Properties of the tip of one-dimensional branching random walks: analytical and numerical results, and motivations from QCD

Stéphane Munier

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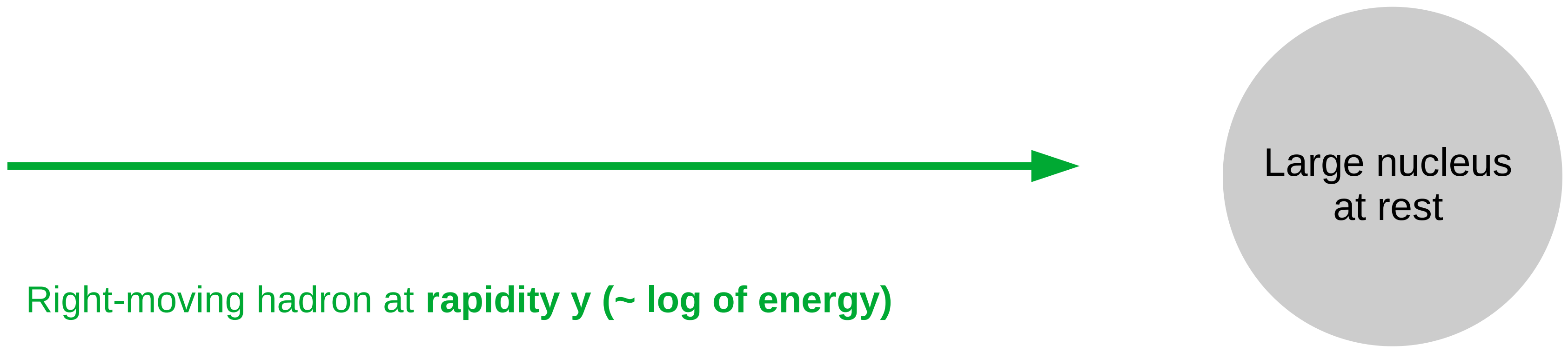
Outline

- From particle physics to branching processes
- Genealogy of particles ending up beyond a predefined position
- An exact Monte Carlo algorithm to generate the tip of BRWs at large times

Work reported here done in collaboration with A.H. Mueller, É. Brunet, A.D. Le

Hadron-nucleus scattering amplitudes



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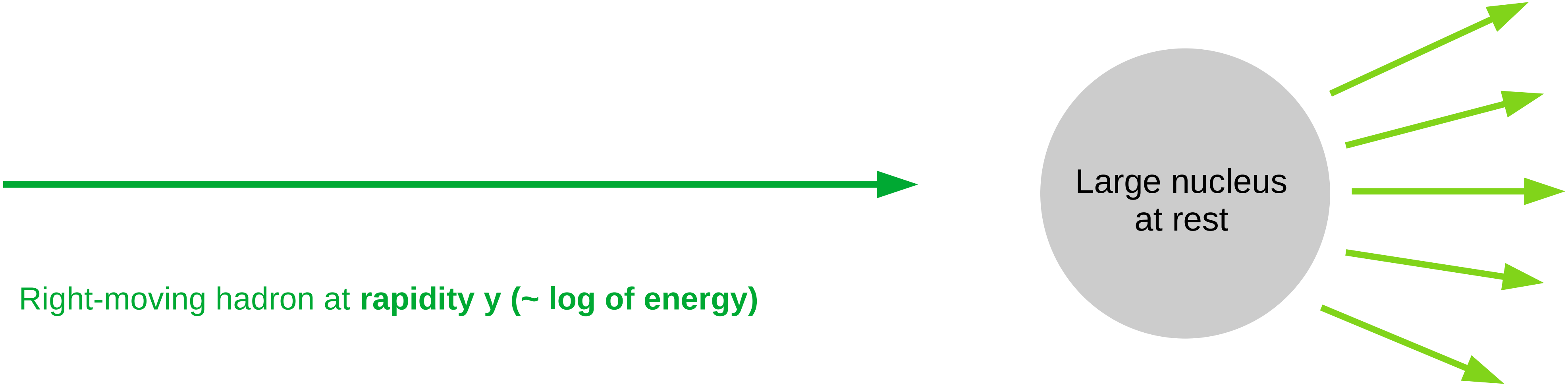

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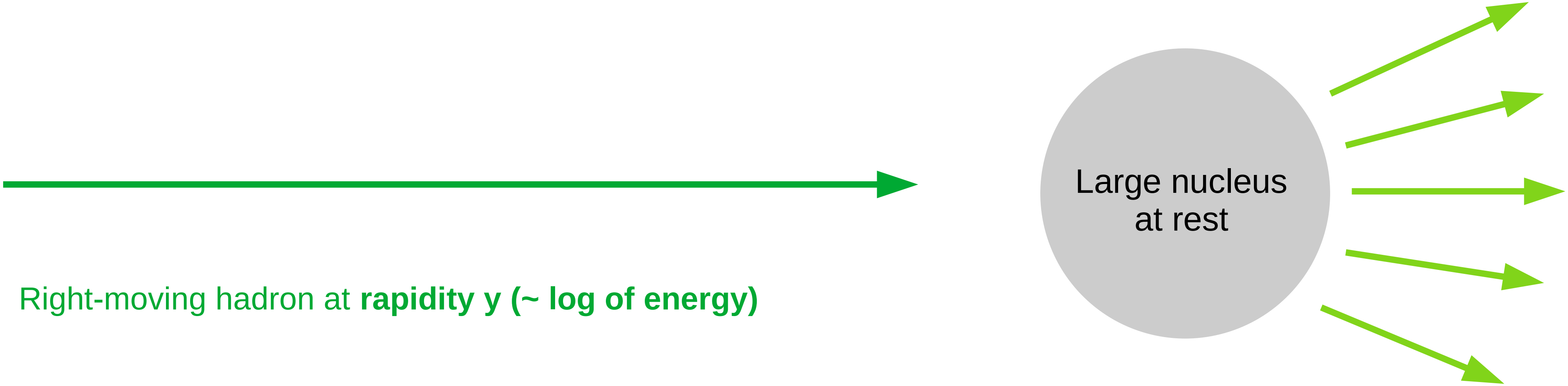


Right-moving hadron at **rapidity y (~ log of energy)**

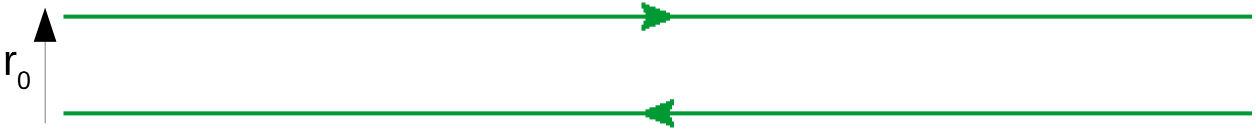
When it interacts with the nucleus, the latter breaks and new particles are seen in the detector (in general, many, covering a large solid angle around the flight direction of the initial hadron).

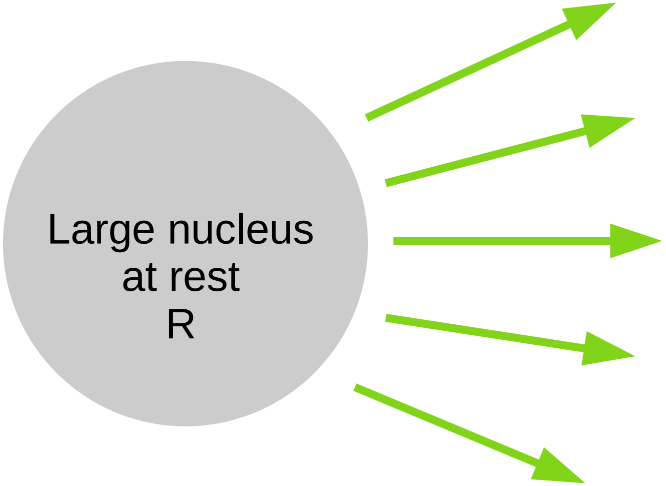
What is the probability that an interaction occurs?

Hadron-nucleus scattering amplitudes: a FKPP problem

Let us consider the simplest hadron: a quark-antiquark pair (= meson)

Characterized by a two-dimensional size vector r_0





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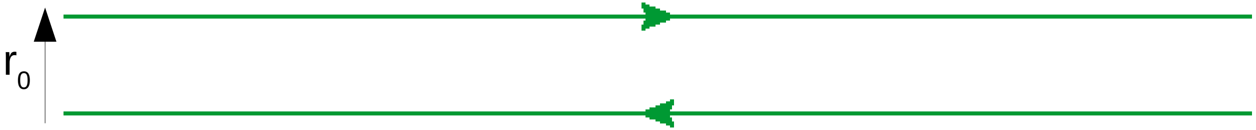

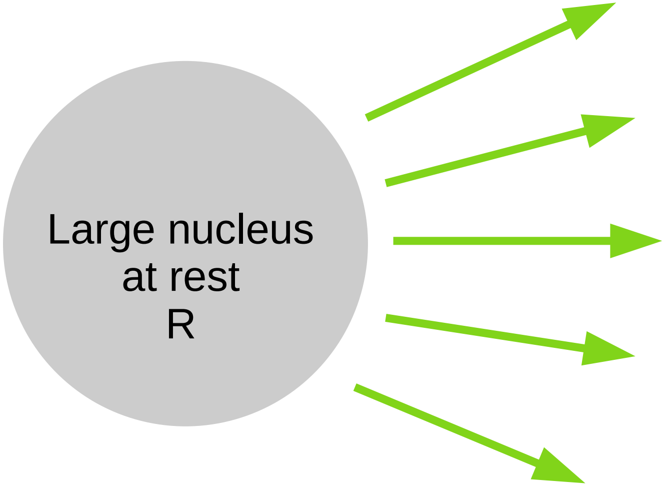

An equation for the probability (amplitude) has been derived from QCD (Balitsky, 1996; Kovchegov, 1999).

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It takes the form of an evolution equation in the rapidity y

$$\partial_y T(r_0,y)=\bar{\alpha}\int\frac{d^2 r_1}{2\pi}\frac{r_0^2}{r_1^2(r_0-r_1)^2}\left[T(r_1,y)+T(r_0-r_1,y)-T(r_0,y)-T(r_1,y)T(r_0-r_1,y)\right]$$

$$T(r_0,y=0)=\Theta(|r_0|-R)$$

which is actually “FKPP-like” i.e. in the same universality class as $\partial_t u(t,x)=\frac{1}{2}\partial_x^2 u(t,x)+u(t,x)[1-u(t,x)]$

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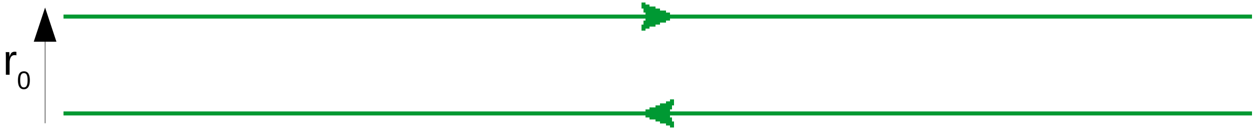

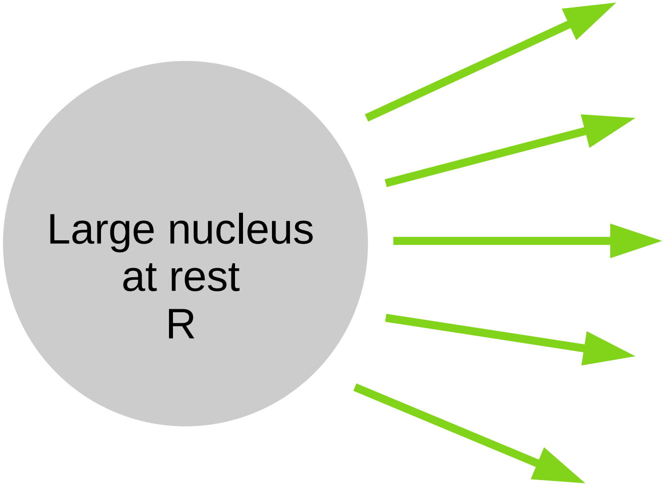

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We are going to interpret this equation physically.

Hadron-nucleus scattering amplitudes: a FKPP problem

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Interaction probability (amplitude):

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$T = 0$ else

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Actually, the hadron is “seen” from the nucleus in an actual quark-antiquark state only if it is *very* slow (rapidity $y \approx 0$), namely almost at rest.

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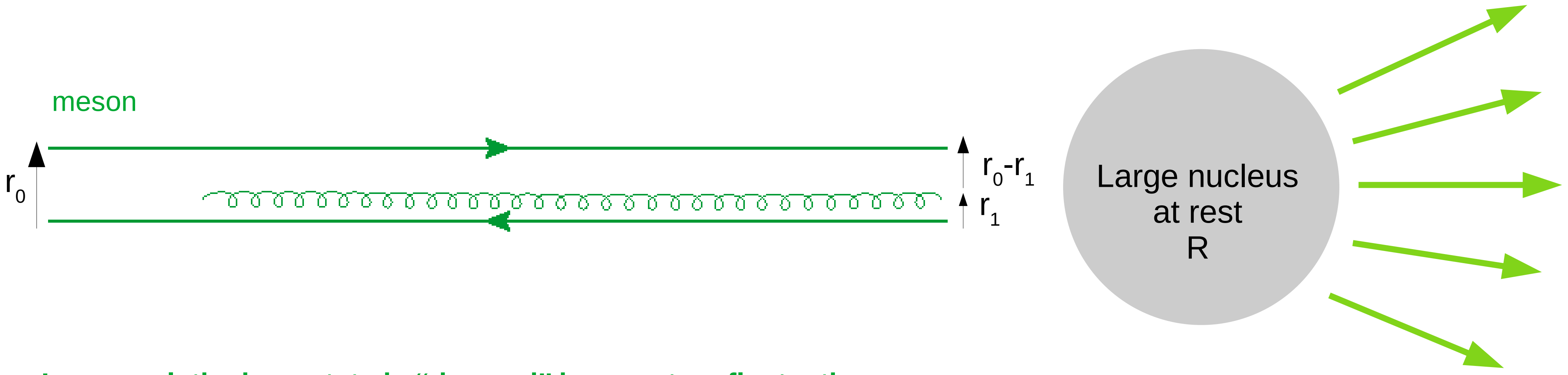

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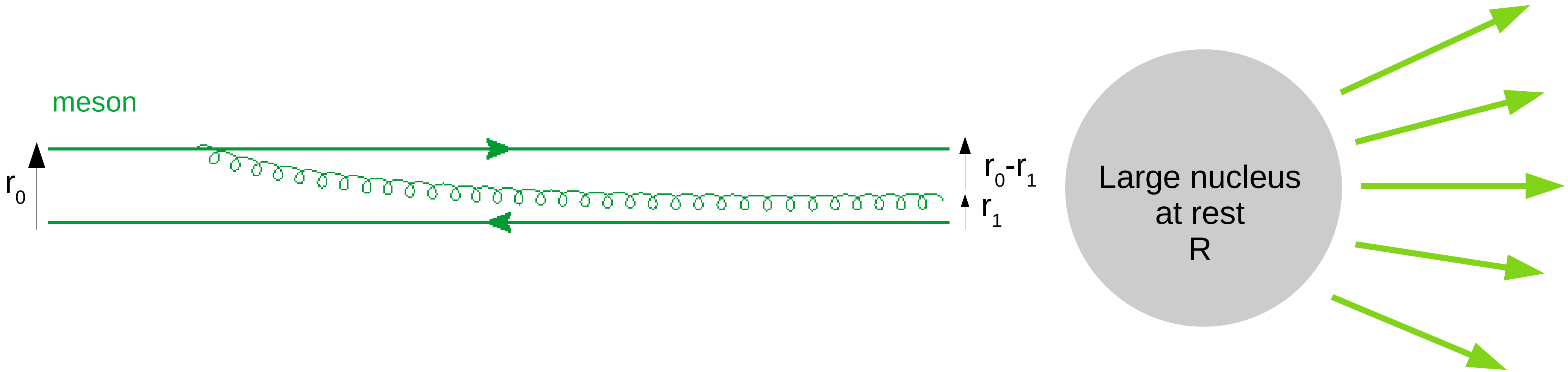

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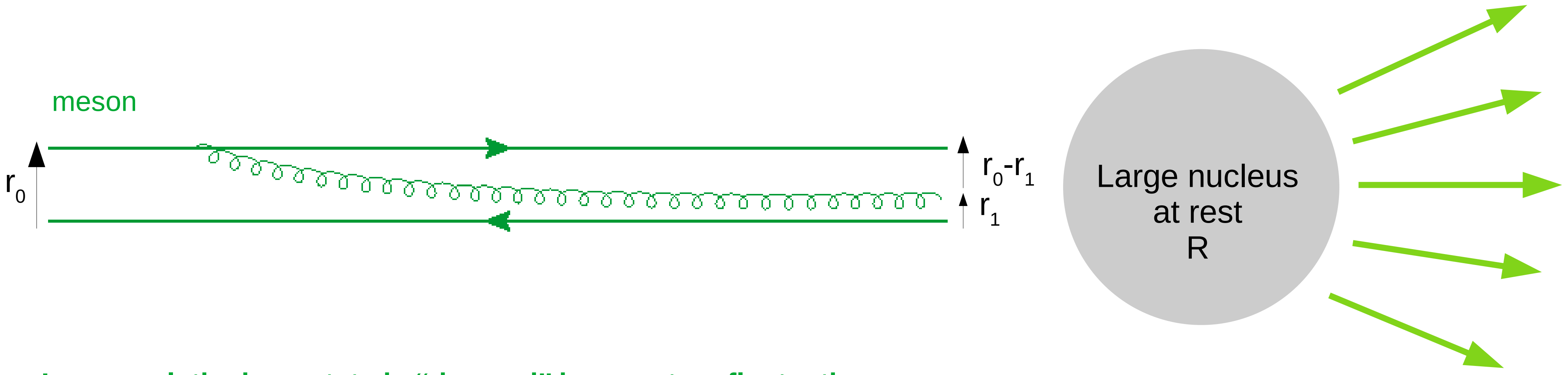

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The probability of a branching is computed from QCD:

$$\bar{\alpha}\, dy \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2}$$

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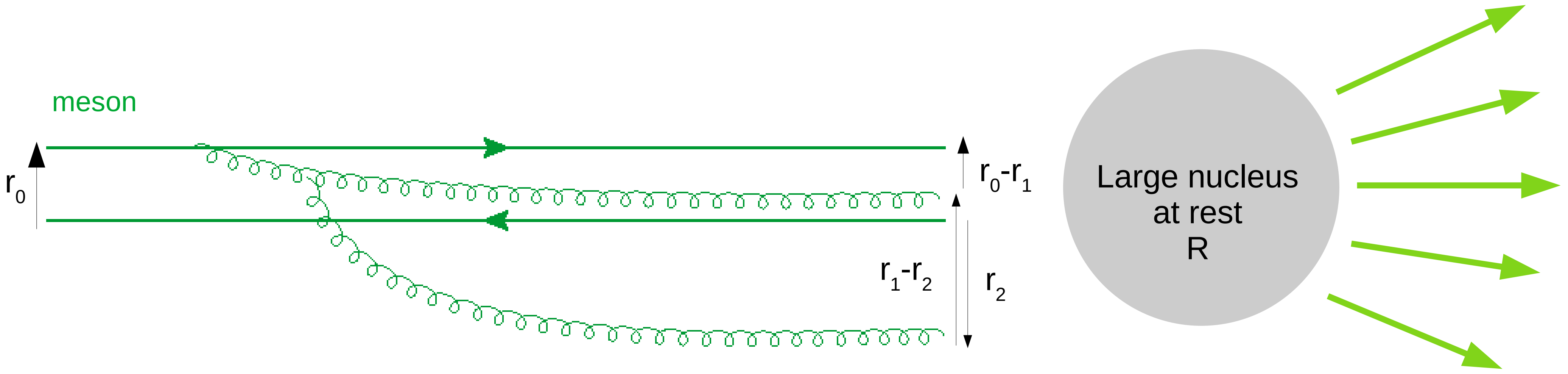

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T = 1 if at least one size > R

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The gluons may also branch into other gluons with the same probability function

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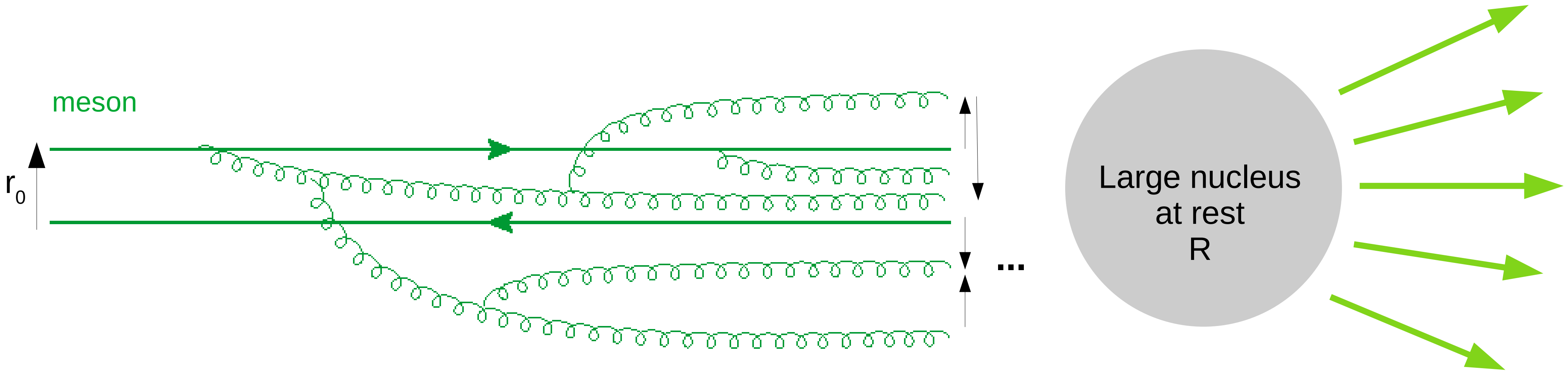

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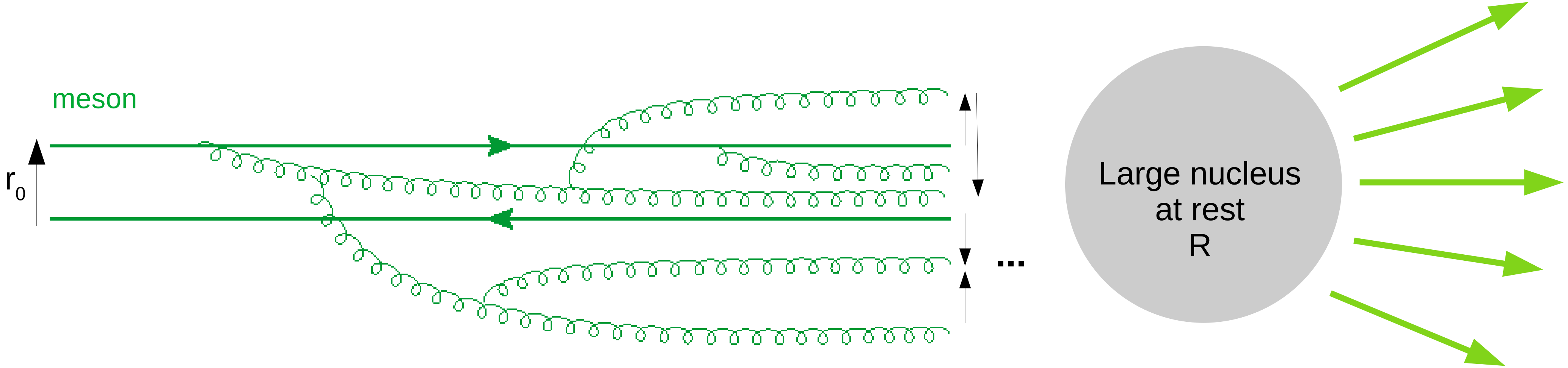

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The Balitsky-Kovchegov equation

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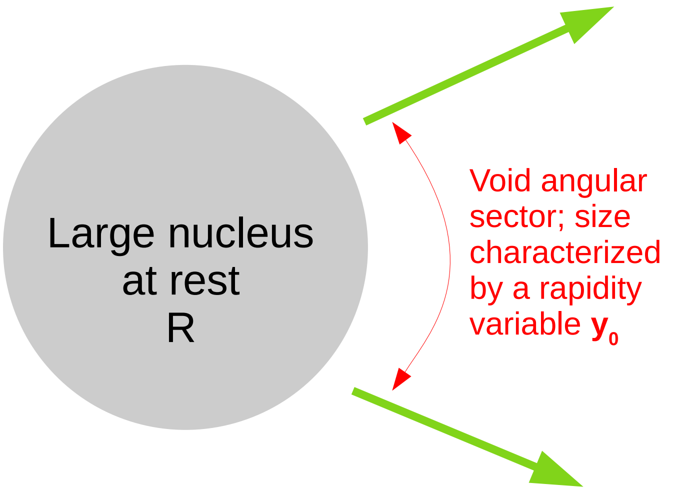
is now understood as an equation for the probability that there is at least one object of size larger than R produced by the branching process iterated to rapidity y.

A genealogy problem in high-energy scattering



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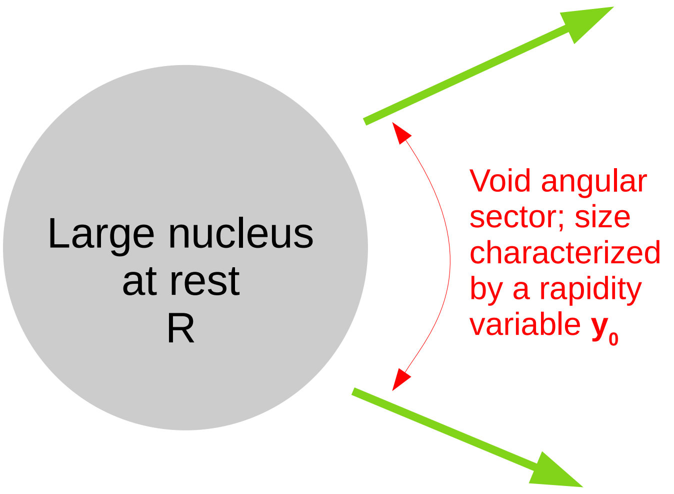

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Equation established in QCD for the distribution of this angle (Kovchegov, Levin, 2001):

Define T_{in} as follows: for $y=y_0$, $T_{in}(r_0,y_0;y_0)=2T(r_0,y_0)-T^2(t_0,y_0)$ and for $y>y_0$:

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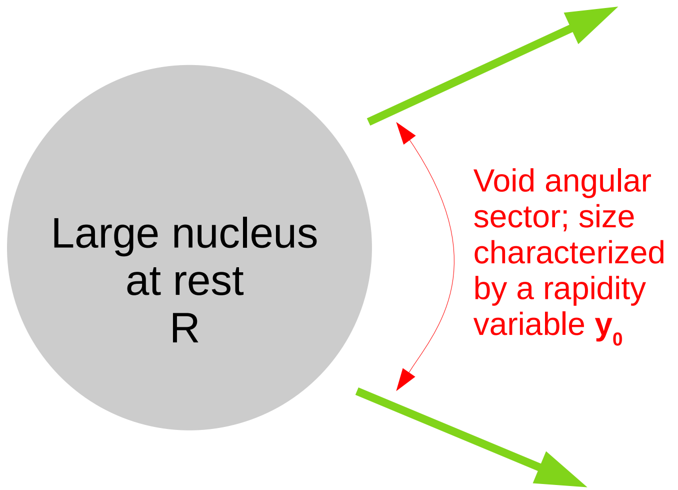

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<u>Interpretation</u>: T_{in} turns out to be twice the probability that the objects of size larger than R at rapidity y had an odd number of ancestors at rapidity Y-y_0

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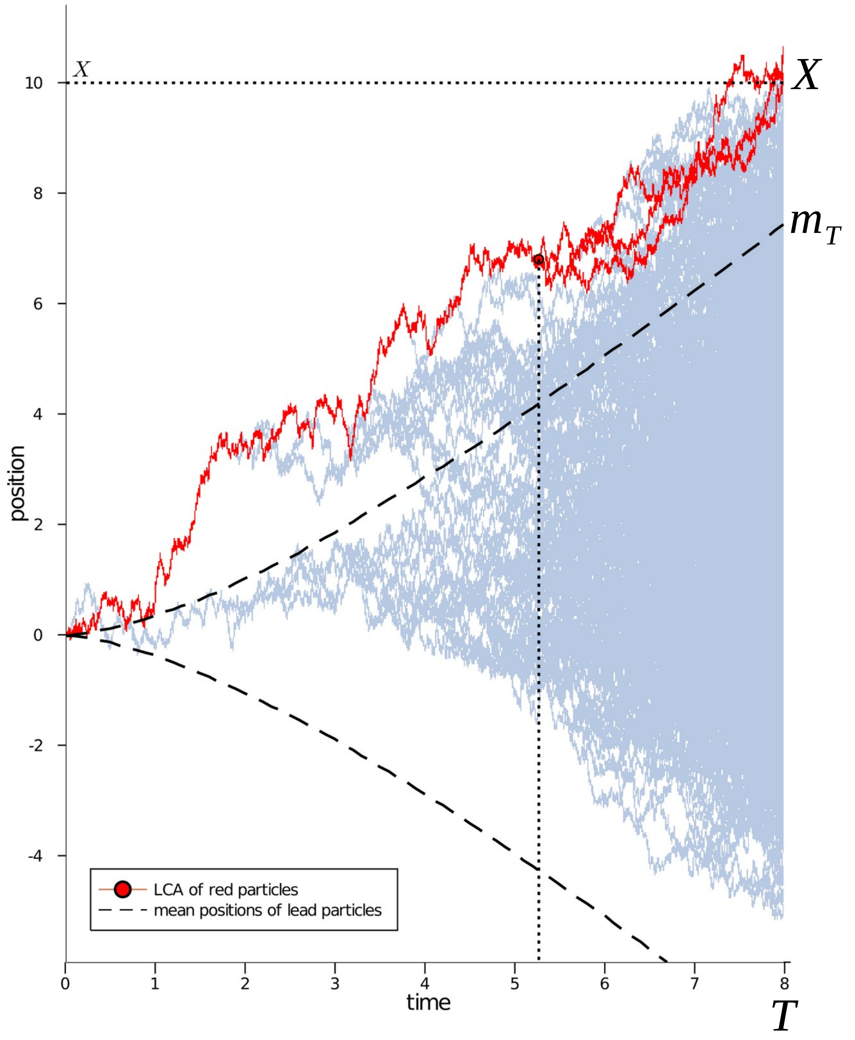
A class of observables on branching processes



We consider realizations of the one-dimensional BBM such that the initial particle at position x=0 **has its rightmost offspring at a position larger than *X* (i.e. it is "red", by definition)** at the final time *T*.

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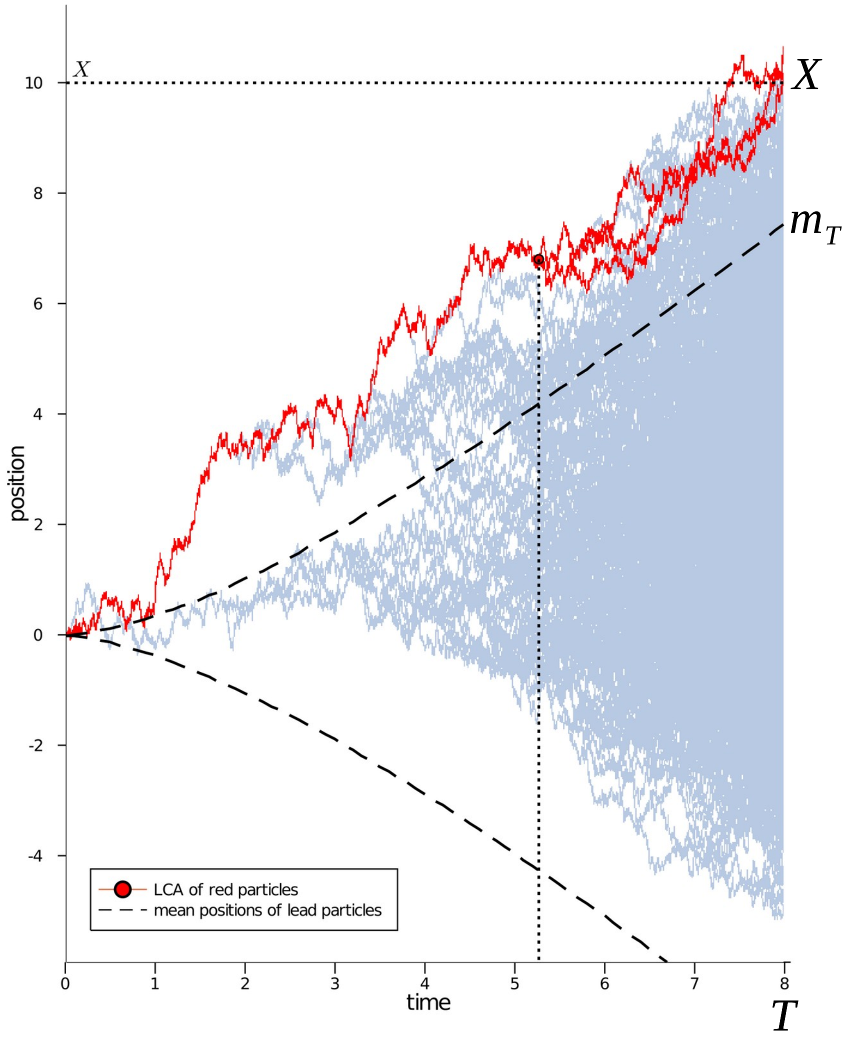

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Expected position of the lead particle at *T*

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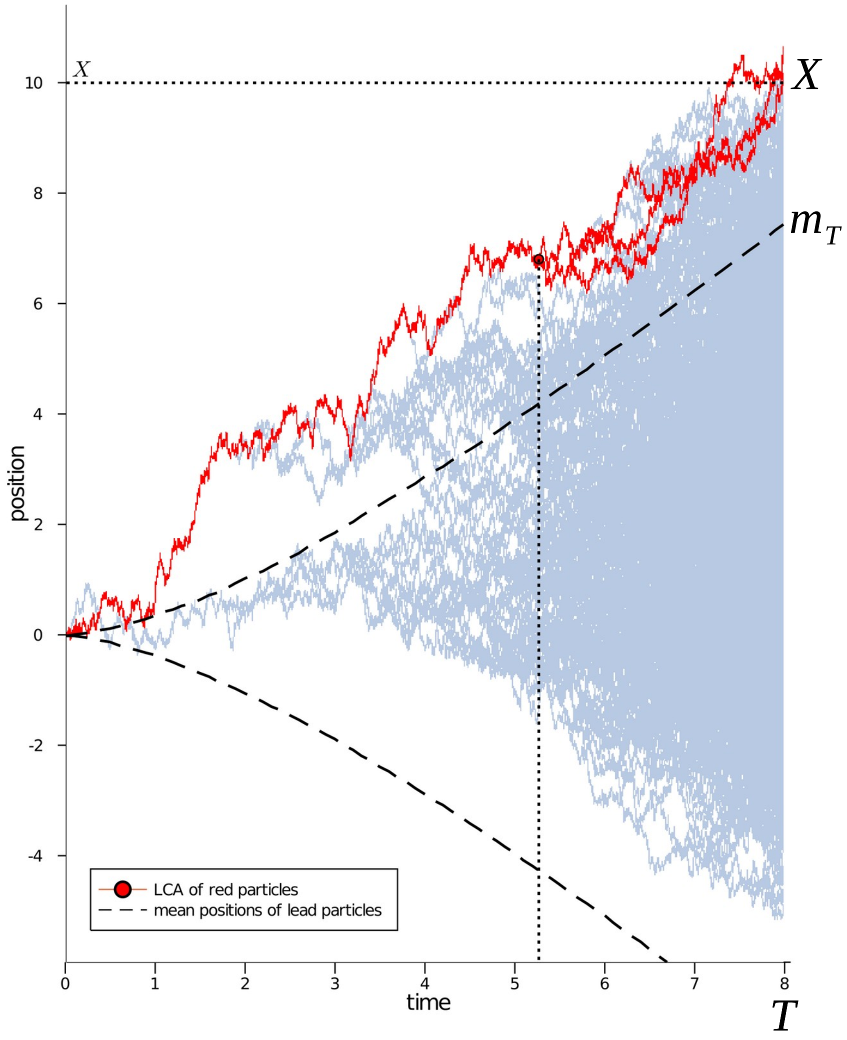

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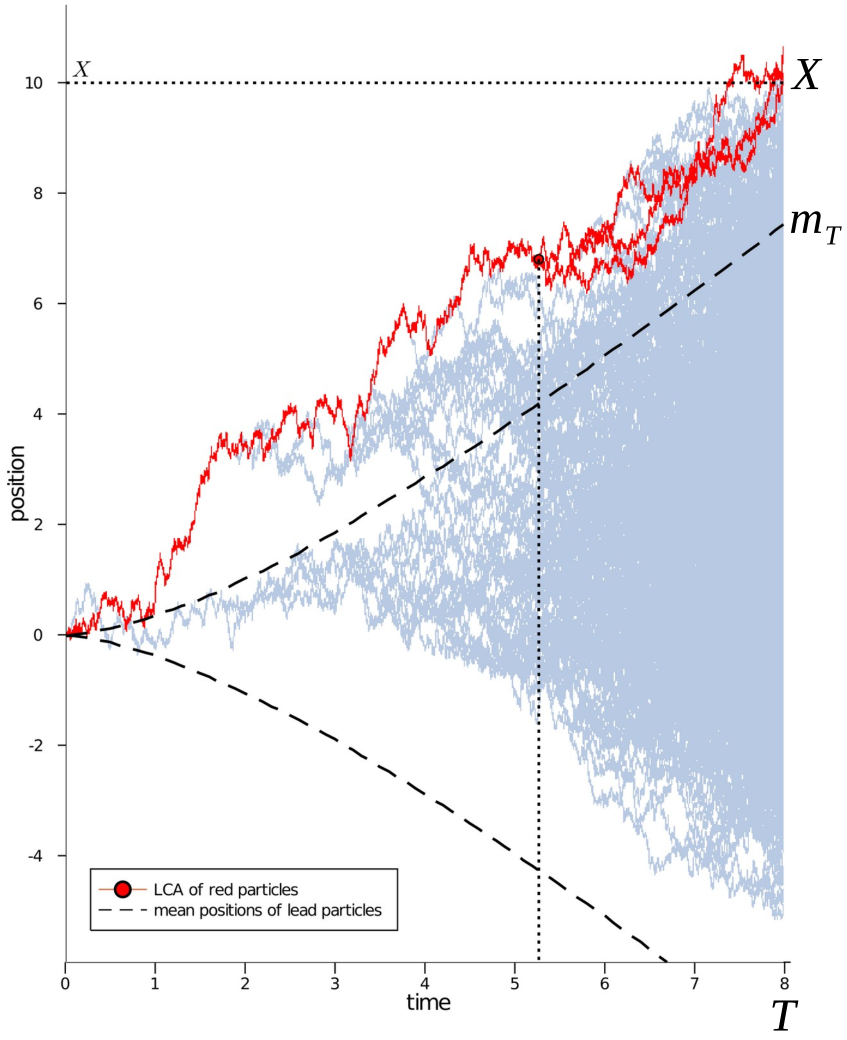

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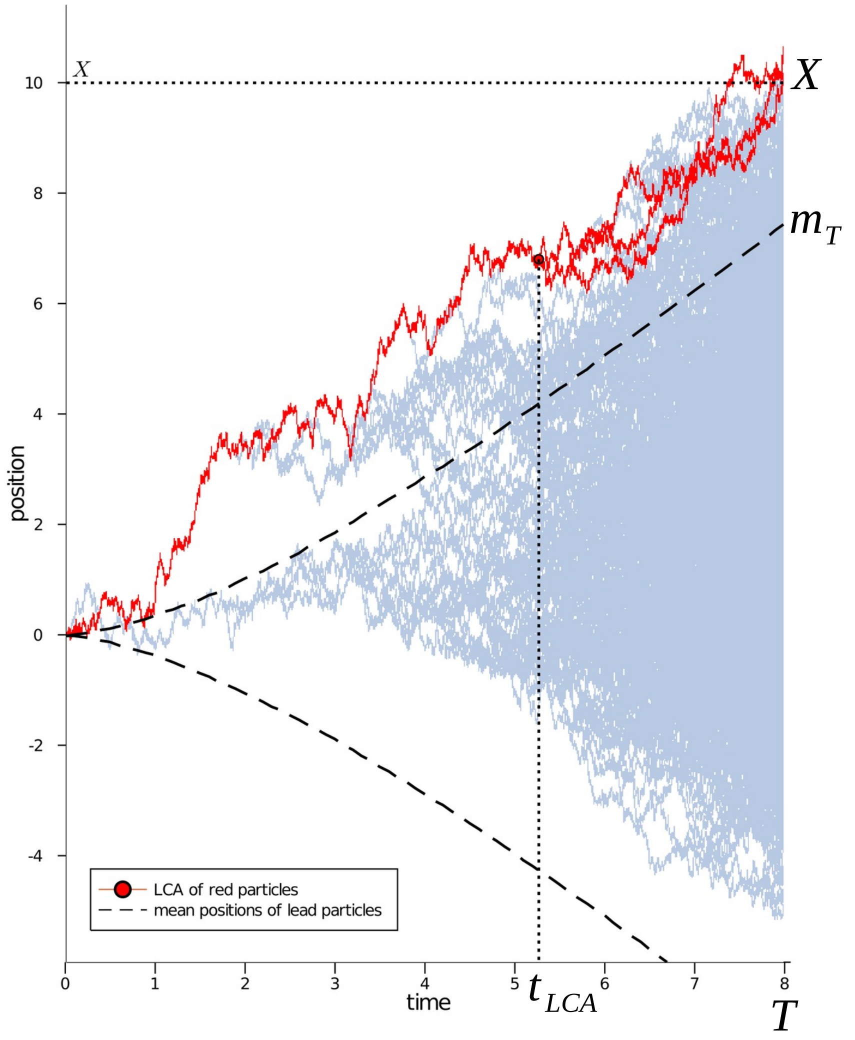
Our BBM is such that the proba *u(t,x)* that the lead particle has position larger than *x* at time *t* obeys the FKPP equation in the form:

$$\partial_t u(t,x) = \frac{1}{2}\partial_x^2 u(t,x) + u(t,x)[1-u(t,x)]$$

$$u(t=0,x) = \Theta(-x)$$

Genealogies: two formulas



Distribution of the branching time of the last common ancestor of all red particles

Probability density of the overlap $q = \frac{t_{LCA}}{T}$

$$\pi(q) \simeq \frac{1}{2\sqrt{\pi T}} \frac{1}{q^{3/2}(1-q)^{3/2}}$$

Le, Mueller, SM, Phys.Rev. D **103** (2021) 054031

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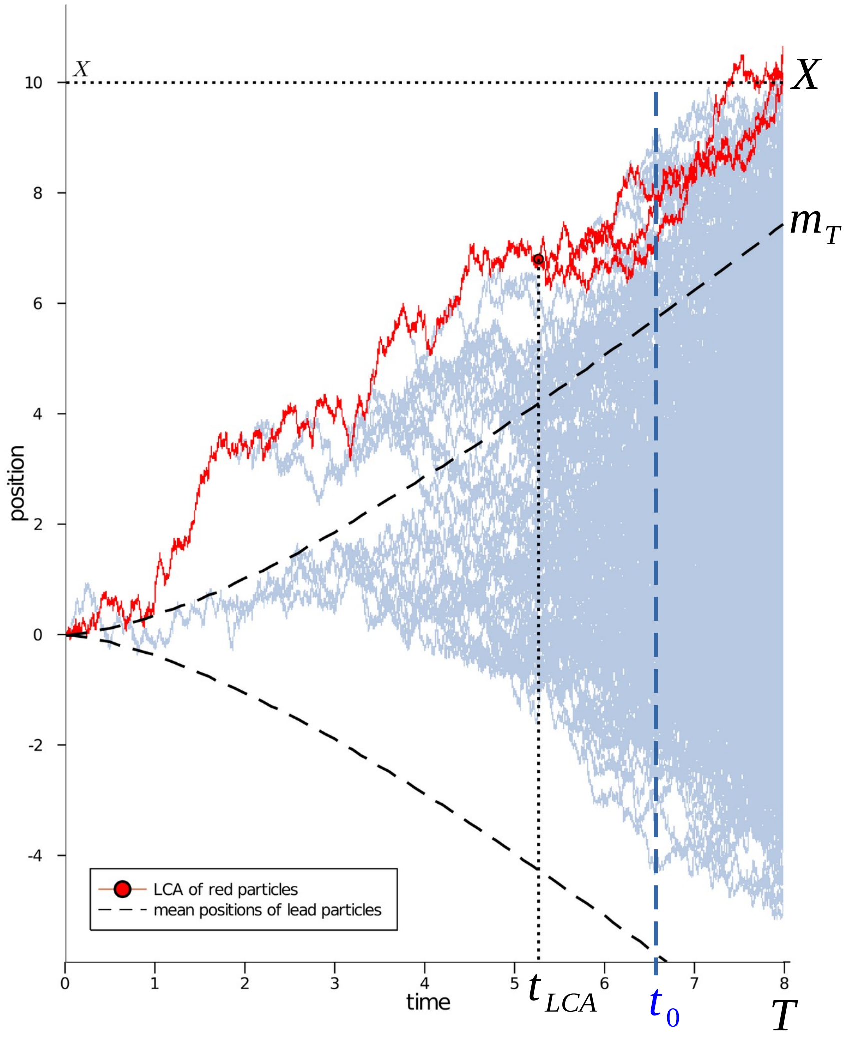

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Distribution of the number of red particles at a given time

Probability to have *k* red particles at t_0 given there is at least one:

$$r_{k\geq 2}(t_0) \simeq \left(\frac{1}{\sqrt{\pi(T-t_0)}} + \frac{1}{\sqrt{2}(X-m_T)} \right) \frac{1}{k(k-1)}$$

Le, Mueller, SM, Phys.Rev. D **104** (2021) 034026

Formulation using a generating function

Probability that a particle at *x* at time *t* is red: $U(t,x)=u(T-t,X-x)$

Call $Q_k(t,x;t_0)$ the probability that the particle at *x* at time *t* has *k* red offspring at time t_0:

$$r_k(t_0)=\frac{Q_k(0,0;t_0)}{U(0,0)}$$

Generating function: $U_\lambda(t,x;t_0)=1-\sum_k \lambda^k Q_k(t,x;t_0)$

Fact: $v(t,x)=U_\lambda(t_0-t,x;t_0)$ obeys the FKPP equation with initial condition $v(0,x)=(1-\lambda)U(t_0,x)$

Hence our observables may be deduced from a solution to the FKPP equation with peculiar initial conditions. It amounts to computing a shift of the large-time position of the TW due to the initial conditions.

There is also a probabilistic method: it requires however to use the phenomenological model for BBM [see Brunet, Derrida, Mueller, SM (2005); Mueller, SM (2014)]

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But our observables use ensembles of rare realizations, evolved to large times… a "naive" implementation would clearly be unuseful.

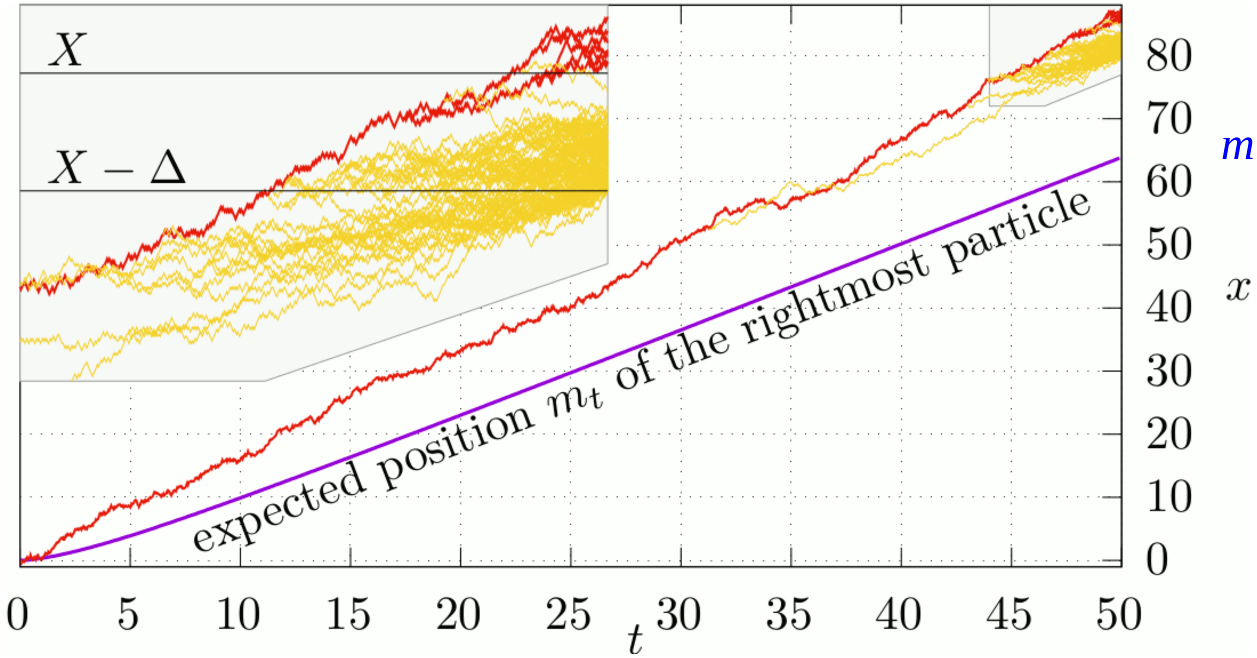
On the other hand, we only care about the particles that arrive near the extremal one.
Is there a way to evolve exactly *only these particles*?

Realization of the tail of a BBM

(actually BRW)

Conditioning: at least one particle beyond X (far from m_T)

Keep all particles arriving in [X-Δ,+∞)



T=50

We can go to much larger times! [Currently, T=O(10000)]

Brunet, Le, Mueller, SM, EPL, **131** (2020) 40002

A simple branching random walk

Consider a set of particles on a lattice in space and time, with respective spacing *dx* and *dt*

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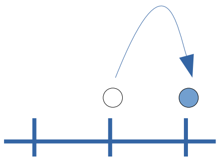
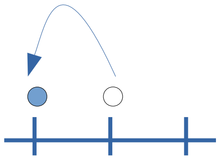
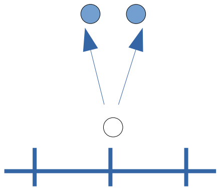
Each particle evolves in time through 3 elementary processes:



Probability p_r



Probability p_l



Probability r

$$p_r + p_l + r = 1$$

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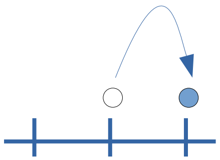

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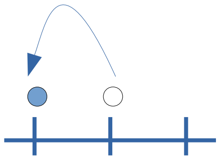

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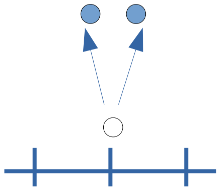

Probability r

$$p_r + p_l + r = 1$$

Start with a single particle at the origin.

The probability that the rightmost particle has a position larger than *x* at time *t* satisfies

$$u(t+dt,x) = p_r u(t,x-dx) + p_l u(t,x+dx) + r\,u(t,x)[2-u(t,x)] \qquad u(t=0,x) = \Theta(-x)$$

The probability that the particle at *x* at time *t* is red reads $U(t,x) = u(T-t, X-x)$

Algorithm

Evolution of the red particles

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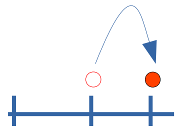

<u>branch:</u>

Algorithm
Evolution of the red particles

Consider generically one particle at position *x* at time *t*.

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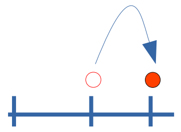

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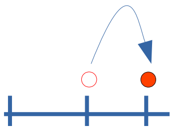

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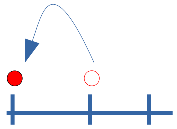

branch:

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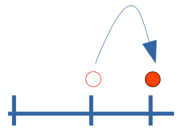

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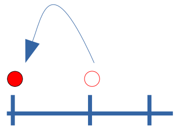
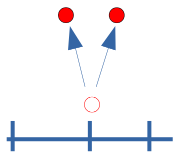


branch:

- Proba that it branches into **two red** *given* that it is red: $P(2\ \text{red}\,|\,\text{red}) = r \dfrac{[U(t+dt\,,x)]^2}{U(t\,,x)}$



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  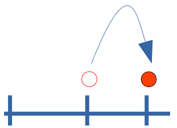

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  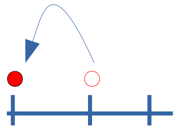

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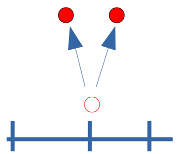
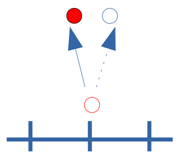
  

- Proba that it branches into **one red** and **one non-red** *given* that it is red:

$$P(\text{red+non-red}\,|\,\text{red}) = r \frac{2U(t+dt\,,x)[1-U(t+dt\,,x)]}{U(t\,,x)}$$



In this case, the nothing actually happens to the red particle.

Algorithm

Evolution of the orange particles

<u>Definitions:</u> **A particle is orange if it has its rightmost offspring in $[X\text{-}\Delta, X)$ at time T.**

A particle is blue if it has its rightmost offspring in $(-\infty, X\text{-}\Delta)$ at time T.

Algorithm

Evolution of the orange particles

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The probability that the particle at *x* at time *t* is orange reads $V_\Delta(t,x) = U(t,x+\Delta) - U(t,x)$

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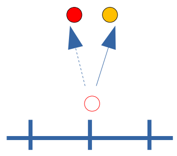
Orange particles are created from branching of red particles

$$P(\text{red+orange} \mid \text{red}) = r\,\frac{2U(t+dt,x)V_\Delta(t+dt,x)}{U(t,x)}$$



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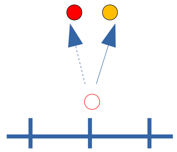
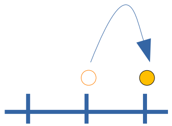
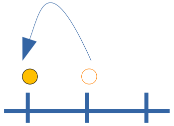


$$P(\text{red+orange | red})=r\frac{2U(t+dt,x)V_\Delta(t+dt,x)}{U(t,x)}$$

<u>Moves:</u> Proba that it jumps **right or left** *given* that it is or left:

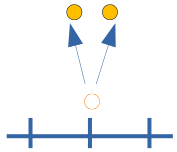
the same as for the red particles, with the substitution $U \to V_\Delta$



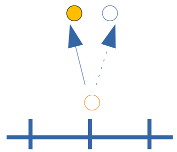


<u>Branchings:</u>

- Proba that it branches in **two orange** *given* that it is orange:
  The same as for the red particles, with the substitution $U \to V_\Delta$



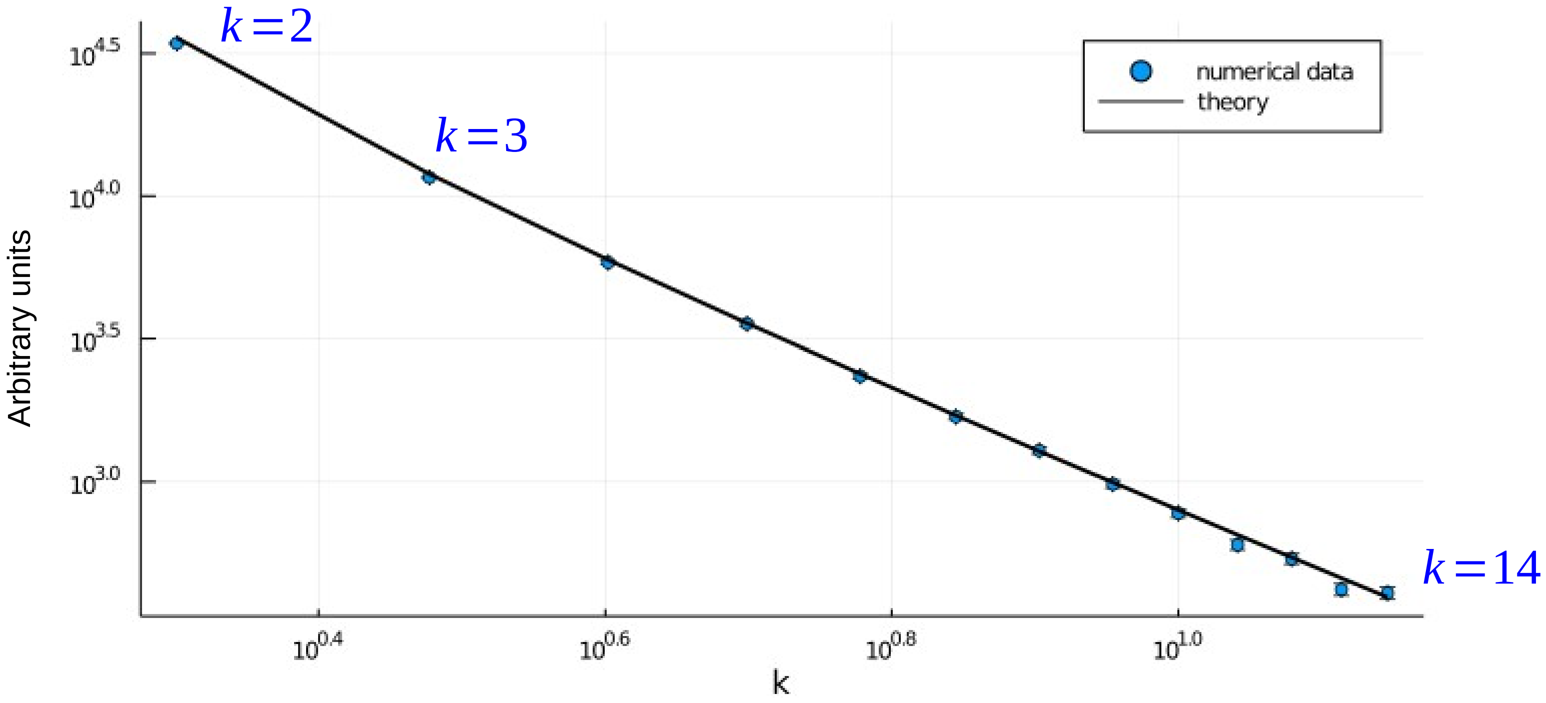
- Proba that it branches in **one orange** and **one blue** *given* that it is orange:



$$P(\text{orange+blue | orange})=r\frac{2V_\Delta(t+dt,x)[1-U(t+dt,x+\Delta)]}{V_\Delta(t,x)}$$

Numerical check of the formula for r_k

$$r_{k\geq 2}(t_0) \simeq \left(\frac{1}{\sqrt{\pi(T-t_0)}} + \frac{1}{\sqrt{2}(X-m_T)} \right) \frac{1}{k(k-1)}$$



$$T=4000,\quad X-m_T=30,\quad T-t_0=200$$

Continuous limit and variants

$$p_r = p_l = \frac{1}{2}(1 - dt), \quad r = dt, \quad dx^2 = dt, \quad dt \to 0$$

$$\partial_t u(t,x) = \frac{1}{2}\partial_x^2 u(t,x) + u(t,x)[1 - u(t,x)]$$

We find that the subtree of red particles is a (unconditioned) BBM with drift $\partial_x \ln U(t,x)$

and branching rate $U(t,x)$

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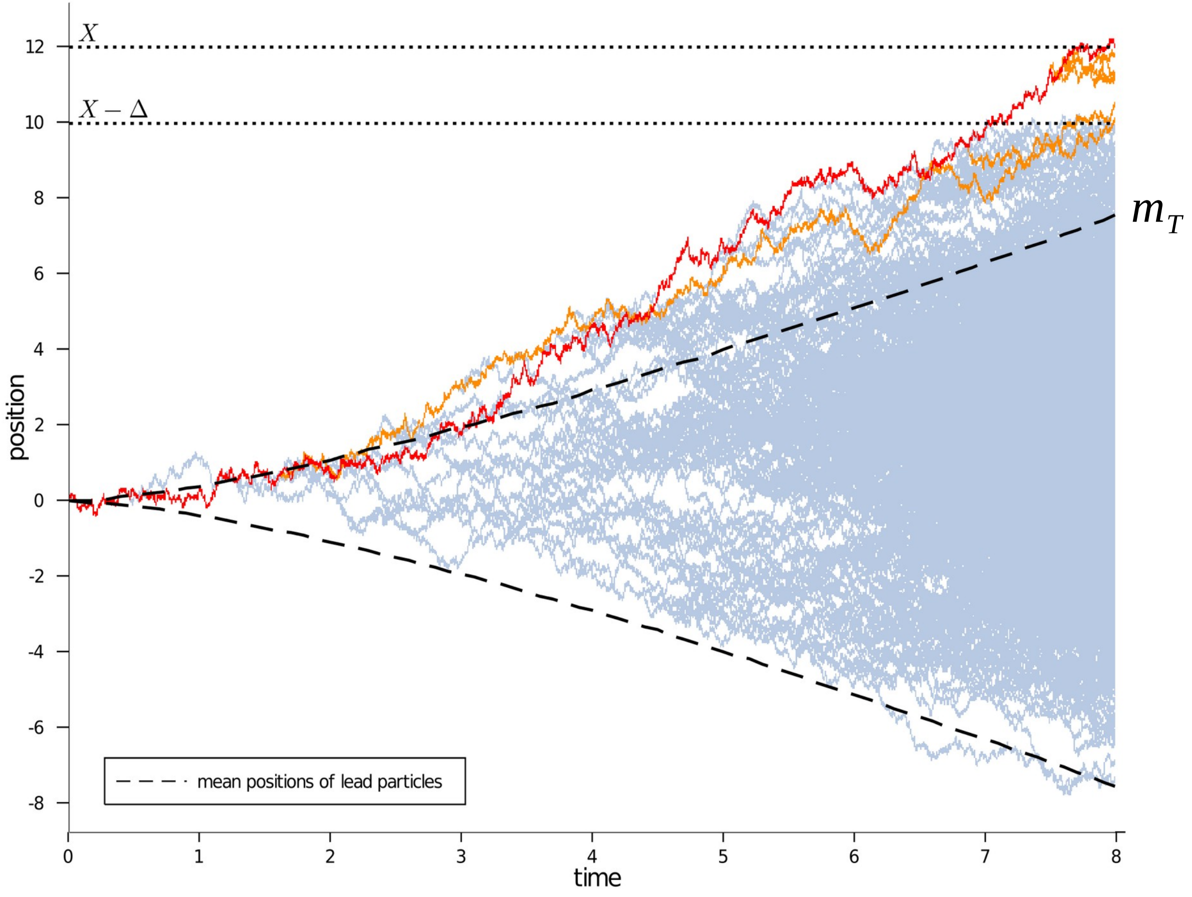
We may ask the lead particle at time *T* to be at *X* ***exactly***.

Then the drift becomes $\partial_x \ln \partial_x U(t,x)$

The trajectory of the red particle probably coincides with the *spine.*

Other uses of the algorithm: one example



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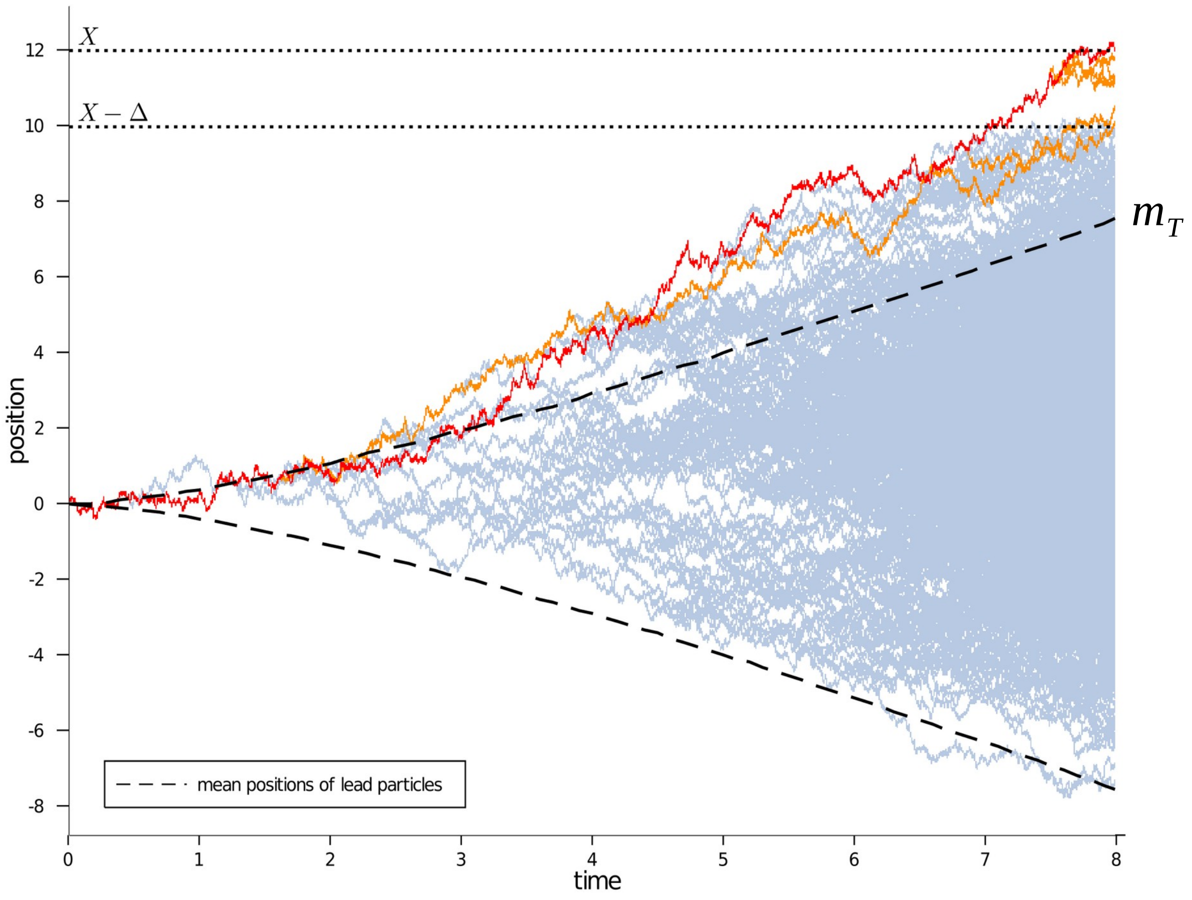

Number of orange particles when the position of the lead particle is fixed

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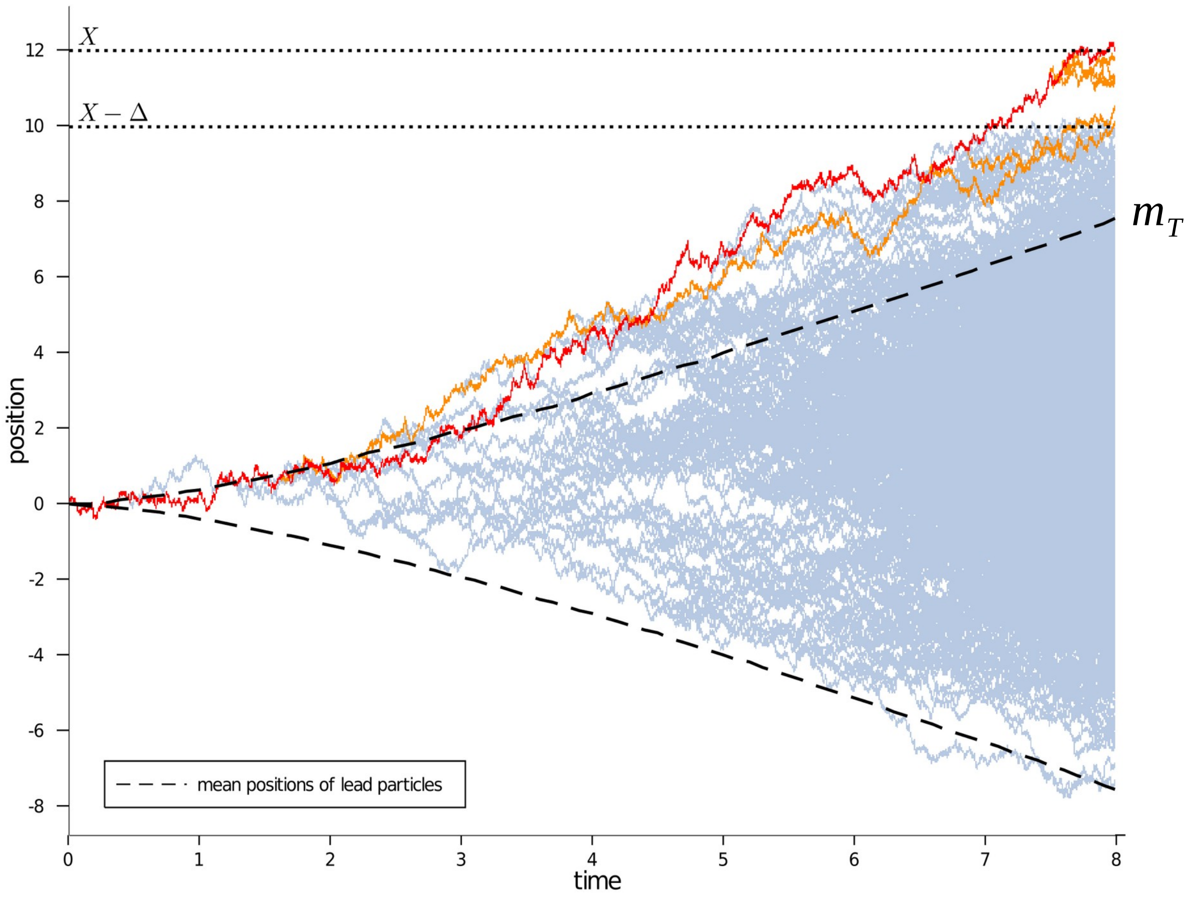

Number of orange particles when the position of the lead particle is fixed (NB: for the BBM)

$$\bar{n}(\Delta) \propto e^{\sqrt{2}\Delta}$$

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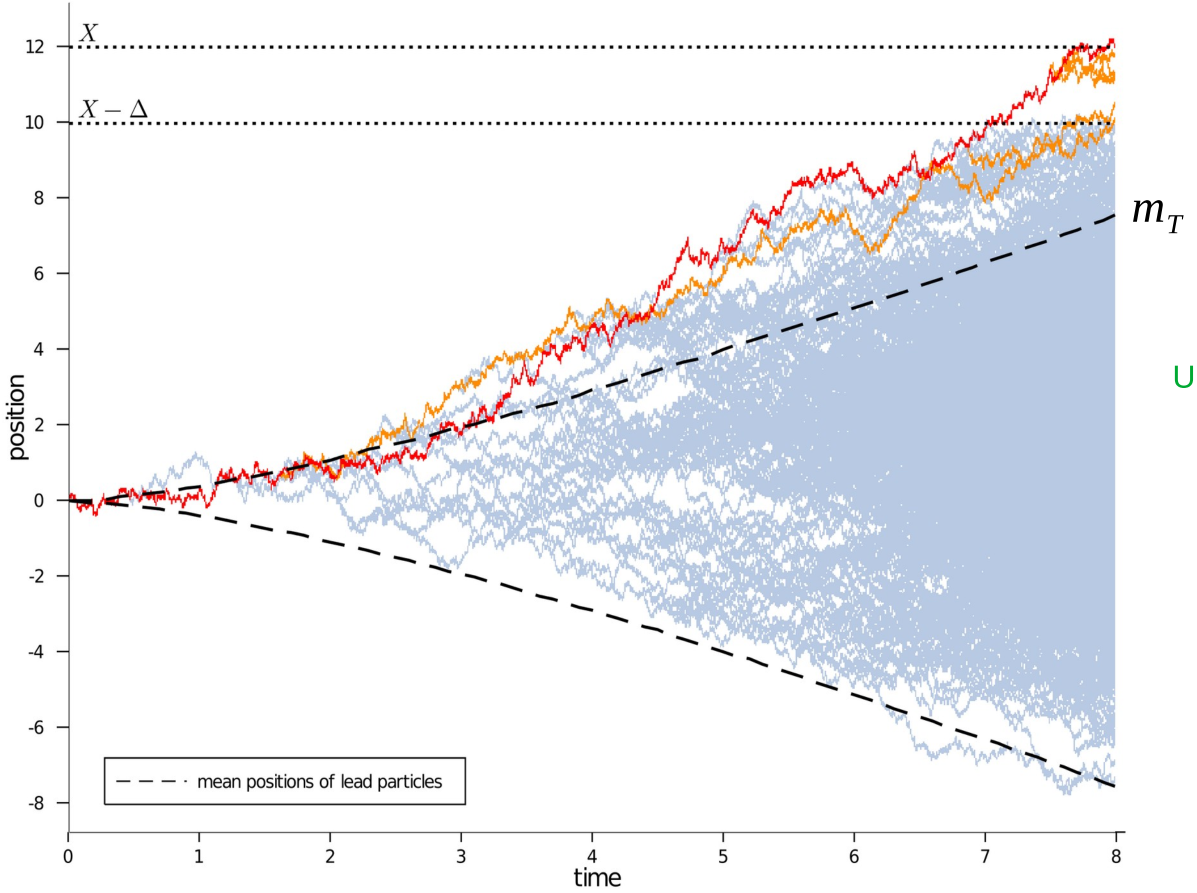

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$$\bar{n}(\Delta) \propto e^{\sqrt{2}\Delta}$$

$$n_{typical}(\Delta) \propto \exp\left(\sqrt{2}\Delta - \zeta\Delta^{2/3}\right)$$

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$$1 \ll X - m_T \ll \sqrt{T}$$

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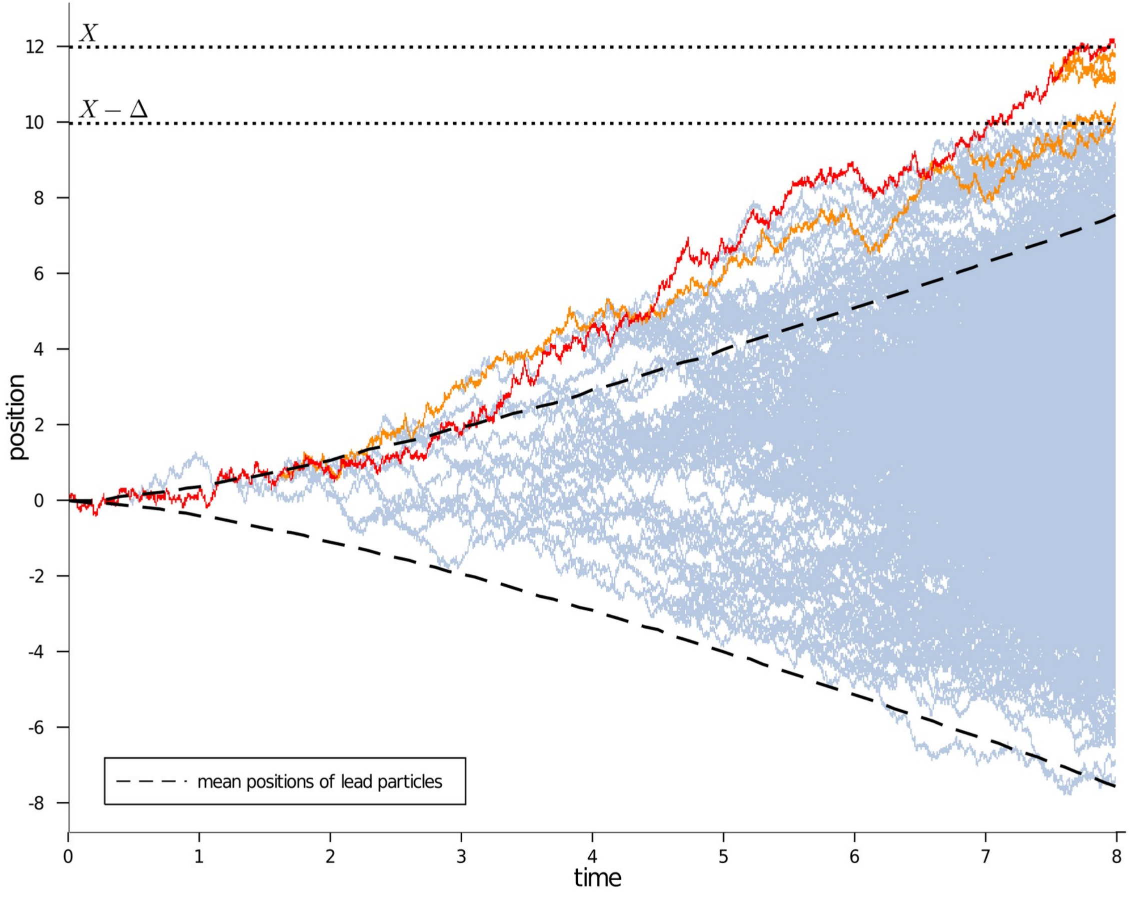

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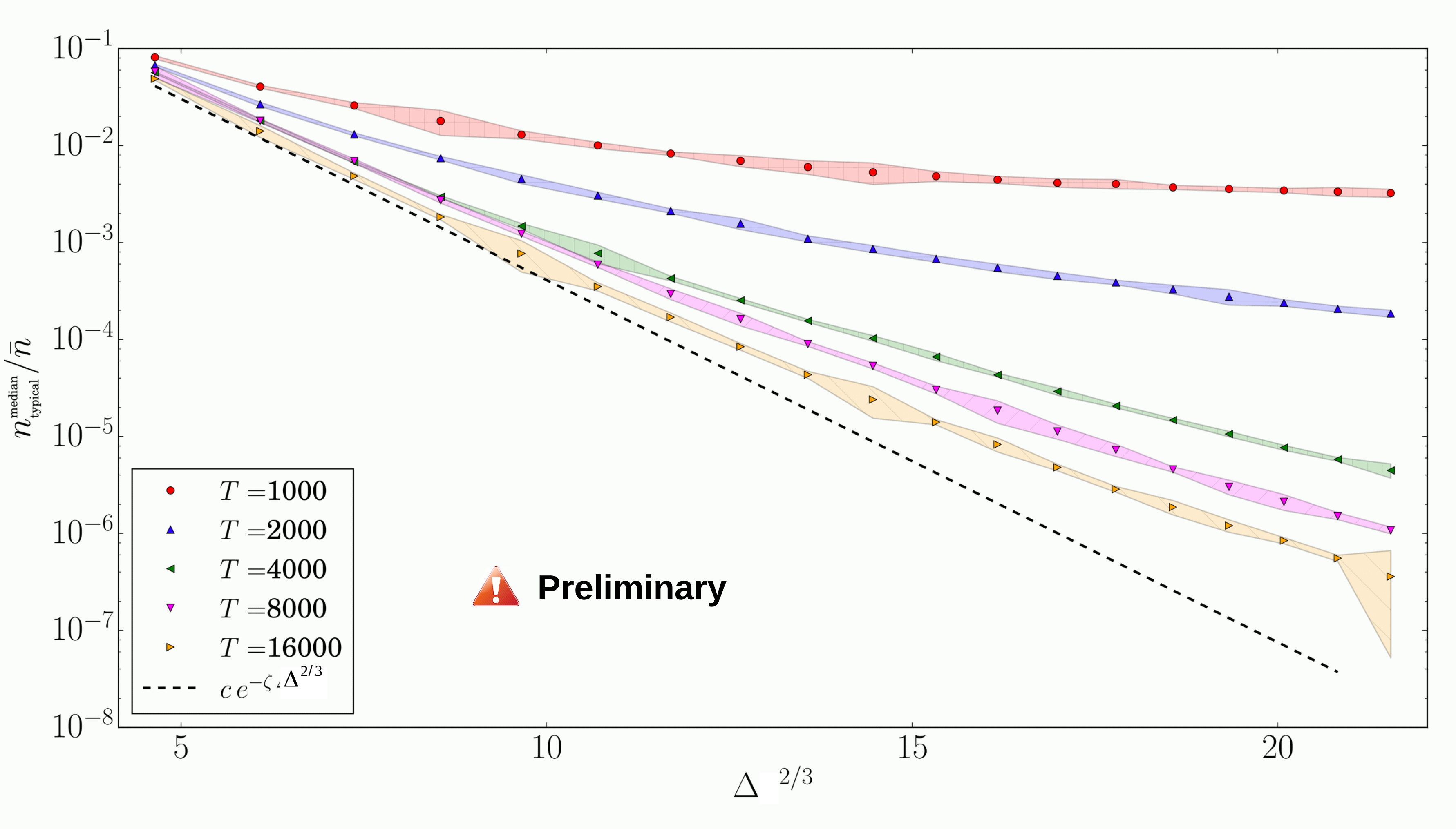
Mueller & SM, Phys.Rev. E **102** (2020) 022104

Formulas obtained from a very, very laborious calculation based on a generating function (Mueller, SM, Phys.Rev. E, 2020), following a formulation of such problems due to Brunet & Derrida (2011).

<u>NB</u>: This formula can be recovered from a probabilistic picture, in a more straightforward way… maybe probabilists will be able to determine the unknown constant?

Numerical check of $\frac{n_{typical}}{\bar{n}} \sim e^{-\zeta \Delta^{2/3}}$



Summary

- Branching processes are found in the formulation of hadronic scattering observables at very high energies. **“FKPP math” is relevant in this context**.

- We have proposed **heuristic** expressions for **properties of particles near the tip of a BRW,** and an exact **Monte Carlo algorithm to generate the tip**.

Summary

- Branching processes are found in the formulation of hadronic scattering observables at very high energies. **“FKPP math” is relevant in this context**.
- We have proposed **heuristic** expressions for **properties of particles near the tip of a BRW,** and an exact **Monte Carlo algorithm to generate the tip**.

Outlook

- Try to understand the full genealogy of the subtree of red particles?
- Understand more completely the density of particles at the tip, possibly in the stochastic picture, which is more “physical”?