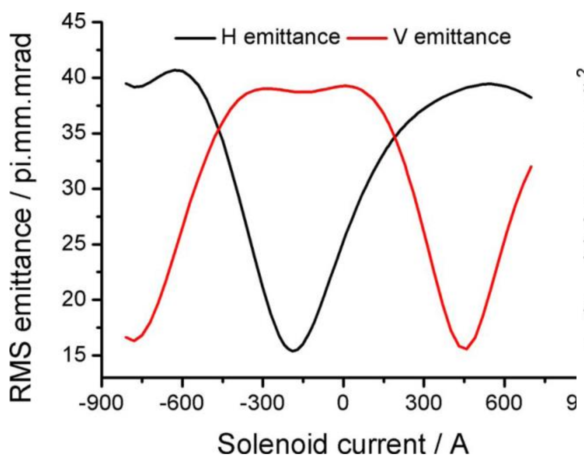
Figures credit [22]

SECRAL experiment at IMP

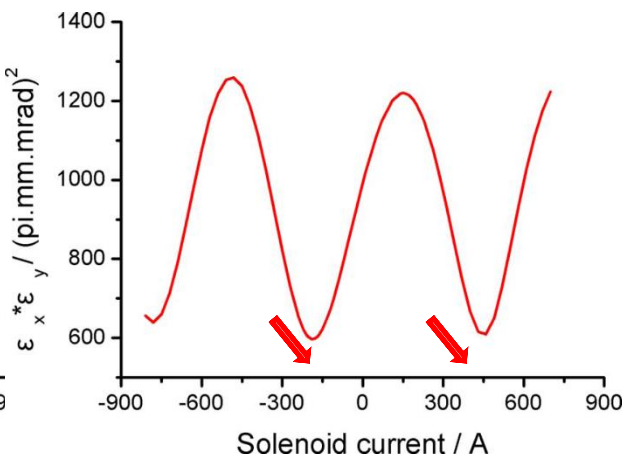
- Initial beam conditions at ECRIS exit required to study coupling are obtained by back-tracking simulation from emittance measurements performed downstream analysis magnet [22]. One can expect possible unknowns, uncertainties and errors specific to that method mixing measurements and simulations
- Axial magnetic field in the extraction region of the ion source generates coupling between transverse planes which can be partly or completely compensated by a solenoid lens. This is consistent with theory as projected 4D transverse phase space (ε_x and ε_y) is modified with solenoid field strength. Then, by reversing current of solenoid beam rotates in opposite direction resulting in a different coupling between planes. Lower emittance has been obtained (0.5 ratio in respect of initial value) resulting of reduced horizontal emittance and a minimum value has been observed on the variation curve for 180 A for which the beam can be considred as decoupled. Considering solenoid current variation, both ε_x and ε_y emittances at the exit of solenoid are obtained from measurements with Allison systems performed after analysing magnet. They show a periodic behaviour also visible if we consider their product, see figures below



Measuremts, experiment 1

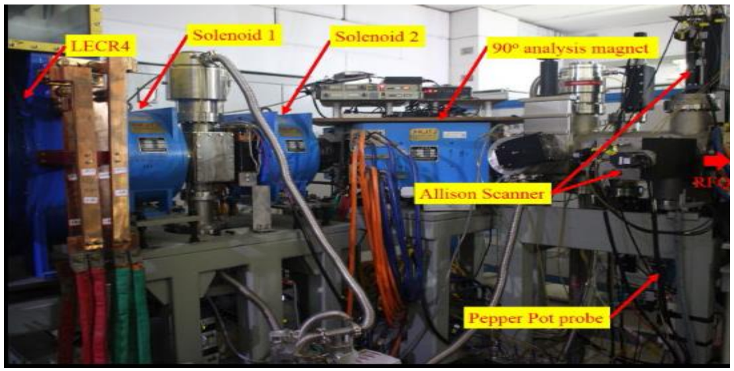


Simulations, experiments 2

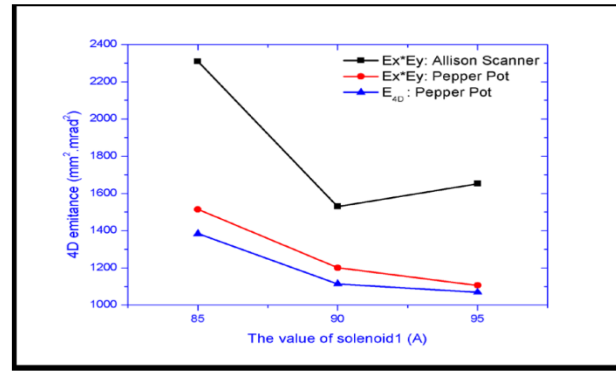


Simulations, experiments 2

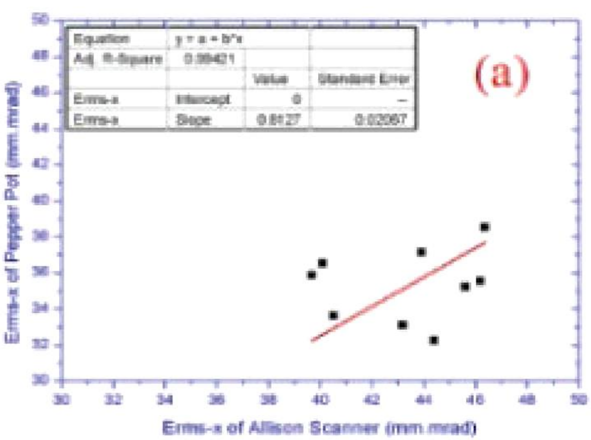
Experiment on LECR4 platform at IMP

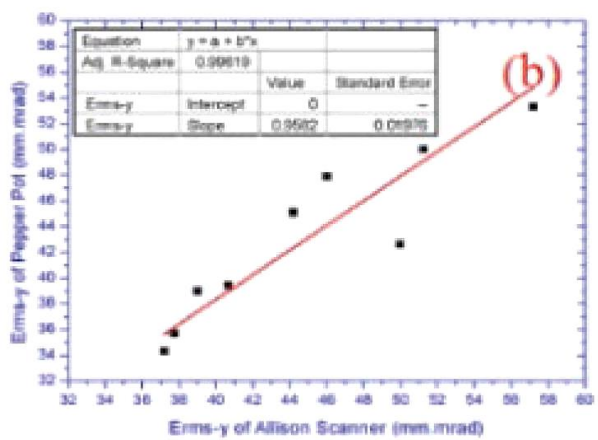


LECR4 usually operates at 18 GHz with maximum injection field of 2.5 T and extraction field of 1.8 T [24]. It shows two solenoids, a 90° analysis magnet. Beams are 560 eμA O^{7+}, 620 eμA Ar^{11}+, and 430 eμA Xe^{11+}



Emittance measurement comparison between 4D pepperpot and 2x2D Allison systems [24]

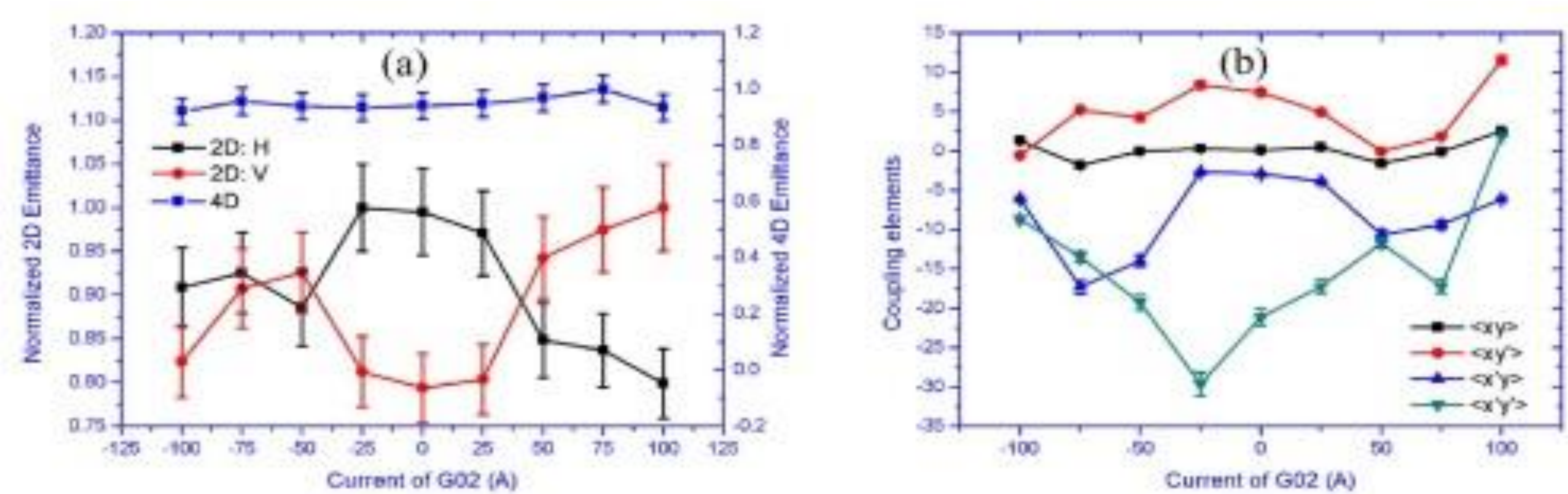




RMS emittance comparison in xx' (a) and yy' (b) planes. Allison emittances are always higher than peppperpot ones and results in yy' plane is better (less dispersion) [24]

Emittance variation with solenoid current

Solenoid 1 current is fixed to 125 A



(a) LECR4 transverse and 4D emittance variation with 2nd solenoid current (normalized values). ε_{4D}, ε_x and ε_y variations are relatively small and can be considered as being in the range of the usual emittance measurement error if 2nd solenoid is powered and absolut current intensity is ≥ 35 A [25]

(b) Coupling elements variation with 2nd solenoid current. Largest variations are observed for xy', x'y and x'y' elements the most usual interplane correlations [25]

NB. Due to the position of the diagnostic box coupling is measured at the end of the beam line.
It demonstrates the superposition of ECRIS and solenoid lenses effects

Field experiment on SPIRAL 2 LEBTs

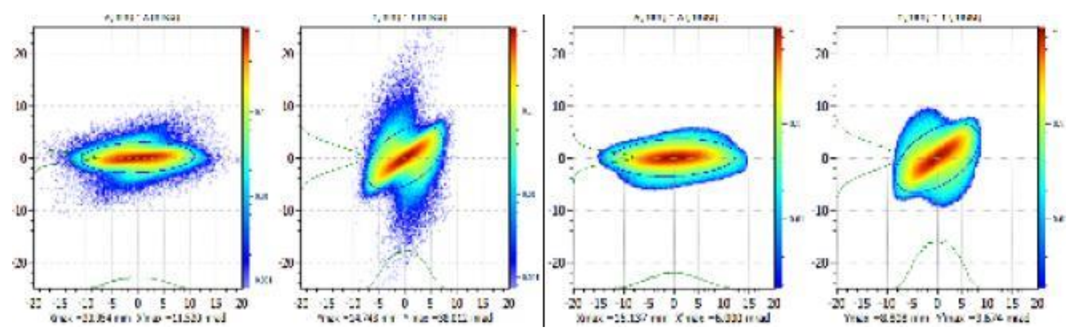
LBE A-B-C LEBTs

2D transverse emittance scanner installed downstream RFQ, 2 quads and 1 rebuncher cell

Commissioning ion beams

Particles	H^+	D^+	ions	option
A/Q	1	2	3	7
Max I (mA)	5	5	1	1
Max energy (MeV/A)	33	20	14	8.5
Max beam power (kW)	165	200	44	51

Emittance figures: simulation (2 on left side) vs scans (xx',yy' planes)



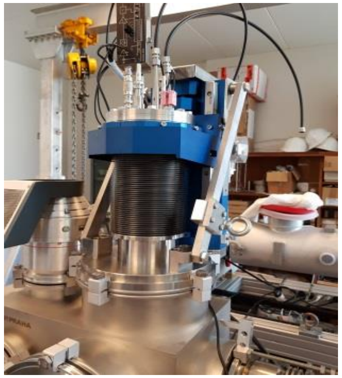
Emittance measurements

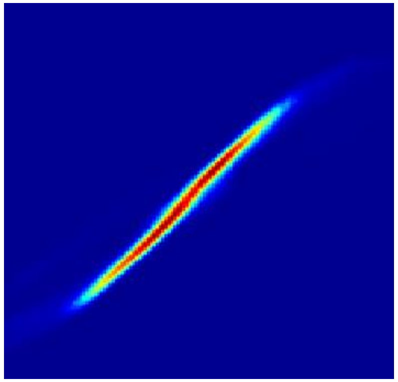
	5 mA H^+		5 mA D^+	
	Simulation	Measurement	Simulation	Measurement
Emit-xx' (rms)	0.20	0.19	0.19	0.17
Beta-xx'	3.75	3.48	4.26	4.02
Alpha-xx'	-0.12	-0.13	-0.23	-0.16
Emit-yy' (rms)	0.26	0.21	0.21	0.20
Beta-yy'	1.38	1.47	0.98	0.98
Alpha-yy'	-0.55	-0.59	-0.33	-0.31

No significant coupling effect has been noticed during beam commissioning. Measured emittance figures are consistent with simulations [8, 9, 27, 28]. Result can be attributed to limited effect on light ion beams used during commissioning (H, D), ECRIS without sextupole, symmetric beam at extraction, tolerance of the beam line, and extended possibilities of beam matching. Opinion shared by LPSC experts with similar ECRIS. Should be confirmed in the future with heavier ions and dedicated experiment

Emittance measurement perspectives at IPHC

- RMS emittance and phase space distribution are measured in 2D with IPHC system using Allison principle. More than a dozen systems are currently in operation around the world. Most of them are used to measure 2D phase space distributions in LEBT of linacs [5, 8, 9, 27, 28]. More recently, IPHC emittance-meter has been successfully installed in injection channel of 70 MeV cyclotron at ARRONAX [12].
- Both transverse planes are scanned separately with same equipment in same plane (requires vacuum break) or alternatively with twin systems installed on different planes (without vacuum break). This configuration raises the issue of measurement consistency/signal fluctuation, signal drifts during prolonged scan, systematic error and correlation between data if one aims to define accurate coupling between both transverse planes





Conclusion

- High transmission efficiency of the beam extracted from ECRIS requires well understanding of transverse beam coupling. Coupling due to ECRIS and solenoid lens may significantly affect transmission in LEBT and matching in accelerator
- Following present review it can be considered that due to transverse beam coupling produced by ECRIS and solenoid product of projected emittance $\varepsilon_x * \varepsilon_y$ becomes larger than hyper-emittance ε_{4D} . According to experiments and simulations related ratio may spread from 1.2 to ≈ 13. Thus, some transfer between both transverse planes has been observed although ε_{4D} remains constant in accordance with Liouville's theorem. The transfer between ε_x and ε_y shows a periodical behaviour depending on solenoid focusing strength (current setting)
- We deduce that in those cases measuring only one or separately both transverse emittances with Allison emittance-meter doesn't enable to characterize beam and perform efficient transmission
- To complete previous points, it is important to notice that:
  - Accurate beam dynamics, beam matching and emittance comparison along beam line according to Liouville's theorem are not always an issue
  - Deleterious effect can be mitigated by light ions, symmetric beam shape and adequat beam line settings
  - Solenoid generates beam coupling which contributes OR counter balances coupling produced by ECRIS depending on its focusing strength and polarity. Optimal settings are possible and solenoid polarity reversal can counter-act 2D emittance increase and lead to minimal $\varepsilon_x * \varepsilon_y$ product
  - Beam coupling can be eliminated by additional solenoid and skew quads (further more skew magnets enable measuring 4D emittance with 2D Allison system but is time consuming)
  - Coupling can be determined with IPHC 2D emittance-meter and mesurements in both transverse planes combined with back-tracking simulations (beam line model dependent accuracy)
  - Coupling can also be determinated with 3 emitm and successive scans (with one rotated at 45°) or one single rotating emitm (assembled on rotating flange, GSI like)

NB. Performances of pepperpot system are usually benchmarked with Allison emittante-meter which serves as a reference. Difference between both measurements is usually of the order of 20 %

Back-up

Axial and radial field components

- Radial fields are used for confinement of plasma and can be complemented by axial magnetic field as for ECRIS at extraction which generates angular momentum an thus coupling between both transverse planes
- 2-D phase space area is no longer conserved in each transverse plane but Liouville's theorem still applies to 4D transverse phase space: 4D hypervolume and 4D emittance are invariant of the motion
- As detailled in the following review in case of beam coupling there are some inter-plane correlations and $\varepsilon_x * \varepsilon_y > \varepsilon_{4D}$

Solenoid study case

In case of solenoid the spiral orbit is created by conjunction of azimuthal and radial forces generated by fringe and focusing fields with additional contribution of initial axial momentum of particles

In LEBT channel $v_z \ll c$

Thus Lorentz factor $\gamma \approx 1$ and equations of motion are

$$\begin{cases} m\, dv_r/dt = -qv_\Theta B_z + mv^2_\Theta/r & (1) \\ m\, dv_\Theta/dt = -qv_z B_z - mv_r v_\Theta/r & (2) \\ m\, dv_z/dt = 0 \text{ as } v_z = cst & \end{cases}$$

With assumption of weak focusing ($r \approx cst$) and small net rotation of particle orbit in the lens, Coriolis force can be neglected. Also,

dv_Θ/dt can be replaced by $v_z(dv_\Theta/dz)$ and B_r by dB_z/dz. According to [1] equation (2) leeds to

$$v_\Theta - qrB_z / 2m = 0 \qquad (3)$$

This relation traduces conservation of angular momentum after complete propagation of the particles through solenoid: the azimuthal velocity increase at the solenoid entrance is cancelled by the decrease produced by the reversed radial field component at the exit, thus resultant azimuthal velocity remains constant (if it is 0 before solenoid it remains 0 after)

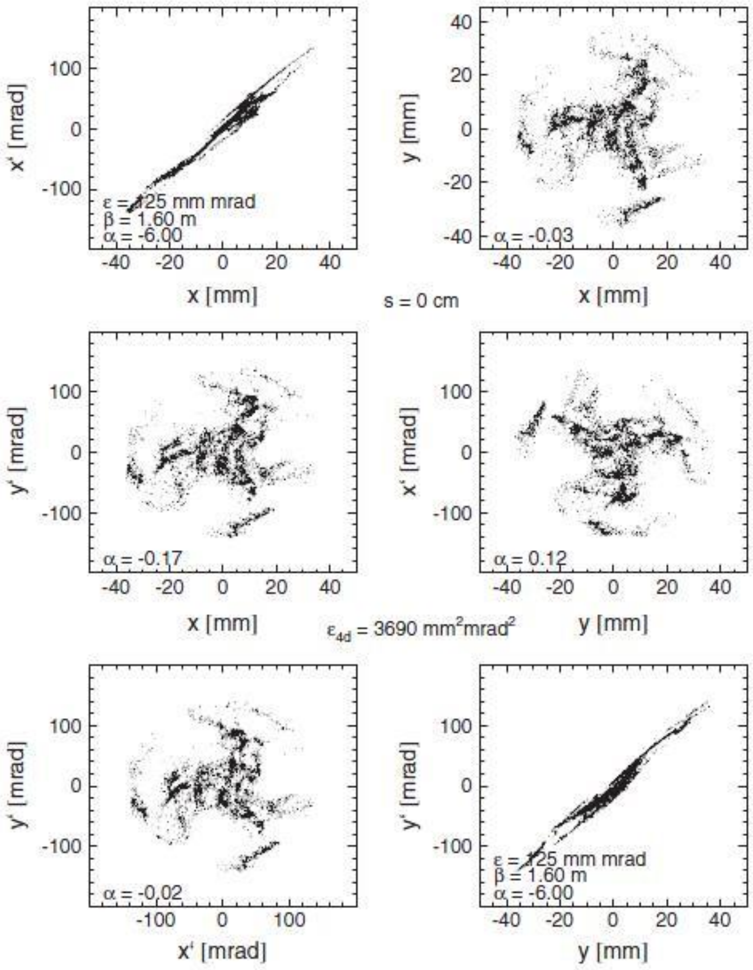
Solenoid study case

Despite net velocity gradient is 0 angular rotation of beam through solenoid can be obtained with angular velocity definition $d\Theta/dt = v_\Theta / r$ and assuming that Bz is constant with radius r (optimistic assumption):

$$\Delta\Theta = (qB_z / 2mv_z)\, \Delta z \qquad (4)$$

Finally, as solenoid introduces coupling between the horizontal and vertical motion of particles moving along the beam line, at least one of the elements of the off-diagonal sub-matrix of resulting beam is non-zero (P. XXX). In order to achieve good beam matching one has to measure again emittances or know the inter-plane correlations

ECRIS study case (end)



- 2D emittances and 4 interplane correlation parameters of $^{16}O^{3+}$ beam at 2.5 keV/u on ECRIS extraction [14]
- Ion density is non-uniform due to sextupolar field generated for plasma confinement and resulting beam is non-circular and non-homogenous increasing coupling effects
- Axial magnetic field in the extraction region of the ion source generates coupling between transverse planes which can be partly or completely compensated by specific settings of solenoid lens [22-25]
- Projected 4D transverse phase space distributions (ε_x and ε_y) are modified with solenoid field strength but determinant of the 4D sigma matrix of the beam will keep constant (according to Liouville theorem 4D emittance is constant with conservative forces)

Step comment

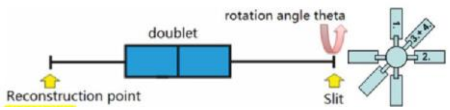
According to Liouville's theorem, emittances is conserved if the particles are subjected to conservative forces such as time-invariant electric and magnetic fields. Therefore if emittance is measured at a point along the beam line it allows for predicting the beam transport, the beam focusing, the losses in restricted beam line apertures at another point. However, some non-conservative forces, such as collisions, scattering, and fluctuations of electric or magnetic fields, increase the emittance. Other non-conservative forces such as E-M radiation decrease the emittance. At low beam energy e.g. in the ECRIS, at extraction and following beam adaptation/matching section there are some non-linear forces, emittance figure is distorted, elliptical approximation is not valid anymore and ellipse figure is not conserved but it remains a useful figure of merit. In case of coupling between both transverse planes the ***beam σ_{4D} matrix*** is defined with the different covariance coefficients and x/y correlations such as

$$\sigma_{4D} = \begin{bmatrix} <x^2> & <xx'> & <xy> & <xy'> \\ <xx'> & <x'^2> & <x'y> & <x'y'> \\ <xy> & <x'y> & <y^2> & <yy'> \\ <xy'> & <x'y'> & <yy'> & <y'^2> \end{bmatrix} = \begin{bmatrix} \sigma_{xx'} & \begin{matrix} <xy> & <xy'> \\ <x'y> & <x'y'> \end{matrix} \\ \begin{matrix} <xy> & <x'y> \\ <xy'> & <x'y'> \end{matrix} & \sigma_{yy'} \end{bmatrix}$$

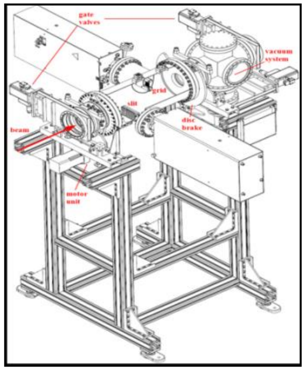
For decoupled planes the surrounded correlation elements are zero, $\varepsilon_x = \sqrt{\det \sigma_{xx'}} = \sqrt{\sigma_{11}\sigma_{22} - \sigma^2_{12}}$, $\varepsilon_{4D} = \varepsilon_x * \varepsilon_y = \sqrt{\det \sigma_{4D}}$

Further experience with ROSE at GSI

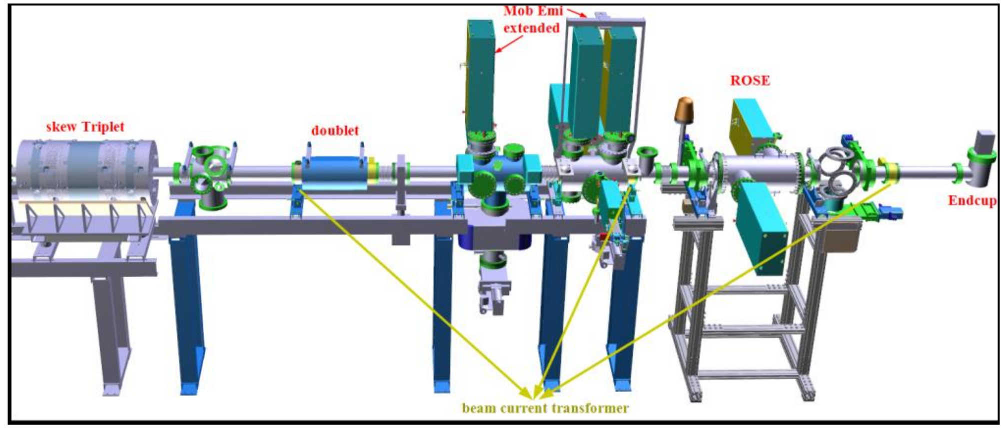
ROSE set-up



Test bench with skew triplet



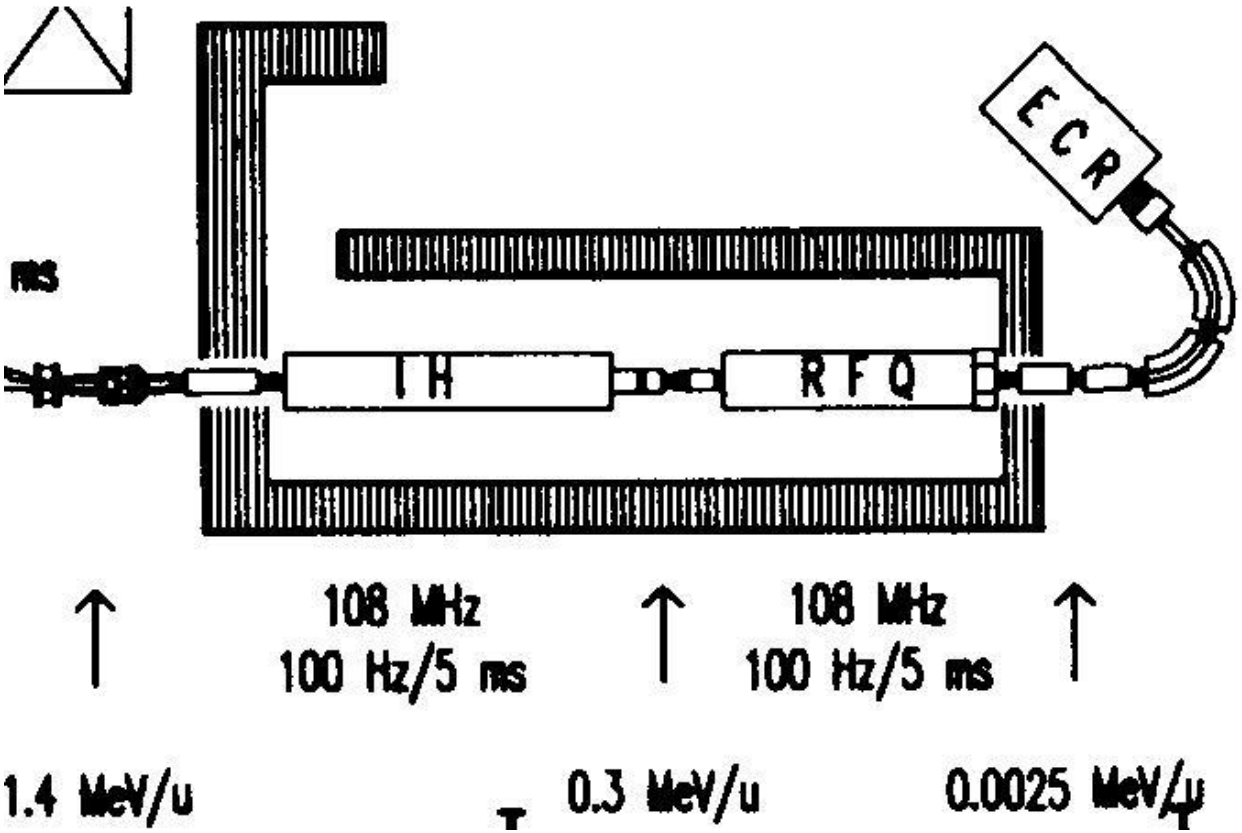
Beam current integrated in the non-scanned plan
(no resolution)



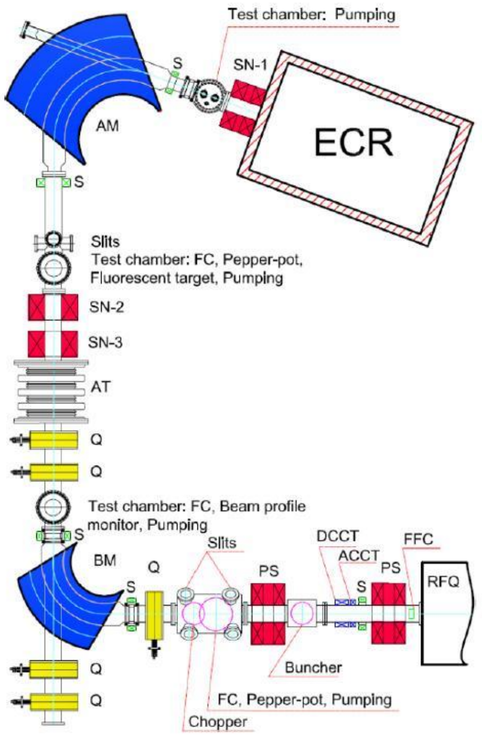
The so-called ROSE emittance-meter has been developed at GSI [18-21]. It is made up of a standard slit grid emittance scanner delivering 2D measurements which is rotatable around the beam axis. In combination with a magnetic doublet used for beam settings it allows to determine the full 4D beam matrix in approximately one hour with a minimum of four emittance measurements at three different angles (45° angular position is performed with two different quad settings).

First experimental set-up used to characterize ROSE's performances is composed of high charge state injector HLI delivering 1.4 MeV/u Ar^{9+} and Xe^{19+} beams. Skew magnetic triplet is used to modify the coupling between planes as no significant initial correlations were found to be present in the HLI beam. The benchmark confirmed the successful measurement of 4D beam matrix by back transformation to a point upstream for different magnetic settings of the doublet. Nevertheless no significant coupling has been identified on original HLI beam line and skew triplet didn't succeed to remove completely coupling on test bench [18, 19]. Another experiment performed on EMTEX demonstated an increase of the projected rms-emittance in the order of 75%. Removing the inter-plane coupling existing before and modified after test bench could increase the beam brilliance and thus the injection efficiency into SIS18 [21]

High charge state injector HLI @ GSI



LEAF experience at IMP



SN solenoid lens, AM analyzing magnet, Q quadrupole, BM bending magnet, S steering magnet, FC Faraday cup, FFC fast Faraday cup, PS paired solenoid [23]

Low-Energy intense-highly charged ion Accelerator Facility program (LEAF) has been launched in 2015 and commissioned in 2018 at IMP [23]. It consists of a 4th generation 45 GHz ECRIS, a LEBT and a RT RFQ for preacceleration of ion beam up to 0.5 MeV/u at 81.25 MHz. High intensity highly charged ion beams of 2 emA $^{238}U^{34+}$ are expected

SN-1 solenoid is used to focuse beam at the entrance of analysing magnet and in addition to SN-2 and SN-3 solenoids to reduce inter-plane coupling for beam matching at the RFQ entrance. Projected ε_x ε_y RMS emittances should be tuned in order to fit both within the RFQ's acceptance [23]