Neutrino Oscillation: an Avenue to Probe the Universe

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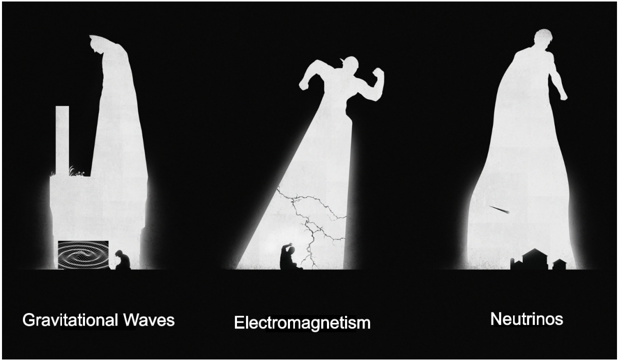
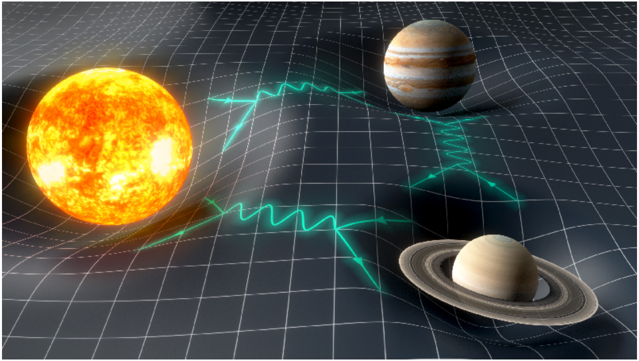
Multimessenger Alliance



Source:downloadwallpapers.net

Gravity at the Quantum Realm



Source: APS

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$$P_{\nu_e \to \nu_\mu} = |\langle \nu_e(t) | \nu_\mu(t) \rangle|^2 = ... = \sin^2(2\theta) \sin^2 \left(\frac{m_2^2 - m_1^2}{2E} L \right)$$

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Dirac Equation in Curved Spacetime

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Part I:General Formalism

- $S = S_{\text{gravity}} + S_{\text{scalar}} + S_{\text{spinor}} + S_{\text{interaction}}$

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Oscillation probability as a function of redshift. Gravity and φ alter neutrino oscillations.

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$$\frac{dp^\alpha}{d\tau} + \frac{1}{m}\Gamma^\alpha_{\ \beta\gamma}p^\beta p^\gamma = mf^\alpha \propto \hbar p^\alpha R_{\mu\beta\gamma\delta}$$

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- In flat FLRW:

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- Change in matter-radiation equality:

$$1+z_{eq} = \frac{\Omega_{m0}}{\Omega_{\gamma 0}}\left[1+N_{\nu}\left(\frac{8}{11}\right)^{1/3}\left(1-\frac{\lambda\varphi^2}{2}\right)^{-1}\right]^{-1}$$

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Solve Dirac equation using WKB approximation at 0^{th} and 1^{st} order in $\hbar$.

Oscillation probability as a function of redshift. Gravity and φ alter neutrino oscillations(NO).

Part II: NO & DE-Interaction term

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Focus later on Λ and Scalar field DE, with Current-Velocity coupling$(\bar{\psi}\gamma^{\mu}\psi\partial_{\mu}\varphi)$

Part II: NO & DE-Dirac Equation

For 2-flavor system (ν_e, ν_μ), define $\boldsymbol{\psi} = \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}$

$$\left(i\gamma^\mu \mathcal{D}_\mu - \mathcal{M}_f\right)\boldsymbol{\psi} = \left(\xi F(\varphi, X^\mu_\varphi) + \xi_f \gamma^\mu G_\mu(\varphi, X^\mu_\varphi)\right)\boldsymbol{\psi}$$

where $\mathcal{M}_f \equiv$ vaccum mass matrix in flavor space;

$$\mathcal{M}_f^2 = U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger$$

and $U \equiv$ mixing matrix $= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$; $\theta \equiv$ mixing angle

Part II: NO & DE-Flavor state

Recall: in flat S.T, $|\nu_\alpha\rangle = \sum_{j=1,2} U_{\alpha j} e^{-i\frac{m_j^2}{2E}L}|\nu_j\rangle$.

Part II: NO & DE-Flavor state

Recall: in flat S.T, $|\nu_\alpha\rangle = \sum_{j=1,2} U_{\alpha j} e^{-i\frac{m_j^2}{2E}L}|\nu_j\rangle$.

In curved S.T, $|\nu_\alpha\rangle = \sum_{j=1,2} U_{\alpha j} e^{i\Phi(\lambda)}|\nu_j\rangle$

where $\Phi(\lambda) = \int_{\lambda_0}^{\lambda} \mathrm{P}.\mathrm{p}_{\mathrm{null}} d\lambda'$; P $\equiv$ 4-momentum operator;

$\mathrm{p}_{\mathrm{null}}$ $\equiv$null vector tangent to worldline.

Part II: NO & DE-Flavor state

$$|\nu_\alpha\rangle = \sum_{j=1,2} U_{\alpha j} e^{i\Phi(\lambda)} |\nu_j\rangle \Leftrightarrow i\frac{d}{d\lambda}|\nu_\alpha(\lambda)\rangle = \Phi(\lambda)|\nu_\alpha(\lambda)\rangle$$

Part II: NO & DE-Transition Amplitude

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$$\Psi_{\alpha\beta} \equiv \langle\nu_\beta|\nu_\alpha(\lambda)\rangle \Longrightarrow i\frac{d}{d\lambda}\Psi_{\alpha\beta} = \left[\frac{1}{2}\tilde{\mathcal{M}}_f^2 + V_I\right]\Psi_{\alpha\beta}$$

$$\tilde{\mathcal{M}}_f^2 = U\begin{pmatrix} \left(m_1 - \xi F\right)^2 & 0 \\ 0 & (m_2 - \xi F)^2 \end{pmatrix} U^\dagger;\ V_I \propto \xi_f G_\mu p^\mu_{\text{null}}$$

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Gravitational MSW effect

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Diagonalize $\left[\frac{1}{2}\tilde{\mathcal{M}}_f^2 + V_I\right]$ by $\tilde{U} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix}$ with eigenvalues $v_\pm$

Part II: NO & DE-Probability

For Λ and scalar DE with linear derivative coupling, we have adiabatic regime, i.e. $\frac{d\tilde{\theta}}{d\lambda} = 0$.

Final Result

$$P_{\nu_e \to \nu_\mu} \equiv |\Psi_{e\mu}|^2 = \mathcal{F}(\xi F, \xi_f G) \sin^2 2\theta \sin^2\left(\frac{\omega_- - \omega_+}{2}\right),$$

$$\omega_- - \omega_+ \approx$$

$$\frac{m_2^2 - m_1^2}{2}(\lambda_0 - \lambda) + V_I \cos 2\theta(\xi_e - \xi_\mu)(\lambda - \lambda_0) + \xi \Delta m \int_{\lambda_0}^{\lambda} F d\lambda'.$$

Compare to flat S.T: $P^{\text{std}}_{\nu_e \to \nu_\mu} = \sin^2 2\theta \sin^2\left(\frac{(m_2^2 - m_1^2)L}{4E}\right)$

Part II: NO & ΛCDM

In Particle Physics (Minkowski spacetime):

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$$\text{FRW} \Big\Downarrow \quad L \to d_L;\, E \to E_0/a$$

$$P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 d_L a}{4E_0}\right)$$

$$d_L = (1+z_e) H_0^{-1} \int_0^{z_e} \left(\Omega_{m0}(1+z)^3 + \Omega_{\Lambda_0}\right)^{-1/2}$$

Part II: NO & ΛCDM

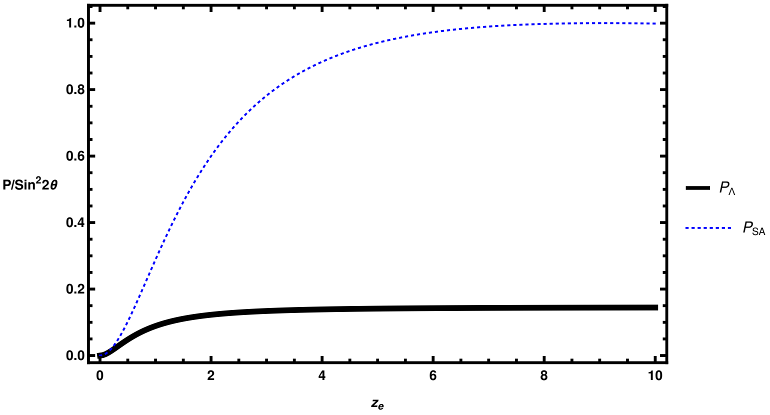
Quantum spinors in flat FLRW universe with cosmological constant DE:

$P_\Lambda = \sin^2 2\theta \sin^2 \omega_\Lambda$;

$$\omega_\Lambda = \frac{\Delta m^2}{2}\int \frac{d\lambda}{E} = \frac{\Delta m^2}{2H_0 E_0}\int_0^{z_e}\left(\Omega_{m_0}(1+z)^7 + \Omega_{\Lambda_0}(1+z)^4\right)^{-1/2} dz$$

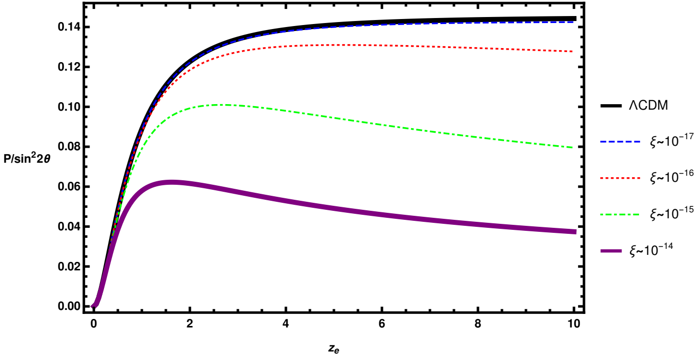
- H_0 ≡Hubble constant today;
- z_e ≡emission redshift;
- $\Omega_{m_0}(\Omega_{\Lambda_0})$ ≡matter(DE) density parameter.

Part II: NO & ΛCDM



Part II: NO & Quintessence

In flat FLRW with scalar field DE(e.g. quintessence, modified gravity) coupled to neutrinos via $\mathcal{L}_{\text{int}} = \xi\bar{\psi}\gamma^{\mu}\psi\partial_{\mu}\varphi$



Part III: NO & the Hubble Tension(HT)-Plan

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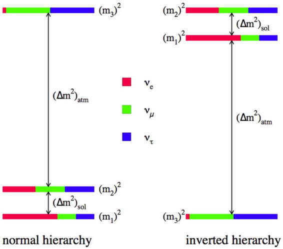

Schematic difference between the two neutrino hierarchies. $m_{1,2,3}$ are eigenvalues for neutrino mass states, and $\Delta m^2_{ij} = m_i^2 - m_j^2$

Part III: NO & HT-Plan

- Generalize to three neutrino flavors in ΛCDM
- Effect of different H_0 values on the oscillation probability.
- Distinguish between Normal Hierarchy(NH) and Inverted Hierarchy(IH)
- Show results in terms of Ternary diagrams and neutrino flux vs. redshift plots.

Part III: NO & HT-Equations Needed

- ΛCDM:
  $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\big(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\big),$
  $H^2(z) = \frac{8\pi G}{3}\big(\rho_m + \rho_\Lambda\big) = H_0^2\left(\Omega_m(1+z)^3 + \Omega_\Lambda\right)$

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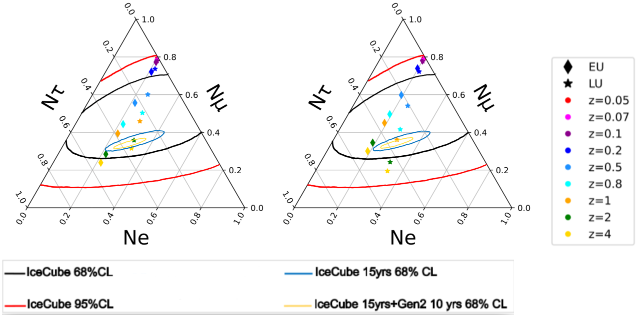
- Transition Probability:

$$P_{\alpha\beta} = \delta_{\alpha\beta} + \sum_{i<j}\left[a_{\alpha\beta;ij}\sin^2\left(\frac{\Delta m_{ij}^2 \Delta\lambda}{4}\right) + b_{\alpha\beta;ij}\sin\left(\frac{\Delta m_{ij}^2 \Delta\lambda}{2}\right)\right]$$

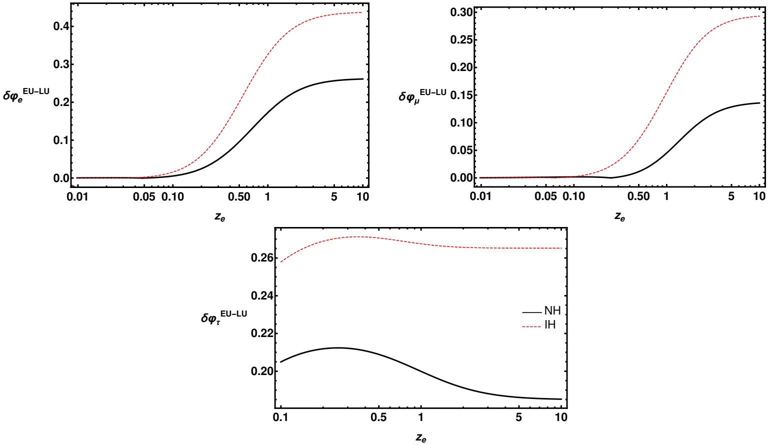
$$\Delta\lambda \equiv \frac{1}{E_0}\int_0^{z_e}\frac{dz}{H(z)(1+z)^2},$$

Part III: NO & HT-Pion Decay I.C.

$$(\nu_e, \nu_\mu, \nu_\tau) = (1/3, 2/3, 0)$$



Part III: NO & HT-Flux EU vs. LU



Part III: NO & HT-Observational Prospects

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- Analysis is made assuming flat spacetime. But gravity now must be included.

What's Next?

- More work on the observational front: MCMCs.
- Look at wave-packets of neutrinos.
- Look at 1^{st} order perturbations and effect of power spectra.
- Neutrinos traveling near Dark Matter halos.
- Apply to other fermionic entities: electrons or DM(?).
- ...

A Take-Home Message

Quantum field theory in curved space-time is a natural generalization to the analysis of our Universe, with many applications still to explore!

The End!

Questions or comments?

Refs:2010.08181, 2105.07973 and 2111.15249

Spinors in Curved Spacetime

- The solution is to introduce tetrad fields, ${e_a}^{\mu}(x)$, that covers the entire spacetime. These fields link local flat coordinates to the global curved ones. Latin indices⇔local coordinates; Greek indices⇔global coordinates.
- $\{\gamma^a, \gamma^b\} = -2\eta^{ab} \Leftrightarrow \{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}; \gamma^\mu(x) = {e_a}^{\mu}(x)\gamma^a$
- $\mathcal{D}_\mu \psi \equiv (\partial_\mu - \Gamma_\mu)\psi; \Gamma_\mu = -\frac{1}{4}\gamma_a\gamma_b e^{a\alpha}(x)\nabla_\mu {e^b}_\alpha(x)$
- $\gamma^a e_a^\mu \Gamma_\mu = \frac{i}{\hbar}\gamma^a e_a^\mu A_\mu; A^\mu = \frac{1}{4}\sqrt{-g}\, e_a^\mu \epsilon^{abcd}(\partial_\sigma e_{b\nu} - \partial_\nu e_{b\sigma})e_c^\nu e_d^\sigma$
- $(i\hbar\gamma^\mu \mathcal{D}_\mu - mc)\psi = 0 \Leftrightarrow \left[i\hbar\gamma^\mu\left(\partial_\mu - \frac{i}{\hbar}A_\mu\right) - m\right]\psi = 0$