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Inflation, preheating, reheating: signatures and codes

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$$\Rightarrow \qquad \dot{\rho}_{\phi} + 3H\rho_{\phi} = -\Gamma_{\phi}\rho_{\phi}$$

radiation

$$\begin{array}{rcl} T^{\mu\nu} &=& T^{\mu\nu}_{\phi} + T^{\mu\nu}_{R} \\ &=& \rho_{\phi} \operatorname{diag}(1,0,0,0) \ + \ \rho_{R} \operatorname{diag}(1,\frac{1}{3},\frac{1}{3},\frac{1}{3}) \end{array}$$

 $\nabla_{\mu} T^{\mu\nu} = 0 \qquad \qquad \Rightarrow \qquad \dot{\rho}_R + 4H\rho_R = \Gamma_{\phi}\rho_{\phi}$





1. Perturbative

reheating

Non-quadratic minimum



$$\begin{split} V(\phi) \;&=\; \lambda M_P^4 \left[\sqrt{6} \tanh\left(\frac{\phi}{\sqrt{6}M_P}\right) \right]^k \\ \stackrel{\phi \ll M_P}{\longrightarrow} \; \lambda \frac{\phi^k}{M_P^{k-4}} \end{split}$$

R. Kallosh and A. Linde, JCAP 07 (2013), 002

$$\phi(t) \simeq \phi_0(t) \mathcal{P}(t)$$

= $\phi_0(t) \sum_n \mathcal{P}_n e^{-in\omega t}$

$$\rho_{\phi} \simeq \frac{k+2}{2} \langle V(\phi) \rangle = V(\phi_0)$$
$$p_{\phi} \simeq \frac{k-2}{k+2} V(\phi_0)$$



quartic minimum \approx radiation



2. Thermalization

3. Preheating



4. Backreaction



Non-quadratic minimum

More careful Boltzmann treatment:

$$\frac{\partial f_R}{\partial t} - H|\mathbf{P}| \frac{\partial f_R}{\partial |\mathbf{P}|} = \frac{1}{P^0} \sum_{n=1}^{\infty} \int \frac{d^3 \mathbf{K}}{(2\pi)^3 n_{\phi}} \frac{d^3 \mathbf{P}'}{(2\pi)^3 2P'^0} (2\pi)^4 \delta^{(4)} (K_n - P - P') |\overline{\mathcal{M}}_n|^2 \times \left[f_{\phi}(K) (1 \pm f_R(P)) (1 \pm f_R(P')) - f_R(P) f_R(P') (1 + f_{\phi}(K)) \right]$$



$$\dot{\rho}_{\phi} + 3H(1+w_{\phi})\rho_{\phi} = -\Gamma_{\phi}(1+w_{\phi})\rho_{\phi}$$
$$\dot{\rho}_{R} + 4H\rho_{R} = \Gamma_{\phi}(1+w_{\phi})\rho_{\phi}$$

$$\Gamma_{\phi} = \frac{1}{8\pi (1+w_{\phi})\rho_{\phi}} \sum_{n=1}^{\infty} \left\langle |\mathcal{M}_{n}|^{2} E_{n} \beta_{n} \right\rangle, \quad \beta_{n} = \sqrt{\left(1 - \frac{(m_{1} + m_{2})^{2}}{E_{n}^{2}}\right) \left(1 - \frac{(m_{1} - m_{2})^{2}}{E_{n}^{2}}\right)}$$







3. Preheating



4. Backreaction



Nature of final state matters!



 $\Gamma_{\phi}(t) \propto \left(rac{
ho_{\phi}}{M_{
m D}^4}
ight)^{\ell}$

MG, K. Kaneta, Y. Mambrini, K. A. Olive, PRD 101 (2020), 123507 MG, K. Kaneta, Y. Mambrini, K. A. Olive, JCAP 04 (2021), 012 MG, M. Pierre, S. Verner, arXiv:2206.08940 [hep-ph]





3. Preheating

4. Backreaction

Nature of final state matters!







$$\rho_{\phi} \propto \begin{cases} e^{-\Gamma_{\phi}t}, & k=2\\ t^{\frac{2k}{2-k}}, & k\neq 2 \end{cases}$$





Nature of final state matters!

























4. Backreaction



Nature of final state matters!





$$\rho_{\phi} \propto \begin{cases} e^{-\Gamma_{\phi}t}, & k=2\\ ??, & k\neq 2 \end{cases}$$





3. Preheating



4. Backreaction



The path to thermal equilibrium



Thermal equilibrium: decay products need to slow-down and multiply







3. Preheating



4. Backreaction



5. Signals

The path to thermal equilibrium

A. Kurkela, G. Moore, 1107.5050



$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_{\chi}] - \mathcal{C}^{``1\leftrightarrow 2"}[f_{\chi}] + \cdots,$$





3. Preheating



4. Backreaction



5. Signals

The path to thermal equilibrium







K. Harigaya, K. Mukaida, 1312.3097; K. Mukaida, M. Yamada, 1506.07661







3. Preheating



4. Backreaction

The path to thermal equilibrium

A. Kurkela, G. Moore, 1107.5050



$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_{\chi}] - \mathcal{C}^{"1\leftrightarrow 2"}[f_{\chi}] + \cdots,$$











3. Preheating

Beyond perturbation theory: fermions





 $(i\gamma^{\mu}\partial_{\mu} - am_{\psi}(t))\Psi = 0$

Introducing $\Psi \equiv a^{3/2}\psi$,

$$\Psi(\tau, \mathbf{x}) = \sum_{r=\pm} \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2}} e^{-i\mathbf{p}\cdot\mathbf{x}} \left[u_p^{(r)}(\tau) \hat{a}_p^{(r)} + v_p^{(r)}(\tau) \hat{b}_{-\mathbf{p}}^{(r)\dagger} \right]$$

w

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		•	••





and

$$U_{1}'(\tau) = -ipU_{2}(\tau) - iam_{\psi}U_{1}(\tau)$$
$$U_{2}'(\tau) = -ipU_{1}(\tau) + iam_{\psi}U_{2}(\tau)$$

 $u_p^{(r)}(\tau) = \frac{1}{\sqrt{2}} \begin{pmatrix} U_1(\tau)\xi_r(\mathbf{p}) \\ U_2(\tau) \underline{\sigma} \cdot \underline{p} \xi_-(\mathbf{p}) \end{pmatrix}$







Beyond perturbation theory: fermions

$${\cal L} \ \supset \ y \, \phi ar \psi y$$

 $b\bar{\psi}\psi$ $\phi = --- \frac{y}{\bar{\psi}}$

$$(i\gamma^\mu\partial_\mu - am_\psi(t))\Psi = 0$$

3. Preheating



4. Backreaction

The UV-convergent energy density is

$$ho_{\psi} = rac{1}{(2\pi)^3 a^4} \int d^3 oldsymbol{p} \, \omega_p n_p$$

~

with the occupation number (PSD)

$$n_p = \frac{1}{2} \left| \left(1 + \frac{am_{\psi}}{\omega_p} \right)^{1/2} U_2 - \left(1 - \frac{am_{\psi}}{\omega_p} \right)^{1/2} U_1 \right|^2 = f_{\psi}(p,t)$$

MG, K. Kaneta, Y. Mambrini, K. A. Olive, S. Verner, JCAP 03 (2022), 016





Beyond perturbation theory: fermions





 $(i\gamma^{\mu}\partial_{\mu} - am_{\psi}(t))\Psi = 0$

3. Preheating



4. Backreaction











3. Preheating

Beyond perturbation theory: scalars





Using the quantization-friendly variable $X_p \equiv a \exp\left(\frac{1}{2} \int a \Gamma_{\chi} d\tau\right) \chi_p$

$$X_p'' + \omega_p^2 X_p = 0$$

$$\rho_f' + 4\mathcal{H}\rho_f = a\Gamma_\chi \dot{\chi}^2$$

where

4. Backreaction

5. Signals

 $\omega_p^2 ~\equiv~ p^2 + a^2 m_\chi^2 - rac{a''}{a} - rac{1}{4} (a \Gamma_\chi)^2 - rac{3}{2} a \mathcal{H} \Gamma_\chi$



and the UV-convergent energy density (and occupation number) are

$$\rho_{\chi} = \frac{e^{-\int a \Gamma_{\chi} d\tau}}{(2\pi)^3 a^4} \int d^3 \boldsymbol{p} \, \omega_p n_p \,, \qquad n_p = \frac{1}{2\omega_p} |\omega_p X_p - i X_p'|^2$$







Beyond perturbation theory: scalars

 $\mathcal{L} \supset \frac{1}{2} \sigma \phi^2 \chi^2$



 $\ddot{\chi} + (3H + \Gamma_{\chi})\dot{\chi} - a^{-2}\nabla^2\chi + m_{\chi}^2(t)\chi = 0$

3. Preheating



4. Backreaction















4. Backreaction





2. Thermalization

Backreaction

For $ho_\chi \lesssim 0.1
ho_\phi$, the Hartree approximation is sufficient

$$\ddot{\phi} + 3H\dot{\phi} + V_{\phi} + \sigma\langle\chi^2
angle \phi \ = \ 0 \,, \qquad \langle\chi^2
angle \ = \ rac{1}{(2\pi)^3a^2}\int d^3oldsymbol{p}\left(|X_p|^2 - rac{1}{2\omega_p}
ight)$$

L. Kofman, A. Linde, A. Starobinsky, PRD 56 (1997) 3258

3. Preheating



4. Backreaction

For $\rho_\chi\gtrsim 0.1 \rho_\phi$ re-scattering of χ in ϕ fragments the inflationary condensate. Spectral methods are insufficient and lattice codes in configuration space are necessary

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2 \phi}{a^2} + V_{,\phi} = 0$$
$$\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2 \chi}{a^2} + V_{,\chi} = 0$$

Software of choice: CosmoLattice (v1.0)

D. Figueroa, et al., arXiv:2102.01031 [astro-ph.CO]

2. Thermalization



3. Preheating



4. Backreaction



5. Signals

Backreaction

For $\rho_\chi\gtrsim 0.1\rho_\phi$ re-scattering of χ in ϕ fragments the inflationary condensate









3. Preheating



4. Backreaction



5. Signals

Backreaction

For $\rho_\chi\gtrsim 0.1\rho_\phi$ re-scattering of χ in ϕ fragments the inflationary condensate

 $f_{\chi} ~\sim~ e^{-lpha(\sigma/\lambda;t)q}$ in the UV



MG, M. Pierre, S. Verner, arXiv:2206.08940 [hep-ph]







3. Preheating



4. Backreaction







3. Preheating

4. Backreaction

Inflaton self-resonance



Non-adiabaticity is encountered for most of the parameter space ⇒ preheating cannot be ignored!

The homogeneous inflaton can pump energy into its own fluctuations (self-resonance)

 \Rightarrow lattice codes are indispensable

5. Signals

R. Kallosh and A. Linde, JCAP 07 (2013), 002

1. Perturbative reheating



3. Preheating



4. Backreaction



5. Signals











2. Thermalization

3. Preheating



4. Backreaction

Signals 1: the CMB

The number of e-folds ($N\equiv \ln a$) after the comoving scale k_* crosses the horizon





3. Preheating



4. Backreaction



5. Signals

Signals 1: the CMB

$$N_{*} = 66.9 - \ln\left(\frac{k_{*}}{a_{0}H_{0}}\right) + \frac{1}{4}\ln\left(\frac{V_{*}^{2}}{\rho_{\text{end}}M_{P}^{4}}\right) + \ln\left[\frac{a_{\text{end}}}{a_{\text{reh}}}\left(\frac{\rho_{\text{end}}}{\rho_{\text{reh}}}\right)^{1/4}\right] - \frac{1}{12}\ln g_{\text{reh}}$$

$$\ln\left(\frac{1}{aH}\right)$$
Present horizon scale
Lambda
Matter ``
Reheating

 $\ln a$







4. Backreaction

5. Signals



 $\mathcal{R}(T_{\rm reh}) > 1$

 $\mathcal{R}(T_{\max}) > 1$

Signals 1: the CMB

k = 4

56 BBN

54

≥ 52

Equation of state during reheating

$$w = \frac{k-2}{k+2}$$

For
$$k=4$$
, $w_{
m int}\simeq 1/3\,\Rightarrow\,N_*=55.9$

For k > 4, $N_* \gtrsim 55.9$, (highly dependent on interactions)









4. Backreaction



 $k^{2}f_{i}(k):$

5. Signals

Signals 2: dark matter

$$\frac{\partial f_{\rm DM}}{\partial t} - Hp \frac{\partial f_{\rm DM}}{\partial p} = \frac{1}{16p} \int \frac{d^3 \mathbf{p}'}{(2\pi)^3 p_0'} \frac{d^3 \mathbf{k}}{(2\pi)^3 k_0} \frac{d^3 \mathbf{k}'}{(2\pi)^3 k_0'} (2\pi)^4 \delta(\mathbf{p} + \mathbf{p}' - \mathbf{k} - \mathbf{k}') |\mathcal{M}|^2_{\to} f_i(k) f_i(k')$$

$$\frac{dn_{\rm DM}}{dt} + 3Hn_{\rm DM} = \int \frac{d^3\mathbf{k}}{(2\pi)^3k_0} \frac{d^3\mathbf{k}'}{(2\pi)^3k'_0} (k \cdot k') \,\sigma(s) \,f_i(k)f_i(k')$$







MG and M. Amin, PRD 98 (2018), 103504









