# Unitarity in Higgs inflation and UV complete models

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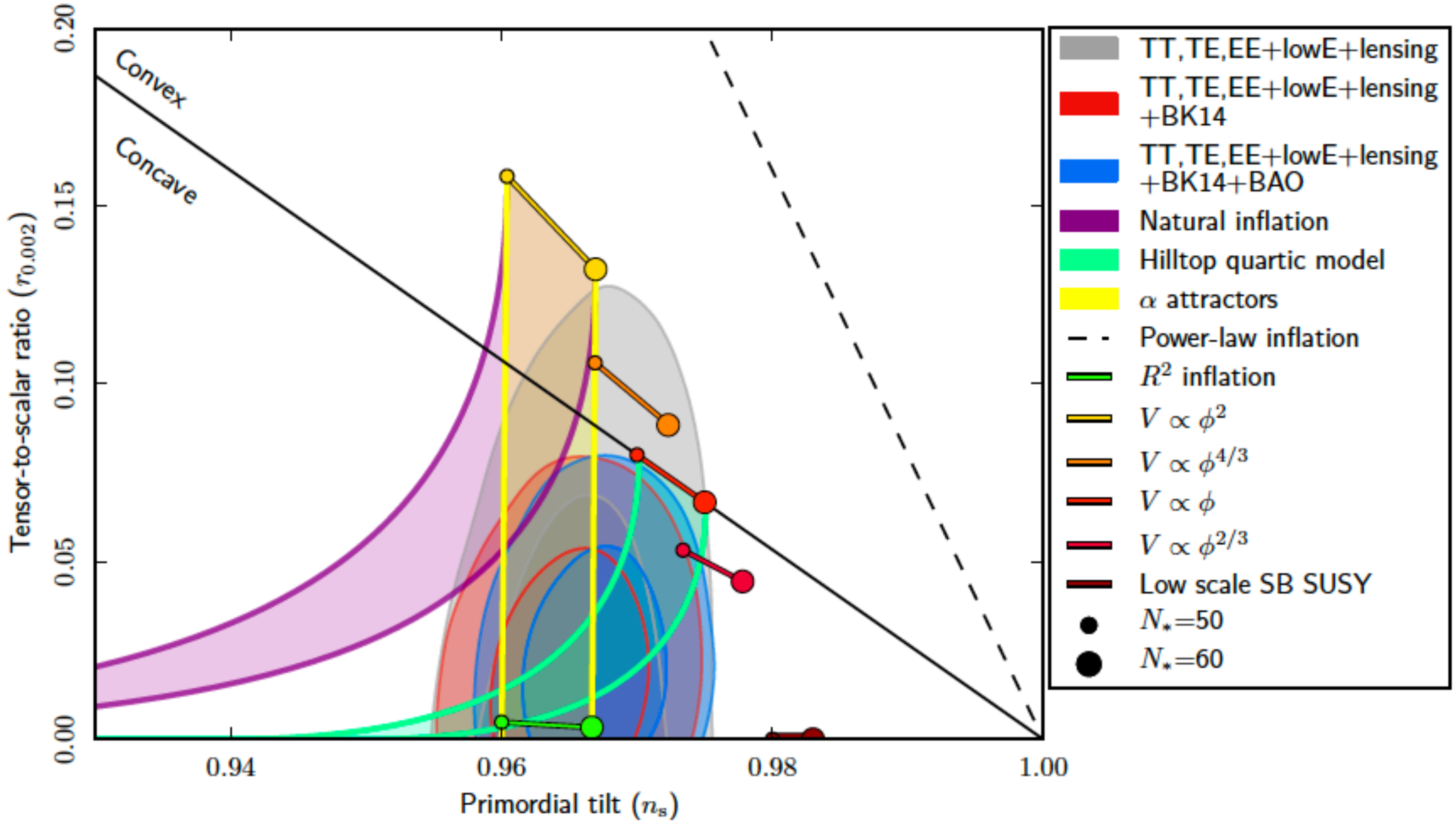
# Outline

- Introduction
- Higgs inflation in sigma models and supergravity extensions
- Higgs inflation in Weyl gravity
- Conclusions

# Natural inflation?






- Explains horizon, homogeneity, flatness, relics, and structures.
- Natural inflation and Higgs($R^2$) inflation are motivated most and compatible with CMB.

# Higgs inflation



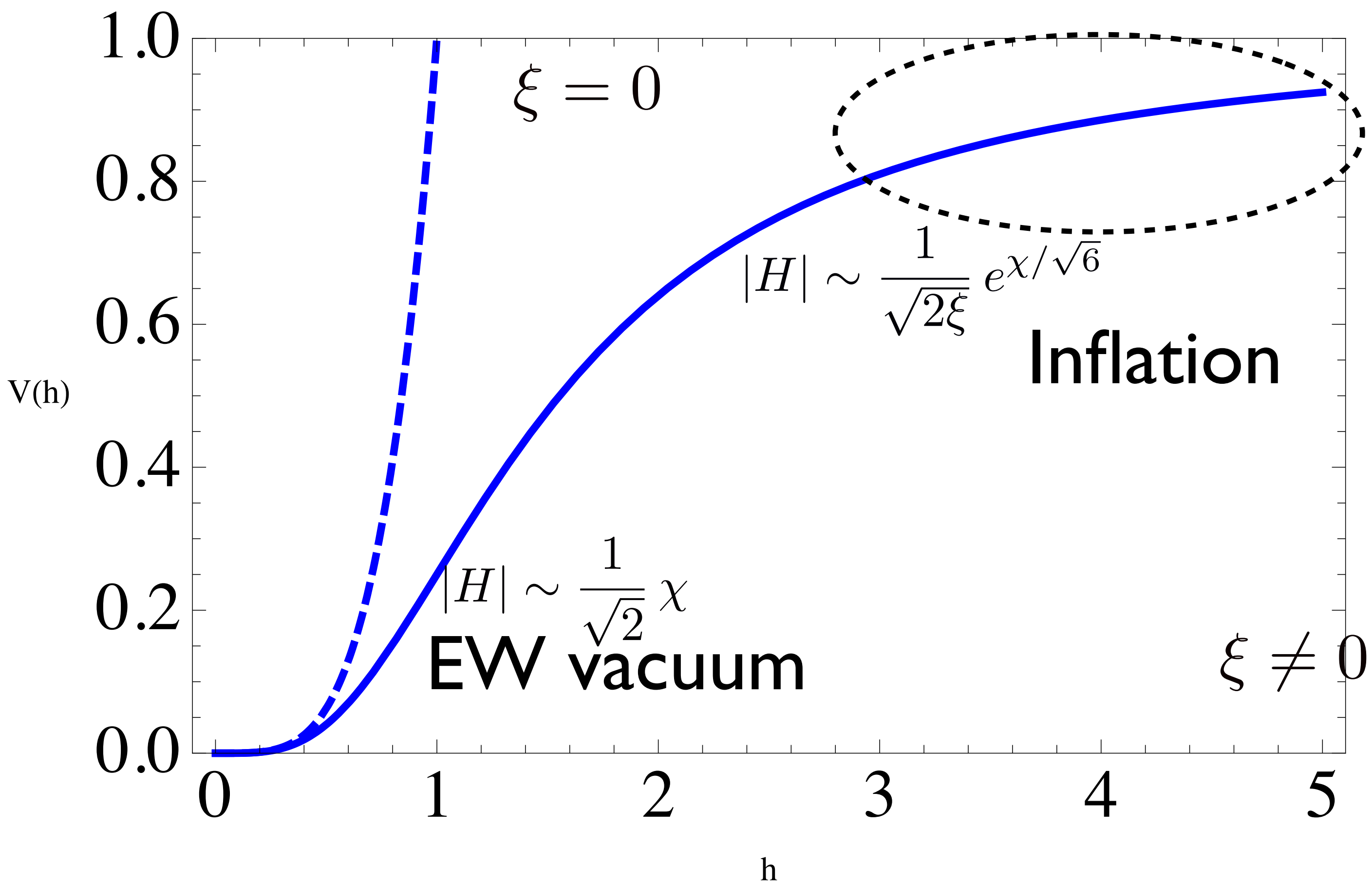
- Higgs non-minimal coupling to gravity is necessary for renormalization on curved background.

[Bezrukov, Shaposhnikov (2007)]

$$\mathcal{L} = \sqrt{-g}\Big(\frac{1}{2}\mathcal{R} + \xi|H|^2\mathcal{R} - |D_\mu H|^2 - \lambda_H(|H|^2 - v^2/2)^2\Big)$$

$$\Rightarrow \quad \mathcal{L}_E = \sqrt{-g_E}\Big(\frac{1}{2}\mathcal{R}(g_E) - \frac{3\xi^2}{\Omega^2}(\partial_\mu|H|^2)^2 - \frac{1}{\Omega}|D_\mu H|^2 - \frac{V}{\Omega^2}\Big)$$

$$g_{\mu\nu} = \Omega^{-1} g_{E,\mu\nu}, \quad \Omega = 1 + 2\xi|H|^2$$



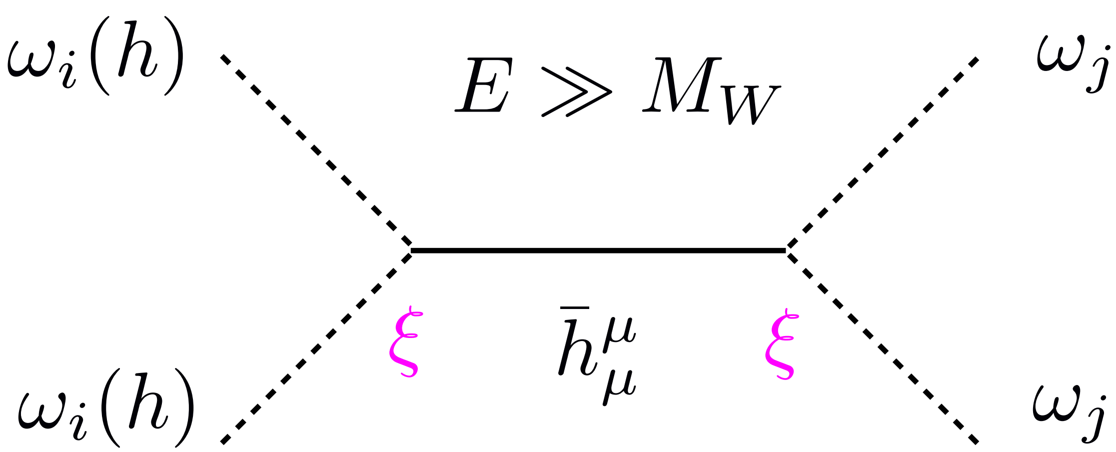

Field-dependent Planck scale drive Higgs potential flat at large fields!

$$\frac{V}{\Omega} = \frac{\lambda_H |H|^4}{(1+2\xi|H|^2)^2} \sim \frac{\lambda_H}{4\xi^2}\Big(1 - e^{-\frac{2}{\sqrt{6}}\chi}\Big)$$

# Troubles with large coupling



- CMB needs a large non-minimal coupling unless tuned.




$$\mathcal{M}_{\omega_i\omega_j\to\omega_i\omega_j} \sim \frac{\xi^2 E^2}{M_P^2}$$

[Burgess, HML, Trott (2009,2010); Barbon, Espinosa (2009); Hertzberg (2010)]

$$\frac{\Delta T}{T} \sim 10^{-4} \Rightarrow \frac{\xi}{\sqrt{\lambda_H}} = 5 \times 10^4$$

Perturbative unitarity is violated at low scale. $E \lesssim \frac{M_P}{\xi}$

- Unitarity scale is larger during inflation. [F. Bezrukov et al, 2010]

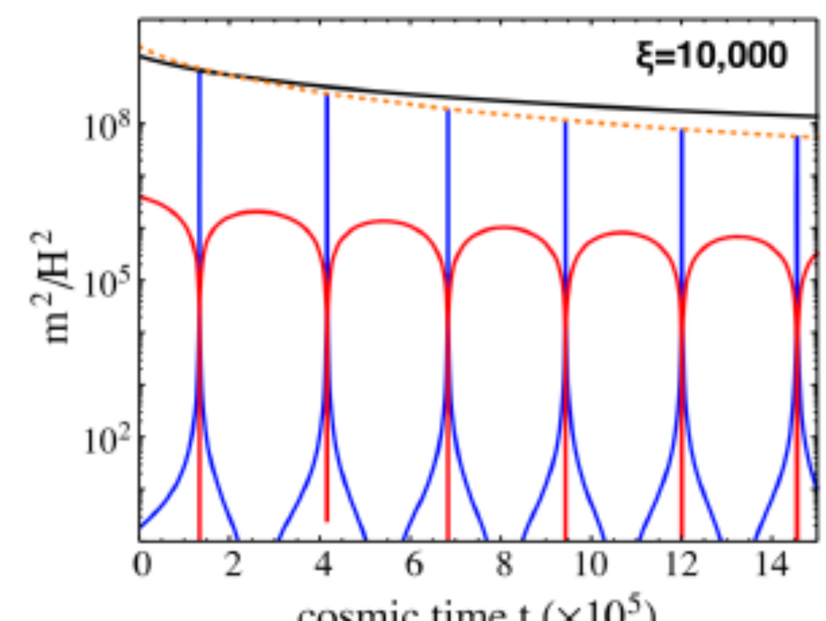
  for saturated W, Z masses: $m_W^2 = \frac{g^2 h^2}{4(1+\xi h^2/M_P^2)} \simeq \frac{1}{4} g^2 \frac{M_P^2}{\xi}.$

- But, preheating violates unitarity.

  Particle momenta beyond cutoff scale:

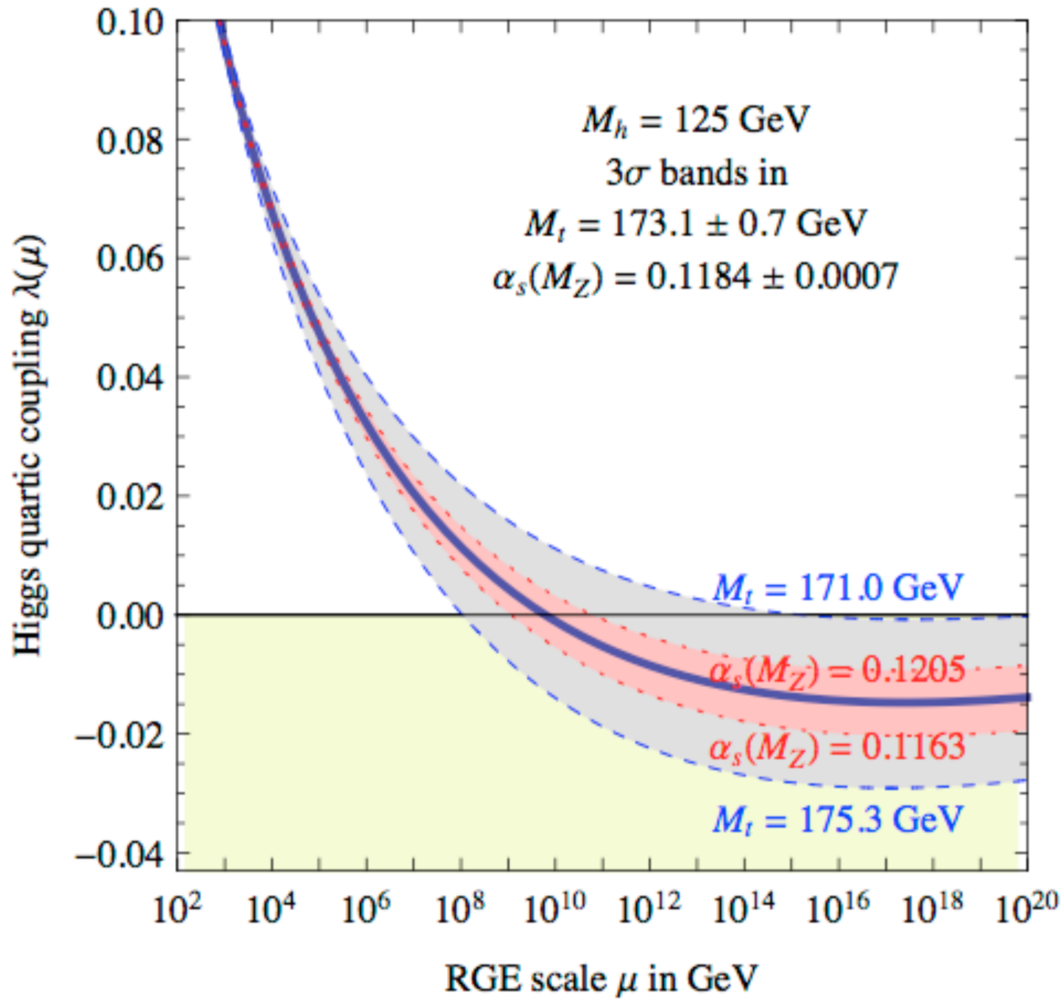
  $$k_{\max} \sim \xi H_{\rm end} \sim \sqrt{\lambda} M_{\rm Pl}$$

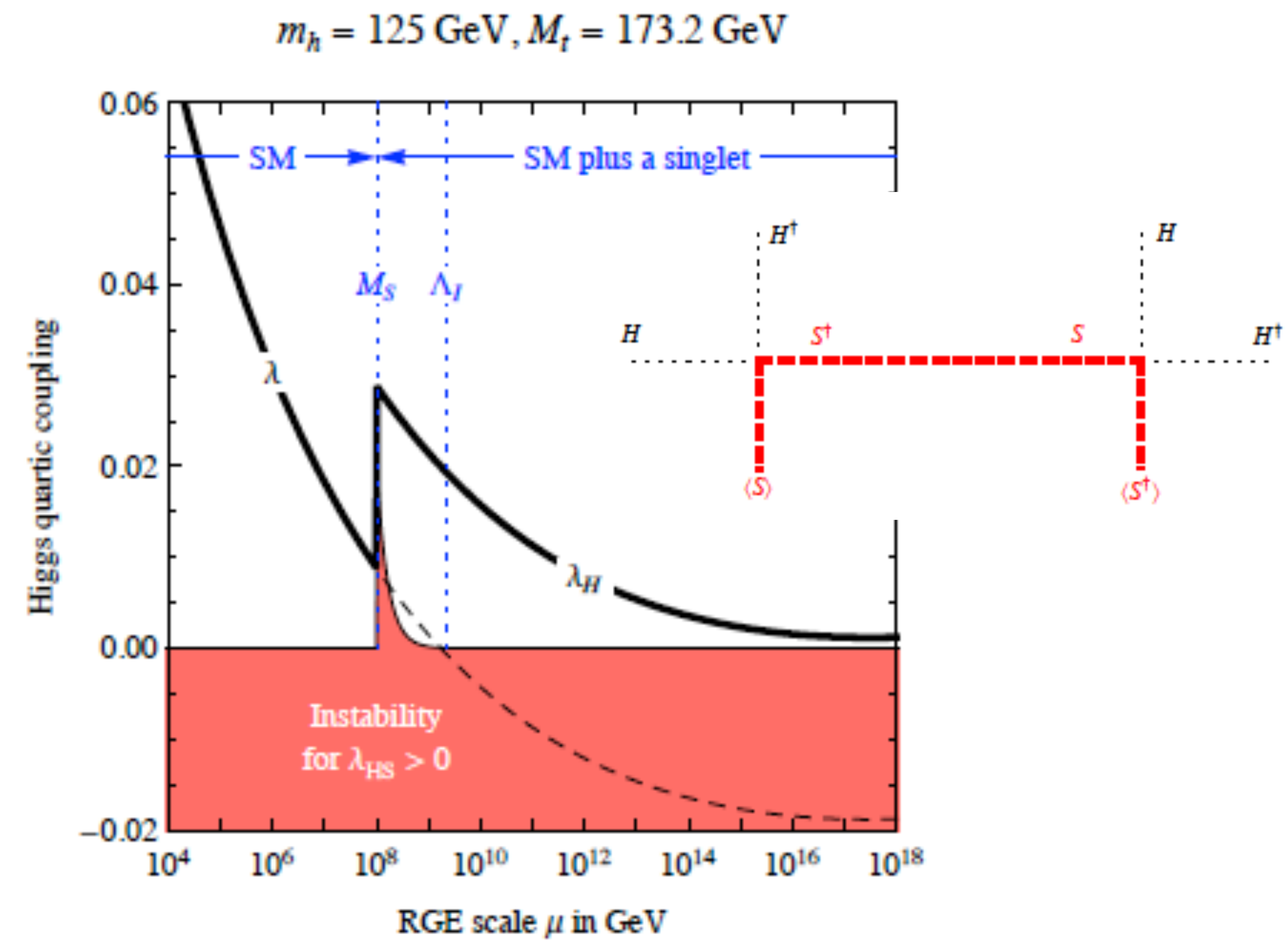
  [e.g. E. Sfakianakis, 2019]



# Vacuum stability






[Giudice, HML, Espinosa et al (2012)]

- Higgs mass needs a small quartic coupling: $m_h^2 = 2v^2\lambda_H$.

  $\lambda_H(m_H) = 0.13$ ➔ SM vacuum would be unstable.

- Higgs mixing with singlet scalar: larger Higgs quartic coupling.

$$m_h^2 = 2v^2\left(\lambda_H - \frac{\lambda_m^2}{\lambda}\right) \quad \Rightarrow \quad \lambda_H = 0.13 + \frac{\lambda_m^2}{\lambda}$$

# Higgs in induced gravity

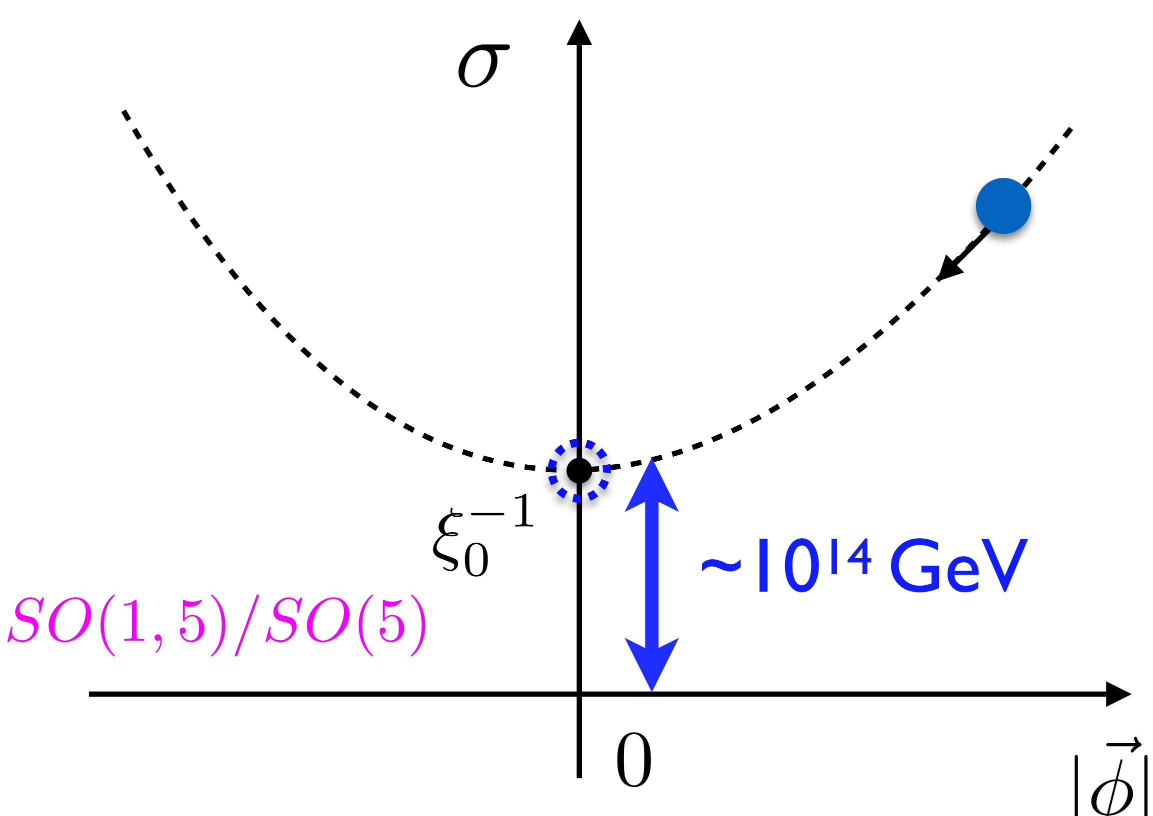


- Integrating in the sigma field with "non-compact" field space, $\sigma^2 - \vec{\phi}^2 = \xi_0^{-1}$, [Giudice,HML (2010)]

$$\mathcal{L}_{\rm kin} = -\frac{1}{2\xi_0 \sigma^2}\left[(\partial_\mu \vec{\phi})^2 + 6\xi_0 (\partial_\mu \sigma)^2\right] : \ SO(1,5)/SO(5).$$

⟷ Induce gravity $\mathcal{L}_J = \sqrt{-g}\left[\xi_0 \sigma^2 \mathcal{R} - \frac{1}{2}(\partial_\mu \sigma)^2 - \lambda\Big(\sigma^2 - \vec{\phi}^2 - \xi_0^{-1}\Big)^2\right]$

$$\int D\psi\, e^{iS(g,\psi)} = e^{i\int d^4x \sqrt{-g}(M_P^2 \mathcal{R} + \cdots)}$$ [Zee, Smolin (1979)]




1) SO(4)→SO(1,5)/SO(5)

2) Higgs inflation-like.

3) Heavy sigma saves vacuum instability.

$$m_\sigma^2 = 4\lambda H^2 \sim (10^{13}\,{\rm GeV})^2$$

# Higgs inflation in sigma models and supergravity extensions

# Higgs in conformal frame



- Original Lagrangian for Higgs inflation.

$$\mathcal{L} = \sqrt{-\hat{g}}\Big[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \frac{\lambda}{4}(\hat{\phi}_i^2)^2\Big]$$

- Frame-independent (conformal) frame for Higgs inflation.

Conformal mode: $\hat{g}_{\mu\nu} = e^{2\varphi}g_{\mu\nu}$

Field redefinition: $\phi_i = e^{\varphi}\hat{\phi}_i, \ \sqrt{6}e^{\varphi} = \phi + \sigma$

$$\Rightarrow \mathcal{L} = \sqrt{-g}\Big[-\frac{1}{2}\Big(1-\frac{1}{6}\phi_i^2 - \frac{1}{6}\sigma^2\Big)R + \frac{1}{2}(\partial_\mu\phi_i)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{\lambda}{4}(\phi_i^2)^2\Big]$$

Gauge fixing

Non-linear sigma model with constraint,

[Giudice, HML (2010); Y. Ema et al (2020)]

$$\Big(\sigma + \frac{\sqrt{6}}{2}\Big)^2 + 3\Big(\xi + \frac{1}{6}\Big)\phi_i^2 - \frac{3}{2} = 0$$

Promote the constraint to a dynamical field with perturbativity

$\Rightarrow$ "Unitarizing Higgs inflation" in conformal frame.

# Starobinsky in conformal frame



- Lagrangian for Starobinsky model + Higgs inflation:

$$\mathcal{L}_{R2} = \sqrt{-\hat{g}}\Big[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \frac{\lambda}{4}(\hat{\phi}_i^2)^2 + \alpha\hat{R}^2\Big]$$

Scalar-dual: $\alpha\hat{R}^2 \Longleftrightarrow -2\alpha\hat{\chi}\hat{R} - \alpha\hat{\chi}^2$

Conformal transform: $\hat{g}_{\mu\nu} = \Omega^{-2}g_{\mu\nu}$ , $\hat{\phi}_i = \Omega\phi_i$ and $\hat{\chi} = \Omega^2\chi$

[Y. Ema et al (2020)] $\Omega^{-2} = \left(1+\frac{\sigma}{\sqrt{6}}\right)^2$

⇒ identical to linear sigma model: "dynamical scalar σ"

$$\frac{\mathcal{L}_{R2}}{\sqrt{-g}} = -\frac{1}{2}R\Big(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\Big) + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\phi_i)^2 - \alpha\chi^2 - \frac{\lambda}{4}\phi_i^4$$

constraint equation, $4\alpha\chi = \frac{1}{2} - \frac{1}{3}\left(\sigma+\frac{\sqrt{6}}{2}\right)^2 - \left(\xi+\frac{1}{6}\right)\phi_i^2,$

sigma quartic coupling, $\kappa = \frac{1}{36\alpha}$

# Generalization to F(R)



- General higher curvature terms + Higgs inflation

$$\sum_k \frac{2(-1)^{k+1}\alpha_k}{k+1}\hat{R}^{k+1} \longleftrightarrow -2\sum_k \alpha_k \hat{\chi}_k \hat{R} - \sum_k 2\left(\frac{k}{k+1}\right)\alpha_k \hat{\chi}_k^{\frac{k+1}{k}}$$

A Lagrange multiplier per each term [HML, Menkara (2021)]

Conformal transform: $\hat{g}_{\mu\nu} = \Omega^{-2} g_{\mu\nu}$ , $\hat{\phi}_i = \Omega\phi_i$ and $\hat{\chi}_k = \Omega^2 \chi_k$

General sigma-model Lagrangian:

$$\frac{\mathcal{L}_{\rm gen}}{\sqrt{-g}} = \underbrace{-\frac{1}{2}R\left(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\right)+\frac{1}{2}(\partial_\mu\sigma)^2+\frac{1}{2}(\partial_\mu\phi_i)^2-\frac{\lambda}{4}\phi_i^4}_{\text{conformal-invariant}} \underbrace{-\sum_k \Omega^{-2+\frac{2}{k}}\left(\frac{2k}{k+1}\right)\alpha_k\chi_k^{1+\frac{1}{k}}}_{\text{sigma potential}}$$

$$+\,y(x)\cdot\left[\sum_k 4\alpha_k\chi_k - \frac{1}{2}+\frac{1}{3}\left(\sigma+\frac{\sqrt{6}}{2}\right)^2+\left(\xi+\frac{1}{6}\right)\phi_i^2\right]$$

$y(x)$: Lagrange multipler

# F(R) on vacuum manifold



- Use equations of motion for $\chi_k$ & $y$. [HML, Menkara (2021)]

$$\frac{\delta \mathcal{L}_{\rm gen}}{\delta \chi_k} = 0 \quad \Rightarrow \quad \chi_k = 2^k \Omega^{2k-2}\, y^k\,, \quad k = 1, 2, \cdots, N$$

$$\frac{\delta \mathcal{L}_{\rm gen}}{\delta y} = 0 \quad \Rightarrow \quad \sum_k 4\alpha_k \chi_k = \frac{1}{2} - \frac{1}{3}\left(\sigma + \frac{\sqrt{6}}{2}\right)^2 - \left(\xi + \frac{1}{6}\right)\phi_i^2$$

$$= \sum_k 4\alpha_k\, 2^k \Omega^{2k-2}\, y^k$$

$y$ : the solution to the $N$-th order algebraic equation.

- General sigma-model potential: a function of $\sigma$ & $\phi_i$

$$U(\sigma, \phi_i) = \sum_k \Omega^{-2+\frac{2}{k}} \left(\frac{2k}{k+1}\right) \alpha_k\, \chi_k^{1+\frac{1}{k}} = \sum_k \left(\frac{2^{k+2} k}{k+1}\right) \alpha_k (\Omega(\sigma))^{2k-2} (y(\sigma, \phi_i))^{k+1}$$

$$\frac{\partial U}{\partial \sigma} = 0 \quad \Rightarrow \quad y = 0: \quad \left(\sigma + \frac{\sqrt{6}}{2}\right)^2 + 3\left(\xi + \frac{1}{6}\right)\phi_i^2 - \frac{3}{2} = 0$$

"Identical" vacuum manifold as for Starobinsky

# Higgs-Starobinsky inflation



- Higgs-Starobinsky Lagrangian in Einstein frame

$$\frac{\mathcal{L}_{R2}}{\sqrt{-g}} = -\frac{1}{2}R\Big(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\Big)+\frac{1}{2}(\partial_\mu\sigma)^2+\frac{1}{2}(\partial_\mu\phi_i)^2-\alpha\chi^2-\frac{\lambda}{4}\phi_i^4$$

$$\Rightarrow \mathcal{L}_E = \sqrt{-g_E}\Big\{-\frac{1}{2}R(g_E)+\frac{1}{2\Omega'^4}\Big[\Big(1-\frac{1}{6}\sigma^2\Big)(\partial_\mu h)^2+\Big(1-\frac{1}{6}h^2\Big)(\partial_\mu\sigma)^2+\frac{1}{3}h\,\sigma\,\partial_\mu h\partial^\mu\sigma\Big]-V(\sigma,h)\Big\}$$

$$V(\sigma,h) = \frac{1}{(1-\frac{1}{6}h^2-\frac{1}{6}\sigma^2)^2}\Big[\frac{1}{4}\kappa_1\Big(\sigma(\sigma+\sqrt{6})+3\Big(\xi+\frac{1}{6}\Big)h^2\Big)^2+\frac{1}{4}\lambda h^4\Big]$$

Perturbativity: $\kappa_1 = \frac{1}{36\alpha_1} \lesssim \mathcal{O}(1)$, $\quad \lambda_{\rm eff} = \lambda+9\kappa_1\Big(\xi+\frac{1}{6}\Big)^2 \lesssim \mathcal{O}(1)$

- Effective theory for inflation: [HML, Menkara (2021)]

$$\frac{\partial V}{\partial h} = 0 \Rightarrow \frac{\mathcal{L}_{\rm eff}}{\sqrt{-g_E}} = -\frac{1}{2}R(g_E)+\underline{\frac{(\partial_\mu\sigma)^2}{2(1-\sigma^2/6)^2}}-V_{\rm eff}(\sigma),$$

$$V_{\rm eff}(\sigma) = 9\lambda\,\kappa_1\sigma^2\Big[\lambda(\sigma-\sqrt{6})^2+\kappa_1\Big(\sigma-3\Big(\xi+\frac{1}{6}\Big)(\sigma-\sqrt{6})\Big)^2\Big]^{-1}$$

# Limits of Higgs inflation

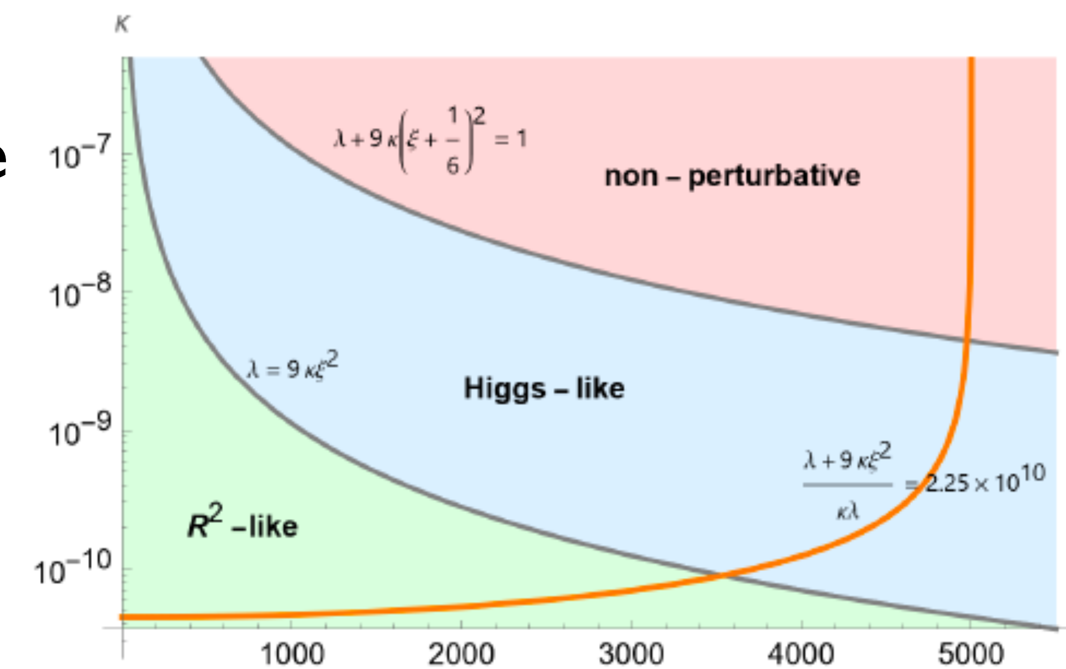


- Inflaton potential: $\sigma = -\sqrt{6}\tanh\left(\frac{\chi}{\sqrt{6}}\right)$

Starobinsky-like

$$V_E \approx \begin{cases} \frac{9\kappa_1}{4}\left(1-e^{-2\chi/\sqrt{6}}\right)^2, & 9\kappa_1\xi^2 \ll \lambda, \\ \frac{\lambda}{4\xi^2}\left(1-e^{-2\chi/\sqrt{6}}\right)^2, & 9\kappa_1\xi^2 \gg \lambda. \end{cases}$$

Higgs-like




- Similar predictions as in Higgs or Starobinsky models.

CMB normalization: $\frac{\sqrt{\lambda+9\kappa_1\xi^2}}{\sqrt{\kappa_1\lambda}} = 1.5\times 10^5$

Spectral index: $n_s = 1 - \frac{2}{N} - \frac{9}{2N^2} + \frac{3\kappa_1}{N^2}\frac{\left(-\lambda+12\lambda\xi+18\kappa_1\xi^2(1+6\xi)\right)}{\left(2\lambda+3\kappa_1\xi(1+6\xi)\right)^2},$

small corrections $(\kappa_1\xi^2 \lesssim 1)$

Tensor-to-scalar: $r = \frac{12}{N^2}$

Reheating: Efficient for large $\xi$. ⟶ A. Menkara's Talk!

# Higgs inflation in supergravity



- Starobinsky model in superconformal supergravity.

[S. Cecotti (1987); Ferrara et al (2013); Kugo et al (1983)]

$$\mathcal{L}_{R^2} = \alpha \bar{\mathcal{R}}\mathcal{R}|_D = \alpha R^2 + \cdots$$

$$\Longleftrightarrow \quad \mathcal{L}_{\rm dual} = \alpha \bar{X}^0 X^0 \bar{C} C|_D + (X^0)^3 T\Big(C - \frac{\mathcal{R}}{X^0}\Big)\Big|_F + {\rm h.c.}$$

$$\mathcal{L}_{\rm dual} = \bar{X}^0 X^0 \Big(\alpha \bar{C} C - (T+\bar{T})\Big)|_D + (X^0)^3 TC|_F + {\rm h.c.}$$

Two superfields needed, $C \,\&\, T$

- SUSY Higgs inflation: Extended Higgs sector $z^\alpha = \{S, H_u, H_d\}$

[D.R.T. Jones et al (2009); Ferrara et al (2010); HML (2010)]

- NMSSM + Starobinsky

[Aoki, HML, Menkara (2021)]

$$\Omega = -3 + |S|^2 + |H_u|^2 + |H_d|^2 + \Big(\frac{3}{2}\chi H_u \cdot H_d + {\rm h.c.}\Big) + |C|^2 - (T+\bar{T})$$

$$W = \lambda S H_u \cdot H_d + \frac{\rho}{3} S^3 \quad + \frac{1}{\sqrt{\alpha}} TC$$

# Jordan vs conformal frames



- Jordan-frame Lagrangian: [Aoki, HML, Menkara (2021)]

$$\mathcal{L}_J/\sqrt{-g} = \left\{\frac{1}{2} - \frac{1}{6}|S|^2 - \frac{1}{6}|H_u|^2 - \frac{1}{6}|H_d|^2 - \frac{1}{6}|C|^2 + \left(-\frac{1}{4}\chi H_u \cdot H_d + \text{h.c.}\right) + \frac{1}{3}\text{Re}T\right\} R$$

Field redefinitions:

$$\hat{z}^i \equiv X^0 z^i, \quad \hat{T} \equiv (X^0)^2 T, \qquad z^i = \{S, H_u, H_d, C\}$$

$$|X^0|^2\Omega(z^I, \bar{z}^{\bar{J}}) = \frac{1}{2} - \frac{1}{6}|\hat{z}^i|^2 - \frac{1}{12}\sigma^2, \qquad X^0 = 1 + \tfrac{1}{\sqrt{6}}\sigma$$

Conformal frame

$$\mathcal{L}_{LS}/\sqrt{-g} = \frac{1}{2}\left(1 - \frac{1}{3}|\hat{S}|^2 - \frac{1}{3}|\hat{H}_u|^2 - \frac{1}{3}|\hat{H}_d|^2 - \frac{1}{3}|\hat{C}|^2 - \frac{1}{6}\sigma^2\right) R$$
$$- |\partial_\mu \hat{S}|^2 - |\partial_\mu \hat{H}_u|^2 - |\partial_\mu \hat{H}_d|^2 - |\partial_\mu \hat{C}|^2 - \frac{1}{2}(\partial_\mu\sigma)^2 + \Omega\mathcal{A}_\mu^2 - V_{LS},$$

manifest symmetry of sigma model & unitarity

$$V^F_{LS} = |\lambda \hat{H}_u \cdot \hat{H}_d + \rho \hat{S}^2|^2 + \lambda^2|\hat{S}|^2(|\hat{H}_u|^2 + |\hat{H}_d|^2) + \frac{1}{\alpha}(\text{Im}\hat{T})^2 + \frac{3}{2}\frac{\chi\lambda}{\sqrt{\alpha}}(\hat{S}\bar{\hat{C}} + \bar{\hat{S}}\hat{C})(|\hat{H}_u|^2 + |\hat{H}_d|^2)$$
$$+ \frac{1}{4\alpha}\left(\sigma^2 + \sqrt{6}\sigma - \left(\frac{3}{2}\chi \hat{H}_u \cdot \hat{H}_d + \text{h.c.}\right)\right)^2 + \frac{1}{\alpha}|\hat{C}|^2\left\{3 + 2\sqrt{6}\sigma + \frac{3}{2}\sigma^2 + \frac{9}{4}\chi^2(|\hat{H}_u|^2 + |\hat{H}_d|^2)\right\}$$

↖ sigma potential ↖ Heavy dual-scalars

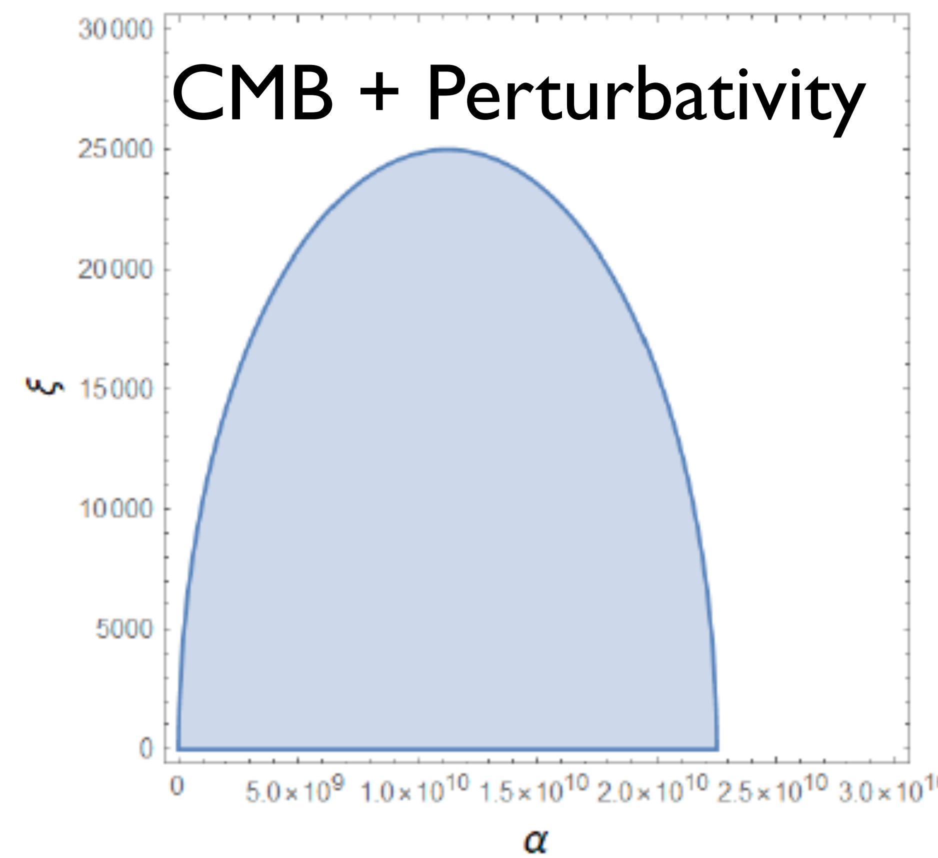
# Stability of inflation



- Inflationary trajectory:

$$H_u^0 = \frac{1}{\sqrt{2}} h \cos\beta\, e^{i\delta_1}, \quad H_d^0 = \frac{1}{\sqrt{2}} h \sin\beta\, e^{i\delta_2} \quad \neq 0,$$

$$\mathrm{Re}T \neq 0, \qquad \text{others} = 0$$




→ "non-SUSY" Higgs-Starobinsky inflation is recovered;

Effective non-minimal coupling, $\xi \equiv -\frac{1}{6} + \frac{\chi}{4}$

- All the scalar directions are decoupled;

[Aoki, HML, Menkara (2021)]

$S \,\&\, C$ fields are tachyonic but they are stabilized with

$$\Delta\Omega = -\zeta_s |S|^4 - \zeta_c |C|^4 - \zeta_{sc} |S|^2 |C|^2\,, \quad \zeta_s, \zeta_c \gtrsim \zeta_{sc}$$

# SUSY breaking



- SUSY is restored at the minimum after inflation: $z^I = 0$
- Dual superfields could lead to SUSY breaking/mediation.

a) Higher curvature terms: $\Omega = -3 + (T+\bar{T}) + |C|^2 - \gamma_c(C+\bar{C}) - \zeta_c|C|^4$

$\langle C\rangle, \langle T\rangle \neq 0$ ⟹ $m_{3/2} \sim M_P/\sqrt{\alpha} \gtrsim 10^{13}\,\mathrm{GeV}$

$F_C, F_T \neq 0$

High-scale SUSY

b) O'Raifeartaigh model: $\Phi$ [Aoki, HML, Menkara (2021)]

$$\Omega = -3 - (T+\bar{T}) + |C|^2 + |\Phi|^2, \qquad W = \frac{1}{\sqrt{\alpha}}TC + \kappa\,\Phi + g\,\Phi C^2$$

$\langle C\rangle = \langle T\rangle = 0$ ⟹ $m_{3/2} = |F_\Phi|/(\sqrt{3}M_P) = \kappa/(\sqrt{3}M_P)$

$F_\Phi = \kappa$

Low-scale SUSY

# Visible soft masses



- Effective Higgsino mass: $W_{\rm eff} = \mu H_u H_d$

Naturalness measure for EWSB: $$m_Z^2 = \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1-\sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2$$

$$\mu = \underbrace{\lambda\langle\tilde{S}\rangle}_{\text{NMSSM}} + \underbrace{\frac{3}{2}\chi m_{3/2} - \frac{1}{2}\chi K_{\bar{I}}\bar{F}^{\bar{I}}}_{}$$

[HML (2010); Aoki, HML, Menkara (2021)]

NMSSM

"Non-minimal coupling" contributes via Giudice-Masiero mechanism.

- Frame function must be extended by contact terms.

$$\Omega = -3 - \underbrace{(T+\bar{T}) + \bar{C}C + \bar{\Phi}\Phi}_{\text{hidden}} + \underbrace{\bar{z}^{\bar{\alpha}} z^{\alpha}}_{\text{visible}} \Rightarrow m^2_{\bar{\alpha}\alpha} = 0$$

$$\Omega_{\rm contact} = C_{\bar{\alpha}\beta} X^\dagger X z^\dagger_{\bar{\alpha}} z_\beta + {\rm c.c}, \quad X = C, \Phi, \Rightarrow m^2_{\bar{\alpha}\beta} \sim C_{\bar{\alpha}\beta}|F_X|^2$$

soft masses

# Higgs inflation in Weyl gravity

# Unitarizing with gauge field



- Non-canonical Higgs kinetic terms = Weyl current-current interactions

$$\frac{\mathcal{L}_{H,\rm eff}}{\sqrt{-g_E}} = \frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{\Omega^2} = \frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega^2}, \qquad K_\mu = \partial_\mu K_H \text{ with } K_H = 12\xi_H|H|^2.$$

- Introduce a Weyl gauge field in Jordan frame.

$$\frac{\mathcal{L}_J}{\sqrt{-g}} = -\frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - \frac{1}{2}g_w w_\mu K^\mu + \frac{1}{2}g_w^2 w_\mu w^\mu K_H, \qquad m_w^2 = 6g_w^2 M_P^2$$

Integrate out Weyl gauge field with $w_\mu = \frac{g_w}{2}\frac{K_\mu}{m_w^2 + g_w^2 K_H}$

$$\Rightarrow \frac{\mathcal{L}_{J,\rm eff}}{\sqrt{-g}} = -\frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega} \Rightarrow \boxed{\frac{\mathcal{L}_{E,\rm eff}}{\sqrt{-g_E}} = -\frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega^2}}$$

cancels non-canonical Higgs kinetic terms!

# Higgs in Weyl gravity



- Weyl geometry

$\tilde{\nabla}_\rho g_{\mu\nu} = 0$, $\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + g_w\Big(\delta^\rho_\mu w_\nu + \delta^\rho_\nu w_\mu - g_{\mu\nu} w^\rho\Big)$ in Weyl gravity

$\longrightarrow$ $\nabla_\rho g_{\mu\nu} = 2 g_w \omega_\rho g_{\mu\nu}$ in Einstein gravity

- Weyl invariant Lagrangian [D. Ghilencea, H. M. Lee (2018)]

Local scale transf: $g_{\mu\nu} \to e^{2\alpha} g_{\mu\nu}, \quad \phi \to e^{-\alpha}\phi, \quad H \to e^{-\alpha} H, \quad w_\mu \to w_\mu - \frac{1}{g_w}\partial_\mu \alpha$

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi \phi^2 + 2\xi_H |H|^2)\tilde{R}(\tilde{\Gamma}) + 3\xi_\phi r_\phi (D_\mu \phi)^2 + 6\xi_H r_H |D_\mu H|^2 - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} - V(H,\phi)$$

$$D_\mu \phi_i = (\partial_\mu - g_w w_\mu)\phi_i$$

- Gauge-fixed Lagrangian: $\langle \phi^2 \rangle = M_P^2/\xi_\phi$ [S. Aoki, H. M. Lee (2022)]

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}(M_P^2 + 2\xi_H |H|^2)R + |\partial_\mu H|^2 - V(H) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} + \frac{1}{2} m_w^2 w_\mu w^\mu - \frac{1}{2} g_w w_\mu K^\mu + \frac{1}{2} g_w^2 w_\mu w^\mu K_H$$

$$\begin{cases} m_w^2 = 6 r_\phi g_w^2 M_P^2 \\ K_H = 12 r_H \xi_H |H|^2 \end{cases}$$

# Unitarity in Weyl gravity



- Higgs kinetic terms in Einstein frame

$$\frac{\mathcal{L}_{\rm kin}}{\sqrt{-g_E}} = \underbrace{6\xi_H(r_H-1)}_{=1}\frac{|\partial_\mu H|^2}{\Omega} + \underline{\frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{\Omega^2}} - \underline{\frac{g_w^2 r_H^2}{8\Omega}\frac{K_\mu K^\mu}{m_w^2+g_w^2 r_H K_H}}$$

Higgs non-minimal coupling — Weyl gauge field

$$\frac{\mathcal{L}_{\rm kin}}{\sqrt{-g_E}} = \frac{1}{\Omega}|\partial_\mu H|^2 + \frac{1}{M_P^2\Omega^2}\frac{3r_\phi\xi_H^2 - 3(\xi_H+\frac{1}{6})^2 - \xi_H(\xi_H+\frac{1}{6})|H|^2/M_P^2}{r_\phi + 2(\xi_H+\frac{1}{6})|H|^2/M_P^2}(\partial_\mu|H|^2)^2$$

$$= |\partial_\mu H|^2 + \frac{1}{\Lambda^2}(\partial_\mu|H|^2)^2 + \cdots$$

[S. Aoki, H. M. Lee (2022)]

High cutoff: $\Lambda = \dfrac{M_P}{\left|\xi_H(3\xi_H+1)(1-\frac{1}{r_\phi}) - \frac{1}{12 r_\phi}\right|^{1/2}}$ $\xrightarrow{r_\phi = 1}$ $\Lambda \sim M_P$

Inflation: $V_E = \dfrac{\lambda_H M_P^4}{4\xi_H^2}\left(1+\dfrac{M_P^2}{\xi_H h^2}\right)^{-2} \simeq \dfrac{\lambda_H M_P^4}{4\xi_H^2}\left(1+\dfrac{\xi_H\chi^2}{M_P^2}\right)^{-2}$, $\chi$ : canonical inflaton

But, Inflation would work only for small $\xi_H$ and $\lambda_H$.

# General Weyl gravity



- Extend with a pair of metrics and Weyl gauge fields:

$$\mathcal{L} = \sum_{i=1,2} \sqrt{-g_i}\Big[-\frac{1}{2}\xi_i\phi_i^2\tilde{R}(\tilde{\Gamma}_i) - \frac{1}{4}w_{i,\mu\nu}w_i^{\mu\nu}\Big] + \Delta\mathcal{L}\,,$$

$$\Delta\mathcal{L} = \sum_{i=1,2}\sqrt{-g_i}\,(-3\xi_i a_i\phi_i^2)\Big(g_{w_1}w_{1,\mu} + \kappa_i g_{w_2}w_{2,\mu}\Big)^2$$

Weyl symmetry: $g_{i,\mu\nu} \to e^{2\alpha_i}g_{i,\mu\nu}, \quad \phi_i \to e^{-\alpha_i}\phi_i, \quad w_{i,\mu} \to w_{i,\mu} - \frac{1}{g_{w_i}}\partial_\mu\alpha_i$

- Effective Weyl gravity: $g_{1,\mu\nu} = g_{2,\mu\nu} \equiv g_{\mu\nu}\,, \quad \phi_1 = \phi_2 = \phi$

$$\mathcal{L}_{\rm eff} = \sqrt{-g}\Big[-\frac{1}{2}\xi_\phi\phi^2\tilde{R}(\tilde{\Gamma}) - \frac{1}{4}w_{\mu\nu}w^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}\Big]$$

[S. Aoki, H. M. Lee(2022)]

$$\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + g_w\Big(\delta^\rho_\mu w_\nu + \delta^\rho_\nu w_\mu - g_{\mu\nu}w^\rho\Big), \quad g_w = \tfrac{1}{2}\sqrt{g_{w_1}^2 + g_{w_2}^2},$$

Two linear combinations of Weyl fields:

$w_\mu = (g_{w_1}w_{1,\mu} + g_{w_2}w_{2,\mu})/\sqrt{g_{w_1}^2 + g_{w_2}^2}\,, \quad X_\mu:$ orthogonal.

# Higgs in general Weyl gravity



- Introduce an extra covariant kinetic term for Higgs:

$$\frac{\mathcal{L}_2}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi \phi^2 + 2\xi_H |H|^2)\tilde{R}(\tilde{\Gamma}) - \frac{1}{4}w_{\mu\nu}w^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \underline{|D'_\mu H|^2} - V(H,\phi)$$

Weyl symmetry: $g_{\mu\nu} \to e^{2\alpha} g_{\mu\nu}, \quad \phi \to e^{-\alpha}\phi, \quad H \to e^{-\alpha} H,$

$$D'_\mu H = (\partial_\mu - g_X X_\mu) H \qquad w_\mu \to w_\mu - \frac{1}{g_w}\partial_\mu \alpha, \quad X_\mu \to X_\mu - \frac{1}{g_X}\partial_\mu \alpha$$

Gauge-fixed Lagrangian: $m_w^2 = 6 g_w^2 M_P^2, \quad K_\mu = 12\xi_H \partial_\mu |H|^2$

$$\frac{\mathcal{L}_2}{\sqrt{-g}} = -\frac{1}{2}(M_P^2 + 2\xi_H |H|^2)R + |D'_\mu H|^2 - V(H) - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}$$
$$\underline{-\frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - \frac{1}{2}g_w w_\mu K^\mu + \frac{1}{2}g_w^2 w_\mu w^\mu K_H}$$

Decoupled Weyl photon w: $\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{M_P^2}{2}R + \frac{|D'_\mu H|^2}{\Omega} - \frac{V(H)}{\Omega^2} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}$

Palatini formulation, but differ by light Weyl photon X.

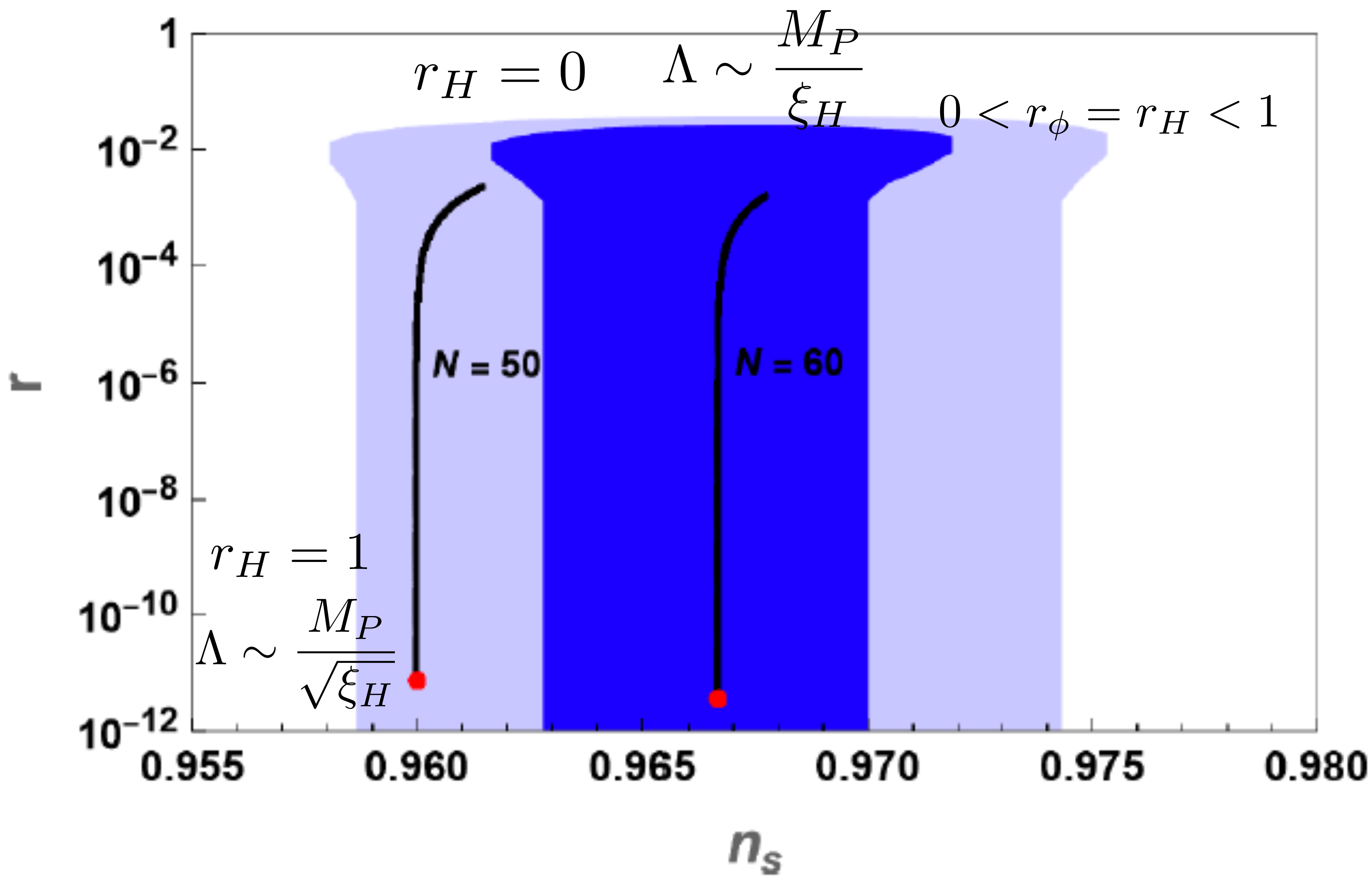
# Metric to Palatini Higgs



- General Weyl-invariant Lagrangian:

$$\frac{\mathcal{L}_G}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi\phi^2+\xi_H|H|^2)\tilde{R} + \underline{3\xi_\phi(r_\phi-1)(D_\mu\phi)^2 + 6\xi_H(r_H-1)|D_\mu H|^2}$$
$$-\frac{1}{4}w_{\mu\nu}w^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + (1-6\xi_H(r_H-1))|D'_\mu H|^2 - V(H,\phi).$$

- Higgs inflation in metric to Palatini cases.




[S. Aoki, H. M. Lee (2022)]

$$\Lambda_1 = \frac{M_P}{\left|3\xi_H^2\left(1-\frac{r_H^2}{r_\phi}\right)+\xi_H\right|^{1/2}}$$

High cutoff scale

→ very small r

# Light Weyl gauge field



- Light Weyl gauge field gets mass at low energy.

$$\mathcal{L}_{X,\text{int}} = a_H\Big(-g_X X_\mu \partial^\mu |H|^2 + g_X^2 X_\mu X^\mu |H|^2\Big) - \frac{1}{2} g_X X_\mu \partial^\mu s^2 + \frac{1}{2} g_X^2 X_\mu X^\mu s^2,$$

$$a_H \equiv 1 - 6\xi_H (r_H - 1)$$

$$H = (0, v+h)^T/\sqrt{2}, \quad s = v_s + \tilde{s}. \longrightarrow m_X^2 = g_X^2 (a_H v^2 + v_s^2)$$

Gauge fixing term: $\mathcal{L}_{gf} = -\frac{1}{2\zeta}\Big(\partial_\mu X^\mu + \zeta g_X (a_H v h + v_s \tilde{s})\Big)^2$

→ Would-be Goldstone is a combination of Higgs and singlet.

- X decays into a Higgs pair; no linear Higgs coupling to X.

$$\begin{pmatrix} h \\ \tilde{s} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_{\rm SM} \\ G_X \end{pmatrix}$$

Higgs data at 10(1)%:

$$\sin\theta \simeq (a_H v)/v_s \lesssim 0.3(0.03)$$

$$\begin{aligned} \mathcal{L}_{X,\text{int}} = & -g_X(a_H\cos^2\theta + \sin^2\theta)\underline{X_\mu h_{\rm SM}\partial^\mu h_{\rm SM}} \\ & -g_X(a_H - 1)\cos\theta\sin\theta X_\mu(G_X\partial^\mu h_{\rm SM} + h_{\rm SM}\partial^\mu G_X) \\ & -g_X(a_H\sin^2\theta + \cos^2\theta)X_\mu G_X\partial^\mu G_X \\ & +g_X^2\sqrt{(a_H v)^2 + v_s^2}\,X_\mu X^\mu G_X + \frac{1}{2}g_X^2(a_H\cos^2\theta + \sin^2\theta)\underline{X_\mu X^\mu h_{\rm SM}^2} \\ & +g_X^2(a_H - 1)\cos\theta\sin\theta\, X_\mu X^\mu G_X h_{\rm SM} \\ & +\frac{1}{2}g_X^2(a_H\sin^2\theta + \cos^2\theta)X_\mu X^\mu G_X^2. \end{aligned}$$

# Light Weyl gauge field



- Gauge kinetic mixing between Weyl and hypercharge:

$$\mathcal{L}_{\rm gmix} = -\frac{1}{2}\sin\xi\, X_{\mu\nu}B^{\mu\nu}$$

Mass eigenstates for Weyl and SM neutral gauge bosons:

$$\begin{pmatrix} B_\mu \\ W_{3\mu} \\ X_\mu \end{pmatrix} = \begin{pmatrix} c_W & t_\xi s_\zeta - s_W c_\zeta & -s_W s_\zeta - t_\xi c_\zeta \\ s_W & c_W c_\zeta & c_W s_\zeta \\ 0 & -s_\zeta/c_\xi & c_\zeta/c_\xi \end{pmatrix} \begin{pmatrix} \tilde{A}_\mu \\ \tilde{Z}_\mu \\ \tilde{X}_\mu \end{pmatrix}$$

Mixing between Z and X: $\tan(2\zeta) = \dfrac{m_Z^2 s_W \sin(2\xi)}{m_X^2 - m_Z^2(c_\xi^2 - s_W^2 s_\xi^2)}$

EM/neutral currents:

$$\mathcal{L}_{\rm EM/NC} \simeq e\tilde{A}_\mu J^\mu_{\rm EM} + \tilde{Z}_\mu\left[\frac{e}{2s_W c_W}J^\mu_Z + \varepsilon g_X t_W J^\mu_X\right] + \tilde{X}_\mu\left[g_X J^\mu_X - e\varepsilon J^\mu_{\rm EM}\right]$$

$J^\mu_X = -a_H\partial_\mu|H|^2 + \frac{1}{2}\partial_\mu s^2$ , Weyl currents

➡ Weyl field can be produced via EM/NC currents.

# Conclusions



- Higgs inflation can be regarded as a non-linear sigma model in non-gravity theory or effective gravity theory with Weyl currents.

- We proposed a UV complete Higgs inflation in linear sigma models in gravity and supergravity: inflaton is the mixed direction of Higgs and sigma fields, and extra scalars from supergravity are safely decoupled.

- For Higgs inflation with a large non-minimal coupling, we introduced a second light Weyl gauge field: Palatini-like inflation with large cutoff, novel phenomenology for Higgs couplings and non-standard gauge interactions.