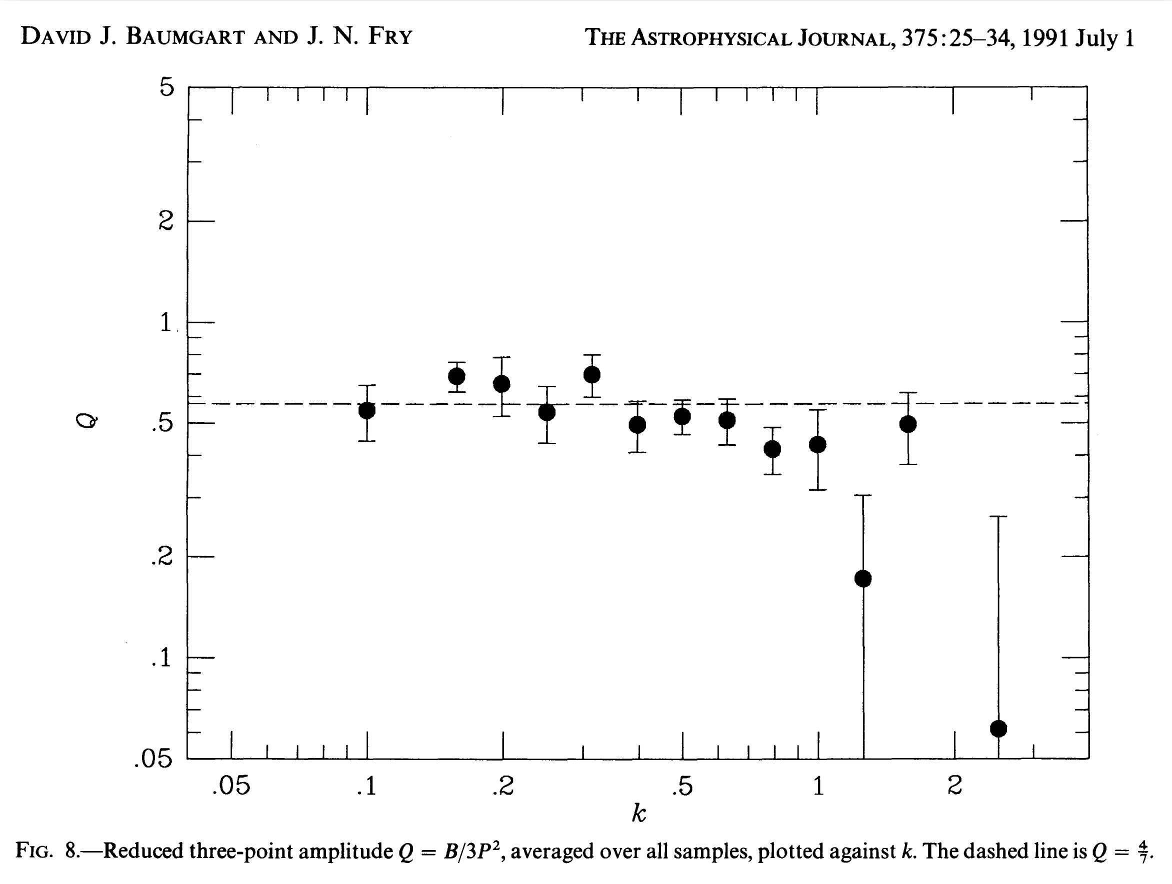
Galaxy Clustering Beyond the Power Spectrum

or

The slow coming-of-age of the galaxy bispectrum



David J. Baumgart and J. N. Fry

The Astrophysical Journal, 375:25–34, 1991 July 1

Fig. 8.—Reduced three-point amplitude $Q = B/3P^2$, averaged over all samples, plotted against k. The dashed line is $Q = \frac{4}{7}$.

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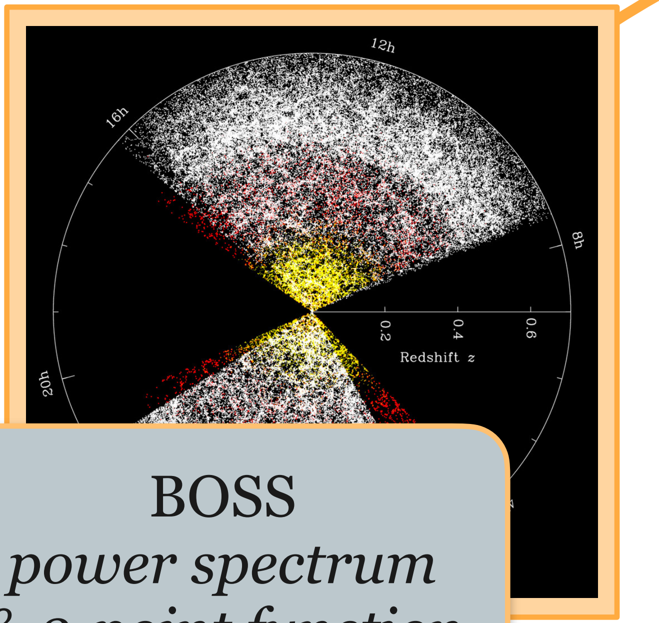
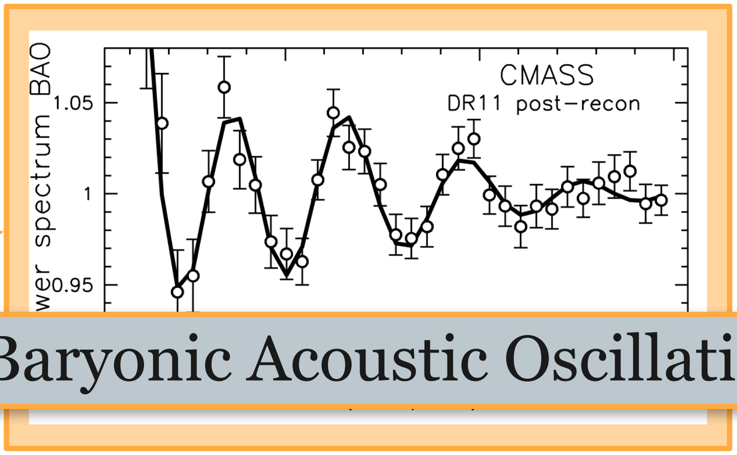
Astronomical Observatory of Trieste

Paris-Saclay Astroparticle Symposium 2022
Wednesday November 9th

Cosmological constraints from the galaxy power spectrum



BOSS
power spectrum
& 2-point function



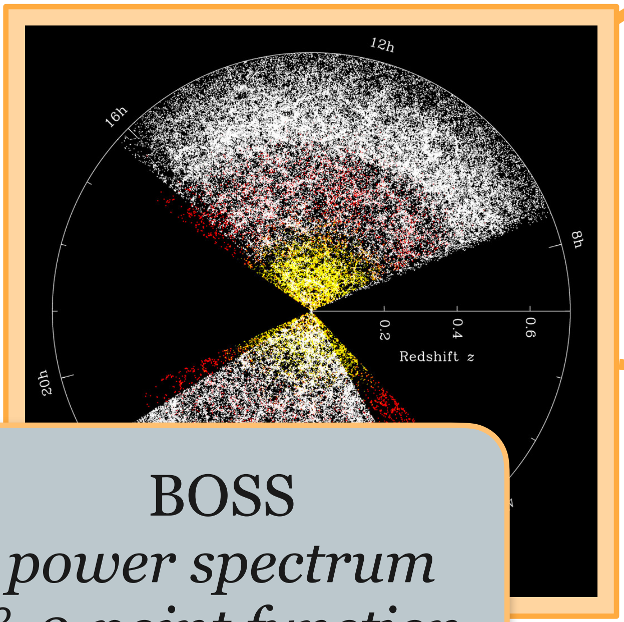
Baryonic Acoustic Oscillations

$$P_g(k) = P_{template}(k/\alpha)$$

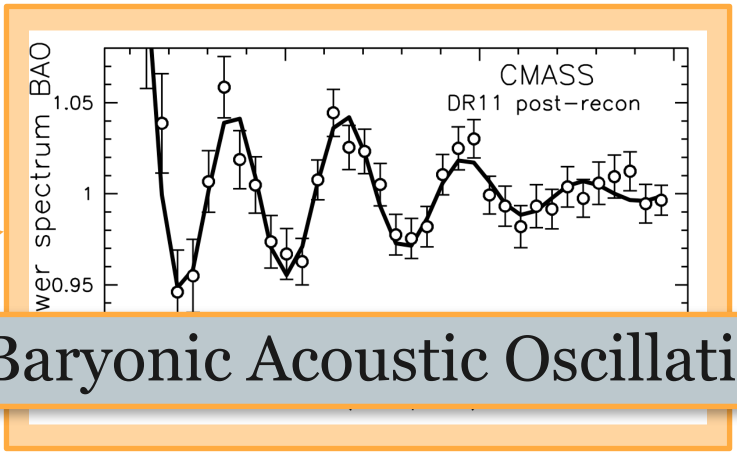
$$\alpha = \frac{D_V(z) r_s^{\mathrm{fid}}(z_d)}{D_V^{\mathrm{fid}}(z) r_s(z_d)} \qquad D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{cz}{H(z)}\right]^{1/3}$$

Geometrical probe of expansion history

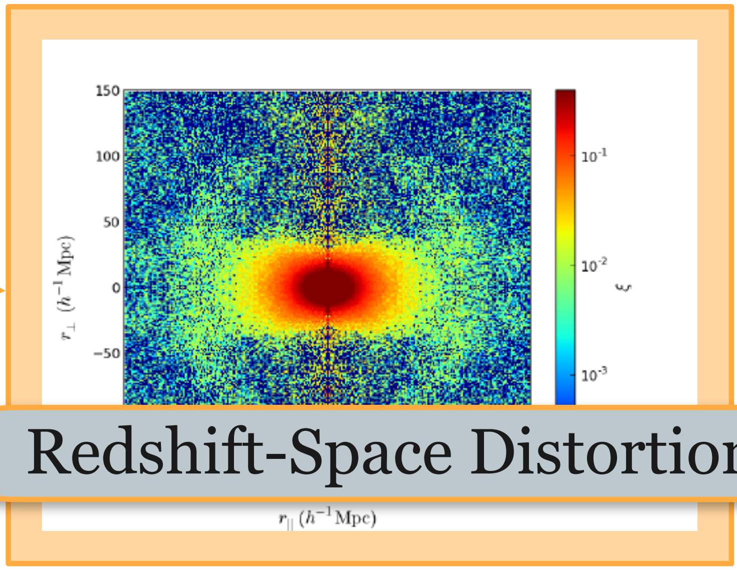
Cosmological constraints from the galaxy power spectrum



BOSS
power spectrum
& 2-point function



Baryonic Acoustic Oscillations

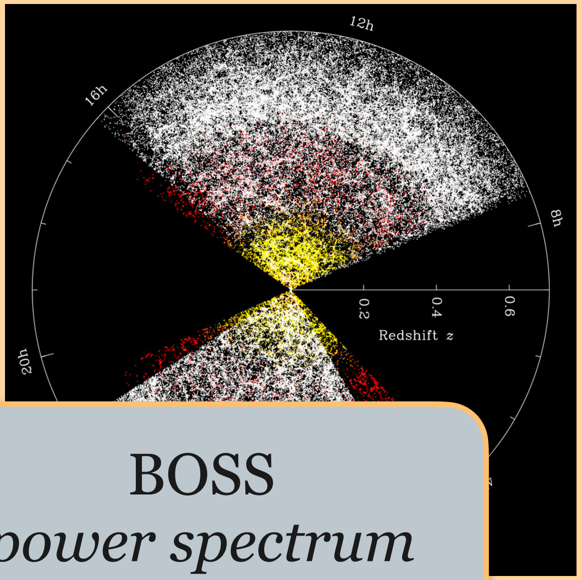


Redshift-Space Distortions

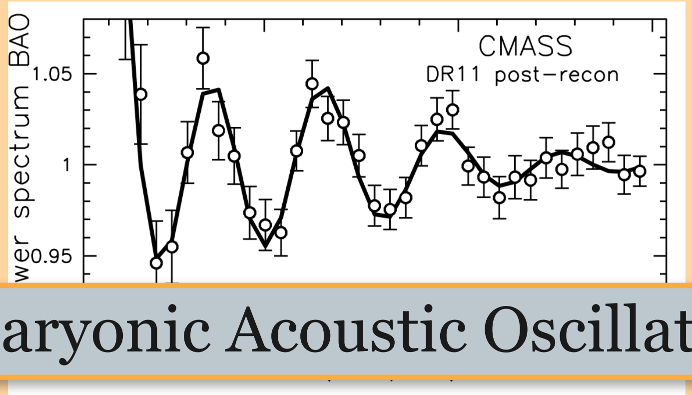
$$P_g(k,\mu) \simeq (b+f\mu^2)^2 \, P_L(k) = \sum_{\ell=0,2,4} P_\ell(k)\,\mathcal{L}_\ell(\mu)$$

Dynamical probe of expansion history

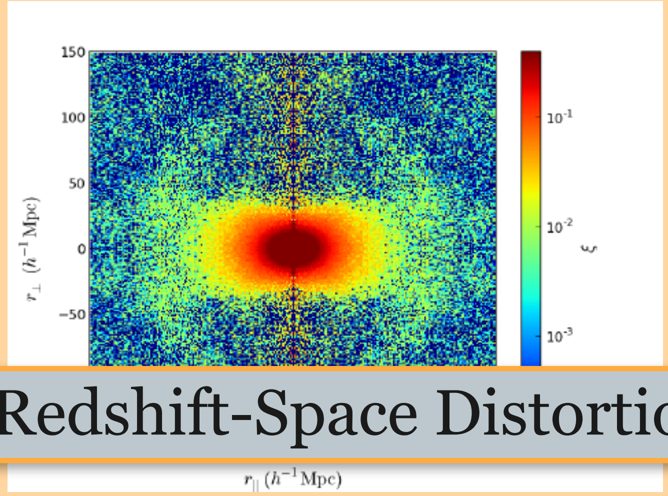
Cosmological constraints from the galaxy power spectrum



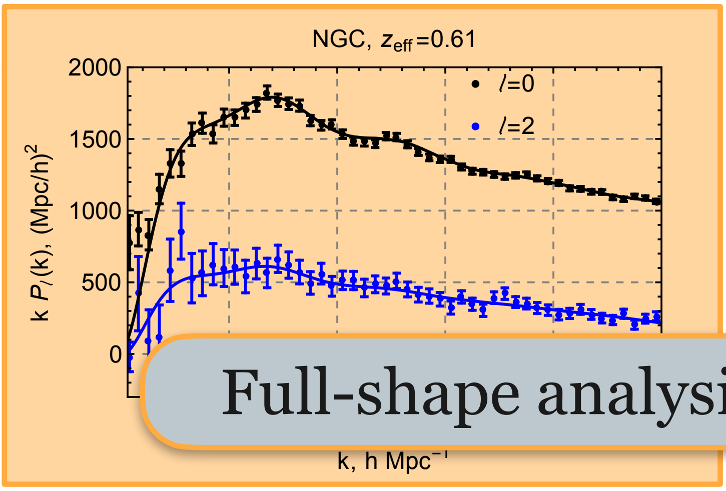
BOSS
power spectrum
& 2-point function



Baryonic Acoustic Oscillations



Redshift-Space Distortions



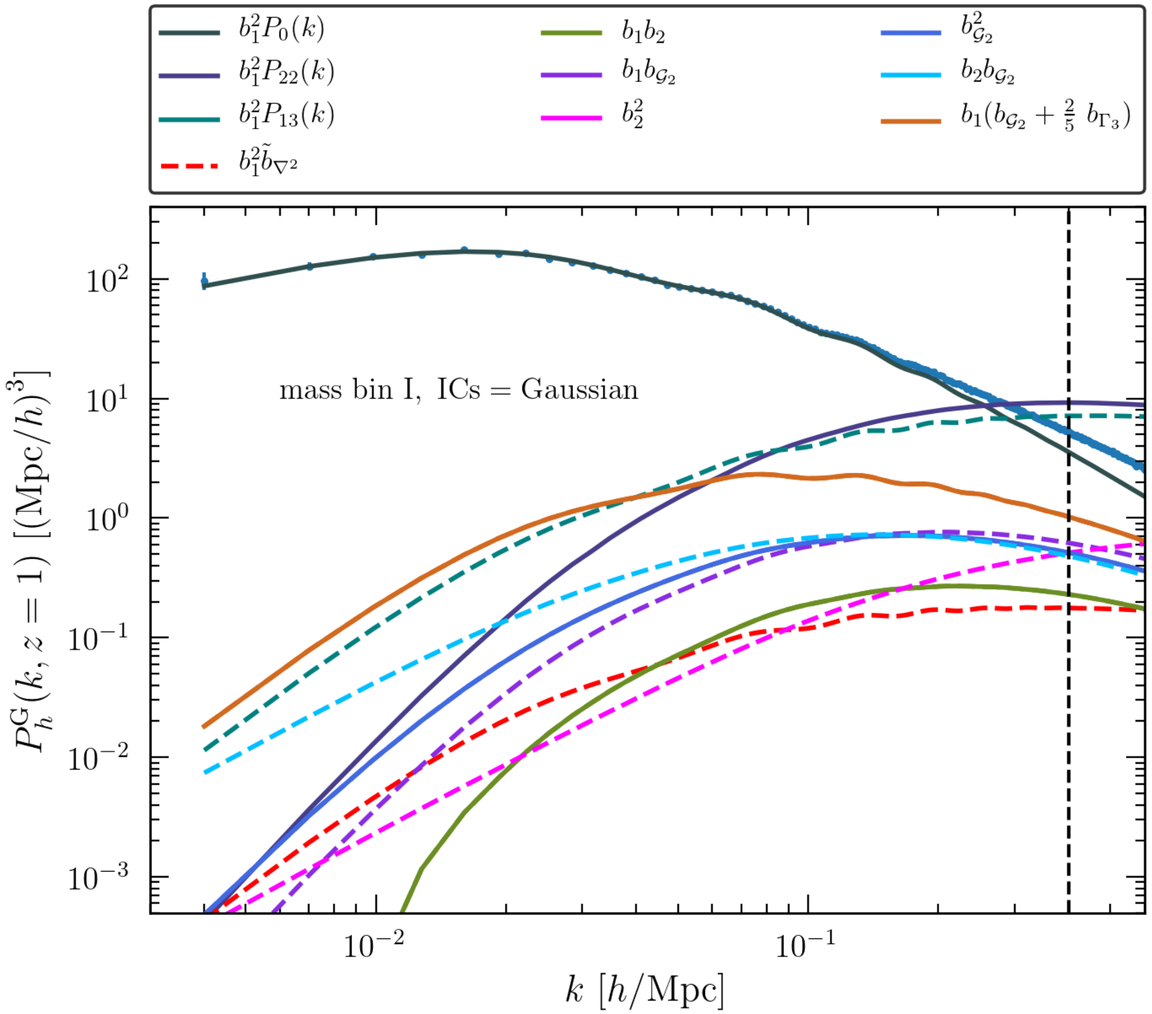
Full-shape analysis

- Dark Energy
- Gravity
- Early Universe
- Neutrinos
- Dark Matter

The galaxy power spectrum in Perturbation Theory

$$\delta_h^{\mathrm{G}}(\mathbf{x}) = b_1\delta(\mathbf{x}) + b_{\nabla^2\delta}\nabla^2\delta(\mathbf{x}) + \epsilon(\mathbf{x}) + \frac{b_2}{2}\delta^2(\mathbf{x}) + b_{\mathcal{G}_2}\mathcal{G}_2(\mathbf{x}) + \epsilon_\delta(\mathbf{x})\delta(\mathbf{x})$$
$$+ \frac{b_3}{6}\delta^3(\mathbf{x}) + b_{\mathcal{G}_3}\mathcal{G}_3(\mathbf{x}) + b_{(\mathcal{G}_2\delta)}\mathcal{G}_2(\mathbf{x})\delta(\mathbf{x}) + b_{\Gamma_3}\Gamma_3(\mathbf{x}) + \epsilon_{\delta^2}(\mathbf{x})\delta^2(\mathbf{x}) + \epsilon_{\mathcal{G}_2}(\mathbf{x})\mathcal{G}_2(\mathbf{x})$$

$$P_h^G(k) = b_1^2\left[P_0(k) + P_m^{1-\mathrm{loop}}(k)\right] + b_1 b_2\mathcal{I}_{\delta^2}(k) + 2b_1 b_{\mathcal{G}_2}\mathcal{I}_{\mathcal{G}_2}(k)$$
$$+ \frac{1}{4}b_2^2\mathcal{I}_{\delta^2\delta^2}(k) + b_{\mathcal{G}_2}^2\ \mathcal{I}_{\mathcal{G}_2\mathcal{G}_2}(k) + b_2 b_{\mathcal{G}_2}\mathcal{I}_{\delta^2\mathcal{G}_2}(k) + 2b_1(b_{\mathcal{G}_2} + \frac{2}{5}b_{\Gamma_3})\mathcal{F}_{\mathcal{G}_2}(k).$$

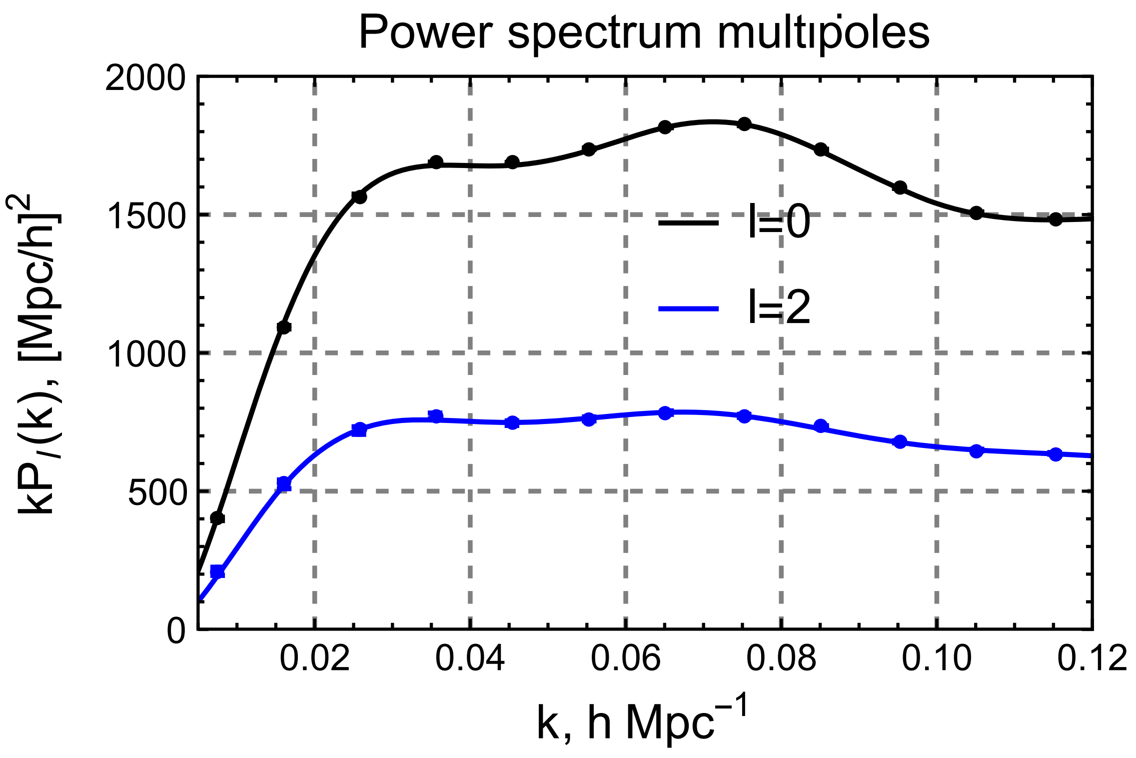


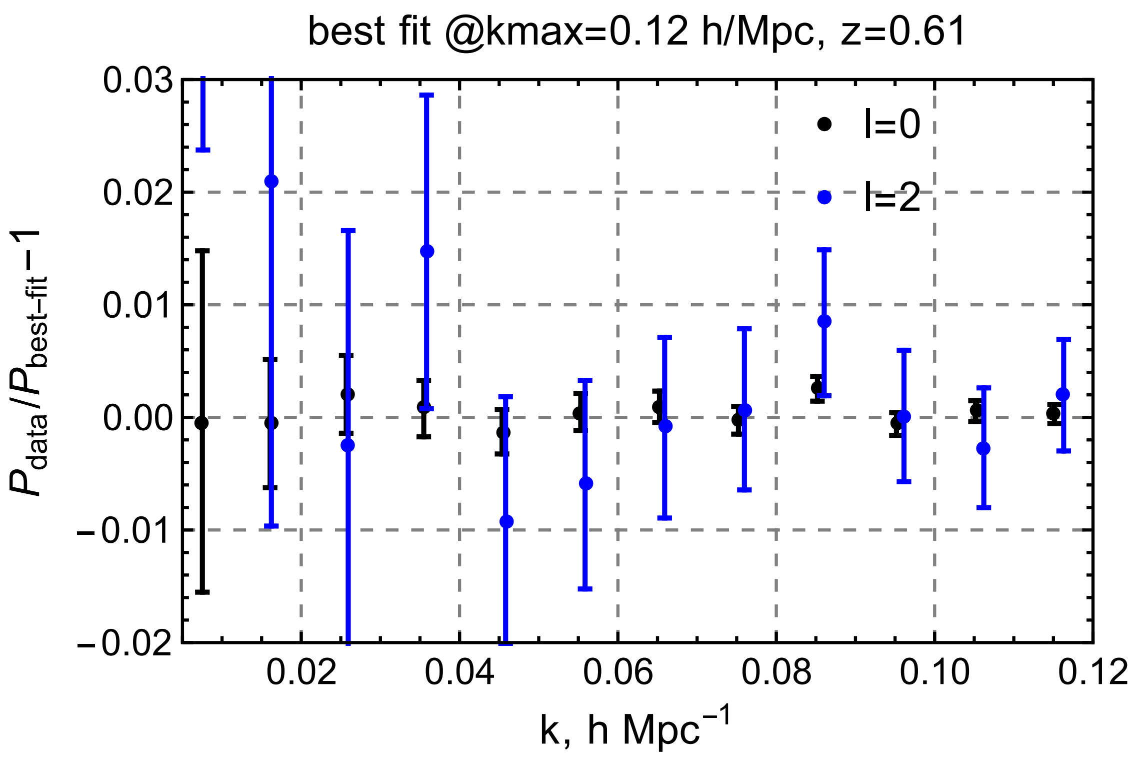
A glimpse …
in real space

The galaxy power spectrum in Perturbation Theory

Test of 1-loop power spectrum in EFTofLSS
HOD galaxies (CMASS, LOWZ)
566 $\mathrm{Gpc}^3 h^{-3}$ ~ 100 times BOSS

$$P_\ell(k) = P_\ell^{\mathrm{tree}}(k) + P_\ell^{\mathrm{loop}}(k) + P_\ell^{\mathrm{ctr}}(k)$$





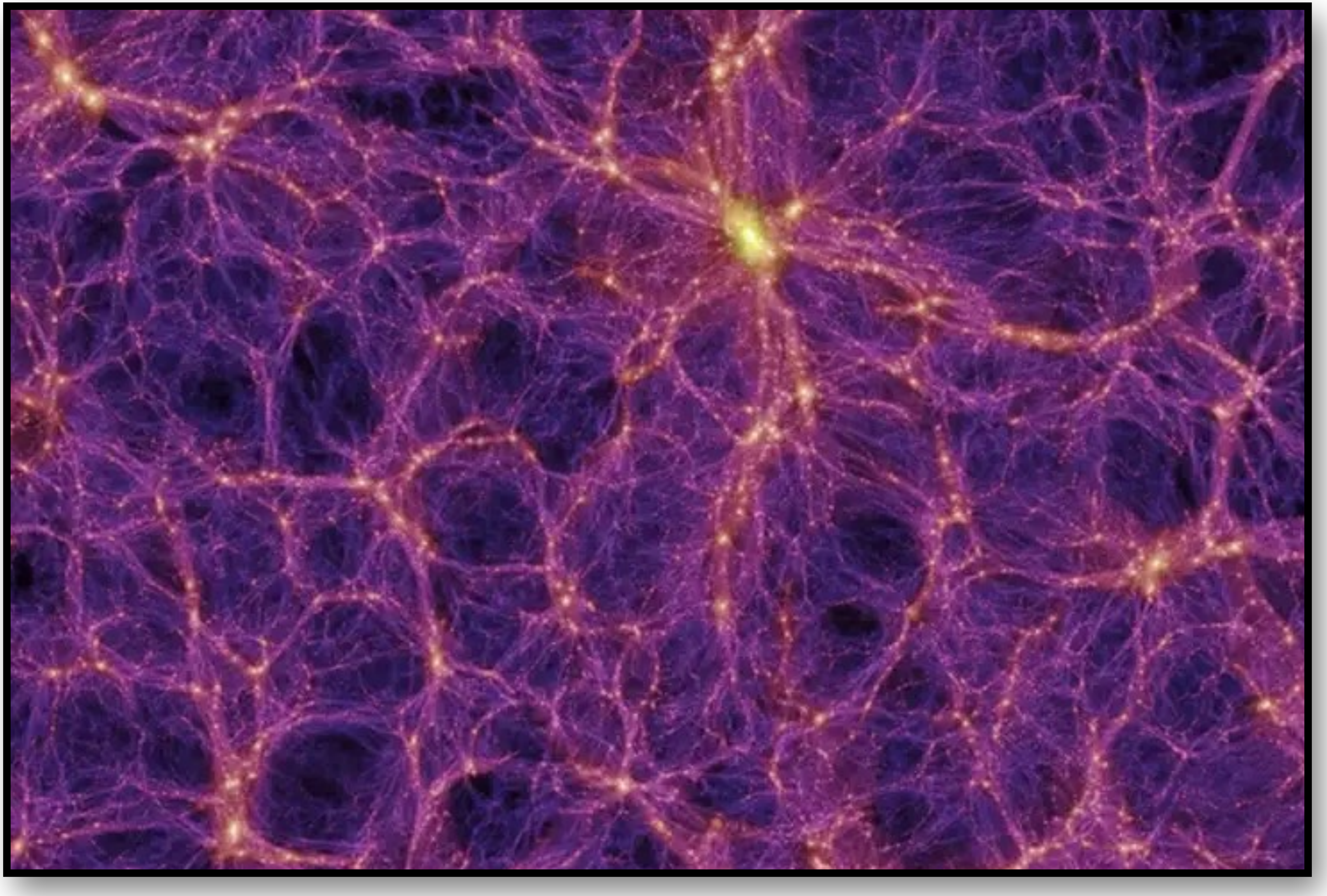
Nishimichi *et al.* (2020)

3 bias parameters
3 counterterms
+ cosmological parameters

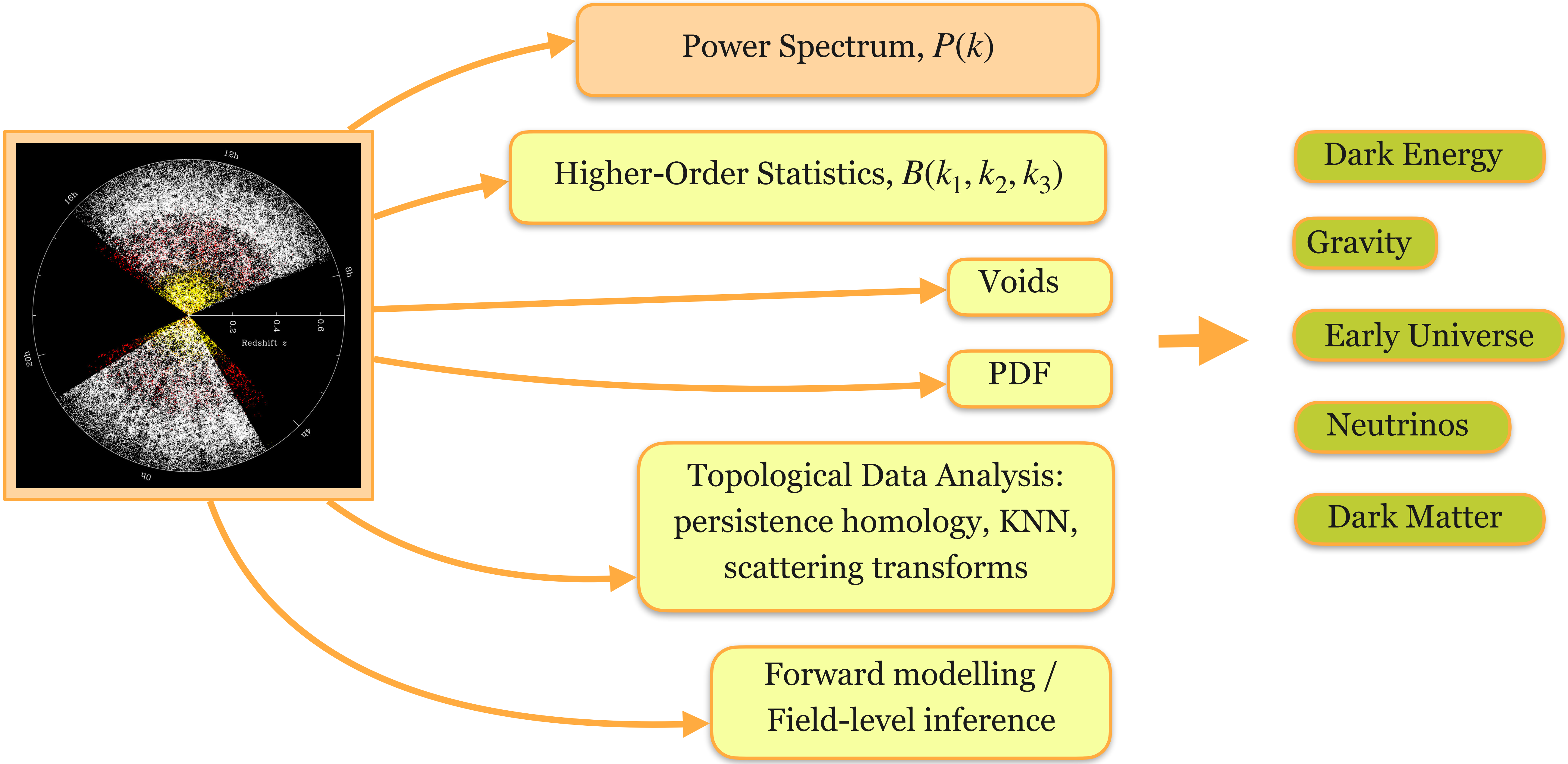
$$\delta_g(\boldsymbol{k}) = b_1\delta(\boldsymbol{k}) + \frac{b_2}{2}\delta^2(\boldsymbol{k}) + b_{\mathcal{G}_2}\mathcal{G}_2(\boldsymbol{k})$$

Too many free parameters …?

Non-Gaussianity



What can we do about it?



Power Spectrum, $P(k)$

Higher-Order Statistics, $B(k_1, k_2, k_3)$

Voids

PDF

Topological Data Analysis: persistence homology, KNN, scattering transforms

Forward modelling / Field-level inference

Dark Energy

Gravity

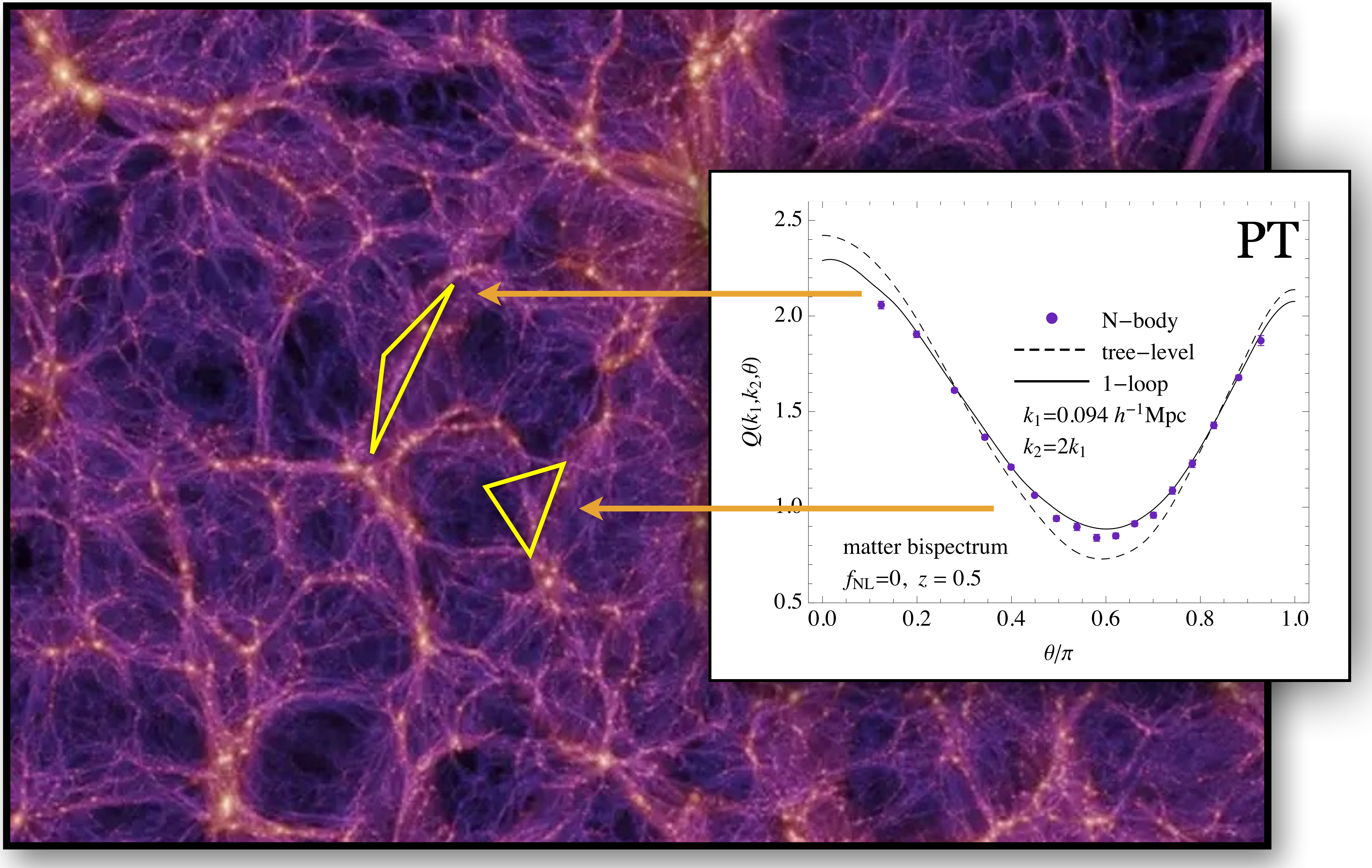
Early Universe

Neutrinos

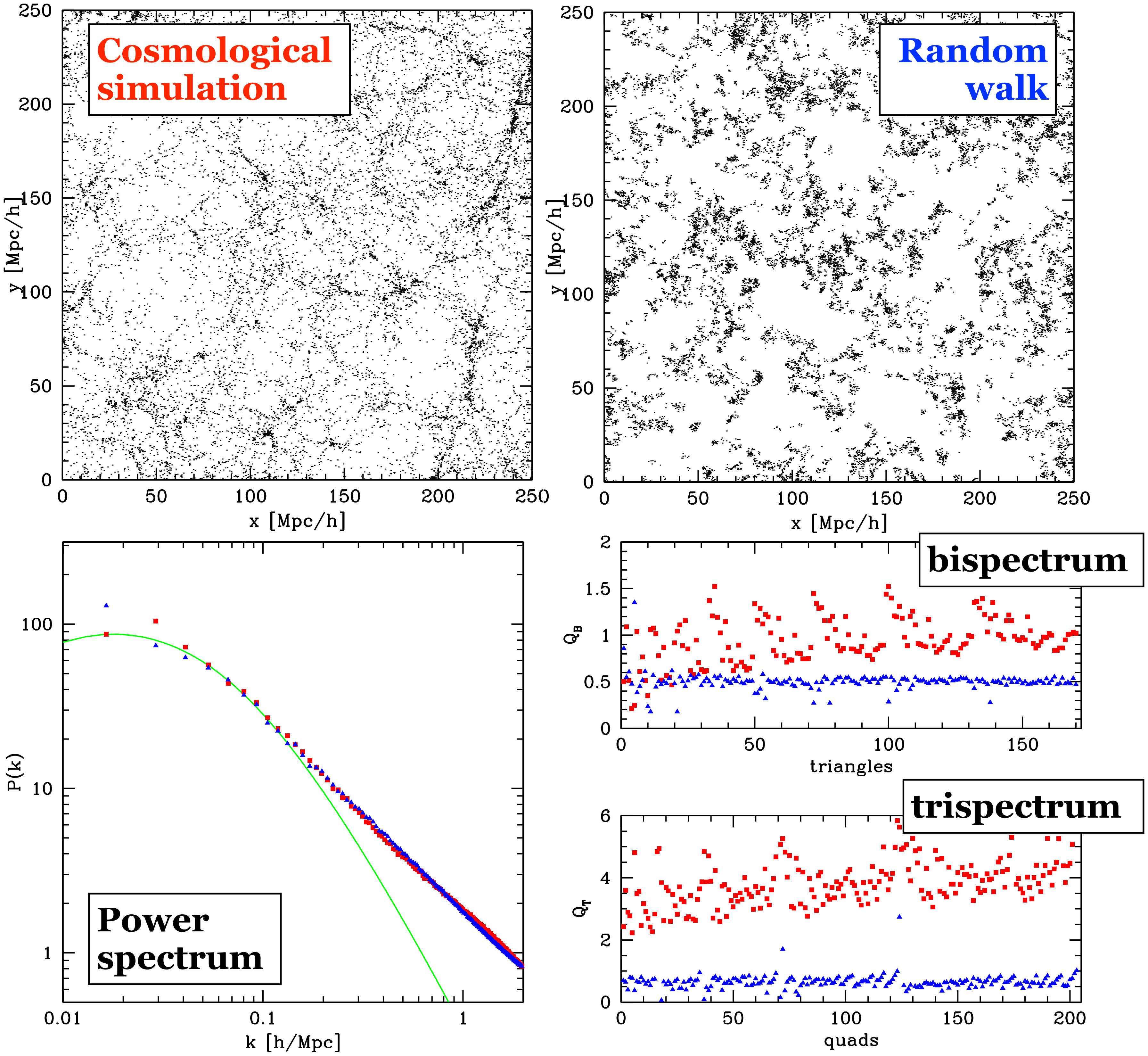
Dark Matter

(An incomplete list)

The galaxy bispectrum: $\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = \delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,B(k_1,k_2,k_3)$



Non-Gaussianity



The power spectrum cannot distinguish a cosmological simulation from a (properly designed) random walk

ES & Scoccimarro (2005)

Outlook

Perturbation Theory model

Anisotropies

Window convolution

Covariance

State-of-the-art: recent BOSS analyses

Beyond ΛCDM

Euclid

Signal-to-noise

Bispectrum S/N is comparable to the power spectrum

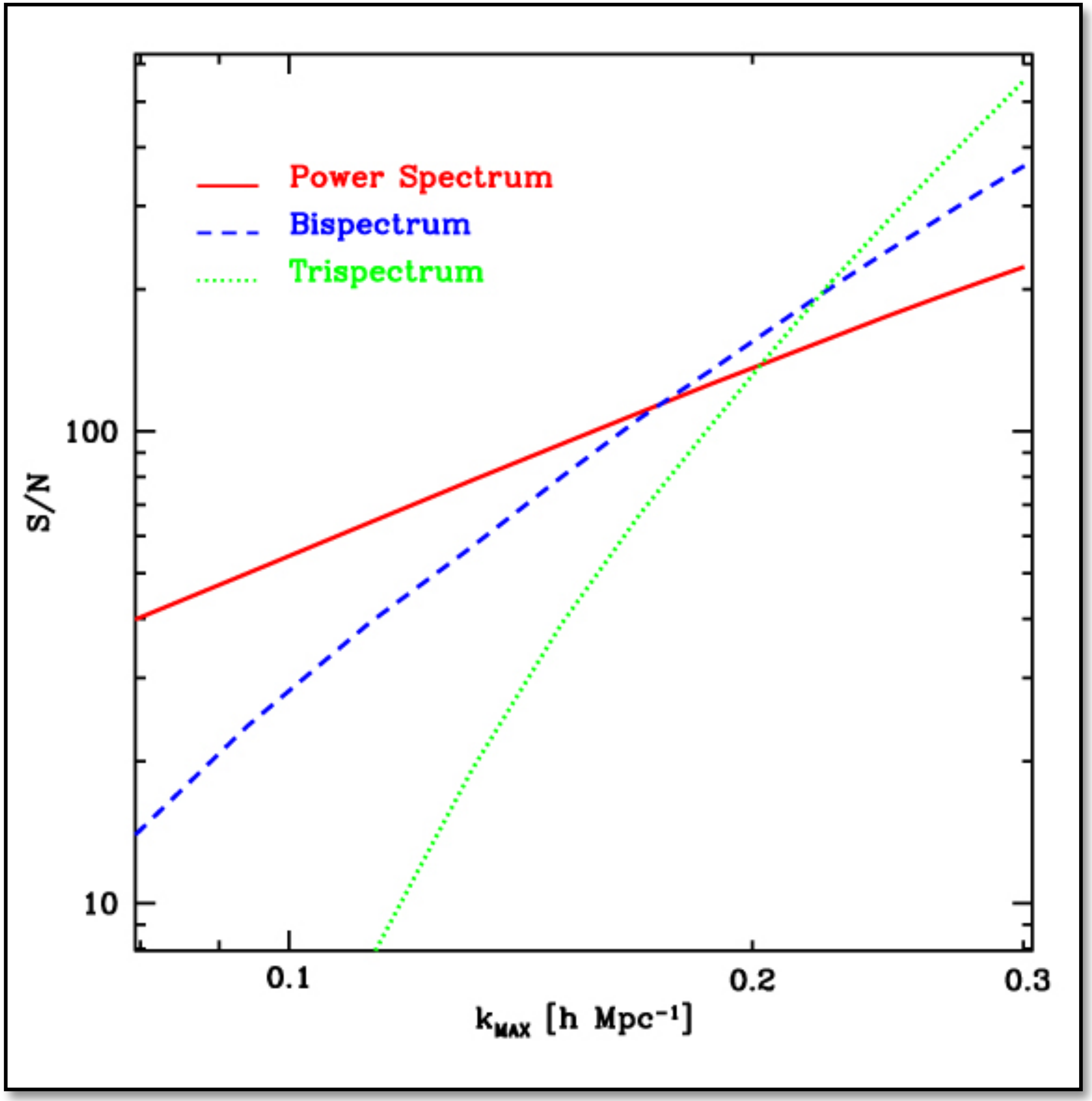
but the signal is distributed over a large number of triangular configurations

To extract enough information we must **get to small scales!**

$$\left(\frac{S}{N}\right)^2_P = \sum_k^{k_{\max}} \frac{P^2(k)}{\Delta P^2(k)}$$

$$\left(\frac{S}{N}\right)^2_B = \sum_{\text{triangles}}^{k_{\max}} \frac{B^2(k_1,k_2,k_3)}{\Delta B^2(k_1,k_2,k_3)}$$

ES & Scoccimarro (2005)



The **matter** bispectrum

PT: $$\underbrace{\delta_{\vec{k}} = \delta^{(1)}_{\vec{k}}}_{\text{Linear solution}} + \underbrace{\delta^{(2)}_{\vec{k}}}_{\text{nonlinear correction}} + \ldots$$

$$\delta^{(2)}_{\vec{k}} = \int d^3q\, F_2(\vec{k}-\vec{q},\vec{q})\, \delta^{(1)}_{\vec{k}-\vec{q}}\, \delta^{(1)}_{\vec{q}}$$

A non-zero bispectrum is a consequence of nonlinear evolution →

$$\langle\delta\delta\delta\rangle = \underbrace{\langle\delta^{(1)}\delta^{(1)}\delta^{(1)}\rangle}_{\text{= 0 for Gaussian initial conditions}} + \langle\delta^{(1)}\delta^{(1)}\delta^{(2)}\rangle + \ldots$$

$$B_G^{tree}(k_1,k_2,k_3) = 2\,F_2(\vec{k}_1,\vec{k}_2)\,P_0(k_1)\,P_0(k_2) + 2\ \text{perm.}$$

Fry (1984)

The **matter** bispectrum

PT: $$\underbrace{\delta_{\vec{k}} = \delta^{(1)}_{\vec{k}}}_{\text{Linear solution}} + \underbrace{\delta^{(2)}_{\vec{k}}}_{\text{nonlinear correction}} + \ldots$$

$$\delta^{(2)}_{\vec{k}} = \int d^3q\, F_2(\vec{k}-\vec{q},\vec{q})\, \delta^{(1)}_{\vec{k}-\vec{q}}\, \delta^{(1)}_{\vec{q}}$$

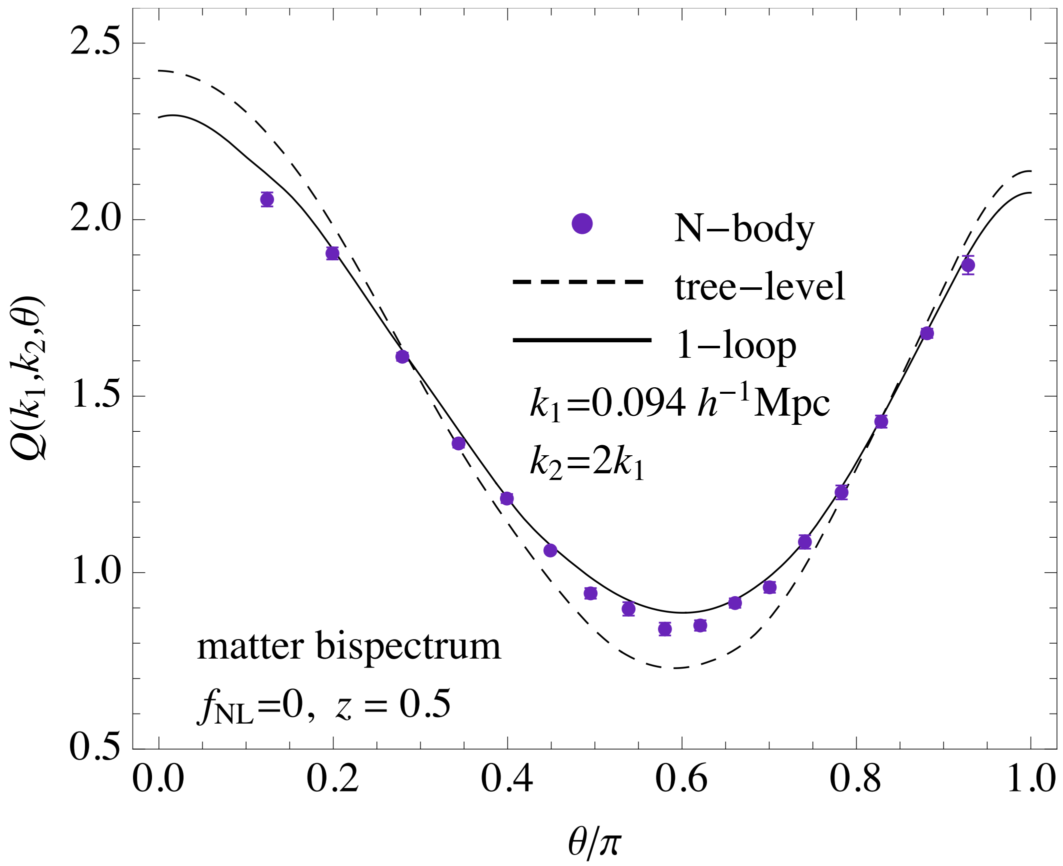
A non-zero bispectrum is a consequence of nonlinear evolution

$$\langle\delta\delta\delta\rangle = \underbrace{\langle\delta^{(1)}\delta^{(1)}\delta^{(1)}\rangle}_{\text{= 0 for Gaussian initial conditions}} + \langle\delta^{(1)}\delta^{(1)}\delta^{(2)}\rangle + \ldots$$

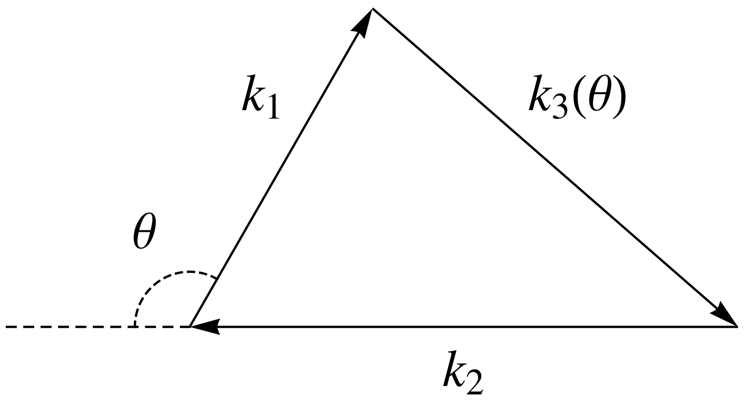
loop corrections

$$B^{tree}_G(k_1,k_2,k_3) = 2\,F_2(\vec{k}_1,\vec{k}_2)\,P_0(k_1)\,P_0(k_2) + 2\ \text{perm.}$$

Fry (1984); Scoccimarro (1997)



ES, Crocce & Desjacques (2010)



$$Q(k_1,k_2,k_3) = \frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+P(k_1)P(k_3)+P(k_2)P(k_3)}$$

The **matter** bispectrum at one-loop

The reach of perturbative models
(as a function of survey volume)

Much to gain to go to one-loop
… but numerically demanding!

Tree-level
(Fry, 1984)

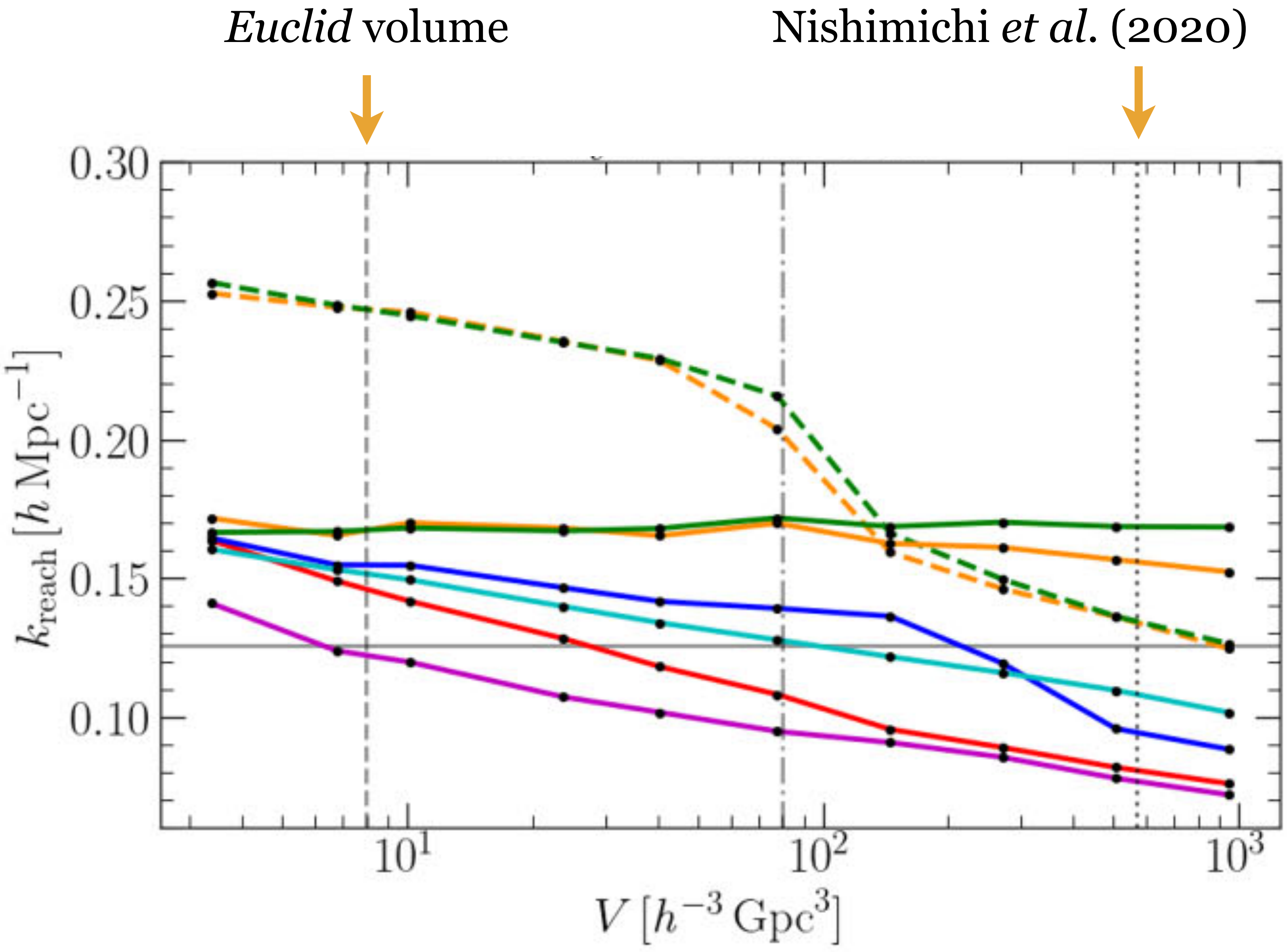
1-loop SPT
(Scoccimarro, 1997; 1998)

Renormalised PT
(Bernardeau, Crocce & Scoccimarro, 2008; 2012)

Lagrangian PT
(Matsubara, 2008)

EFTofLSS
EFTofLSS (IR-res)
(Angulo *et al.*, 2015;
Baldauf *et al.*, 2015)

But also more phenomenological models are available:
Scoccimarro & Couchmann (2001);
Gil-Marín *et al.* (2012)



Alkhanishvili *et al.* (2019)

Galaxy bias

Non-Gaussianity
from nonlinear bias

$$\delta_g(\boldsymbol{k}) = b_1 \delta(\boldsymbol{k}) + \frac{b_2}{2}\delta^2(\boldsymbol{k}) + b_{\mathcal{G}_2}\mathcal{G}_2(\boldsymbol{k})$$

quadratic bias, local & nonlocal

$$\rightarrow \quad \langle \delta_g \delta_g \delta_g \rangle = b_1^3 \, \langle \delta\delta\delta \rangle + b_1^2 \, b_2 \, \langle \delta\delta\delta^2 \rangle + \ldots$$

$$B_g(k_1, k_2, k_3) = b_1^3 \, B_m(k_1, k_2, k_3) + b_1^2 \, b_2 \, P_L(k_1) \, P_L(k_2) +$$

$$+ 2 \, b_1^2 \, b_{\mathcal{G}_2} \, S(\mathbf{k_1}, \mathbf{k_2}) \, P_L(k_1) \, P_L(k_2) + \text{perm.} + \text{loop corrections}$$

Galaxy bias

Non-Gaussianity from nonlinear bias

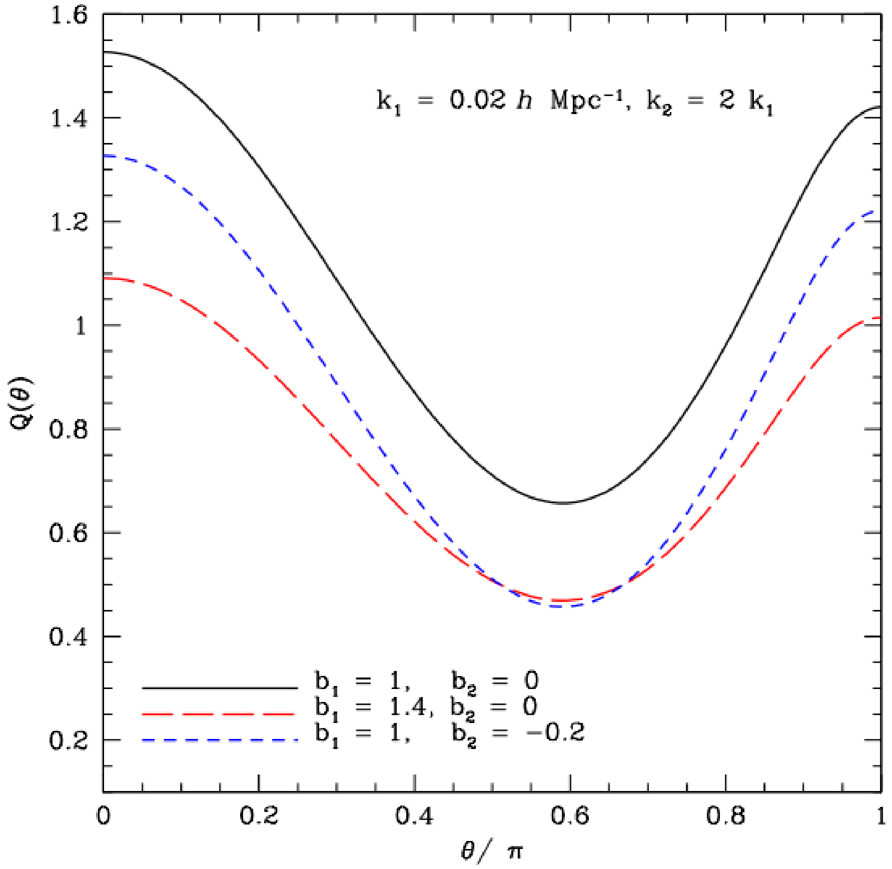
$$\delta_g(\boldsymbol{k}) = b_1\delta(\boldsymbol{k}) + \frac{b_2}{2}\delta^2(\boldsymbol{k}) + b_{\mathcal{G}_2}\mathcal{G}_2(\boldsymbol{k})$$

quadratic bias, local & nonlocal

→ $$\langle\delta_g\delta_g\delta_g\rangle = b_1^3\,\langle\delta\delta\delta\rangle + b_1^2\,b_2\,\langle\delta\delta\delta^2\rangle + \ldots$$

$$B_g(k_1,k_2,k_3) = b_1^3\,B_m(k_1,k_2,k_3) + b_1^2\,b_2\,P_L(k_1)\,P_L(k_2) +$$

$$+2\,b_1^2\,b_{\mathcal{G}_2}\,S(\mathbf{k_1},\mathbf{k_2})\,P_L(k_1)\,P_L(k_2) + \text{perm.} + \text{loop corrections}$$



$$Q_g(k_1,k_2,k_3) = \frac{1}{b_1}Q(k_1,k_2,k_3) + \frac{b_2}{b_1^2}$$

This allows to break the degeneracy between b_1 and A_s in the power spectrum, as $P_g(k) \simeq b_1^2\,P_L(k) \sim b_1^2\,A_s$ but also to determine b_2

Galaxy bias, nonlocal

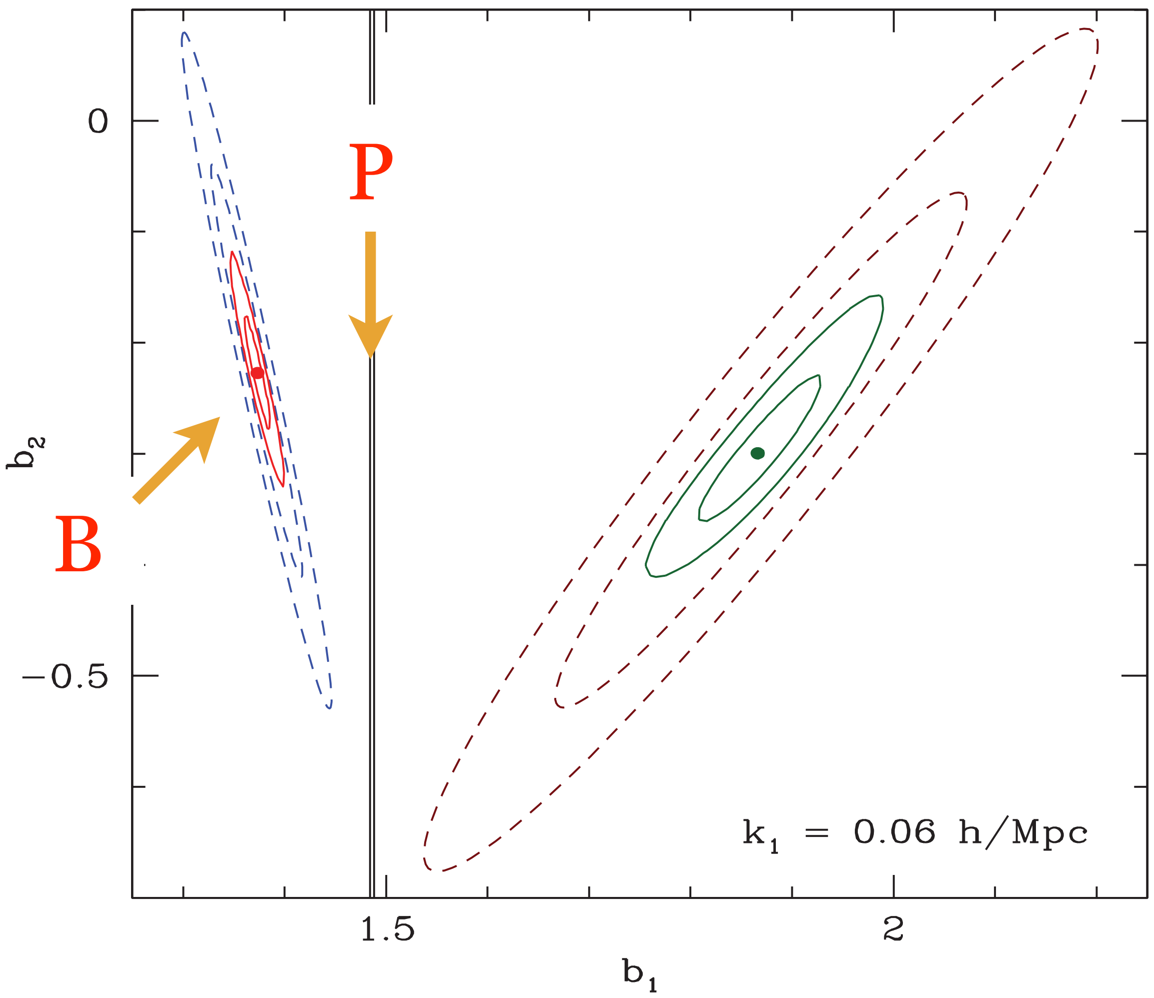
Measurements of the galaxy bispectrum in N-body simulations identify a problem in our understanding of galaxy bias:

In a **local bias** model, the linear b_1 bias determined from the power spectrum was inconsistent with the one determined from the bispectrum

$$P_g(k) = b_1^2\, P_L(k)$$

$$B_g(k_1,k_2,k_3) = b_1^3\, B_m(k_1,k_2,k_3) + \\ + b_1^2\, b_2\, P_L(k_1)\, P_L(k_2) + \text{perm.}$$

Pollack *et al.* (2012)

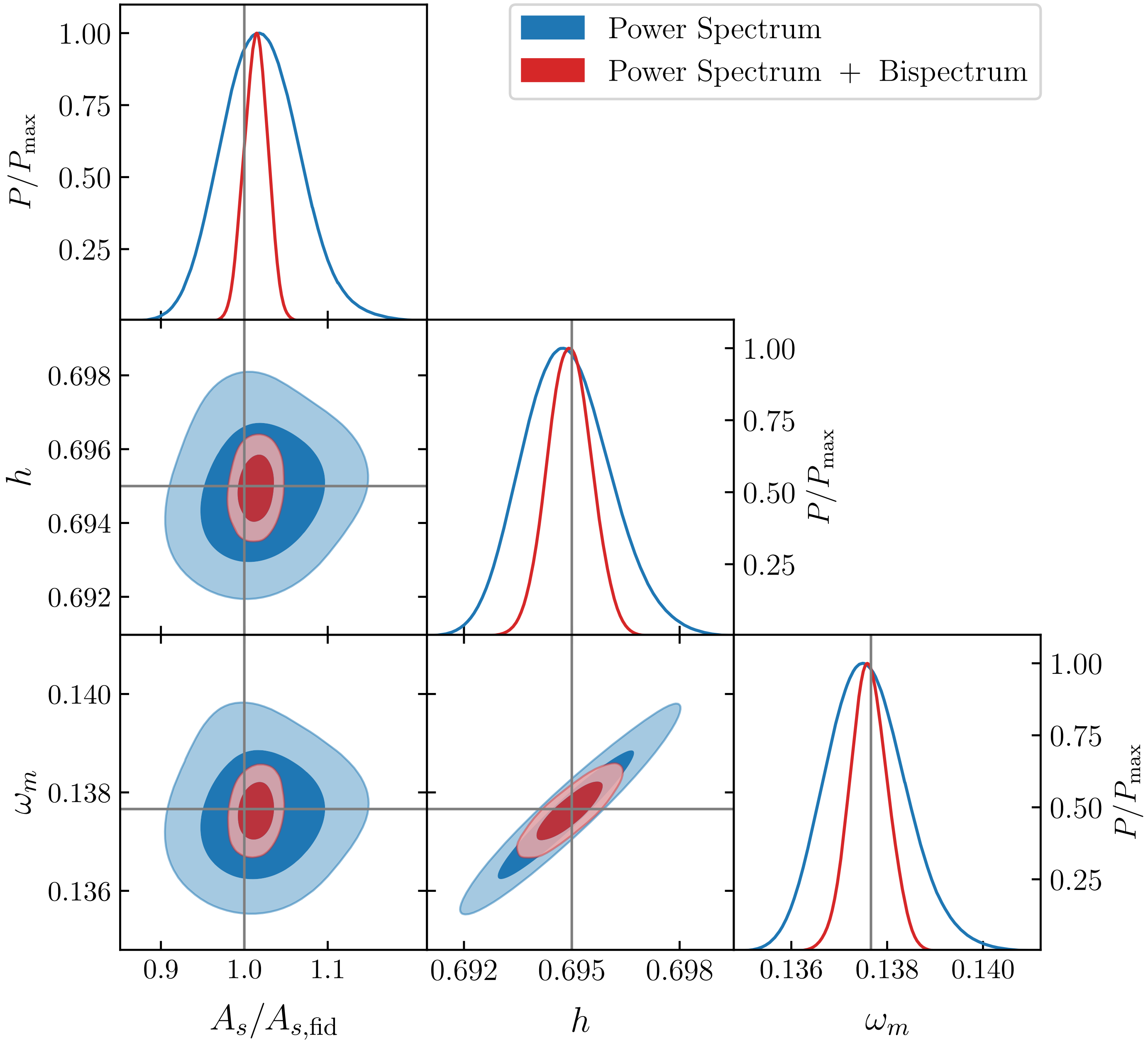


Nonlocal bias
Chan *et al.* (2012), Baldauf *et al.* (2012)

$$\delta_g(\boldsymbol{k}) = b_1\delta(\boldsymbol{k}) + \frac{b_2}{2}\delta^2(\boldsymbol{k}) + \boxed{b_{\mathcal{G}_2}\mathcal{G}_2(\boldsymbol{k})}$$

Galaxy bias & *tree-level* bispectrum

P at 1-loop, B tree-level
Halos
test on $1000\, h^{-3}\mathrm{Gpc}^3$ of cumulative volume



Oddo, Rizzo *et al.* (2021)

The bispectrum is expected to reduce degeneracies in the power spectrum loop corrections

$$b_1, b_2, b_{\mathcal{G}_2}$$

$$\downarrow$$

$$P_\ell(k) = P_\ell^{\rm tree}(k) + P_\ell^{\rm loop}(k) + P_\ell^{\rm ctr}(k)$$

Limited reach on such a large volume: $k^B_{max} \simeq 0.09\, h\mathrm{Mpc}^{-1}$

Significant improvement over *P*, *but in real space!*

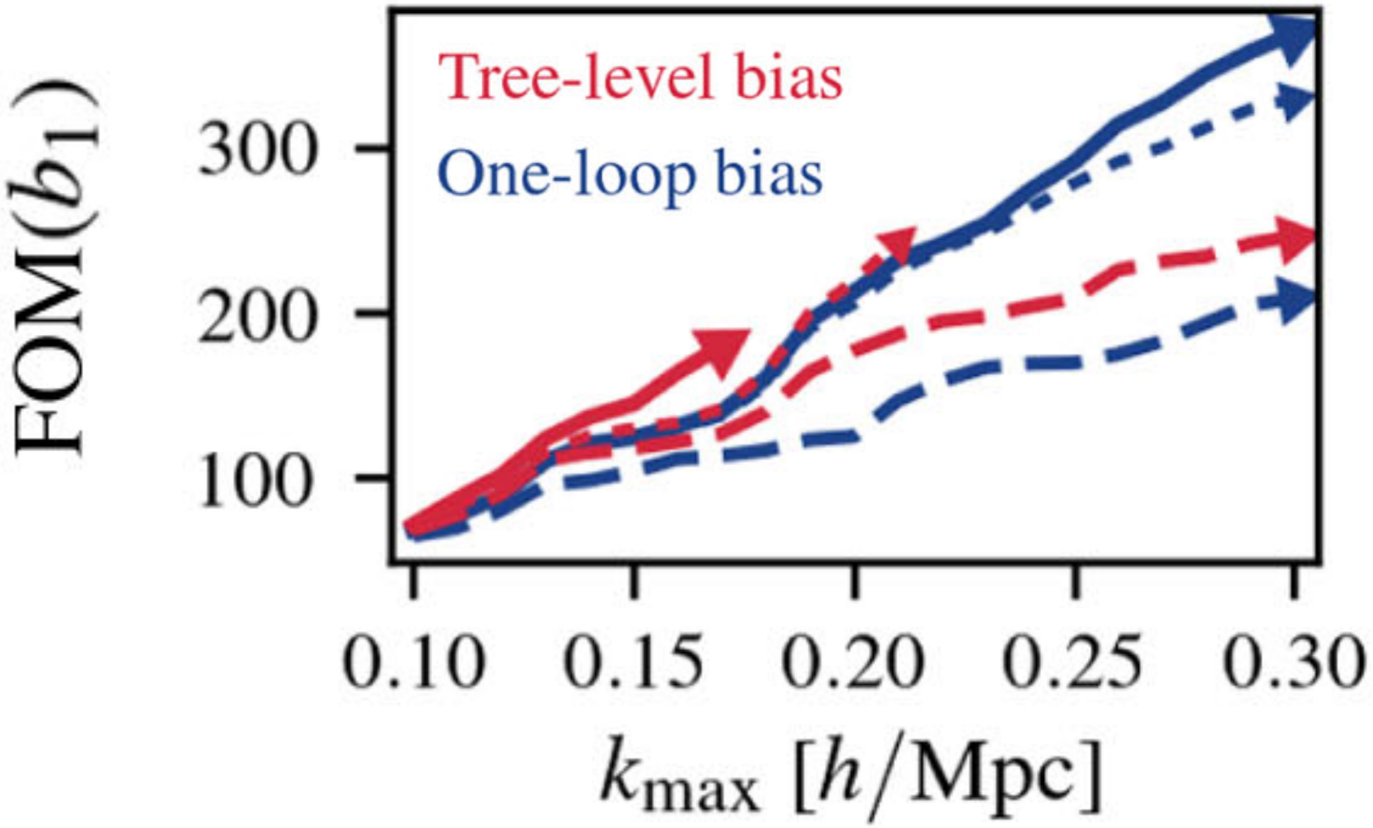
See also Ivanov *et al.* (2022)

Galaxy bias & *one-loop* bispectrum

Test of 1-loop bispectrum
bias model in real space

HOD galaxies (CMASS, LOWZ)
& halos
$6\,\mathrm{Gpc}^3 h^{-3}$

8 parameters (tree-level B)
15 parameters (one-loop B)



Eggemeier *et al.* (2021)

One-loop corrections greatly extend the reach of the model and its potential to constrain its parameters
(despite their larger number)

but, again, this is still real space ...

Redshift-space

Galaxy density in redshift space: more nonlinearity

$$\delta_s(\mathbf{k}) = Z_1(\mathbf{k})\delta_L(\mathbf{k}) + \int d^3q\, Z_2(\mathbf{q}, \mathbf{k}-\mathbf{q})\delta_L(\mathbf{q})\delta_L(\mathbf{k}-\mathbf{q}) + \dots$$

$$\begin{aligned} Z_1(\mathbf{k}) &= b_1 + f\mu^2\,, \\ Z_2(\mathbf{k}_1, \mathbf{k}_2) &= \frac{b_2}{2} + b_1 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_{\mathcal{G}_2} S(\mathbf{k}_1, \mathbf{k}_2) + f\mu_{12}^2 G_2(\mathbf{k}_1, \mathbf{k}_2) + \\ &\quad + \frac{f\mu_{12}k_{12}}{2}\left[\frac{\mu_1}{k_1} Z_1(\mathbf{k}_2) + \frac{\mu_2}{k_2} Z_1(\mathbf{k}_1)\right] \end{aligned}$$

Redshift-space
PT kernels

$$B_s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2\, Z_1(\mathbf{k}_1)\, Z_1(\mathbf{k}_2) Z_2(\mathbf{k}_1, \mathbf{k}_2) P_L(k_1) P_L(k_2) + 2 \text{ perm.}$$

Galaxy bispectrum in redshift space

Redshift-space

Galaxy density in redshift space: more nonlinearity

$$\delta_s(\mathbf{k}) = Z_1(\mathbf{k})\delta_L(\mathbf{k}) + \int d^3q\, Z_2(\mathbf{q}, \mathbf{k}-\mathbf{q})\delta_L(\mathbf{q})\delta_L(\mathbf{k}-\mathbf{q}) + \dots$$

$$\begin{aligned} Z_1(\mathbf{k}) &= b_1 + f\mu^2\,, \\ Z_2(\mathbf{k}_1, \mathbf{k}_2) &= \frac{b_2}{2} + b_1 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_{\mathcal{G}_2} S(\mathbf{k}_1, \mathbf{k}_2) + f\mu_{12}^2 G_2(\mathbf{k}_1, \mathbf{k}_2) + \\ &\quad + \frac{f\mu_{12}k_{12}}{2}\left[\frac{\mu_1}{k_1} Z_1(\mathbf{k}_2) + \frac{\mu_2}{k_2} Z_1(\mathbf{k}_1)\right] \end{aligned}$$

Redshift-space
PT kernels

$$B_s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = B_s^{(\mathrm{det})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + B_s^{(\mathrm{stoch})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

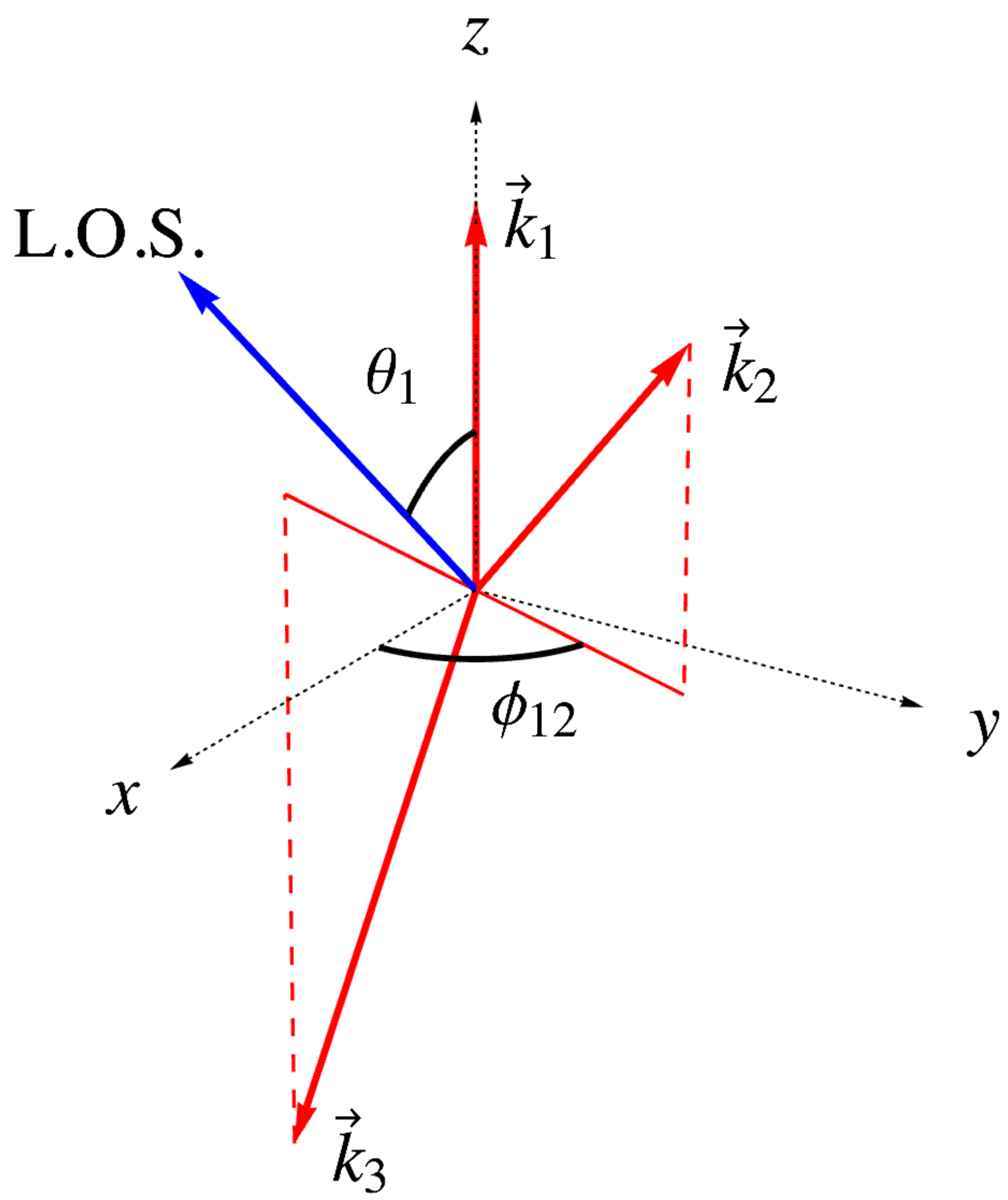
$$B_s^{(\mathrm{det})}(\mathbf{k}_1, \mathbf{k}_2, \hat{n}) = 2\, Z_1(\mathbf{k}_1)\, Z_1(\mathbf{k}_2) Z_2(\mathbf{k}_1, \mathbf{k}_2) P_L(k_1) P_L(k_2) + 2 \text{ perm.}$$

$$B_s^{(\mathrm{stoch})}(\mathbf{k}_1, \mathbf{k}_2, \hat{n}) = \frac{1}{\bar{n}}\left[(1+\alpha_1)\, b_1 + (1+\alpha_3)\, f\, \mu^2\right]\, Z_1(\mathbf{k}_1)\, P_L(k_1) + 2 \text{ perm.} + \frac{1+\alpha_2}{\bar{n}^2}$$

Lot's of fun here!

Redshift-space: bispectrum *multipoles*

$$B_s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = B_s(k_1, k_2, k_3, \theta_1, \phi_{12})$$



The orientation of the triangle w.r.t. the line-of-sight now matters

Different choices are possible (see e.g. Hashimoto *et al.*, 2017, Gualdi & Verde, 2020)

We follow Scoccimarro *et al.* (1999), with the FFT-based estimator of Scoccimarro (2015).

$$B_s(k_1, k_2, k_3, \theta_1, \phi_{12}) = \sum_{\ell,m} B_{\ell,m}(k_1, k_2, k_3) Y_{\ell,m}(\theta_1, \phi_{12})$$

$$\mu_1 \equiv \mu \equiv \cos\theta_1$$

Redshift-space: bispectrum *monopole*

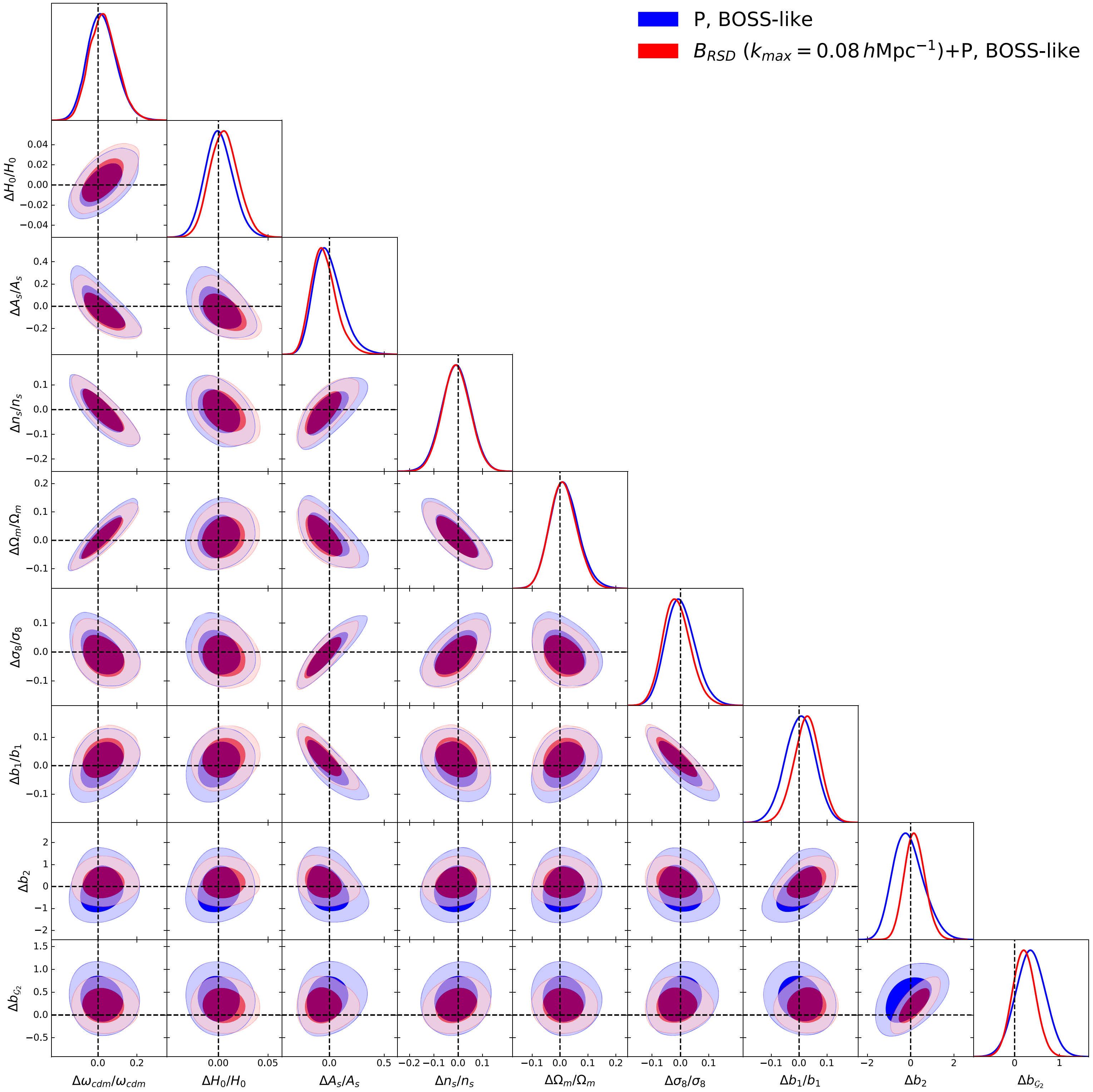


FIG. 7. Same as Fig. 5 but with the covariance rescaled by 100 to match the BOSS survey volume.

Test of tree-level bispectrum
in redshift space
EFTofLSS

BOSS-like HOD

Some (10%) improvement
on amplitude parameters (A_s, σ_8)
on BOSS-like volume

On full volume, $566\,h^{-3}\mathrm{Gpc}^3$:

$$\frac{\sigma_{\mathrm{P+B}}}{\sigma_{\mathrm{P}}}\{\omega_{\mathrm{cdm}}, h, n_s, A_s, \Omega_m, \sigma_8\}$$
$$= \{0.88, 0.94, 0.86, 0.95, 0.89, 0.96\},$$
$$\frac{\sigma_{\mathrm{P+B}}}{\sigma_{\mathrm{P}}}\{b_1, b_2, b_{\mathcal{G}_2}, P_{\mathrm{shot}}\}$$
$$= \{0.84, 0.18, 0.09, 0.65\}.$$

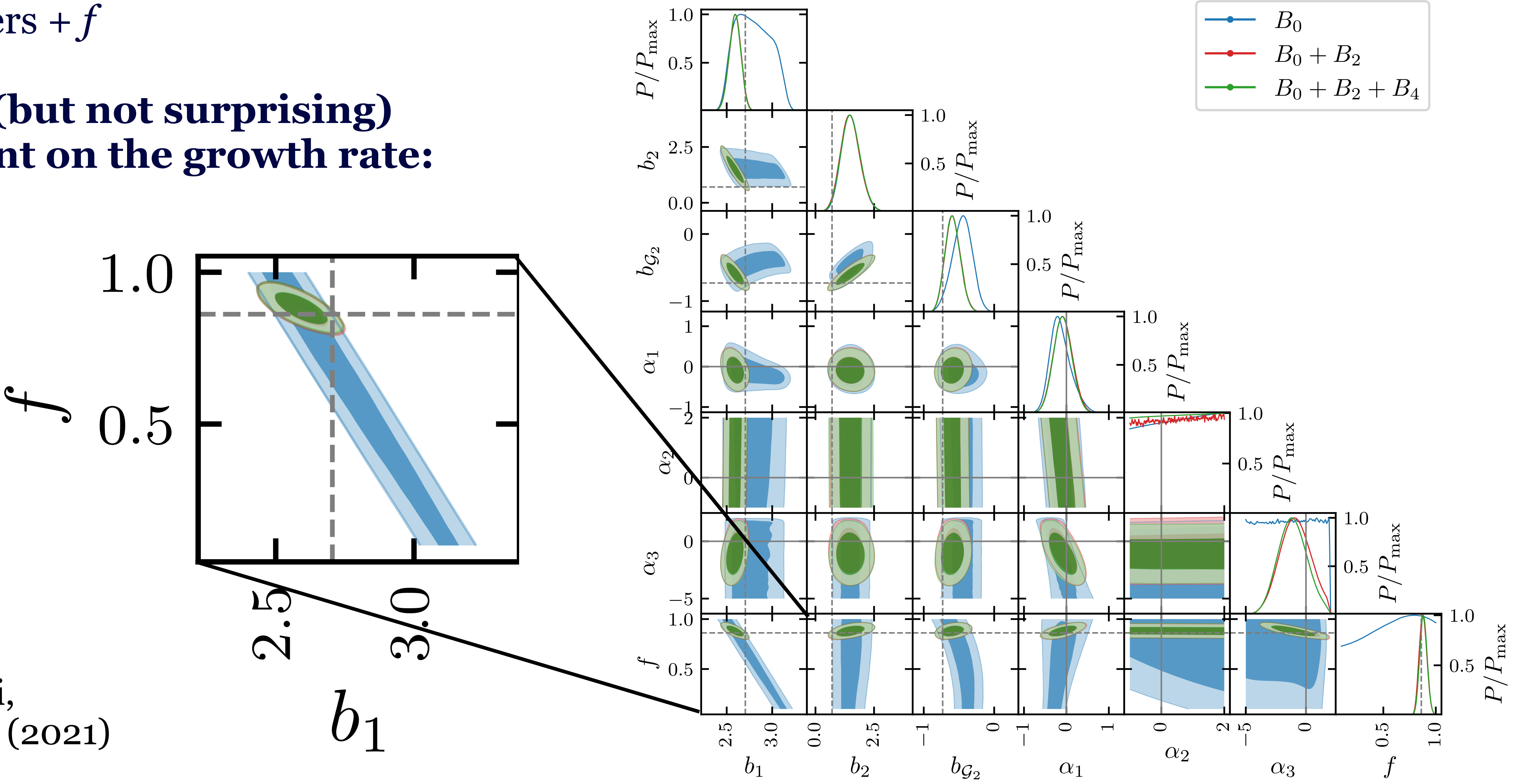
Ivanov *et al.* (2022)

Redshift-space: bispectrum *monopole & quadrupole*

Test of bispectrum multipoles: halos on $1000\,h^{-3}\mathrm{Gpc}^3$ of cumulative volume

bias parameters + f

Significant (but not surprising) improvement on the growth rate:



Rizzo, Moretti,
Pardede *et al.* (2021)

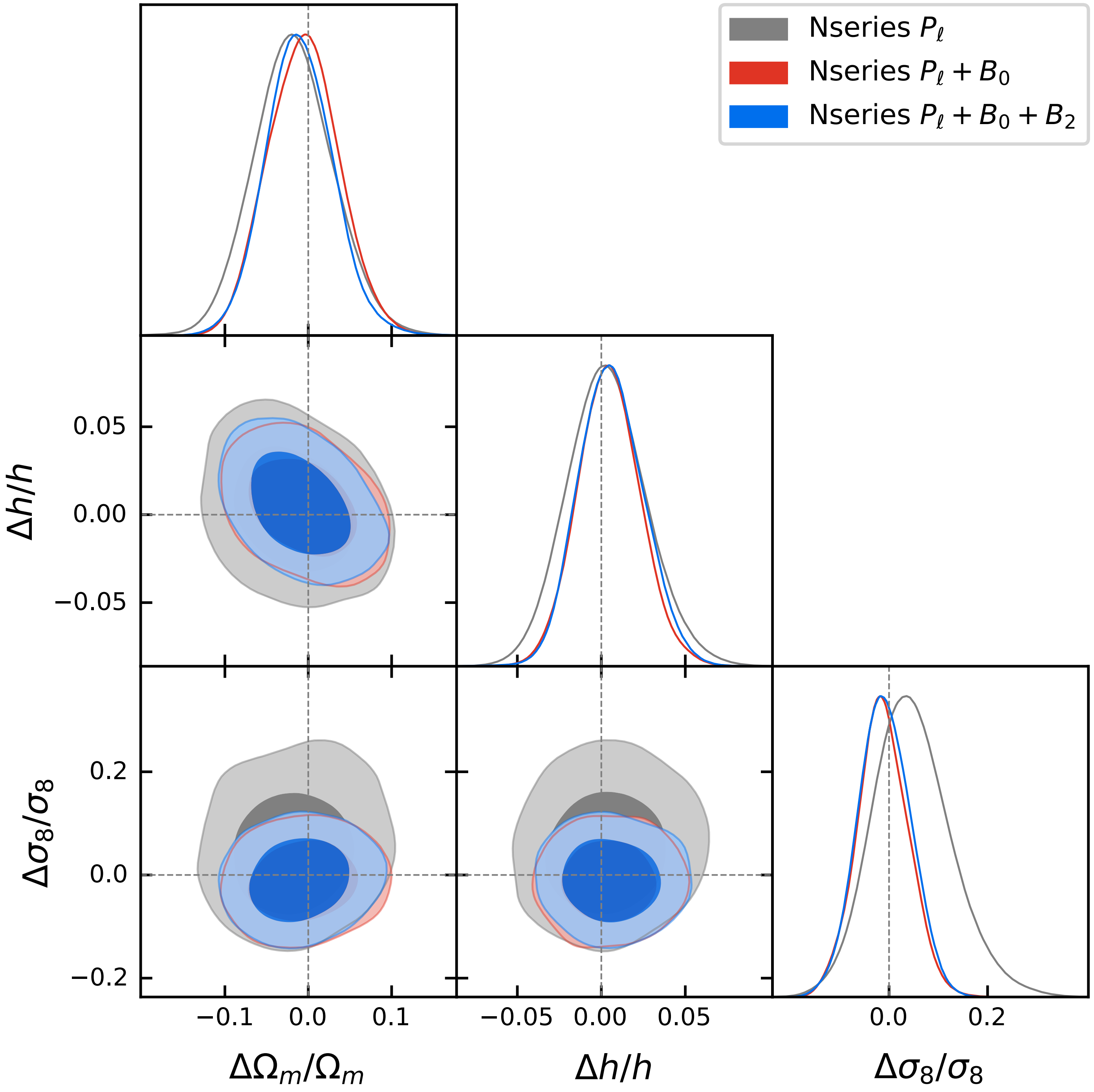
See also Gualdi & Verde (2020), Gualdi *et al.* (2021), D'Amico et al. (2022)

Redshift-space: bispectrum *monopole & quadrupole*

Test of the bispectrum model:
B_0 at 1-loop
B_2 tree-level

CMASS HOD mocks + window

Significant improvement adding B_0 at one-loop, much less adding B_2 tree-level
(but very limited number triangles in this case ...)



D'Amico *et al.* (2022)

Window convolution

The convolution of the bispectrum prediction with the window function is a problem ..

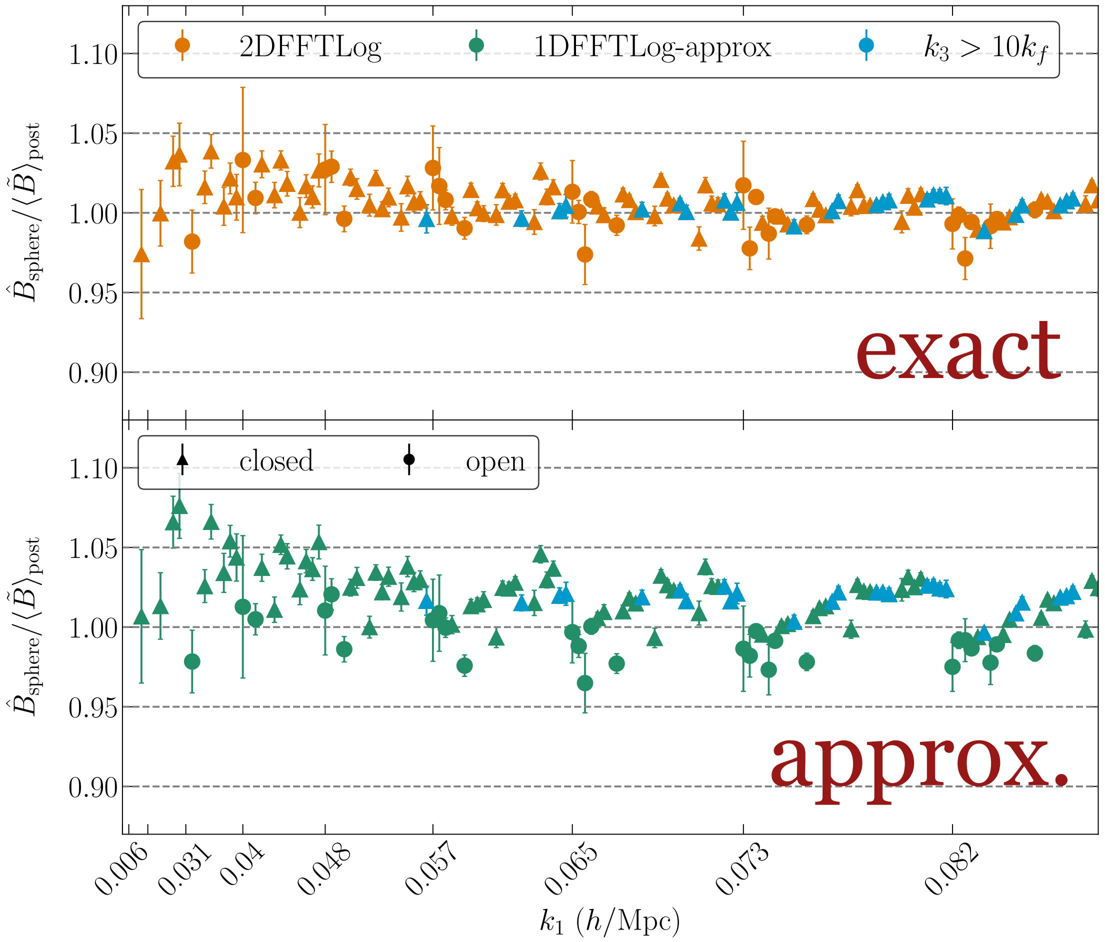
$$\tilde{B}(\mathbf{k}_1,\mathbf{k}_2) = \int \frac{d^3p_1}{(2\pi)^3}\int \frac{d^3p_2}{(2\pi)^3} B_W(\mathbf{k}_1-\mathbf{p}_1,\mathbf{k}_2-\mathbf{p}_2)B(\mathbf{p}_1,\mathbf{p}_2)$$

Three approaches so far:

- “Tree-level” approximation (Gil-Marín *et al.*, 2015)

  $$\tilde{B} \simeq 2Z_1(\mathbf{k}_1)Z_1(\mathbf{k}_2)Z_2(\mathbf{k}_1,\mathbf{k}_2)\tilde{P}(k_1)\tilde{P}(k_2) + \text{perm}\,.$$

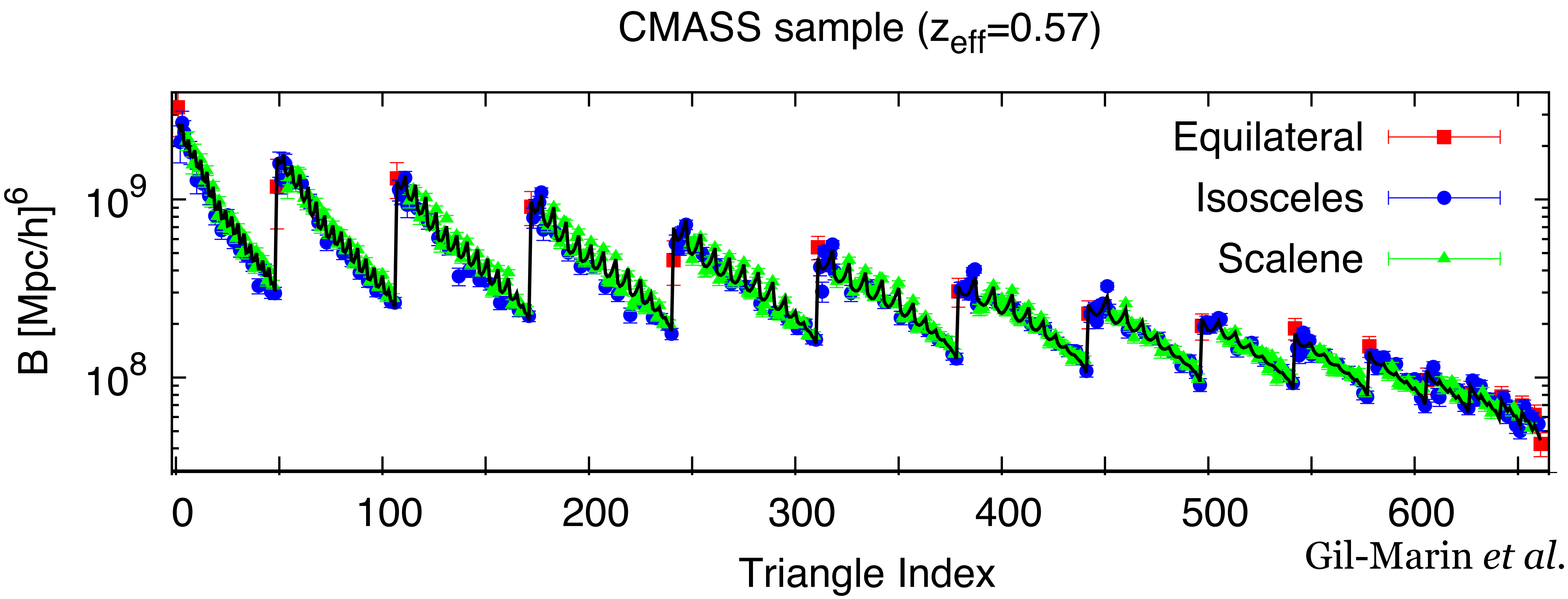
- Windowless estimator (Philcox, 2021)
- Exact convolution (Pardede *et al.*, 2022)



Pardede, Rizzo *et al.* (2022)

Covariance

The bispectrum signal is **distributed over a large number of configurations**



Gil-Marin *et al.* (2017)

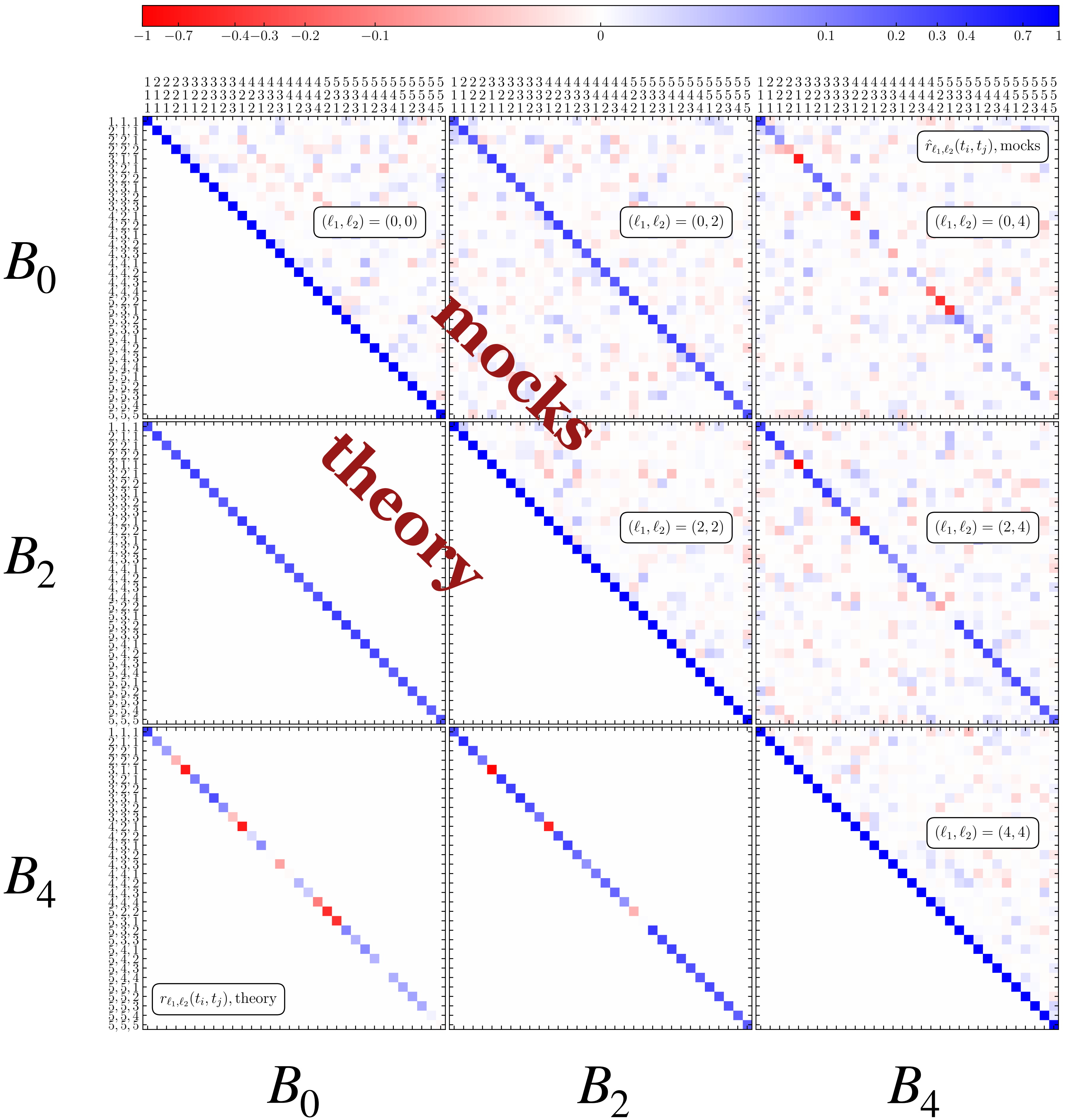
A robust, numerical estimates of such a **large covariance matrix** requires a **large number of mocks**

Covariance

As an alternative, a
theoretical covariance
should be considered ...

Gaussian covariance of
bispectrum multipoles
in a box (no window)
Rizzo, Moretti, Pardede *et al.* (2022)

See also Sugiyama *et al.* (2022)



Or **compression methods**
Gualdi *et al.* (2018; 2019)

To sum up

- The model
  tree-level *vs* **one-loop** *vs* **phenomenological**

- Anisotropy
  monopole *vs* **monopole + quadrupole**

- Window function convolution
  approximated *vs* **exact** *vs* **windowless**

- Covariance
  numerical *vs* theoretical (?)

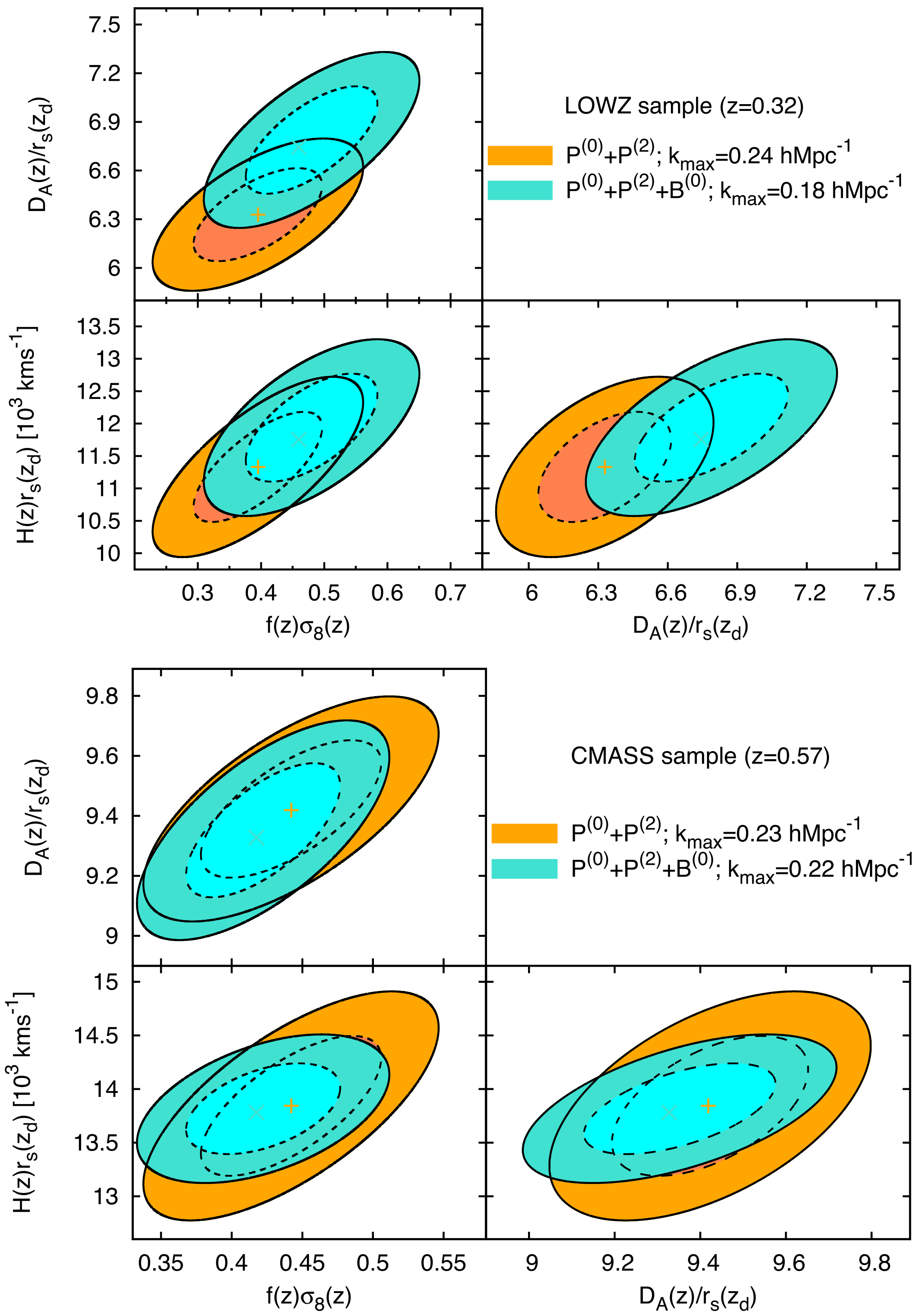
Analysis of BOSS data: Gil-Marin *et al.* (2017)

- *data:* monopole (825 triangles, $\Delta k = 0.01 h\,\mathrm{Mpc}^{-1}$)
- *model*: fit to N-body + tree-level bias & RSD (+AP)

  $0.03\,h\,\mathrm{Mpc}^{-1} \leq k \leq 0.18\,h\,\mathrm{Mpc}^{-1}$
  $0.03\,h\,\mathrm{Mpc}^{-1} \leq k \leq 0.22\,h\,\mathrm{Mpc}^{-1}$
- *window*: approximation

  $$\widetilde{B} \simeq Z_1(\mathbf{k}_1) Z_1(\mathbf{k}_2) Z_2(\mathbf{k}_1, \mathbf{k}_2) \widetilde{P}(k_1) \widetilde{P}(k_2)$$
- *covariance*: numerical (2048 Patchy mocks)
- *analysis*: template fitting

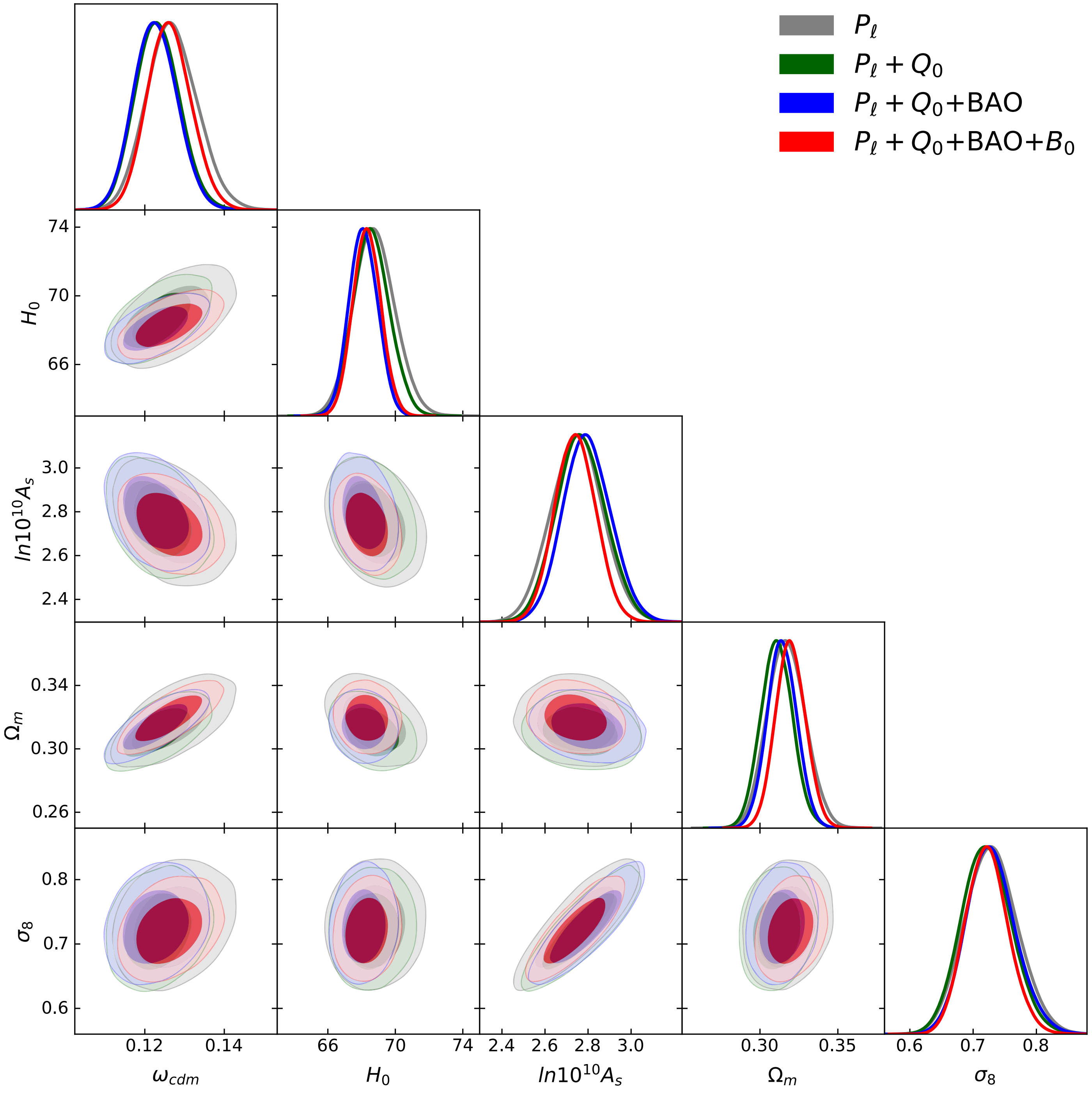
  $$\{b_1, b_2, A_{\mathrm{noise}}, \sigma^P_{\mathrm{FoG}}, \sigma^B_{\mathrm{FoG}}, f, \sigma_8, \alpha_\parallel, \alpha_\perp\}.$$



Significant improvement, up to 50%
(for CMASS) on RSD parameters

Analysis of BOSS data: Philcox & Ivanov (2022)

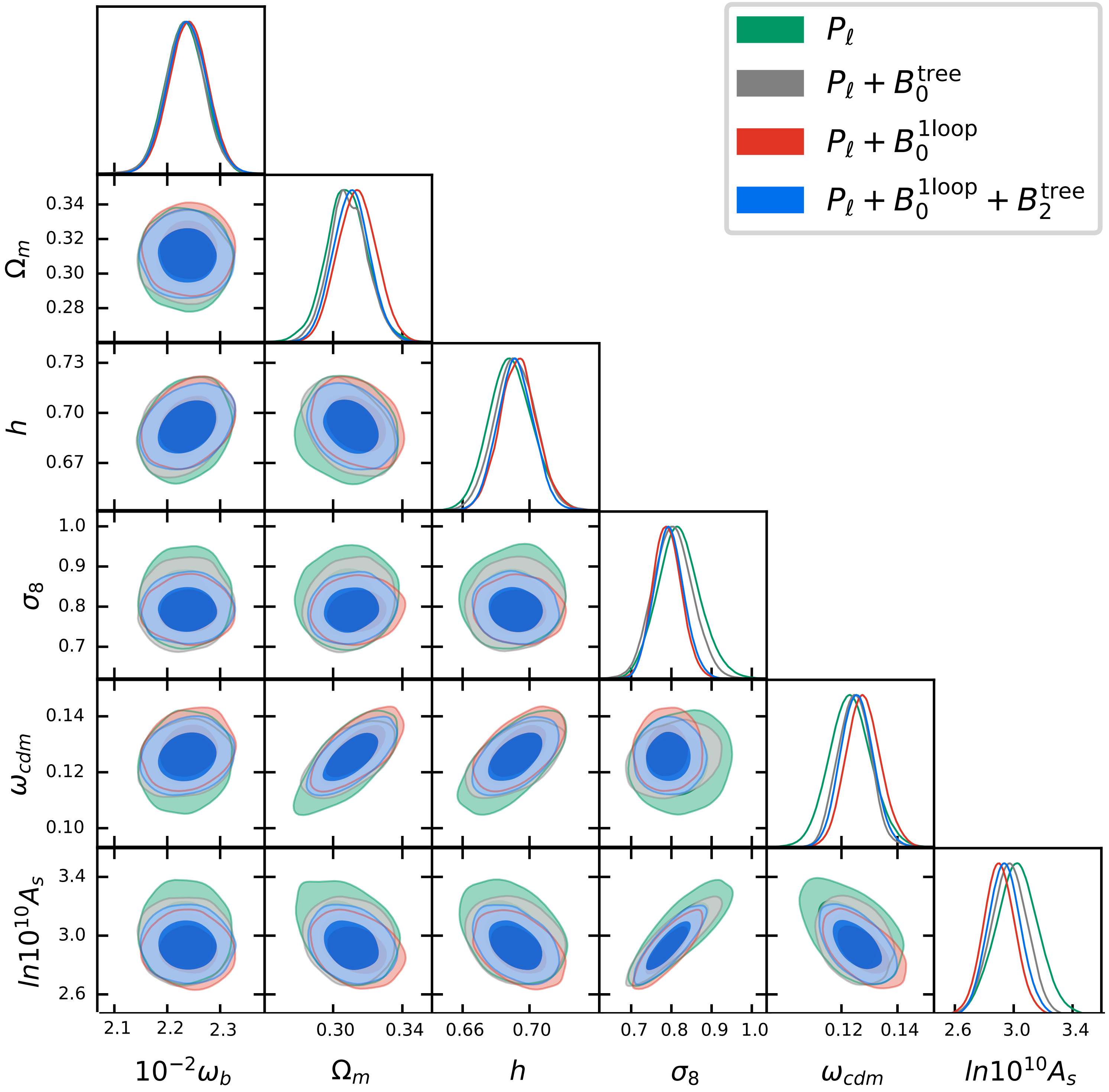
- *data:* monopole
  (62 triangles, $\Delta k = 0.01 h\mathrm{Mpc}^{-1}$,
  $0.01 \leq k \leq 0.08\, h\mathrm{Mpc}^{-1}$)
- *model*: tree-level
- *window*: windowless estimator
- *covariance*: numerical
  (2048 Patchy mocks)
- *analysis*: full-shape
  3/4 cosmo + 13 bias/noise
  parameters



13% improvement on σ_8

Analysis of BOSS data: D'Amico *et al.* (2022)

- *data:* monopole & quadrupole (150 triangles for B_0, 9 for B_2, $\Delta k = 0.02\,h\mathrm{Mpc}^{-1}$, $0.02 \leq k \leq 0.21\,h\mathrm{Mpc}^{-1}$ for CMASS)
- *model*: 1-loop for B_0, tree-level for B_2
- *window*: approximation
- *covariance*: numerical (2048 Patchy mocks)
- *analysis*: full-shape 3 cosmo + 12 bias/noise parameters



Significant improvement (30% for σ_8) from one-loop B_0, rather than B_2

Beyond ΛCDM

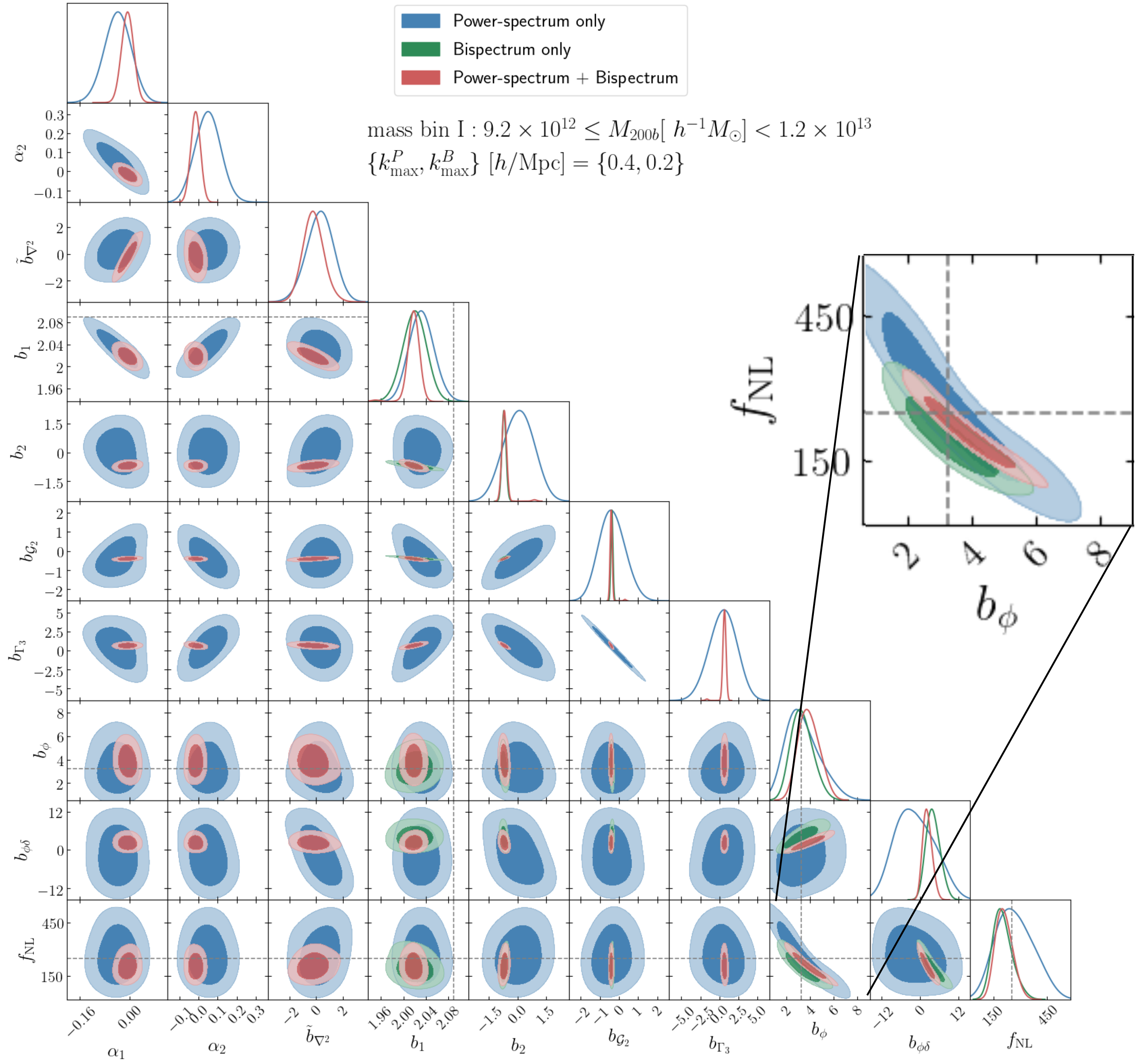
Beyond ΛCDM: Tests with Primordial non-Gaussainity

Test of the power spectrum & bispectrum model in real space

Eos simulations, 80 h^{-3} Gpc^3
Halo catalogs

Significant improvement (factor of 5) over power spectrum only

Also from the reduction of the $f_{\rm NL} - b_\phi$ degeneracy



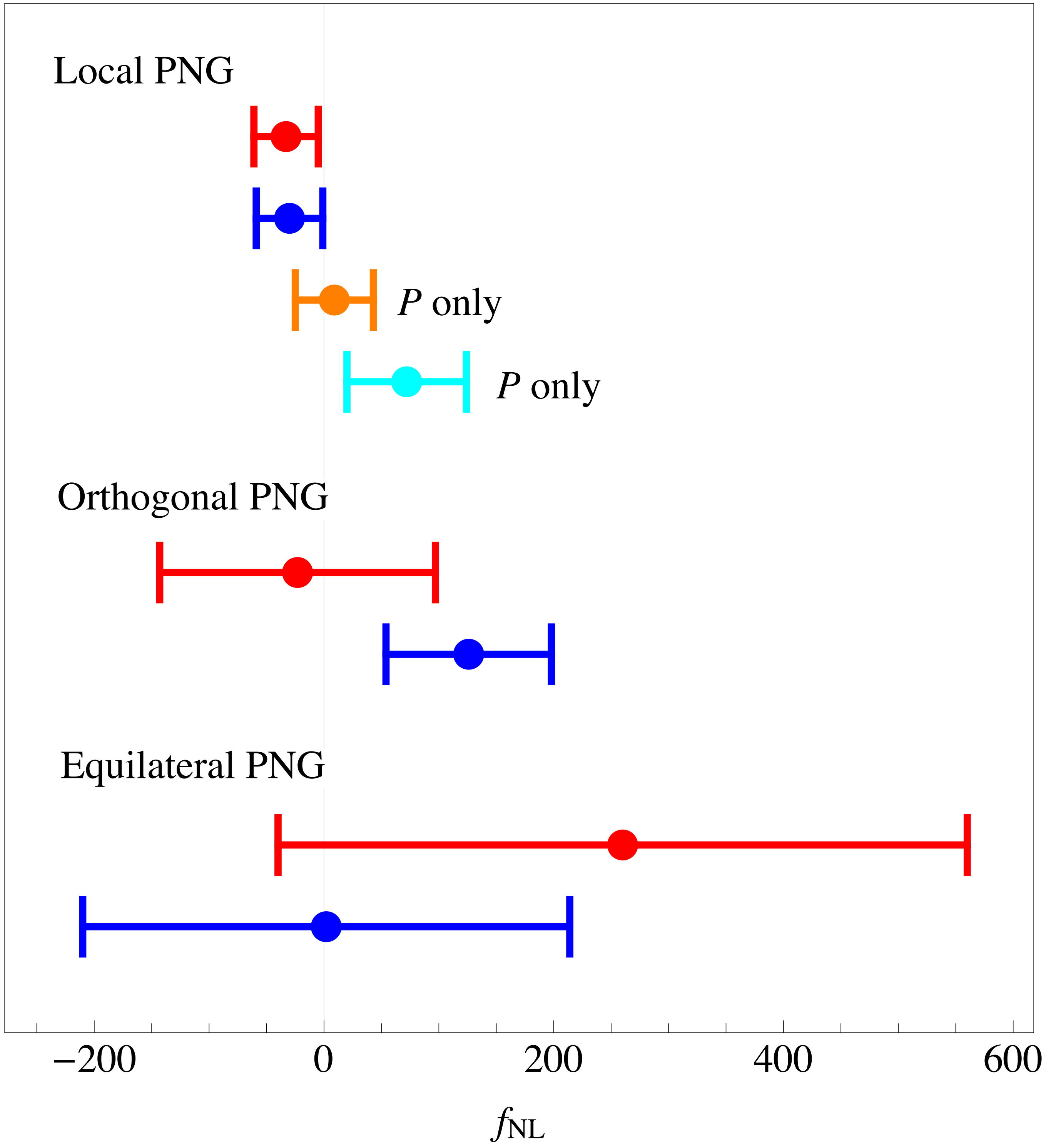
Moradinezhad *et al.* (2021)

BOSS analysis beyond ΛCDM: Primordial non-Gaussainity

1. The bispectrum greatly improves constraints on local PNG and …

2. … it *allows* those on single-field inflation models

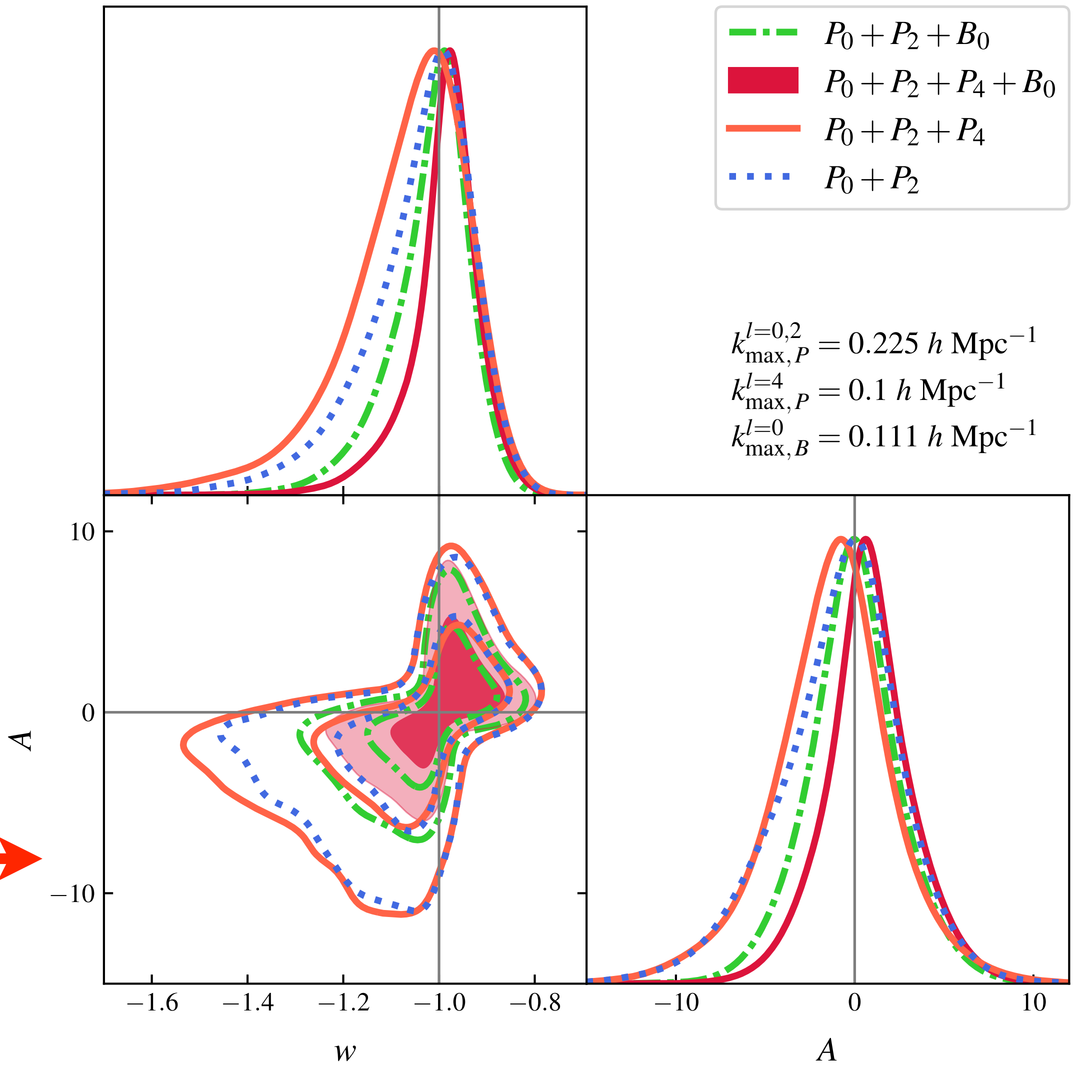
D'Amico *et al.* (2022)
Cabass *et al.* (2022A, 2022B)



BOSS analysis beyond ΛCDM: Interacting Dark Energy

Extensions to ΛCDM are beginning to be explored …

e.g. Interacting Dark Energy constraints from BOSS
Tsedrik *et al.* (2022)



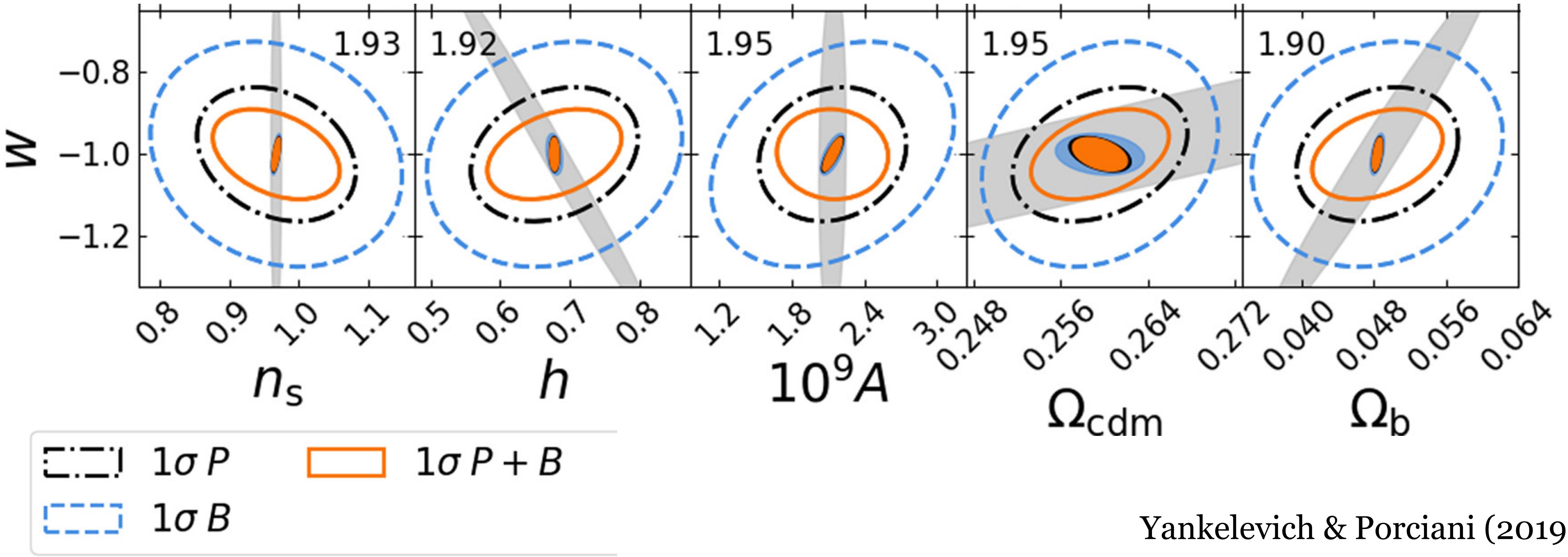
Other directions include, for instance, a bispectrum dipole from GR effects
See e.g. Clarkson *et al.* (2019)

Euclid

Euclid-like forecasts

Fully anisotropic bispectrum: $B_s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$

$k_{\rm max} = 0.15\, h\, {\rm Mpc}^{-1}$



Yankelevich & Porciani (2019)

Significant improvement on dark energy equation of state parameter

The *Euclid* to do list

- The model
  tree-level PT *vs* **one-loop PT** *vs* **phenomenological**
  we probably need to go beyond tree-level, but loop + AP integrations are challenging

- Anisotropy
  monopole *vs* **monopole + quadrupole**
  we already have multipoles estimators, so …

- Window function
  approximated *vs* **exact** *vs* **windowless**
  it would be very *nice to test both exact convolution and windowless*

- Covariance
  numerical *vs* theoretical
  probably a theoretical approach or a compression method are inevitable

- Alternative estimators
  Skew-spectra (Schmittfull *et al*., 2015; Moradinezhad *et al*. 2020; …)
  Tri-polar Spherical Harmonic Decomposition (Sugiyama *et al*., 2017)
  Modal estimator (Fergusson *et al*., 2012; Byun *et al*., 2021) … and more …