Reheating and dark matter freeze-in in the Higgs-R^2 inflation model

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Overview

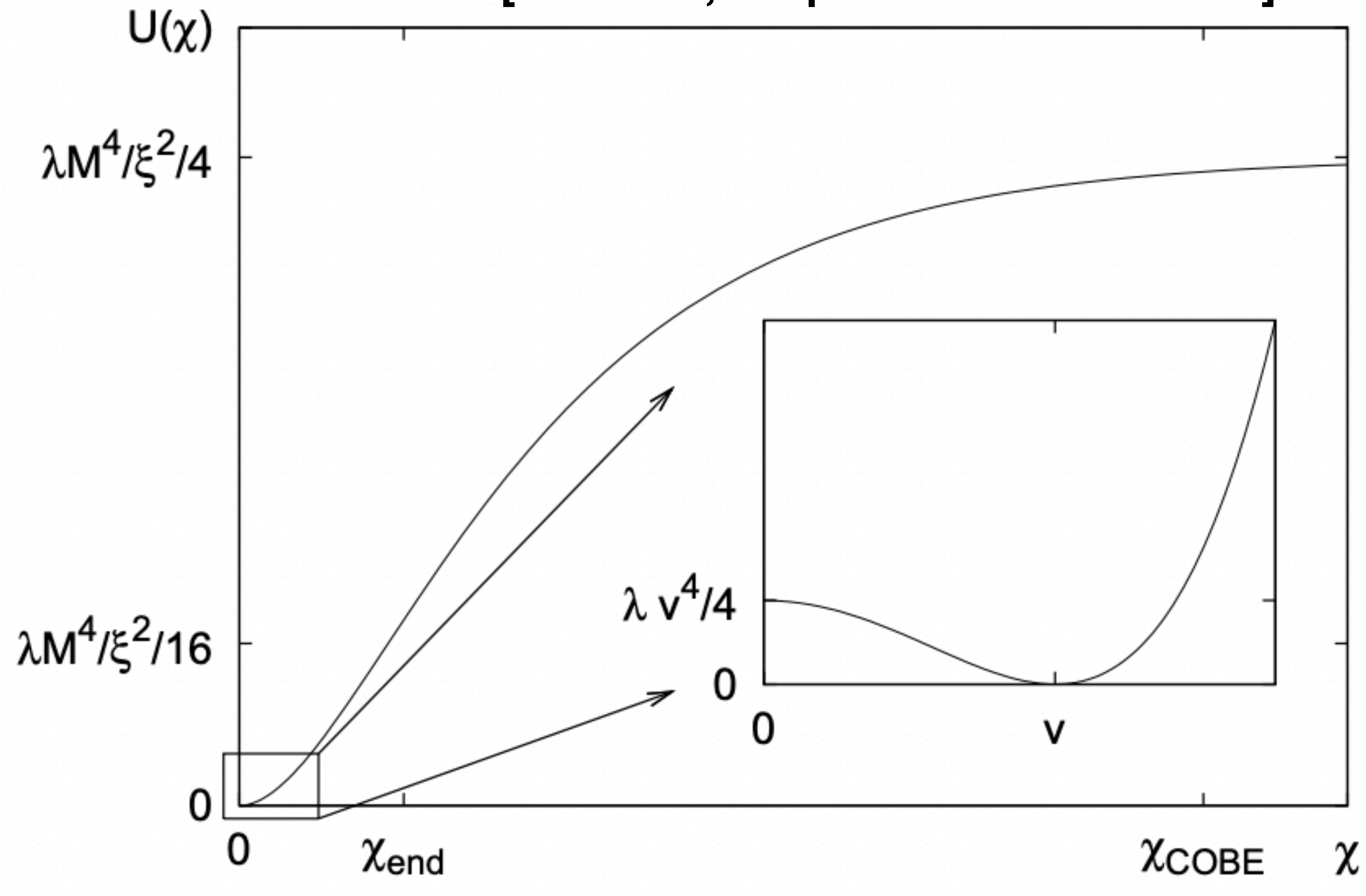
- Higgs Inflation
- Reheating
- Dark Matter Production
- Summary

Higgs Inflation

Higgs Inflation

$$\mathscr{L} = \sqrt{-g_J}\left[-\frac{1}{2}(1+\xi h^2)R_J + \frac{1}{2}g_J^{\mu\nu}\partial_\mu h\partial_\nu h - \frac{\lambda}{4}h^4\right]$$

[Bezrukov, Shaposhnikov - 0710.3755]



- $\xi \simeq 10^4$ in order to match observations.
- $H \simeq \sqrt{\lambda_H} M_P/\xi \implies 1 \gg H/M \gg \sqrt{\lambda_H}$.
- The remaining window of validity for the EFT is very narrow.
- The plateau in the inflationary potential is very sensitive to the UV physics, making the model lose its naturalness.

[Burgess, HML,Trott(2009,2010); Barbon, Espinosa (2009); Hertzberg(2010)]

Higgs-R^2

$$\mathscr{L}/\sqrt{-g_J} = \frac{1}{2}(M_p^2 + \xi\hat{h}^2)R_J - \frac{1}{2}(\partial_\mu\hat{h})^2 - \frac{\lambda}{4}\hat{h}^4 + \underline{\alpha R_J^2}$$

Lagrangian in the Jordan frame

Recast R into an auxiliary field $\hat{\chi}$ 2002.11739

$$\mathscr{L}/\sqrt{-g_J} = \frac{1}{2}(M_p^2 + \xi\hat{h}^2 + \underline{4\alpha\hat{\chi}})R_J - \frac{1}{2}(\partial_\mu\hat{h})^2 - \frac{\lambda}{4}\hat{h}^4 - \underline{\alpha\hat{\chi}^2}$$

$$\Lambda_{\rm cutoff} = M_P$$

The new χ field linearizes Higgs inflation

Higgs-sigma models are a UV completion of Higgs inflation **[Giudice, Lee, 1010.1417]**

Inflationary potential

$$V = \frac{1}{\Omega^4}\left[\frac{1}{4}\kappa\left(\sigma(\sigma+\sqrt{6}M_p)+3\left(\xi+\frac{1}{6}\right)h^2\right)^2+\frac{1}{4}\lambda h^4\right]$$

Higgs much heavier than Hubble scale

$$h^2 = \frac{\kappa_1\sigma(\sigma+\sqrt{6})\left(\sigma-3\left(\xi+\frac{1}{6}\right)(\sigma-\sqrt{6})\right)}{\lambda(\sigma-\sqrt{6})-3\kappa_1\left(\xi+\frac{1}{6}\right)\left(\sigma-3\left(\xi+\frac{1}{6}\right)(\sigma-\sqrt{6})\right)}$$

Canonical field $\sigma = -\sqrt{6}\tanh\left(\frac{\chi}{\sqrt{6}}\right)$

$$V_{\rm eff}(\chi) = \frac{9\kappa}{4}\left(1-e^{-2\chi/\sqrt{6}}\right)^2\left[1+\frac{\kappa_1}{4\lambda}\left(6\xi+e^{-2\chi/\sqrt{6}}\right)^2\right]^{-1}$$

χ **drives inflation while the Higgs is frozen at a non-zero VEV**

Reheating

Reheating in the Higgs-R^2 model

Boltzmann equations

$$\ddot{\sigma} + \frac{\sigma}{3\Omega^2 M_p^2}\dot{\sigma}^2 + \frac{h}{3\Omega^2 M_p^2}\dot{\sigma}\dot{h} + (3H + \Gamma_{\sigma_0})\dot{\sigma} + \frac{2\sigma}{3\Omega^2 M_p^2}U + \frac{1}{\Omega^2}\left(1 - \frac{\sigma^2}{6M_p^2}\right)U_\sigma - \frac{h\sigma}{6\Omega^2 M_p^2}U_h = 0,$$

$$\ddot{h} + \frac{h}{3\Omega^2 M_p^2}\dot{h}^2 + \frac{\sigma}{3\Omega^2 M_p^2}\dot{\sigma}\dot{h} + (3H + \Gamma_{h_{\rm osc}})\dot{h} + \frac{2h}{3\Omega^2 M_p^2}U + \frac{1}{\Omega^2}\left(1 - \frac{h^2}{6M_p^2}\right)U_h - \frac{h\sigma}{6\Omega^2 M_p^2}U_\sigma = 0,$$

$$\dot{\rho}_r + 4H\rho_r - \frac{\Gamma_{\sigma_0}}{\Omega^4}\left[\left(1 - \frac{h^2}{6M_p^2}\right)\dot{\sigma}^2 + \frac{h\sigma}{6M_p^2}\dot{\sigma}\dot{h}\right] - \frac{\Gamma_{h_{\rm osc}}}{\Omega^4}\left[\left(1 - \frac{\sigma^2}{6M_p^2}\right)\dot{h}^2 + \frac{h\sigma}{6M_p^2}\dot{\sigma}\dot{h}\right] = 0,$$

Our approach for reheating can be compared to the oscillation condensate with dissipation in non-equilibrium thermodynamics 2108.00254

Background field evolution

We divide the fields into a background and an oscillating part

$$\sigma(t) = \sigma_0, \qquad h(t) = \underline{h_0(\sigma_0)} + h_{\text{osc}}(t),$$

[M. He - 2010.11717]

The Higgs is released from background value it had during inflation

From the inflationary potential $V \supset \kappa\xi\sigma h^2$

$$(h_0(\sigma_0))^2 \simeq -\frac{3\sqrt{6}\kappa\left(\xi + \frac{1}{6}\right)}{\lambda + 9\kappa\left(\xi + \frac{1}{6}\right)^2} M_p \sigma_0$$

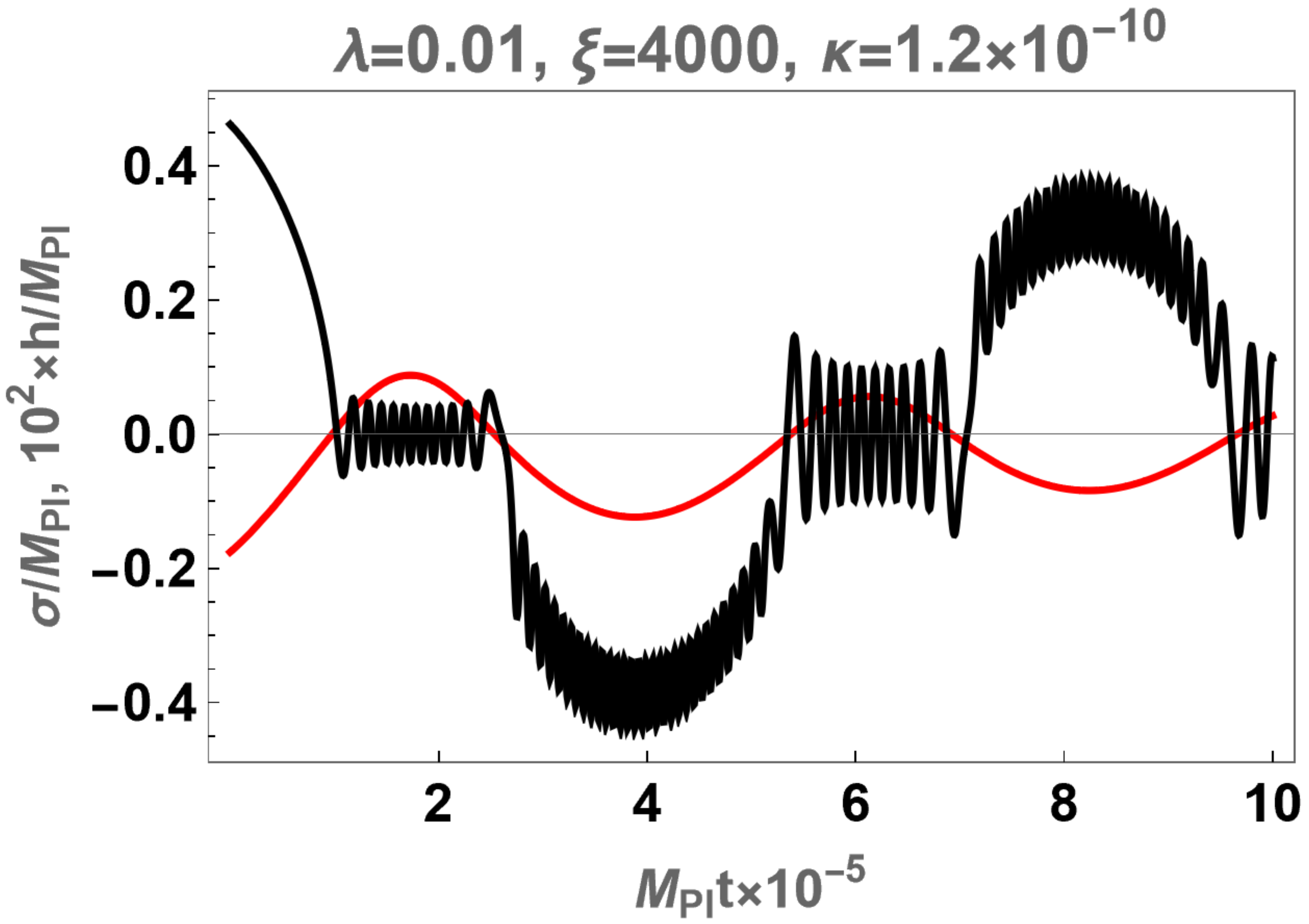
For $\sigma < 0$

$h_0 = 0$ **for** $\sigma_0 > 0$

Background field evolution

$$m_\sigma^2 = \begin{cases} 3\kappa M_{\rm Pl}^2 & , \quad \sigma_0 > 0, \\ \frac{3\kappa\lambda M_{\rm Pl}^2}{\lambda + 9\kappa\left(\xi + \frac{1}{6}\right)^2} & , \quad \sigma_0 < 0 \end{cases}$$

$$m_h^2 = \begin{cases} 3\sqrt{6}\kappa\left(\xi + \frac{1}{6}\right) M_{\rm Pl}\sigma_0 & , \quad \sigma_0 > 0, \\ 6\sqrt{6}\kappa\left(\xi + \frac{1}{6}\right)(-M_{\rm Pl}\sigma_0) & , \quad \sigma_0 < 0, \end{cases}$$



The highly oscillating part becomes prominent for $\sigma_0 > 0$, when the Higgs background vanishes.
It turns out that $h_{\rm osc}$ is the dominant source for reheating due to both rapid oscillations and a large Yukawa coupling.

Decay rates of σ condensates

$$\sigma = \sigma_0(t) + \delta\sigma,$$

Couplings to the SM particles are conformal $\Longrightarrow$ suppressed by the Planck scale

$$\mathscr{L} \supset c\sigma_0(\delta h)^2$$

$$\Gamma_{\sigma_0\to\delta h\delta h} = \begin{cases} \frac{9\sqrt{3}}{16\pi} M_p \kappa^{3/2}\left(\xi+\frac{1}{6}\right)^2\left(1-4\sqrt{6}\left(\xi+\frac{1}{6}\right)\frac{\sigma_0}{M_p}\right)^{1/2} & , \sigma_0>0, \\ \frac{9\sqrt{3}}{4\pi} M_p \kappa^{3/2}\left(\xi+\frac{1}{6}\right)^2\sqrt{\frac{\lambda_{\rm eff}}{\lambda}}\left(1+8\sqrt{6}\left(\xi+\frac{1}{6}\right)\frac{\lambda_{\rm eff}}{\lambda}\frac{\sigma_0}{M_p}\right)^{1/2} & , \sigma_0<0. \end{cases}$$

$$c = \begin{cases} -\frac{3}{2}\sqrt{6}\kappa\left(\xi+1/6\right)M_p l & , \quad \sigma_0>0, \\ 3\sqrt{6}\kappa\left(\xi+1/6\right)M_p & , \quad \sigma_0<0. \end{cases}$$

In both cases, for $\xi \gtrsim 1$ this decay mode is kinematically blocked.

The factor $|\sigma_0|M_P \lesssim 0.1\left(\xi+\frac{1}{6}\right)^{-1}$ makes it subdominant after a few oscillations.

Decay rates of Higgs condensates

$$h = h_0(\sigma_0) + h_{\rm osc}(t) + \delta h$$

$$\mathscr{L} \supset -\frac{y_t}{\sqrt{2}} h \bar{t} t = -\frac{y_t}{\sqrt{2}} (h_0(\sigma_0) + h_{\rm osc}) \bar{t} t$$

top mass →

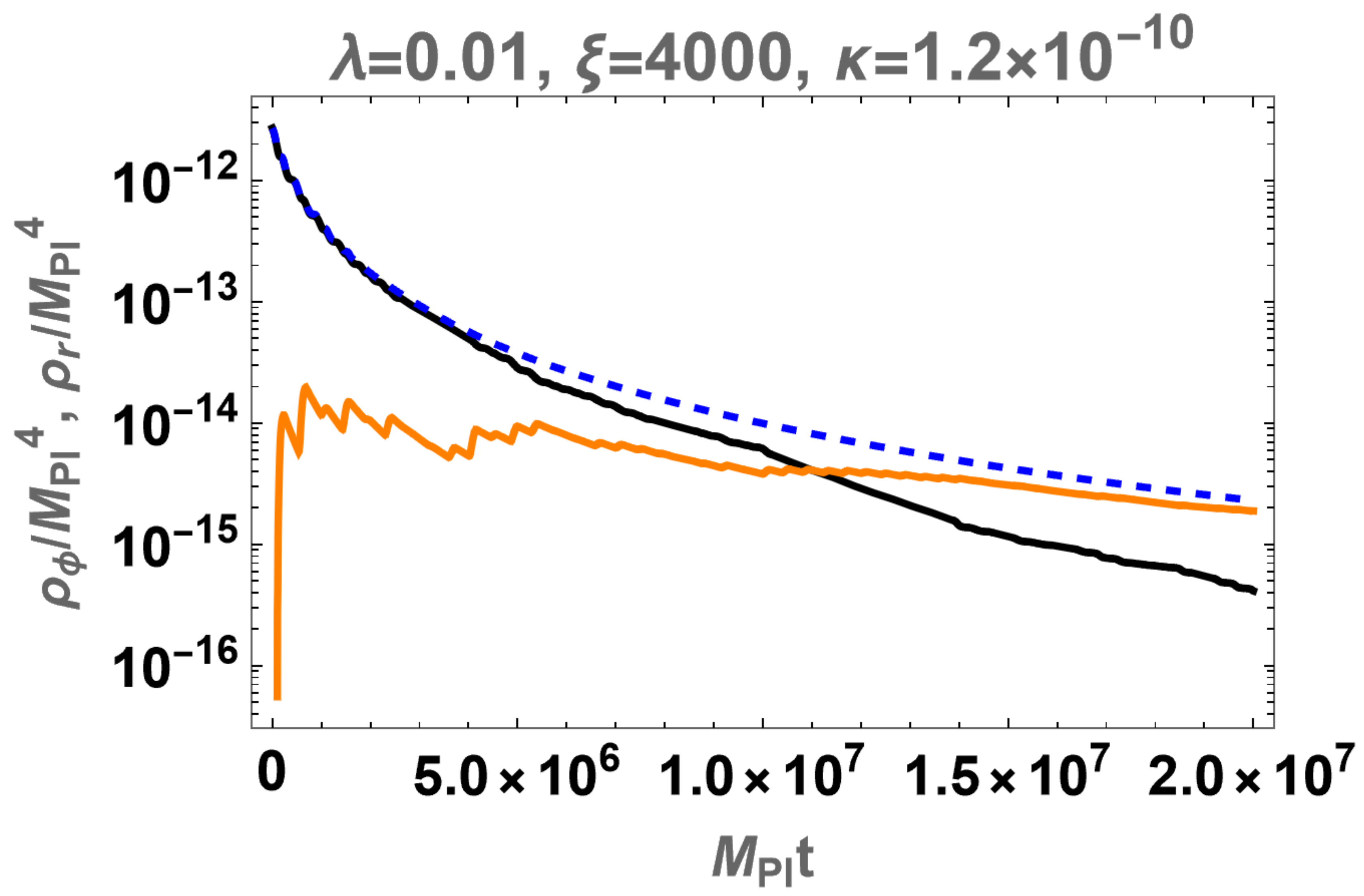
$$m_t = y_t \sqrt{\frac{3\sqrt{6}\kappa}{2\lambda_{\rm eff}} \left(\xi + \frac{1}{6}\right) (-M_P \sigma_0)}$$

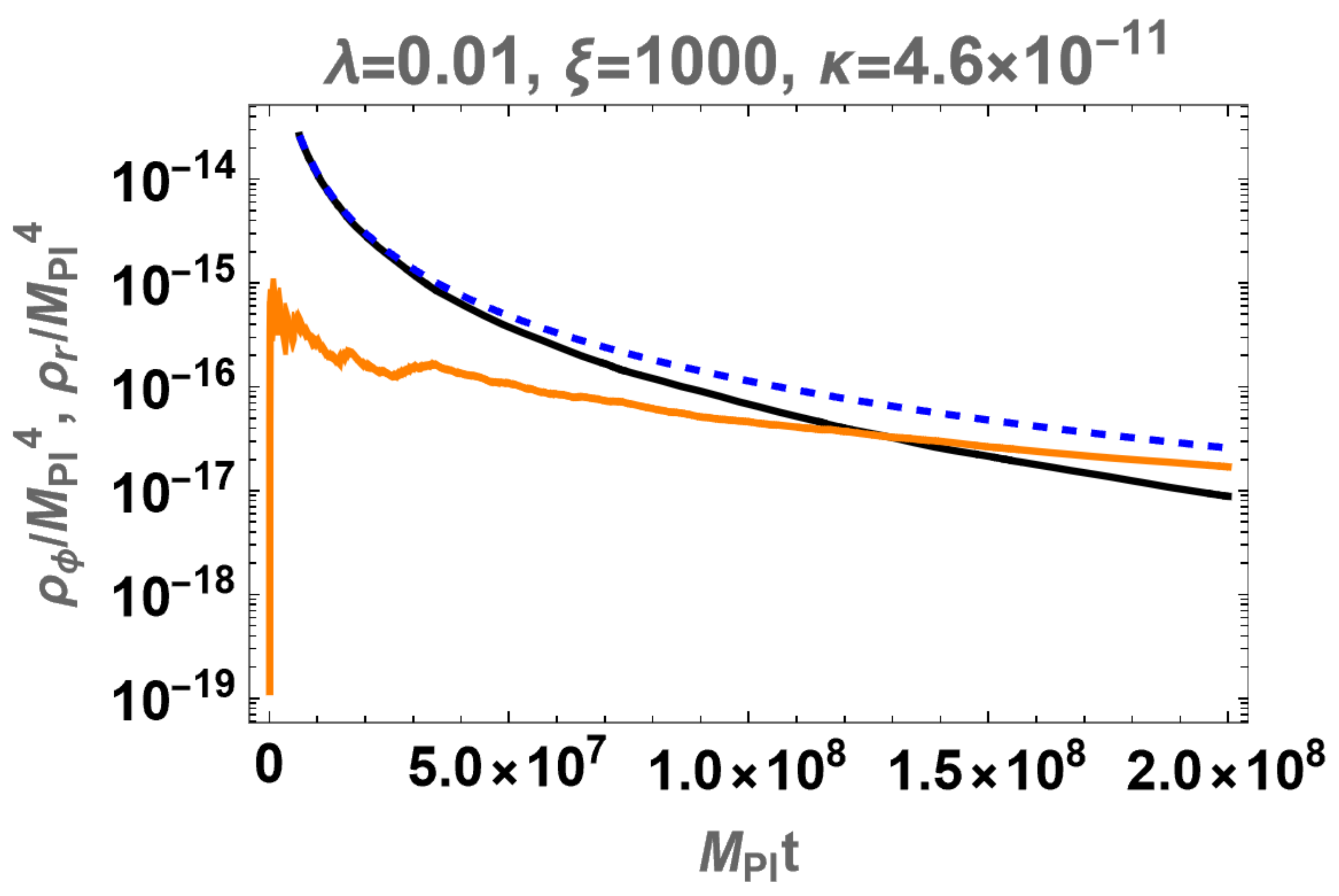
$m_t = 0$ **for** $\sigma_0 > 0$

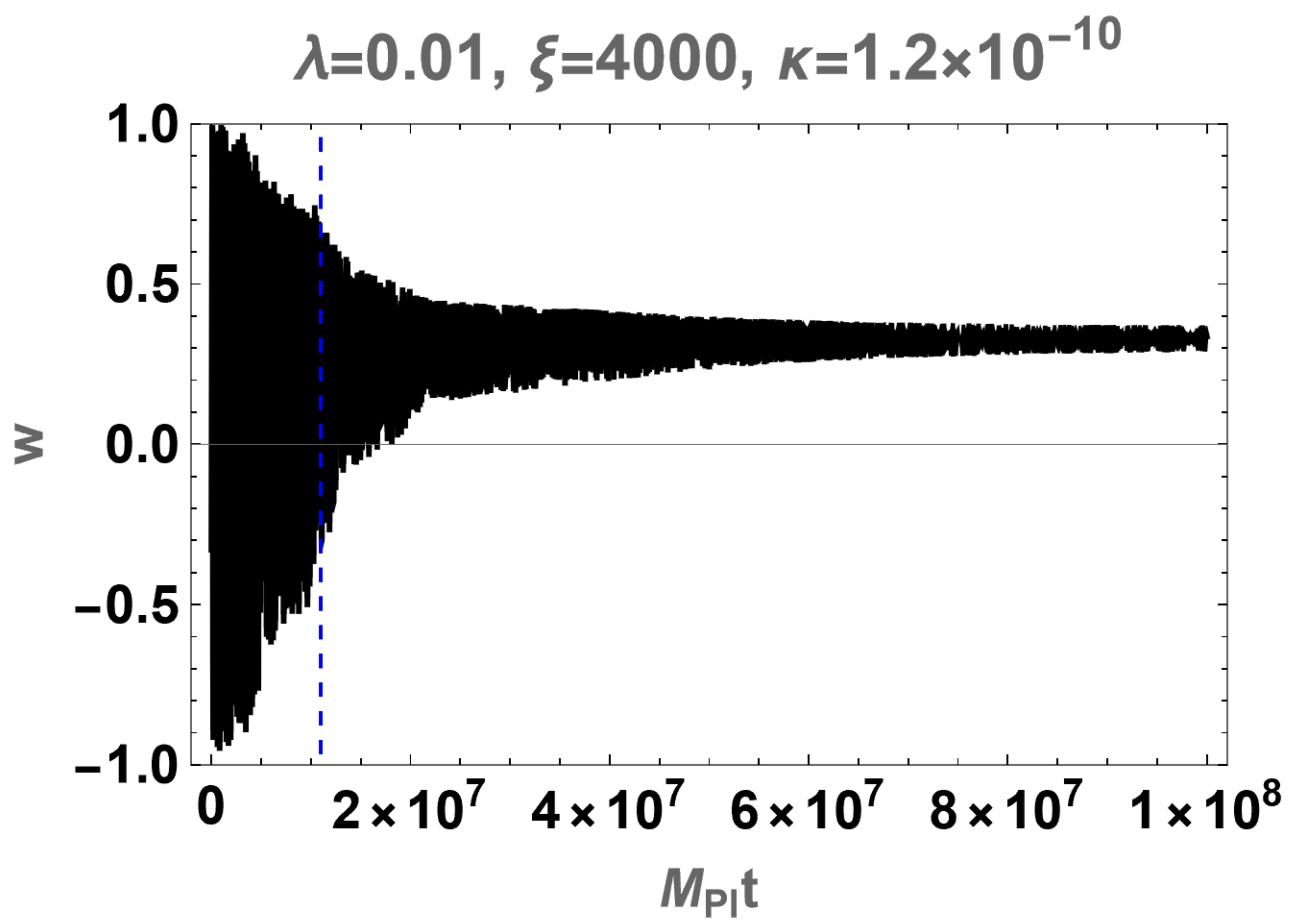
$$\Gamma_{h_{\rm osc} \to t\bar{t}} = \begin{cases} \frac{3y_t^2}{16\pi} M_p \left(3\sqrt{6}\kappa \left(\xi + \frac{1}{6}\right) \frac{\sigma_0}{M_p}\right)^{1/2} & , \quad \sigma_0 > 0, \\ \frac{3y_t^2}{16\pi} M_p \left(-6\sqrt{6}\kappa \left(\xi + \frac{1}{6}\right) \frac{\sigma_0}{M_p l}\right)^{1/2} \left(1 - \frac{y_t^2}{\lambda_{\rm eff}}\right)^{3/2} & , \quad \sigma_0 < 0. \end{cases}$$

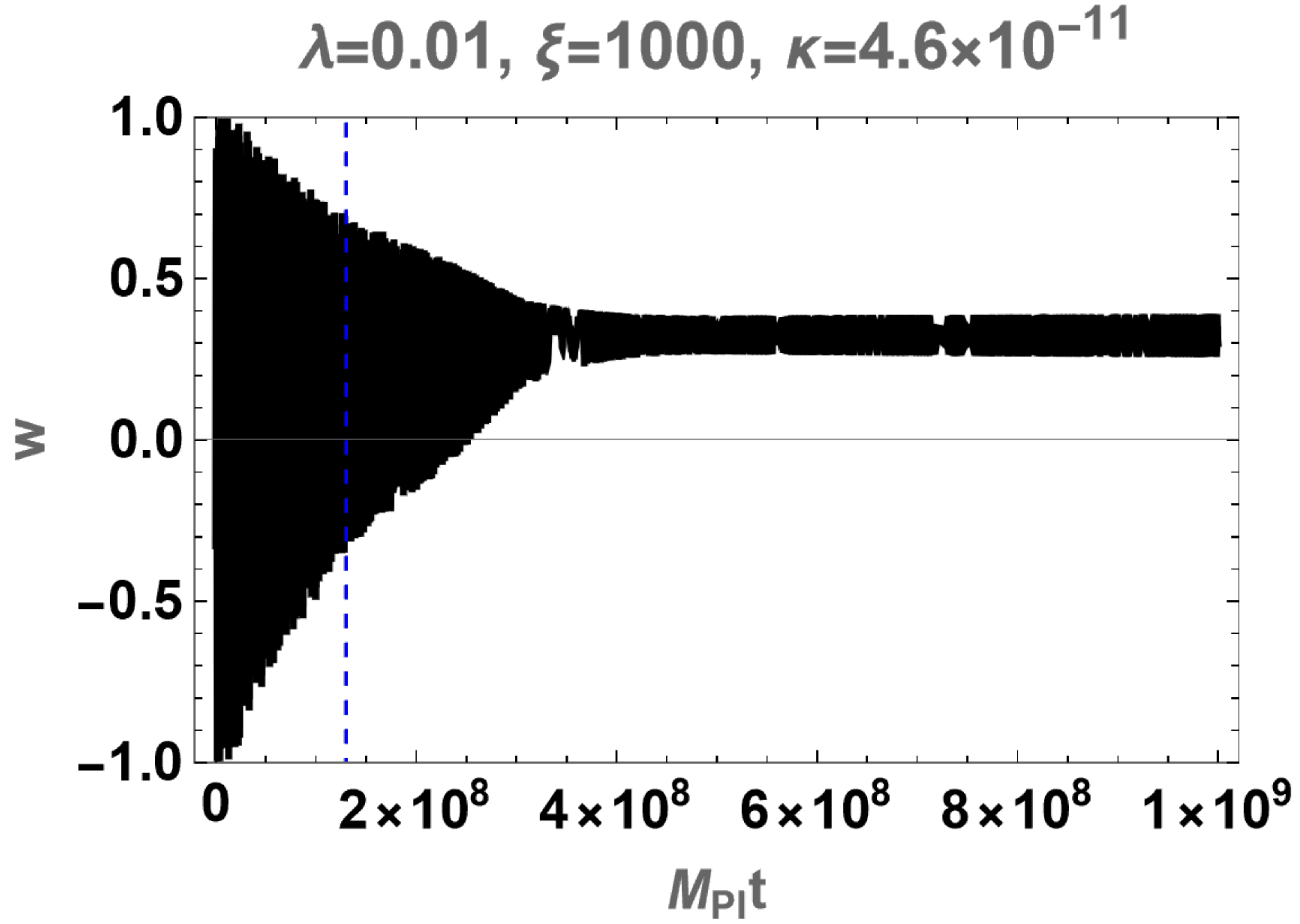
For $\sigma_0 < 0$ the decay $h_{\rm osc} \to t\bar{t}$ is kinematically allowed only for $\xi \gtrsim 5000$ for $y_t = 0.5$
For $\sigma_0 > 0$ $h_{\rm osc} \to t\bar{t}$ is always open. Thus, is the dominant decay mode for the Higgs condensate.

Numerical solutions





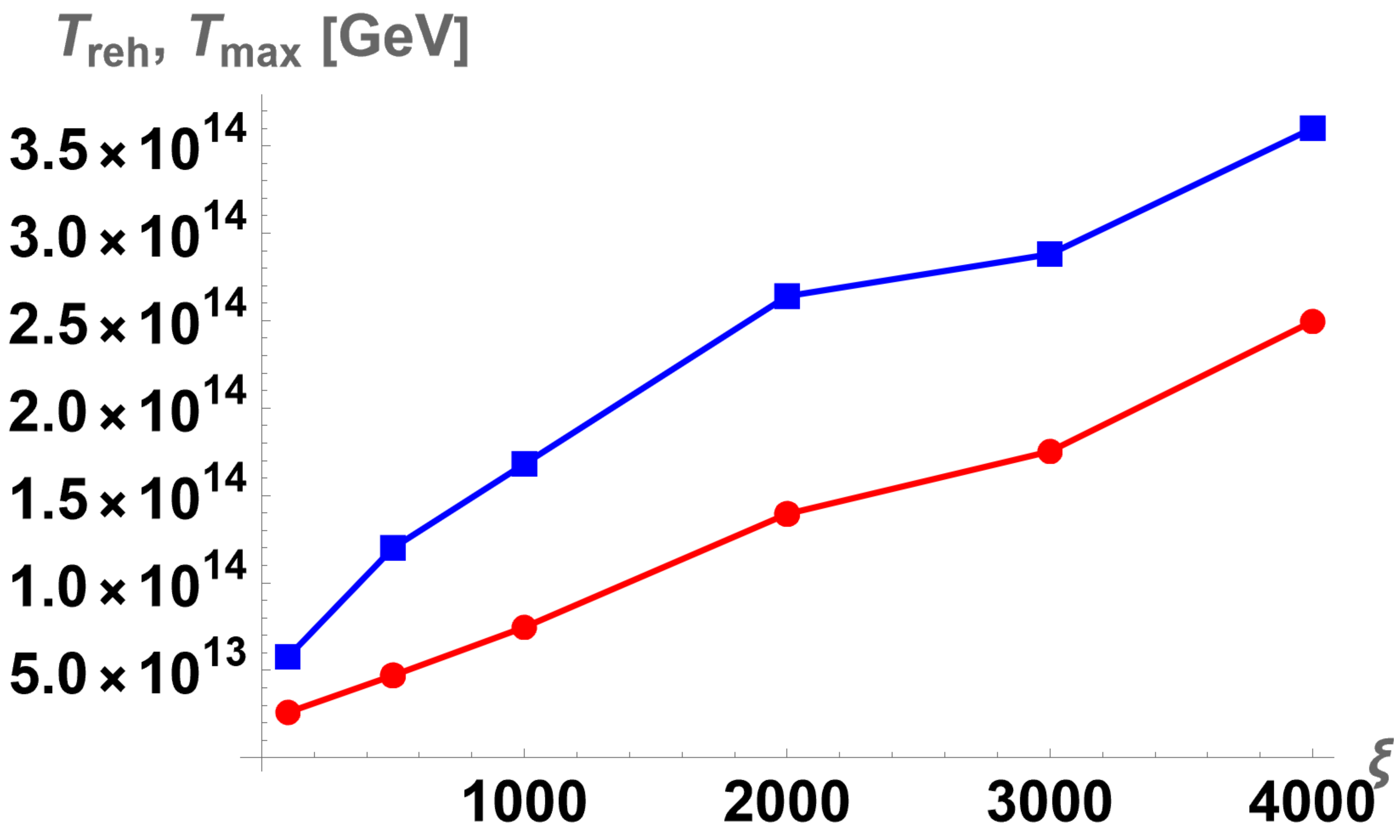




Reheating and max temperature

$$T_{\rm reh}^4 = \frac{72 M_p^2}{5\pi^2 g_{\rm reh}} \left(\frac{\Gamma_\sigma \rho_{\sigma,\rm end} + \Gamma_h \rho_{h,\rm end}}{\rho_{\sigma,\rm end} + \rho_{h,\rm end}} \right)^2$$

$$T_{\rm max}^4 = \frac{12\sqrt{3}}{\pi^2 g_{\rm reh}} \left(\frac{3}{8}\right)^{\frac{3}{5}} M_p \frac{\Gamma_\sigma \rho_{\sigma,\rm end} + \Gamma_h \rho_{h,\rm end}}{\sqrt{\rho_{\rm end}}}$$



Inflationary predictions

Instant thermalization $T = \left(\frac{30}{\pi^2 g_{\rm reh}}\rho_r\right)^{1/4}$ **with** $g_{\rm reh} = 106.75$

For $100 \leq \xi \leq 4000$**,** $2.6 \times 10^{13}\,{\rm GeV} \leq T_{\rm reh} \leq 2.5 \times 10^{14}\,{\rm GeV}$

$T_{\max}$ **varies by** $5.8 \times 10^{13}\,{\rm GeV} \leq T_{\max} \leq 3.6 \times 10^{14}\,{\rm GeV}$

$$n_s = 0.9608 - 0.9614,$$
$$r = 0.0041 - 0.0042$$

$N = 53.2 - 54.0$

Correction in the number of e-foldings due to delayed reheating $-\Delta n_s = 0.00064 - 0.0012.$

Freeze-in Dark Matter

Dark Matter Model

$$\mathscr{L}/\sqrt{-g_E} = \frac{M_p^2}{2}R_E - \frac{1}{2\Omega^4}\left(1 - \frac{h^2}{6M_p^2} - \frac{X^2}{6M_p^2}\right)\left(\partial_\mu\sigma\right)^2 - \frac{1}{2\Omega^4}\left(1 - \frac{\sigma^2}{6M_p^2} - \frac{X^2}{6M_p^2}\right)\left(\partial_\mu h\right)^2$$

$$\Omega^2 = 1 - \frac{h^2}{6M_p^2} - \frac{X^2}{6M_p^2} - \frac{\sigma^2}{6M_p^2}$$

$$-\frac{1}{2\Omega^4}\left(1 - \frac{\sigma^2}{6M_p^2} - \frac{h^2}{6M_p^2}\right)\left(\partial_\mu X\right)^2 - \frac{hX}{6M_p^2\Omega^4}\partial_\mu h\partial^\mu X - \frac{h\sigma}{6M_p^2\Omega^4}\partial_\mu h\partial^\mu\sigma - \frac{X\sigma}{6M_p^2\Omega^4}\partial_\mu X\partial^\mu\sigma - V + \frac{1}{(\Omega\Delta)^4}\mathscr{L}_{\rm SM}$$

$$V = \frac{1}{\Omega^4}\left[\frac{1}{4}\kappa\left(\sigma(\sigma+\sqrt{6}M_p) + 3\tilde{\xi}h^2 + 3\tilde{\eta}X^2\right)^2 + \frac{\lambda}{4}h^4 + \frac{m_X^2\Delta^{-2}}{2}X^2 + \frac{\lambda_X}{4}X^4 + \frac{\lambda_{hX}}{4}h^2X^2\right]$$

When $|\tilde{\eta}| \ll 1$ DM couples feebly to σ and h through gravity.

We have a vanishing Higgs portal $|\lambda_{hX}| \ll 1$

Conformal couplings

$$\tilde{\eta} \equiv \eta + \frac{1}{6}$$

$$\tilde{\xi} = \xi + \frac{1}{6}$$

Dark Matter Production

During Reheating

-During the matter domination era
-Consider both thermal and non thermal scattering

$$Y(T_{\rm reh}) = Y_{\rm thermal}(T_{\rm reh}) + Y_{\rm non-thermal}(T_{\rm reh})$$

-Thermal scattering = SM bath
-Non-thermal scattering = Inflaton condensates

After Reheating

-During radiation domination
-Dark Matter is produced thermally from the SM bath

Production mechanisms

-Contact interactions
-Graviton exchanges

Production after reheating

Thermal production from contact terms

The inflation fields are already settled around the origin

We can neglect their VEVs

The only direct interactions with the SM is through derivatives couplings of the Higgs.

$$\mathscr{L}_X/\sqrt{-g} \supset -\frac{X^2}{12M_p^2}(\partial_\mu h)^2 - \frac{hX}{6M_p^2}\partial_\mu h \partial^\mu X$$

The couplings coming from $g^{\mu\nu}T^{SM}_{\mu\nu}$ vanish because fermions and gauge bosons are massless.

$$\mathscr{M}_{h+h\to X+X} = -\frac{s+2m_X^2}{6M_{\rm Pl}^2} - 18\kappa\tilde{\eta}\left(\xi+\frac{1}{6}\right) - \lambda_{hX}$$

Thermal production from gravitational interactions

$$h\,h \to X\,X$$

$$\mathcal{L} \supset \frac{1}{M_p} h^{\mu\nu}\left(T^{\rm SM}_{\mu\nu} + T^{h}_{\mu\nu} + T^{\sigma}_{\mu\nu} + T^{X}_{\mu\nu}\right)$$

$$\mathcal{M}^{G}_{h+h\to X+X} = -\frac{1}{M_p^2}\frac{(t-m_X^2)(s+t-m_X^2)}{s}$$

$$|\mathcal{M}^{\rm total}_{h+h\to X+X}|^2 = \left(\frac{s+2m_X^2}{6M_{\rm P}^2} + 18\kappa\tilde{\eta}\tilde{\xi} + \lambda_{hX} + \frac{(t-m_X^2)(s+t-m_X^2)}{sM_p^2}\right)^2$$

$$Y(T_*) \simeq Y(T_{\rm reh}) + \frac{\sqrt{10}}{20480\pi^4 g_{\rm reh}^{1/2}}\frac{4m_X^4 + 45M_p^4\left(\lambda_{hX} + 18\kappa\tilde{\eta}\tilde{\xi}\right)^2}{m_X M_p^3} + \frac{209\sqrt{10}}{240\pi^6 g_{\rm reh}^{1/2}}\frac{T_{\rm reh}^3}{M_{\rm Pl}^3}$$

Dependence on the reheating dynamics

IR Freeze-in

UV Freeze-in

Production during reheating

$$Y(T_{\rm reh}) = Y_{\rm thermal}(T_{\rm reh}) + Y_{\rm non-thermal}(T_{\rm reh})$$

Thermal production (SM bath)

We will have contributions from both σ and h

SM particles interact with DM only through **graviton exchanges**

$$|\mathcal{M}^{G}_{f+f\to X+X}|^2 = \frac{-1}{2M_p^4 s^2}\left(s+2t-2m_X^2\right)^2\left(\left(t-m_X^2\right)^2+st\right)$$

$$|\mathcal{M}^{G}_{V+V\to X+X}|^2 = \frac{2}{M_p^4 s^2}\left(m_X^4-2m_X^2 t+t(s+t)\right)^2$$

$$Y_{\text{thermal}}(T_{\text{reh}}) \simeq \frac{69\sqrt{10}}{40\pi^6 g_{\text{reh}}^{1/2}}\frac{T_{\text{reh}}^3}{M_p^3}$$

Non-thermal production

(In the conformal case with vanishing Higgs portal)

We cannot neglect the VEVs of the fields.

$$\mathscr{L} \supset -\frac{1}{12M_p^2}X^2(\partial_\mu\sigma_0) - \frac{1}{6M_p^2}X\sigma\partial_\mu X\partial^\mu\sigma_0 - \frac{1}{12M_p^2}\sigma_0^2(\partial_\mu X) \longrightarrow$$

Contribute to $\sigma_0 + \sigma_0 \to X + X$

$$\mathscr{M}_1^{\text{non-der}} = -\frac{\kappa}{4}\sigma_e^2$$

$$\mathscr{M}_1^{\text{der}} = -\frac{\kappa}{8}\sigma_e^2\left(1 - \frac{\sigma_0^2}{3M_p^2}\right)$$

$$\mathscr{M}_1^{G} = \frac{3}{8}\kappa\sigma_e^2\left(1 + \frac{\sigma_0^2}{6M_p^2}\right)$$

Leading order cancellation

$$\mathscr{M}_1^{\text{total}} = \frac{5}{48}\kappa\sigma_e^2\frac{\sigma_0^2}{M_p^2}$$

$$Y_{\text{non-thermal}}(T_{\text{reh}}) \simeq \frac{\sqrt{3}\pi g_{\text{reh}}}{2239488}\frac{T_{\text{reh}}}{\kappa^2 M_p^{11}}\frac{\rho_{\sigma,\text{end}}^4}{\rho_{\text{end}}^{3/2}}$$

Dark Matter Abundance

Combining thermal and non-thermal results

$$\Omega h^2 = 1.6\times 10^8 \left(\frac{m_X}{1\mathrm{GeV}}\right)\left(\frac{g_0}{g_{\rm reh}}\right) Y(T_*),$$

Non-thermal Thermal

$$\simeq 1.6\times 10^8 \left(\frac{m_X}{1\mathrm{GeV}}\right)\left(\frac{g_0}{g_{\rm reh}}\right)\left[\frac{\sqrt{3}\pi g_{\rm reh}}{2239488}\frac{T_{\rm reh}}{\kappa^2 M_p^{11}}\frac{\rho^4_{\sigma,\rm end}}{\rho^{3/2}_{\rm end}} + \frac{623\sqrt{10}}{240\pi^6 g^{1/2}_{\rm reh}}\frac{T^3_{\rm reh}}{M_p^3}\right.$$

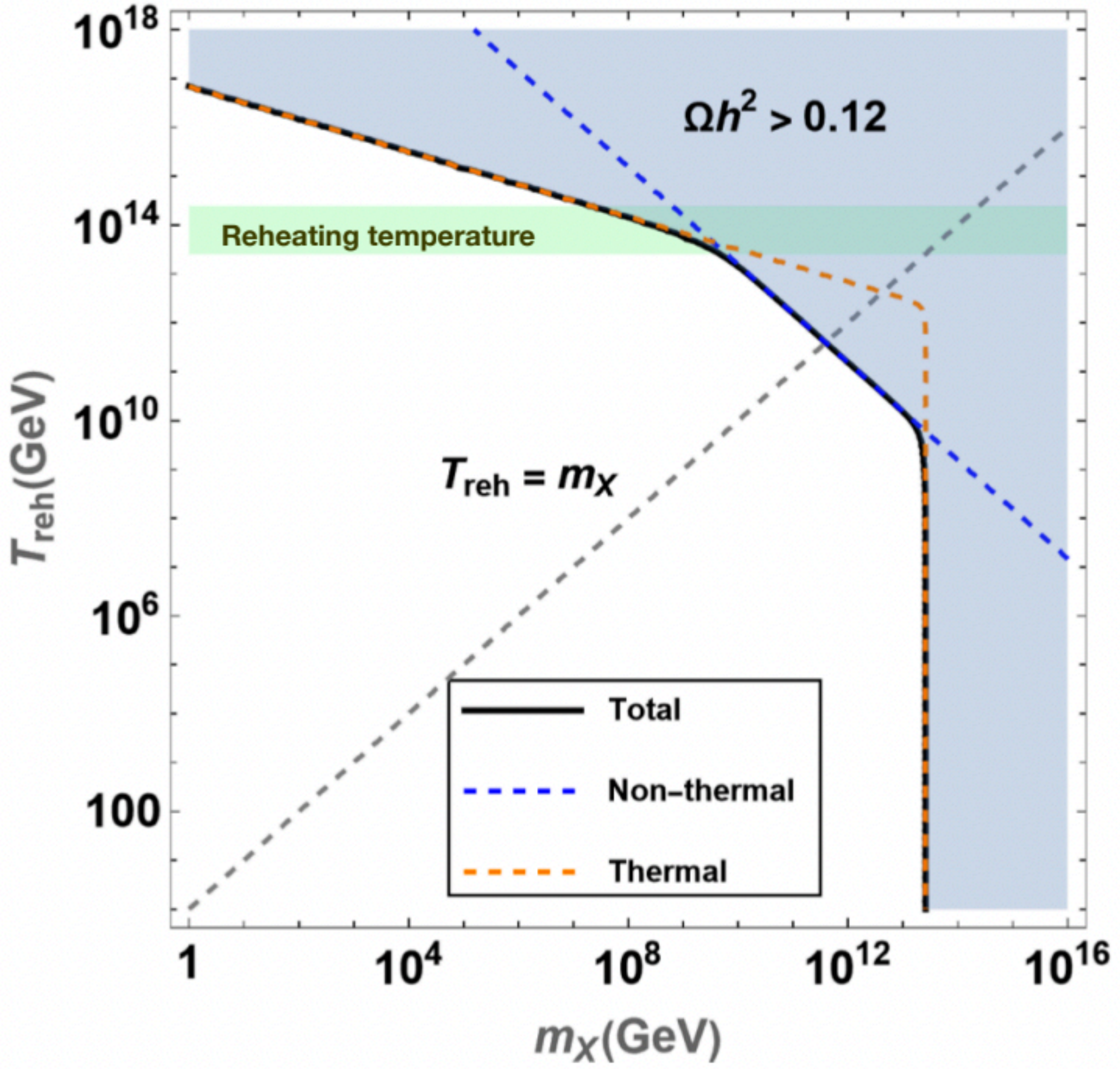
$$\left. + \frac{\sqrt{10}}{20480\pi^4 g^{1/2}_{\rm reh}}\frac{4m_X^4 + 45M_p^4\left(\lambda_{hX} + 18\kappa\tilde{\eta}\tilde{\xi}\right)^2}{m_X M_p^3}\right]$$

In the nearly conformal coupling thermal scattering is the most effective way to produce dark matter.

DM with conformal coupling

$$2.1 \times 10^7\,\mathrm{GeV} \leq m_X \leq 4.6 \times 10^9\,\mathrm{GeV}$$

The lower limit doesn't reproduce all DM content, additional mechanism needed.



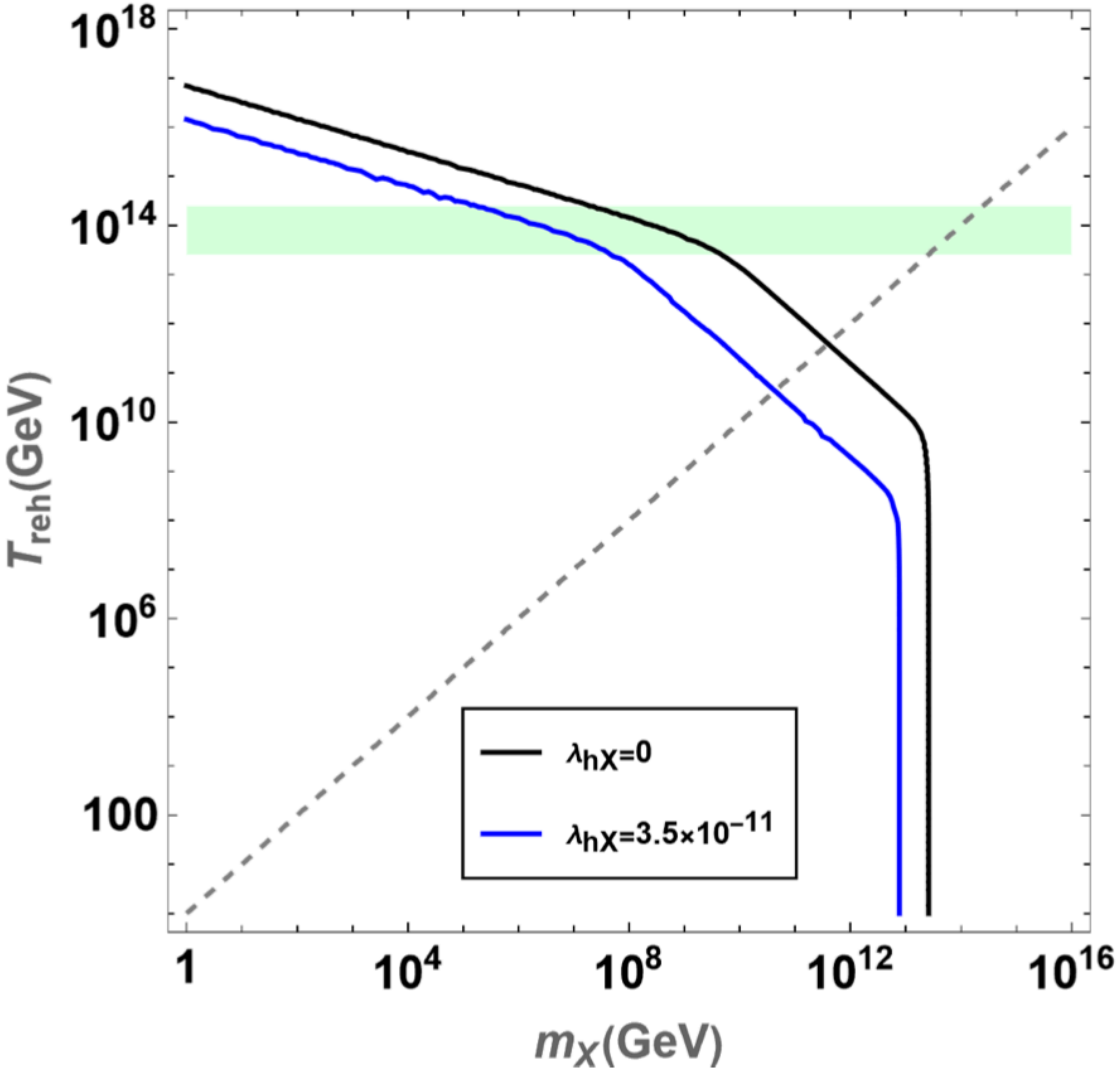
Dark Matter with non-conformal interactions

For thermal production we have additional couplings λ_{hX} and $\kappa\tilde{\eta}\tilde{\xi}$.

These must be constrain in order to avoid over abundance $|\lambda_{hX}| \lesssim 10^{-12}$ and $|\tilde{\eta}| \lesssim 10^{-6}$ for $\tilde{\xi}\kappa \sim 10^{-7}$.

For non thermal production:

$$\mathscr{L} \supset \begin{cases} -3\sqrt{\frac{3}{2}}M_p\tilde{\eta}\kappa\sigma_0 X^2 \quad , & \sigma_0 > 0, \\ -3\sqrt{\frac{3}{2}}M_p\kappa\frac{\tilde{\eta}\lambda - \lambda_{hX}\tilde{\xi}/2}{\lambda + 9\kappa\tilde{\xi}^2}\sigma_0 X^2, & \sigma_0 < 0. \end{cases}$$



Smaller DM masses can be obtained.

Summary

Goal of this work:

- Provide a UV complete model for Higgs inflation and subsequent reheating.
- Study the option of FIMP DM reproducing the observed energy density.

Reheating

- In the beginning of the oscillation stage, we have a mix of Higgs and σ condensates.
- The Higgs condensate is dominant for reheating due to **kinematic blocking**.
- Reheating temperature varies in the range $10^{13}\,\mathrm{GeV}$ and $10^{14}\,\mathrm{GeV}$ depending on ξ

Dark Matter production

- We studied a model for scalar Dark Matter production.
- In the nearly conformal coupling thermal scattering is the most effective way to produce dark matter.
- The mass of Dark Matter is in the range $10^7 - 10^9$ GeV.
- Including non-conformal effects leads to even lighter Dark Matter.

Back ups

Analytic solutions

Separate the dynamics of h and σ

$$\rho_\sigma = \frac{1}{2}\dot\sigma^2 + \frac{1}{2}m_{\sigma,+}^2\sigma^2, \quad \rho_h = \frac{1}{2}\dot h^2 + \frac{1}{2}m_h^2 h^2 + \frac{\lambda_{\text{eff}}}{4}h^4,$$
$$p_\sigma = \frac{1}{2}\dot\sigma^2 - \frac{1}{2}m_{\sigma,+}^2\sigma^2, \quad p_h = \frac{1}{2}\dot h^2 - \frac{1}{2}m_h^2 h^2 - \frac{\lambda_{\text{eff}}}{4}h^4,$$

$$m_{\sigma,+}^2 = 3\kappa M_p^2,$$
$$m_h^2 = 3\sqrt{6}\kappa\left(\xi + \frac{1}{6}\right)M_p\sigma_0\,.$$

These simplifies Boltzmann equations

$$\dot\rho_\sigma + 3H(\rho_\sigma + p_\sigma) + \Gamma_\sigma(\rho_\sigma + p_\sigma) = 0,$$
$$\dot\rho_h + 3H(\rho_h + p_h) + \Gamma_h(\rho_h + p_h) = 0,$$
$$\dot\rho_r + 4H\rho_r - \Gamma_\sigma(\rho_\sigma + p_\sigma) - \Gamma_h(\rho_h + p_h) = 0,$$
$$3M_p^2H^2 = \rho_\sigma + \rho_h + \rho_r$$

$$p_\sigma = p_h = \Gamma_\sigma = \Gamma_h = 0$$

$$\rho_r = \frac{2\sqrt{3}}{5}M_p\frac{\Gamma_\sigma\rho_{\sigma,\text{end}} + \Gamma_h\rho_{h,\text{end}}}{\sqrt{\rho_{\text{end}}}}\left(\left(\frac{a}{a_{\text{end}}}\right)^{-\frac{3}{2}} - \left(\frac{a}{a_{\text{end}}}\right)^{-4}\right)$$

Reheating and max temperature

$$\rho_\sigma = \rho_{\sigma,\mathrm{end}} \left(\frac{a}{a_{\mathrm{end}}}\right)^{-3}, \quad \rho_h = \rho_{h,\mathrm{end}} \left(\frac{a}{a_{\mathrm{end}}}\right)^{-3}$$

$$\rho_r(a_{\mathrm{reh}}) = \frac{\pi^2 g_{\mathrm{reh}}}{30} T_{\mathrm{reh}}^4$$

$$\rho_\sigma(a_{\mathrm{reh}}) + \rho_h(a_{\mathrm{reh}}) = \rho_r(a_{\mathrm{reh}})$$

$$\rho_r = \frac{2\sqrt{3}}{5} M_{\mathrm{Pl}} \frac{\Gamma_\sigma \rho_{\sigma,\mathrm{end}} + \Gamma_h \rho_{h,\mathrm{end}}}{\sqrt{\rho_{\mathrm{end}}}} \left(\left(\frac{a}{a_{\mathrm{end}}}\right)^{-\frac{3}{2}} - \left(\frac{a}{a_{\mathrm{end}}}\right)^{-4} \right)$$

$$T_{\mathrm{max}}^4 = \frac{12\sqrt{3}}{\pi^2 g_{\mathrm{reh}}} \left(\frac{3}{8}\right)^{\frac{3}{5}} M_{\mathrm{Pl}} \frac{\Gamma_\sigma \rho_{\sigma,\mathrm{end}} + \Gamma_h \rho_{h,\mathrm{end}}}{\sqrt{\rho_{\mathrm{end}}}}$$

$$\left(\frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}\right)^3 = \frac{25}{12} \frac{\rho_{\mathrm{end}}^3}{M_{\mathrm{Pl}}^2 \left(\Gamma_\sigma \rho_{\sigma,\mathrm{end}} + \Gamma_h \rho_{h,\mathrm{end}}\right)^2}$$