Bispectrum and finite volume effects: window-convolution

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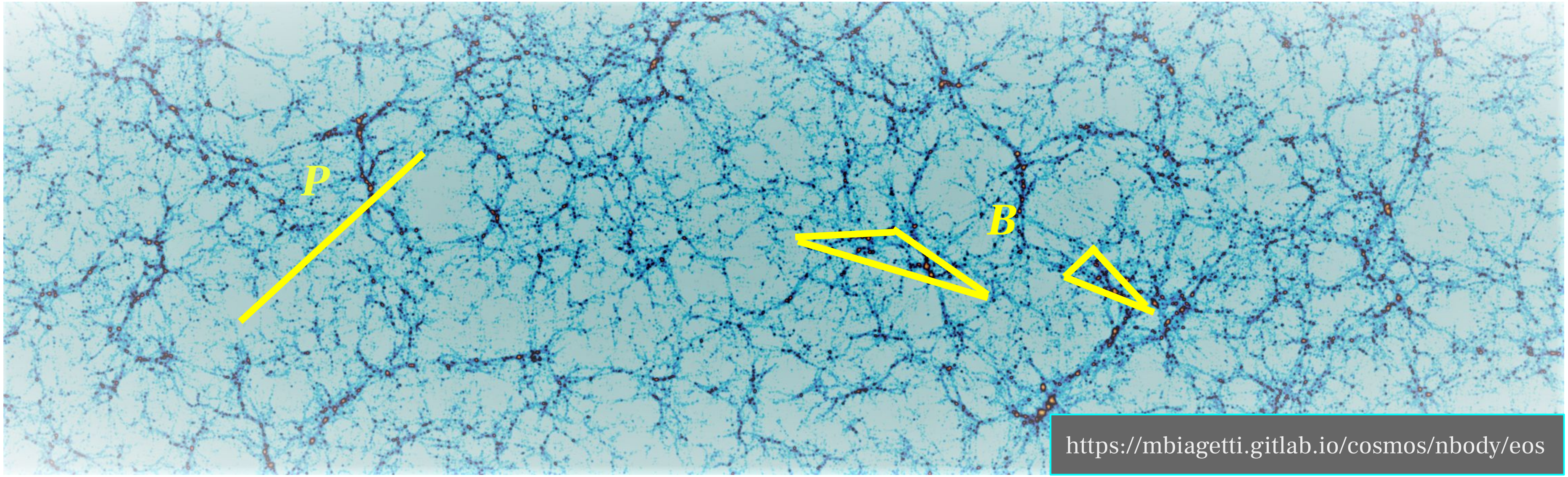
Bispectrum captures the non-Gaussianity

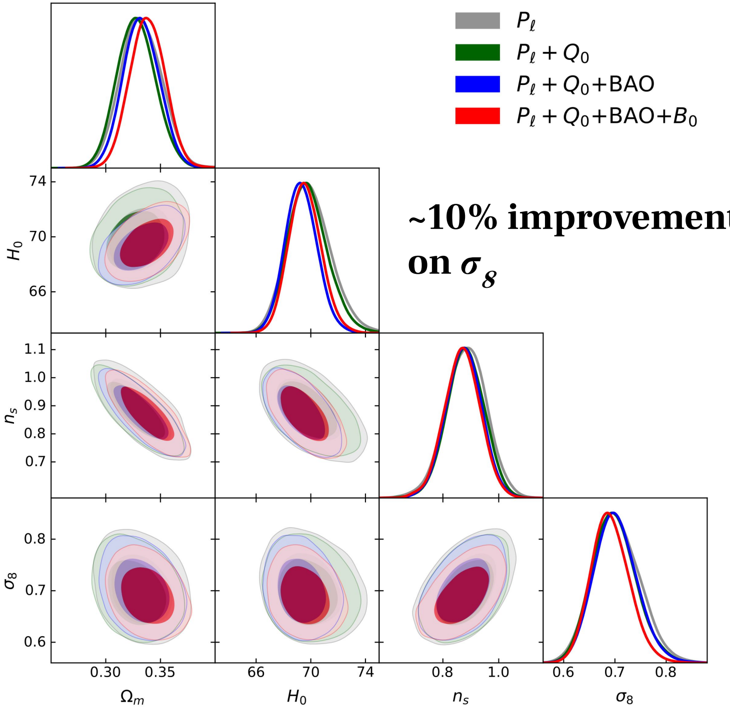
Power Spectrum (P) + **Bispectrum** (B) + …



$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(\mathbf{k}_1, \mathbf{k}_2)$$

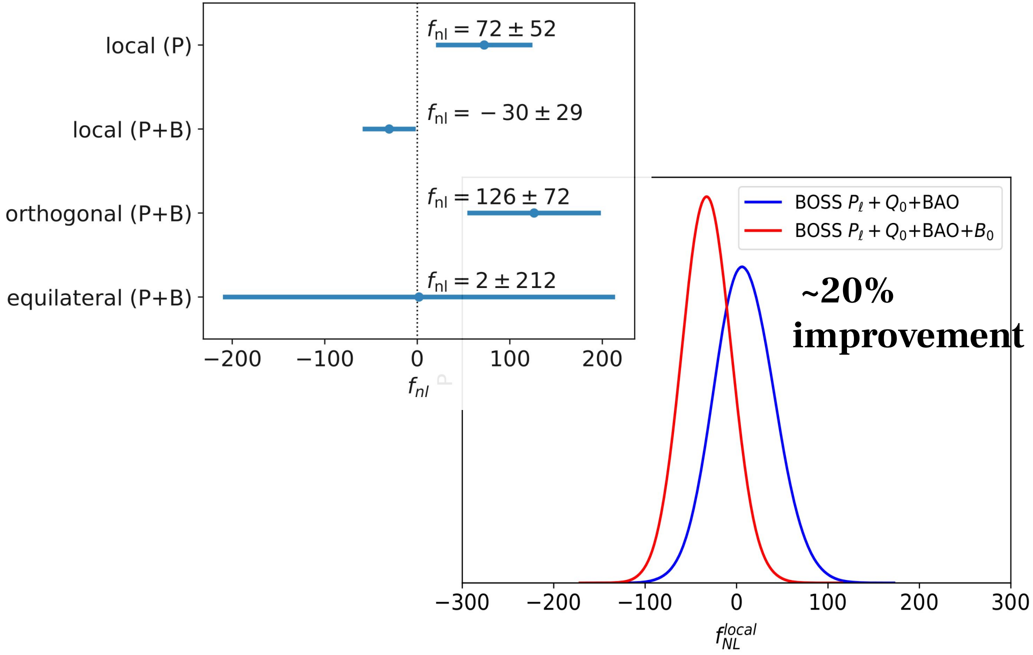
Including bispectrum monopole (BOSS DR12)

Constraint on cosmological params:



~10% improvement on σ_8

Constraint on primordial non-Gaussianity:



~20% improvement

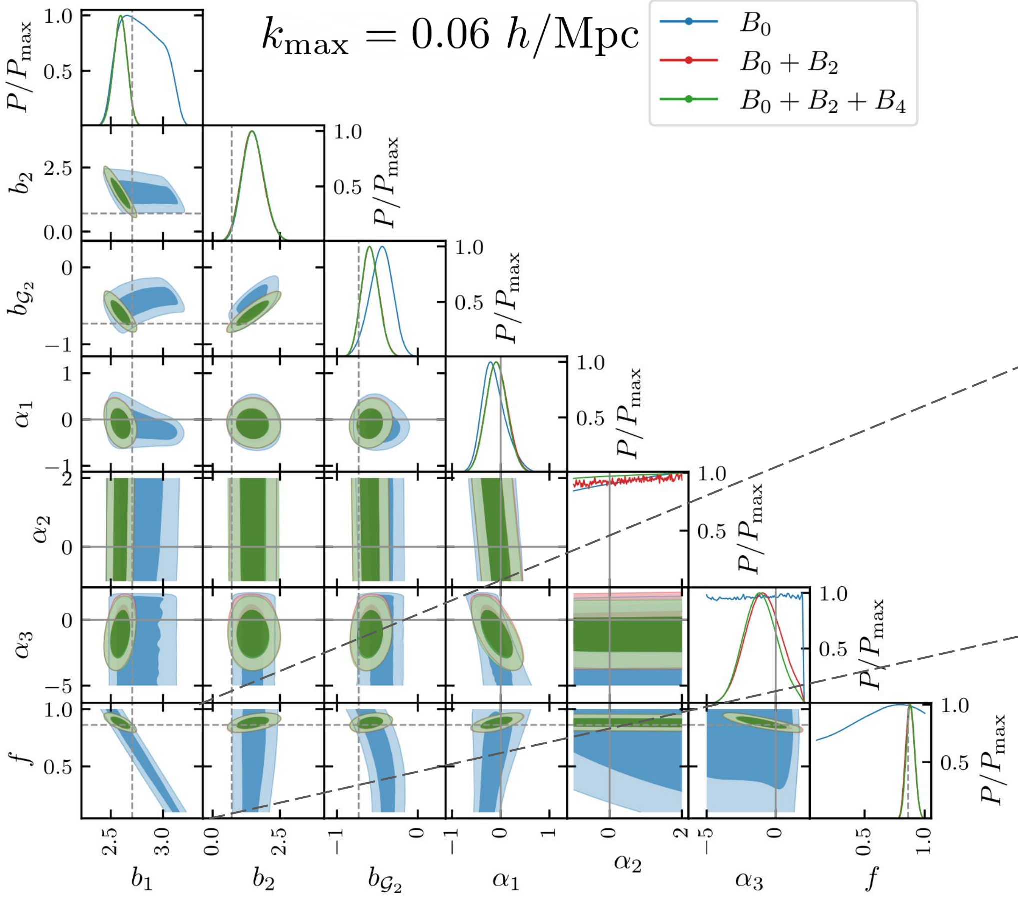
Philcox&Ivanov,21 (also: D'Amico+19)

also one-loop bispectrum: Philcox+22, D'Amico+22b

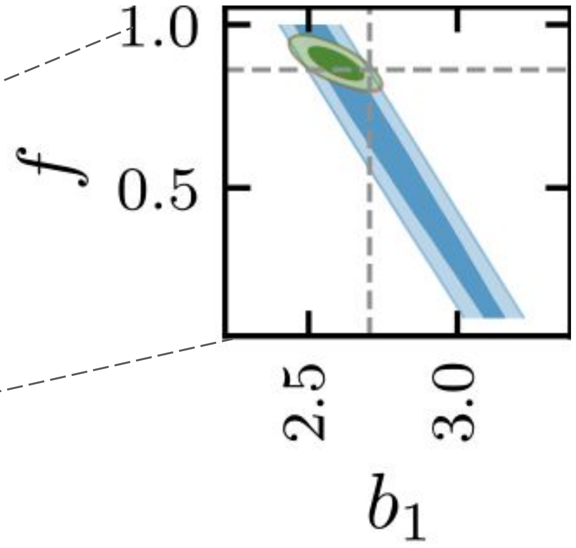
Cabass+22b, D'Amico+22a

non-local PNG: Cabass+22a, bispectrum is **necessary**

Inclusion of the bispectrum multipoles

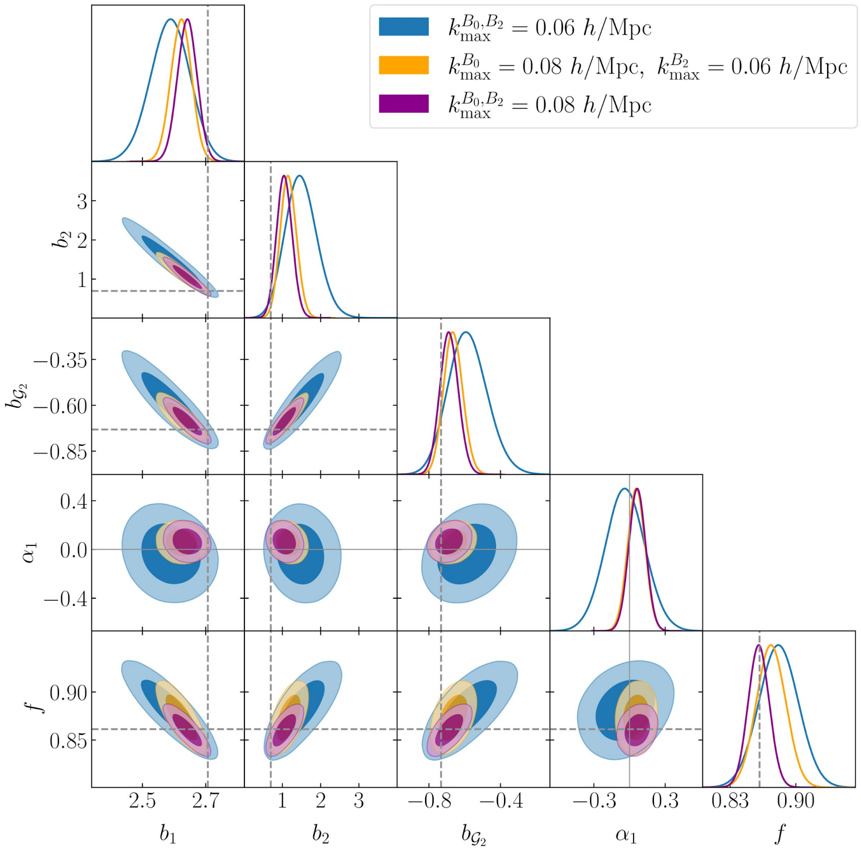


B_0
$+\quad B_2$ (significant information)
$+\quad B_4$ (negligible information)



Rizzo, Moretti, Pardede+ (arXiv: 2204.13628)

Exploring different scale-cuts



Rizzo, Moretti, Pardede+ (arXiv: 2204.13628)

Measurement vs. theory: large-scale systematics

Scoccimarro estimator* Scoccimarro +15

FFT-based, optimal on small-scale

$$\hat{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B} \int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \delta_D(\mathbf{q}_{123}) \tilde{\delta}_L(\mathbf{q}_1) \tilde{\delta}(\mathbf{q}_2) \tilde{\delta}(\mathbf{q}_3)$$

$$\int_{k_1} d^3q_1 \equiv \int_{|k_1 - \Delta k/2| \leq k_1 \leq |k_1 + \Delta k/2|} d^3q_1$$

$$V_B \equiv \int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \delta_D(\mathbf{q}_{123})$$

*window-free estimator Tegmark+97, has been revived recently: Philcox20, Philcox21

Measurement vs. theory: large-scale systematics

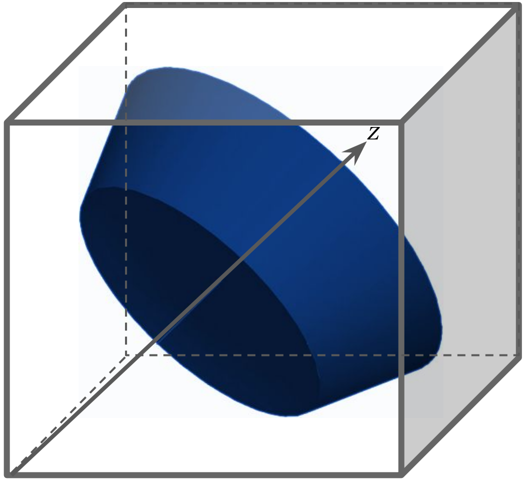
biased on large-scale

$$\hat{B}_L(k_1,k_2,k_3) = \frac{2L+1}{V_B}\int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \delta_D(\mathbf{q}_{123})\tilde{\delta}_L(\mathbf{q}_1)\tilde{\delta}(\mathbf{q}_2)\tilde{\delta}(\mathbf{q}_3)$$

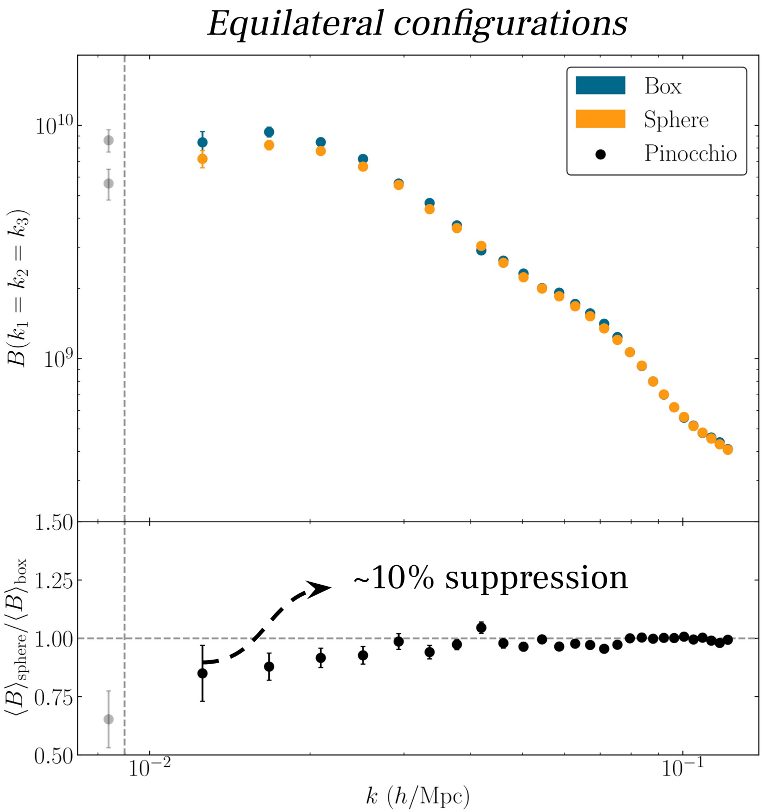
binning operator

window function $\tilde{\delta}(\mathbf{x}) = W(\mathbf{x})\delta(\mathbf{x})$

variation of the **LOS** $\tilde{\delta}_L(\mathbf{q}) \equiv \int d^3x\ \tilde{\delta}(\mathbf{x})\mathcal{L}_L(\mathbf{q}\cdot\mathbf{x})e^{-i\mathbf{q}\cdot\mathbf{x}}$



In bispectrum …



window convolution will mix modes

10000 Pinocchio sphere catalogue

Note: this is a huge volume ≈ 3500 [Gpc/h]^3

… main effect is on large scale

To include window effect in bispectrum

Schematically (monopole with no binning operator)

$$\tilde{B}(\vec{k}_1, \vec{k}_2) = \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} B_W(\vec{k}_1 - \vec{p}_1, \vec{k}_2 - \vec{p}_2) B(\vec{p}_1, \vec{p}_2)$$

- 6D integral
- Time ~ **hours**/evaluation
- Not feasible for likelihood analysis

An approximation

$$\tilde{B}[P_L] \simeq B[\tilde{P}_L]$$

1DFFTLog-approx

$$\tilde{B}(\mathbf{k}_1, \mathbf{k}_2) \simeq Z(\mathbf{k}_1, \mathbf{k}_2)\tilde{P}_L(k_1)\tilde{P}_L(k_2) + \text{cyc.}$$

- Reduced to power spectrum-window convolution
  see e.g. Wilson+15, Castorina+17, d'Amico+19
- BOSS DR 11/12 Gil-Marin+14a, b and +16a, b
- Recently used in d'Amico +19,+22
- Doesn't work for squeezed triangles

Computed via (1D) FFTLog

Exact: as a matrix multiplication

We showed that bispectrum-window convolution can be casted into a 1D integral

2DFFTLog

$$\tilde{B}_\ell[T_i] = \sum_{j,\ell'} \mathcal{M}_{\ell\ell'}[T_i, T'_j] B_{\ell'}[T'_j]$$

Mixing matrix

Computable via (2D) FFTLog

e.g. `2D-FFTLog` (Fang+20)

of the window 3PCF multipoles

Bispectrum

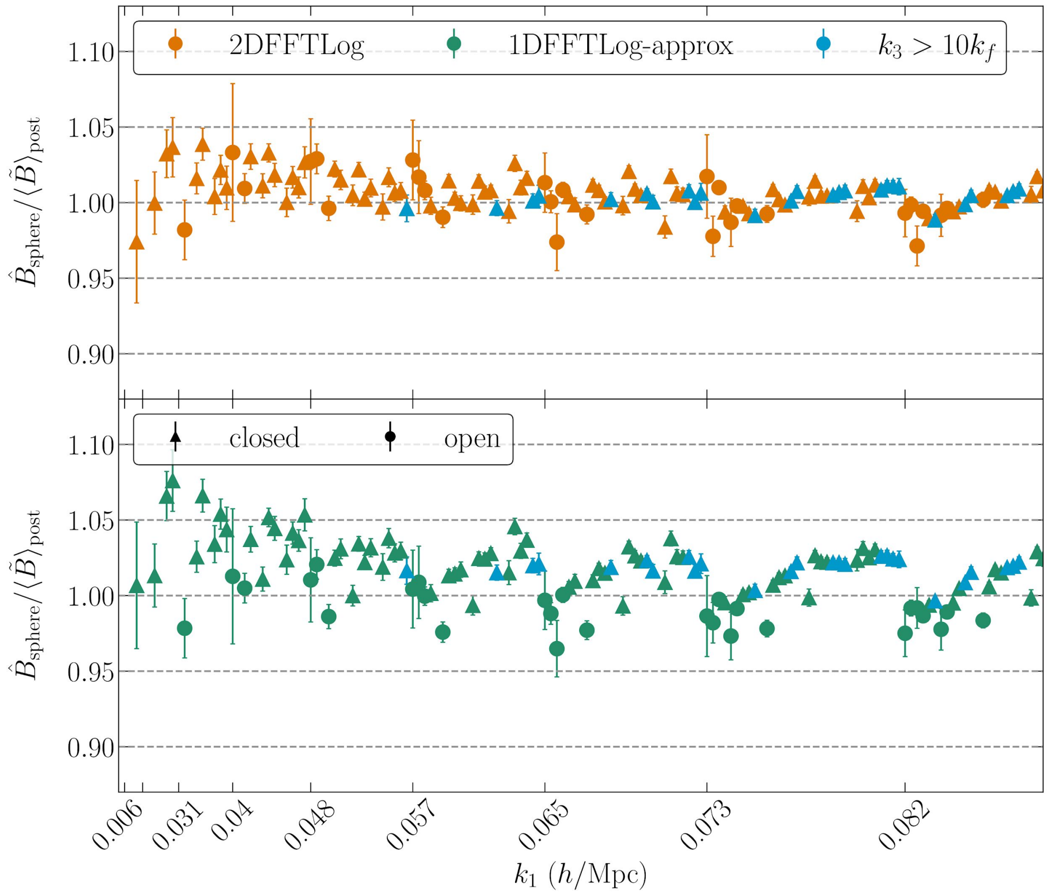
Function of three sides (k_1, k_2, k_3)

Pardede, Rizzo, Biagetti, Castorina, Sefusatti, Monaco (arXiv: 2203.04174)

Spherical window convolution in real-space

Fit on **Pinocchio** mocks

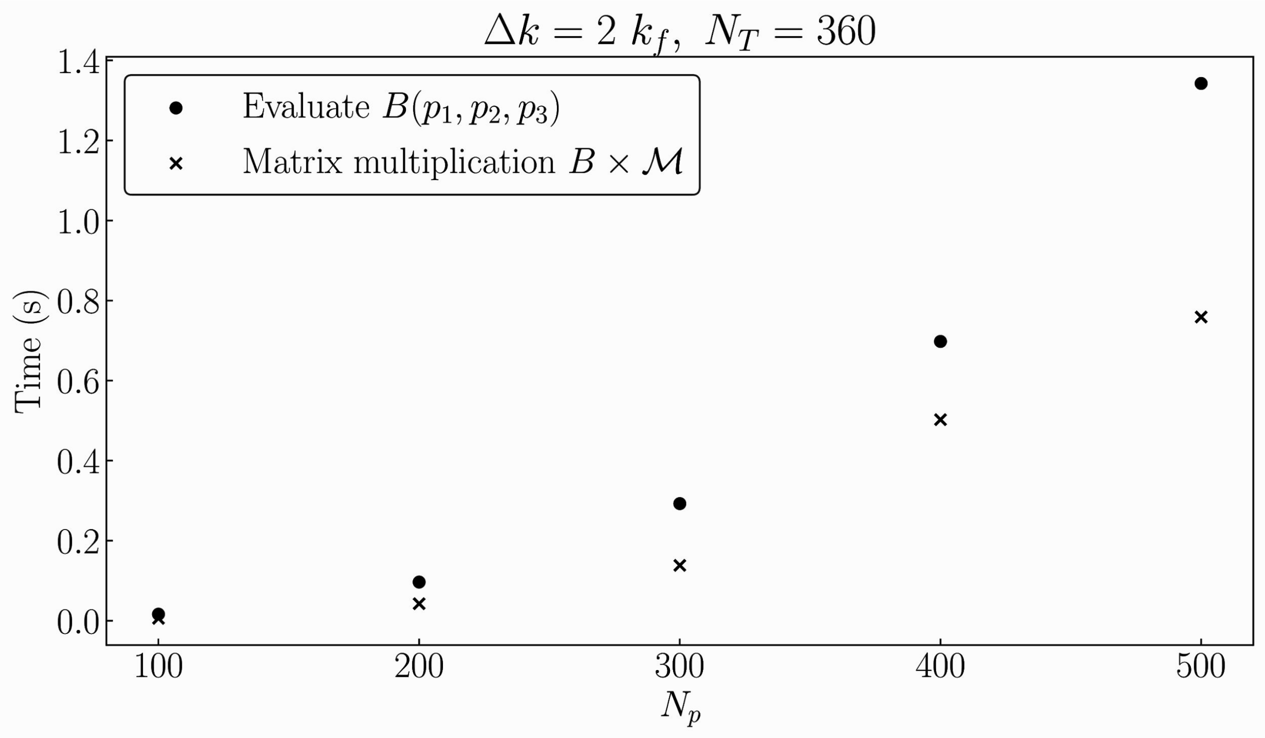
volume ≈ 3500 $[\mathrm{Gpc}/h]^3$



Window convolution computation time

Takes ~ **2 seconds**

⇒ comparable to a typical Boltzmann solver call



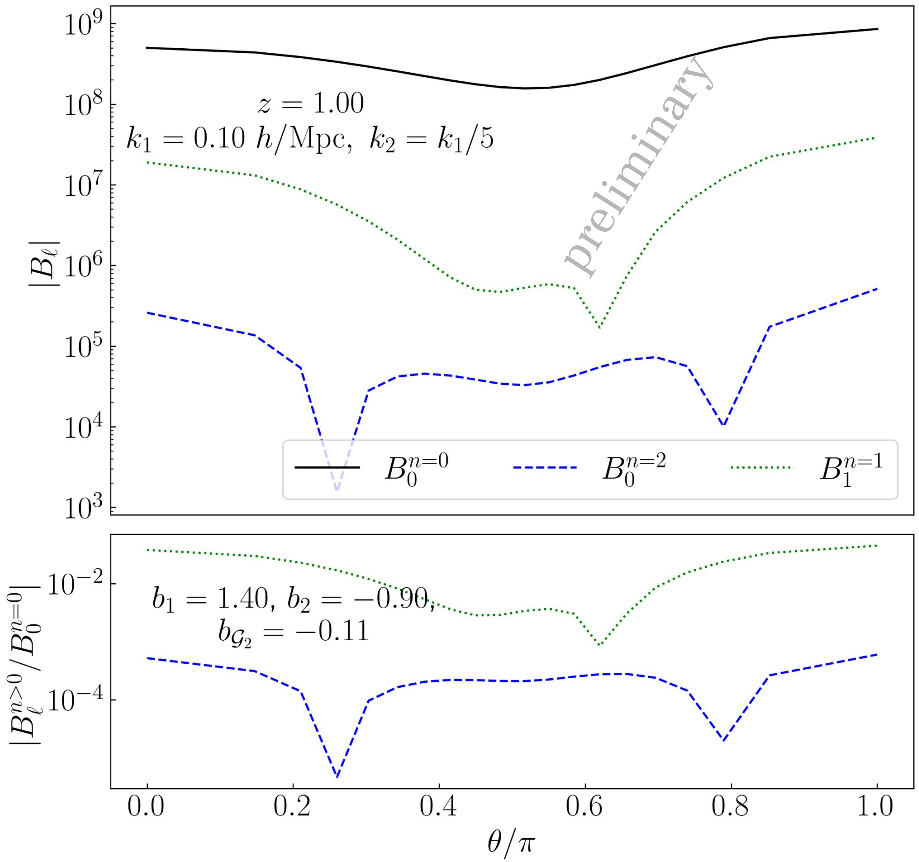
Finally: wide-angle effects in bispectrum

$$\hat{B}_L(k_1,k_2,k_3) = \frac{2L+1}{V_B}\int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \delta_D(\mathbf{q}_{123}) \boxed{\tilde{\delta}_L(\mathbf{q}_1)} \tilde{\delta}(\mathbf{q}_2)\tilde{\delta}(\mathbf{q}_3)$$

choice of **LOS** $\tilde{\delta}_L(\mathbf{q}) \equiv \int d^3x\, \tilde{\delta}(\mathbf{x})\mathcal{L}_L(\mathbf{q}\cdot\mathbf{x})e^{-i\mathbf{q}\cdot\mathbf{x}}$

- correction to plane-parallel approximation
- coupled to the window function

… WA correction in monopole and dipole



- a ~ 0.01% k^{-2} correction for monopole (systematics for PNG, …)
- induce a nonzero dipole ~1% monopole (systematics for GR effects, …)

Pardede, Di Dio, Castorina (in prep.)
see also Noorikuhani, Scoccimarro, 2022

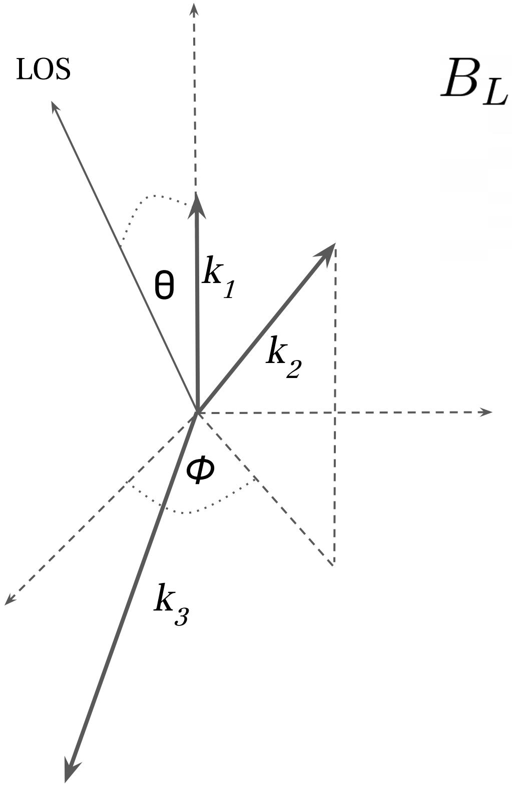
Summary

1. Including bispectrum multipoles analysis is important but come with extra modelling complexity, ex: survey window effects, wide-angle effects
2. We gave an efficient formulation for bispectrum window convolution
3. We tested the formulation in ideal case of spherical window convolution in real space
4. Wide-angle effects have some sizable corrections
5. These effects are important for large-volume surveys, e.g. Euclid/DESI when you want to extract signal, free from large-scale systematic effects

Thank you!

-Extras-

The bispectrum multipoles



$$B_L(k_1,k_2,k_3) = \frac{2L+1}{4\pi}\int d\cos\theta \int d\phi B(k_1,k_2,k_3,\theta,\phi)\,\mathcal{L}_L(\cos\theta)\,.$$

PT (perturbation theory) model

angles w.r.t line of sight

- galaxies are not in their rest frame
- $m \neq 0$ contains negligible information Gagrani+16
- **tree-level: only even multipoles exist B_0, B_2, B_4, …**

Tree-level bispectrum

$$B(\mathbf{k}_1,\mathbf{k}_2,\hat{\mathbf{x}}) = B^{(\mathrm{det})}(\mathbf{k}_1,\mathbf{k}_2,\hat{\mathbf{x}}) + B^{(\mathrm{stoch})}(\mathbf{k}_1,\mathbf{k}_2,\hat{\mathbf{x}})$$

$$B^{(\mathrm{det})}(\mathbf{k}_1,\mathbf{k}_2,\hat{\mathbf{x}}) = 2Z_2(\mathbf{k}_1,\mathbf{k}_2,\hat{\mathbf{x}})Z_1(\mathbf{k}_1,\hat{\mathbf{x}})Z_1(\mathbf{k}_2,\hat{\mathbf{x}})P_L(k_1)P_L(k_2) + \mathrm{cyc.}$$

$$B^{(\mathrm{stoch})}(\mathbf{k}_1,\mathbf{k}_2,\hat{\mathbf{x}}) = \frac{1}{\bar{n}}[(1+\alpha_1)b_1 + (1+\alpha_3)f(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{x}})^2]Z_1(\mathbf{k}_1,\hat{\mathbf{x}})P_L(k_1) + \mathrm{cyc.} + \frac{1+\alpha_2}{\bar{n}^2}$$

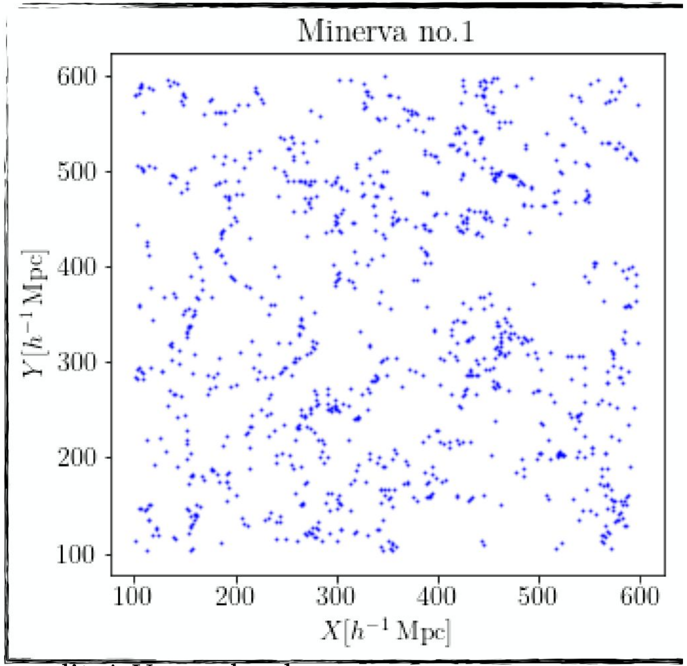
$$Z_1(\mathbf{k},\hat{\mathbf{x}}) = b_1 + f(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{x}})^2$$

$$Z_2(\mathbf{k}_1,\mathbf{k}_2,\hat{\mathbf{x}}) = \frac{b_2}{2} + b_1 F_2(\mathbf{k}_1,\mathbf{k}_2) + b_{\mathcal{G}_2}S(\mathbf{k}_1,\mathbf{k}_2)$$
$$f(\hat{\mathbf{k}}_{12}\cdot\hat{\mathbf{x}})^2 G(\mathbf{k}_1,\mathbf{k}_2) + \frac{f(\mathbf{k}_{12}\cdot\hat{\mathbf{x}})}{2}\left[\frac{\hat{\mathbf{k}}_1\cdot\hat{\mathbf{x}}}{k_1}Z_1(\mathbf{k}_2,\hat{\mathbf{x}}) + \frac{\hat{\mathbf{k}}_2\cdot\hat{\mathbf{x}}}{k_2}Z_1(\mathbf{k}_1,\hat{\mathbf{x}})\right]$$

$$\mathbf{k}_{12} \equiv \mathbf{k}_1 + \mathbf{k}_2$$

The bispectrum multipoles: test on simulations

1. 298 **Minerva** (N-body) Grieb+16
2. 10000 **Pinocchio** (3LPT) Monaco+02



credit: A.Veropalumbo

Rizzo, Moretti, Pardede+ (arXiv: 2204.13628)

@$z = 1$

ΛCDM cosmology

$L_{box} = 1500$ Mpc/h

$V_{eff} \simeq 1000\ (\mathrm{Gpc}/h)^3$

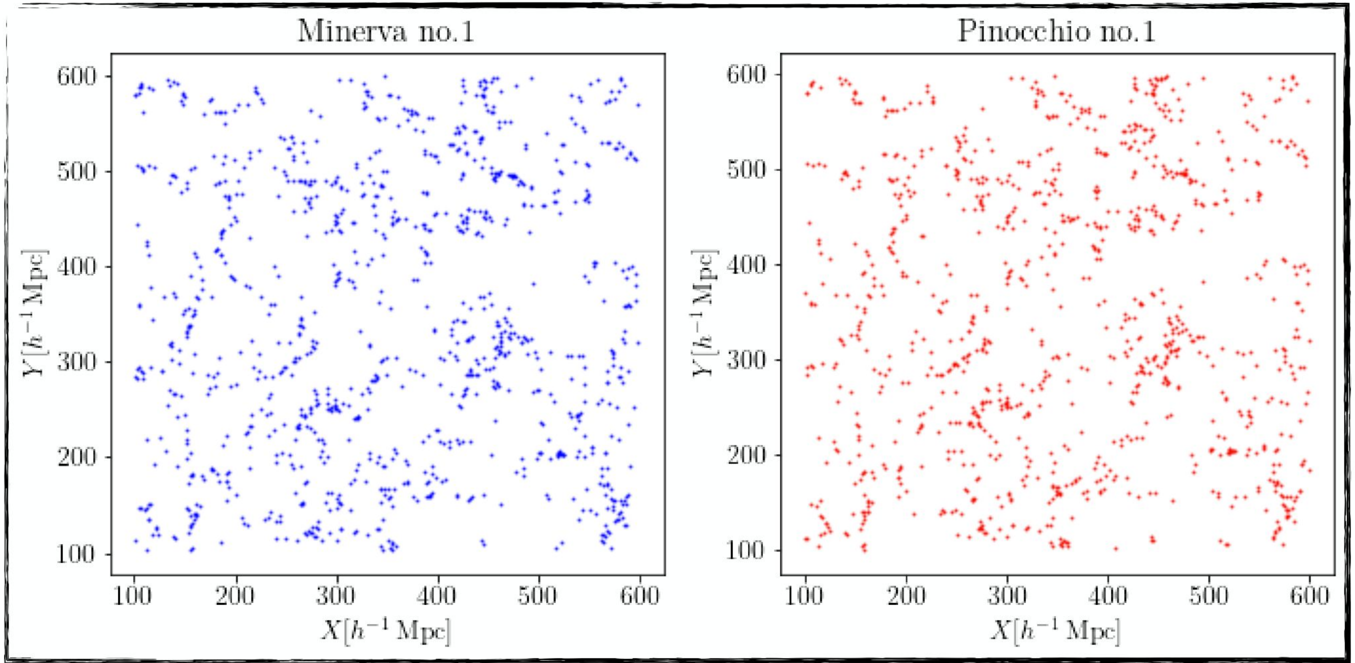
$\simeq$ 2x volume in PT-challenge Nishimichi+20

… the numerical covariance

1. 298 **Minerva** (N-body) Grieb+16
2. 10000 **Pinocchio** (3LPT) Monaco+02

- approx. based on Lagrangian pert. theory
- relatively fast and accurate

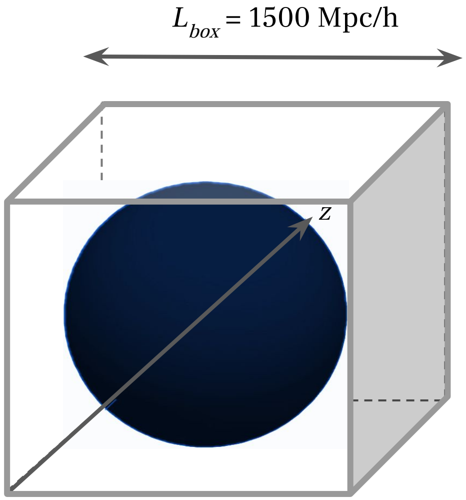
provide a robust estimate of the covariance



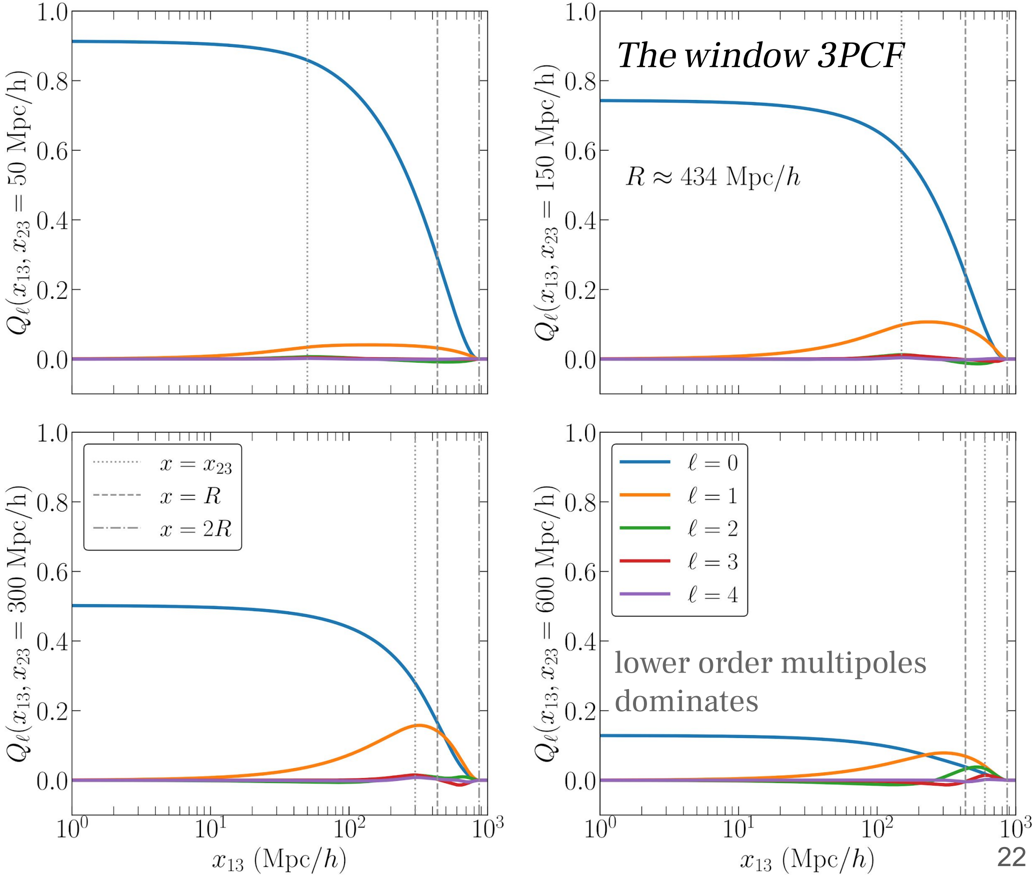
credit: A.Veropalumbo

Spherical window convolution in real-space

Sphere catalogue:
Minerva/Pinocchio carved on a sphere of $R \sim 434$ Mpc/h



Total vol = 700^3 (Mpc/h)$]^3$

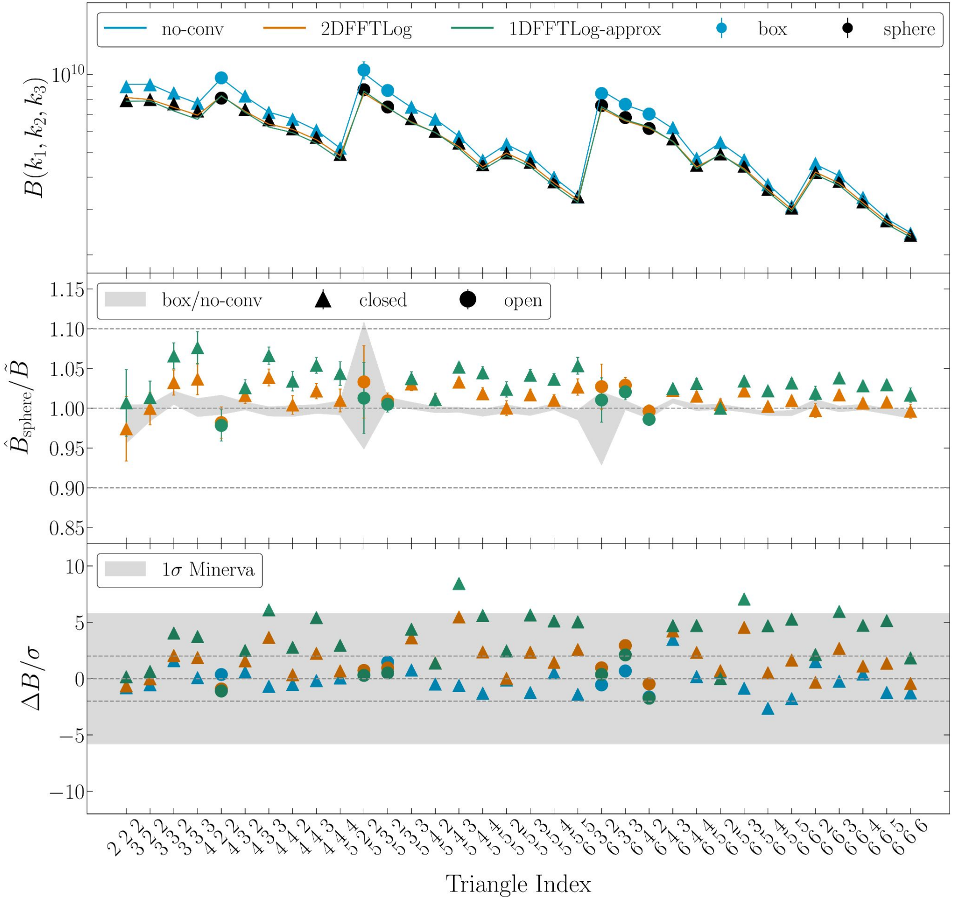


First few triangles

Fit on **Pinocchio** mocks

volume ≈ 3500 [Gpc/h]^3

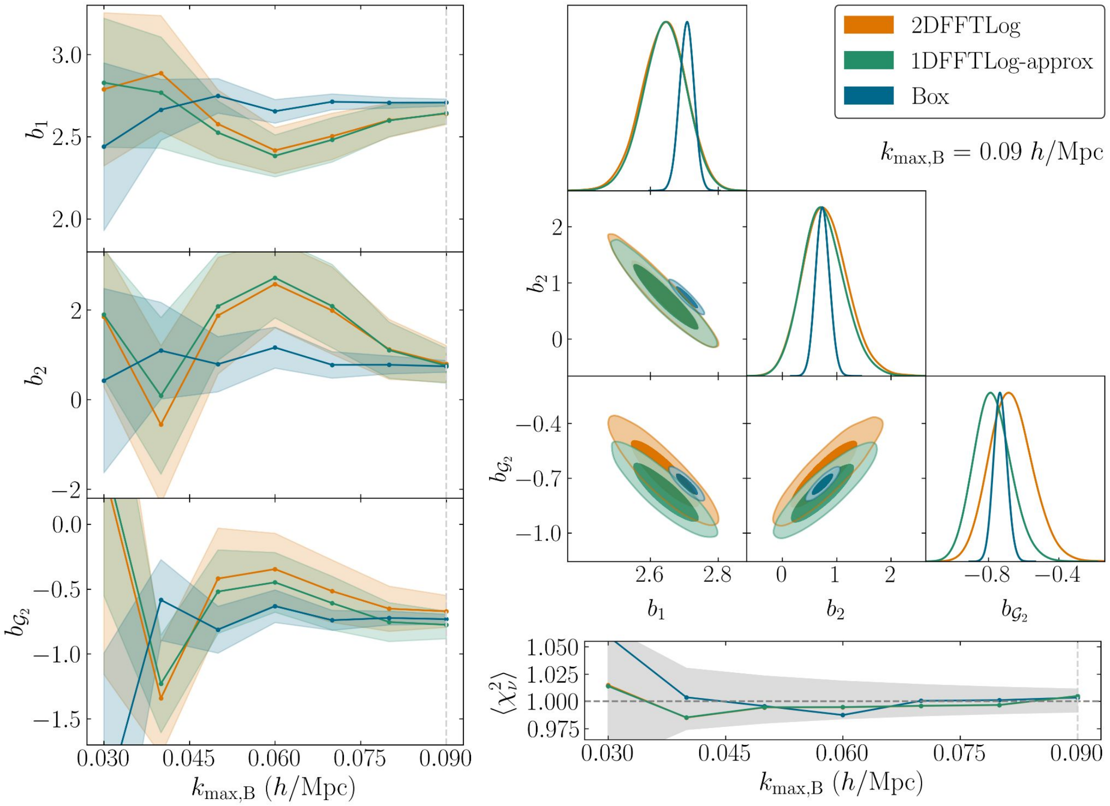
Minerva volume:
consistent within 1-sigma



Recovering bias parameters

Analysis on **Minerva** data

≈ 1/6 times volume in Nishimichi+20
≈ 6 times $z \in [1.5, 1.8]$ *Euclid* volume



Exact bispectrum window convolution

Taking $\langle \hat{B}_L \rangle = \tilde{B}_L$

$$\tilde{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B} \int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \, \delta_D(\vec{q}_{123})$$
$$\times \int d^3x_3 \int d^3x_{13} \int d^3x_{23} \, e^{-i\vec{q}_1 \cdot \vec{x}_{13}} e^{-i\vec{q}_2 \cdot \vec{x}_{23}} \zeta(\vec{x}_{13}, \vec{x}_{23}, \hat{x}_3)$$
$$\times W(\vec{x}_1) W(\vec{x}_2) W(\vec{x}_3) \mathcal{L}_L(\hat{q}_1 \cdot \hat{x}_3)$$

Need to: systematically reduce the angular integration

… the final expression

Some form of integral between
the unconvolved bisp. and the mixing matrix

$$\tilde{B}_L(k_1, k_2, k_3) = \int \frac{dp_1}{2\pi^2} p_1^2 \int \frac{dp_2}{2\pi^2} p_2^2 \int \frac{dp_3}{2\pi^2} p_3^2 \sum_{L' M'} B_{L' M'}(p_1, p_2, p_3)$$

$$\times \sum_{\ell} \underbrace{I_{\ell \ell 0}(p_1, p_2, p_3) \mathcal{Q}^L_{L', -M', \ell}(k_1, k_2, k_3; p_1, p_2)}_{\text{the mixing matrix}}$$

Window convolution ~ matrix mult.

One part of the mixing matrix is a known function

$$\tilde{B}_L(k_1,k_2,k_3) = \int \frac{dp_1}{2\pi^2}p_1^2 \int \frac{dp_2}{2\pi^2}p_2^2 \int \frac{dp_3}{2\pi^2}p_3^2 \sum_{L'M'} B_{L'M'}(p_1,p_2,p_3)$$
$$\times \sum_{\ell} I_{\ell\ell 0}(p_1,p_2,p_3)\,\mathcal{Q}^{L}_{L',-M',\ell}(k_1,k_2,k_3;p_1,p_2)$$

enforce the triangle condition

$$\tilde{B}_\ell[T_i] = \sum_{j,\ell'} \mathcal{M}_{\ell\ell'}[T_i,T'_j]B_{\ell'}[T'_j]$$

$$I_{\ell\ell 0}(x,y,z) = (-1)^\ell \frac{\pi^2}{xyz}\theta(1-\hat{x}\cdot\hat{y})\theta(1+\hat{x}\cdot\hat{y})\mathcal{L}_\ell(\hat{x}\cdot\hat{y})$$

Contribution from the the window 3PCF

The other contribution is coming from the random catalogue

$$\tilde{B}_L(k_1, k_2, k_3) = \int \frac{dp_1}{2\pi^2} p_1^2 \int \frac{dp_2}{2\pi^2} p_2^2 \int \frac{dp_3}{2\pi^2} p_3^2 \sum_{L'M'} B_{L'M'}(p_1, p_2, p_3)$$
$$\times \sum_{\ell} I_{\ell\ell 0}(p_1, p_2, p_3) \mathcal{Q}^{L}_{L', -M', \ell}(k_1, k_2, k_3; p_1, p_2)$$

require several steps of computations

How to compute the 3PCF contribution?

$W(\mathbf{x})$ → the random catalogue

3PCF of the window e.g. by direct-counting

$$Q^{L}_{L'M'\ell\ell'\ell_1\ell_2}(x_{13}, x_{23})$$

(2D) Hankel transform e.g. 2D-FFTLog (Fang+20)

$$\mathcal{W}^{L}_{L'M'\ell\ell'\ell_1\ell_2}(q_1, q_2, p_1, p_2)$$

linear combination

$$\mathcal{Q}^{L}_{L'M'\ell}(k_1, k_2, k_3; p_1, p_2)$$

The window 3PCF - **measurement**

$$Q^{L}_{L'M'\ell\ell'\ell_1\ell_2}(x_{13}, x_{23}) \equiv (-1)^{M'} \sum_{\tilde{\ell}_1,\tilde{\ell}_2} \sum_{\substack{M,m_1,m_2\\ m,m',\tilde{m}_1,\tilde{m}_2}} 4\pi i^{\ell'-\ell+\ell_2-\ell_1}\, \mathcal{G}^{Mm_1m_2}_{L\ell_1\ell_2}\, \mathcal{G}^{M'mm'}_{L'\ell\ell'}\, \mathcal{G}^{m_1m'\tilde{m}_1}_{\ell_1\ell'\tilde{\ell}_1}\, \mathcal{G}^{m_2m\tilde{m}_2}_{\ell_2\ell\tilde{\ell}_2}$$

$$\times \int d^3x_3 \int \frac{d^2\hat{x}_{13}}{4\pi} \int \frac{d^2\hat{x}_{23}}{4\pi} Y^*_{LM}(\hat{x}_3) Y_{\tilde{\ell}_1\tilde{m}_1}(\hat{x}_{13}) Y_{\tilde{\ell}_2\tilde{m}_2}(\hat{x}_{23}) \times W(\vec{x}_3 + \vec{x}_{13}) W(\vec{x}_3 + \vec{x}_{23}) W(\vec{x}_3).$$

Computed via e.g. direct counting, FFT-based, etc.

The window 3PCF – **Hankel transf.**

Combination of two dimensional Hankel transforms

$$\mathcal{W}^{L}_{L'M'\ell\ell'\ell_1\ell_2}(q_1,q_2;p_1,p_2) \equiv (4\pi)^2 \int dx_{13}\, x_{13}^2 \int dx_{23}\, x_{23}^2\, j_{\ell'}(p_1 x_{13})\, j_{\ell}(p_2 x_{23})$$
$$\times \left[j_{\ell_1}(q_1 x_{13})\, j_{\ell_2}(q_2 x_{23})\, Q^{L}_{L'M'\ell\ell'\ell_1\ell_2}(x_{13},x_{23}) \right],$$

A two dimensional Hankel transform
e.g. `2DFFTLog` Fang+20

Window 3PCF multipoles

The window 3PCF – **binning**

How to handle the binning operator?

$$\mathcal{Q}^{L}_{L'M'\ell}(k_1,k_2,k_3;p_1,p_2) \simeq \sum_{\ell_1,\ell_2,\ell'} 16\pi^2 \frac{I_{\ell_2\ell_2 0}(k_1,k_2,k_3)}{I_{000}(k_1,k_2,k_3)} \mathcal{W}^{L}_{L'M'\ell\ell'\ell_1\ell_2}(k_1,k_2;p_1,p_2)\,.$$

- Evaluated at the center of the bin
- Bin numerically later

The window 3PCF - **FFT-based**

$$Q^{L}_{L'M'\ell\ell'\ell_1\ell_2}(x_{13},x_{23}) = (-1)^{M'} \sum_{\substack{M,m_1,m_2\\ m,m'}} \sum_{\substack{\tilde{\ell}_1,\tilde{\ell}_2\\ \tilde{m}_1,\tilde{m}_2}} 4\pi i^{\ell'-\ell+\ell_2-\ell_1}\ \mathcal{G}^{Mm_1m_2}_{L\ell_1\ell_2}\mathcal{G}^{M'mm'}_{L'\ell\ell'}\mathcal{G}^{\tilde{m}_1m_1m'}_{\tilde{\ell}_1\ell_1\ell'}\ \mathcal{G}^{\tilde{m}_2m_2m}_{\tilde{\ell}_2\ell_2\ell}$$

$$\times \int\ d^3x_3\ W_{\tilde{\ell}_1\tilde{m}_1}(\vec{x}_3;x_{13})W_{\tilde{\ell}_2\tilde{m}_2}(\vec{x}_3;x_{23})W_{LM}(\vec{x}_3)$$

$$W_{\ell m}(\vec{x}_3;x_{ij}) \equiv\ i^{\ell}\int \frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{x}_3}j_{\ell}(qx_{ij})Y_{\ell m}(\hat{q})W(\vec{q})$$

$$W_{LM}(\vec{x}_3) \equiv\ W(\vec{x_3})Y^{*}_{LM}(\vec{x}_3)\,.$$

Default parameters

N_p	ℓ_{max}	ℓ'_{max}	P_{min} [h Mpc^{-1}]	p_{max} [h Mpc^{-1}]
512	30	2	10^{-5}	0.5