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Primordial Black Holes from Dissipation During Inflation

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2. Background





4. GWs

Black holes from inflation



Quantum fluctuations in ϕ , g, are strechted by expansion

$$\langle \mathcal{R}(k)\mathcal{R}^*(k')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}\delta(k-k') \longrightarrow \text{single field:} \mathcal{P}_{\mathcal{R}}(k_*) \simeq \frac{H_*^4}{4\pi^2\dot{\phi}_*^2}$$



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3. (Random) fluctuations



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Black holes from inflation



(Y. Mambrini)

Quantum fluctuations in $\phi,\,g,$ are strechted by expansion

$$\langle \mathcal{R}(k)\mathcal{R}^*(k')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}\delta(k-k') \longrightarrow \text{single field:} \mathcal{P}_{\mathcal{R}}(k_*) \simeq \frac{H_*^4}{4\pi^2\dot{\phi}_*^2}$$

Given critical threshold, $\delta_c \approx 0.5$, the fraction of BHs of mass M is

$$\beta(M) = \int_{\delta_c}^{\infty} d\delta P(\delta, M)$$

Typically
$$P \propto e^{-\delta^2/2\sigma^2(M)}$$
, and

overdense

$$\sigma^2(M(k)) = \frac{16}{81} \int \frac{dq}{q} \left(\frac{q}{k}\right)^4 \mathcal{P}_{\mathcal{R}}(q) W(q/k)^2$$

W. Press, P. Schechter, Astrophys. J. 187 (1974), 425





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1. PBHs from inflation

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Peaks in the power spectrum



A. Green, B. Kavanagh, J. Phys. G 48 (2021), 043001

Simplest mechanism: ultra slow-roll, $\,{\cal P}_{\cal R}\propto \dot{\phi}^{-2}\,$







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Slow roll from particle production

Non-adiabatic, non-thermalized production

$$\mathcal{L} \supset -\frac{1}{2}g^2\sum_i(\phi-\phi_i)^2\chi_i^2$$

 $\boldsymbol{\chi}$

(trapped inflation)

 ϕ

D. Green *et al.*, PRD 80 (2009), 063533 L. Pearce, M. Peloso, L. Sorbo, JCAP 11 (2016), 058

 $\boldsymbol{\chi}$



Slow roll from particle production

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 ϕ



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where

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Dissipation during inflation

The total (inflaton+radiation) stress-energy tensor is conserved, $\nabla_{\mu} T^{\mu\nu} = 0$, but individually,

$$\nabla_{\mu} T^{\mu\nu}_{(\phi)} = Q^{\nu}$$
$$\nabla_{\mu} T^{\mu\nu}_{(r)} = -Q^{\nu}$$

(M. Gleiser, R. Ramos, PRD 50 (1994) 2441; M. Bastero-Gil et.al., JCAP 05 (2014) 004)

$$Q_{\mu} = \underbrace{-\Gamma u^{\nu} \nabla_{\nu} \phi \nabla_{\mu} \phi}_{\text{dissipation}} + \underbrace{\sqrt{\frac{2\Gamma T}{a^{3}}} \xi_{t} \nabla_{\mu} \phi}_{\text{fluctuation}}$$

 ξ_t denotes a normalized white-noise process, $\xi_t = dW_t/dt$, with dW_t a Wiener increment,

$$\langle \xi_t(\mathbf{x})\xi_{t'}(\mathbf{x}')\rangle = \delta^{(3)}(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

Only dissipation affects the background

inflation 0

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Background dynamics

 $\ddot{\phi}$

$$\begin{aligned} + (3H+\Gamma)\dot{\phi} + V_{\phi} &= 0\\ \dot{\rho}_r + 4H\rho_r &= \Gamma\dot{\phi}^2\\ \rho_r + \frac{1}{2}\dot{\phi}^2 + V &= 3M_P^2H^2 \end{aligned}$$

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Localized dissipation?

Inflaton coupled to a d.o.f. χ connected or part of the thermal bath

 $V(\phi, \chi) = V(\phi) + f(\phi)g(\chi)$

Integration over χ and an ensamble averaging,

with

$$\Gamma = -i \int d^4x \left(\frac{\partial f}{\partial \phi}\right)^2 \theta(t-t') \left\langle \left[g(\chi(x)) - g(\chi(x'))\right] \right\rangle (t-t')$$

A. Hosoya, M. Sakagami, PRD 29 (1984), 2228; M. Bastero-Gil, A. Berera, R. Ramos, JCAP 09 (2011), 033

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Localized dissipation?

Thermal corrections to the potential can be suppressed via heavy field exchange, $\,\phi o \chi o \sigma$

$$\mathcal{L} = \frac{1}{2\mathcal{K}(\varphi)}\partial^{\mu}\varphi\partial_{\mu}\varphi + \frac{1}{2}\partial^{\mu}\chi\partial_{\mu}\chi + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - \frac{1}{2}g^{2}\varphi^{2}\chi^{2} - \frac{1}{2}\tilde{g}^{2}\varphi\chi\sigma^{2} - V(\varphi)$$

With ϕ the canonically normalized field,

$$\Gamma = \frac{g^4 (\partial_\phi \varphi^2)^2}{2T} \int \frac{d^4 p}{(2\pi)^4} n(\omega) [n(\omega) + 1] \rho_{\chi}^2(\omega)$$

$$\simeq \frac{40 g^4 (\partial_\phi \varphi^2)^2 \Gamma_{\chi,0}^2 T^3}{(2\pi)^3 m_{\chi}^6} \qquad (T \ll m_{\chi})$$

where

$$\rho_{\chi} = \frac{4\omega_p \Gamma_{\chi}}{(\omega^2 - \omega_p^2)^2 + 4\omega_p^2 \Gamma_{\chi}^2}$$
$$n(\omega) = \left(e^{\omega T} - 1\right)^{-1}$$
$$\Gamma_{\chi} \simeq \frac{\tilde{g}^4 \varphi^2}{8\pi \omega_p(\mathbf{p})}$$

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Localized dissipation?

Thermal corrections to the potential can be suppressed via heavy field exchange, $\phi \to \chi \to \sigma$ $\mathcal{L} = \frac{1}{2\mathcal{K}(\omega)}\partial^{\mu}\varphi\partial_{\mu}\varphi + \frac{1}{2}\partial^{\mu}\chi\partial_{\mu}\chi + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - \frac{1}{2}g^{2}\varphi^{2}\chi^{2} - \frac{1}{2}\tilde{g}^{2}\varphi\chi\sigma^{2} - V(\varphi)$ $\mathcal{K}(\varphi) = \frac{4\varphi^4}{(1+\varphi^2)^2}$ $V(\varphi) = \lambda(\varphi + \beta)^2$ 6 $/M_P^4 \ [\times 10^{-11}]$ ϕ_{end} $V(\phi) = \lambda \left(\phi - \sqrt{(\phi - \phi_\star)^2 + 1} + \sqrt{1 + \phi_\star^2} \right)^2$ $\Gamma = \frac{5}{16\pi^5} \left(\frac{\tilde{g}}{c}\right)^4 \frac{T^3}{1+(\phi-\phi_*)^2}$ -50 5 10 15 ϕ/M_P

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Scalar fluctuations

Introduce the set of linear fluctuations in the Newtonian gauge, $\delta\phi$, $\delta\rho_r$, $\delta q_r = \frac{4}{3}\rho_r\delta v_r$, and $ds^2 = (1+2\psi)dt^2 - a^2(1-2\psi)\delta_{ij}dx^i dx^j$

Einstein's equations lead to

r

$$\begin{aligned} 3H(\dot{\psi} + H\psi) + \frac{k^2}{a^2}\psi &= -\frac{1}{2M_p^2} \left[\delta\rho_r + \dot{\phi}(\delta\dot{\phi} - \dot{\phi}\psi) + V_{\phi}\delta\phi \right] \\ \dot{\psi} + H\psi &= -\frac{1}{2M_p^2} \left(\delta q_r - \dot{\phi}\delta\phi \right) \\ \ddot{\psi} + 4H\dot{\psi} + (2\dot{H} + 3H^2)\psi &= \frac{1}{2M_p^2} \left[\frac{1}{3}\delta\rho_r + \dot{\phi}(\delta\dot{\phi} + \dot{\phi}\psi) - V_{\phi}\delta\phi \right] \end{aligned}$$

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Scalar fluctuations

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Continuity equations in turn give

(M. Bastero-Gil et.al., JCAP 05 (2014) 004)

$$\begin{split} \delta\ddot{\phi} + (3H+\Gamma)\delta\dot{\phi} + \left(\frac{k^2}{a^2} + V_{\phi\phi} + \dot{\phi}\Gamma_{\phi}\right)\delta\phi + \Gamma_T\frac{\dot{\phi}T}{4\rho_r}\delta\rho_r - 4\dot{\psi}\dot{\phi} + (2V_{\phi} + \Gamma\dot{\phi})\psi &= \sqrt{\frac{2\Gamma T}{a^3}}\xi_t\\ \delta\dot{\rho}_r + \left(4H - \Gamma_T\frac{\dot{\phi}^2 T}{4\rho_r}\right)\delta\rho_r - \frac{k^2}{a^2}\delta q_r + \Gamma\dot{\phi}^2\psi - 4\rho_r\dot{\psi} - (\Gamma_{\phi}\delta\phi - 2\Gamma\delta\dot{\phi})\dot{\phi} &= -\sqrt{\frac{2\Gamma T}{a^3}}\dot{\phi}\xi_t\\ \delta\dot{q}_r + \frac{4}{3}\rho_r\psi + 3H\delta q_r + \frac{1}{3}\delta\rho_r + \Gamma\dot{\phi}\delta\phi &= 0 \end{split}$$

Solutions reach an attractor, so initial conditions may be chosen as

$$\delta q_r = 0, \qquad \delta \rho_r = 0, \qquad \psi = 0, \qquad \delta \phi = -\frac{\dot{\phi} e^{-ik\tau}}{2M_p \ aH\sqrt{k\epsilon}}$$

Difficult to find their gauge-invariant versions!

5. Conclusion

(we use a stochastic RK method)

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Frequentist vs deterministic

ltô's lemma:

$$dX = a X dt + b dW_t$$

$$dX^2 = (2a X^2 + b^2) dt + 2b X dW_t$$

$$\Rightarrow \qquad \frac{d\langle X \rangle}{dt} = a \langle X \rangle$$

$$\frac{d\langle X^2 \rangle}{dt} = 2a \langle X^2 \rangle + b^2$$

J/V

For cosmological perturbations,

$$rac{d\langle oldsymbol{\Phi} oldsymbol{\Phi}^\dagger
angle - oldsymbol{A} \langle oldsymbol{\Phi} oldsymbol{\Phi}^\dagger
angle - \langle oldsymbol{\Phi} oldsymbol{\Phi}^\dagger
angle oldsymbol{A}^{
m T} + oldsymbol{B} oldsymbol{B}^{
m T}$$

The curvature power spectrum is

(

$$\langle \mathcal{P}_{\mathcal{R}}
angle \; = \; \left. rac{k^3}{2\pi^2} oldsymbol{C}^{\mathrm{T}} \langle oldsymbol{\Phi} oldsymbol{\Phi}^{\dagger}
angle oldsymbol{C}
ight|_{k \ll aH}$$

with $\mathcal{R} = oldsymbol{C}^T oldsymbol{\Phi}$

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Analytical approximation

- Decouple $\delta \phi$: $\dot{\delta \phi} + (\dot{\delta \phi}, \delta \phi, \mathsf{background}) = f_{\phi}(t) \xi_t$
- Approximate ${\cal R}~pprox~-rac{\delta\phi}{\phi'}$
- Parametrize background as piecewise constants
- Solve homogeneous equation, $\,\ddot{\delta\phi}+(\dot{\delta\phi},\delta\phi,{\sf background})\,=\,0\,,\,\,{\sf and}\,\,{\sf find}\,\,{\sf Green's}\,\,{\sf function}$

• Formally solve,
$$\delta \phi(t) = \delta \phi^{(h)}(t) + \int dt' \ G(t,t') f_{\phi}(t') \xi_{t'}$$

•
$$\mathcal{P}_{\delta\phi} = \mathcal{P}_{\delta\phi}^{(h)}(t) + \int dt' G(t,t')^2 f_{\phi}(t')^2 \longrightarrow \mathcal{P}_{\mathcal{R}}$$

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Induced gravitational waves

Second order induced GWs,

$$h_k^{s\prime\prime} + 2\mathcal{H}h_k^{s\prime} + k^2 h_k^s = S_k^s$$

$$S_{k}^{s} = \int \frac{3p}{(2\pi)^{3}} \mathbf{e}_{ij}^{s}(\mathbf{k}) p_{i} p_{j} \left[8\psi_{p}\psi_{k-p} + \frac{16\rho}{3(\rho+p)} \left(\psi_{p} + \frac{1}{\mathcal{H}}\psi_{p}'\right) \left(\psi_{k-p} + \frac{1}{\mathcal{H}}\psi_{k-p}'\right) \right]$$

Y. Lu et al., PRD 102 (2020), 083503

The source needs to be tracked during and after inflation,

$$S_{k(\text{pre})}^{s} = \frac{4}{3} \left(\frac{\rho}{\rho+p} \right) \left(\frac{\phi'^{2}}{\mathcal{H}^{2} M_{p}^{4}} \right) \int \frac{3p}{(2\pi)^{3}} \mathbf{e}^{s}(\mathbf{k}, \mathbf{p}) \delta \phi_{p} \delta \phi_{k-p}$$

$$S_{k(\text{post})}^{s} = \int \frac{3p}{(2\pi)^{3}} \mathbf{e}^{s}(\mathbf{k}, \mathbf{p}) \left[8\psi_{p} \psi_{k-p} + 4 \left(\psi_{p} + \frac{1}{\mathcal{H}} \psi_{p}' \right) \left(\psi_{k-p} + \frac{1}{\mathcal{H}} \psi_{k-p}' \right) \right]$$

$$(s = (+, \times))$$

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- Localized dissipation can be realized with non-minimal kinetic terms
- Dissipation affects the background, but does not drive the enhancement in $\mathcal{P}_\mathcal{R}$
- Thermal fluctuations are the main source for a peaked $\mathcal{P}_\mathcal{R}$
- Thermal attractor allows for analytical estimates
- First full determination of the stochastic GW signal
- Induced growth of fluctuations during reheating?

Thank you