Simulating the universe at home

- Gravitational production in Python
- 3D visualization of the primordial chaos

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Prerequisites

- A laptop and the internet

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- Computer languages (C, Python, ...), command line based OS (Linux/Mac)

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- Google

Gravitational scalar production

- Equations of motion (Φ, χ)

$$V(\phi) = \lambda M_P^4 \left| \sqrt{6} \tanh\left(\frac{\phi}{\sqrt{6} M_P} \right) \right|^k$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

Gravitational scalar production

- Equations of motion (Φ, χ)

$$X_k'' + \omega_k^2 X_k = 0 \qquad X \equiv a\chi$$

$$\omega_k^2 = k^2 - \frac{a''}{a} + a^2 m_\chi^2 \qquad -\frac{a''}{a} = -\frac{1}{6}R = -\frac{1}{6M_P^2}(4V - \dot{\phi}^2)$$

Gravitational scalar production

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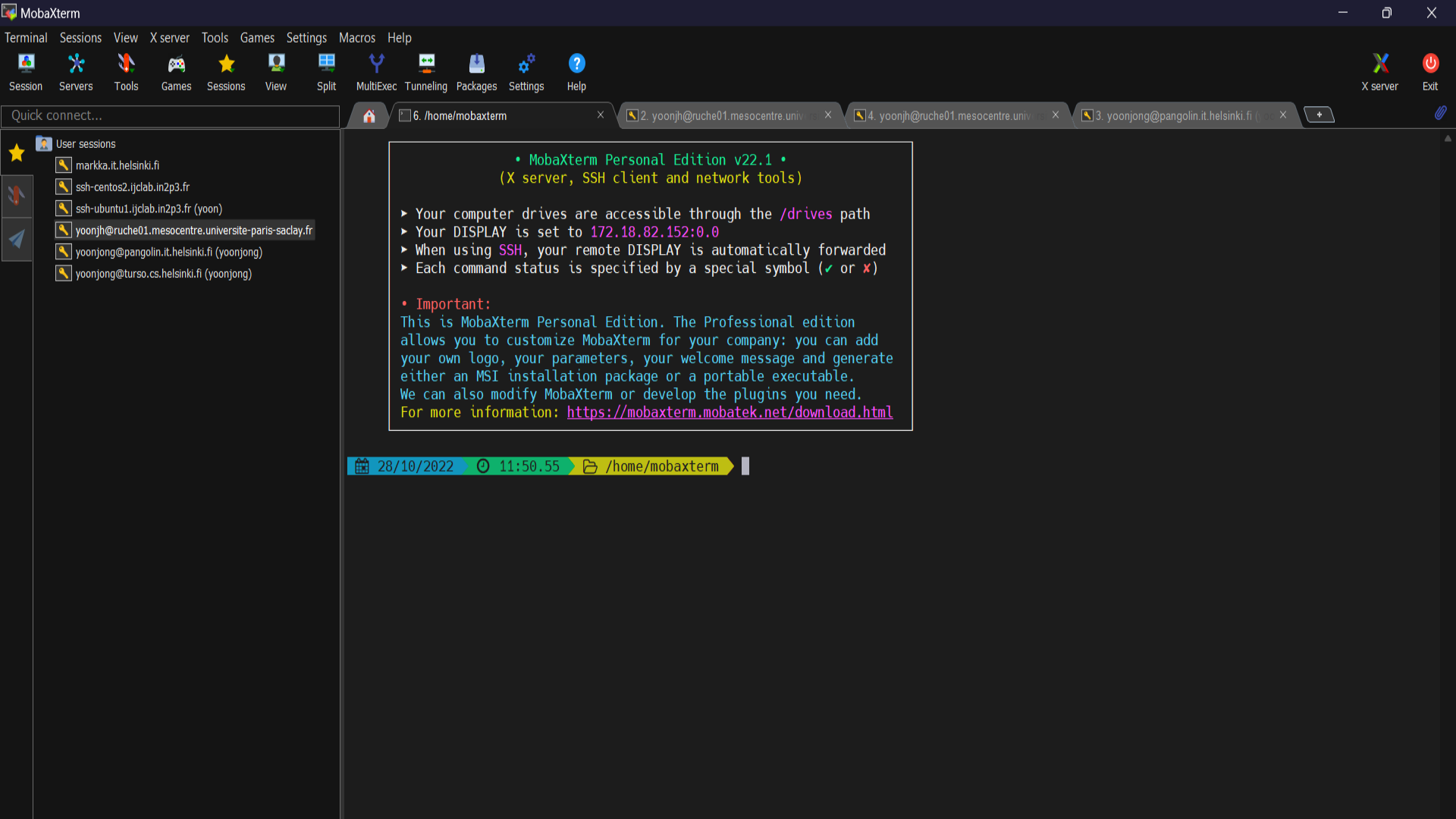
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$\ddot{X}_k + H\dot{X}_k + \left[\frac{k^2}{a^2} - \frac{1}{6}R\right] X_k = 0$$

Flowchart

- Remote access a cluster center via SSH
- A simple Python code with MPI
- Process data with Mathematica


MobaXterm
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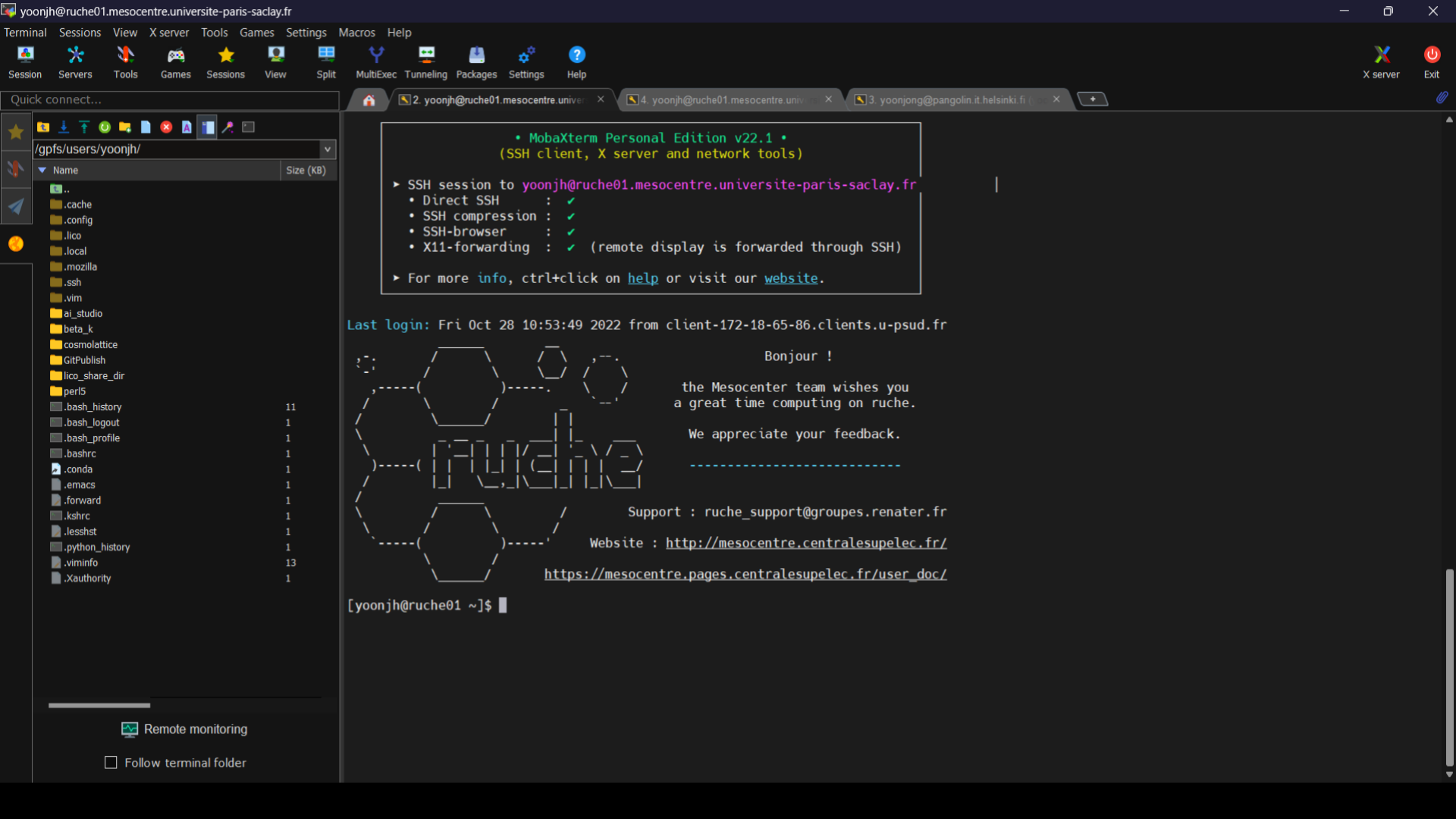
X server Exit
Quick connect...
6. /home/mobaxterm
2. yoonjh@ruche01.mesocentre.univ
4. yoonjh@ruche01.mesocentre.univ
3. yoonjong@pangolin.it.helsinki.fi
User sessions
markka.it.helsinki.fi
ssh-centos2.ijclab.in2p3.fr
ssh-ubuntu1.ijclab.in2p3.fr (yoon)
yoonjh@ruche01.mesocentre.universite-paris-saclay.fr
yoonjong@pangolin.it.helsinki.fi (yoonjong)
yoonjong@turso.cs.helsinki.fi (yoonjong)
• MobaXterm Personal Edition v22.1 •
(X server, SSH client and network tools)
▸ Your computer drives are accessible through the /drives path
▸ Your DISPLAY is set to 172.18.82.152:0.0
▸ When using SSH, your remote DISPLAY is automatically forwarded
▸ Each command status is specified by a special symbol (✔ or ✘)
• Important:
This is MobaXterm Personal Edition. The Professional edition
allows you to customize MobaXterm for your company: you can add
your own logo, your parameters, your welcome message and generate
either an MSI installation package or a portable executable.
We can also modify MobaXterm or develop the plugins you need.
For more information: https://mobaxterm.mobatek.net/download.html
28/10/2022 11:50.55 /home/mobaxterm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X server Exit
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4. yoonjh@ruche01.mesocentre.univ
3. yoonjong@pangolin.it.helsinki.fi
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Name Size (KB)
..
.cache
.config
.lico
.local
.mozilla
.ssh
.vim
ai_studio
beta_k
cosmolattice
GitPublish
lico_share_dir
perl5
.bash_history 11
.bash_logout 1
.bash_profile 1
.bashrc 1
.conda 1
.emacs 1
.forward 1
.kshrc 1
.lesshst 1
.python_history 1
.viminfo 13
.Xauthority 1
Remote monitoring
Follow terminal folder
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(SSH client, X server and network tools)
▸ SSH session to yoonjh@ruche01.mesocentre.universite-paris-saclay.fr
• Direct SSH : ✔
• SSH compression : ✔
• SSH-browser : ✔
• X11-forwarding : ✔ (remote display is forwarded through SSH)
▸ For more info, ctrl+click on help or visit our website.
Last login: Fri Oct 28 10:53:49 2022 from client-172-18-65-86.clients.u-psud.fr
Bonjour !
the Mesocenter team wishes you
a great time computing on ruche.
We appreciate your feedback.
Support : ruche_support@groupes.renater.fr
Website : http://mesocentre.centralesupelec.fr/
https://mesocentre.pages.centralesupelec.fr/user_doc/
[yoonjh@ruche01 ~]$

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* beta_k_mpi.py

```
import time
import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt
from mpi4py import MPI


#############
##
##
##        V=Lambda Mpl^4 (sqrt(6)*tanh(phi/sqrt(6)))^k
##        dz=sqrt(Lambda)*Mpl dt
##        fr=fp/Mpl
##
##
#############

k=int(2)

def V(phi):
    phi=np.real(phi)
    return (np.sqrt(6.)*np.tanh(phi/np.sqrt(6.)))**k

def dV(phi):
    phi=np.real(phi)
    return k*np.sqrt(6.)**(k-1)/np.cosh(phi/np.sqrt(6.))**2*np.tanh(phi/np.sqrt(6.))**(k-1)

def H(phi, phip):
    phi=np.real(phi)
    phip=np.real(phip)
    return np.sqrt( 0.5*phip**2. + V(phi)) / (np.sqrt(3.))



# y[0]=phi y[1]=phi' y[2]=a y[3]=chi y[4]=chi'

def f(t, y):
    return np.array([ np.real(y[1]),  - dV(y[0]) - 3*H(y[0], y[1])*np.real(y[1]) , y[2]*H(y[0], y[1]),y[4], -y[4]*H(y[0], y[1])-y[3]*((kc/np.real(y[2]))**2.+Mchi**2.+(-1./6.)*(4.*V(y[0])-np.real(y[1])**2.))])

def end(t,y):
    phi=np.real(y[0])
    phip=np.real(y[1])
    return V(phi)-phip**2.

#
def beta_k(chi, chip, a, kc, Mchi):
    wc=np.sqrt(kc**2.+a**2.*Mchi**2.)
    return 0.5*np.abs(a*chip)**2./wc+0.5*wc*np.abs(chi)**2.+complex(0.,1.)/2.*(np.conj(a*chip)*chi-np.conj(chi)*a*chip)
```

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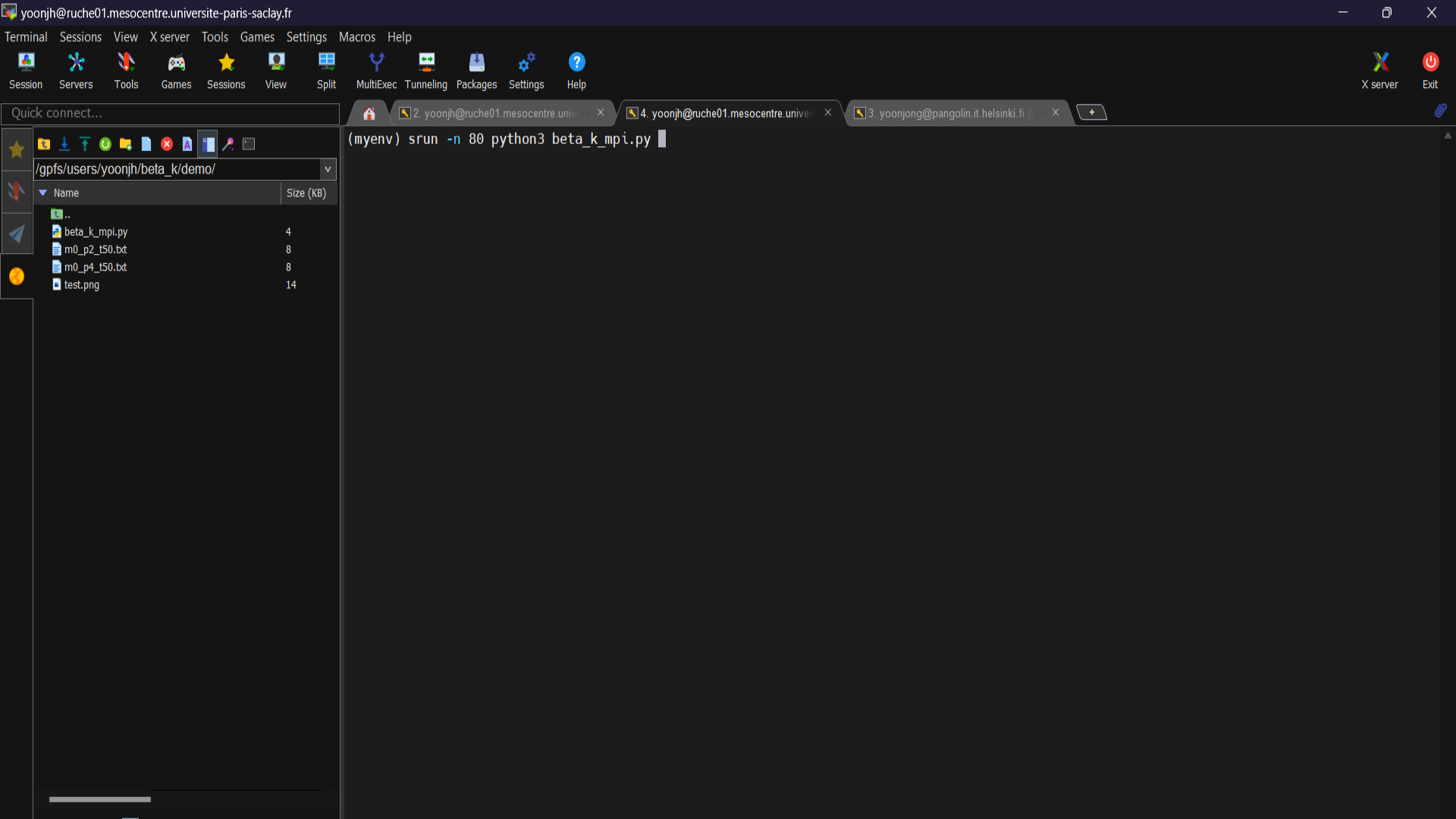
* beta_k_mpi.py

```
108
109 comm = MPI.COMM_WORLD
110 rank = comm.Get_rank()
111
112 phi0 = float(3.5)
113 phip0 = float(- 0.)
114 a0=float(1.)
115
116 start_time = 0.
117 stop_time = 50
118
119 time_eval = np.linspace(start_time, stop_time, 20000)
120
121 nq=int(80)
122
123 if rank == 0:
124     q = np.logspace(-2.0,2.0,nq)
125 else:
126     q = None
127
128
129 q = comm.scatter(q, root=0)
130
131 result = []
132 beta_k_array = []
133 beta_k_array0 = []
134
135 kc=float(q*aend*Hend)
136 Mchi=float(np.sqrt(2.0)*0.0)
137
138 chi0=complex(1./np.sqrt(2.*kc),0.)
139 chip0=complex(0.,-np.sqrt(kc/2.))
140
141 y0 = np.array([phi0, phip0, a0, chi0, chip0])
142
143 sol2 = solve_ivp(f, [start_time, stop_time], y0, method='BDF', t_eval=time_eval,events=end)
144 result.append(q)
145 result.append(kc)
146 result.append(H(np.real(sol2.y_events[0][0][0]),np.real(sol2.y_events[0][0][1]) ))
147 result.append(beta_k(sol2.y[3,-1],sol2.y[4,-1],sol2.y[2,-1],kc,Mchi)*q**3/(2.*np.pi**2))
148 result.append(beta_k(sol2.y[3,0],sol2.y[4,0],sol2.y[2,0],kc,Mchi)*q**3/(2.*np.pi**2))
149
150
151 newData = comm.gather(result,root=0)
152
153 if rank == 0:
154     arr=np.array([])
155     for kn in range(nq):
156         arr=np.append(arr,[np.real(newData[kn][0]),newData[kn][3]])
157     np.savetxt('beta_k.txt', np.reshape(arr,(-1,2)), delimiter=' ')
158
```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4. yoonjh@ruche01.mesocentre.univer
3. yoonjong@pangolin.it.helsinki.fi
/gpfs/users/yoonjh/beta_k/demo/
Name
Size (KB)
..
beta_k_mpi.py 4
m0_p2_t50.txt 8
m0_p4_t50.txt 8
test.png 14
(myenv) srun -n 80 python3 beta_k_mpi.py

read_pyout.nb * - Wolfram Mathematica

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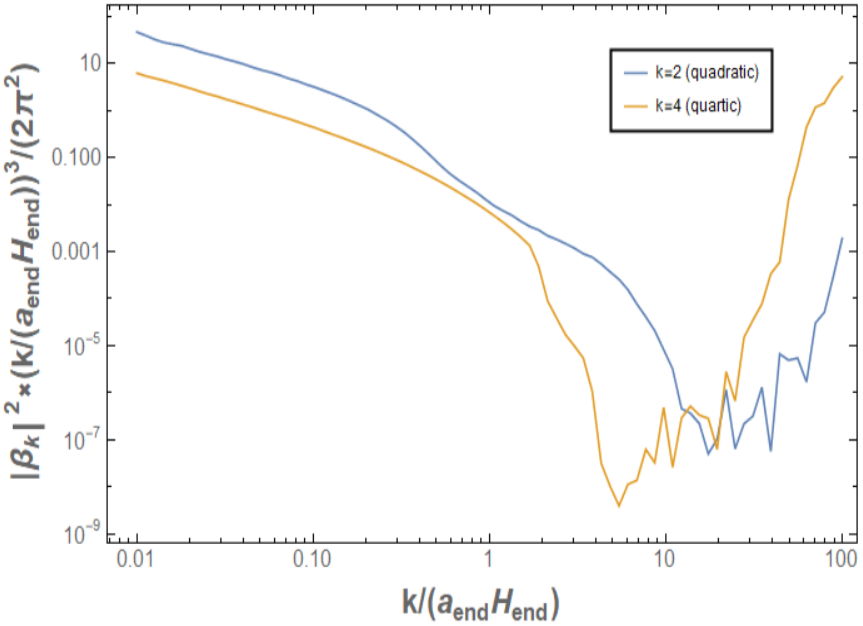
In[11]:=
```
PSS[0] = Map[ToExpression@StringReplace[#, "e" → "*^"] &, Import[NotebookDirectory[] <> "m0_p2_t50.txt", "Table"], 1] /. j → I;
PSS[1] = Map[ToExpression@StringReplace[#, "e" → "*^"] &, Import[NotebookDirectory[] <> "m0_p4_t50.txt", "Table"], 1] /. j → I;

ListLogLogPlot[{PSS[0], PSS[1]}, Joined → True, Frame -> True, FrameLabel → {Style["k/(aendHend)", Bold], Style["|βk|^2 × (k/(aendHend))^3/(2π^2)", Bold]}, ImagePadding → {{100, 20}, {70, 10}},
 ImageSize → 720, PlotLegends → Placed[LineLegend[{"k=2 (quadratic)", "k=4 (quartic)"}, LegendFunction → (Framed[#, RoundingRadius → 0] &)], {0.78, 0.84}], FrameTicksStyle → Directive[16],
 FrameStyle → Directive[24], PlotRange → All]
```

Out[13]=



Discussion

- Limitations
- How to improve?

Virtual discrete universe

- Solving EOMs on discrete space
- Required computer resources VS Advancing technology

Lattice simulations

- LATTICEEASY, CLUSTEREASY, Defrost, CUDAEasy, HLattice, PyCOOL, ...

- CosmoLattice (2021~)

CosmoLattice (2021~)

- Scalar and gauge field dynamics in an expanding universe

- Developed by Daniel G. Figueroa, Adrien Florio, Francisco Torrenti, and Wessel Valkenburg

- MPI-based, GW, 3-D visualization, etc.

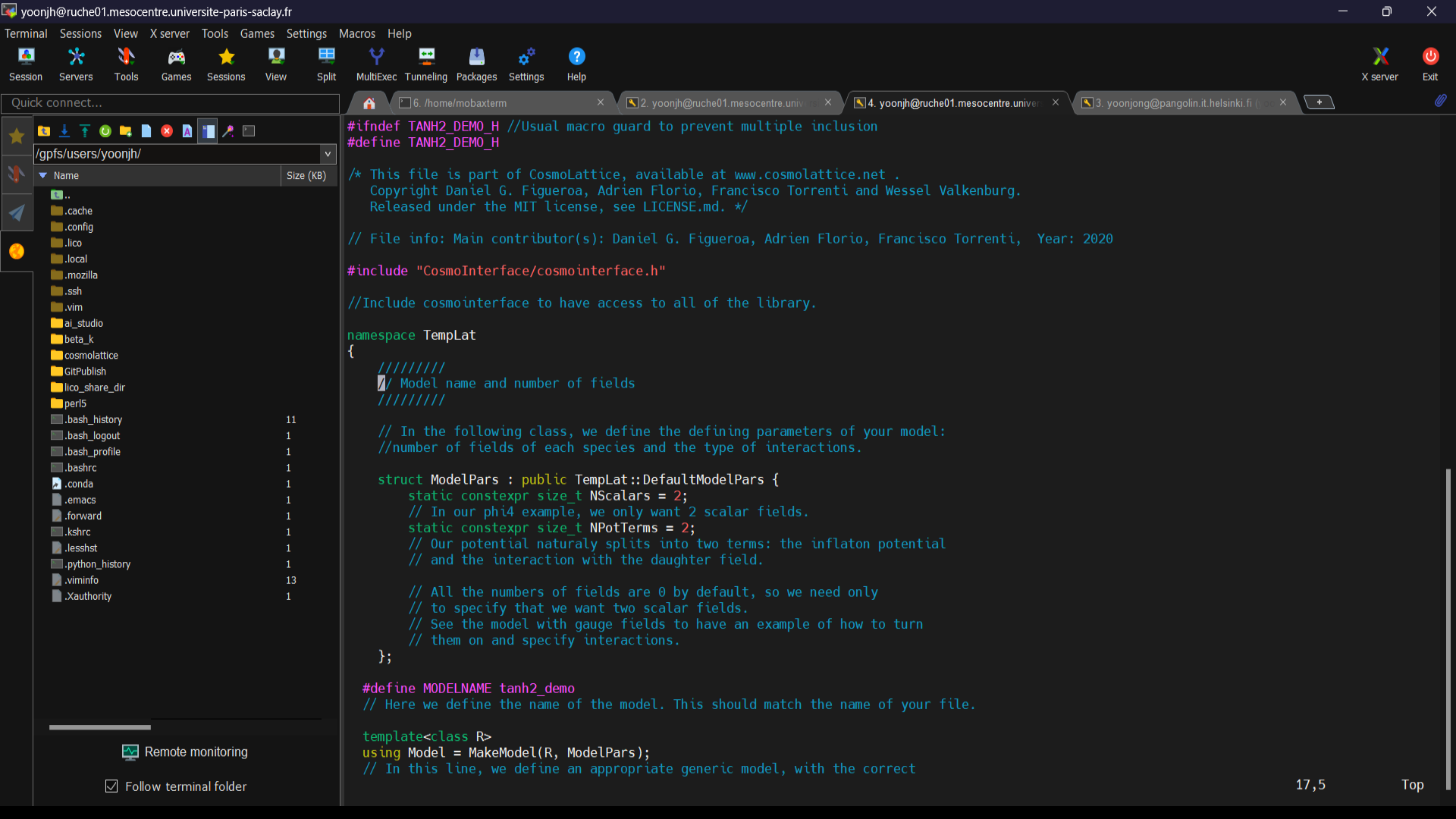
Flowchart

- Write a model (V, dV, ...)

- Compile and execute

(set the simulation and initial parameters carefully)

- Refine output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3. yoonjong@pangolin.it.helsinki.fi
/gpfs/users/yoonjh/
Name
Size (KB)
..
.cache
.config
.lico
.local
.mozilla
.ssh
.vim
ai_studio
beta_k
cosmolattice
GitPublish
lico_share_dir
perl5
.bash_history 11
.bash_logout 1
.bash_profile 1
.bashrc 1
.conda 1
.emacs 1
.forward 1
.kshrc 1
.lesshst 1
.python_history 1
.viminfo 13
.Xauthority 1
Remote monitoring
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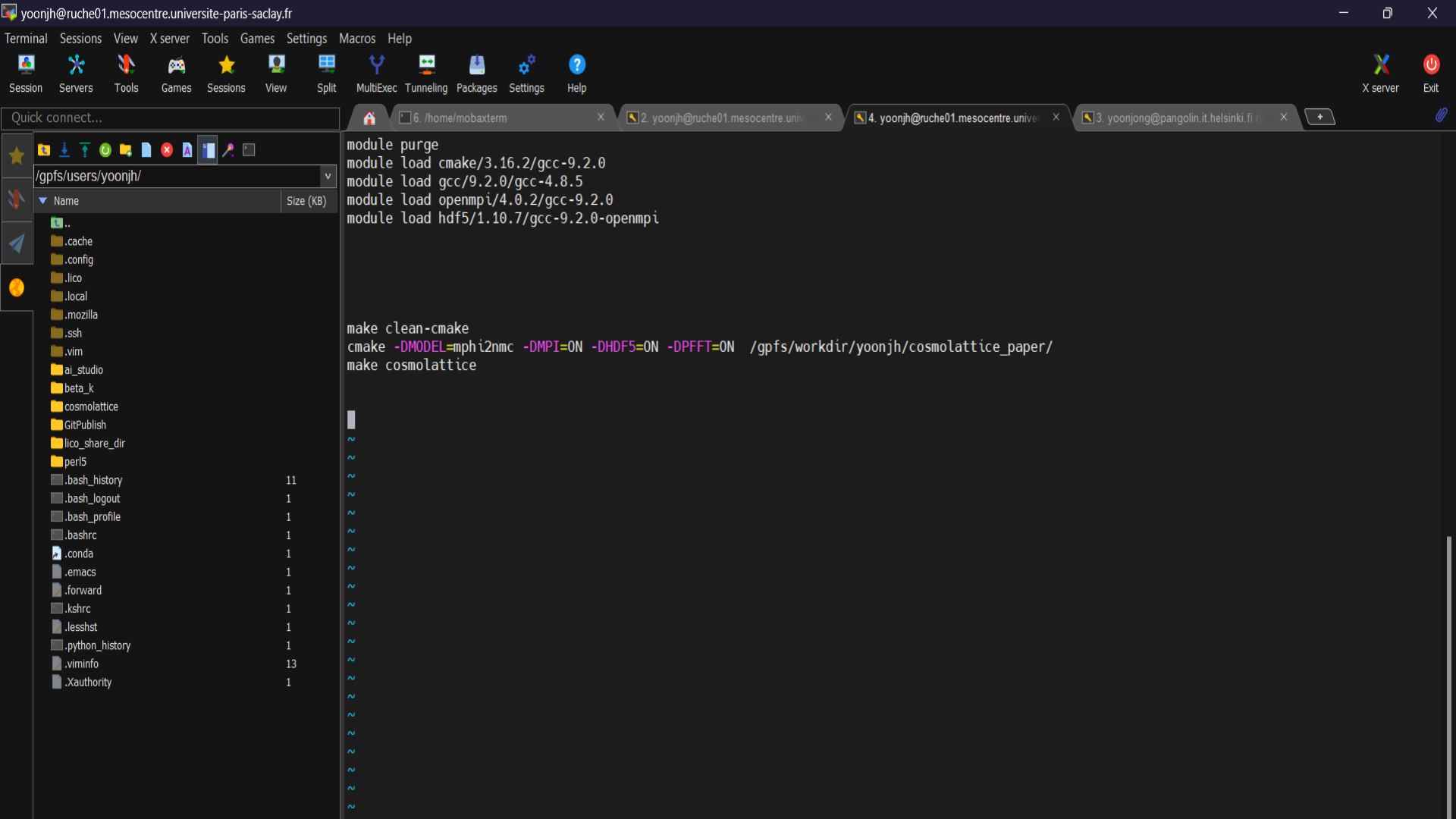
#ifndef TANH2_DEMO_H //Usual macro guard to prevent multiple inclusion
#define TANH2_DEMO_H
/* This file is part of CosmoLattice, available at www.cosmolattice.net .
Copyright Daniel G. Figueroa, Adrien Florio, Francisco Torrenti and Wessel Valkenburg.
Released under the MIT license, see LICENSE.md. */
// File info: Main contributor(s): Daniel G. Figueroa, Adrien Florio, Francisco Torrenti, Year: 2020
#include "CosmoInterface/cosmointerface.h"
//Include cosmointerface to have access to all of the library.
namespace TempLat
{
////////
// Model name and number of fields
////////
// In the following class, we define the defining parameters of your model:
//number of fields of each species and the type of interactions.
struct ModelPars : public TempLat::DefaultModelPars {
static constexpr size_t NScalars = 2;
// In our phi4 example, we only want 2 scalar fields.
static constexpr size_t NPotTerms = 2;
// Our potential naturaly splits into two terms: the inflaton potential
// and the interaction with the daughter field.
// All the numbers of fields are 0 by default, so we need only
// to specify that we want two scalar fields.
// See the model with gauge fields to have an example of how to turn
// them on and specify interactions.
};
#define MODELNAME tanh2_demo
// Here we define the name of the model. This should match the name of your file.
template<class R>
using Model = MakeModel(R, ModelPars);
// In this line, we define an appropriate generic model, with the correct
17,5 Top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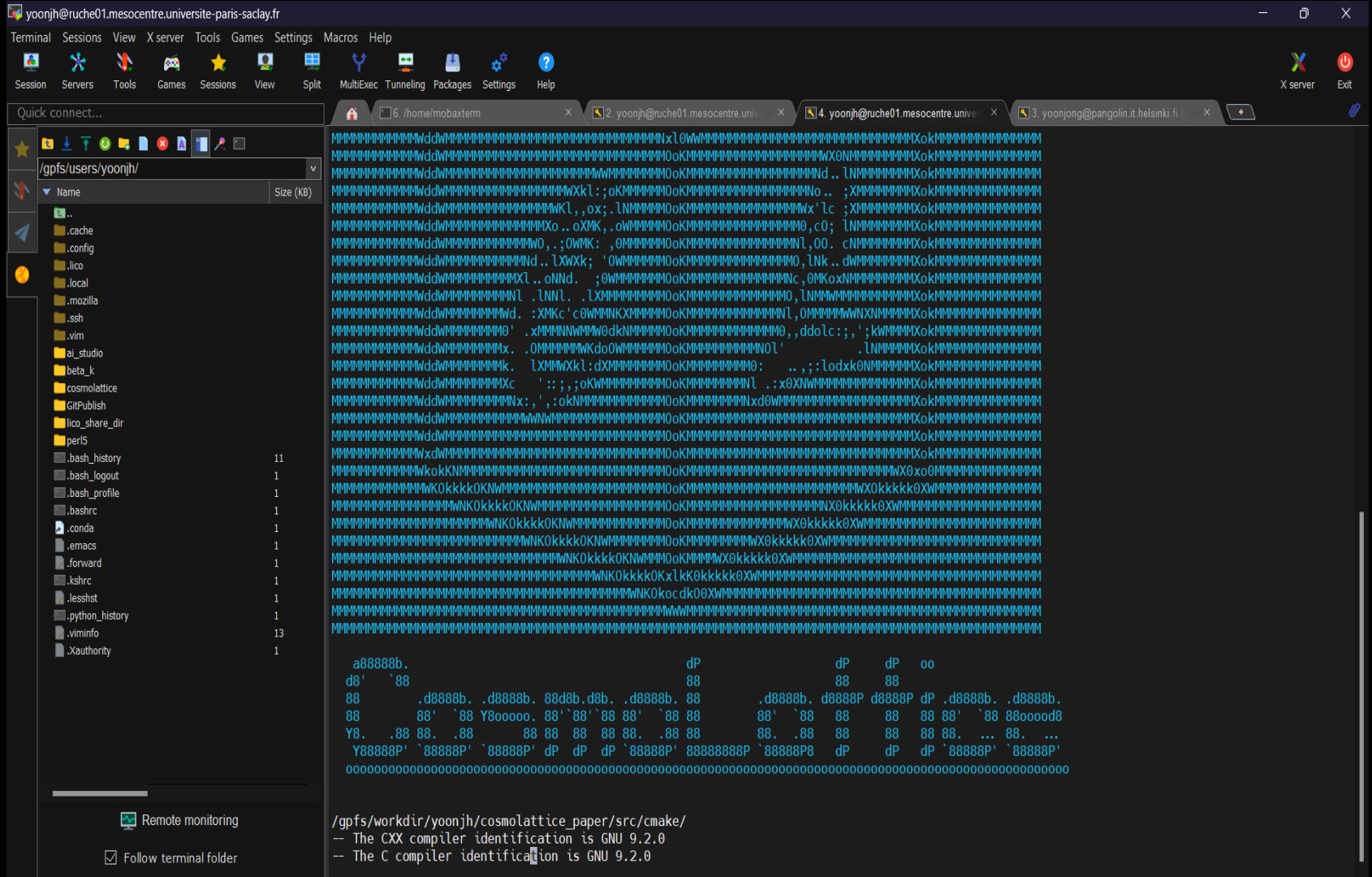
/gpfs/users/yoonjh/
Name
Size (KB)
..
.cache
.config
.lico
.local
.mozilla
.ssh
.vim
ai_studio
beta_k
cosmolattice
GitPublish
lico_share_dir
perl5
.bash_history 11
.bash_logout 1
.bash_profile 1
.bashrc 1
.conda 1
.emacs 1
.forward 1
.kshrc 1
.lesshst 1
.python_history 1
.viminfo 13
.Xauthority 1
module purge
module load cmake/3.16.2/gcc-9.2.0
module load gcc/9.2.0/gcc-4.8.5
module load openmpi/4.0.2/gcc-9.2.0
module load hdf5/1.10.7/gcc-9.2.0-openmpi
make clean-cmake
cmake -DMODEL=mphi2nmc -DMPI=ON -DHDF5=ON -DPFFT=ON /gpfs/workdir/yoonjh/cosmolattice_paper/
make cosmolattice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perl5
.bash_history 11
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.bashrc 1
.conda 1
.emacs 1
.forward 1
.kshrc 1
.lesshst 1
.python_history 1
.viminfo 13
.Xauthority 1
Remote monitoring
Follow terminal folder
/gpfs/workdir/yoonjh/cosmolattice_paper/src/cmake/
-- The CXX compiler identification is GNU 9.2.0
-- The C compiler identification is GNU 9.2.0

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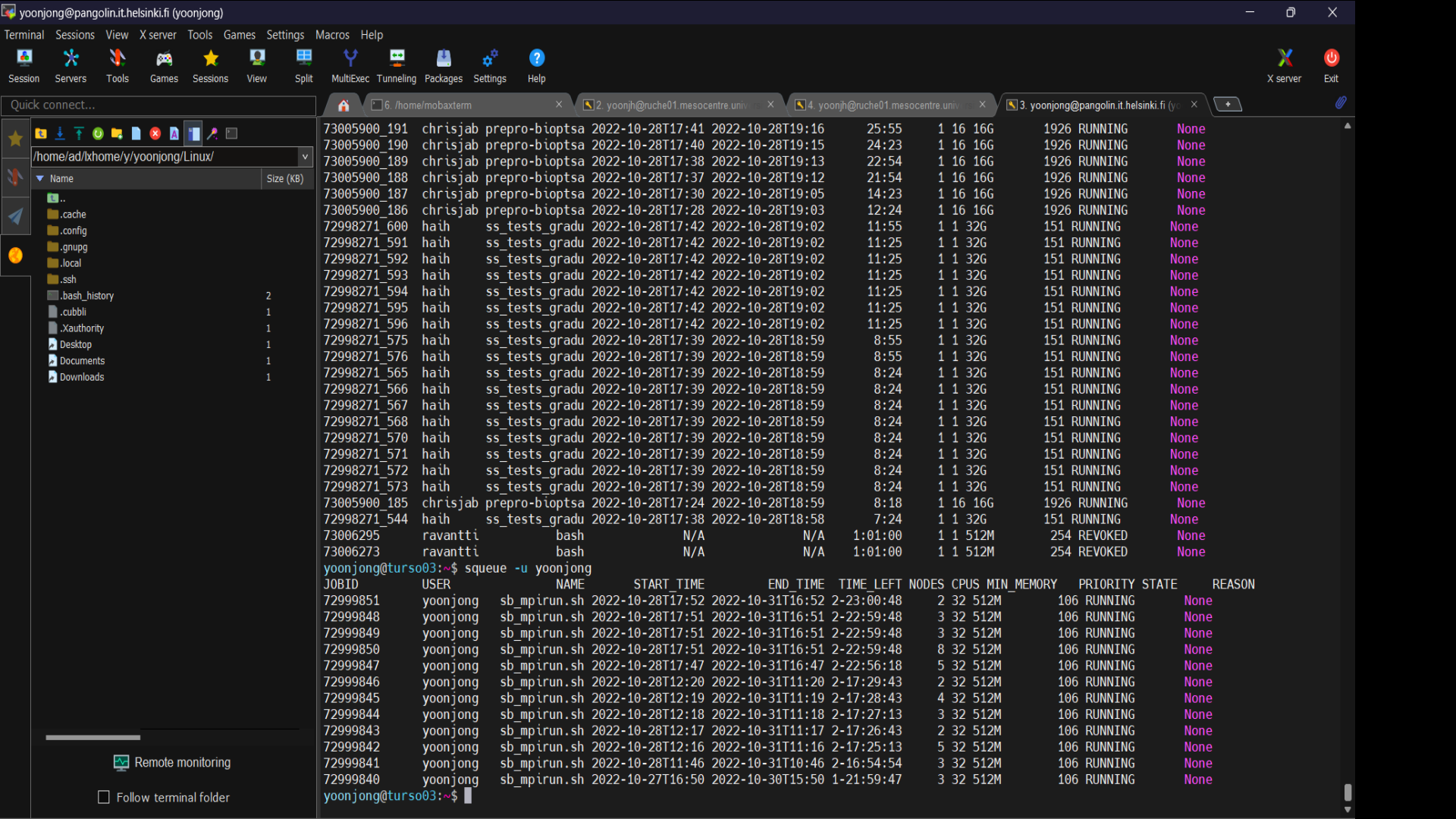
4. yoonjh@ruche01.mesocentre.univer
3. yoonjong@pangolin.it.helsinki.fi
/gpfs/users/yoonjh/
Name Size (KB)
.. .cache .config .lico .local .mozilla .ssh .vim ai_studio beta_k cosmolattice GitPublish lico_share_dir perl5
.bash_history 11
.bash_logout 1
.bash_profile 1
.bashrc 1
.conda 1
.emacs 1
.forward 1
.kshrc 1
.lesshst 1
.python_history 1
.viminfo 13
.Xauthority 1
Remote monitoring
Follow terminal folder
#!/bin/bash
#SBATCH --job-name=cosmolattice
#SBATCH --time=04:00:00
#SBATCH --ntasks=64
#SBATCH --cpus-per-task=1
#SBATCH --partition=cpu_med
#SBATCH -o log.txt
module purge
module load openmpi/4.0.2/gcc-9.2.0
module load hdf5/1.10.7/gcc-9.2.0-openmpi
srun tanh2_demo input=./tanh2_demo.in
-- INSERT -- 13,1 All


yoonjong@pangolin.it.helsinki.fi (yoonjong)
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Name
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..
.cache
.config
.gnupg
.local
.ssh
.bash_history 2
.cubbli 1
.Xauthority 1
Desktop 1
Documents 1
Downloads 1
Remote monitoring
Follow terminal folder
73005900_191 chrisjab prepro-bioptsa 2022-10-28T17:41 2022-10-28T19:16 25:55 1 16 16G 1926 RUNNING None
73005900_190 chrisjab prepro-bioptsa 2022-10-28T17:40 2022-10-28T19:15 24:23 1 16 16G 1926 RUNNING None
73005900_189 chrisjab prepro-bioptsa 2022-10-28T17:38 2022-10-28T19:13 22:54 1 16 16G 1926 RUNNING None
73005900_188 chrisjab prepro-bioptsa 2022-10-28T17:37 2022-10-28T19:12 21:54 1 16 16G 1926 RUNNING None
73005900_187 chrisjab prepro-bioptsa 2022-10-28T17:30 2022-10-28T19:05 14:23 1 16 16G 1926 RUNNING None
73005900_186 chrisjab prepro-bioptsa 2022-10-28T17:28 2022-10-28T19:03 12:24 1 16 16G 1926 RUNNING None
72998271_600 haih ss_tests_gradu 2022-10-28T17:42 2022-10-28T19:02 11:55 1 1 32G 151 RUNNING None
72998271_591 haih ss_tests_gradu 2022-10-28T17:42 2022-10-28T19:02 11:25 1 1 32G 151 RUNNING None
72998271_592 haih ss_tests_gradu 2022-10-28T17:42 2022-10-28T19:02 11:25 1 1 32G 151 RUNNING None
72998271_593 haih ss_tests_gradu 2022-10-28T17:42 2022-10-28T19:02 11:25 1 1 32G 151 RUNNING None
72998271_594 haih ss_tests_gradu 2022-10-28T17:42 2022-10-28T19:02 11:25 1 1 32G 151 RUNNING None
72998271_595 haih ss_tests_gradu 2022-10-28T17:42 2022-10-28T19:02 11:25 1 1 32G 151 RUNNING None
72998271_596 haih ss_tests_gradu 2022-10-28T17:42 2022-10-28T19:02 11:25 1 1 32G 151 RUNNING None
72998271_575 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:55 1 1 32G 151 RUNNING None
72998271_576 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:55 1 1 32G 151 RUNNING None
72998271_565 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:24 1 1 32G 151 RUNNING None
72998271_566 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:24 1 1 32G 151 RUNNING None
72998271_567 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:24 1 1 32G 151 RUNNING None
72998271_568 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:24 1 1 32G 151 RUNNING None
72998271_570 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:24 1 1 32G 151 RUNNING None
72998271_571 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:24 1 1 32G 151 RUNNING None
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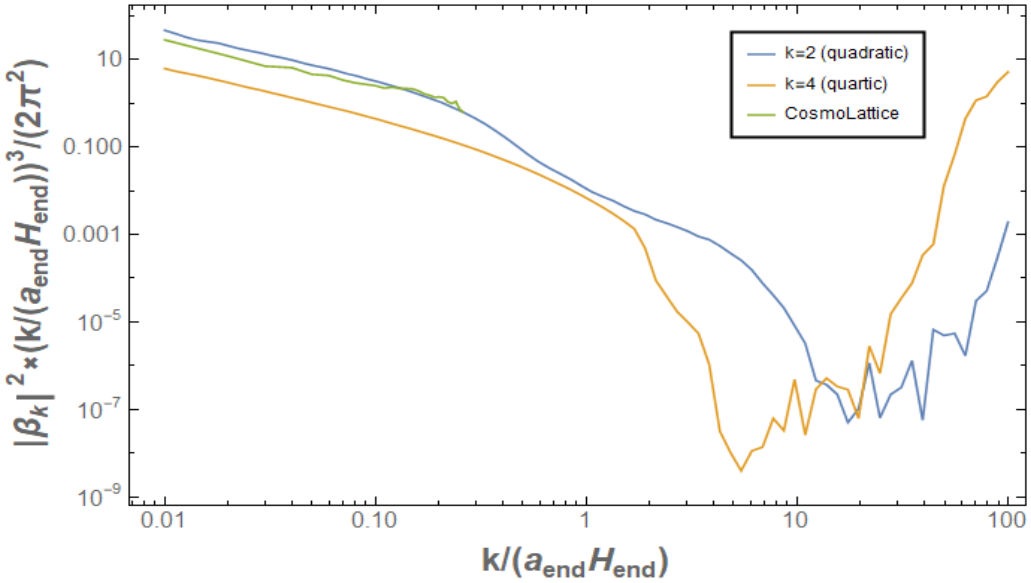
72998271_573 haih ss_tests_gradu 2022-10-28T17:39 2022-10-28T18:59 8:24 1 1 32G 151 RUNNING None
73005900_185 chrisjab prepro-bioptsa 2022-10-28T17:24 2022-10-28T18:59 8:18 1 16 16G 1926 RUNNING None
72998271_544 haih ss_tests_gradu 2022-10-28T17:38 2022-10-28T18:58 7:24 1 1 32G 151 RUNNING None
73006295 ravantti bash N/A N/A 1:01:00 1 1 512M 254 REVOKED None
73006273 ravantti bash N/A N/A 1:01:00 1 1 512M 254 REVOKED None
yoonjong@turso03:~$ squeue -u yoonjong
JOBID USER NAME START_TIME END_TIME TIME_LEFT NODES CPUS MIN_MEMORY PRIORITY STATE REASON
72999851 yoonjong sb_mpirun.sh 2022-10-28T17:52 2022-10-31T16:52 2-23:00:48 2 32 512M 106 RUNNING None
72999848 yoonjong sb_mpirun.sh 2022-10-28T17:51 2022-10-31T16:51 2-22:59:48 3 32 512M 106 RUNNING None
72999849 yoonjong sb_mpirun.sh 2022-10-28T17:51 2022-10-31T16:51 2-22:59:48 3 32 512M 106 RUNNING None
72999850 yoonjong sb_mpirun.sh 2022-10-28T17:51 2022-10-31T16:51 2-22:59:48 8 32 512M 106 RUNNING None
72999847 yoonjong sb_mpirun.sh 2022-10-28T17:47 2022-10-31T16:47 2-22:56:18 5 32 512M 106 RUNNING None
72999846 yoonjong sb_mpirun.sh 2022-10-28T12:20 2022-10-31T11:20 2-17:29:43 2 32 512M 106 RUNNING None
72999845 yoonjong sb_mpirun.sh 2022-10-28T12:19 2022-10-31T11:19 2-17:28:43 4 32 512M 106 RUNNING None
72999844 yoonjong sb_mpirun.sh 2022-10-28T12:18 2022-10-31T11:18 2-17:27:13 3 32 512M 106 RUNNING None
72999843 yoonjong sb_mpirun.sh 2022-10-28T12:17 2022-10-31T11:17 2-17:26:43 2 32 512M 106 RUNNING None
72999842 yoonjong sb_mpirun.sh 2022-10-28T12:16 2022-10-31T11:16 2-17:25:13 5 32 512M 106 RUNNING None
72999841 yoonjong sb_mpirun.sh 2022-10-28T11:46 2022-10-31T10:46 2-16:54:54 3 32 512M 106 RUNNING None
72999840 yoonjong sb_mpirun.sh 2022-10-27T16:50 2022-10-30T15:50 1-21:59:47 3 32 512M 106 RUNNING None
yoonjong@turso03:~$

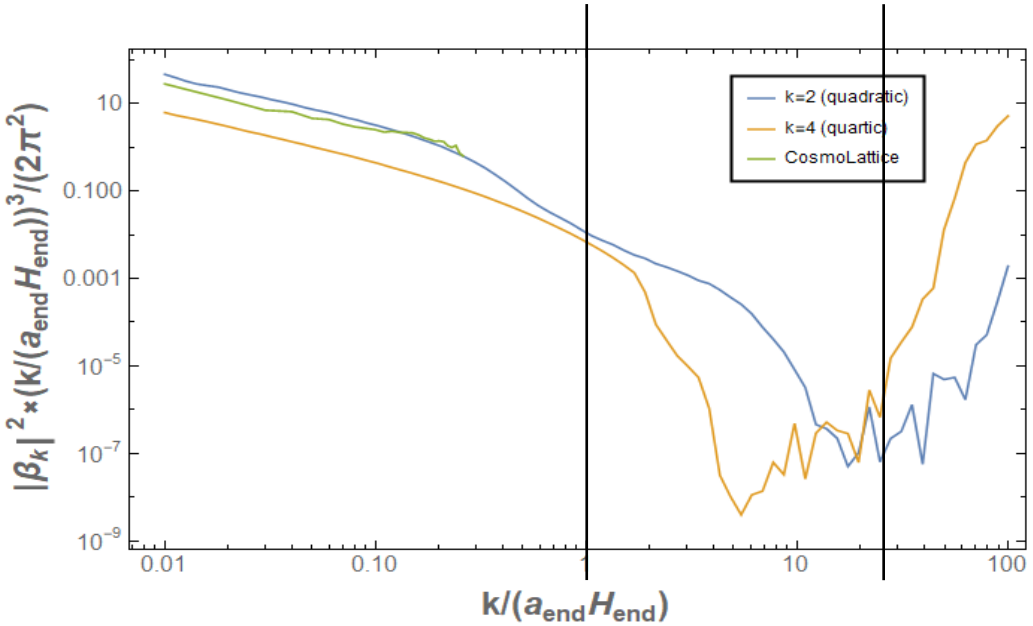
Simulation output

- (Space-averaged) energy densities, field values, ...
- (K-binned) power spectra, frequency, occupation number
- The scale factor, Hubble paramter, energy conservation

$0.01 < k/a_e H_e < 0.27$



$1 < k/a_e H_e < 27$


k=2 (quadratic)
k=4 (quartic)
CosmoLattice
10
0.100
0.001
10^{-5}
10^{-7}
10^{-9}
0.01
0.10
1
10
100
$|\beta_k|^2 * (k/(a_{end} H_{end}))^3/(2\pi^2)$
$k/(a_{end} H_{end})$

$1 < k/a_e H_e < 27$

- k >> aH initially. Soon k ~ aH, tachyonic resonance expected to prevail

- After inflation, production still goes on as R/H^2 oscillates and adibaticity can be violated

- For t >> t_{end}, turbulance over, fluctuations freeze

3D Visualization

- $t_0 \sim 0$
- $t_{end} \sim 6$
- $t_{max} \sim 50$