Non-perturbative gravitational production of Vector Dark Matter

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based on: A. Ahmed, B. Grządkowski, AS 2005.01766

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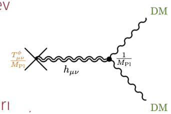

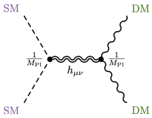

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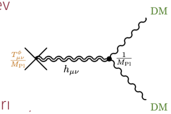

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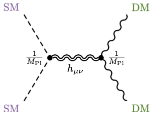

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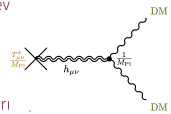

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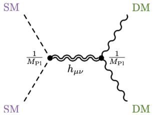

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The action for a massive spin-1 DM spectator field X_μ in a background metric $g_{\mu\nu}$ is given by

$X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$

$$S_{\rm DM} = \int d^4x \sqrt{-g}\mathcal{L}_{\rm DM}, \quad \mathcal{L}_{\rm DM} = -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} X_{\mu\nu} X_{\alpha\beta} - \frac{1}{2} m_X^2 g^{\mu\nu} X_\mu X_\nu .$$

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Gravitational coupling

only through $g^{\mu\nu}$, $\sqrt{-g}$

without no-minimal coupling, i.e.,

$\xi_1 R\, g^{\mu\nu} X_\mu X_\nu, \xi_2 R^{\mu\nu} X_\mu X_\nu$

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Non-zero mass

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Remarks

⇨ X_0 does not have a kinetic term; it is an auxiliary field.

$$\mathcal{X}_0 = \frac{-i\vec{k}\cdot\vec{\mathcal{X}}'}{k^2 + a^2 m_X^2}$$

⇨ Massless vectors with a minimal coupling to gravity, i.e., $\xi_1 = 0 = \xi_2$ are conformally coupled to gravity.

Not populated by expansion

EoMs for DM fields

Fourier decomposition

$$[\hat{a}_\lambda(\vec{p}), \hat{a}^\dagger_\sigma(\vec{q})] = (2\pi)^3 \delta_{\lambda,\sigma}\delta^{(3)}(\vec{p} - \vec{q})$$

$$\hat{\vec{X}}(\tau, \vec{x}) = \sum_{\lambda=\pm,L} \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \vec{\epsilon}_\lambda(\vec{k}) \{\mathcal{X}_\lambda(\tau, k)\hat{a}_\lambda(\vec{k}) + \mathcal{X}^*_\lambda(\tau, k)\hat{a}^\dagger_\lambda(\vec{k})\}$$

Harmonic oscillator equation, Wronskian

$$\mathcal{X}''_\lambda + \omega^2_\lambda(\tau)\mathcal{X}_\lambda = 0, \qquad \mathcal{X}'_\lambda\mathcal{X}^*_\lambda - \mathcal{X}^{*\prime}_\lambda\mathcal{X}_\lambda = -\mathrm{i}$$

Time-dependent frequencies

Transverse modes

$$\omega^2_\pm(\tau) \equiv k^2 + a^2(\tau)m^2_X,$$

Redefined longitudinal mode*

$$\omega^2_L(\tau) \equiv k^2 + a^2(\tau)m^2_X - \frac{k^2}{k^2 + a^2(\tau)m^2_X}\left[\frac{a''(\tau)}{a(\tau)} - \frac{3a^2(\tau)m^2_X}{k^2 + a^2(\tau)m^2_X}\left(\frac{a'(\tau)}{a(\tau)}\right)^2\right]$$

Longitudinal mode $\mathcal{A}_L$ is not canonically normalized

$$\mathcal{X}_L \equiv \frac{am_X}{\sqrt{k^2 + a^2m^2_X}}\mathcal{A}_L$$

The vacuum expectation value of the total energy density od vector DM field decomposes as,

$$\langle \hat{\rho}_X \rangle \equiv \langle 0 | \hat{\rho}_X | 0 \rangle = \langle \hat{\rho}_L \rangle + \langle \hat{\rho}_\pm \rangle,$$

Bunch-Davies vacuum

with

Transverse modes

$$\langle \hat{\rho}_\pm \rangle = \frac{1}{2a^4} \int \frac{d^3\vec{k}}{(2\pi)^3} \left\{ |\mathcal{X}_\pm'|^2 + |\mathcal{X}_\pm|^2 \left[k^2 + a^2 m_X^2 \right] \right\},$$

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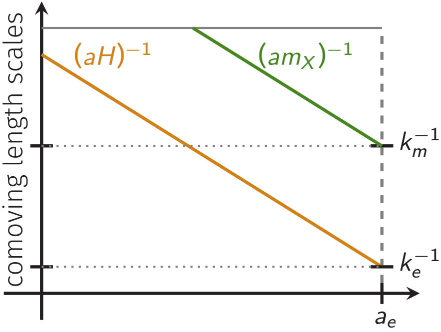
Note that $\langle \hat{\rho}_L \rangle \gg \langle \hat{\rho}_\pm \rangle$

This could change if one considers direct X_μ coupling to the inflaton ϕ of the form $\phi X_{\mu\nu} \tilde{X}^{\mu\nu}$, see e.g. arXiv:1810.07208

Evolution of the longitudinal modes during inflation



de Sitter Inflation

$(a \leq a_e)$

$$a_{\mathrm{I}} \simeq -\frac{1}{H_e(\tau - 3\tau_e(1+\overline{w})/2)}, \quad H_{\mathrm{I}} \simeq H_e,$$

$$\left(\frac{a'}{a}\right)^2 = a_{\mathrm{I}}^2 H_e^2, \qquad \frac{a''}{a} = 2a_{\mathrm{I}}^2 H_e^2.$$

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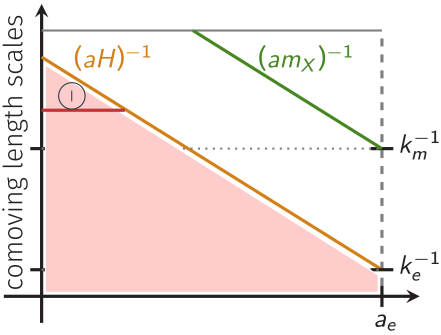

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$\omega_L^2 \simeq k^2,$

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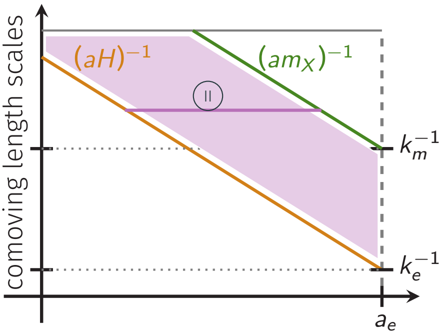

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(II) **Super-horizon R**

$$\omega_L^2 \simeq k^2 - 2a_{\rm I}^2 H_e^2 < 0,$$

tachyonic enhancement

$$\mathcal{X}_L^{\rm II} \simeq \frac{i}{2\sqrt{k}} \frac{a_{\rm I} H_e}{k} e^{-ik\tau},$$

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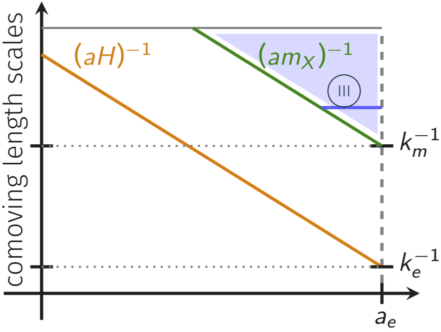

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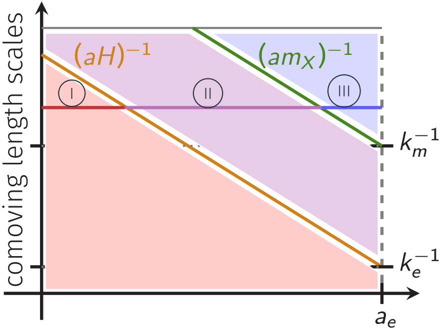
III Super-horizon NR

$$\omega_L^2 \simeq a_{\mathrm{I}}^2 m_X^2 + k^2 \frac{H_I^2}{m_X^2},$$

constant solution

$$\mathcal{X}_L^{(\mathrm{III})} \simeq \mathrm{const.}$$

Evolution of the longitudinal modes during inflation



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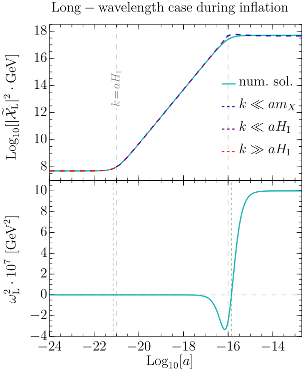
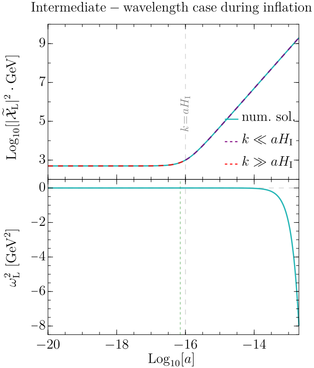
Short-wavelength (I) — Intermediate-wavelength (I–II) — Long-wavelength (I–III)

Evolution of the logitudinal modes during inflation





at the end of inflation

sub-horizon modes $k \gg k_e$

$$\frac{d\langle\hat{\rho}_L^e\rangle}{d\ln k} \approx \frac{1}{4\pi^2}\frac{k^4}{a_e^4}$$

UV divergent, cut-off scale $\Lambda_{\mathrm{UV}} = k_e$

super-horizon modes $k \ll k_e$

$$\frac{d\langle\hat{\rho}_L^e\rangle}{d\ln k} \approx \frac{1}{2}\frac{k^2}{a_e^2}\frac{H_e^2}{4\pi^2}$$

$$\frac{d\langle\hat{\rho}_L\rangle}{d\ln k} = \frac{k^3}{4\pi^2 a^4}\left\{|\mathcal{X}_L'|^2 - \frac{a'}{a}(\mathcal{X}_L'\mathcal{X}_L^* + \mathcal{X}_L^{*\prime}\mathcal{X}_L)\frac{k^2}{k^2+a^2m_X^2} + |\mathcal{X}_L|^2\left[k^2+a^2m_X^2+\left(\frac{a'}{a}\right)^2\frac{k^4}{(k^2+a^2m_X^2)^2}\right]\right\}$$

Post-inflationary evolution

Boltzmann equations

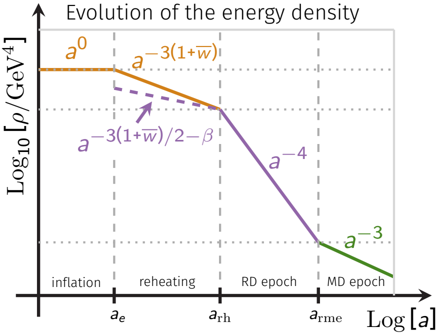
$$\dot{\rho}_\phi + 3H(1+\overline{w})\rho_\phi = -\Gamma_\phi \rho_\phi$$

$$\dot{\rho}_{\rm SM} + 4H\rho_{\rm SM} = \Gamma_\phi \rho_\phi$$

$$\Gamma_\phi \propto a^{-\beta}$$

Friedmann equations

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\rho_\phi + \rho_{\rm SM}\right)$$



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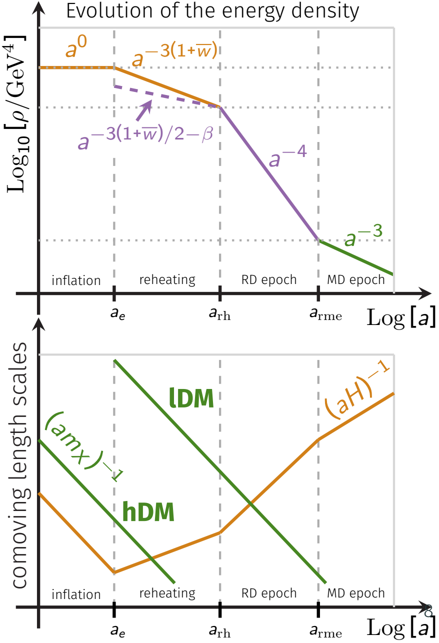
Evolution of a, H and the Hubble radius

Reheating

$$a(\tau) = a_e\,(\tau/\tau_e)^{\frac{2}{1+3\overline{w}}}\,, \qquad H(a) = H_e(a/a_e)^{-\frac{3(1+\overline{w})}{2}}$$

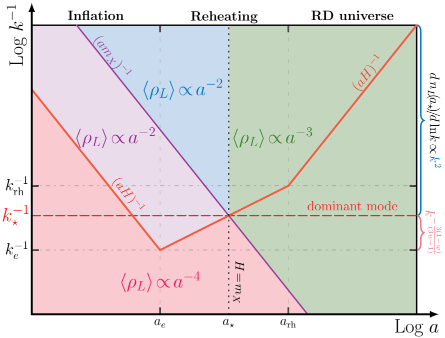
RD epoch

$$a(\tau) = a_{\rm rh}\,(\tau/\tau_{\rm rh})\,, \qquad H(a) = H_{\rm rh}(a/a_{\rm rh})^{-2}$$



heavy DM vectors (hDM)

$m_X > H_{\rm rh}$



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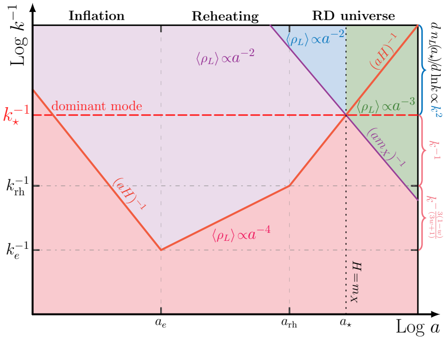

dominant mode

$$k_\star \equiv a_\star m_X, \qquad H(a_\star) = m_X$$

- hDM: $H_{\rm rh} \leq m_X < H_e$,

$$\frac{d\langle n_{\rm L}^{\rm hDM}(a_\star)\rangle}{d\ln k} = \frac{H_e^3}{8\pi}\begin{cases}\left(\frac{m_X}{H_e}\right)^{\frac{2}{1+\overline{w}}}\left(\frac{k_e}{k}\right)^{\frac{3(1-\overline{w})}{(1+3\overline{w})}}, & k_\star < k < k_e,\\ \left(\frac{m_X}{H_e}\right)^{\frac{1-3\overline{w}}{3(1+\overline{w})}}\left(\frac{k}{k_e}\right)^2, & k < k_\star.\end{cases}$$

- lDM: $m_X < H_{\rm rh}$,

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$k_{\rm e} \equiv a_e H_e$

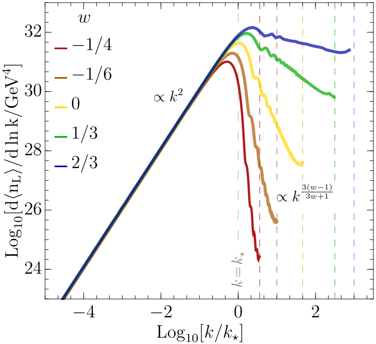
$k_\star \equiv a_\star m_X$

reheating efficiency

$\gamma^2 \equiv H_{\rm rh}/H_e$

$k_{\rm rh} \equiv a_{\rm rh} H_{\rm rh}$

$\frac{d\langle n_{\rm L}(a_\star)\rangle}{d\ln k}(k)$ for $H_{\rm rh} < m_X < H_I$



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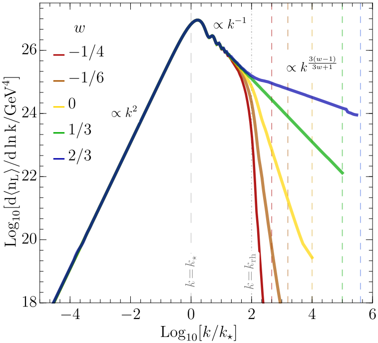


Relic abundance

The present day value of the DM relic density

$$\Omega_X^{\rm obs} h^2 \equiv \frac{\rho_X(a_0)}{\rho_c} h^2 \simeq 0.1198 \pm 0.0012$$

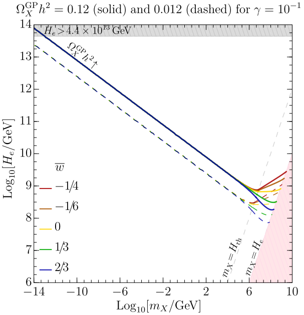
Predicted value

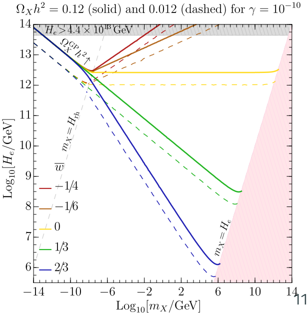
$$\Omega_X^{\rm GP} h^2 \simeq \frac{s_0\, h^2}{4 M_{\rm Pl}^2\, \rho_c} \frac{T_{\rm rh}}{m_X} \begin{cases} \left(\frac{m_X}{H_{\rm rh}}\right)^{\frac{2\overline{w}}{1+\overline{w}}} \langle n_L^{\rm hDM}(a_\star)\rangle, & \text{hDM} \\ \sqrt{\frac{m_X}{H_{\rm rh}}}\, \langle n_L^{\rm lDM}(a_\star)\rangle, & \text{lDM} \end{cases}$$

Sensitive to the details of reheating

$$\Omega_X^{\rm GP} h^2 \approx 0.12 \times \begin{cases} \left(\frac{m_X}{0.33\,\text{GeV}}\right)^{\frac{2\overline{w}}{1+\overline{w}}} \left(\frac{H_e}{3.3\times 10^{10}\,\text{GeV}}\right)^{\frac{5+\overline{w}}{2(1+\overline{w})}} \left(\frac{10^{-5}}{\gamma}\right)^{\frac{3\overline{w}-1}{1+\overline{w}}}, & \text{hDM} \\ \left(\frac{m_X}{2\times 10^{-14}\,\text{GeV}}\right)^{1/2} \left(\frac{H_e}{6.6\times 10^{13}\,\text{GeV}}\right)^{2}, & \text{lDM} \end{cases}$$

Independent of the details of reheating





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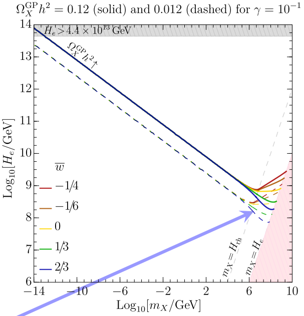

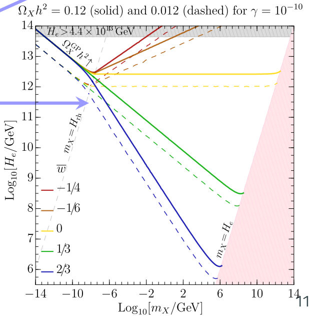

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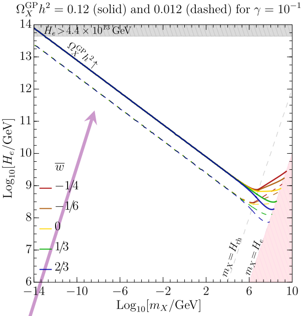

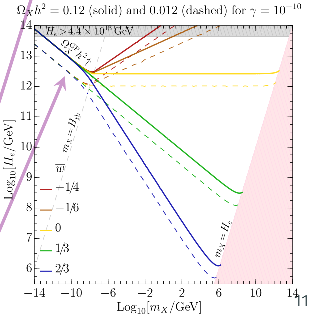

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- We have demonstrated that accounting for the finite duration of reheating has a significant impact on the production of heavy DM vectors, i.e., with mass $m_X > H_{\rm rh}$.
- Finally, we have shown that the non-standard expansion history play a crucial role in the gravitational producation of heavy DM species.

Thank you for your attention!

Back-up slides

The isocurvature constraints are suppressed if

$$k_\star \ll k_{\rm CMB} \approx 0.05\,\mathrm{Mpc}^{-1}.$$

$$k_\star \approx 1400\,\mathrm{pc}^{-1}\sqrt{\frac{m_X}{10^{-14}\mathrm{GeV}}}\begin{cases}\left(\frac{H_{\rm rh}}{m_X}\right)^{\frac{1-3\overline{w}}{6(1+\overline{w})}}, & H_{\rm rh}\leq m_X < H_e\,,\\ 1\,, & H_{\rm mre}\leq m_X < H_{\rm rh}\,.\end{cases}$$

lDM vectors with mass $m_X \geq 10^{-14}\mathrm{GeV}$ are safe

$$\left(\frac{H_{\rm rh}}{m_X}\right)^{\frac{1-3w}{6(1+w)}} = \begin{cases}\left(\frac{m_X}{H_{\rm rh}}\right)^{[0,\,1/6)} \geq 1\,, & w=[1/3,\ 1)\,,\\ \left(\frac{H_{\rm rh}}{m_X}\right)^{(1/2,\,0)} < 1\,, & w=(-1/3,\ 1/3)\,.\end{cases}$$

lower bound on the reheating scale

$$m_X \geq H_{\rm rh} \geq 10^{-14}\mathrm{GeV}\left(\frac{10^{-14}\mathrm{GeV}}{\mathrm{m_X}}\right)^{\frac{2(1+3\mathrm{w})}{(1-3\mathrm{w})}}, \qquad w = (-1/3, 1/3).$$

Time-averaged Boltzmann equations

Expansion | **Interactions**

$$\dot{\rho}_\phi + 3(1 + \overline{w})H\rho_\phi = -\langle \Gamma_\phi \rangle \rho_\phi$$

$$\dot{\rho}_\mathcal{R} + 4H\rho_\mathcal{R} = \langle \Gamma_\phi^\mathcal{R} \rangle \rho_\phi$$

$$\overline{w} \equiv \langle p_\phi \rangle / \langle \rho_\phi \rangle$$

$$H^2 = \frac{\rho_\phi + \rho_\mathcal{R}}{3M_{pl}^2}$$

Time-dependent decay rate

$$\langle \Gamma_\phi^\mathcal{R} \rangle \simeq \langle \Gamma_\phi \rangle = \Gamma_\phi^e \left(\frac{a_e}{a} \right)^\beta$$

constant parameter

Non-instantaneous reheating

$$\rho_\phi(a) \overset{H \gg \Gamma_\phi}{\simeq} 3 M_{\rm Pl}^2 H_e^2 \left(\frac{a_e}{a}\right)^{3(1+\overline{w})}$$

$$\rho_{\mathcal{R}}(a) = \frac{6 M_{Pl}^2 H_e \Gamma_\phi^e}{5 - 3\overline{w} - 2\beta} \left[\left(\frac{a_e}{a}\right)^{\beta + 3(1+\overline{w})/2} - \left(\frac{a_e}{a}\right)^4 \right]$$

the dominant term
for $\beta \leq (n+4)/(n+1)$