Dark matter production from preheating and structure formation constraints

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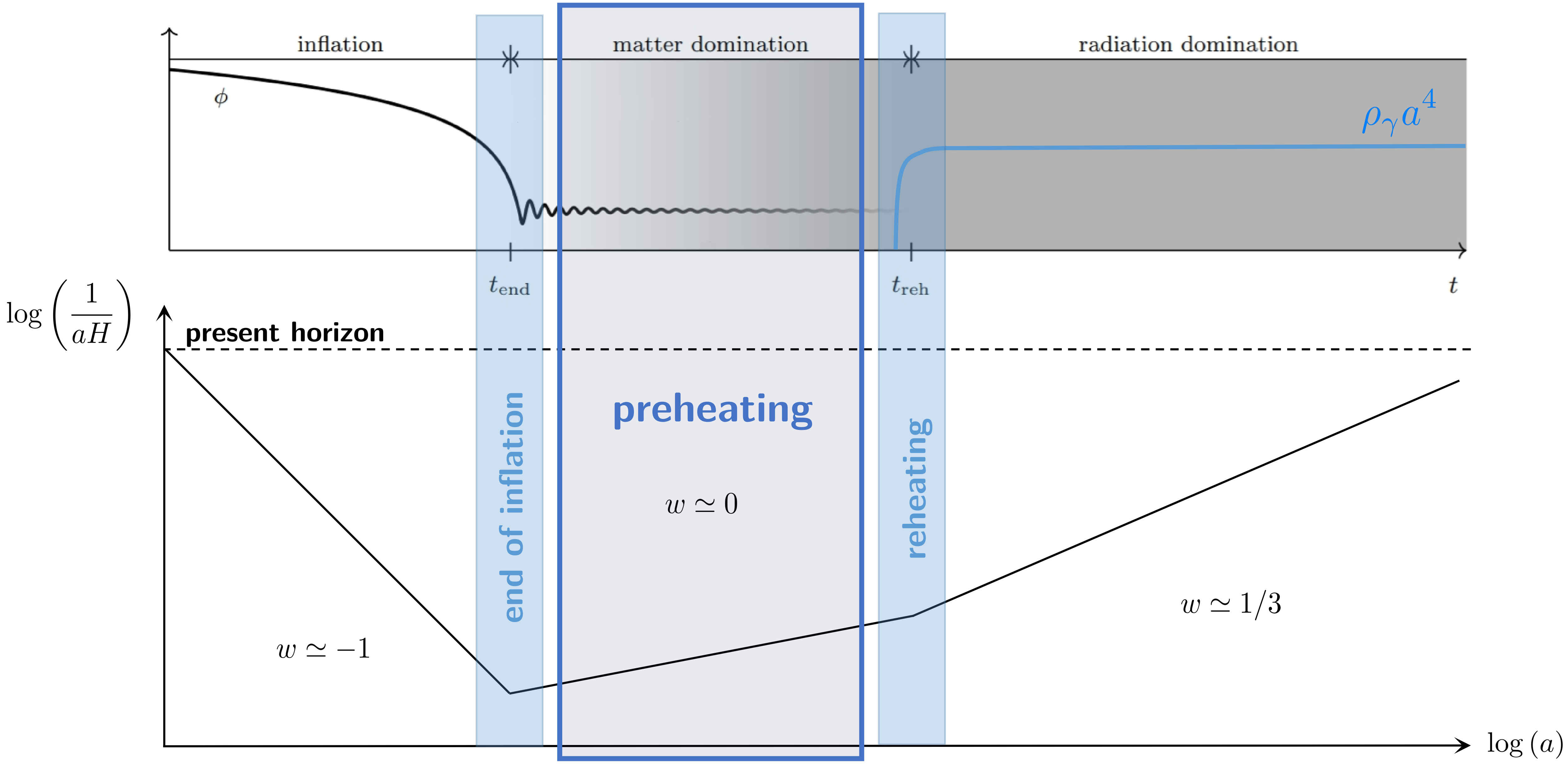
Paris-Saclay Astroparticle Symposium

Based on
[**arXiv:2206.08940**] with **S. Verner** & **M. A. G. Garcia**
& [**arXiv:2011.13458**] with **G. Ballesteros** & **M. A. G. Garcia**

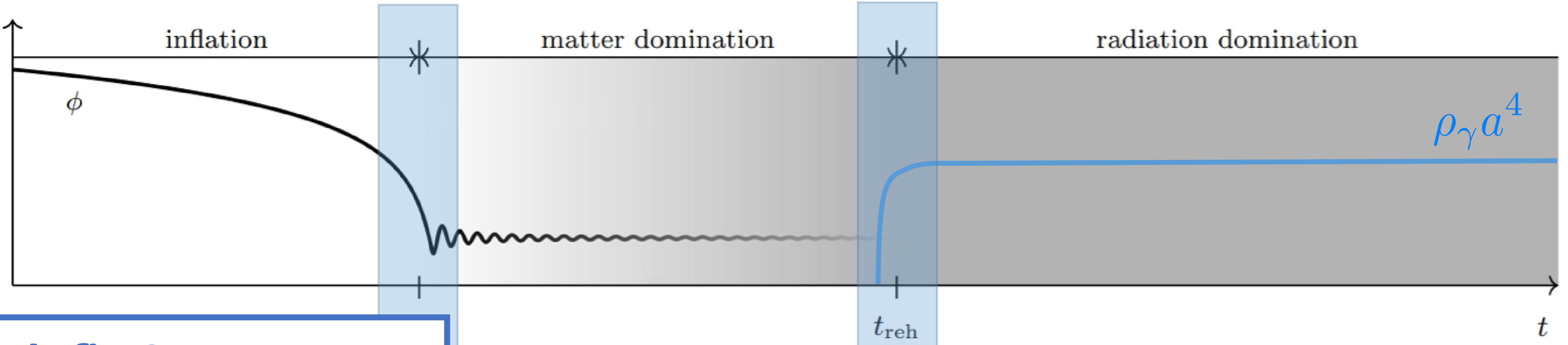
What could source dark matter production in the early universe?

What are the associated signatures/constraints?

The early universe

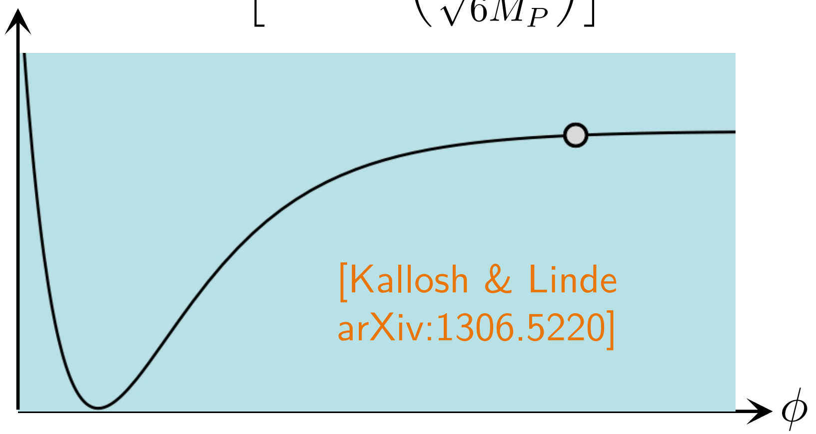


The early universe

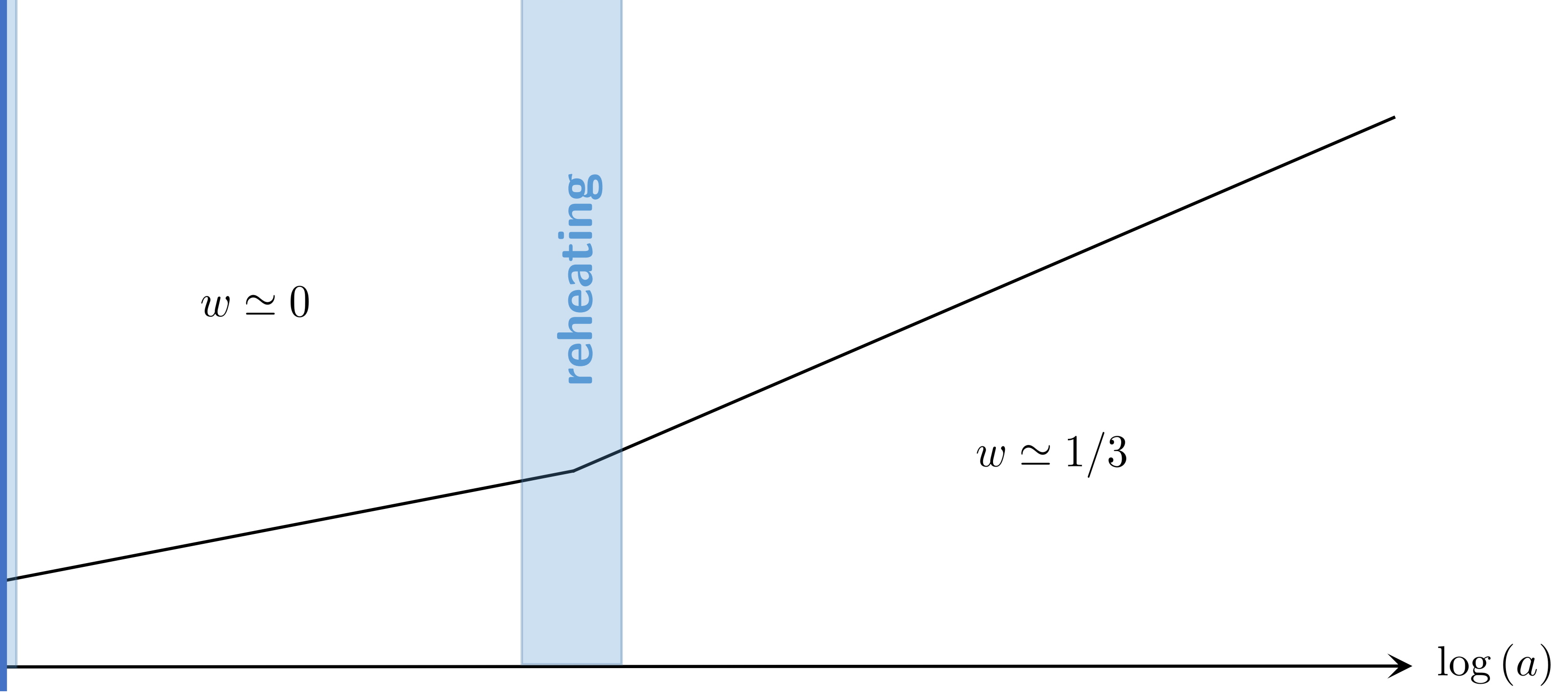


Inflation

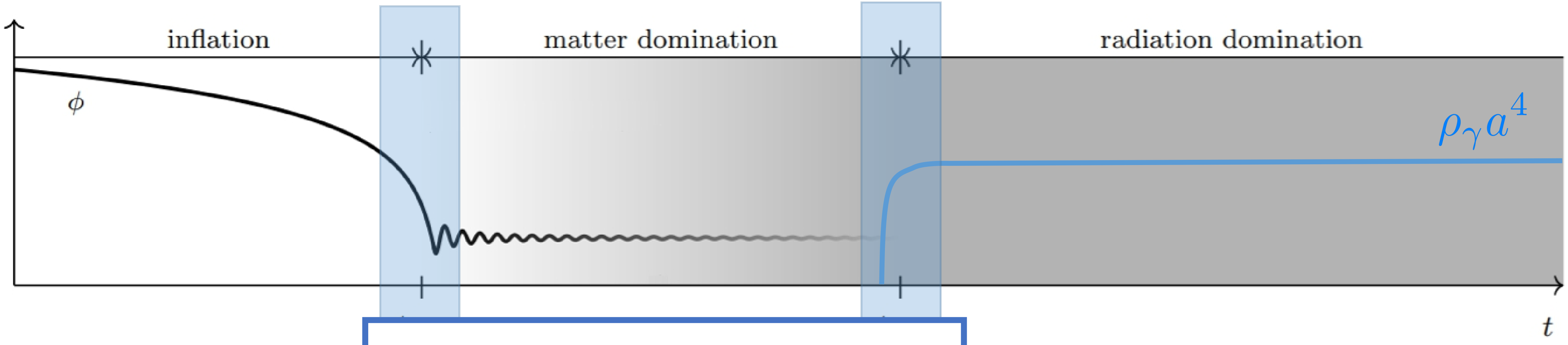
$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^2$$

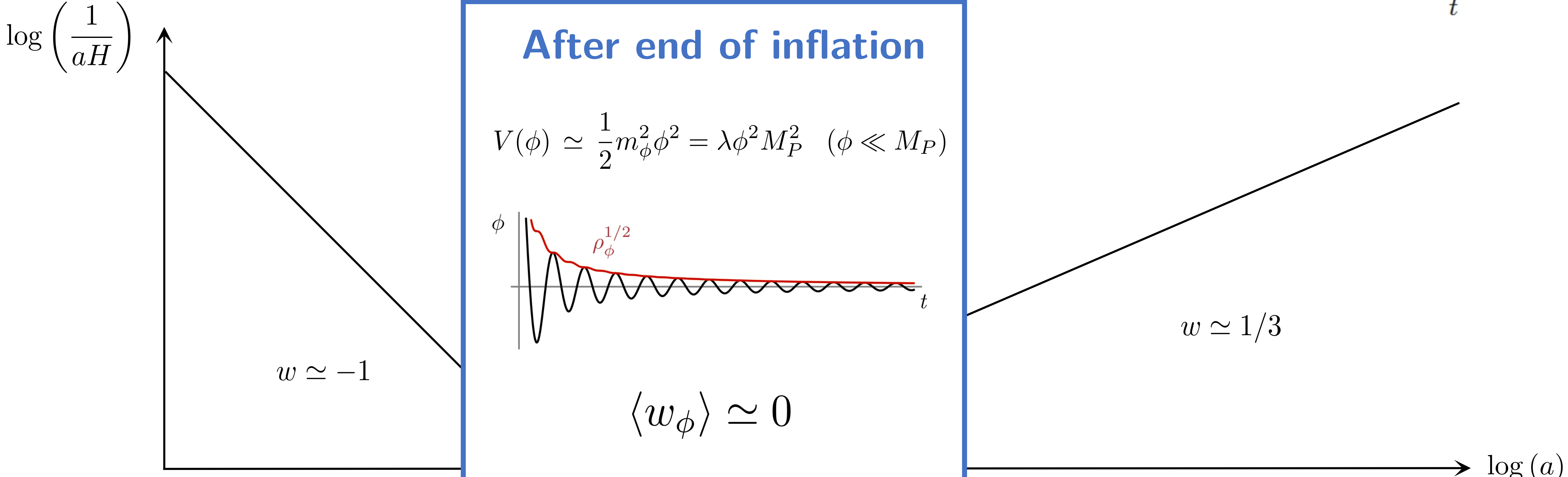


$$\lambda \simeq \frac{18\pi^2 A_s(k_\star)}{6N_*^2} \sim 10^{-11}$$



The early universe



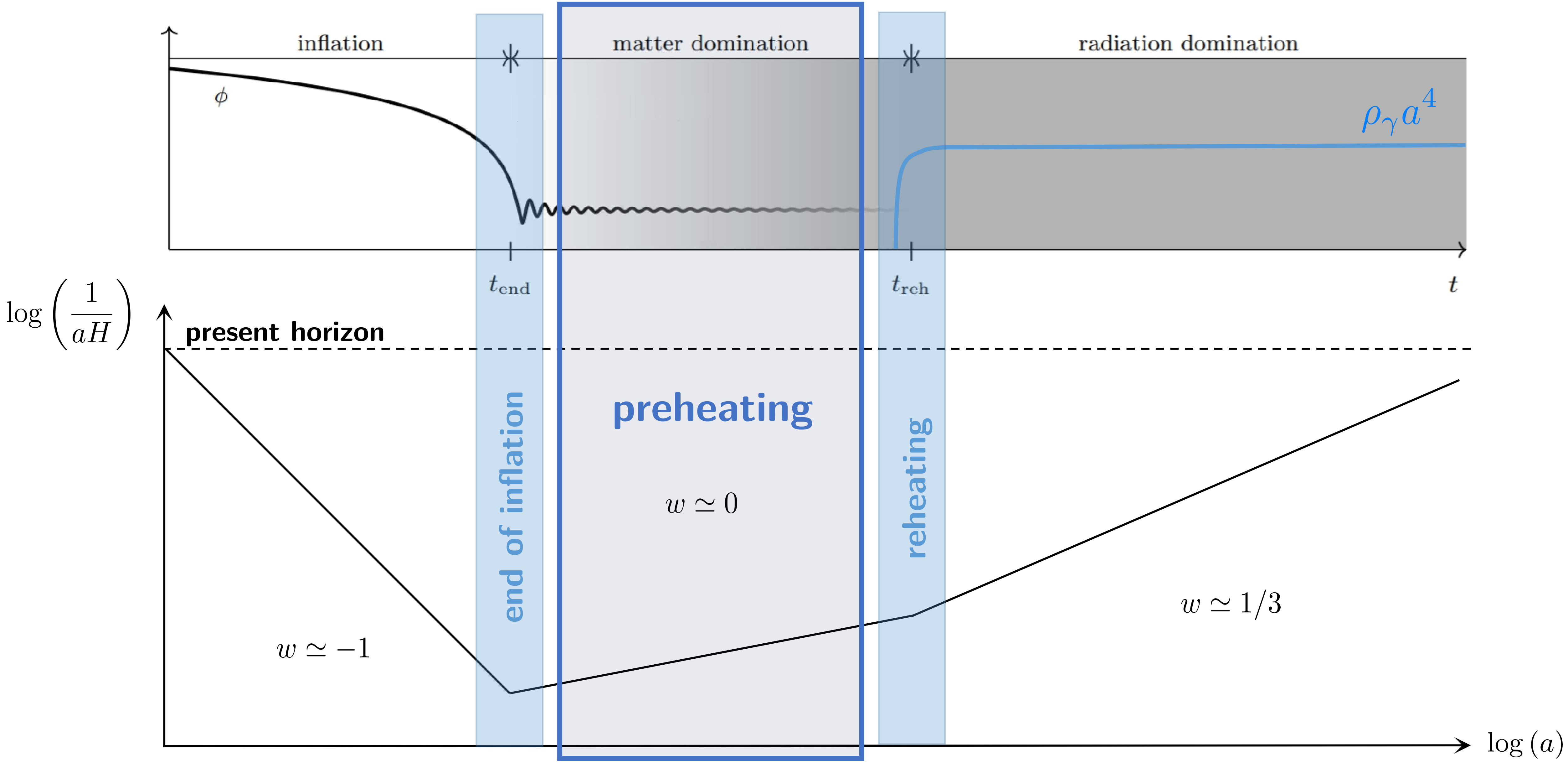


After end of inflation

$$V(\phi) \simeq \frac{1}{2} m_\phi^2 \phi^2 = \lambda \phi^2 M_P^2 \quad (\phi \ll M_P)$$

$$\langle w_\phi \rangle \simeq 0$$

The early universe



Dark matter production from preheating

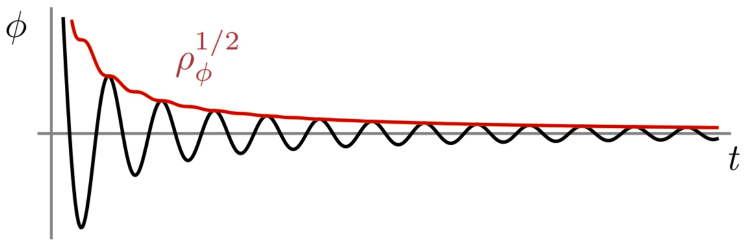
Particle production during preheating

- Consider **coupling** to **scalar dark matter**

$$\mathcal{S} = \int d^4x\,\sqrt{-g}\Big(R + \frac{1}{2}(\partial_\mu\phi)^2 - \lambda\phi^2 M_P^2$$ inflaton

$$+\frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}\sigma\phi^2\chi^2 - \frac{1}{2}m_\chi^2\chi^2\Big)$$ **scalar**

minimal coupling to gravity



$$\Rightarrow\quad m_{\chi,\mathrm{eff}}^2(t) = m_\chi^2 + \sigma\phi^2(t)$$

Goal: Estimate DM production for all regimes of σ/λ

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int d^3\boldsymbol{P}\, f_\chi(P_0, t)$$

Scalar preheating: field and Boltzmann picture

Boltzmann approach

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\chi}{\partial |\boldsymbol{P}|} = \frac{\pi^2}{\beta^2 m_\phi^3}\rho_\phi \Gamma_{\phi\phi\to\chi\chi}\delta\left(|\boldsymbol{P}| - m_\phi \beta(t)\right)(1 + 2f_\chi(|\boldsymbol{P}|))$$

Bose enhancement

$$\beta(t) \equiv \sqrt{1 - \frac{m_{\text{eff}}^2(t)}{m_\phi^2}}$$

: kinematic blocking

The field picture

- **Mode function equation: harmonic oscillator** with **time-dependent** frequency

$$X_p'' + \omega_p^2 X_p = 0 \qquad \omega_p^2(t) = p^2 + a^2(t)\hat{m}_{\text{eff}}^2(t) \qquad \hat{m}_{\text{eff}}^2(t) = m_\chi^2 + \sigma\phi^2 + \frac{R}{6}$$

Gravity!

- **Distribution function** from **occupation number**

$$n_p = \frac{1}{2\omega_p}\left|\omega_p X_p - iX_p'\right|^2$$

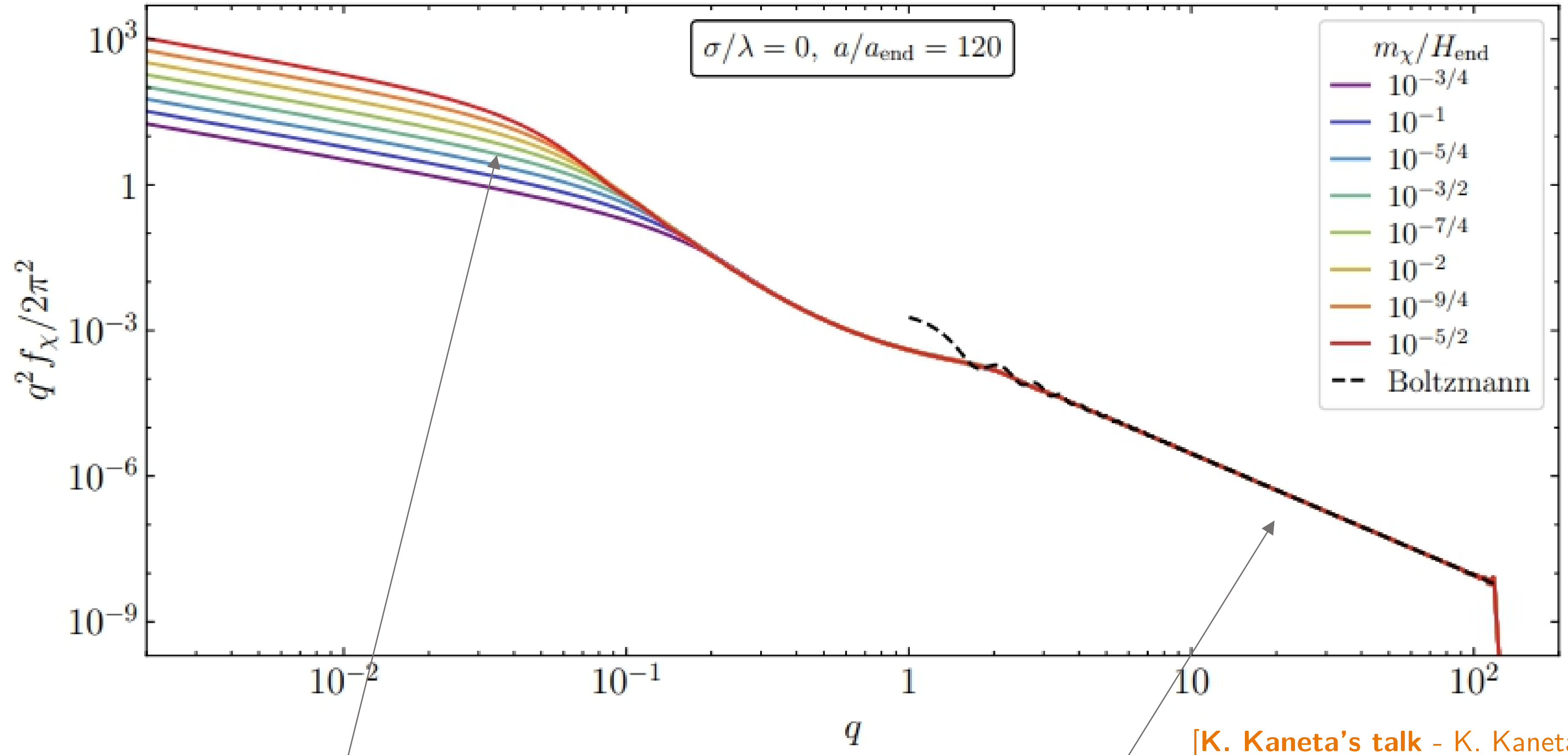
\+ **Bunch-Davies** initial conditions

$\Rightarrow$ $f_\chi(P,t) = n_{aP}(t)$

[**K. Kaneta's talk** - K. Kaneta, S. M. Lee, K. Oda - arXiv: 2206.10929]
[L. Kofman, A. Linde, A. Starobinsky - arXiv:9704452 – arXiv:9405187]
[M. A. G. Garcia, K. Kaneta, K. Olive, Y. Mambrini, S. Verner - arXiv: 2109.13280]

Gravitational production $\sigma/\lambda \ll 1$

[N. Herring, D. Boyanovsky & A. R. Zentner - arXiv:1912.10859]
[S. Ling & A. J. Long - arXiv:2101.11621]



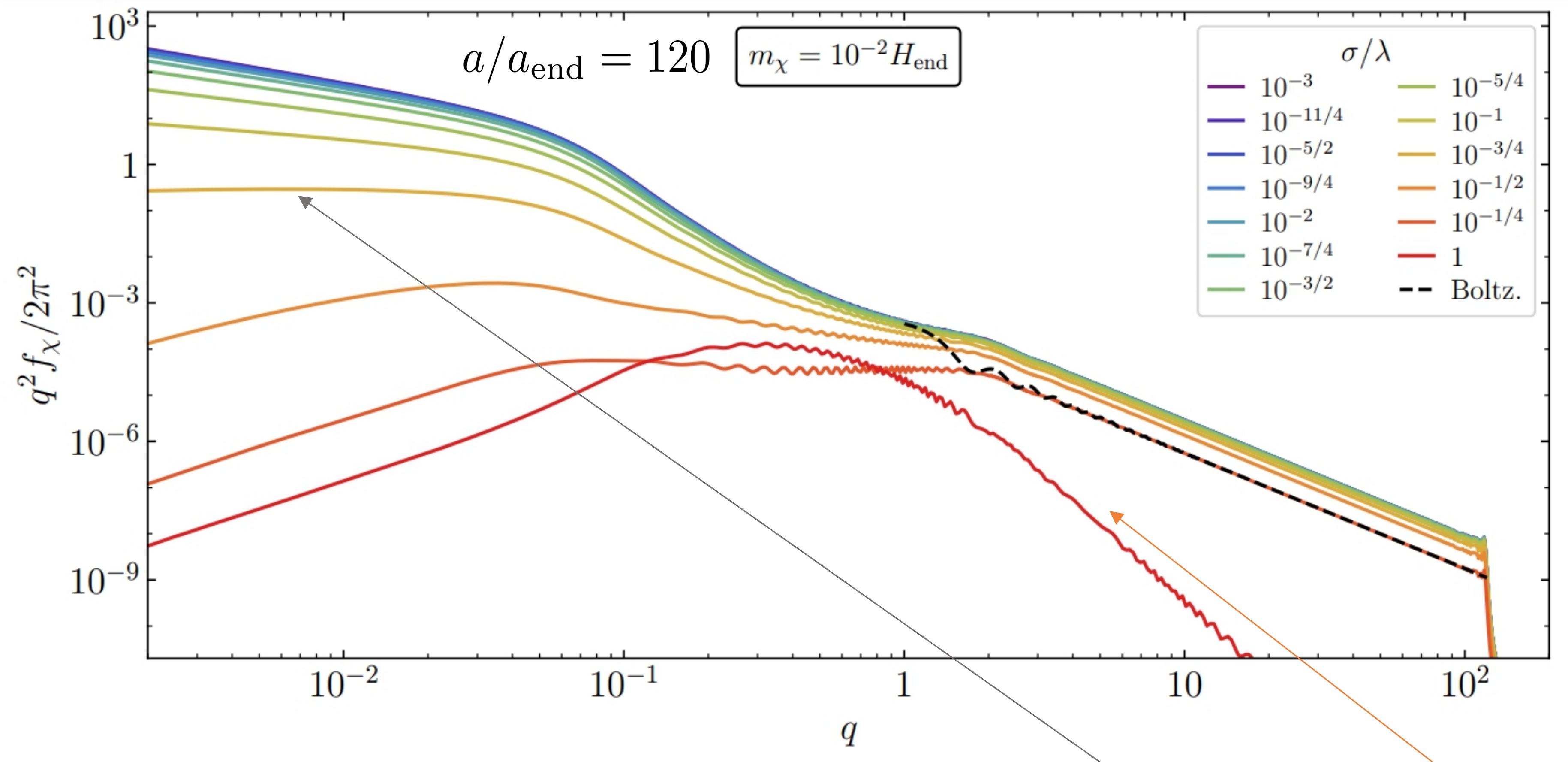
$$q \equiv \frac{P}{T_\star}\left(\frac{a}{a_0}\right)$$

$$T_\star \equiv m_\phi \left(\frac{a_{\rm end}}{a_0}\right)$$

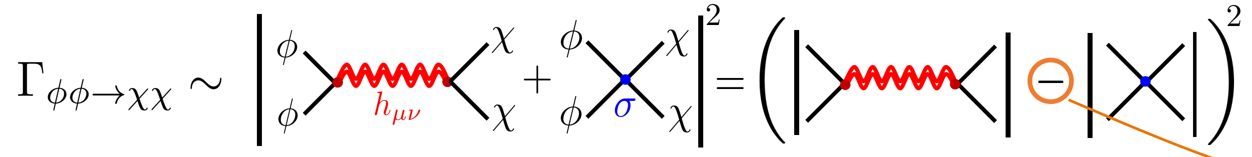
[**K. Kaneta's talk** - K. Kaneta, S. M. Lee, K. Oda - arXiv: 2206.10929]

- **Excitation of light field in quasi De-Sitter space:** increases as mass decreases
- Recover fluid **(Boltzmann) regime** at **large** q $\quad f_\chi \sim q^{-9/2}$

Gravitational interferences $0 < \sigma/\lambda < 1$



➡ **Effective mass** behaves a **IR modes regulator** and **suppresses density production!**

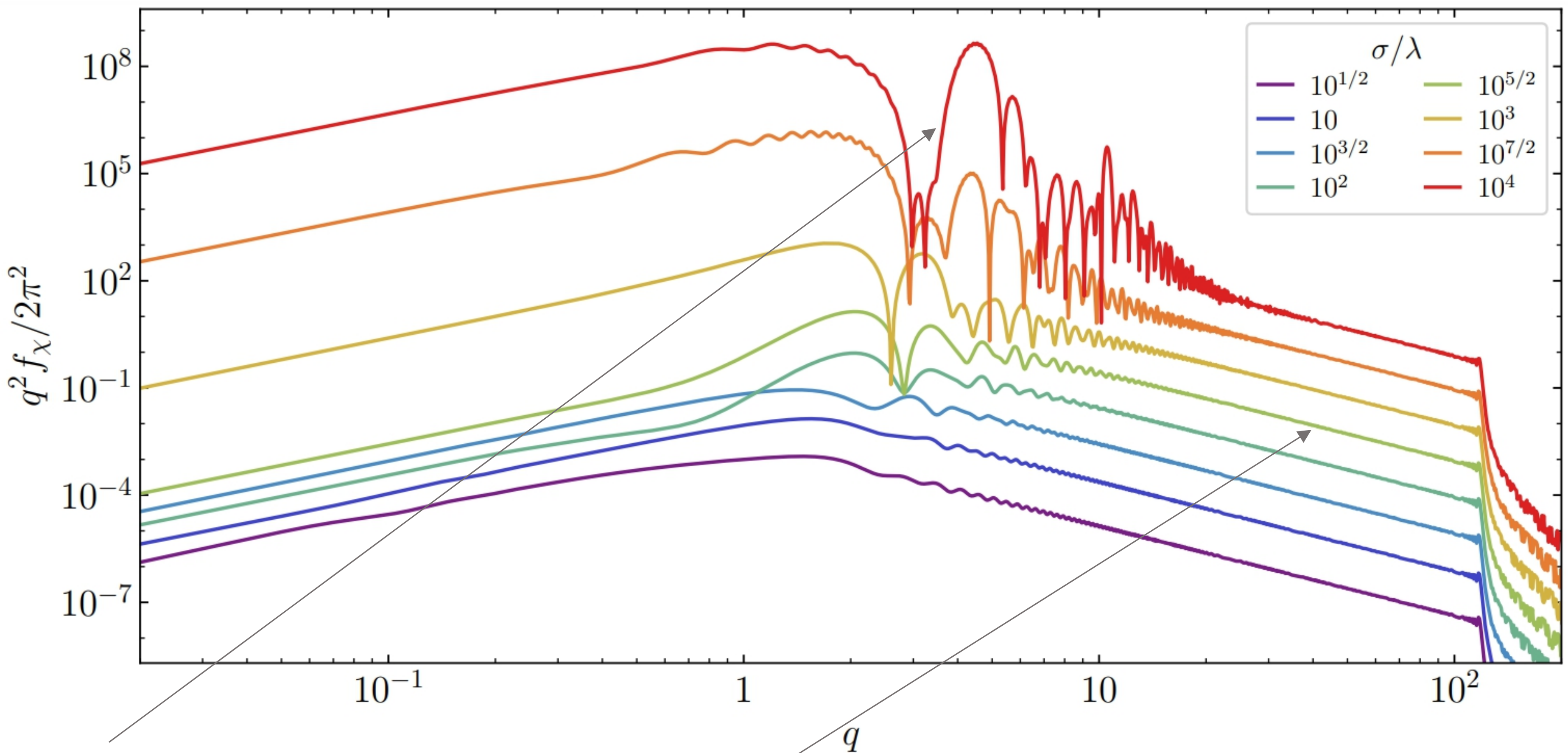


➡ $\Gamma_{\phi\phi\to\chi\chi} \sim \frac{1}{32\pi}\frac{\rho_\phi^2(t)}{m_\phi^3}\left[\sigma - \lambda\left(1 + \frac{m_\chi^2}{2m_\phi^2}\right)\right]^2$

Interferences!

Small couplings $1 < \sigma/\lambda < 10^4$

$a/a_{\mathrm{end}} = 120$



- **Broad** resonances at **large** coupling from instabilities! **Mode equation** $\Longleftrightarrow$ **Mathieu equation**
- Recover **Boltzmann regime** at **large** ***q*** $f_\chi \sim q^{-9/2}$

[L. Kofman, A. Linde, A. Starobinsky arXiv:9704452 – arXiv:9405187]

Large couplings $\sigma/\lambda > 10^4$

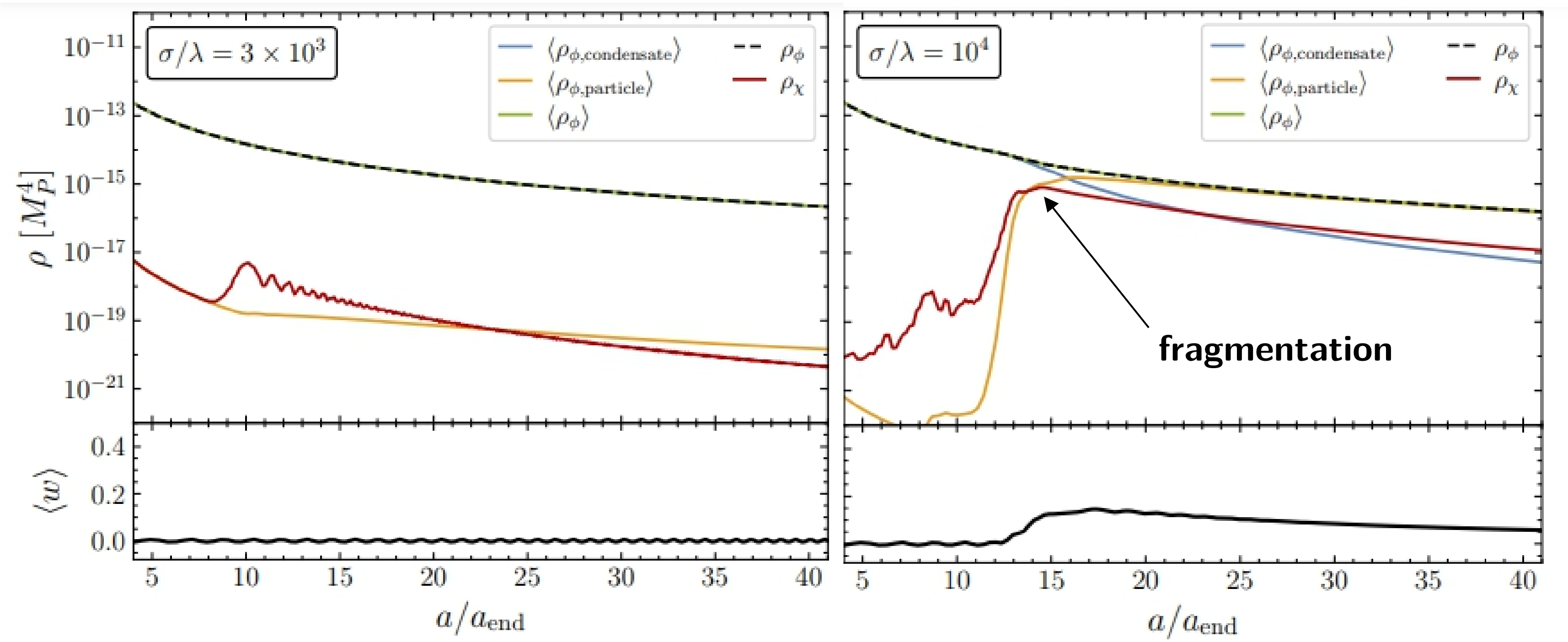
Copiously produced **dark matter disrupts inflaton condensate**

➡ **Hartree approximation** $\ddot{\phi} + 3H\dot{\phi} + V_\phi + \sigma\phi^2\langle\chi^2\rangle = 0$

➡ **Real space lattice simulations**

CosmoLattice
A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

[D. G. Figueroa, A. Florio, F. Torrenti, W. Valkenburg, arXiv:2102.01031]

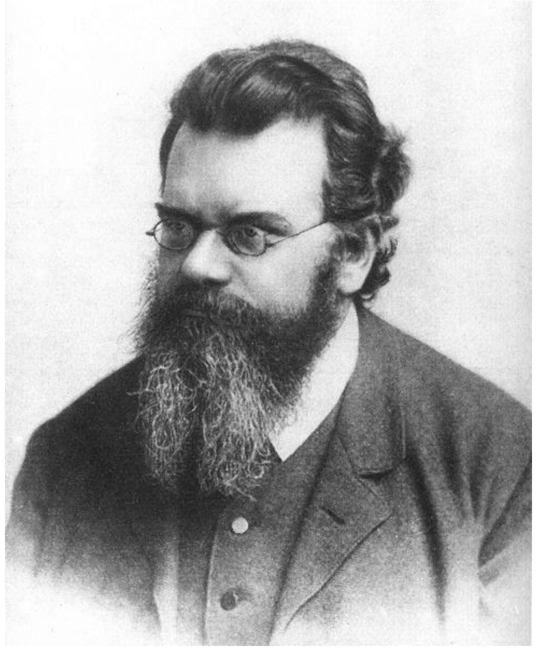


$$\rho_{\phi,\text{condensate}} \equiv \frac{1}{2}\dot{\bar{\phi}}^2 + V(\bar{\phi})$$

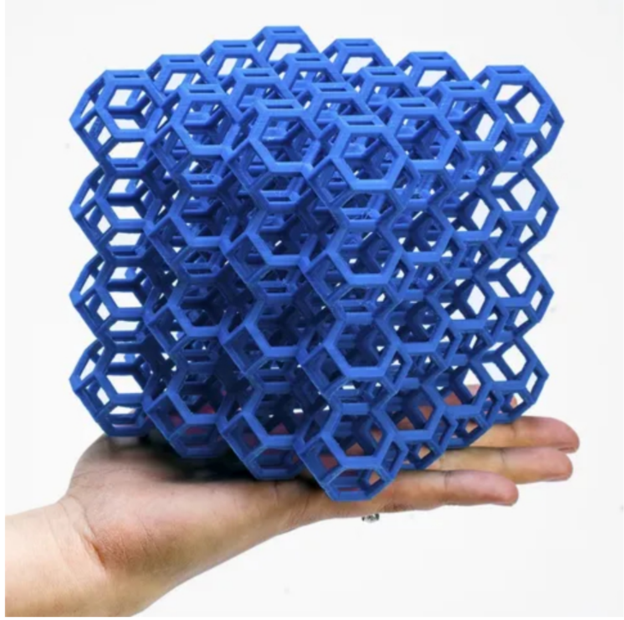
$$\rho_{\phi,\text{particle}} \equiv \rho_\phi - \rho_{\phi,\text{condensate}}$$

Large couplings $\sigma/\lambda > 10^4$

Hartree

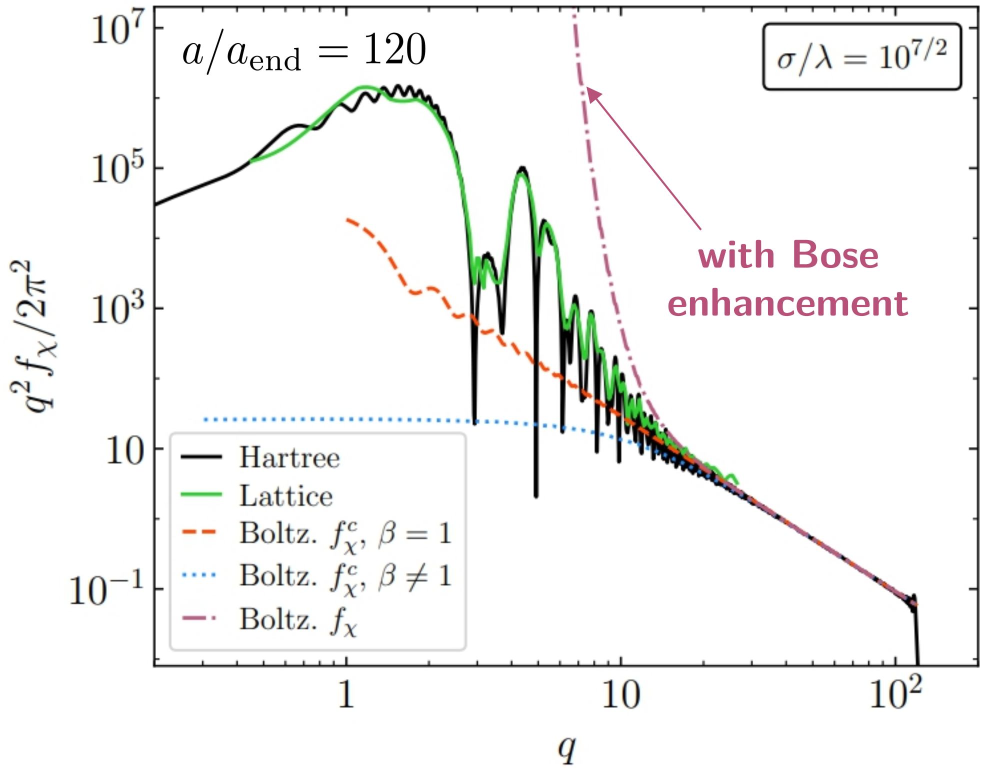


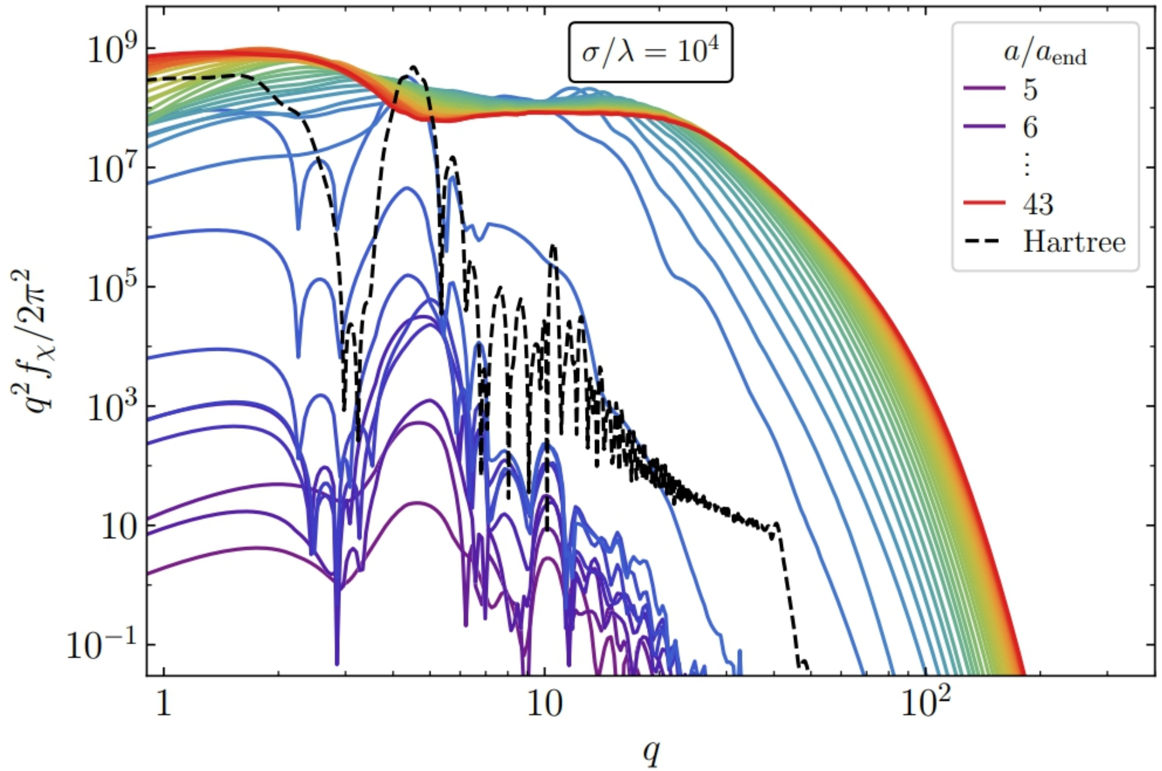
Boltzmann



or lattice?

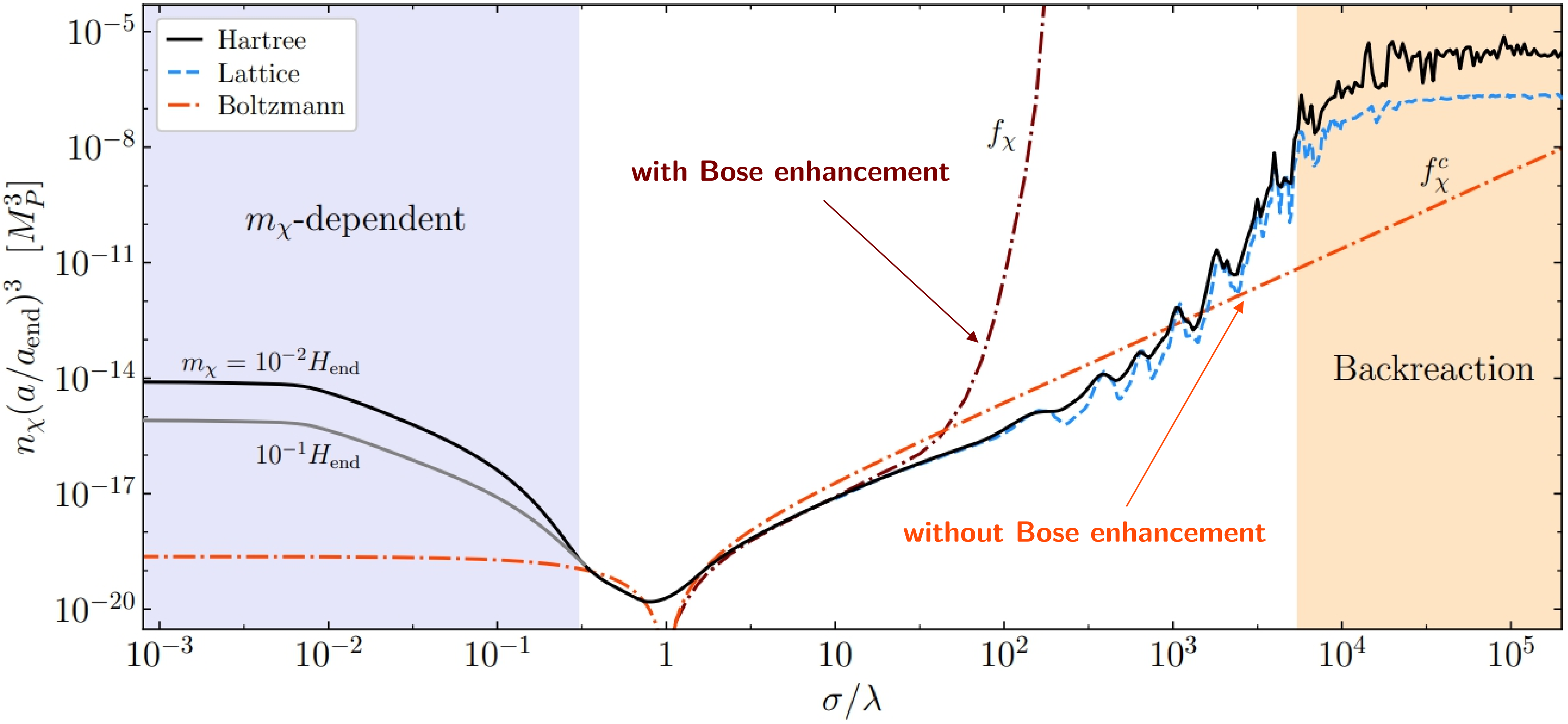
Large couplings $\sigma/\lambda > 10^4$





- ➡ **Hartree** and **lattice consistent**
- ➡ **Boltzmann fails** at capturing IR w±o Bose or kinematic β factors
- ➡ **Quasi-termal** distribution from lattice
- ➡ **Hartree fails** if backreaction too large

Scalar preheating phases

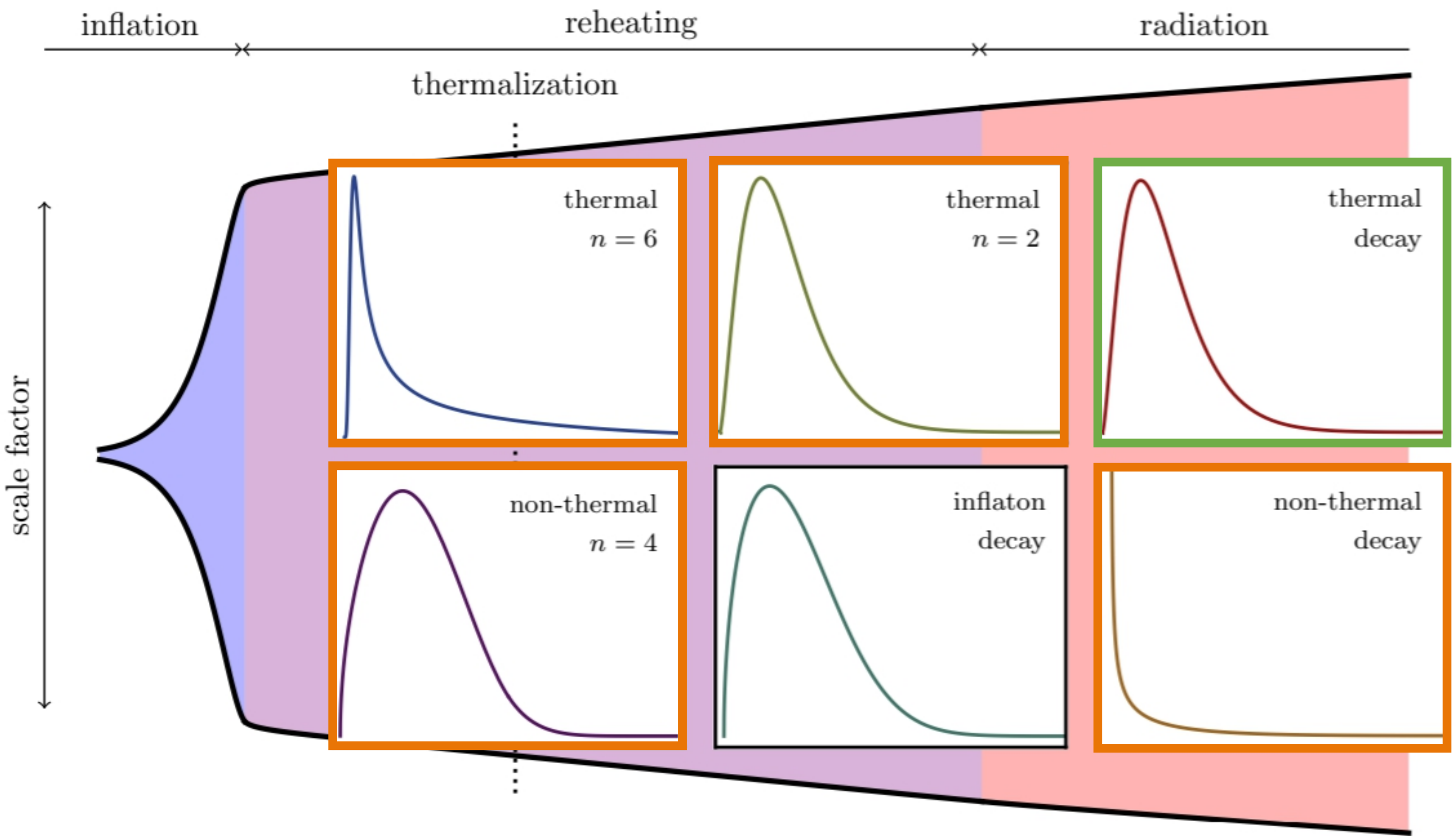


➡ Applicable to **generic light scalar**, not just **dark matter**

Cosmological signatures

DM phase space distribution from freeze-in scenarios

- **Previously, in Paris-Saclay Astroparticle Symposium '21**



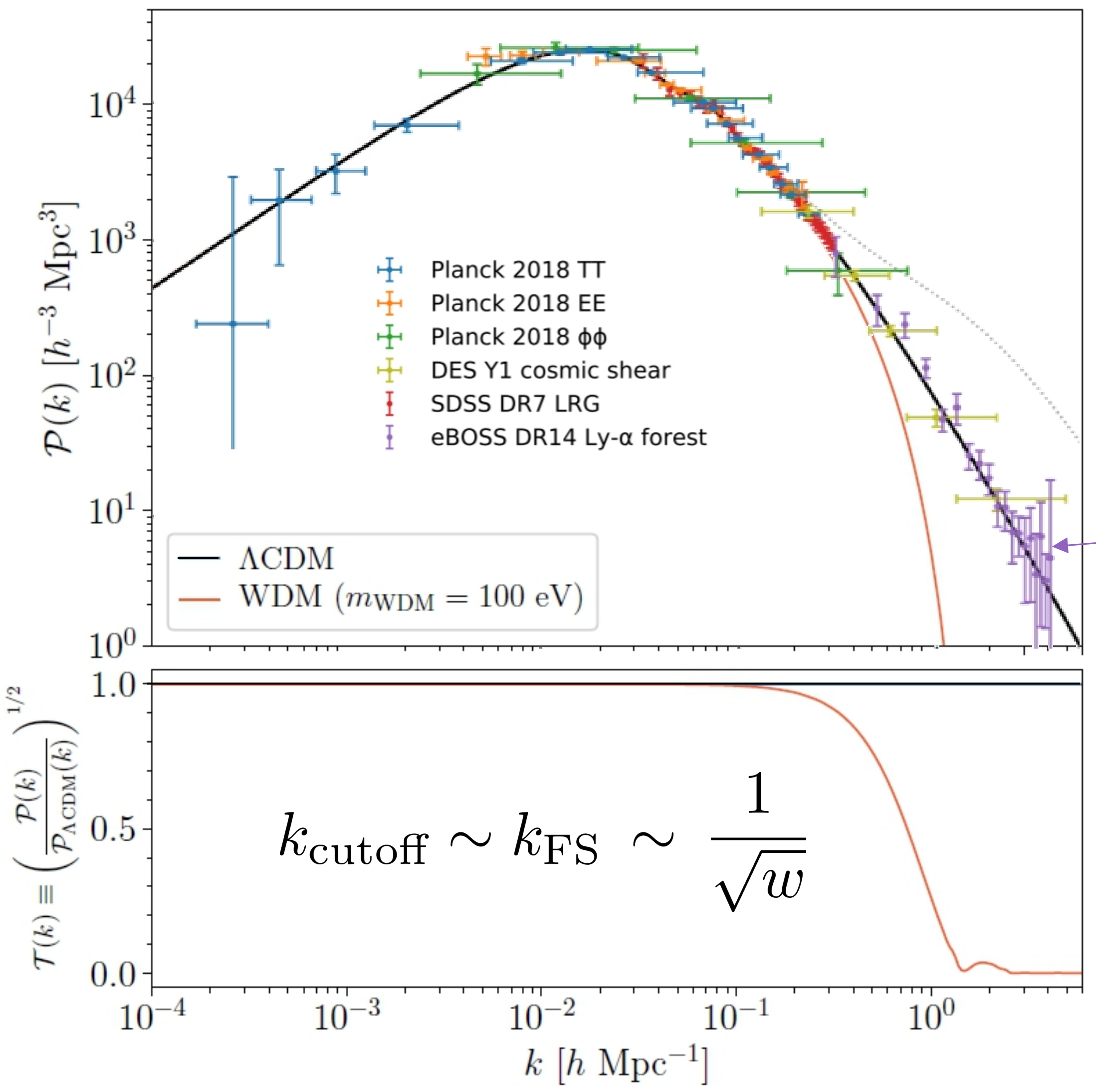
[G. Ballesteros, M A. G. Garcia, **MP**, JCAP 03 (2021) 101]

Freeze-in via scattering

Freeze-in via decay

- Previous analysis for cases with "**well-behaved**" distributions $f(q) \propto q^{\alpha} \exp\left(-\beta\, q^{\gamma}\right)$

Translate constraints on non-cold dark matter



[S. Chabanier, M. Millea, N. Palanque-Delabrouille, MNRAS 489 (2019) 2, 2247-2253]

- **Cutoff** determined by **equation-of-state** parameter

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_\star^2}{3m_\chi^2}\frac{\langle q^2\rangle}{a^2}$$

- **Find mass** that reproduces **cutoff** constrained by **Lyman-α**

[G. Ballesteros, M A. G. Garcia, **MP**, JCAP 03 (2021) 101]

$$m_\chi > 7.5\ \text{keV}\left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ \text{keV}}\right)^{4/3}\left(\frac{T_\star}{T_\gamma^0}\right)\sqrt{\langle q^2\rangle}$$

w – matching

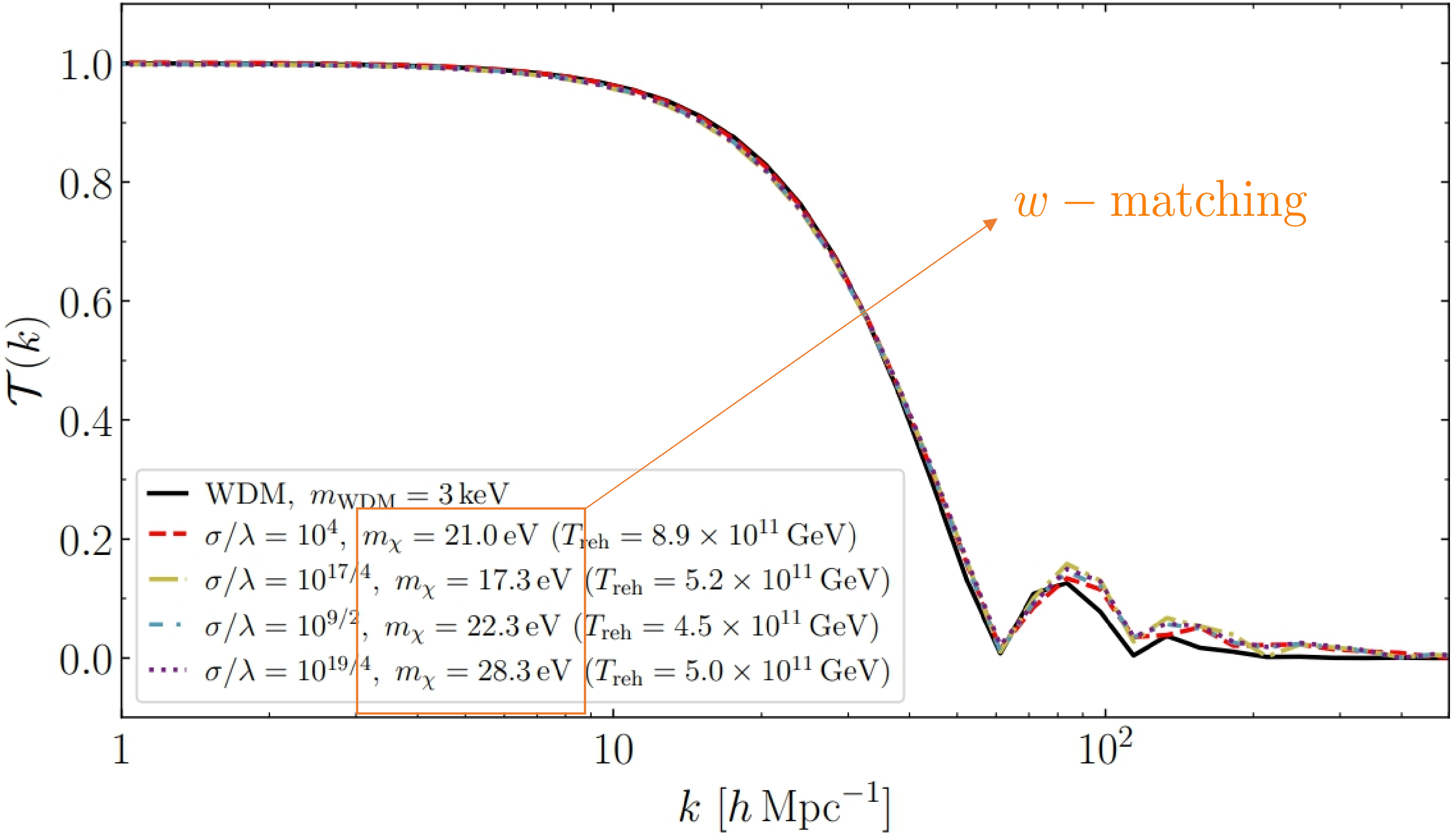
$$\langle q^2\rangle \equiv \frac{\int dq q^4 f(q)}{\int dq q^2 f(q)}$$

Photon temperature now

Constraints on preheating production $10^4 < \sigma/\lambda < 10^5$

- **Power spectrum** computed **numerically with CLASS**

[D. Blas, J. Lesgourgues & T. Tram JCAP 07 (2011) 034 - J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]



- **Excellent agreement** with $w-$matching for **all distributions**! Even the nasty ones!

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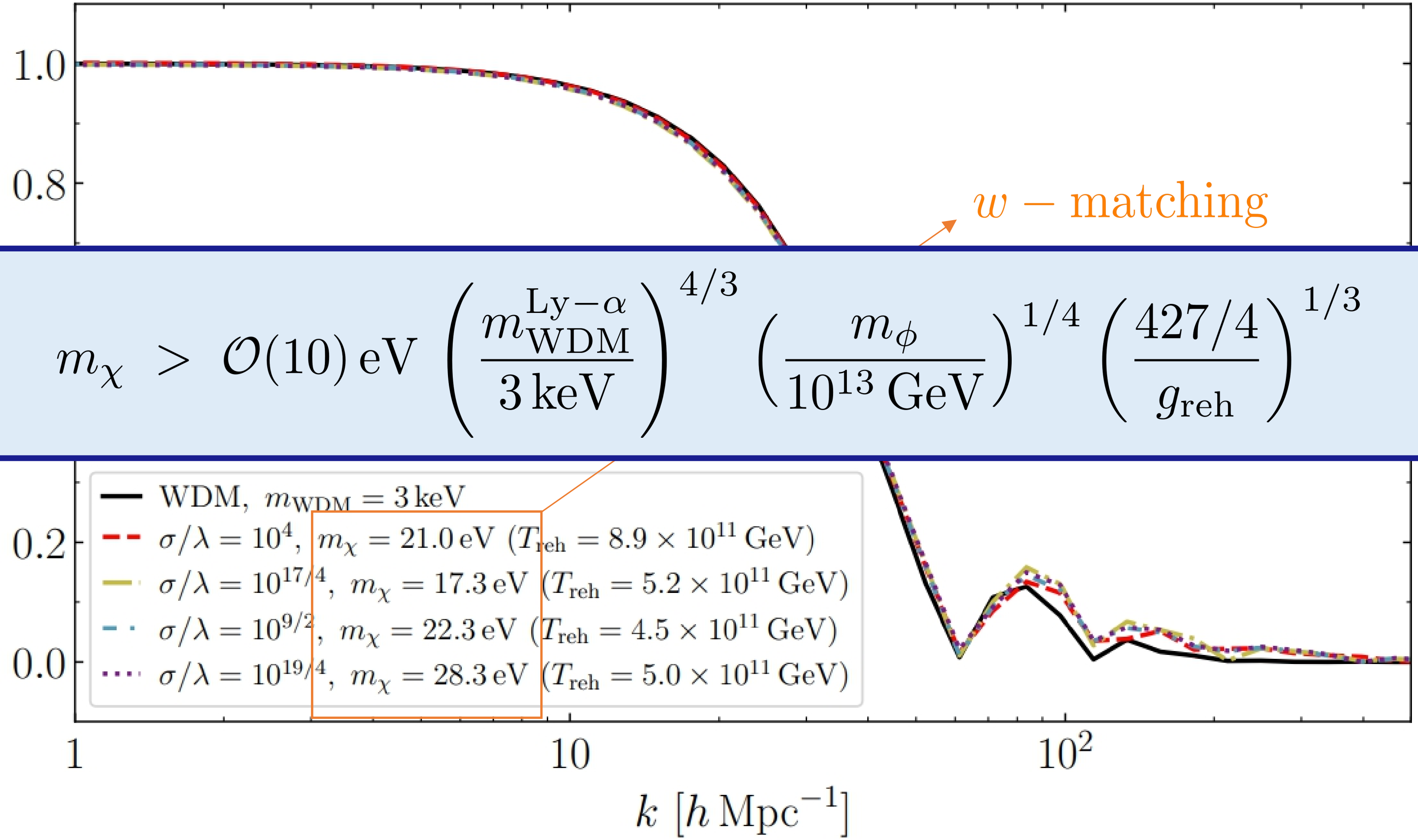
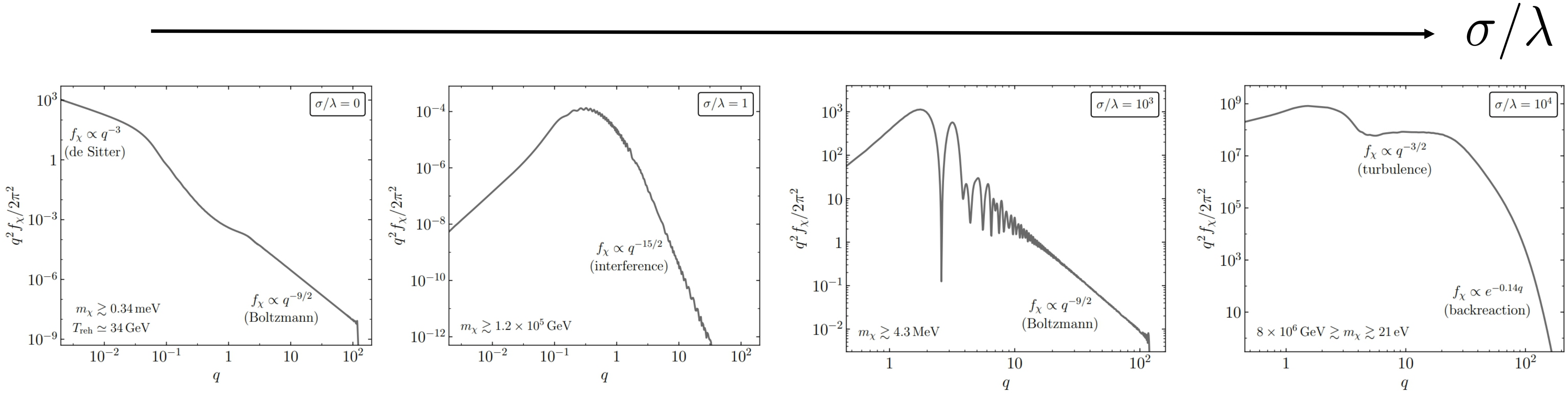


$$m_\chi \;>\; \mathcal{O}(10)\,\mathrm{eV}\left(\frac{m_{\mathrm{WDM}}^{\mathrm{Ly}-\alpha}}{3\,\mathrm{keV}}\right)^{4/3}\left(\frac{m_\phi}{10^{13}\,\mathrm{GeV}}\right)^{1/4}\left(\frac{427/4}{g_{\mathrm{reh}}}\right)^{1/3}$$

- **Excellent agreement** with $w-\mathrm{matching}$ for **all distributions**! Even the nasty ones!

Take home message

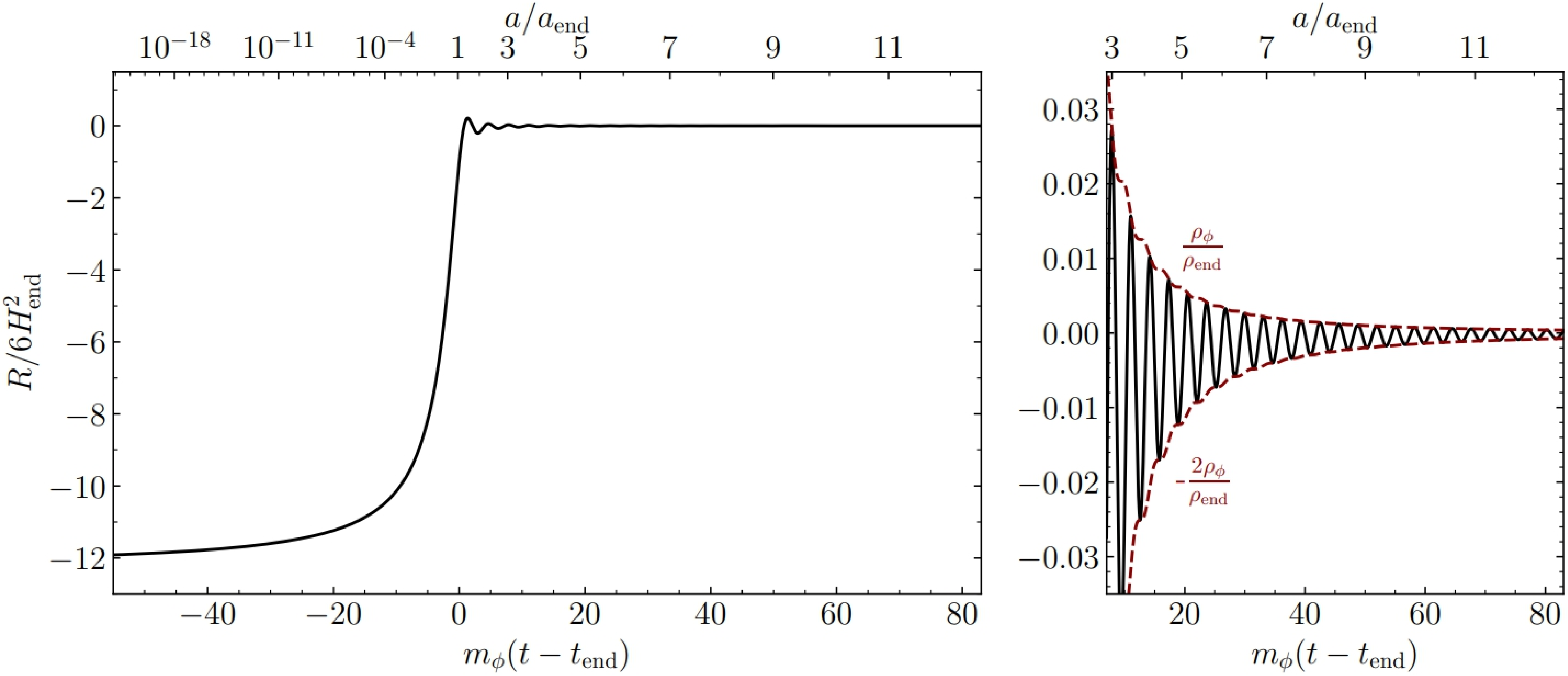


- Characterization of **scalar preheating** over **large spectrum** of **couplings**
- Check **validity** of **several approaches**
- **Dark matter produced from preheating constrained by Lyman-** α

Thank you for your attention

Back-up slides

Gravitational contribution to the effective mass

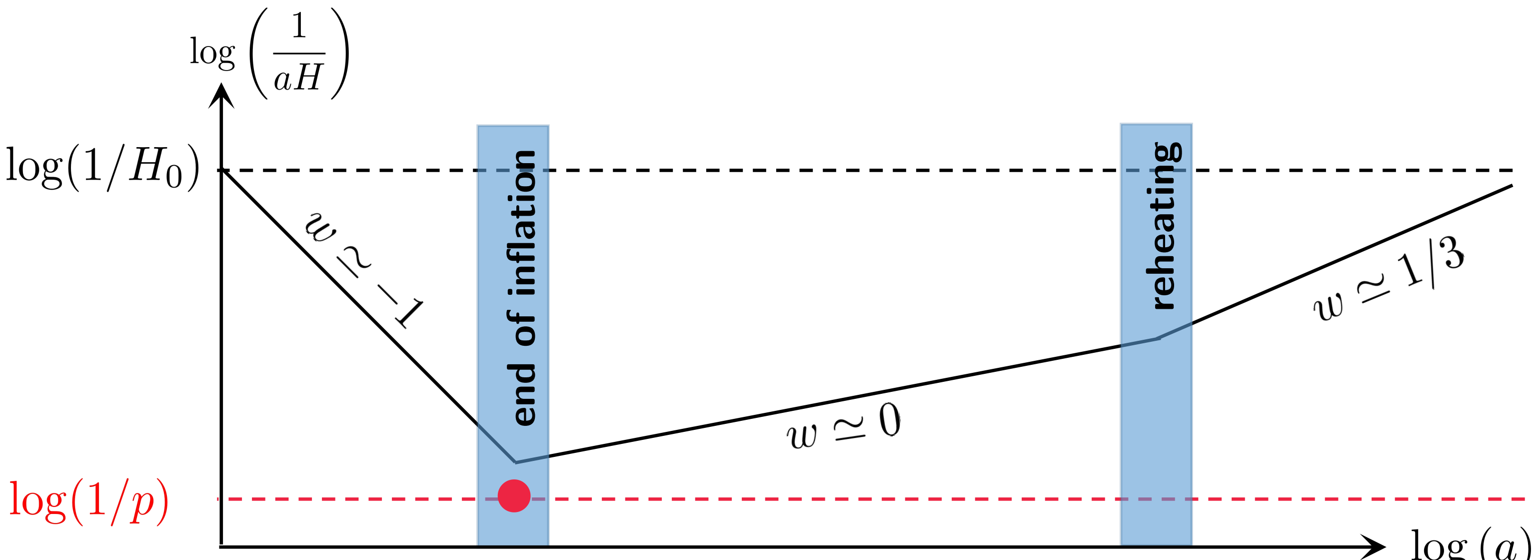


Scalar preheating: the field picture

- Set **initial condition** for **mode functions** in **Bunch-Davies** vacuum $X_p(\tau_0) = \frac{1}{\sqrt{2\omega_p}}$ $X'_p(\tau_0) = -\frac{i\omega_p}{\sqrt{2\omega_p}}$

$$\omega_p^2(t_{\text{end}}) = p^2 + a_{\text{end}}^2 \left(m_\chi^2 + \sigma\phi^2 - H_{\text{end}}^2\right)$$

- **For small physical scales** **modes always inside horizon** $\omega_p^2 > 0$ ● $\tau_0 = \tau_{\text{end}}$



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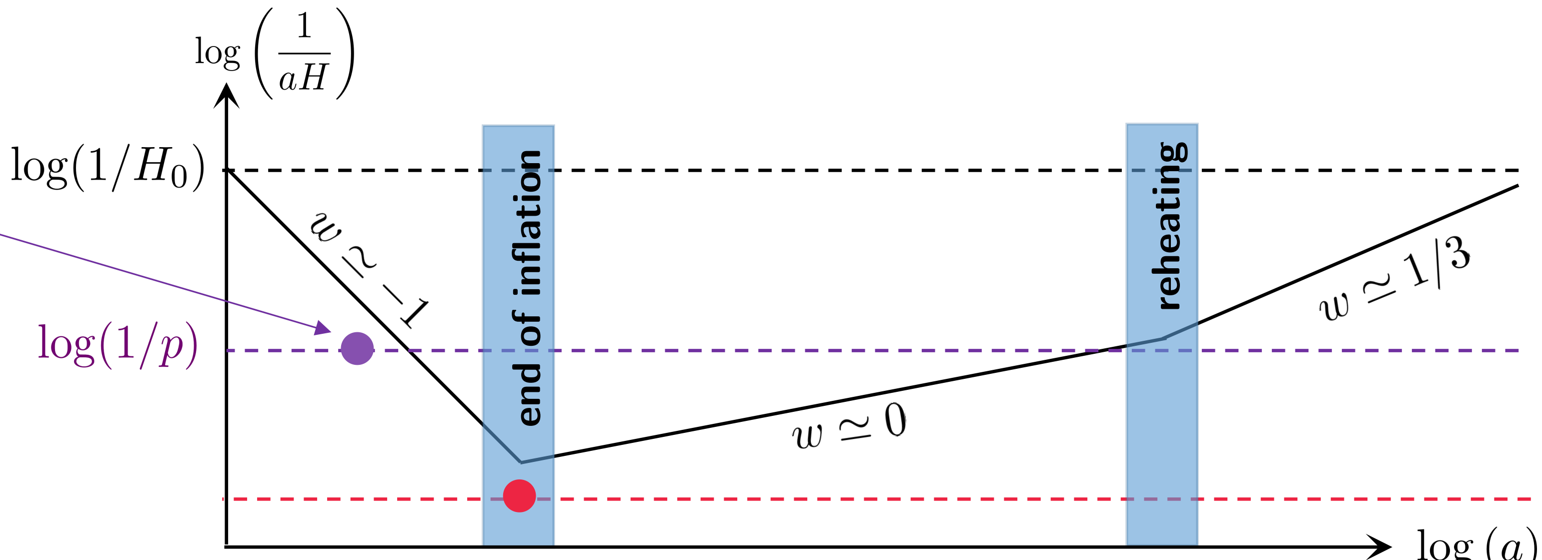
$$\omega_p^2(t_{\rm end}) = p^2 + a_{\rm end}^2\left(m_\chi^2 + \sigma\phi^2 - H_{\rm end}^2\right)$$

- **For small physical scales** **modes always inside horizon** $\omega_p^2 > 0$ ● $\tau_0 = \tau_{\rm end}$
- **For** $m_\chi^2 + \sigma\phi^2 < H_{\rm end}^2$ **modes** with $p^2 < a_{\rm end}^2\left(H_{\rm end}^2 - m_\chi^2 - \sigma\phi^2\right)$ $\omega_p^2(t_{\rm end}) < 0$

● $\tau_0 < \tau_{\rm end}$

$$p^2 \gg a(\tau_0)^2 H(\tau_0)^2$$

➡ **Particle production!**
Red-tilt of the spectrum!



Reheating

- In **fluid picture**: transition to radiation era via **dissipation** term $\equiv \Gamma_\phi \rho_\phi (1 + w_\phi)$

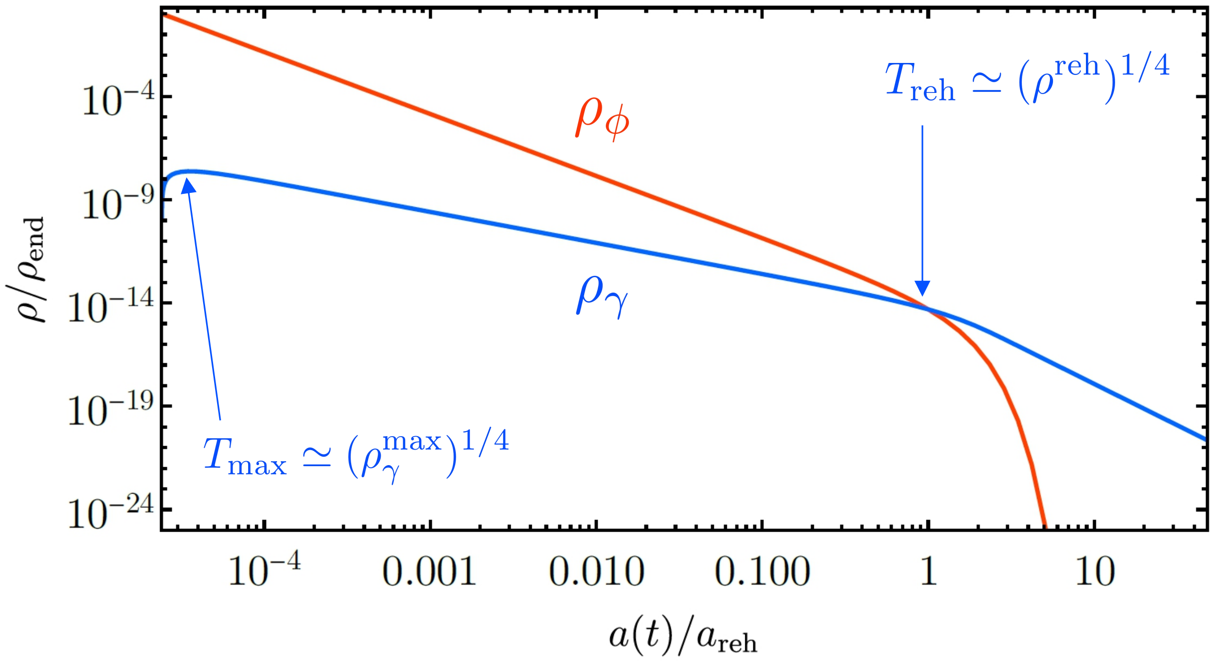
$$T^{\mu\nu}_{\rm tot} = T^{\mu\nu}_\phi + T^{\mu\nu}_\gamma \qquad \nabla_\mu T^{\mu\nu}_{\rm tot} = 0 \qquad \nabla_\mu T^{\mu\nu}_\phi = -\nabla_\mu T^{\mu\nu}_\gamma$$

- System of **Friedmann-Boltzmann equations**

$$\begin{aligned} \dot\rho_\phi + 3H(1+w_\phi)\rho_\phi &= -\Gamma_\phi \rho_\phi (1+w_\phi) \\ \dot\rho_\gamma + 4H\rho_\gamma &= \Gamma_\phi \rho_\phi (1+w_\phi) \\ H^2 &= \frac{1}{3M_{\rm Pl}}(\rho_\phi + \rho_\gamma) \end{aligned}$$

$$\Rightarrow \quad \rho_\phi(t) \simeq \rho_{\rm end} \left(\frac{a}{a_{\rm end}}\right)^{-3} e^{-\Gamma_\phi (t - t_{\rm end})}$$

$\Gamma_\phi \ll m_\phi$

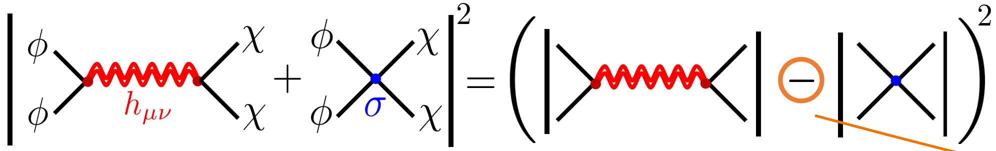


Scalar preheating: the fluid approach

- Treat **inflaton** as **coherent oscillating condensate**

$$\phi(t) \simeq \phi_0(t) \sum_{n=-\infty}^{\infty} \mathcal{P}_n e^{-in\omega_\phi t} \qquad E_n = n\omega_\phi \quad \Rightarrow \quad \Gamma_{\phi\phi\to\chi\chi} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^{\infty} E_n \beta_n |\mathcal{M}_n|^2$$

$|\mathcal{M}|^2_{\phi\phi\to\chi\chi} =$



$$\Rightarrow \quad \Gamma_{\phi\phi\to\chi\chi} = \frac{1}{32\pi}\frac{\rho_\phi^2(t)}{m_\phi^3}\left[\sigma - \lambda\left(1 + \frac{m_\chi^2}{2m_\phi^2}\right)\right]^2 \beta_2$$

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + 2\frac{h_{\mu\nu}}{M_P}$$

- **Approximate solution for** $\beta \simeq 1,\ t_{\rm end} < t < t_{\rm reh}$

$$f_\chi(q,t) \sim q^{-9/2}\theta(q-1)\theta\left(\frac{a}{a_{\rm end}} - q\right)$$

$$q \equiv \frac{P}{T_\star}\left(\frac{a}{a_0}\right) \qquad T_\star \equiv m_\phi\left(\frac{a_{\rm end}}{a_0}\right)$$

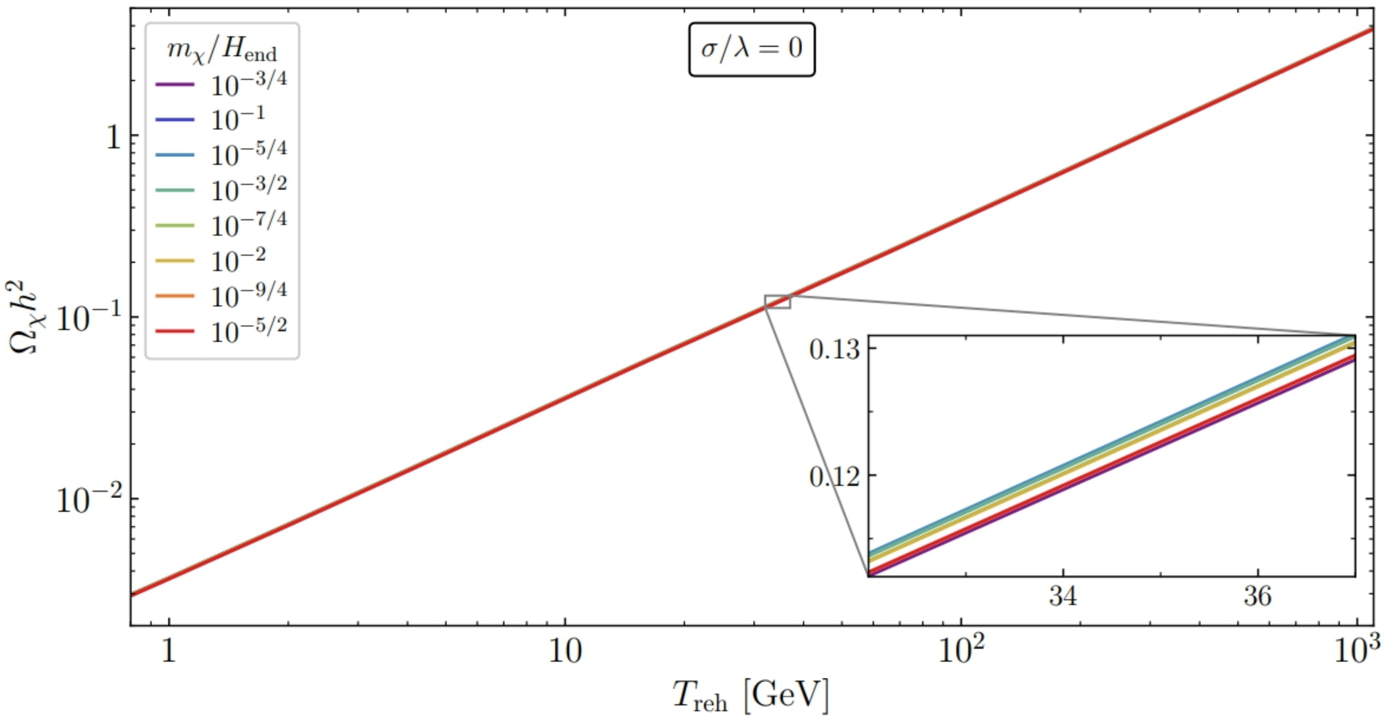
- $n_\chi \left(\frac{a}{a_{\rm end}}\right)^3 = \frac{m_\phi^3}{2\pi^2}\int {\rm d}q\, q^2 f_\chi(q,t)$ **: time independent** when DM production **stops**

Gravitational production $\sigma/\lambda \ll 1$

[N. Herring, D. Boyanovsky & A. R. Zentner - arXiv:1912.10859]
[S. Ling and A. J. Long - arXiv:2101.11621]

➡ **Take IR cutoff** as **present horizon**

$$q_{\rm IR} = q_0 = \left(\frac{90}{\pi^2}\right)^{1/4} \left(\frac{11}{43}\right)^{1/3} g_{\rm reh}^{1/12} \left(\frac{H_{\rm end} M_P}{m_\phi^2}\right)^{1/2} \frac{H_0}{T_0} \left(\frac{a_{\rm reh}}{a_{\rm end}}\right)^{1/4} \quad \Leftrightarrow \quad p_0 = a_0 H_0$$



For small DM mass

For $T_{\rm reh} > 30$ GeV

Overclose the universe!

Backreaction

Backreaction important at **large coupling** $\sigma/\lambda > 10^4$

- **Hartree** approximation $\ddot{\phi} + 3H\dot{\phi} + V_\phi + \sigma\phi^2\langle\chi^2\rangle = 0$
  - ➡ To simulate **energy transferred** back to inflaton
  - ➡ Ok but neglects **rescattering** and **disruption** of the **condensate**
- **Lattice simulations** in real space
  - ➡ Occupation numbers are **large**, **classical approach** justified
  - ➡ Cannot be **used** for **smaller couplings**
  - ➡ Does not **account** for **metric perturbations**

CosmoLattice

A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

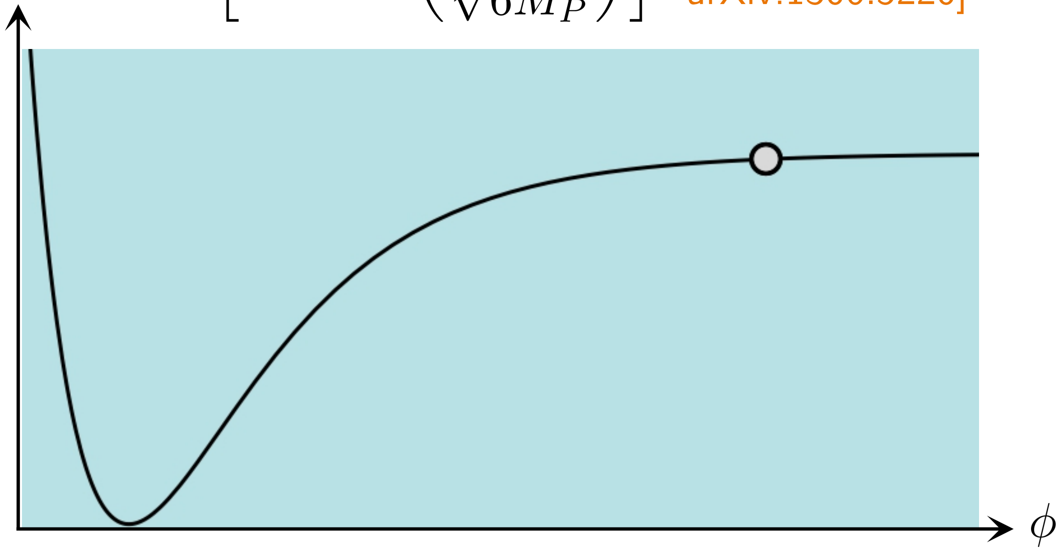
[D. G. Figueroa, A. Florio, F. Torrenti, W. Valkenburg, arXiv:2102.01031]

End of inflation

For concreteness, **consider**

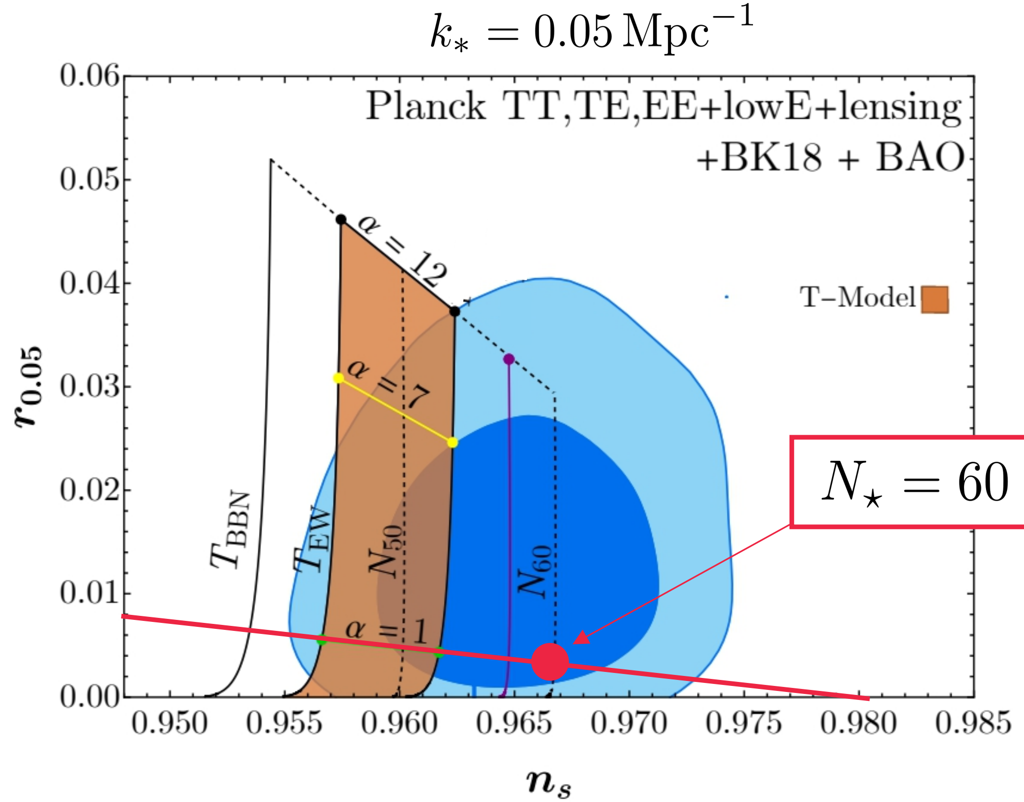
$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0$$

$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^2$$ [Kallosh & Linde arXiv:1306.5220]



$$\lambda \simeq \frac{18\pi^2 A_s(k_\star)}{6N_*^2} \sim 10^{-11}$$

[J. Ellis, M. A. G. Garcia, D. V. Nanopoulos, K. A. Olive, and S. Verner - arXiv:2112.04466]



$$r \simeq 16\epsilon_\star \simeq \frac{12}{N_\star^2} \qquad n_s \simeq 1 - 6\epsilon_\star + 2\eta_\star \simeq 1 - \frac{2}{N_\star}$$

$$A_s(k_\star) \simeq 2.1 \times 10^{-9}$$ [Planck 18']

ϕ **inflaton**

End of inflation

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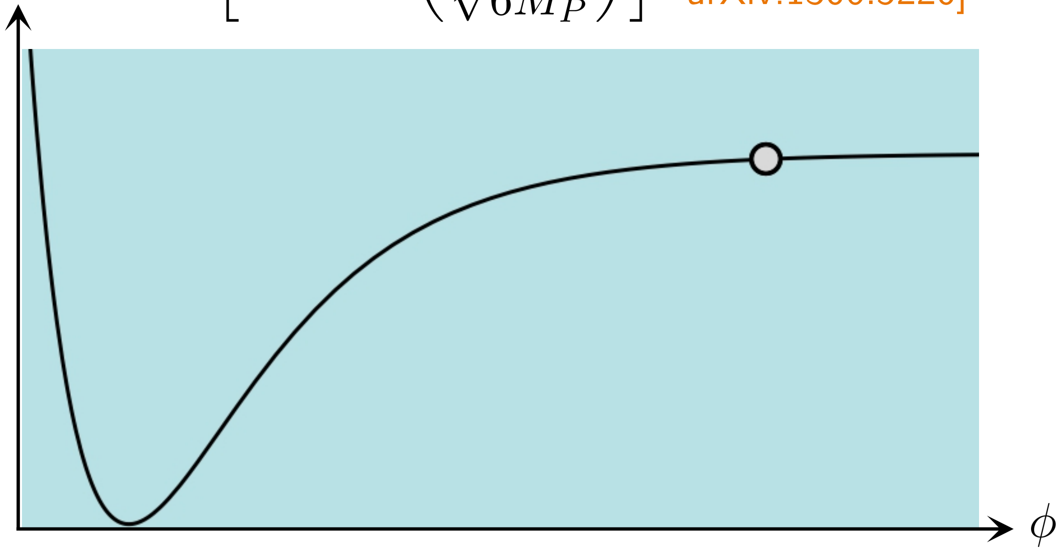
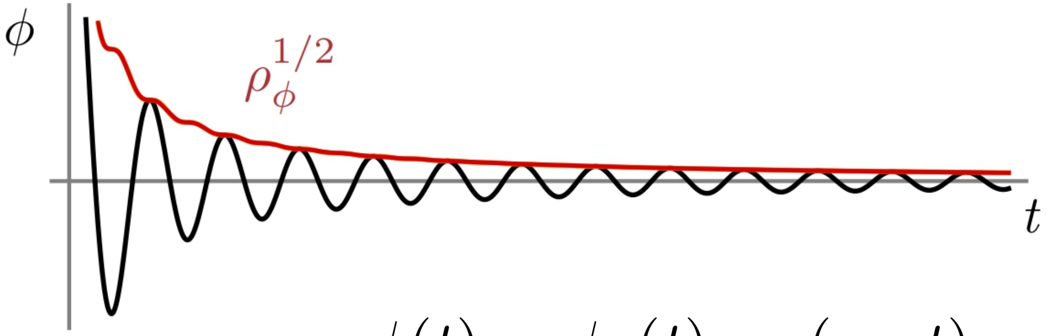


$$\lambda \simeq \frac{18\pi^2 A_s(k_\star)}{6N_*^2} \sim 10^{-11}$$

Close to the minimum

$$V(\phi) \simeq \frac{1}{2}m_\phi^2\phi^2 = \lambda\phi^2 M_P^2 \quad (\phi \ll M_P)$$



$$\phi(t) \simeq \phi_0(t)\cos(m_\phi t)$$

$$\langle P_\phi \rangle = \frac{1}{2}\langle\dot{\phi}^2\rangle - \langle V(\phi)\rangle \simeq 0$$

$$\langle \rho_\phi \rangle = \frac{1}{2}\langle\dot{\phi}^2\rangle + \langle V(\phi)\rangle \simeq V(\phi_0)$$

$$\langle w_\phi \rangle \simeq 0$$

Perturbative reheating $\rho_\phi(t) \simeq \rho_{\mathrm{end}}\left(\frac{a}{a_{\mathrm{end}}}\right)^{-3} e^{-\Gamma_\phi(t-t_{\mathrm{end}})}$ $\quad \Gamma_\phi \ll m_\phi$

ϕ **: inflaton**

Scalar production: resonance effects

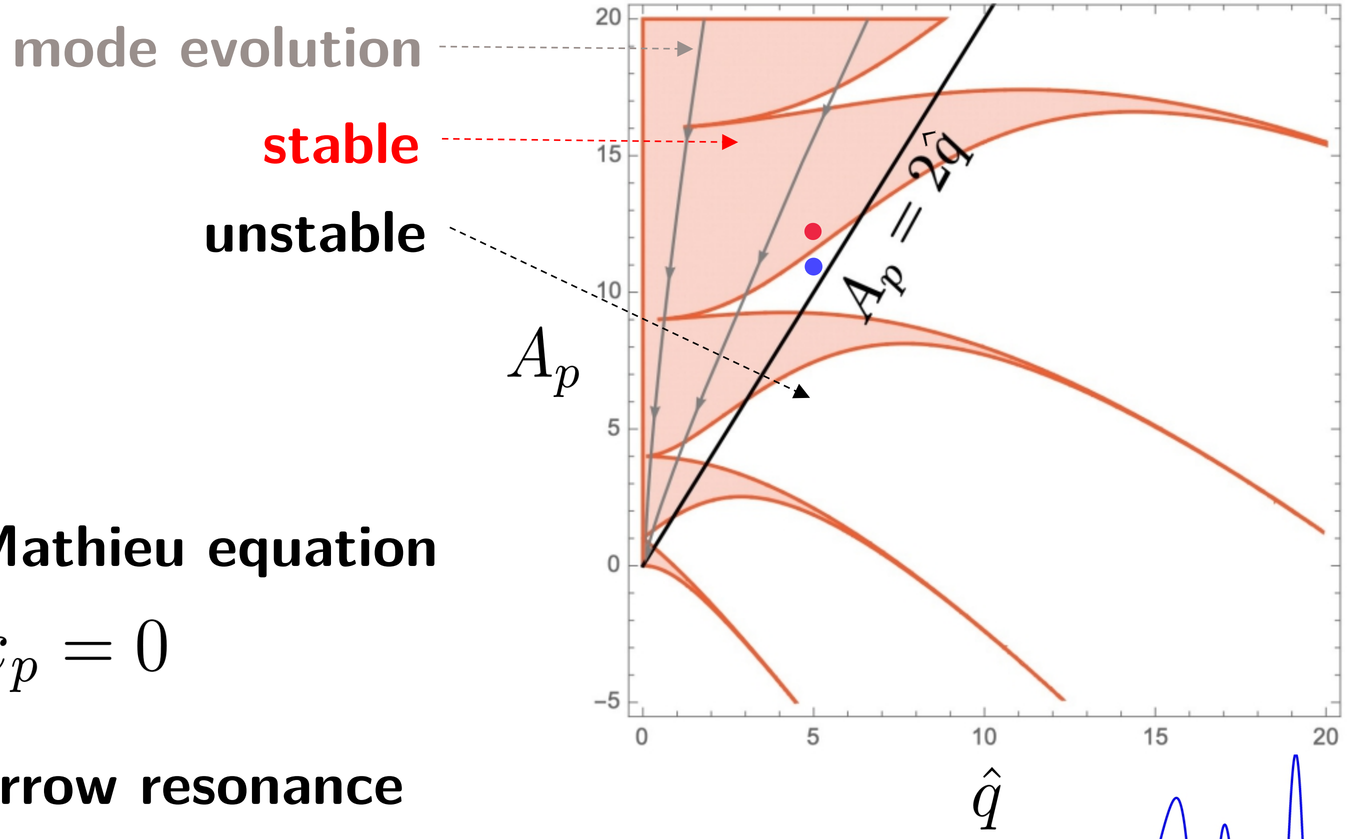
[M. A. G. Garcia, K. Kaneta, Y. Mambrini, K. A. Olive, S. Verner, arXiv:2109.13280]

- **Changing variables**

$$x_p \equiv a^{1/2} X_p \qquad z \equiv m_\phi t + \frac{\pi}{2}$$

$$A_p = \frac{p^2}{m_\phi^2 a^2} + 2\hat{q}$$
("energy²")

$$\hat{q} = \frac{\sigma \phi_0^2}{4 m_\phi^2}$$
("mass²")

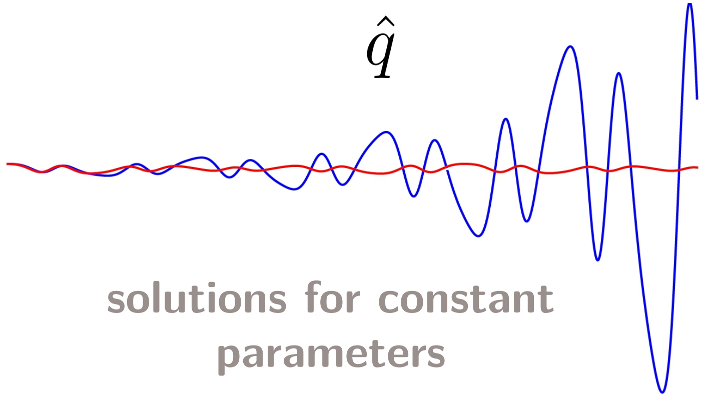


- **Equation** for **mode functions** $\Leftrightarrow$ **Mathieu equation**

$$\ddot{x}_p + (A_p - 2\hat{q}\cos(2z))x_p = 0$$

- For **large** momentum, i.e. $A_p \gg \hat{q}$ **narrow resonance**
  $A_p \sim 1 \pm \hat{q}$ ➡ **stable**

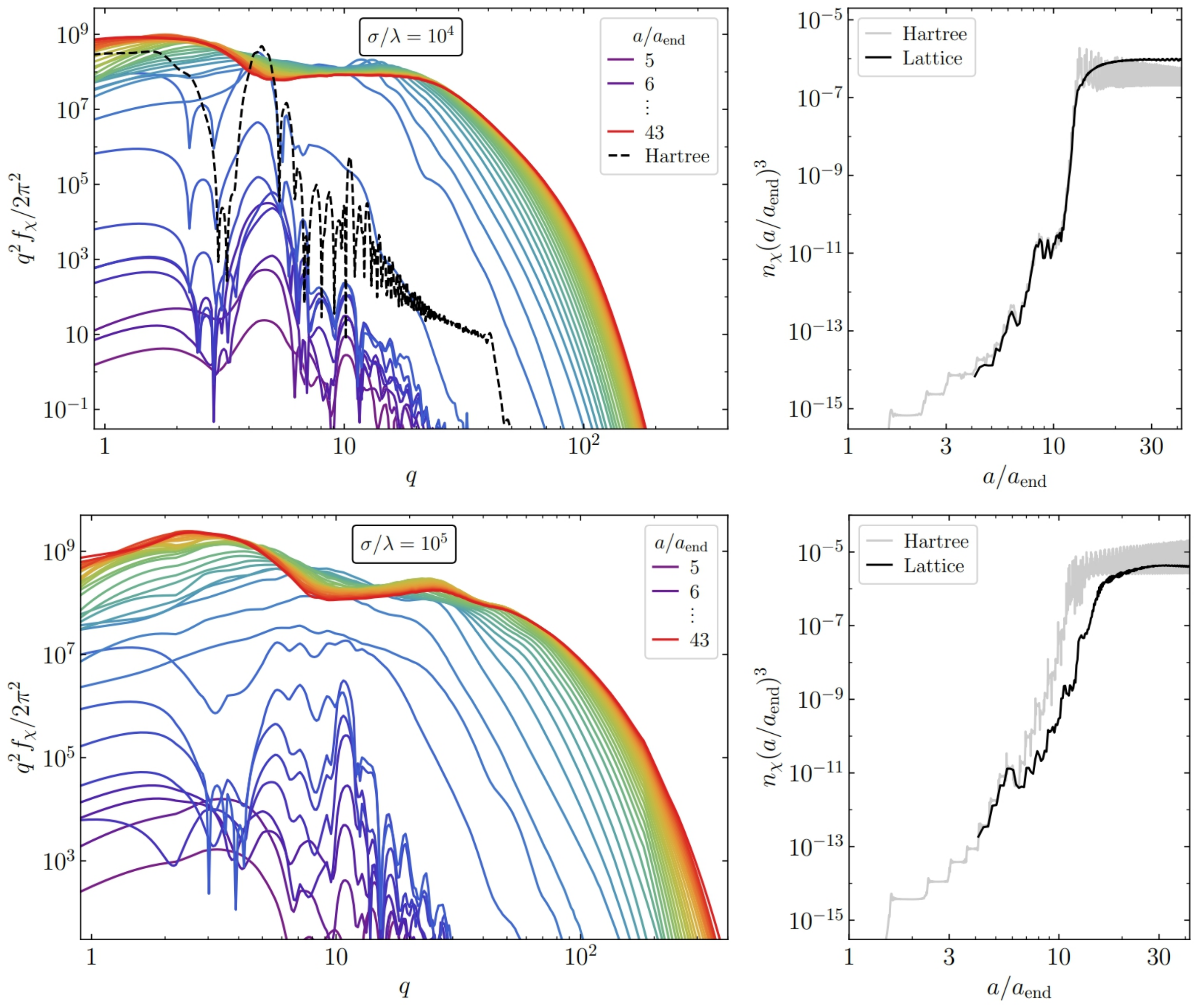
- Close to $A_p \sim 2\hat{q}$, **efficient** particle production
  as **crossing** several **instability bands over** $\Delta t \sim \hat{q}/H$



solutions for constant parameters

[L. Kofman, A. Linde, A. Starobinsky - arXiv:9704452]

Lattice simulations: phase space distribution

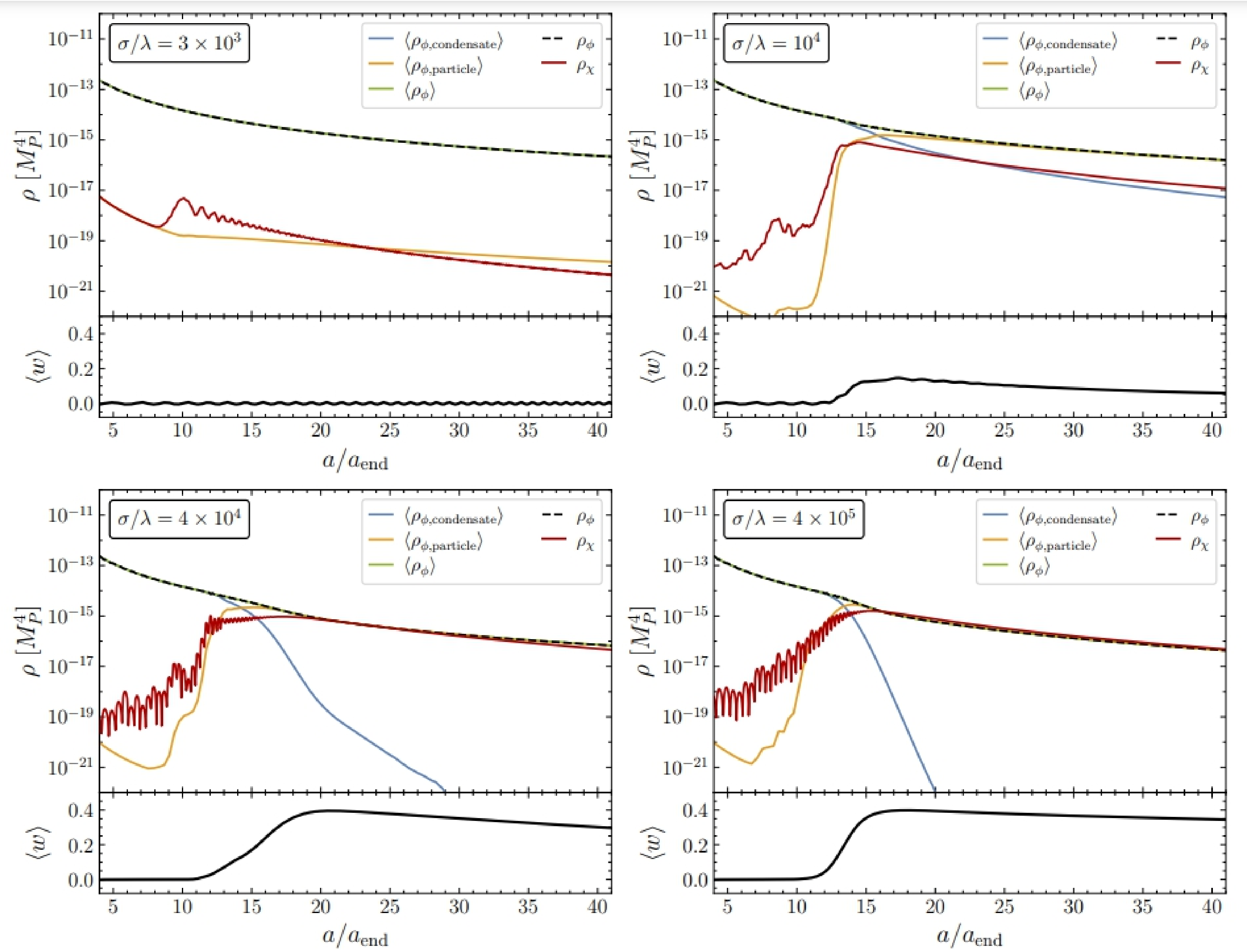


CosmoLattice

A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

[D. G. Figueroa, A. Florio, F. Torrenti, W. Valkenburg, arXiv:2102.01031]

Lattice simulations: energy density and fragmentation



$$\rho_{\phi,\text{condensate}} \equiv \frac{1}{2}\dot{\bar{\phi}}^2 + V(\bar{\phi})$$

$$\rho_{\phi,\text{particle}} \equiv \rho_\phi - \rho_{\phi,\text{condensate}}$$

$$\langle w(a) \rangle \equiv \frac{1}{\Delta a} \int_a^{a+\Delta a} w(\tilde{a})\, d\tilde{a}$$

CosmoLattice

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Constraints on parameter space