Study of saturation physics in hard processes

PhD thesis under the supervision of Samuel Wallon and Lech Szymanowski

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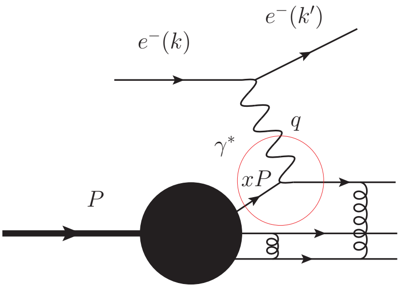
Outline

1. The example of DIS
   - Theoretical tools
   - DGLAP evolution equations
   - Phase-space diagram for parton evolution in DIS
2. Shockwave formalism
   - Introducing the formalism
   - Current project: hadron production with large $p_\perp$

DIS and factorisation theorem



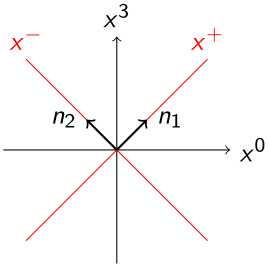
- e^- source of photon γ^* that probes the internal structure of the proton
- Kinematic invariants

$$Q^2 = -q^2 \gg \Lambda^2_{QCD}$$
$$s = (P+q)^2$$
$$x = \frac{Q^2}{2p \cdot q}$$

- Q^2 is the hard scale $\Rightarrow \alpha_S(Q^2) \ll 1$ and DIS is a hard process separation short-distance process (parton-γ^* scattering) from the long-distance non-perturbative process, describing the partonic content of the hadron
  $\rightarrow$ cross-section/scattering amplitude = convolution of a hard coefficient and non-perturbative function
- $r_\perp \sim 1/Q$: transverse resolution of the probe $\sim$ size of parton

Light cone variable

$$x^\mu = (x^0, x^1, x^2, x^3)$$
$$\to x^\mu = (x^+, x^-, x_\perp)$$
$$x^\pm = \frac{x^0 \mp x^3}{\sqrt{2}}, x_\perp = (x^1, x^2)$$



Light-cone vectors $n_1^\mu = (1, 0, 0_\perp)$ and $n_2^\mu = (0, 1, 0_\perp)$ that define the +/- directions and

$$x^\mu = x^+ n_1^\mu + x^- n_2^\mu + x_\perp$$
$$x \cdot y = x^+ y^- + x^- y^+ + x_\perp \cdot y_\perp = x^+ y^- + x^- y^+ - \vec{x} \cdot \vec{y}$$

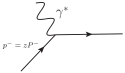
Boost along +z simplified to a simple exponential:

$$x'^0 = \gamma(x^0 + \beta x^3), \ x'^3 = \gamma(\beta x^0 + x^3)$$
$$\to x'^+ = e^\omega x^+, \ x'^- = e^{-\omega}, \ x'_\perp = x_\perp$$

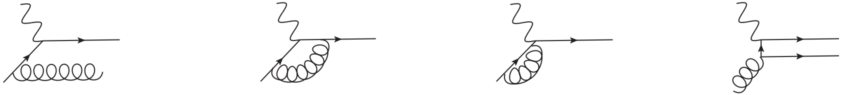
with $\omega = \frac{1}{2} \ln \frac{1+\beta}{1-\beta}$ the rapidity

DGLAP equations

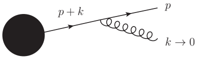
- Bjorken limit/ Moderate-x region: $Q^2 \sim s$
- Partonic content encoded into PDF (Parton distribution functions) $q_f(z, Q^2) \sim$ probability of extracting a quark with a fraction of the proton longitudinal momentum equal to z and at scale Q^2.
- LO: Naive parton model



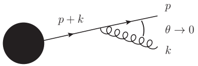
- NLO (order α_s): add QCD corrections to the naive picture $\Rightarrow$ Divergences



- Soft divergences cancel between real and virtual corrections



- Collinear divergences absorbed into bare divergent PDF



$\Rightarrow$ Left with large $\ln(Q^2)$ to resum to all orders in perturbation theory as $\alpha_S \ln(Q^2) \sim 1$

DGLAP equations

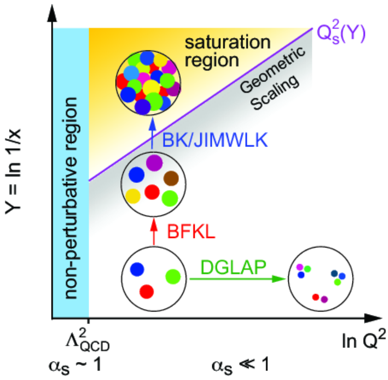
The resummation of the large $\ln Q^2$ leads to PDF that are scale dependant as encoded in the DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi) equations:

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q_f(z,Q^2) \\ G(z,Q^2) \end{pmatrix} = \int_z^1 \frac{\mathrm{d}\beta}{\beta}\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}\left(\frac{z}{\beta}\right)\begin{pmatrix} q_f(\beta,Q^2) \\ G(\beta,Q^2) \end{pmatrix}$$

P_{ji} are the splitting functions $\sim$ probability of parton i to radiate into j
$q_f(z,Q^2)$, $G(z,Q^2)$ are the quark and gluon PDF.

The number of partons increases with Q^2. But their sizes $\sim 1/Q$ decrease.
$\Rightarrow$ proton $=$ dilute system of almost point-like partons.

Phase-space diagram for parton evolution in DIS



Source: arXiv:1212.3482 [hep-ph]

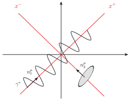
- Bjorken limit/moderate-x region ($Q^2 \sim s$)
  DGLAP : Resummation of ln Q^2
- Regge-Gribov limit/small-x region :
  $Q^2 \ll s \leftrightarrow x \sim Q^2/s \to 0$
  Resummation of $\ln(1/x)$ via BFKL and then BK/JIMWLK, non-linear extension of BFKL (saturation physics).
- $Q_s(x, A)$: saturation scale.
  Simple estimation :
  $\frac{(xG_A(x,Q_s^2)/Q_s^2)}{\pi R_A^2} \sim 1 \Rightarrow Q_s^2 \propto A^{1/3}x^{-0.3}$
  as

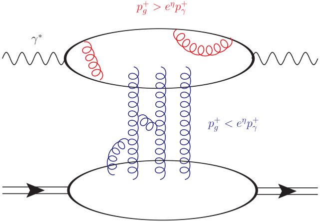
$$R_A \sim A^{1/3}$$

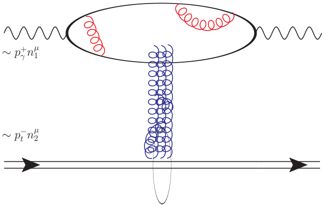
$$G_A(x, Q^2) \sim AG_p(x, Q^) \sim Ax^{-0.3}$$

Shockwave approximation

High-energy limit: $s = (p_\gamma + p_t)^2 \gg |p_\gamma|^2, M_t^2, |p'_{00}|^2$







- Separation of gluons in the Lagrangian into "fast gluons" and "slow gluons" with a cut-off defined by an arbitrary rapidity parameter $\eta < 0$:

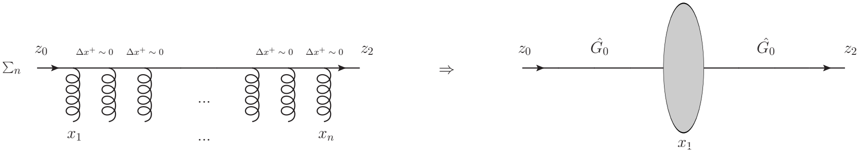
$$\mathcal{A}^\mu(p_g^+, p_g^-, \vec{p_g}) = A^\mu(p_g^+ > e^\eta p_\gamma^+, p_g^-, \vec{p_g}) + b_0^\mu(p_g{}^+ < e^\eta p_\gamma^+, p_g^-, \vec{p_g})$$

- Boost from target rest frame to a frame where $p_\gamma^+ \sim p_t^- \sim \sqrt{s}$

$$b_0^\mu(x^+, x^-, \vec{x}) \xrightarrow{\Lambda} b^\mu(x^+, x^-, \vec{x}) = b^-(x^+, \vec{x}) n_2^\mu = \delta(x^+) B(\vec{x}) n_2^\mu$$

with $\Lambda = e^\omega \sim \frac{\sqrt{s}}{M_t}$

Wilson line



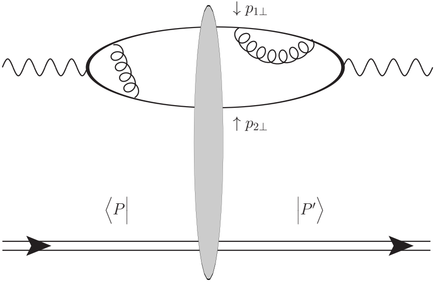
Resummation of interactions with external field $b_\mu(x^+, x^-, \vec{x})$ into a Wilson line :

$$\mathcal{U}(\vec{x_1}) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz^+ b^-\left(z^+, \vec{z}\right)\right]$$

$$\mathcal{U}(\vec{p}) = \int d^d\vec{z}\, e^{-i\vec{p}\cdot\vec{z}}\,\mathcal{U}(\vec{z})$$

Phase-space dimension in calculations: $D = 2 + d =$ with $d = 2 + 2\epsilon$, the transverse dimension.
Dimension necessary for dimensional regularization.

Factorization in the shock wave approximation

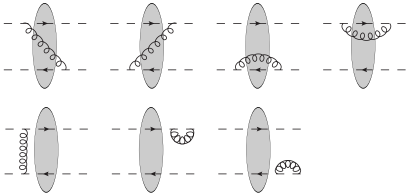


$$\mathcal{A}^\eta = \int \mathrm{d}^d p_{1\perp}\, \mathrm{d}^d p_{2\perp}\, \Phi^\eta(p_{1\perp}, p_{2\perp}) \left\langle P' \middle| \left[\mathrm{Tr}\left(\mathcal{U}_1^\eta \mathcal{U}_2^{\eta\dagger} \right) - N_c \right] (\vec{p_1}, \vec{p_2}) \middle| P \right\rangle$$

Dipole operator:

$$\mathcal{U}_{ij}^\eta = 1 - \frac{1}{N_c} \mathrm{Tr}\left(\mathcal{U}_{\vec{z}_i}^\eta \mathcal{U}_{\vec{z}_j}^{\eta\dagger} \right)$$

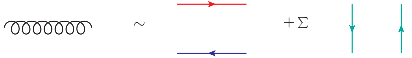
$$\tilde{\mathcal{U}}_{ij}^\eta(\vec{p_1}, \vec{p_2}) = \int_{-\infty}^{+\infty} d^d \vec{z_1}\, d^d \vec{z_2}\; e^{-i\vec{p_1}\cdot\vec{z_1}} e^{-i\vec{p_2}\cdot\vec{z_2}}\, \mathcal{U}_{ij}^\eta$$

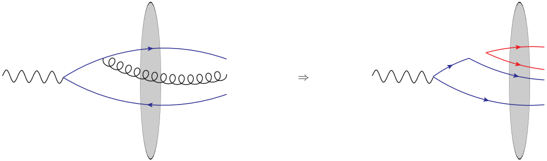


B-JIMWLK (Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner) equations:

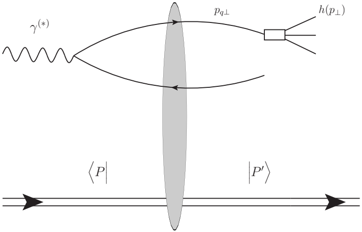
$$\frac{\partial \mathcal{U}^{\eta}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2\vec{z}_3 \left(\frac{\vec{z}^{\,2}_{12}}{\vec{z}^{\,2}_{23}\vec{z}^{\,2}_{31}} \right) \left[\mathcal{U}^{\eta}_{13} + \mathcal{U}^{\eta}_{32} - \mathcal{U}^{\eta}_{12} - \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32} \right]$$

Presence of a double dipole operator because gluons cross the shockwave.





Current project: hadron production with large $p_\perp$



Photon can be virtual (produced via electron) or real so from a heavy ion (case of interest).
$p_{h_\perp}$ is large, it is the hard scale of the process, necessary if the photon is real.
$\Rightarrow$ Factorisation theorem : photon impact factor and fragmentation function (FF).
Objective: Find the analytical expression for $\mathrm{d}\sigma/\mathrm{d}^2 p_{h\perp}$, which is exempt of all divergences.

- Rapidity divergences: eliminated via the B-JIMWLK evolution equations
- Soft divergences: cancelled between real and virtual corrections of the impact factors
- Collinear divergences, cancelled with the evolution equation of FF.

Thank you for your attention !

Backup slides

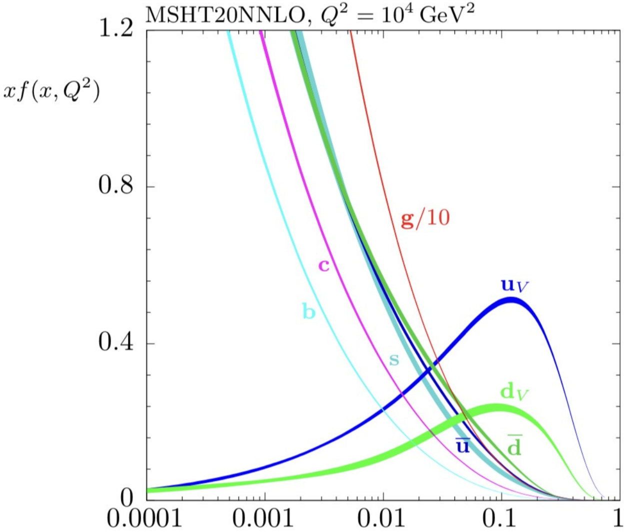
Dipole picture in the high energy limit

Supposing that the scattering is studied in the target rest frame. The axis can be chosen in a way that $q^\mu = \left(q^+, -Q^2/q^+, 0_\perp\right)$, with $q^+ \sim s/M_t$. Using the Heisenberg inequality adapted to the light-cone language,

$$x^+ \sim \frac{1}{|q^-|} \sim \frac{q^+}{Q^2} \gg R_A$$

If the photon fluctuates into a colorless dipole, meaning a pair of quark-antiquark, this dipole has a lifetime much larger than the target size. Hence, it is the dipole that will interact with the target.
Also the dipole transverse size does not change with the interaction with the target.

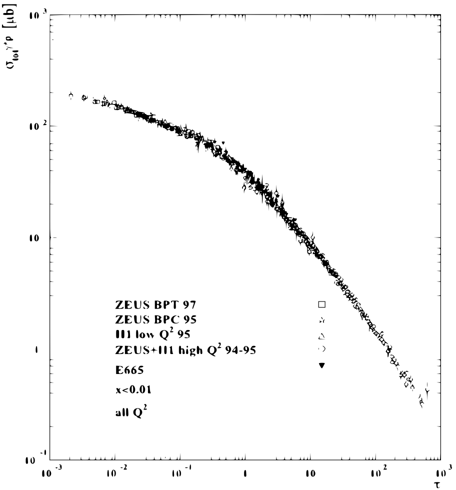
PDF fits



Source: arXiv:2012.04684 [hep-ph]

Hint of saturation at HERA

A hint that saturation physics is a valid theory in the high-energy limit is the observation of geometric scaling by Stasto, Golec-Biernat and Kwiecindki (2001). Using saturation model known at the time applied for DIS, they predicted that the total cross-section for $\gamma^* p$ at small-x only depend on a dimensionless variable $\tau = Q^2 R_0^2(x)$, with $R_0^2(x)$ the saturation radius. This prediction was observed in HERA data.



Source: arXiv:hep-ph/0007192

BK equation

Mean field approximation or large-N_c limit (all gluons become a pair quark-antiquark):

$$\left\langle \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}\right\rangle \to \left\langle \mathcal{U}_{13}^{\eta}\right\rangle \left\langle \mathcal{U}_{32}^{\eta}\right\rangle$$

With this approximation, the hierarchy of equations is broken and one obtains the BK (Balitsky, Kovchegov) equation:

$$\frac{\partial\left\langle \mathcal{U}_{12}^{\eta}\right\rangle}{\partial\eta} = \frac{\alpha_s N_c}{2\pi^2}\int d^2\vec{z}_3 \left(\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2}\vec{z}_{31}^{\,2}}\right)\left[\left\langle \mathcal{U}_{13}^{\eta}\right\rangle + \left\langle \mathcal{U}_{32}^{\eta}\right\rangle - \left\langle \mathcal{U}_{12}^{\eta}\right\rangle - \left\langle \mathcal{U}_{13}^{\eta}\right\rangle\left\langle \mathcal{U}_{32}^{\eta}\right\rangle\right]$$

Model for the dipole operator: $b_\perp$-independent MV (McLerran and Venugopalan) model

The model is valid for a large nucleus ($A \gg 1$) but it is still also usually applied for a nucleon target.
The scattering is viewed "from the point of view" of the ultra-relativistic target, moving in the $-$ direction. There is also a division of gluon in terms of their LC longitudinal momentum:

- Large-x gluons or valence partons are frozen sources of colour charges leading to color charge density $\rho^a(x^+, x_\perp)$ such that $\left\langle \rho^a(x^+, x_\perp)\rho^b(y^+, y_\perp) \right\rangle = g^2 \delta^{ab} \delta(x^+ - y^+)\delta(x_\perp - y_\perp)\lambda_A(x^+)$
- Small-x gluons or wee partons, produced via the classical Yang-Mills equation $\mathcal{D}_\mu F^{\mu\nu} = J^{\nu a} = \delta^{\nu -}\rho^a(x^+, x_\perp)$

To calculate any physical observable, one needs to calculate the observable in terms of a given ρ^a and then average over all the possible configuration :

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\rho W_\eta[\rho]\mathcal{O}[\rho]$$

$W_\eta[\rho]$ is the gauge-invariant weight function at the rapidity cut-off η. Its simplest chosen form is $W_\eta[\rho] = \mathcal{P}\exp\left\{ -\frac{1}{2}\int dx^+ d^2\vec{x} \frac{\rho_a(x^+, x_\perp)\rho_a(x^+, x_\perp)}{\lambda_A(x^-)} \right\}$

Model for the dipole operator: GBW model

Modelisation of the averaged dipole operator , inspired by GBW model :

$$D(r_\perp, b_\perp) = 1 - \frac{1}{N_c} \operatorname{Tr}\left[\mathcal{U}(b_\perp + \frac{r_\perp}{2})\mathcal{U}^\dagger(b_\perp - \frac{r_\perp}{2})\right]$$

$$\left\langle D(r_\perp, b_\perp)\right\rangle = 1 - \exp\left(-\frac{-r_\perp^2 Q_{s0}^2}{4} e^{-b_\perp^2/2B} C_\phi(r_\perp, b_\perp)\right)$$

$$= 1 - \exp\left(-\frac{-r_\perp^2 Q_s^2(b_\perp)}{4} C_\phi(r_\perp, b_\perp)\right)$$

$$C_\phi(r_\perp, b_\perp) = 1 + \frac{\tilde{c}}{2}\cos(2\phi_{rb})$$

$\tilde{c} = 0$ if one does not want to add any dependence on the the dipole orientation.