

IMPRINTS OF  
MINI PRIMORDIAL BLACK HOLES  
IN  
COSMOLOGICAL DATA

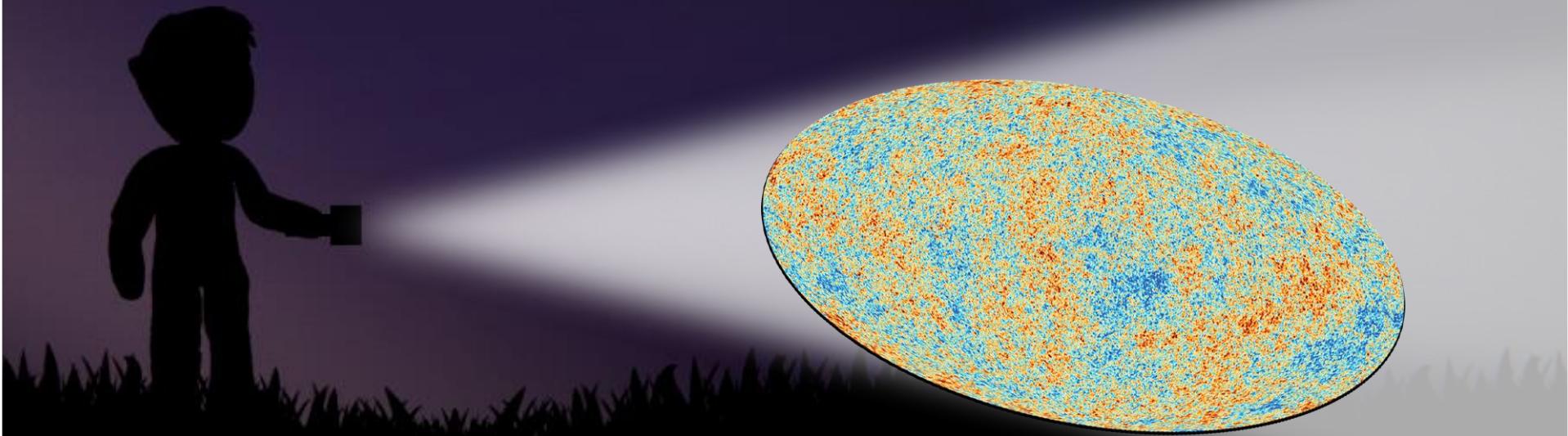
A. Cheek, L.H., Y. F. Perez-Gonzalez, and J. Turner  
*Phys.Rev.D* 105 (2022) 1, 015022. and *Phys.Rev.D* 105 (2022) 1, 015023 [PRD Highlights]

A. Cheek, L.H., Y. F. Perez-Gonzalez, and J. Turner  
*To appear* [ArXiv: 2211.XXXX]

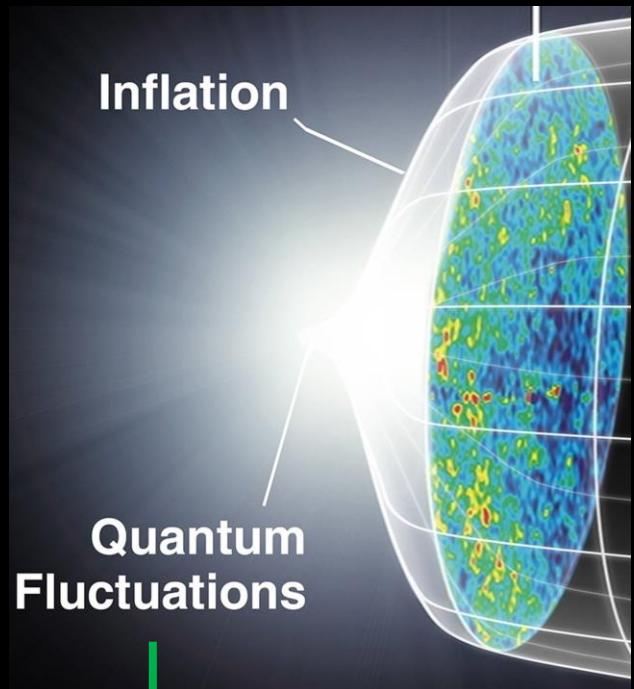
K.R. Dienes, L.H., F. Huang, D.Kim, B. Thomas, and T.M.P. Tait  
*To appear* [ArXiv: 2211.XXXX]

# Why Mini Primordial Black Holes?

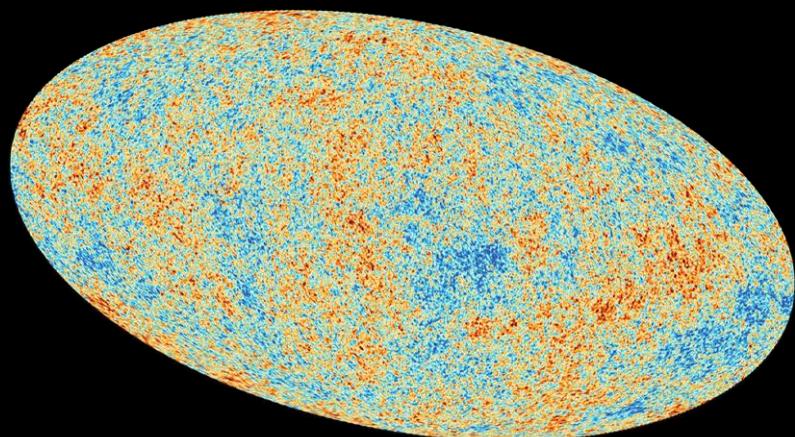
CMB  
Perturbations



# Why Mini Primordial Black Holes?

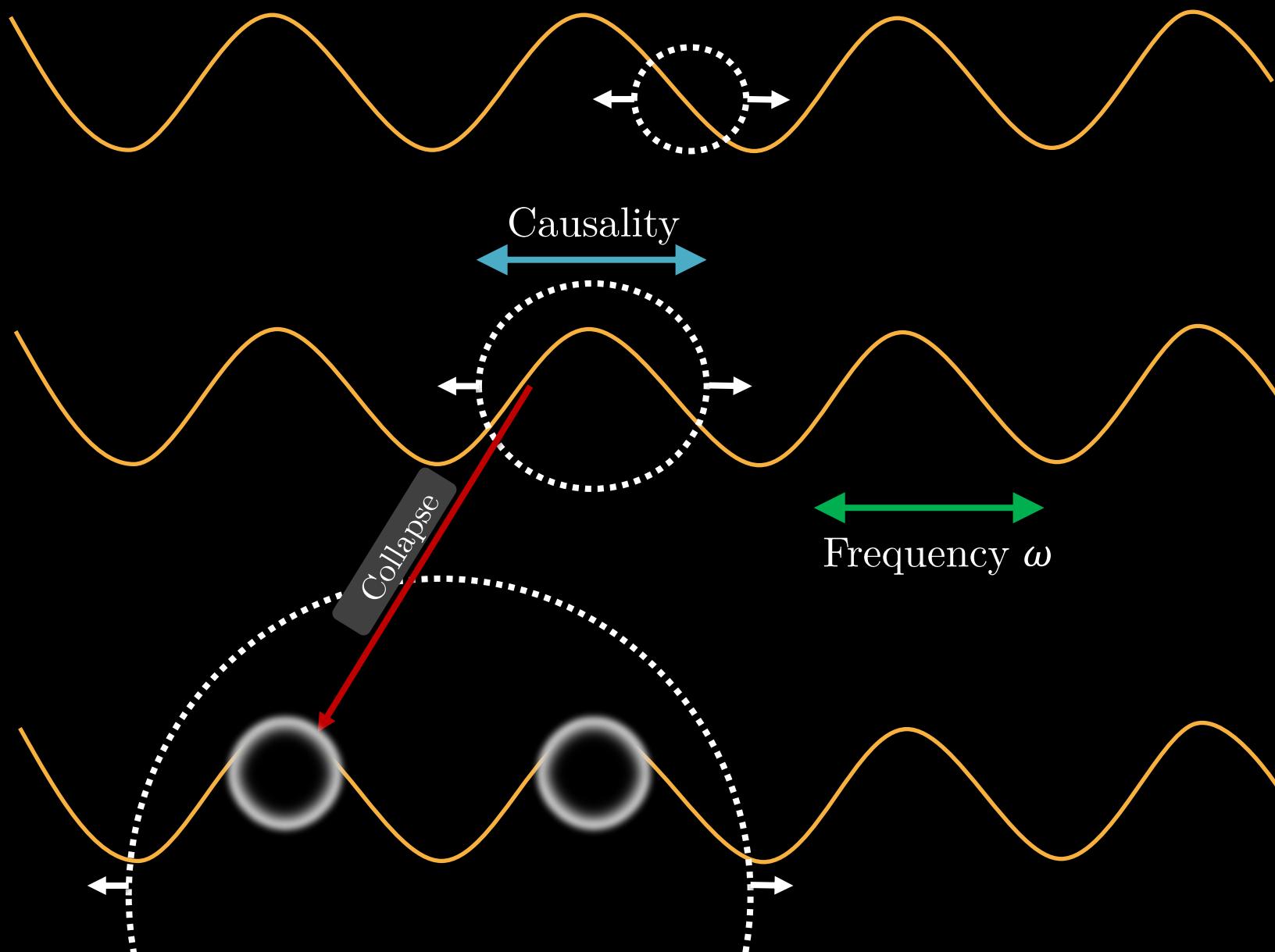


CMB  
Perturbations

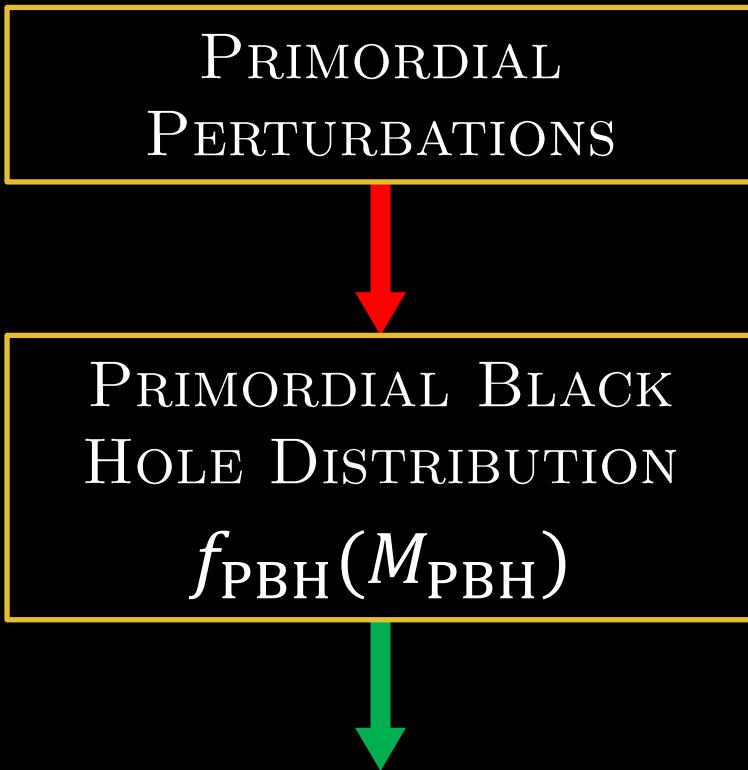


Primordial  
Perturbations

# Why Mini Primordial Black Holes?



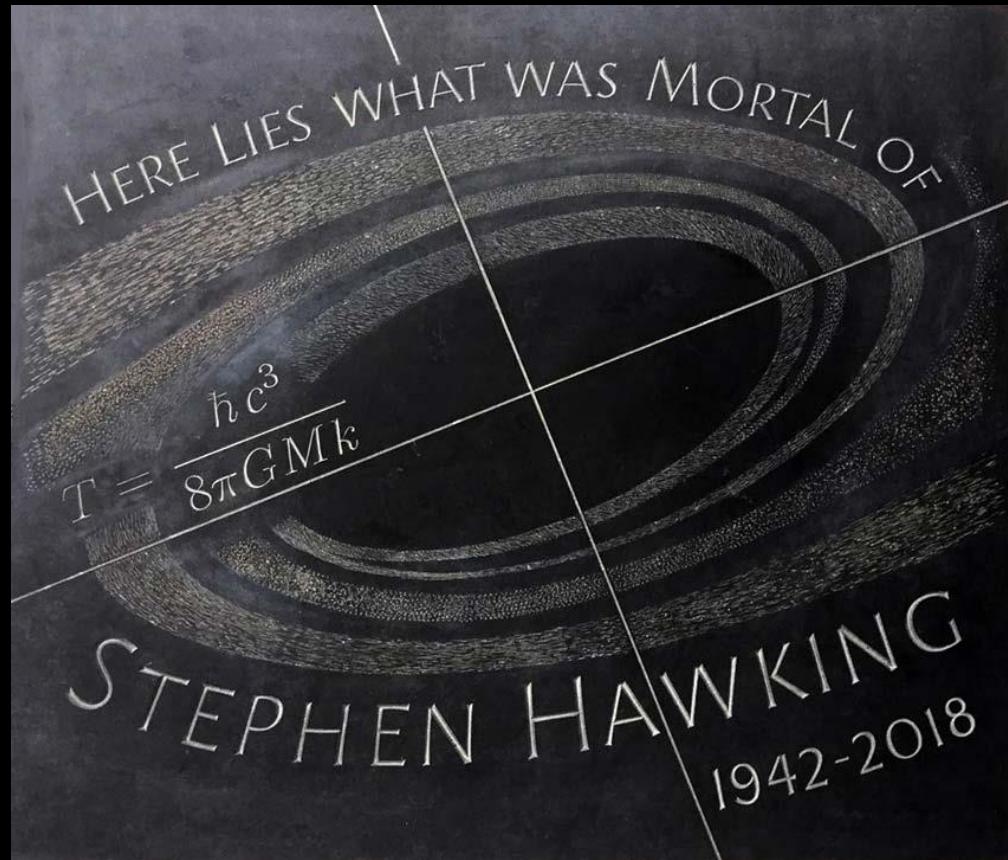
# Why Mini Primordial Black Holes?



Observable Imprints ?

# BLACK HOLES EVAPORATE...

S. HAWKING, 1974



# Why Mini Primordial Black Holes?

PRIMORDIAL BLACK  
HOLE DISTRIBUTION

$$f_{\text{PBH}}(M_{\text{PBH}})$$

- Some may be stable and participate to the DM relic abundance ( $M_{\text{PBH}} \gtrsim 10^{15} \text{ g}$ )
- Some may be unstable and evaporate after BBN ( $10^{15} \text{ g} \lesssim M_{\text{PBH}} \lesssim 10^9 \text{ g}$ )
- Some may be unstable and evaporate before BBN ( $M_{\text{PBH}} \lesssim 10^9 \text{ g}$ )

The topic of this talk ...

# OUTLINE

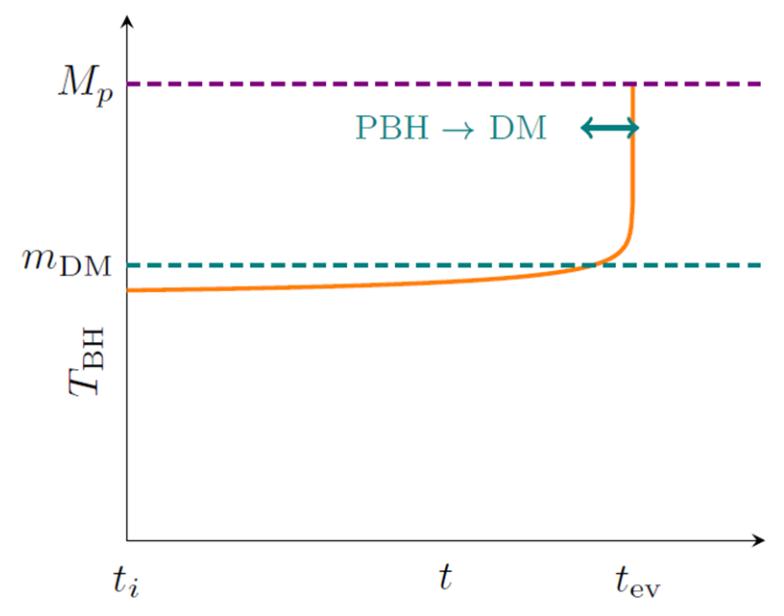
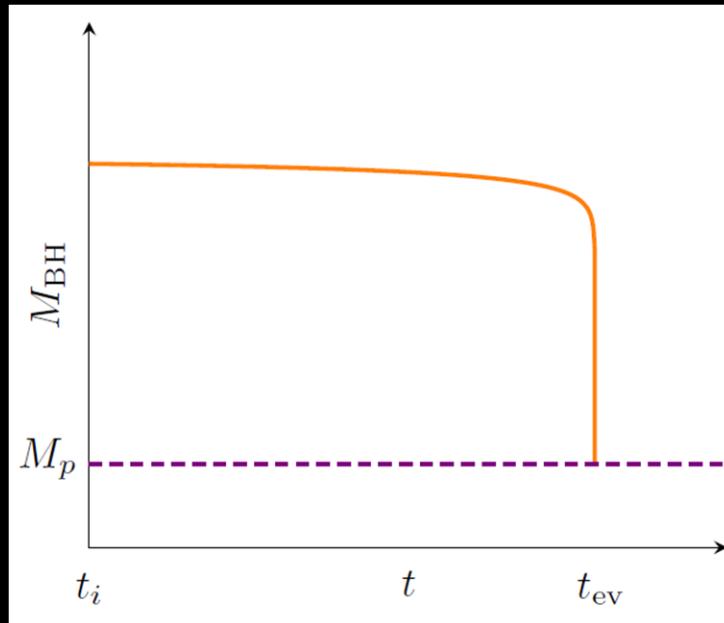
- I. Effect of PBH evaporation on Dark-Matter Phenomenology
- II. Kerr PBHs and Dark Radiation
- III. Kerr PBHs and Warm Dark Matter
- IV. Evaporation of Extended Distributions

## I. Effect of PBH evaporation on Dark-Matter Phenomenology

*If the DM relic density is made, at least partially, of particles, PBHs would contribute to its production.*

# PBH EVAPORATION

$$T_{\text{BH}} = \frac{1}{8\pi G M_{\text{BH}}} \sim 1.06 \text{ GeV} \left( \frac{10^{13} \text{ g}}{M_{\text{BH}}} \right)$$



→ More and more particles contribute to the evaporation

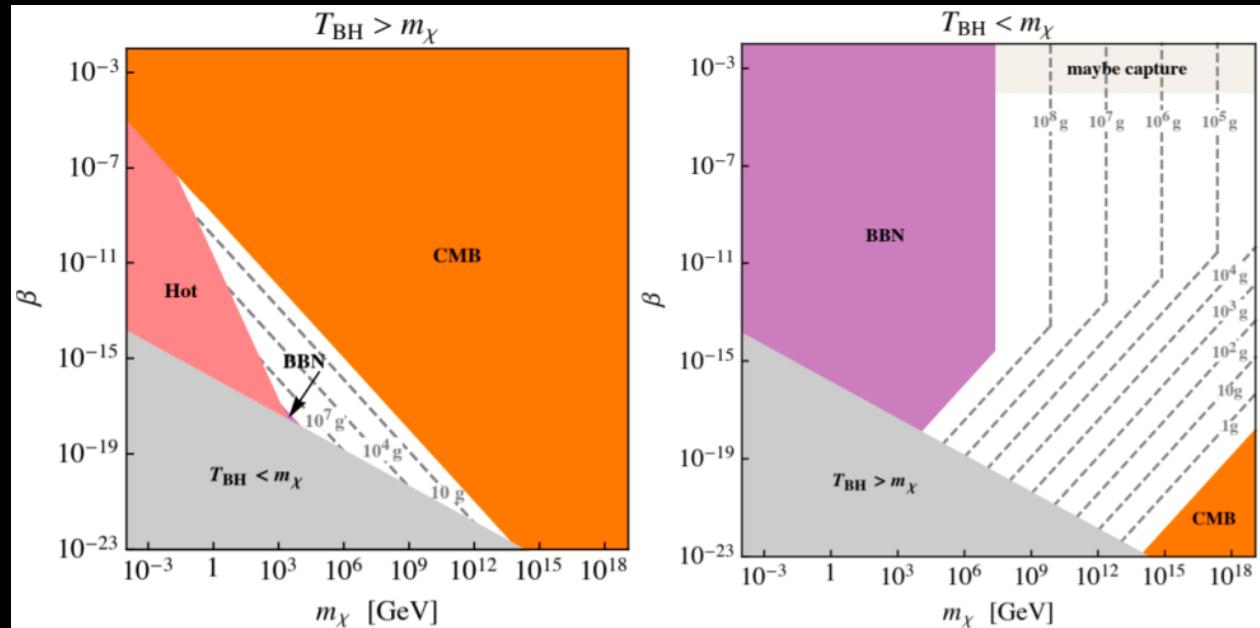
# DM FROM EVAPORATION

$$\frac{d^2 \mathcal{N}_i}{dp dt} = \frac{g_i}{2\pi^2} \frac{\sigma_{s_i}(M_{\text{BH}}, \mu_i, p)}{\exp [E_i(p)/T_{\text{BH}}] - (-1)^{2s_i}} \frac{p^3}{E_i(p)}$$

Very much used in the literature: the **geometrical-optics limit**

$$GM_{\text{BH}}p \gg 1$$

$$\sigma_{s_i}(E, \mu)|_{\text{GO}} = 27\pi G^2 M_{\text{BH}}^2$$



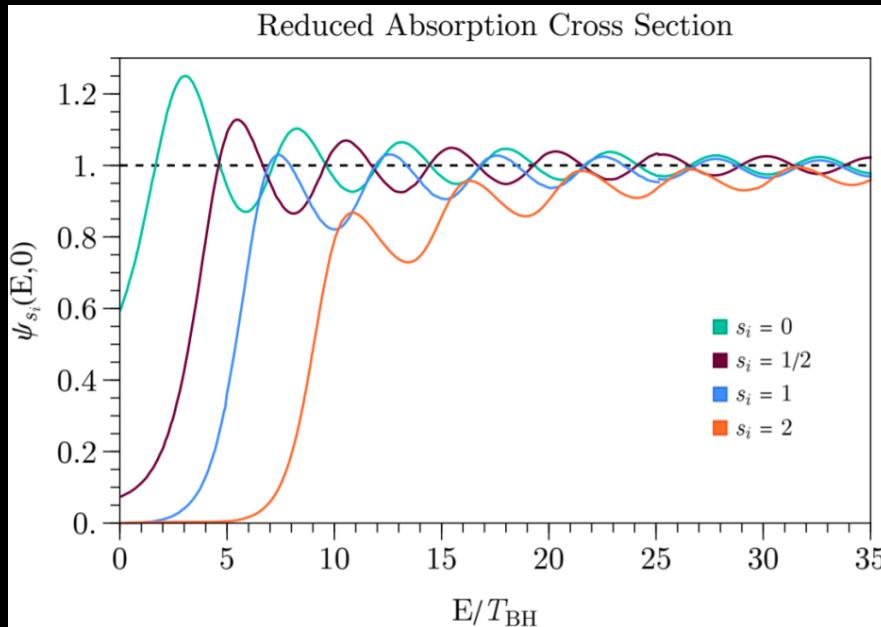
[Gondolo, Sandick and Shams Es Haghi '20]

# DM FROM EVAPORATION

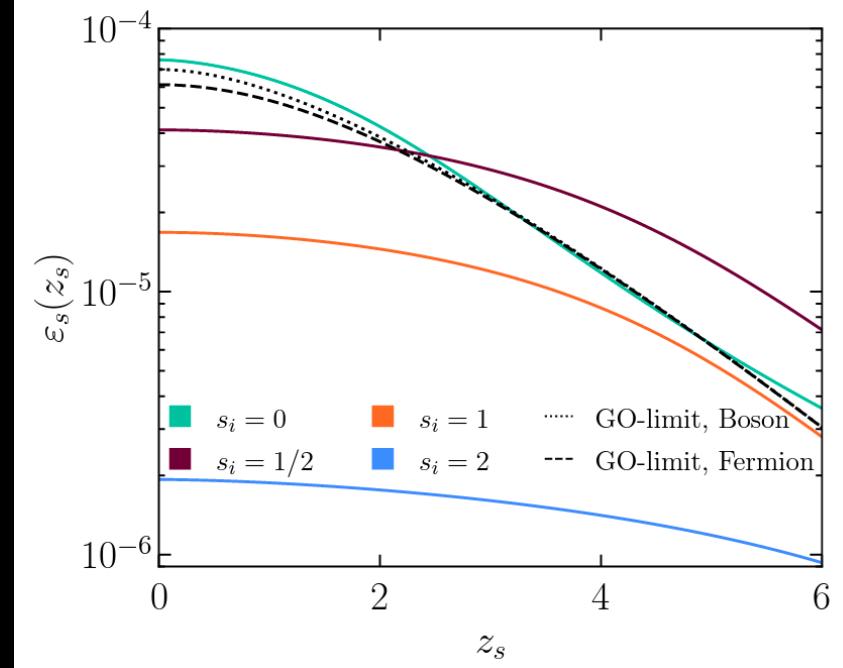
$$\frac{d^2 \mathcal{N}_i}{dp dt} = \frac{g_i}{2\pi^2} \frac{\sigma_{s_i}(M_{\text{BH}}, \mu_i, p)}{\exp [E_i(p)/T_{\text{BH}}] - (-1)^{2s_i}} \frac{p^3}{E_i(p)}$$

Very bad approximation at (not too) low momentum...

$$\psi_{s_i}(E, \mu) \equiv \frac{\sigma_{s_i}(E, \mu)}{27\pi G^2 M_{\text{BH}}^2}$$

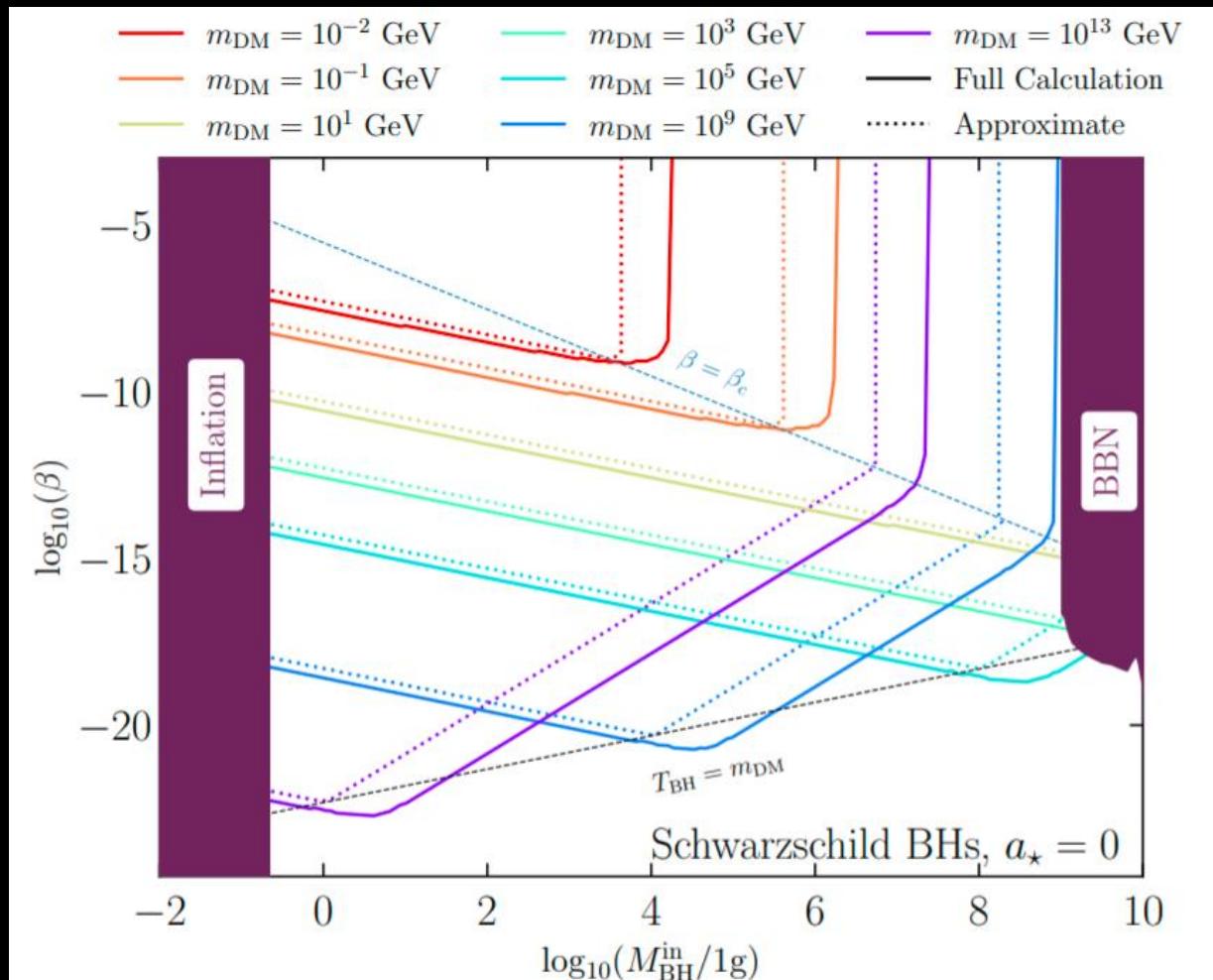


$$\varepsilon_i(z_i) = \frac{27}{8192\pi^5} \int_{z_i}^{\infty} \frac{\psi_{s_i}(x)(x^2 - z_i^2)}{\exp(x) - (-1)^{2s_i}} x \, dx$$



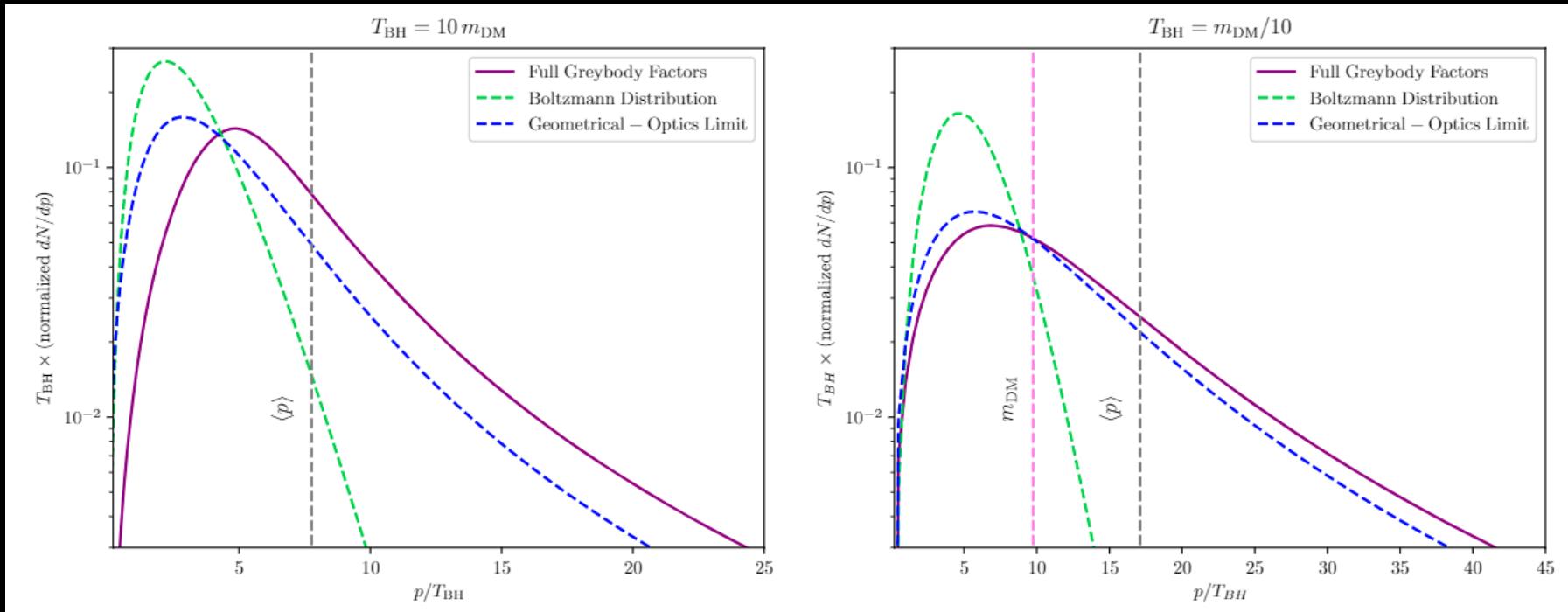
# DM FROM EVAPORATION

$$f_{\text{PBH}}(M) = \delta(M - M_{\text{PBH}})$$



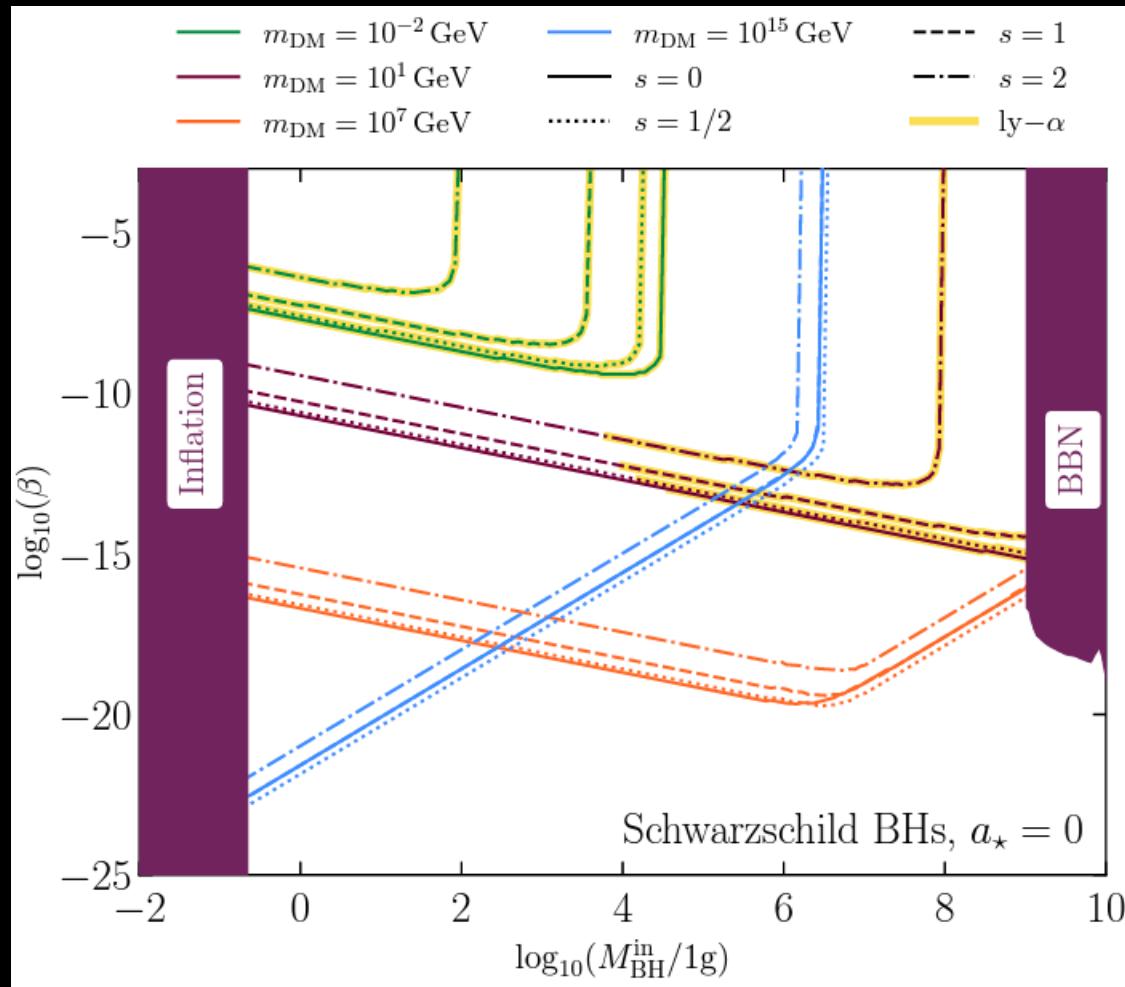
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# DM FROM EVAPORATION

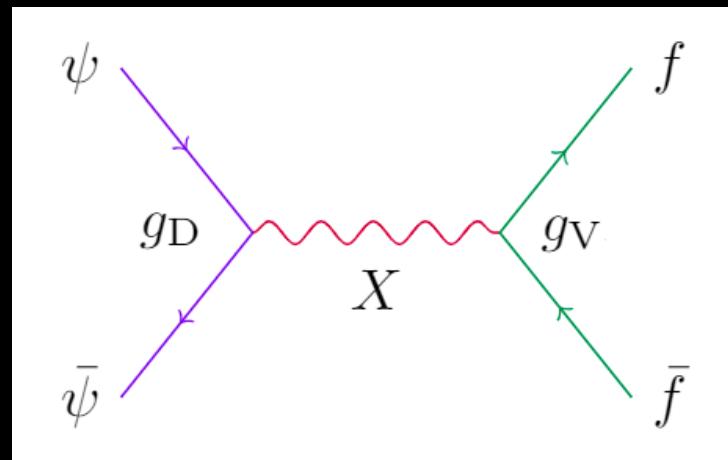
$$f_{\text{PBH}}(M) = \delta(M - M_{\text{PBH}})$$



# THERMAL PRODUCTION OF DM

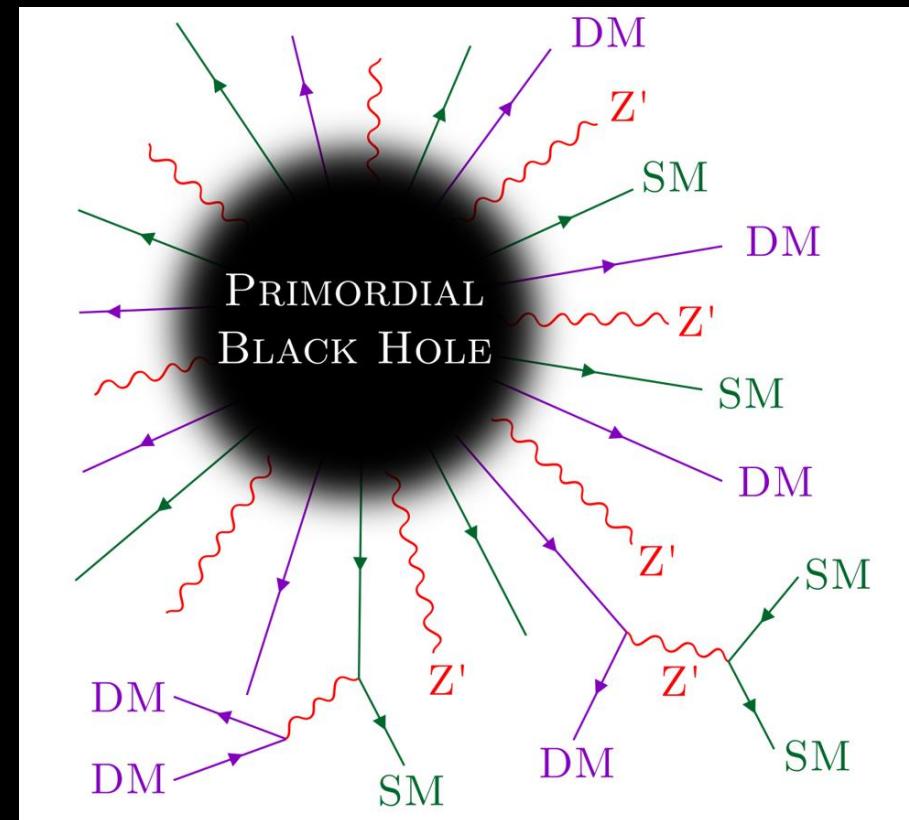
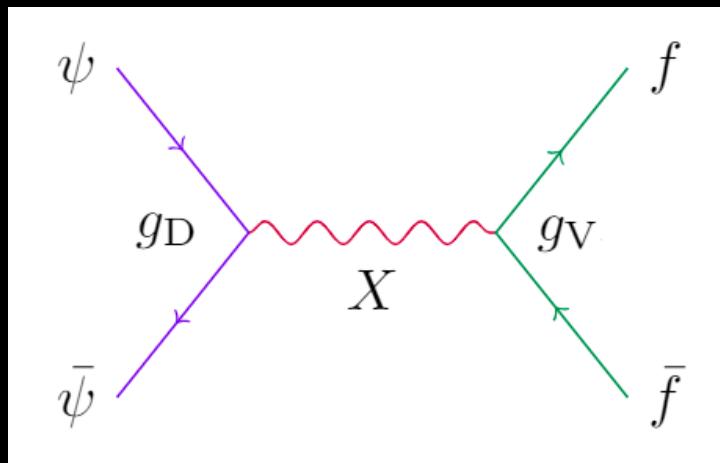
- DM may interact with SM particles and be produced in the early universe through thermal processes...
- Freeze-In or Freeze-Out

$$\dot{n}_{\text{DM}}^{\text{th}} + 3Hn_{\text{DM}}^{\text{th}} = \langle \sigma v \rangle_{\text{th}} (n_{\text{DM,eq}}^2 - n_{\text{DM}}^{\text{th}2})$$

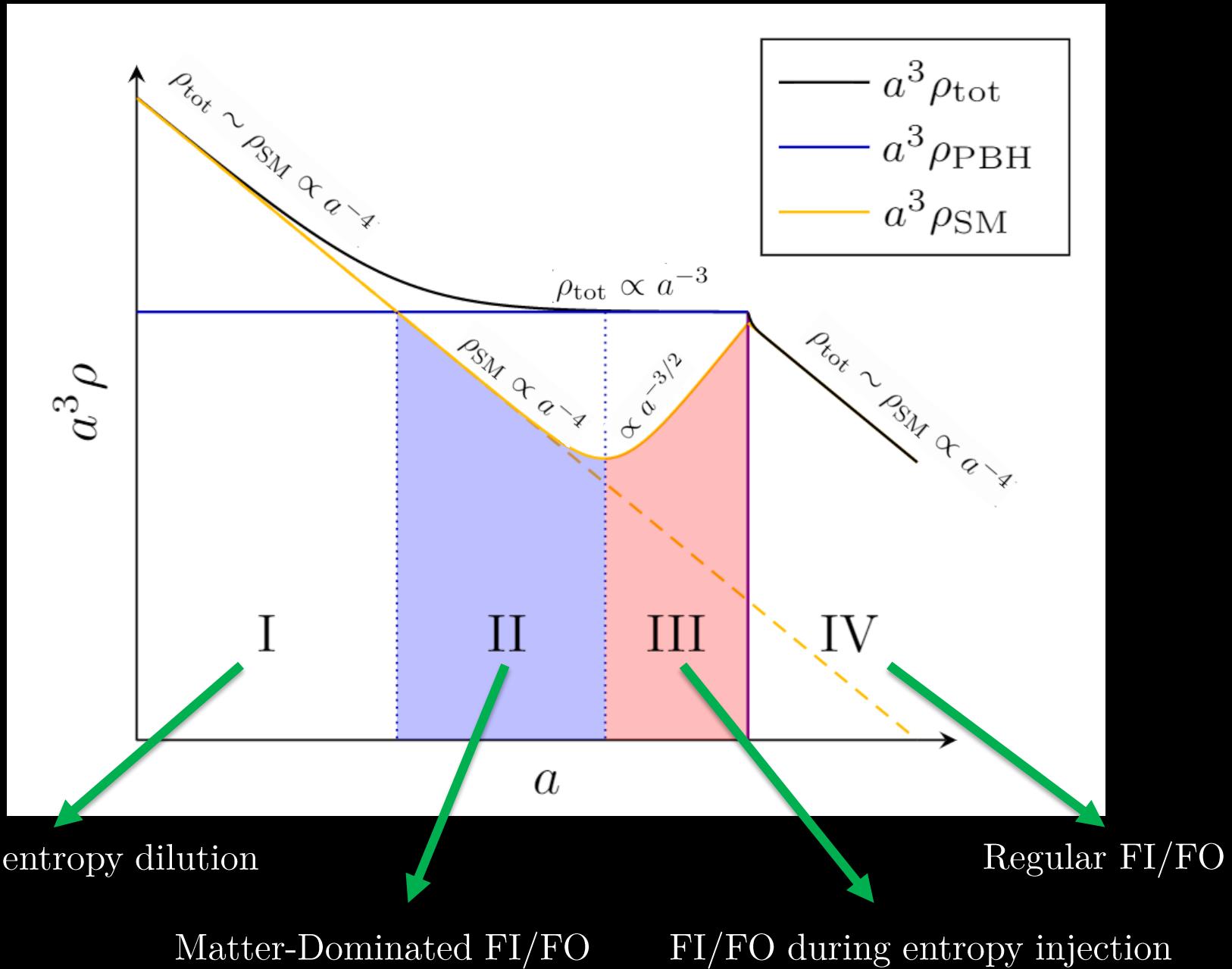


# THERMAL PRODUCTION OF DM

- DM may interact with SM particles and be produced in the early universe through thermal processes...
- Freeze-In or Freeze-Out



# MODIFIED COSMOLOGY



# ANALYTICAL RESULTS

## Freeze-In contribution

$$\begin{aligned}
\Omega_{\text{I}} &= \alpha m_X^3 \frac{m_{\text{DM}}}{\rho_c} \frac{36\sqrt{10}}{\pi\sqrt{g_{\star,\rho}(m_X)}} \frac{g_{\star,s}(T_{\text{eq}})}{g_{\star,s}(m_X)} \frac{T_{\text{eq}}^3 m_p}{m_X^4} \frac{a_{\text{eq}}^3}{a_0^3} G_{1,3}^{2,1}\left(\left.\frac{1}{2}, \frac{5}{2}, 0\right| \frac{m_X}{T_{\text{eq}}}, \frac{1}{2}\right), \\
\Omega_{\text{II}} &= \frac{\alpha m_X^3}{4} \frac{m_{\text{DM}}}{\rho_c} \sqrt{\frac{3m_p^2}{\rho_{\text{PBH}}^c}} \left(\frac{a_c}{a_0}\right)^3 T_c \left(\frac{g_{\star,s}(T_c)}{g_{\star,s}(m_X)}\right)^{\frac{1}{3}} G_{1,3}^{2,1}\left(\left.-\frac{3}{4}, \frac{1}{2}, -\frac{7}{4}\right| \frac{m_X}{2T_c} \left(\frac{g_{\star,s}(m_X)}{g_{\star,s}(T_c)}\right)^{\frac{1}{3}}, \frac{1}{2}\right), \\
\Omega_{\text{III}} &= 2\alpha m_X^3 \frac{m_{\text{DM}}}{\rho_c} \sqrt{\frac{3m_p^2}{\rho_{\text{PBH}}^{\text{ev}}}} \left(\frac{a_{\text{ev}}}{a_0}\right)^3 T_{\text{ev}} G_{1,3}^{2,1}\left(\left.-\frac{9}{2}, \frac{1}{2}, -\frac{11}{2}\right| \frac{m_X}{2T_{\text{ev}}}, \frac{1}{2}\right), \\
\Omega_{\text{IV}} &= \alpha m_X^3 \frac{m_{\text{DM}}}{\rho_c} \frac{36\sqrt{10}}{\pi\sqrt{g_{\star,\rho}(m_X)}} \frac{g_{\star,s}(T_0)}{g_{\star,s}(m_X)} \frac{T_0^3 m_p}{m_X^4} G_{1,3}^{2,1}\left(\left.\frac{3}{2}, \frac{5}{2}, 0\right| \frac{m_X}{T_0}, \frac{1}{2}\right),
\end{aligned}$$

## Freeze-Out contribution

- Regime I and IV:

$$x_{\text{FO}} = \ln \left[ \frac{3}{2} \sqrt{\frac{5}{\pi^5 g_{\star}(T_{\text{FO}})}} g_{\text{DM}} m_{\text{DM}} m_p \langle \sigma v \rangle \sqrt{x_{\text{FO}}} \right]$$

- Regime II:

$$x_{\text{FO}} = \ln \left[ \frac{3}{2} \sqrt{\frac{5}{\pi^5 g_{\star}(T_{\text{FO}})}} \frac{g_{\text{DM}} m_{\text{DM}} m_p \langle \sigma v \rangle}{\sqrt{\kappa}} \right],$$

- Regime III:

$$x_{\text{FO}} = \ln \left[ \frac{3}{2} \sqrt{\frac{5}{\pi^5 g_{\star}(T_{\text{FO}})}} \frac{g_{\text{DM}} m_p \langle \sigma v \rangle}{m_{\text{DM}}} T_{\text{ev}}^2 x_{\text{FO}}^{5/2} \right].$$

$$\begin{aligned}
\Omega_{\text{I}} &= \frac{15}{2\pi} \frac{x_{\text{FO}}}{\sqrt{10g_{\star}(T_{\text{FO}})}} \frac{s_{\text{eq}}}{m_p \langle \sigma v \rangle \rho_c} \left(\frac{a_{\text{eq}}}{a_0}\right)^3, \\
\Omega_{\text{II}} &= \frac{45}{4\pi} \frac{1}{m_{\text{DM}} m_p \langle \sigma v \rangle} \sqrt{\frac{\kappa}{10g_{\star}(T_{\text{FO}})}} x_{\text{FO}}^{3/2}, \\
\Omega_{\text{III}} &= \frac{\pi}{2} \sqrt{\frac{g_{\star}(T_{\text{FO}})}{10}} \frac{m_{\text{DM}}^2}{m_p \langle \sigma v \rangle} \kappa \left(\frac{m_{\text{DM}} T_{\text{ev}}}{T_{\text{FO}}^2}\right)^2, \\
\Omega_{\text{IV}} &= \frac{15}{2\pi} \frac{x_{\text{FO}}}{\sqrt{10g_{\star}(T_{\text{FO}})}} \frac{s_0}{m_p \langle \sigma v \rangle \rho_c},
\end{aligned}$$

# RESULTS

## Freeze-Out [Cheek, LH, Perez-Gonzalez and Turner '22]

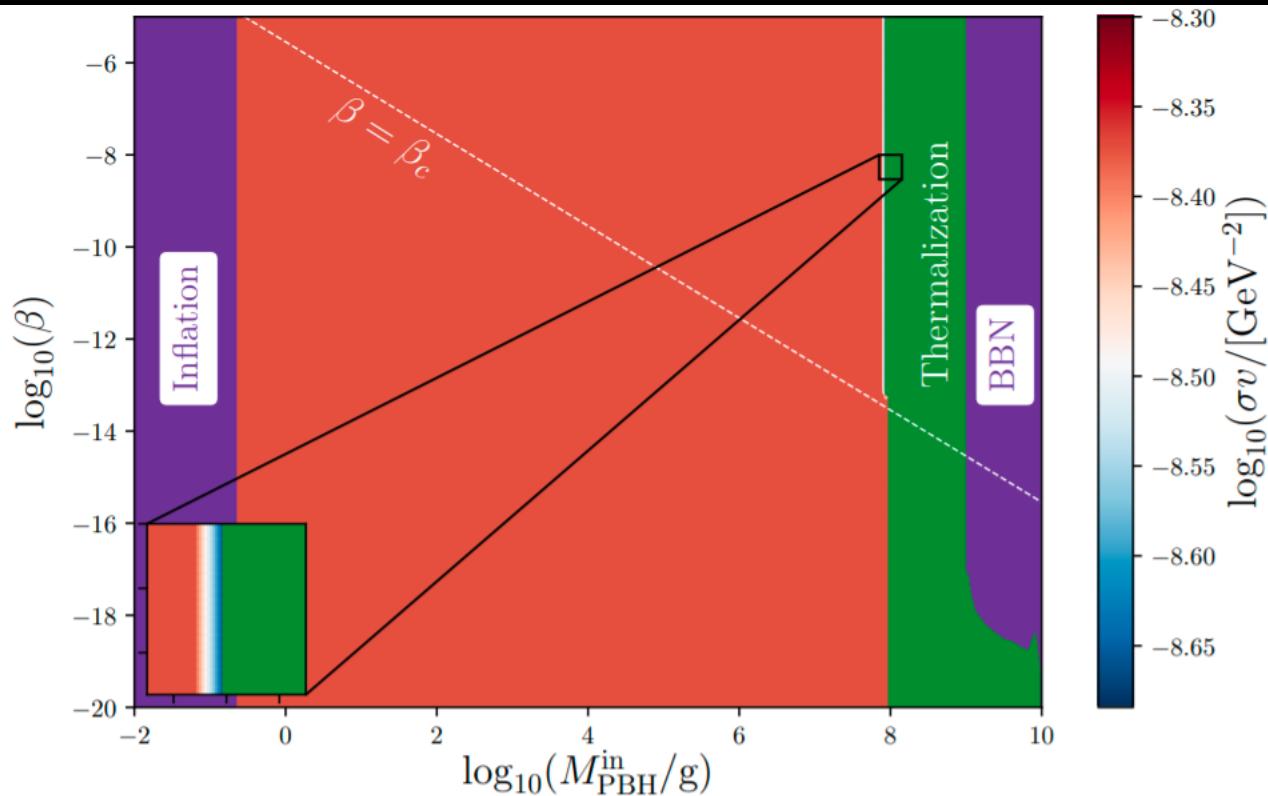


Fig. 7. Two-dimensional scan over the PBH fraction  $\beta$  and mass  $M_{\text{BH}}$  for a mediator mass  $m_X = 10 \text{ GeV}$  and a dark matter mass  $m_{\text{DM}} = 1 \text{ GeV}$ , and  $\text{Br}(X \rightarrow \text{DM}) = 0.5$ . The color map indicates the value of the non-relativistic cross-section of DM annihilation leading to the correct relic abundance in the Freeze-Out case. See the main text for a description of the different constraints.

# RESULTS

Freeze-In

[Cheek, LH, Perez-Gonzalez and Turner '22]

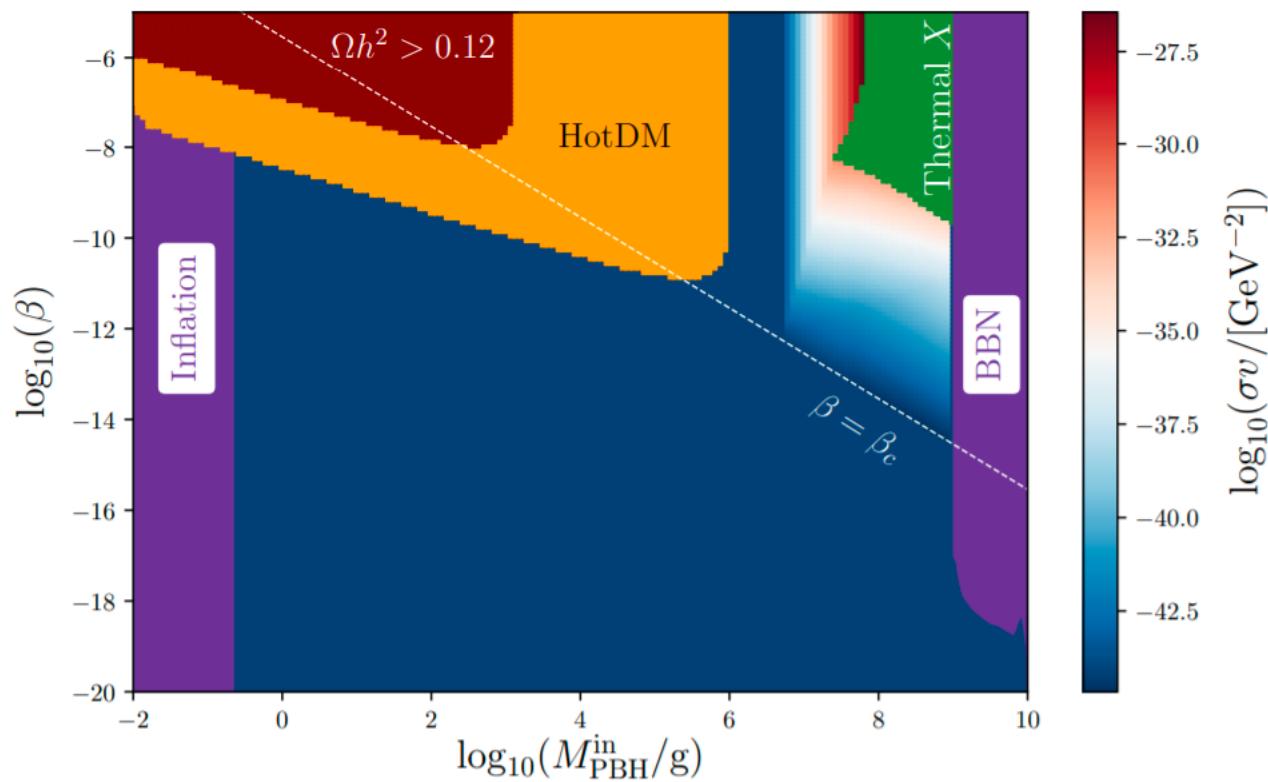


Fig. 11. Two-dimensional scan over the PBH fraction  $\beta$  and mass  $M_{\text{PBH}}$  for a mediator mass  $m_X = 1 \text{ TeV}$ , a dark matter mass  $m_{\text{DM}} = 1 \text{ MeV}$ , and  $\text{Br}(X \rightarrow \text{SM}) = 10^{-7}$ . The color map indicates the value of the non-relativistic cross-section of DM annihilation leading to the correct relic abundance in the Freeze-In case. See the main text for a description of the different constraints.

## II. Kerr PBHs and Dark Radiation

$$\frac{\mathrm{d}^2\mathcal{N}_{ilm}}{\mathrm{d}p\mathrm{d}t}=\frac{\sigma_{s_i}^{lm}(M_{\rm BH},p,a_\star)}{\exp\left[(E_i-m\Omega)/T_{\rm BH}\right]-(-1)^{2s_i}}\frac{p^3}{E_i}$$

$$\varepsilon_i(M_{\rm BH},a_\star)=\frac{g_i}{2\pi^2}\int_0^\infty \sum_{l=s_i}\sum_{m=-l}^l\frac{\mathrm{d}^2\mathcal{N}_{ilm}}{\mathrm{d}p\mathrm{d}t}\,EdE\,,$$
$$\gamma_i(M_{\rm BH},a_\star)=\frac{g_i}{2\pi^2}\int_0^\infty \sum_{l=s_i}\sum_{m=-l}^lm\frac{\mathrm{d}^2\mathcal{N}_{ilm}}{\mathrm{d}p\mathrm{d}t}\,dE\,,$$

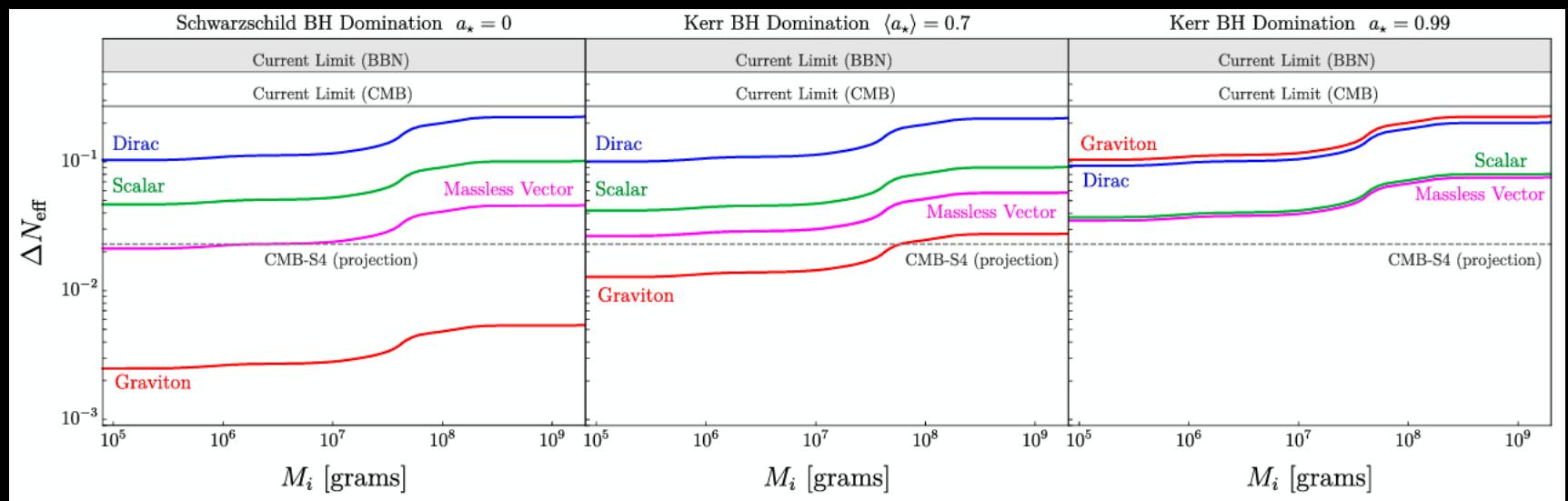
$$\frac{d M_{\rm BH}}{dt} = - \epsilon(M_{\rm BH},a_\star) \frac{M_p^4}{M_{\rm BH}^2} \,,$$
$$\frac{da_\star}{dt} = - a_\star [\gamma(M_{\rm BH},a_\star) - 2 \epsilon(M_{\rm BH},a_\star)] \frac{M_p^4}{M_{\rm BH}^3} \,.$$

# Kerr PBHs and Dark Radiation

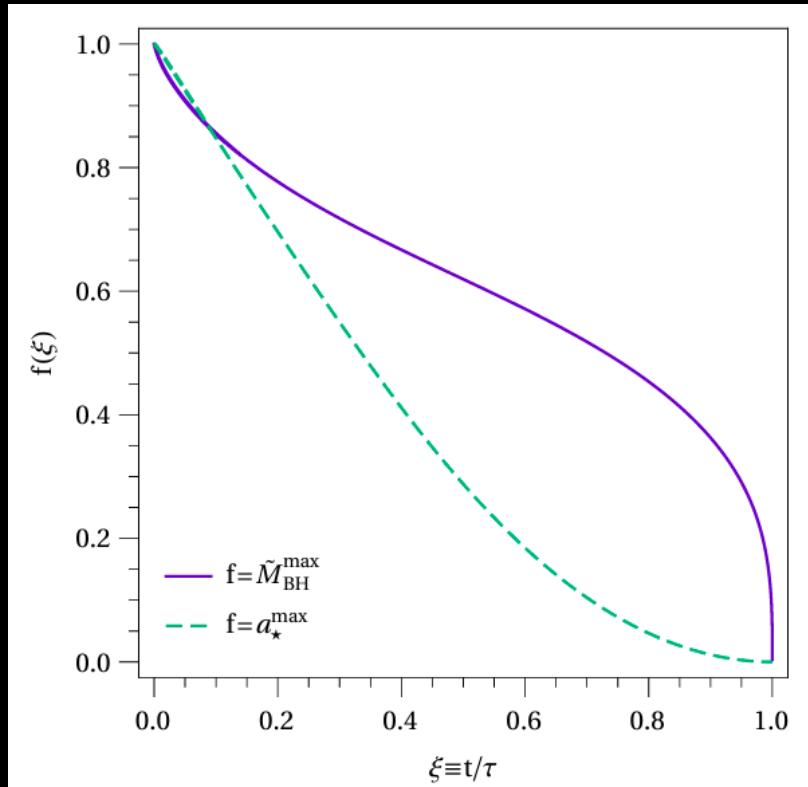
Dark particles with small masses can contribute to  $\Delta N_{\text{eff}}$

Schwarzschild PBH → Negligible

Kerr PBH → Argued to be critical



# Kerr PBHs and Dark Radiation



Major effects:

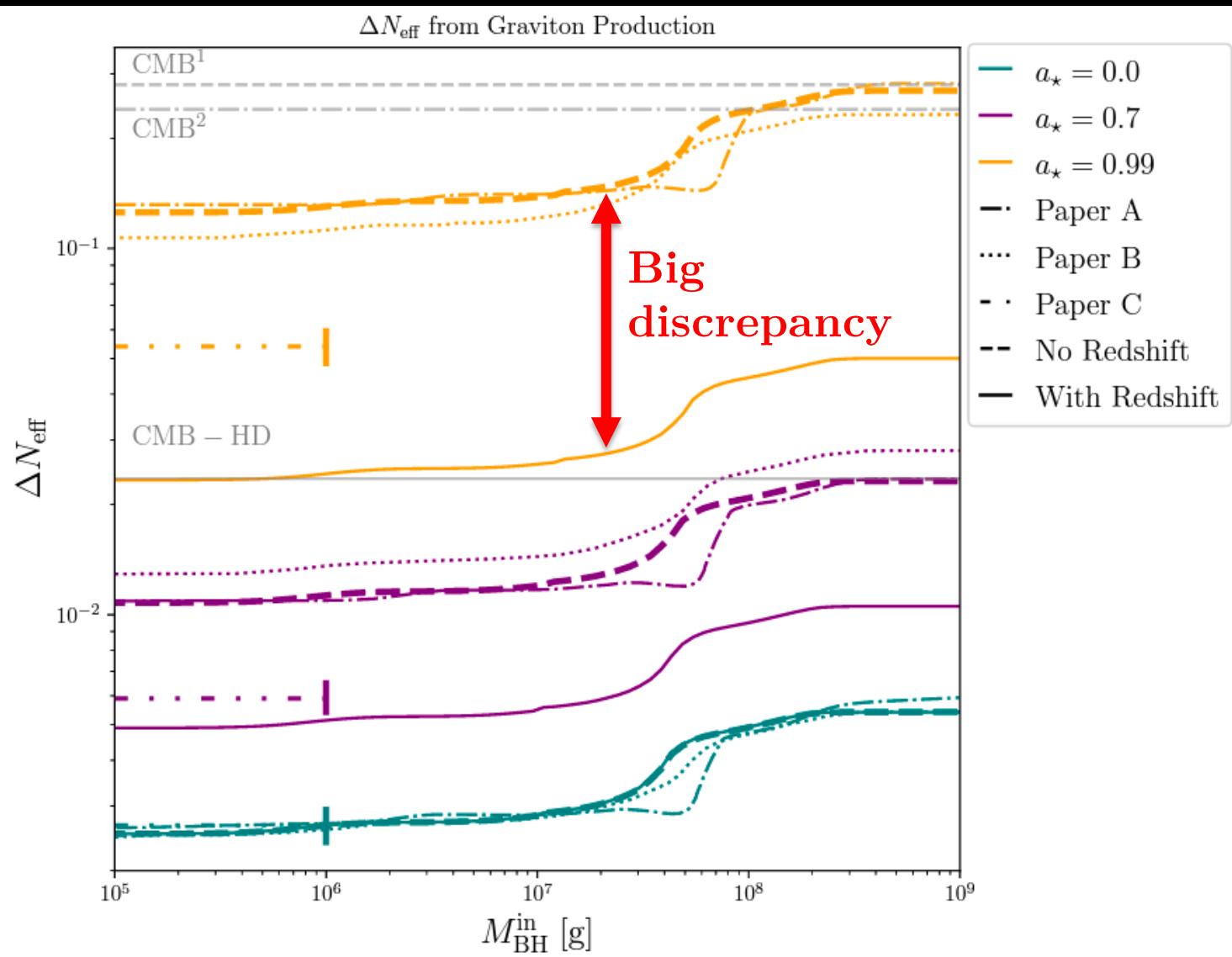
Spin loss faster than mass loss

→ Shorter lifetime

→ Different ratio Dark  
Radiation / Radiation

How to calculate  $\Delta N_{\text{eff}}$  ?

# Kerr PBHs and Dark Radiation



# Kerr PBHs and Dark Radiation

Why ?

# Kerr PBHs and Dark Radiation

Why ?

$$\rho_{\text{R}}^{\text{SM}} = \rho_{\gamma} \left[ 1 + \frac{7}{8} \left( \frac{T_{\nu}}{T_{\gamma}} \right) N_{\text{eff}}^{\text{SM}} \right],$$

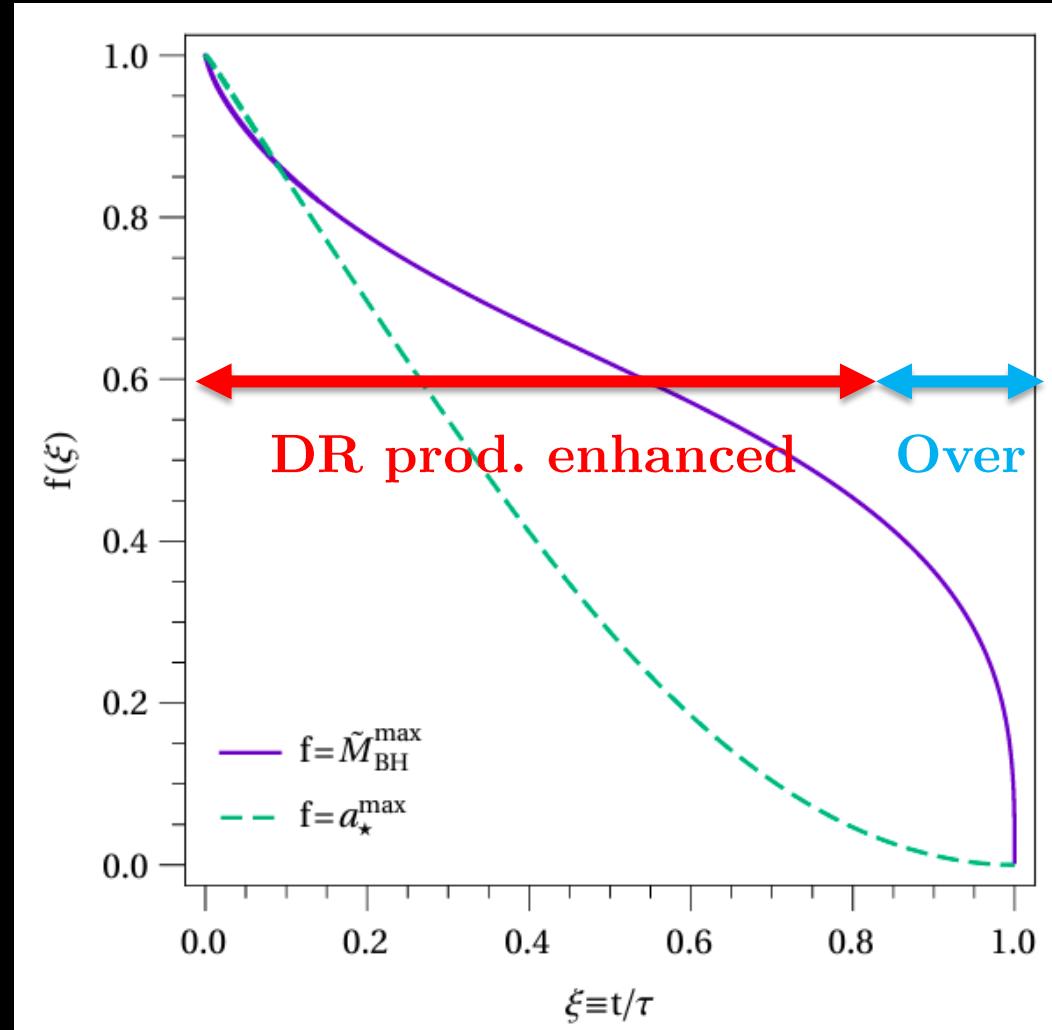


$$N_{\text{eff}} \approx 3.045 \quad (\text{not just } 3 \ldots)$$

The neutrino decoupling is NOT instantaneous  
+ Temperature-dependent entropy transfer from electrons

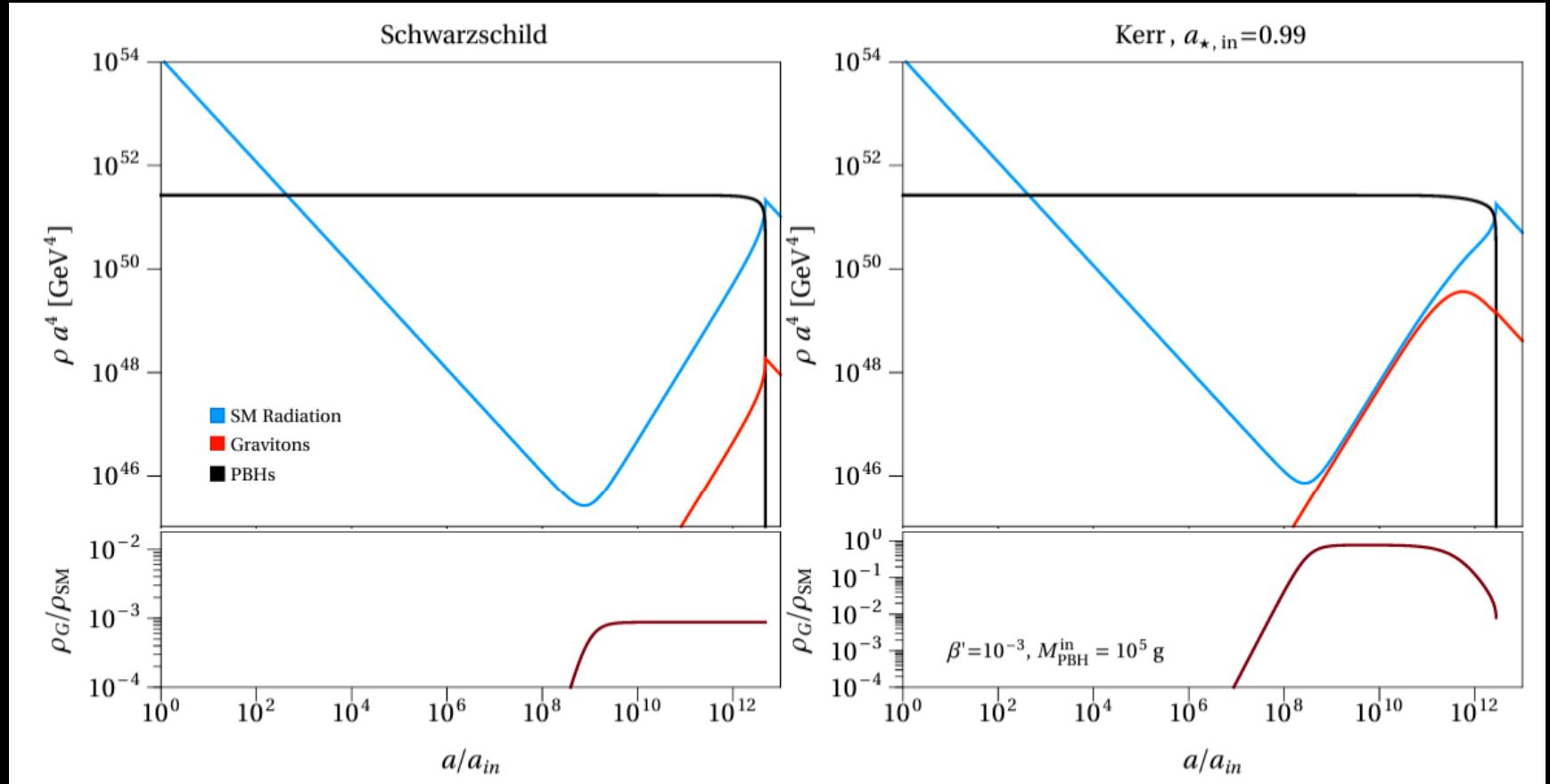
# Kerr PBHs and Dark Radiation

Why ?



# Kerr PBHs and Dark Radiation

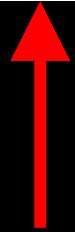
Why ?



# Kerr PBHs and Dark Radiation

Why ?

$$\frac{d\mathcal{N}_{DM}}{dp} = \int_0^\tau dt' \frac{a(\tau)}{a(t')} \times \frac{d^2\mathcal{N}_{DM}}{dp'dt'} \left( p \frac{a(\tau)}{a(t')}, t' \right)$$

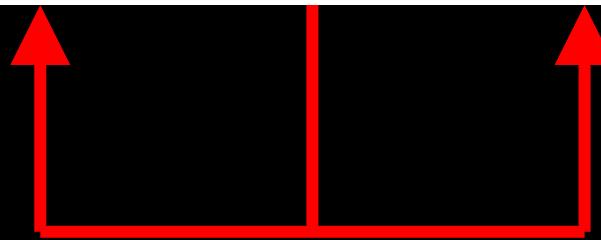


some redshift is good

# Kerr PBHs and Dark Radiation

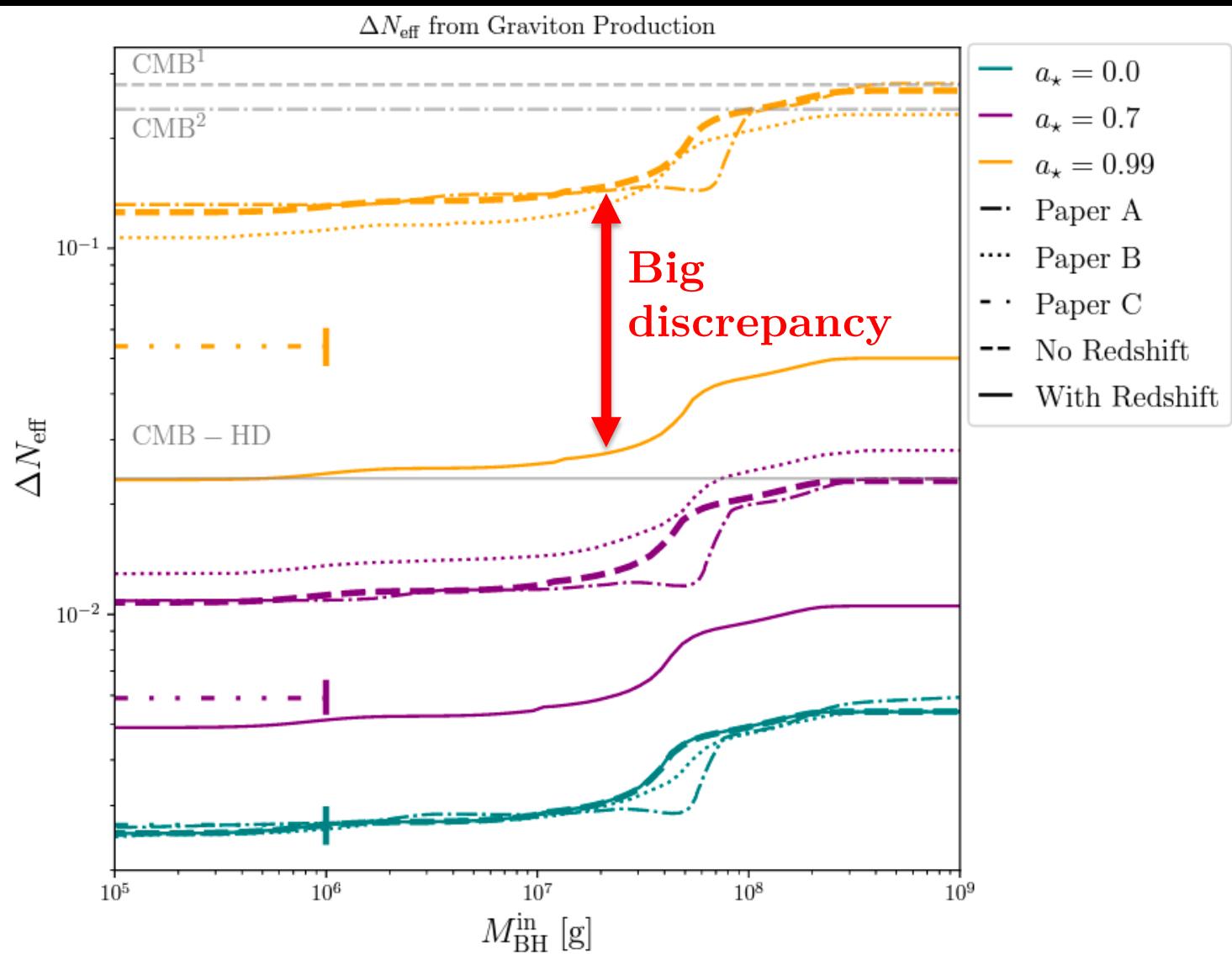
Why ?

$$\frac{d\mathcal{N}_{DM}}{dp} = \int_0^\tau dt' \frac{a(\tau)}{a(t')} \times \frac{d^2\mathcal{N}_{DM}}{dp'dt'} \left( p \frac{a(\tau)}{a(t')}, t' \right)$$

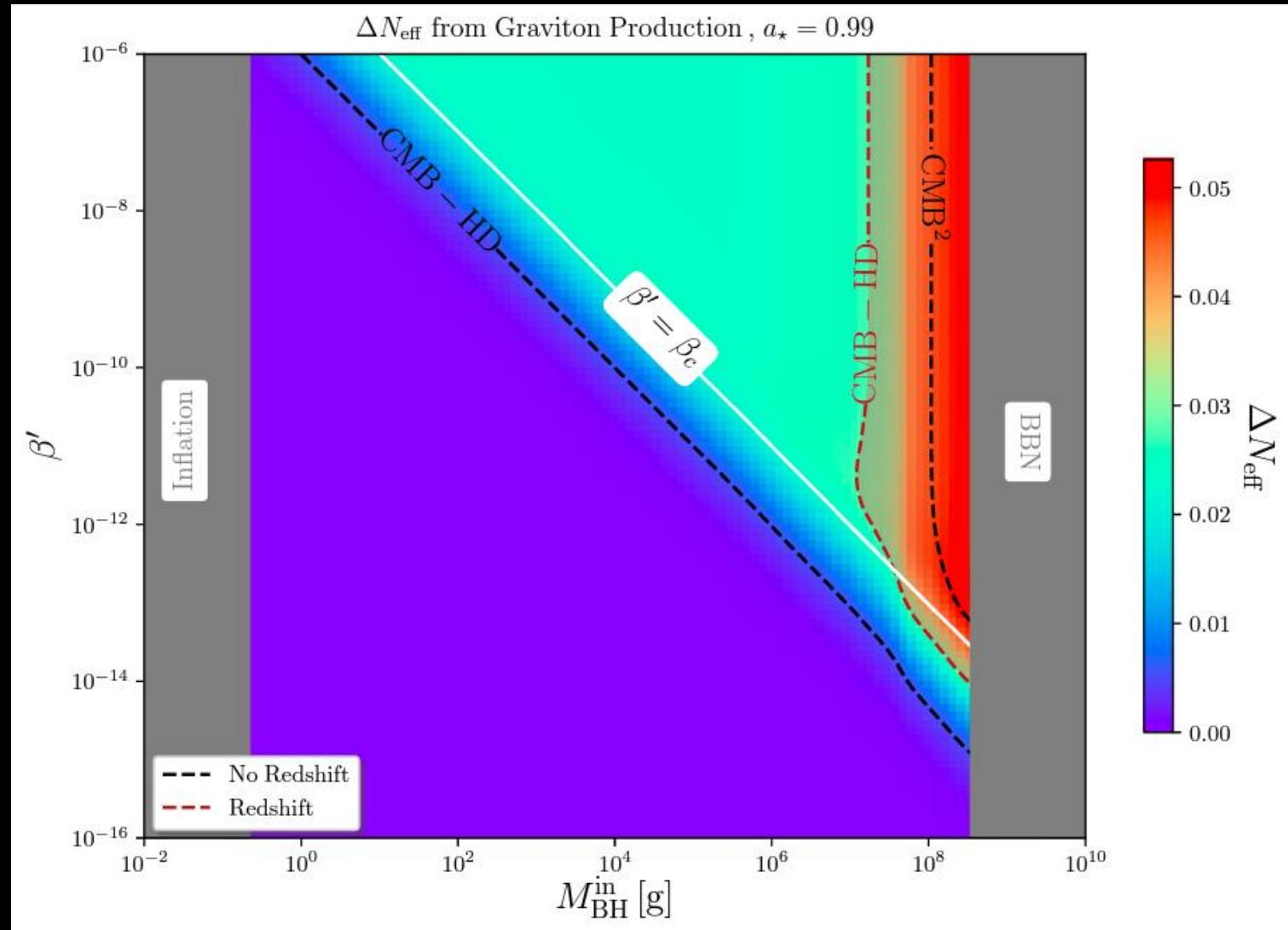


The correct one is better!

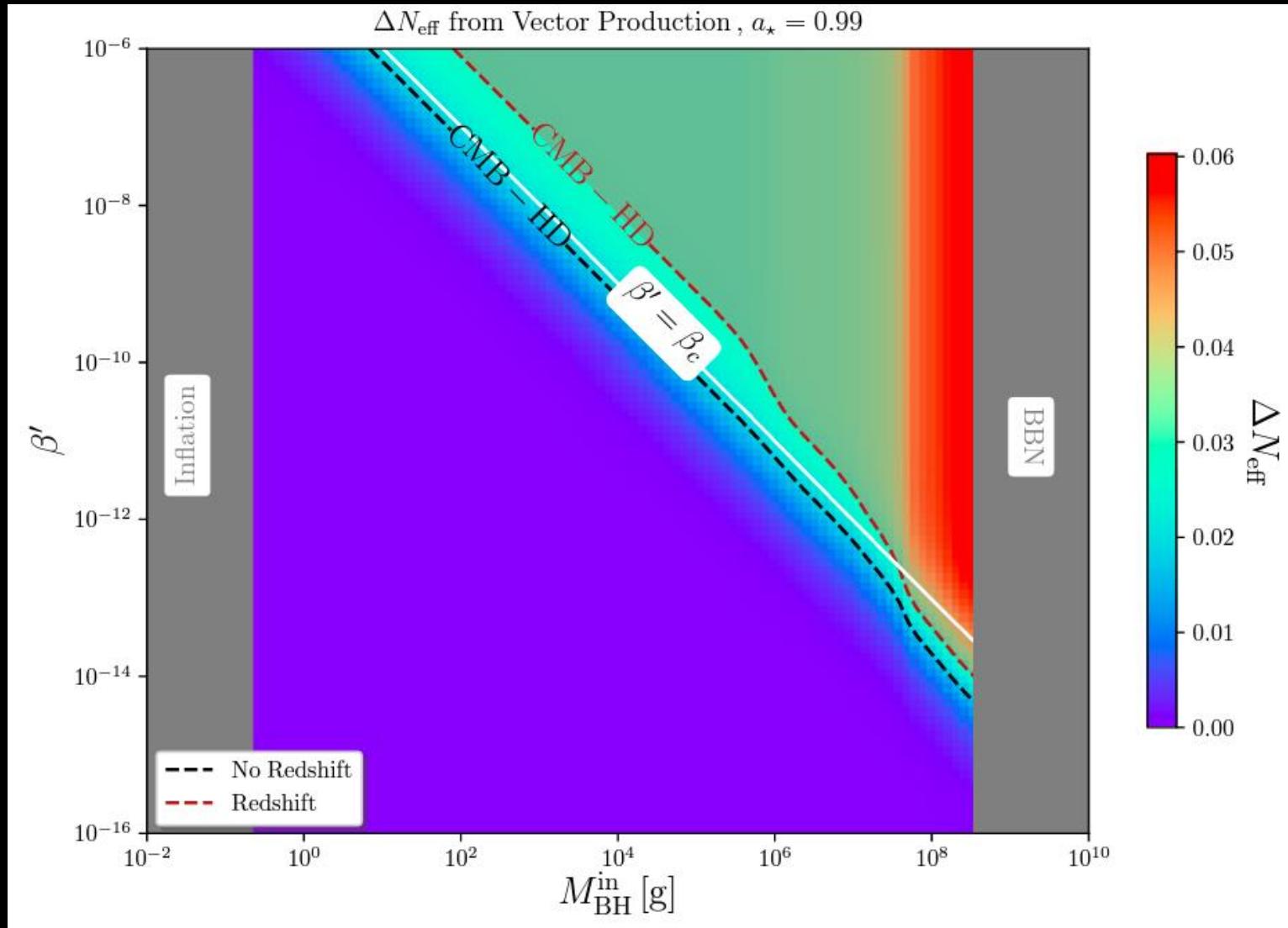
# Kerr PBHs and Dark Radiation



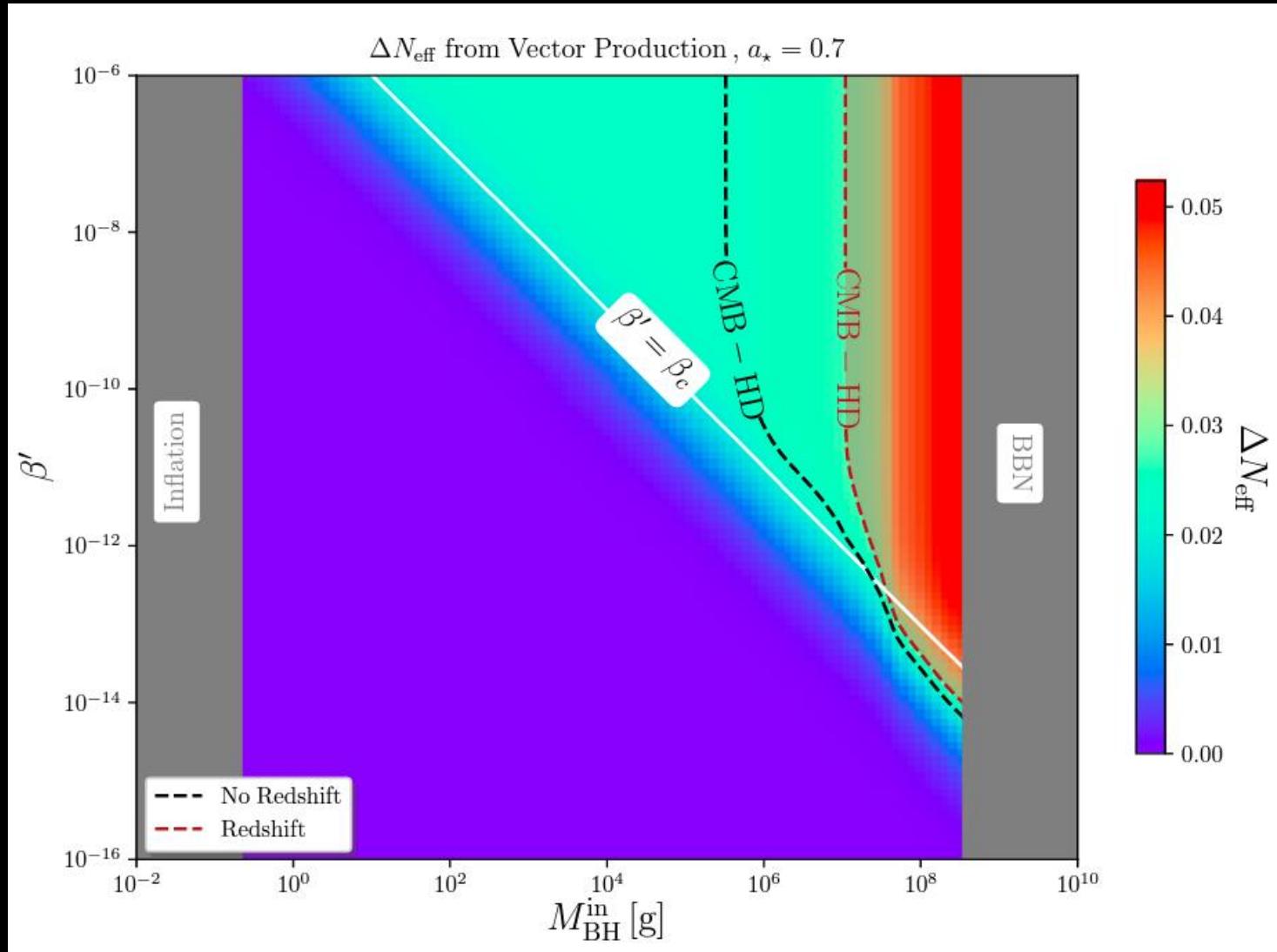
# Kerr PBHs and Dark Radiation



# Kerr PBHs and Dark Radiation



# Kerr PBHs and Dark Radiation



### III. Kerr PBHs and Warm Dark Matter

# Kerr PBHs and Warm Dark Matter

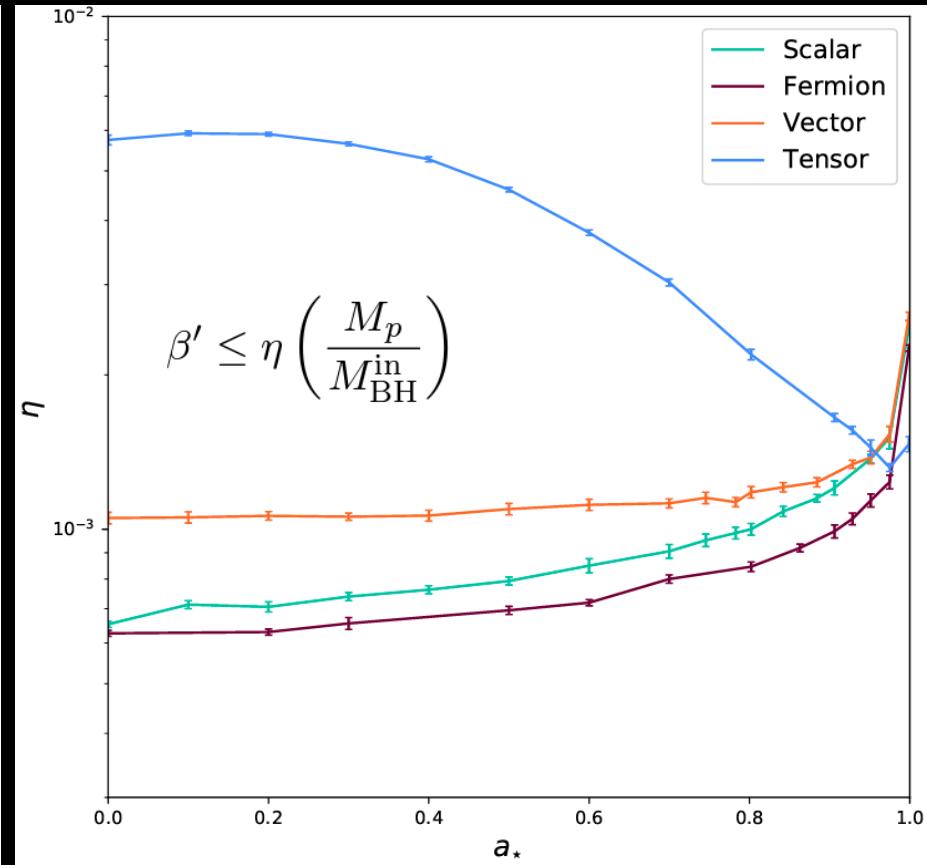
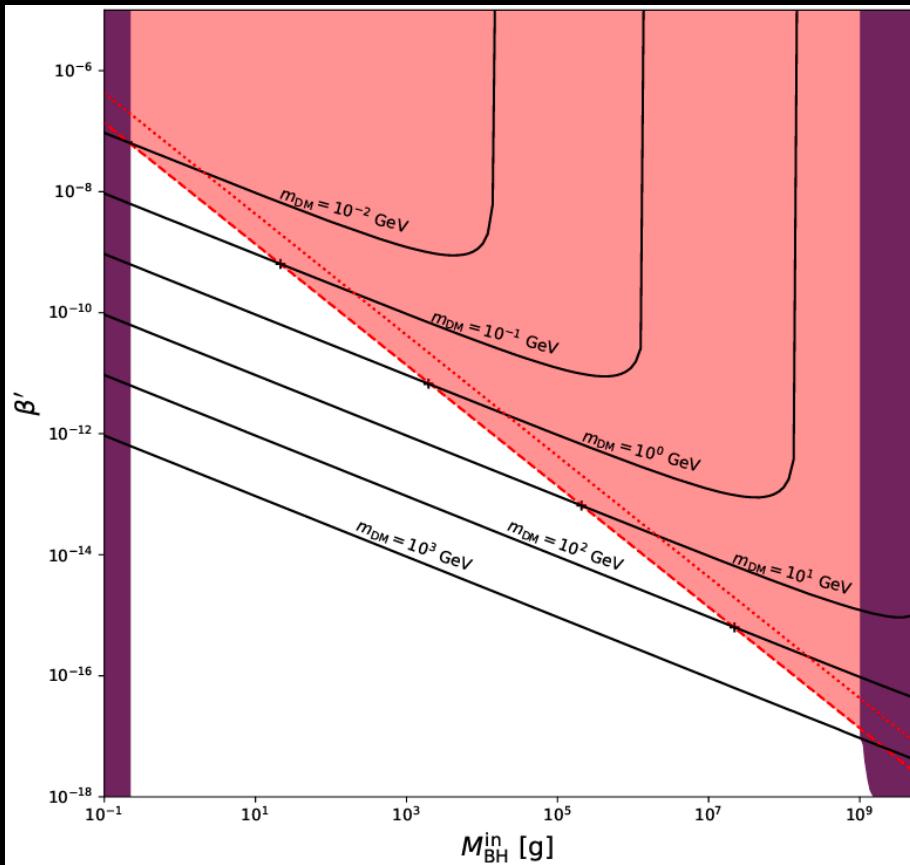
Using CLASS: expected matter power spectrum

$$P(k) = P_{\text{CDM}}(k)T^2(k)$$

$$T(k) = (1 + (\alpha k)^{2\mu})^{-5/\mu}$$

Saturated at

$$\alpha = 1.3 \times 10^{-2} \text{ Mpc } h^{-1}$$



## IV. Evaporation of Extended Distributions

### III. Evaporation of Extended Distributions

In reality, PBHs don't all have the same mass...

$$f_{\text{PBH}}(M, a) = \delta(M - M_{\text{PBH}}) \times \delta(a - a_*)$$



$$f_{\text{PBH}}(M, a) = F(M - M_{\text{PBH}}) \times A(a - a_*)$$

### III. Evaporation of Extended Distributions

$$\begin{aligned} dn_{\text{BH}} &= f_{\text{BH}}(M, a, t) dM da \\ &= f_{\text{BH}}(M_i, a_i, t_i) dM_i da_i . \end{aligned}$$

$$\frac{d\varrho_{\text{BH}}}{dt} = \int_0^\infty \frac{dM}{dt} \Theta(M) f_{\text{BH}}(M_i, a_i, t_i) dM_i da_i$$

Dynamics of the evaporation + Friedmann equations



Included in **FRISBHEE**

<https://github.com/yfperezg/frisbhee>

### III. Evaporation of Extended Distributions

Examples:

$$\frac{dn}{dM} \propto \frac{1}{M^2} \exp\left[-\frac{(\log M - \log M_c)^2}{2 \sigma^2}\right]$$

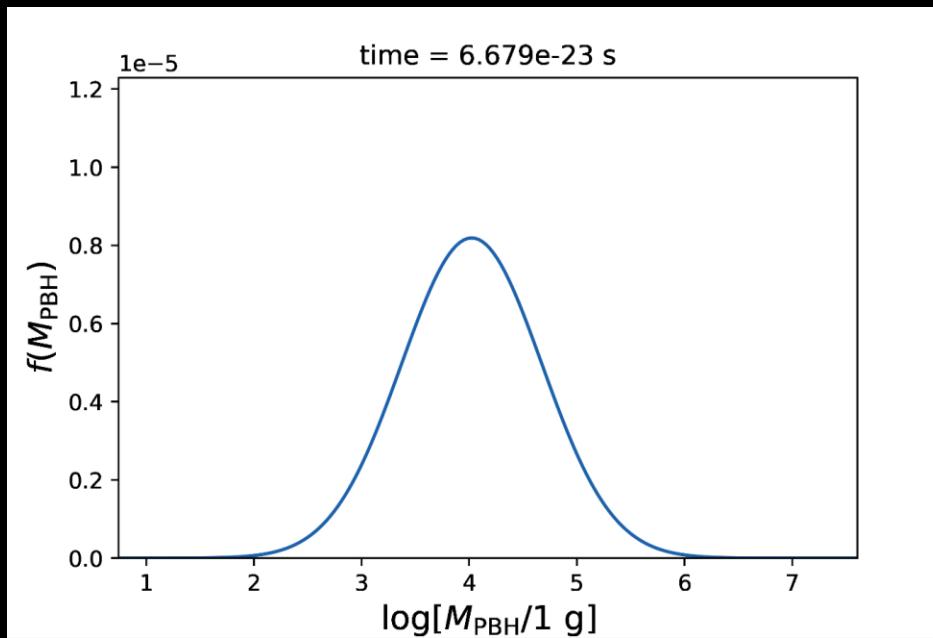
Evaporation smeared around  $\tau(M_c)$

$$\frac{dn}{dM} \propto M^{-\alpha} \quad \text{with} \quad \alpha = \frac{2(1+2w)}{1+w}$$

Regime of ‘Cosmological Stasis’

### III. Evaporation of Extended Distributions

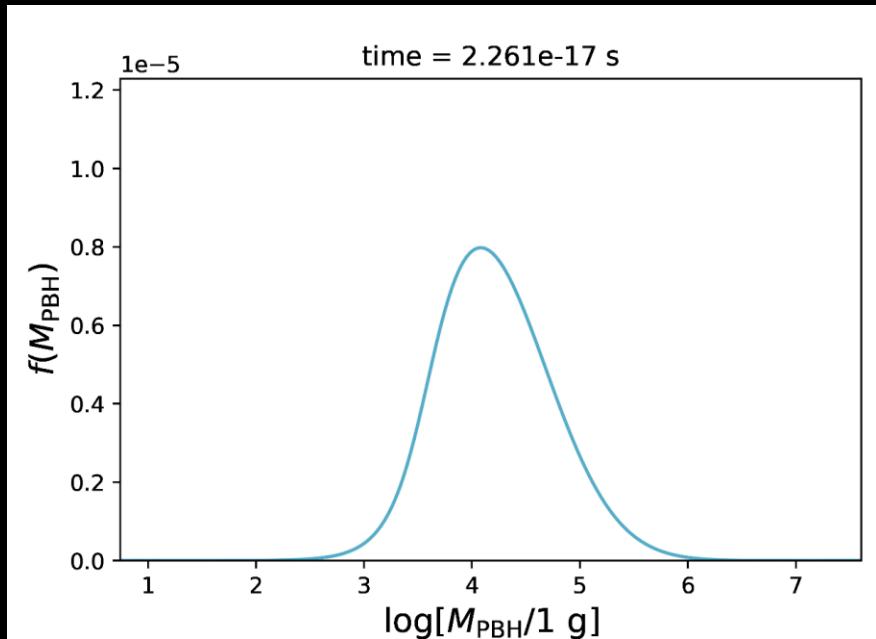
$$n_{\text{PBH}} = \int dM f(M) \quad f(M) = \frac{n_{\text{BH}}}{\sqrt{2\pi}\sigma M} \exp\left(-\frac{\log^2(M/M_c)}{2\sigma^2}\right)$$



Log-normal distribution  
Dolgov, 93  
Green, 2016  
Kannike, 2017

### III. Evaporation of Extended Distributions

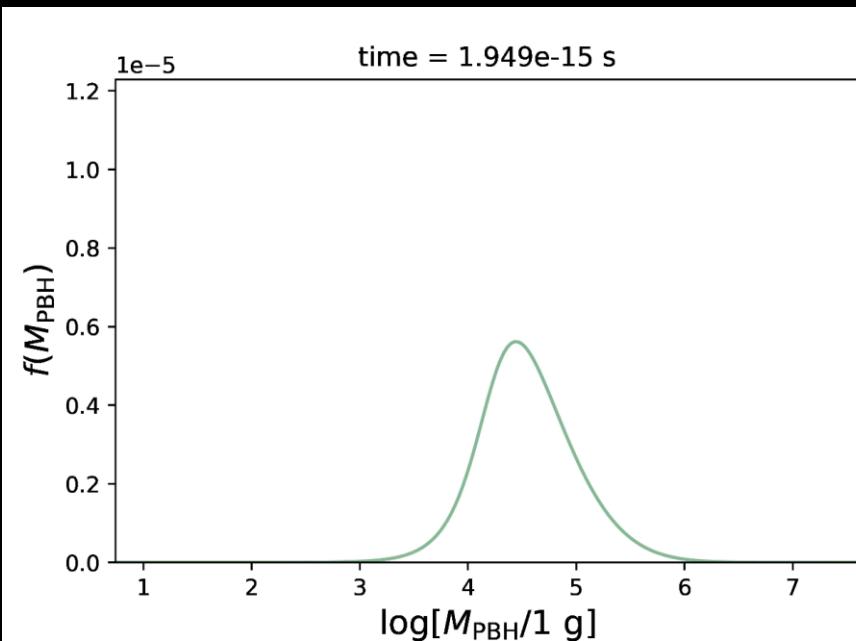
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### III. Evaporation of Extended Distributions

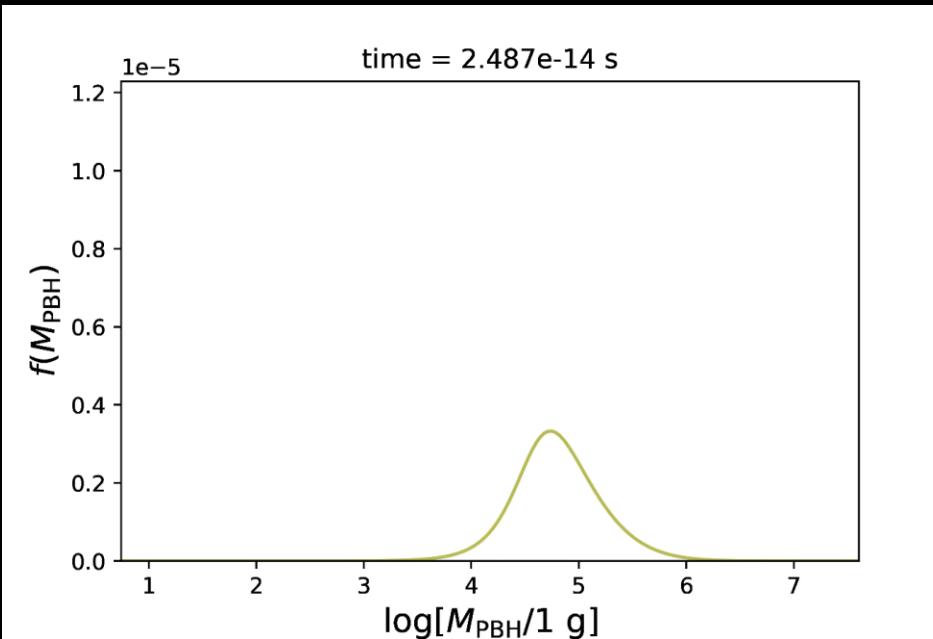
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### III. Evaporation of Extended Distributions

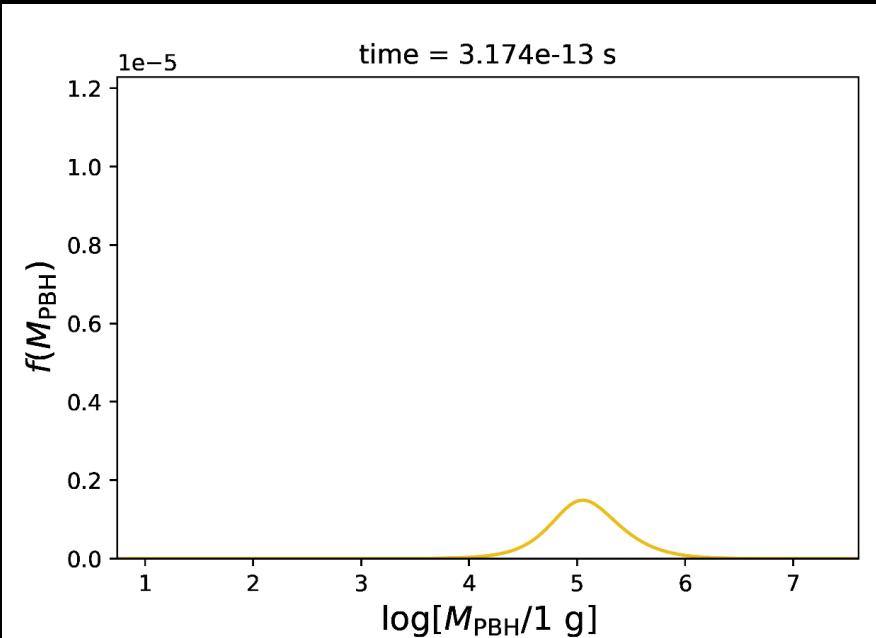
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### III. Evaporation of Extended Distributions

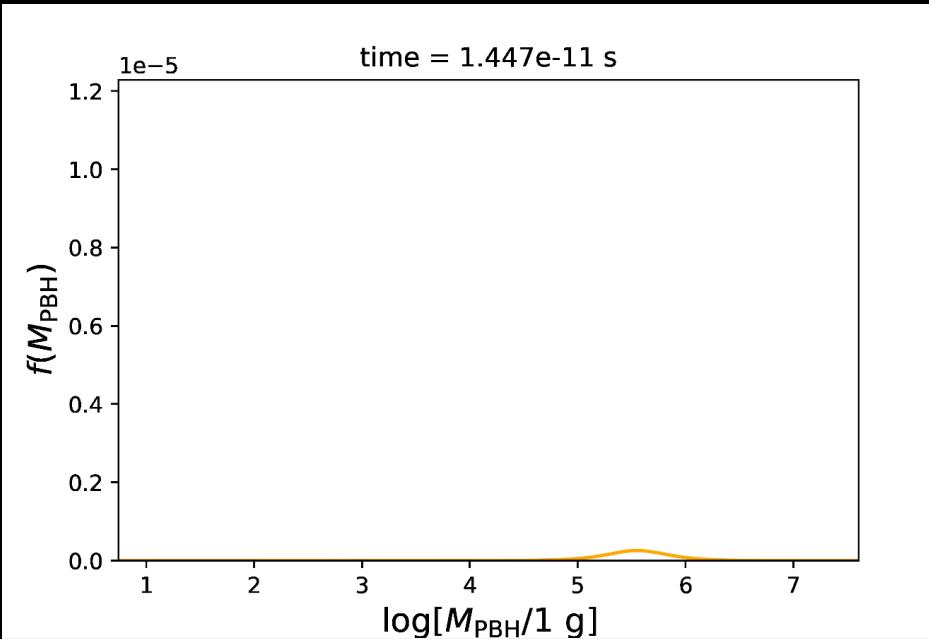
$$n_{\text{PBH}} = \int dM f(M) \quad f(M) = \frac{n_{\text{BH}}}{\sqrt{2\pi}\sigma M} \exp\left(-\frac{\log^2(M/M_c)}{2\sigma^2}\right)$$



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Green, 2016  
Kannike, 2017

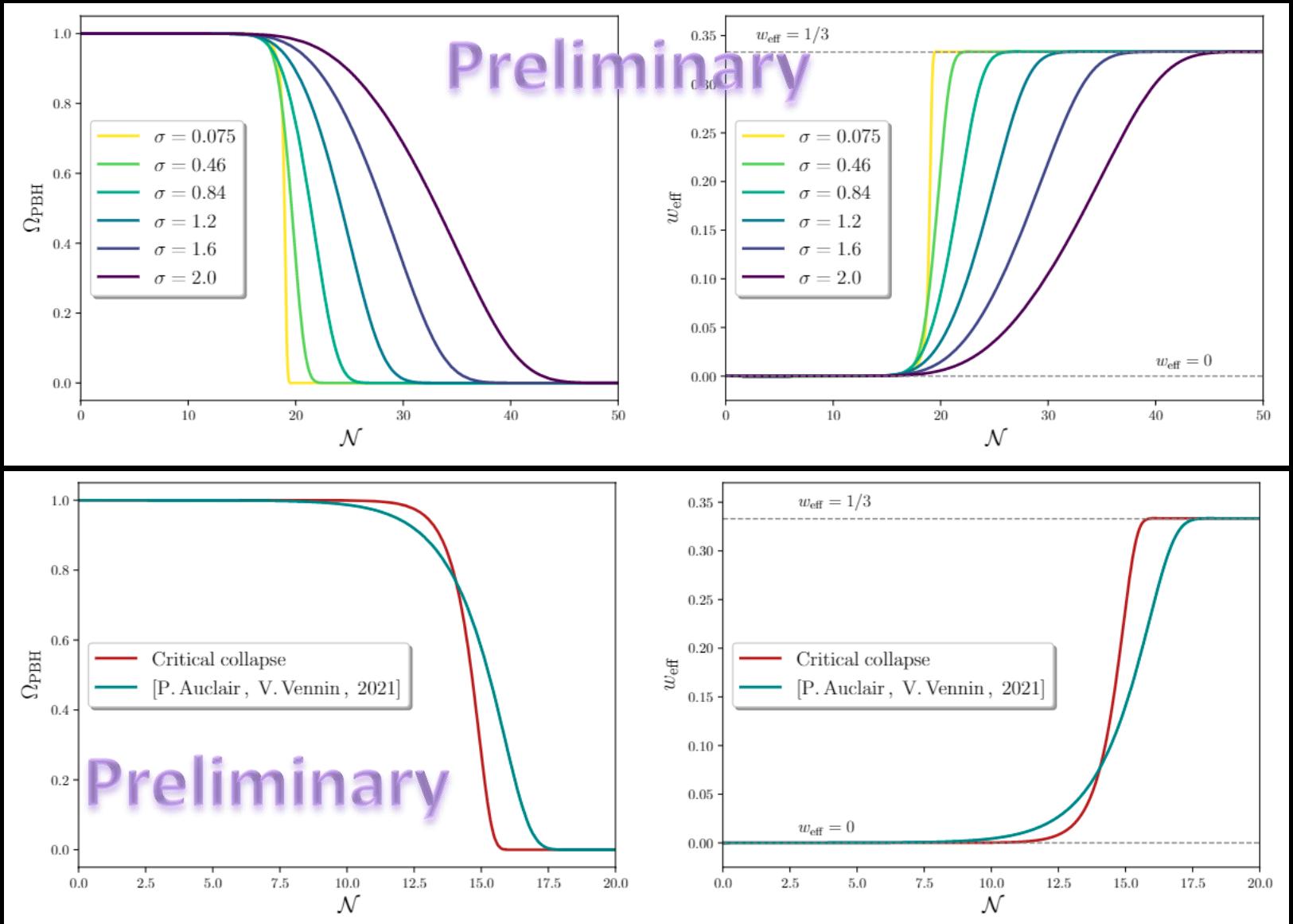
### III. Evaporation of Extended Distributions

$$n_{\text{PBH}} = \int dM f(M) \quad f(M) = \frac{n_{\text{BH}}}{\sqrt{2\pi}\sigma M} \exp\left(-\frac{\log^2(M/M_c)}{2\sigma^2}\right)$$



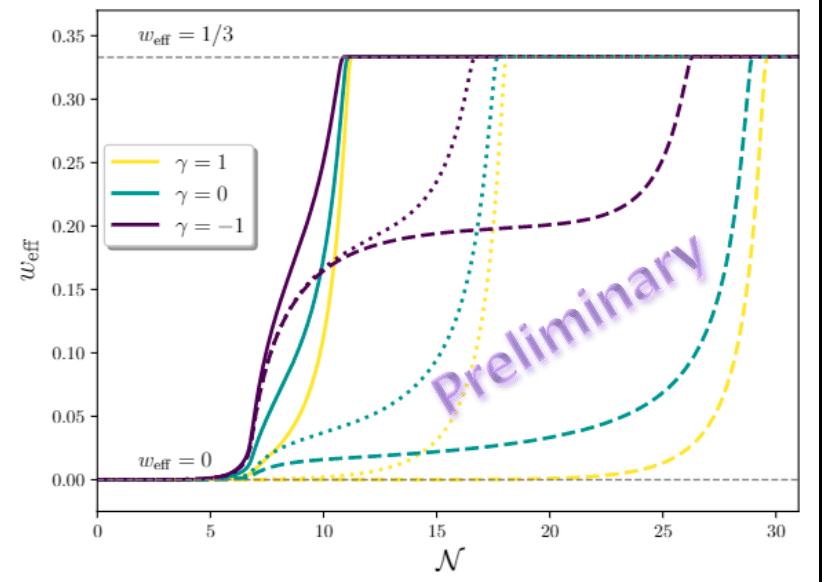
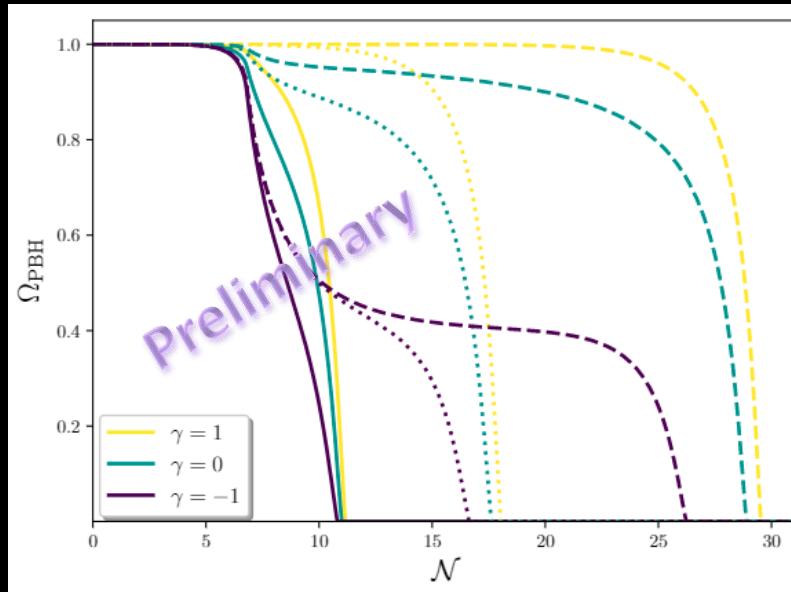
Log-normal distribution  
Dolgov, 93  
Green, 2016  
Kannike, 2017

### III. Evaporation of Extended Distributions



### III. Evaporation of Extended Distributions

$$\frac{dn}{dM} \propto M^{-\alpha} \quad \text{with} \quad \alpha = \frac{2(1+2w)}{1+w}$$



‘Stasis’ regime reached for  $0 < w \leq 1$

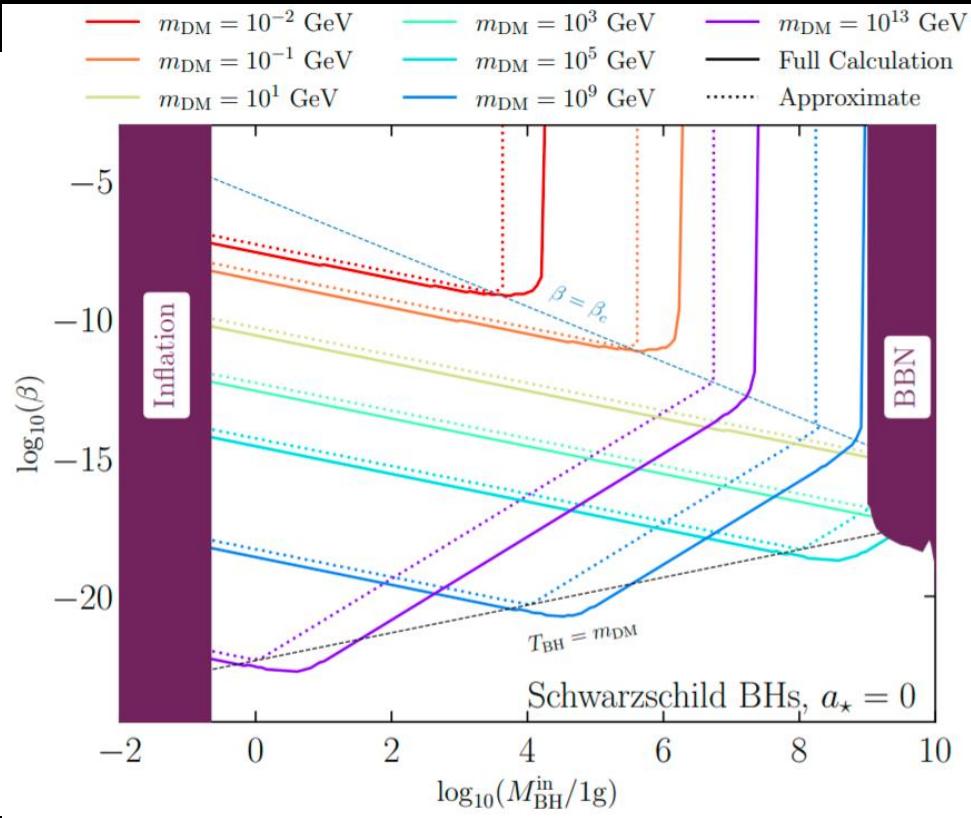
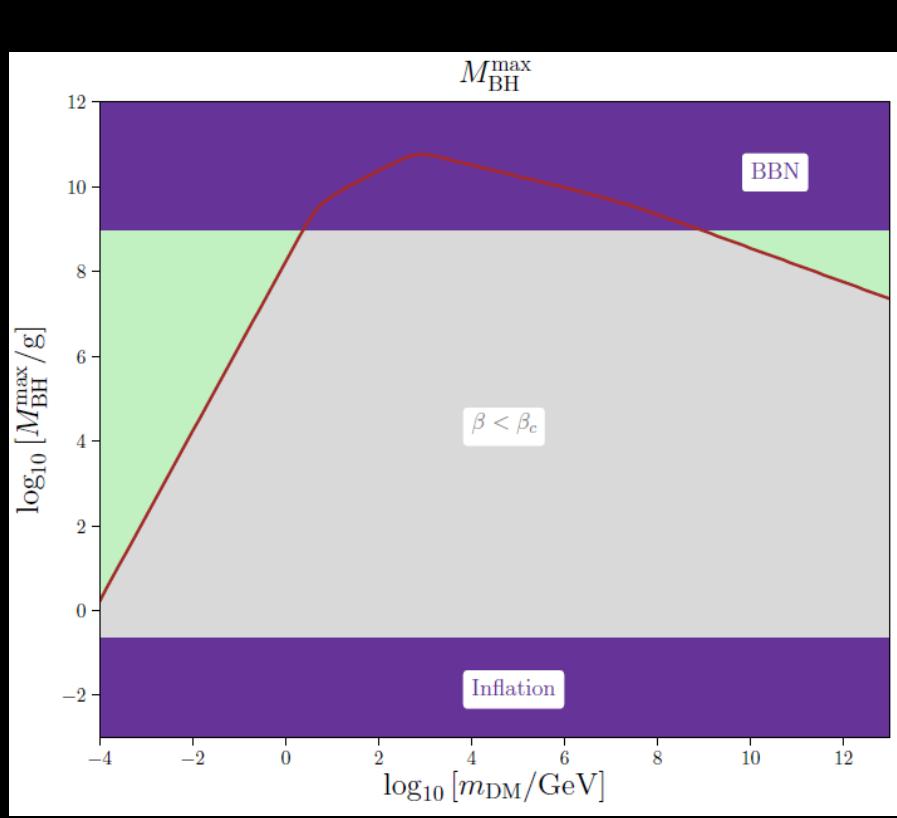
# CONCLUSION

Mini PBHs can leave several imprints in the early Universe

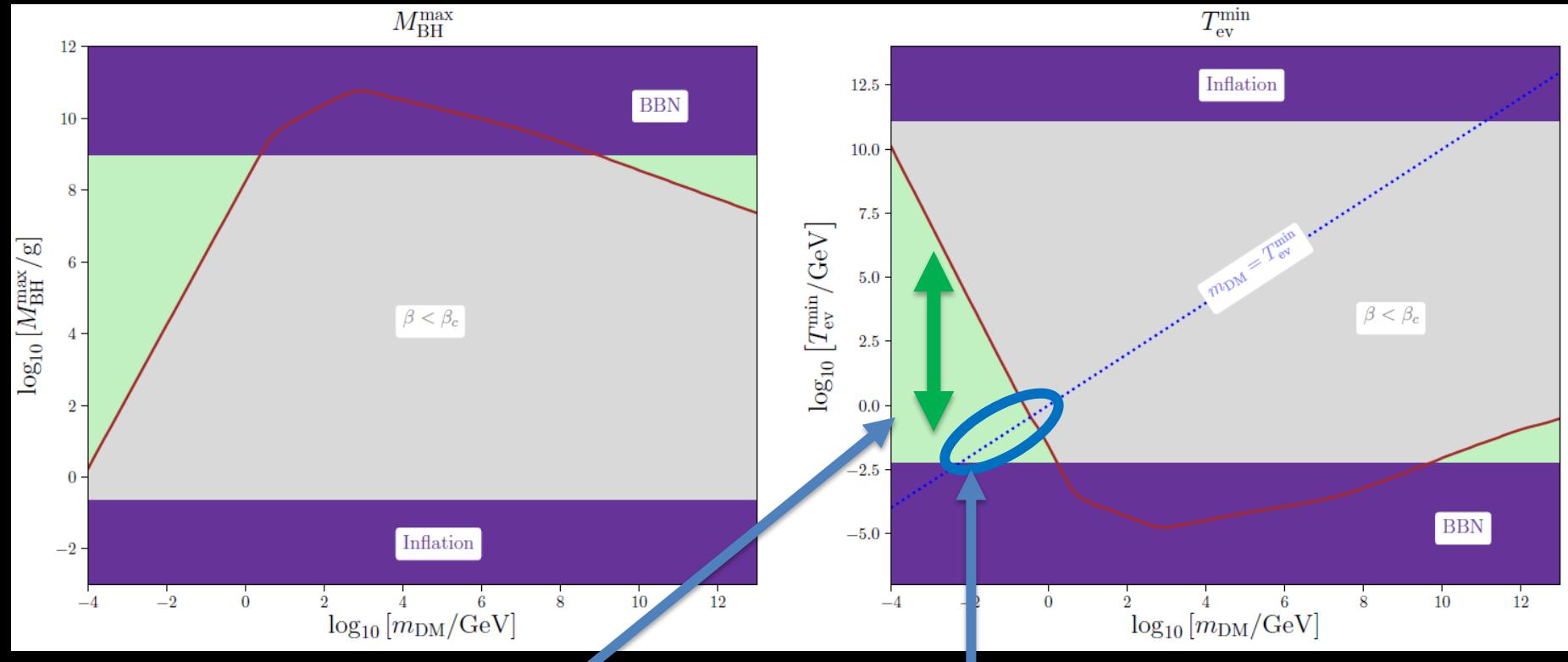
- Modify cosmology (EMD+ entropy inj.)
- Produce dark matter, leading to modified predictions for particle searches
- Particles produced from evaporation can be extremely boosted, which can lead to additionnal constraints from structure formation
- Kerr PBHs can lead to a large production of gravitons – existing results were refined
- Our code is accessible online: [?](#)  
<https://github.com/yfperezg/frisbee>

Back up

# MODIFIED COSMOLOGY



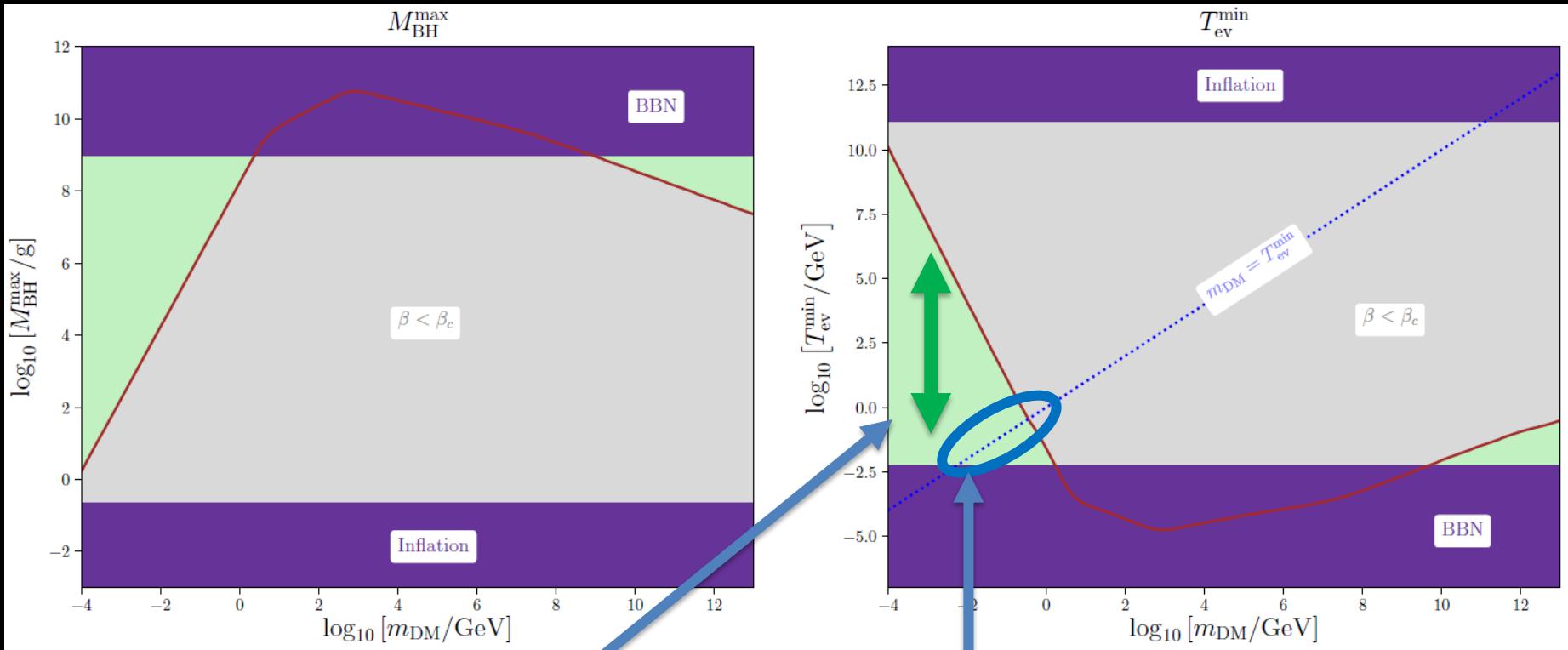
# MODIFIED COSMOLOGY



Region of interest  
for Freeze-In

Region of interest  
for Freeze-Out

# MODIFIED COSMOLOGY

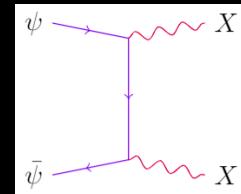


Region of interest  
for Freeze-In

~~Region of interest  
for Freeze-Out~~

**Thermalization  
Of PBHs products...**

TBH large +



# BOLTZMANN EQUATIONS

Freeze-In case:

$$\dot{n}_{\text{DM}}^{\text{th}} + 3Hn_{\text{DM}}^{\text{th}} = \langle \sigma v \rangle_{\text{th}} (n_{\text{DM,eq}}^2 - n_{\text{DM}}^{\text{th} 2})$$

$$\dot{n}_{\text{DM}}^{\text{ev}} + 3Hn_{\text{DM}}^{\text{ev}} = \left. \frac{dn_{\text{DM}}^{\text{ev}}}{dt} \right|_{\text{BH}}$$

$$\dot{n}_X + 3Hn_X = \left. \frac{dn_X}{dt} \right|_{\text{BH}}$$

$$\dot{\rho}_{\text{SM}} + 4H\rho_{\text{SM}} = \left. \frac{dM}{dt} \right|_{\text{SM}}$$

# BOLTZMANN EQUATIONS

Freeze-In case:

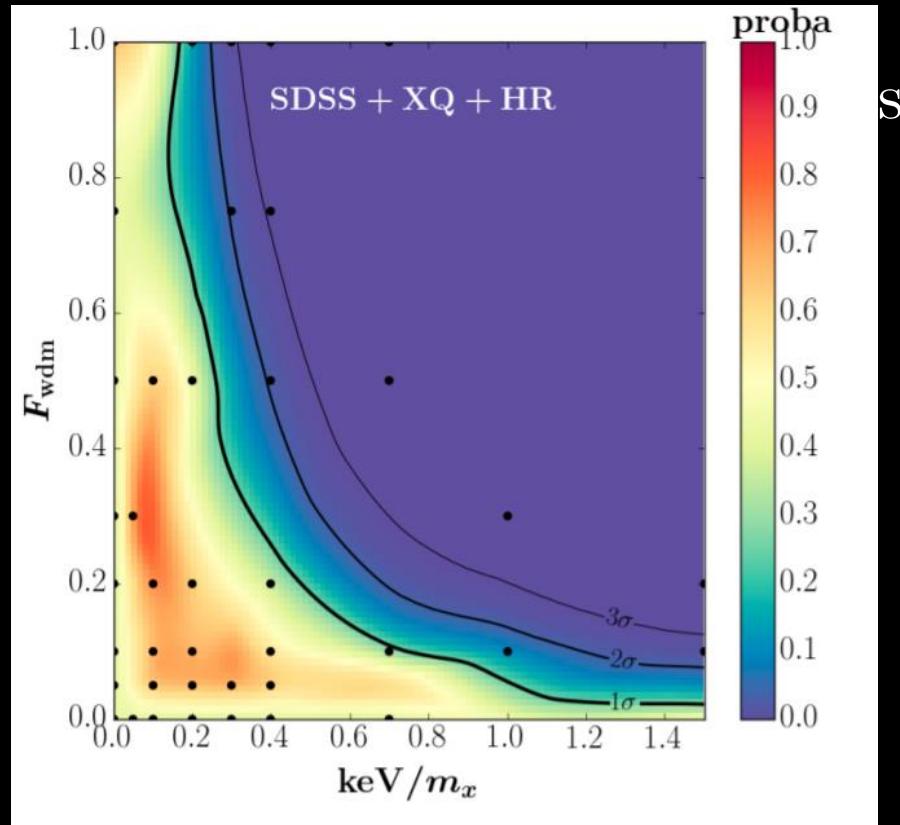
$$\dot{n}_{\text{DM}}^{\text{th}} + 3Hn_{\text{DM}}^{\text{th}} = \langle \sigma v \rangle_{\text{th}} (n_{\text{DM,eq}}^2 - n_{\text{DM}}^{\text{th} 2})$$

$$\dot{n}_{\text{DM}}^{\text{ev}} + 3Hn_{\text{DM}}^{\text{ev}} = \left. \frac{dn_{\text{DM}}^{\text{ev}}}{dt} \right|_{\text{BH}} + 2\Gamma_{X \rightarrow \text{DM}} \left\langle \frac{m_X}{E_X} \right\rangle_{\text{ev}} n_X$$

$$\dot{n}_X + 3Hn_X = \left. \frac{dn_X}{dt} \right|_{\text{BH}} - \Gamma_X \left\langle \frac{m_X}{E_X} \right\rangle_{\text{ev}} n_X$$

$$\dot{\rho}_{\text{SM}} + 4H\rho_{\text{SM}} = \left. \frac{dM}{dt} \right|_{\text{SM}} + 2m_X \Gamma_{X \rightarrow \text{SM}} n_X$$

# NON-COLD DARK MATTER

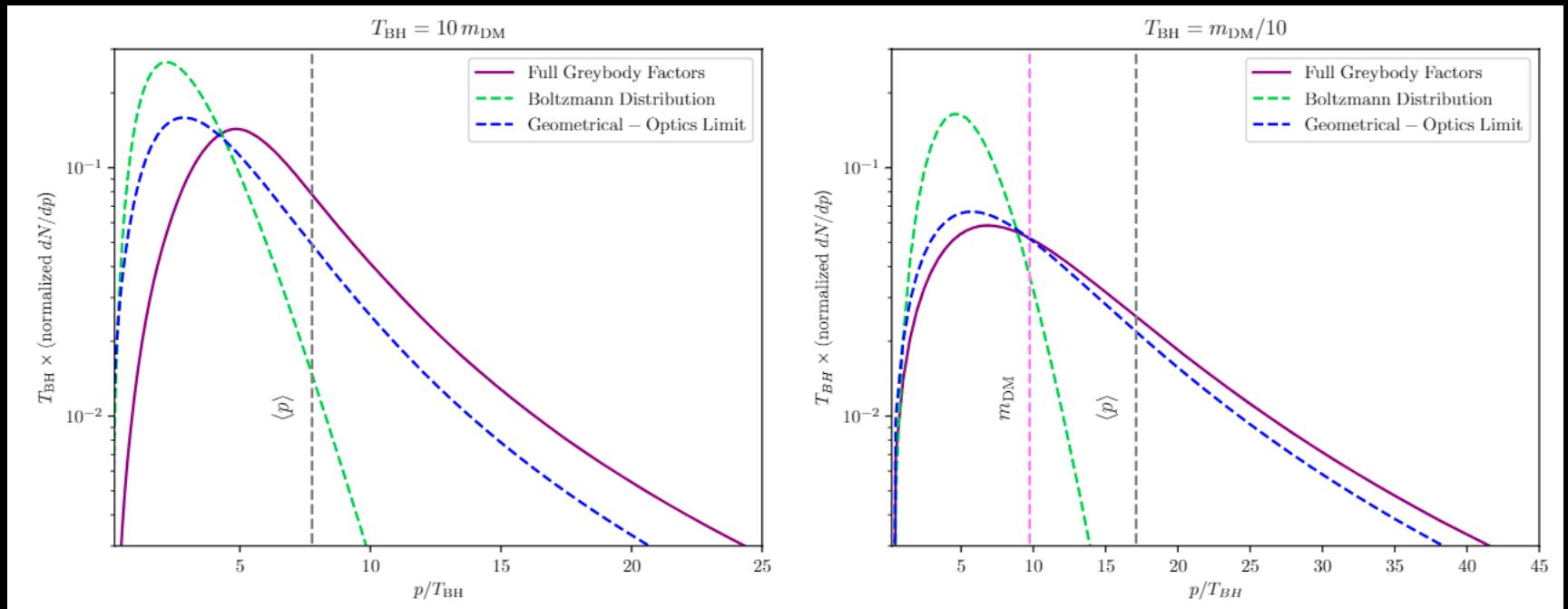


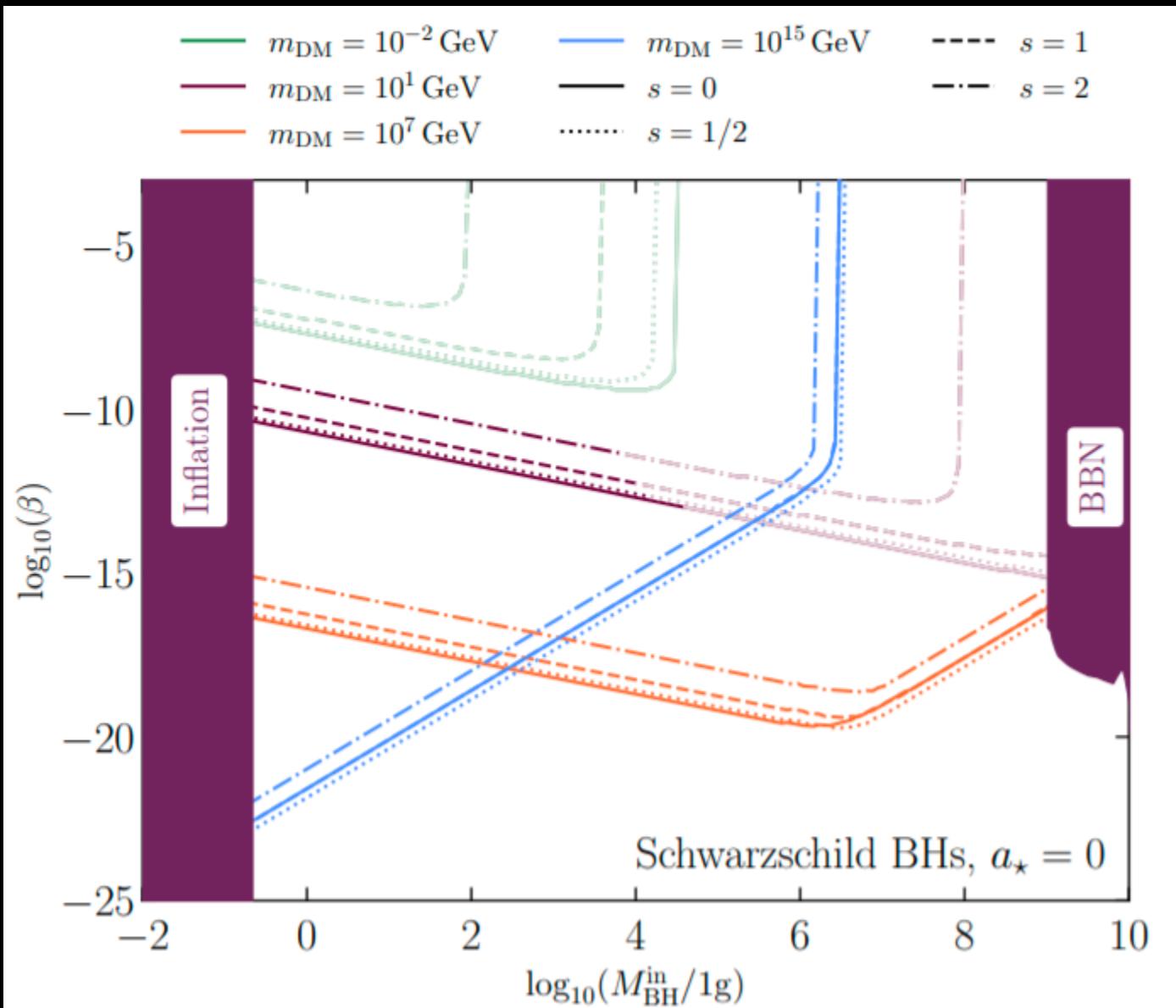
[Baur *et al.* 2017]

$$\langle v \rangle|_{t=t_0} = a_{\text{ev}} \times \frac{\langle p \rangle|_{t=t_{\text{ev}}}}{m_{\text{DM}}}$$

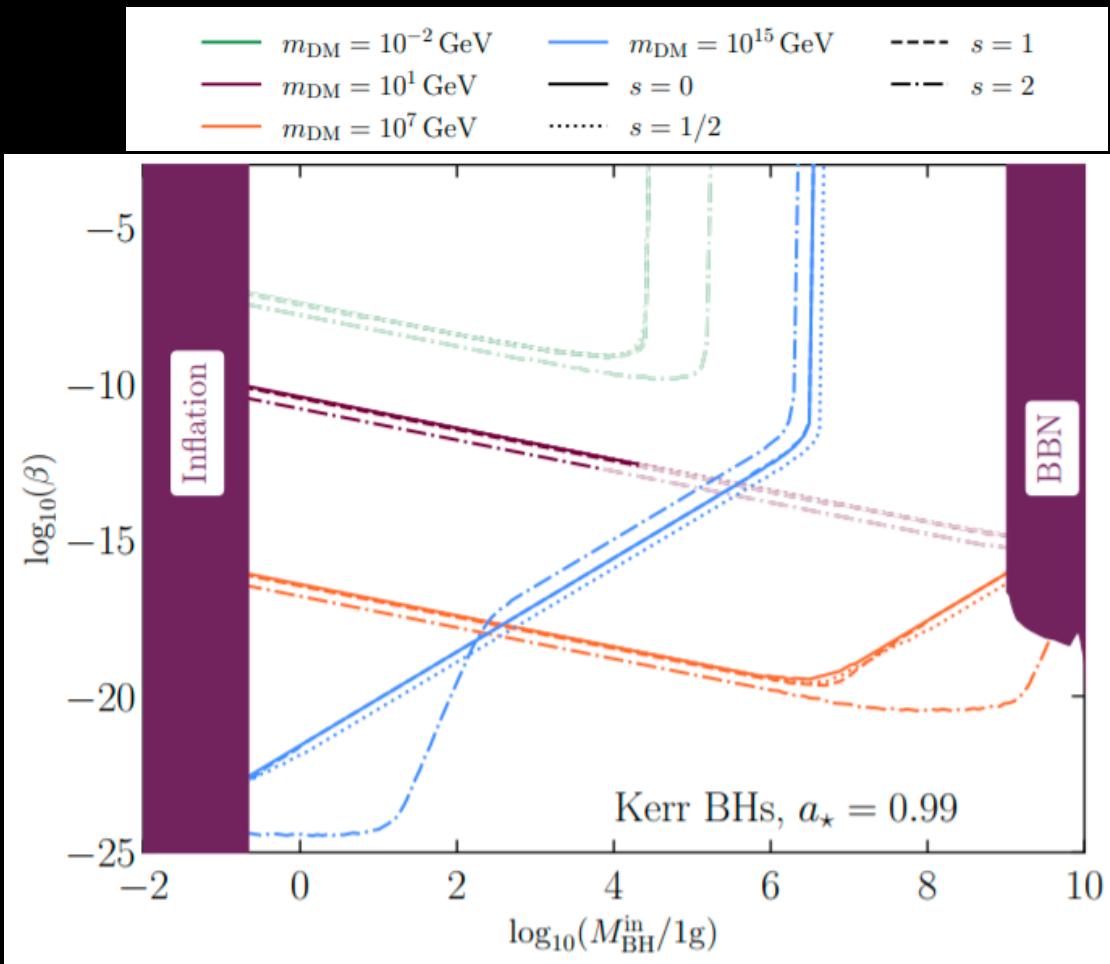
# DM FROM EVAPORATION

- Peculiar spectrum of evaporated DM particles
- Non-negligible difference between geometrical-optics limit and full distributions





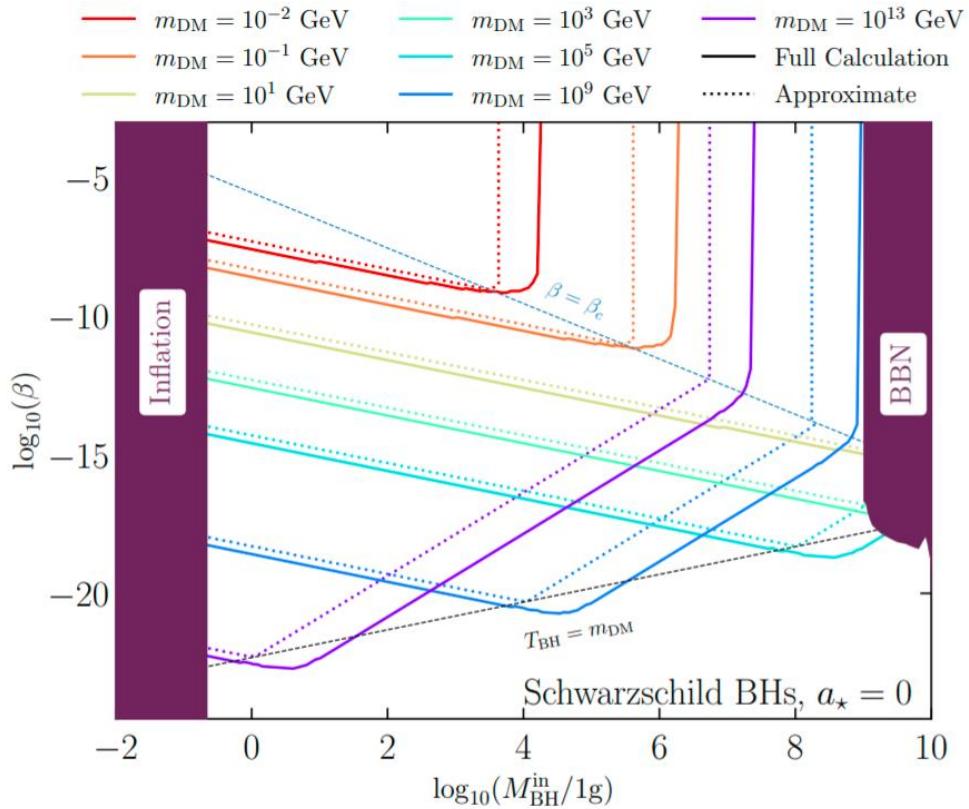
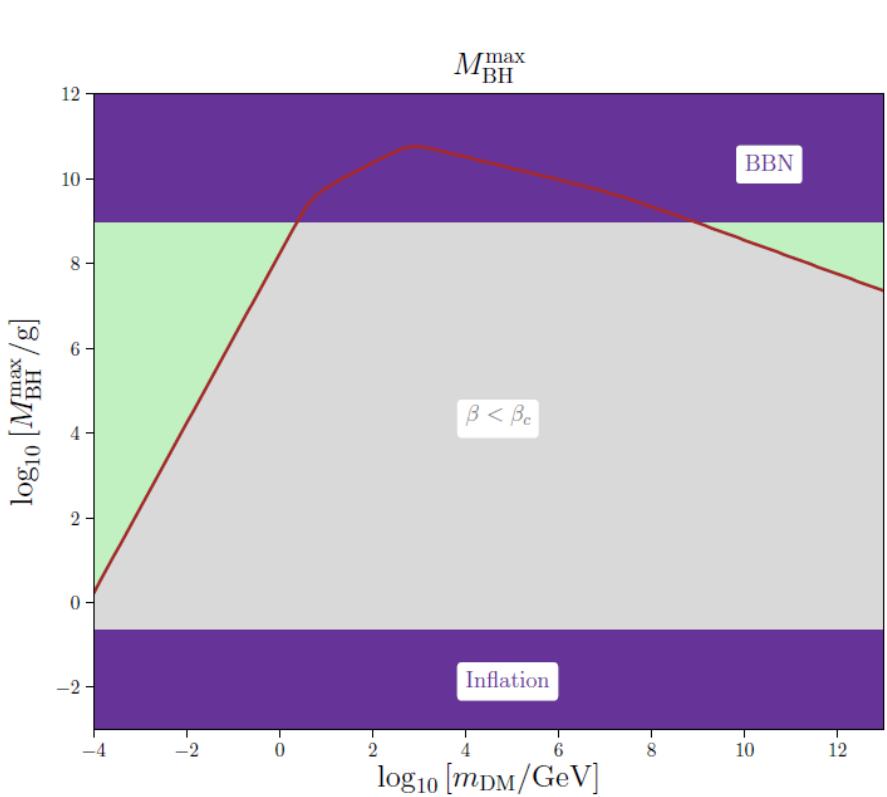
$$T_{\text{BH}} = \frac{1}{4\pi G M_{\text{BH}}} \frac{\sqrt{1 - a_*^2}}{1 + \sqrt{1 - a_*^2}},$$



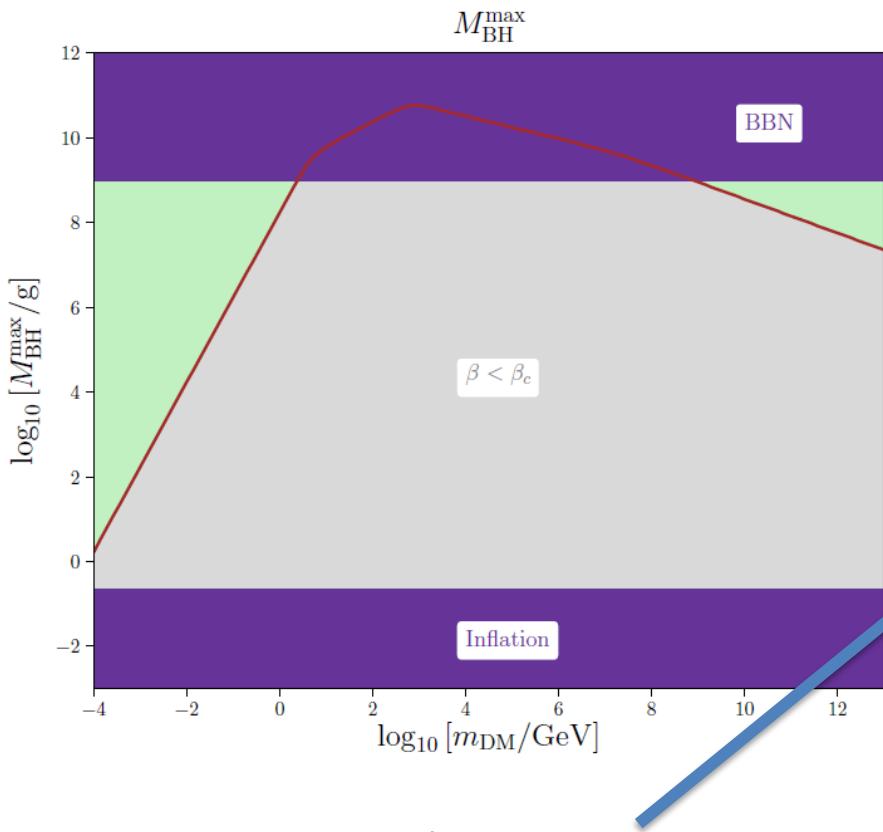
$$\frac{d^2 \mathcal{N}_{ilm}}{dp dt} = \frac{\sigma_{s_i}^{lm}(M_{\text{BH}}, p, a_*)}{\exp[(E_i - m\Omega)/T_{\text{BH}}] - (-1)^{2s_i}} \frac{p^3}{E_i}$$

where  $\Omega = (a_*/2GM_{\text{BH}})(1/(1 + \sqrt{1 - a_*^2}))$

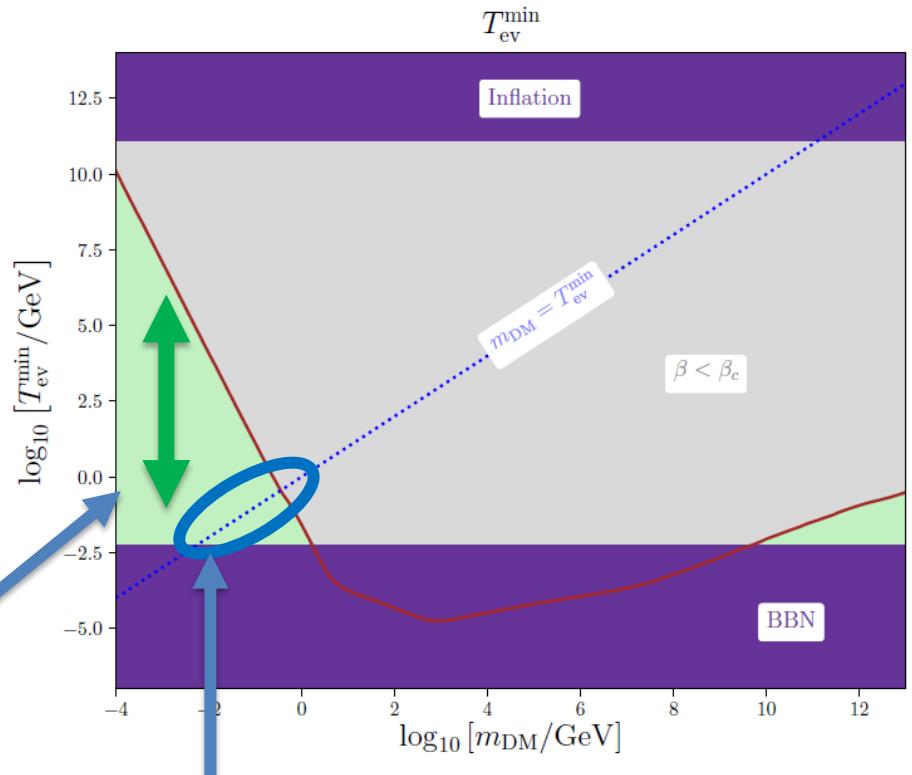
# MODIFIED COSMOLOGY



# MODIFIED COSMOLOGY

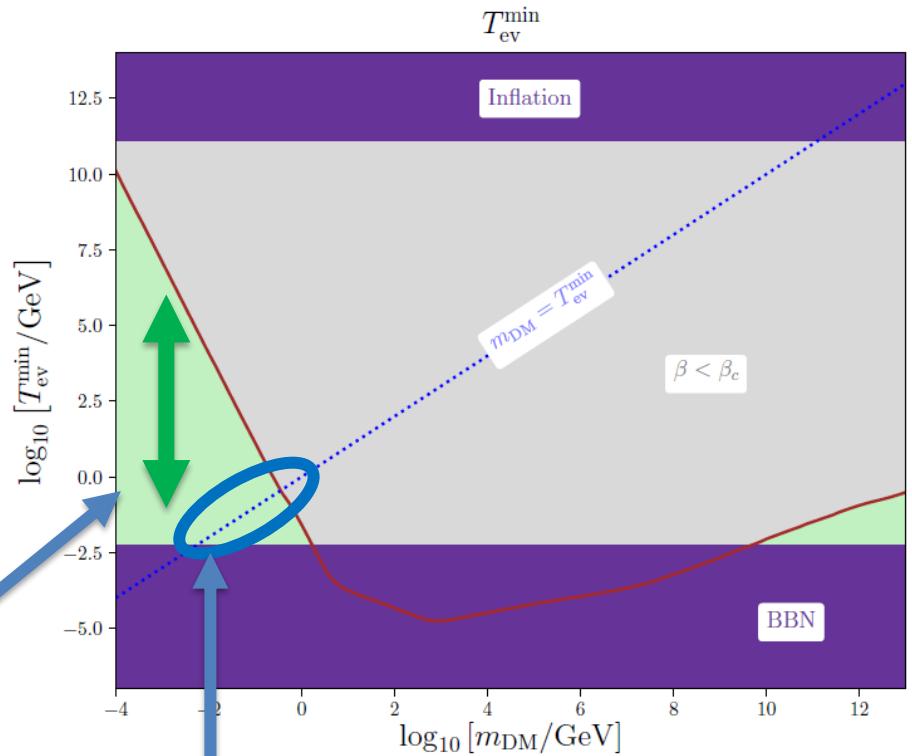
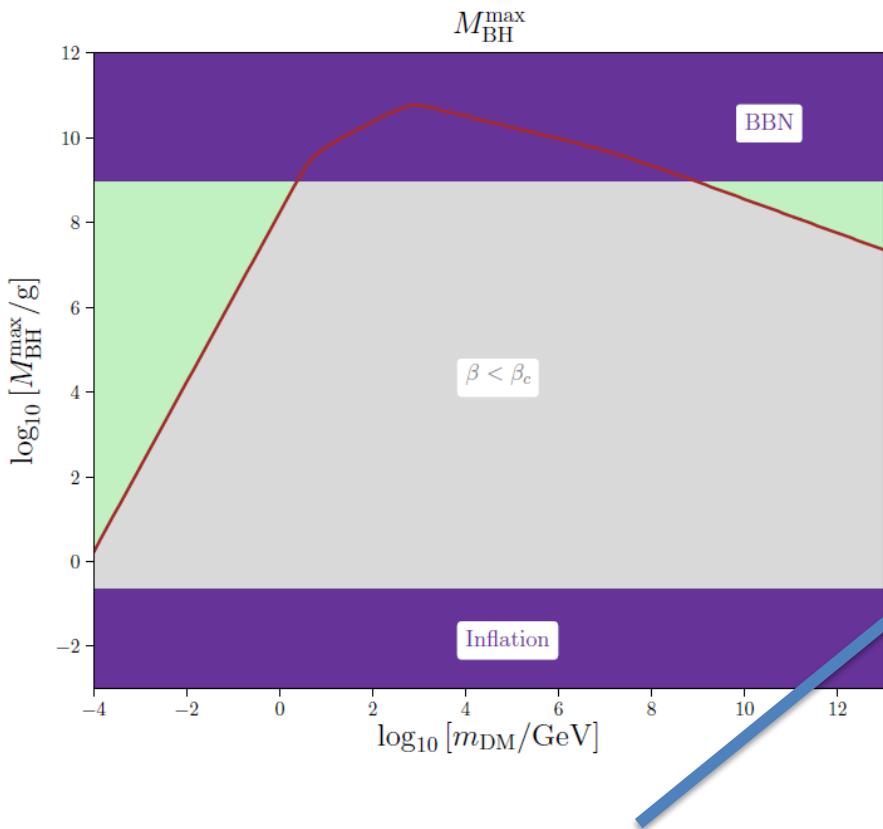


Region of interest  
for Freeze-In



Region of interest  
for Freeze-Out

# MODIFIED COSMOLOGY

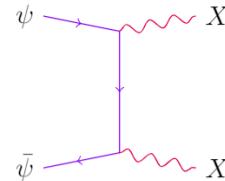


Region of interest  
for Freeze-In

~~Region of interest  
for Freeze-Out~~

**Thermalization  
Of PBHs products...**

TBH large +



# PBH EVAPORATION

$$\frac{dM_{\text{BH}}}{dt} \equiv \sum_i \left. \frac{dM_{\text{BH}}}{dt} \right|_i = - \sum_i \int_0^\infty E_i \frac{d^2 \mathcal{N}_i}{dp dt} dp = -\varepsilon(M_{\text{BH}}) \frac{M_p^4}{M_{\text{BH}}^2}$$

$$\frac{d^2 \mathcal{N}_i}{dp dt} = \frac{g_i}{2\pi^2} \frac{\sigma_{s_i}(M_{\text{BH}}, \mu_i, p)}{\exp [E_i(p)/T_{\text{BH}}] - (-1)^{2s_i}} \frac{p^3}{E_i(p)}$$

$$\varepsilon(M_{\text{BH}}) \equiv \sum_i g_i \varepsilon_i(z_i) \quad z_i = \mu_i/T_{\text{BH}}$$

BSM  
Contributions?

$$T_{\text{BH}} = \frac{1}{8\pi G M_{\text{BH}}} \sim 1.06 \text{ GeV} \left( \frac{10^{13} \text{ g}}{M_{\text{BH}}} \right)$$

# Kerr PBHs and Dark Radiation

$$\frac{\mathrm{d}^2\mathcal{N}_{ilm}}{\mathrm{d}p\mathrm{d}t}=\frac{\sigma_{s_i}^{lm}(M_\mathrm{BH},p,a_\star)}{\exp\left[(E_i-m\Omega)/T_\mathrm{BH}\right]-(-1)^{2s_i}}\frac{p^3}{E_i}$$

$$\frac{d M_{\rm BH}}{dt} = - \epsilon(M_{\rm BH}, a_\star) \frac{M_p^4}{M_{\rm BH}^2} \,, \\ \frac{da_\star}{dt} = - a_\star [\gamma(M_{\rm BH}, a_\star) - 2 \epsilon(M_{\rm BH}, a_\star)] \frac{M_p^4}{M_{\rm BH}^3} \,,$$

$$\varepsilon_i(M_{\rm BH}, a_\star) = \frac{g_i}{2\pi^2} \int_0^\infty \sum_l \sum_{m=-l}^l \frac{\mathrm{d}^2\mathcal{N}_{ilm}}{\mathrm{d}p\mathrm{d}t} \, EdE \,, \\ \gamma_i(M_{\rm BH}, a_\star) = \frac{g_i}{2\pi^2} \int_0^\infty \sum_l \sum_{m=-l}^l m \frac{\mathrm{d}^2\mathcal{N}_{ilm}}{\mathrm{d}p\mathrm{d}t} \, dE \,,$$

# THERMAL PRODUCTION OF DM

$$\dot{n}_{\text{DM}} + 3Hn_{\text{DM}} = g_{\text{DM}} \int C[f_{\text{DM}}] \frac{d^3 p}{(2\pi)^3} + \left. \frac{dn_{\text{DM}}}{dt} \right|_{\text{BH}}$$

$$\dot{n}_X + 3Hn_X = g_X \int C[f_X] \frac{d^3 p}{(2\pi)^3} + \left. \frac{dn_X}{dt} \right|_{\text{BH}},$$

$$\dot{\rho}_{\text{SM}} + 4H\rho_{\text{SM}} = \left. \frac{dM}{dt} \right|_{\text{SM}}.$$

$$\left. \frac{dn_i}{dt} \right|_{\text{BH}} = n_{\text{BH}} g_i \int \left. \frac{\partial f_i}{\partial t} \right|_{\text{BH}} \frac{p^2 dp}{2\pi^2}$$

DM Annihilation, X decay

PBH evaporation

PBHs evaporate **non-trivial distributions** of DM and X particles



Non-trivial evolution of the full distributions  $f_X(p)$  and  $f_{\text{DM}}(p)$

Simplified approach...

# THERMAL PRODUCTION OF DM

- If PBHs evaporate **before FO**:  
→ Assume **INSTANTANEOUS** thermalization
  - If PBHs evaporate **after FO**:  
→ Assume **No** thermalization
  - **FI case**: assume **No** thermalization
- Check those assumptions by evaluating at all time

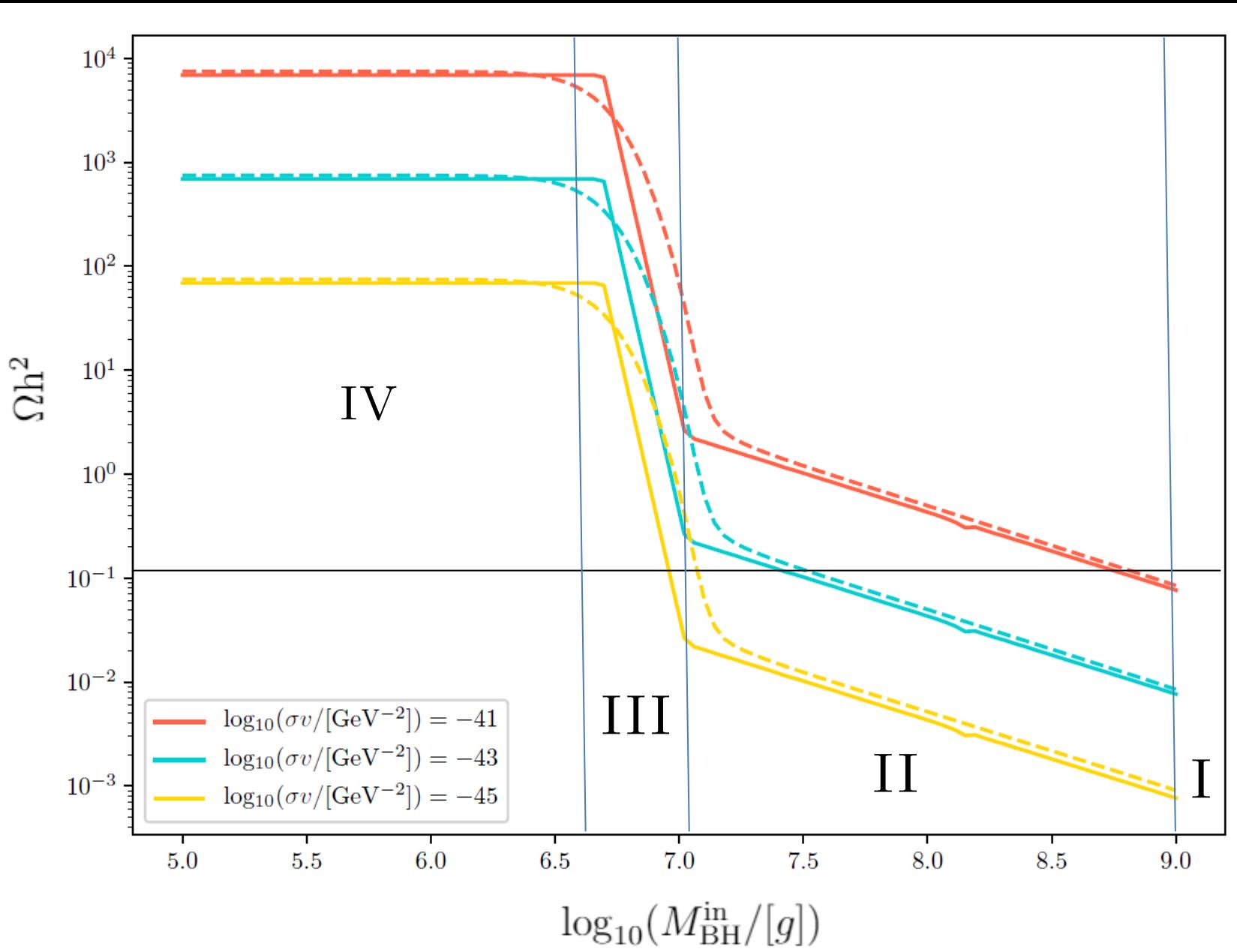
$$\Gamma_{\text{th+ev}} \equiv \frac{\langle \sigma \cdot v \rangle_{\text{th+ev}} \times n^{\text{th}}}{H}$$

$$\langle \sigma \cdot v \rangle_{\text{th+ev}} \equiv \frac{\int \sigma \cdot v_{\text{moll}} f_{\text{ev}} f_{\text{th}} d^3 \vec{p}_1 d^3 \vec{p}_2}{\left[ \int d^3 \vec{p}_1 f_{\text{ev}} \right] \left[ \int d^3 \vec{p}_2 f_{\text{th}} \right]}.$$

# EFFECTS OF PBH EVAPORATION

1. PBHs produce additional DM particles  
[Gondolo *et al* 2020, Bernal *et al* 2020]
2. PBHs produce mediator particles  $X$
3. The evaporation of PBHs can inject entropy in the SM bath *after* the thermal production of DM
4. The evaporation of PBHs can modify the cosmological evolution *during* the thermal production of DM
5. Particles with energy  $E \sim T_{\text{BH}}$  may be warm...

# COMPARISON WITH NUMERICS



# Kerr PBHs and Dark Radiation

In the Standard Model

$$\rho_R^{\text{SM}} = \rho_\gamma \left[ 1 + \frac{7}{8} \left( \frac{T_\nu}{T_\gamma} \right) N_{\text{eff}}^{\text{SM}} \right],$$

$$T_\nu = (4/11)^{1/3} T_\gamma$$

In the presence of Dark Radiation

$$\rho_R \equiv \rho_\gamma \left[ 1 + \frac{7}{8} \left( \frac{T_\nu}{T_\gamma} \right) (N_{\text{eff}}^{\text{SM}} + \Delta N_{\text{eff}}) \right]$$

$$\Delta N_{\text{eff}} = \left\{ \frac{8}{7} \left( \frac{4}{11} \right)^{-\frac{4}{3}} + N_{\text{eff}}^{\text{SM}} \right\} \frac{\rho_{\text{DR}}(T_{\text{ev}})}{\rho_R^{\text{SM}}(T_{\text{ev}})} \left( \frac{g_*(T_{\text{ev}})}{g_*(T_{\text{eq}})} \right) \left( \frac{g_{*S}(T_{\text{eq}})}{g_{*S}(T_{\text{ev}})} \right)^{\frac{4}{3}}$$



The quantity to evaluate