Testing Superstring Theory with Cosmological Observations

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Goals of Early Universe Cosmology

- Understand origin and early evolution of the universe.
- Explain observed large-scale structure.
- Make predictions for future observations.

Challenge:

- **What does superstring theory predict?**
- Can observations be used to test superstring theory?

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Outline

Plan

Optical Telescopes: Gemini Telescope

Galaxies: Building Blocks of the Cosmology

Large-Scale Structure

String Cosmology

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Data

Scenarios

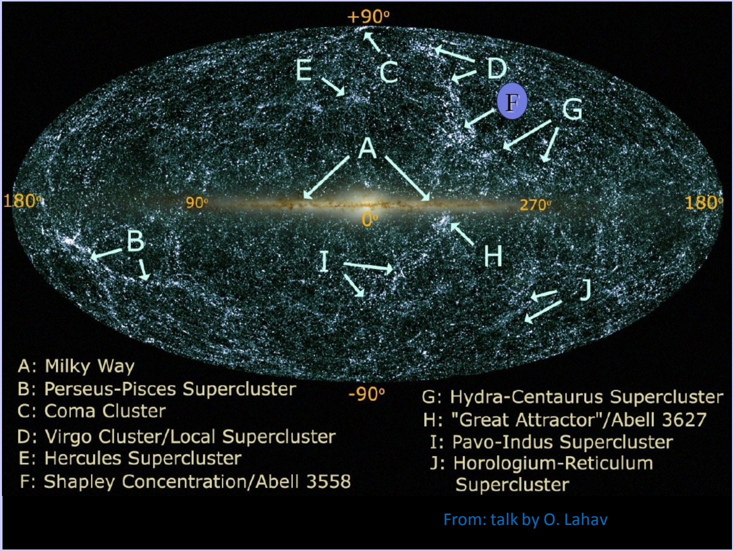
Challenges

Paradigms

String Gas Cosmology

Matrix Theory

Conclusions

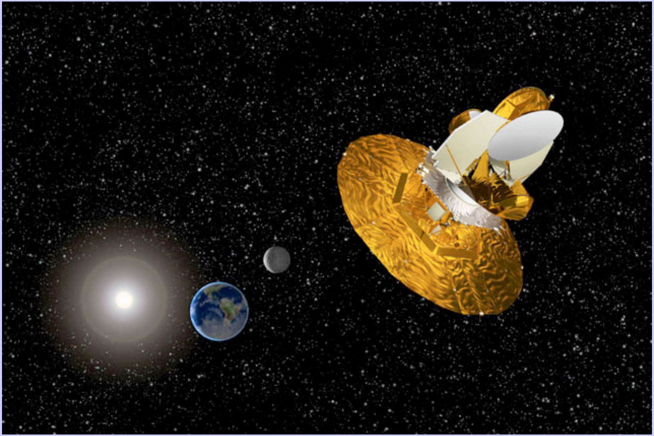


Lessons

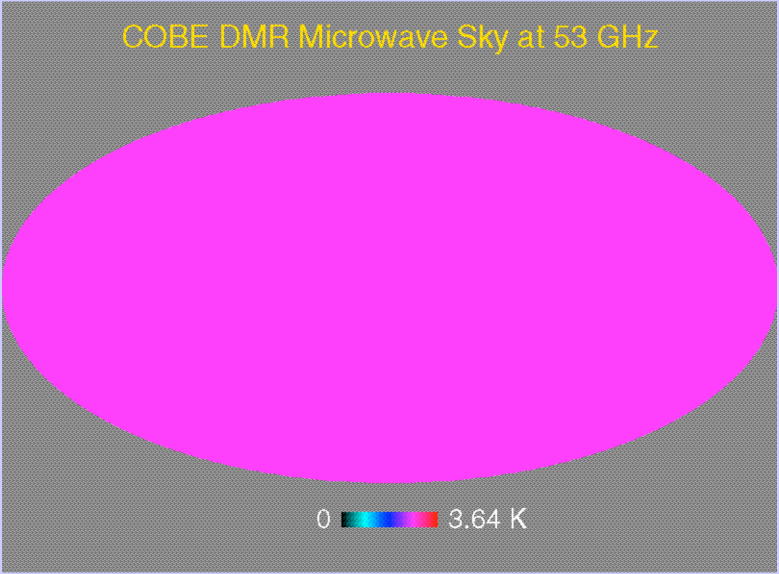
- The universe is immense ($\sim 13 \times 10^9$ light years).
- There is structure in the universe.
- Convergence to homogeneity on large scales (support for the cosmological principle).

Microwave Telescopes on the Earth: SPT Telescope

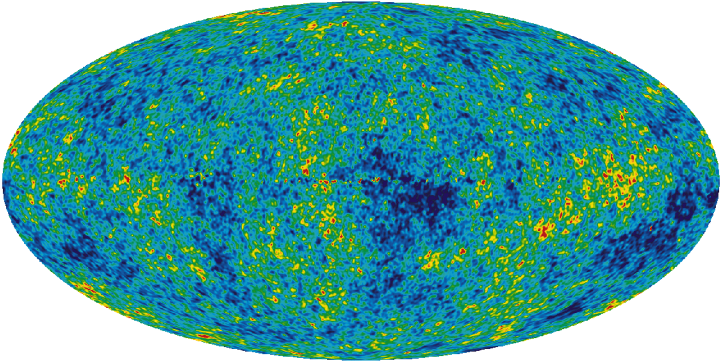
Microwave Telescopes in Space: WMAP Telescope



Isotropic CMB Background

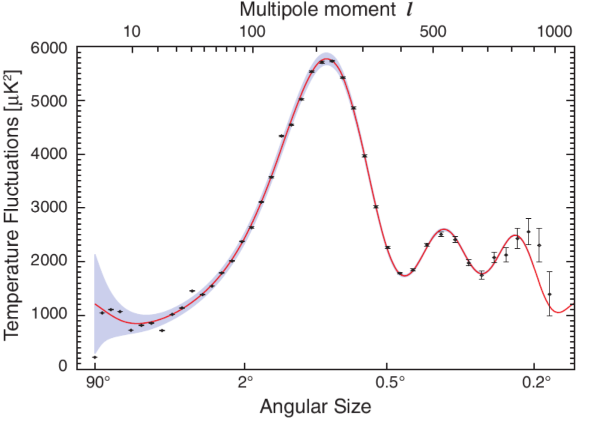


Map of the Cosmic Microwave Background (CMB)



Credit: NASA/WMAP Science Team

Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Lessons

- The CMB is isotropic to excellent accuracy. (strong support for the cosmological principle).
- Small amplitude anisotropies with **distinctive structure.**

New Windows to Probe the Universe are Opening Up

String Cosmology

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CHIME 21cm Telescope

New Windows to Probe the Universe are Opening Up

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HIRAX 21cm Telescope

Plan

Scenario 1: Standard Big Bang Cosmology (SBB)

Assumptions

- 1. Space-Time described by **classical General Relativity**
- 2. Matter described by **classical matter**: superposition of ideal gases
- Cold matter: $p = 0$
- Radiation: $p = \frac{1}{3}\rho$
- 3. Space is **homogeneous** and **isotropic** (Cosmological Principle)

Space-time metric:

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

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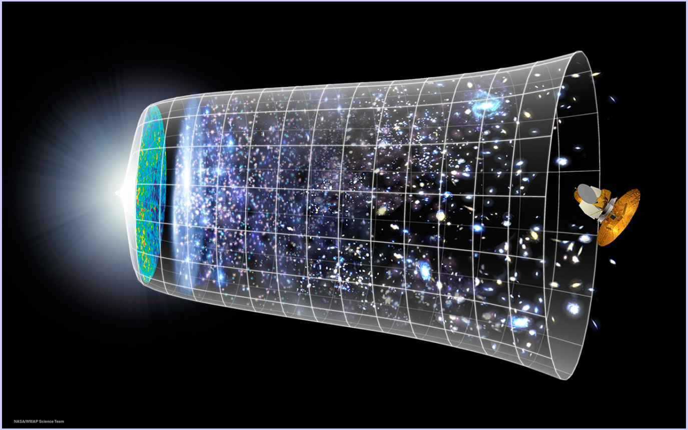
Successes

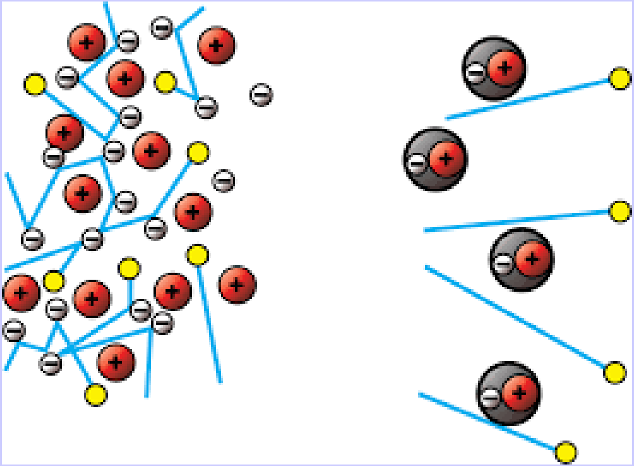
- Hubble law - Space is expanding
- Existence and Black Body nature of the Cosmic Microwave Background
- Nucleosynthesis - Abundances of light elements

Time Line: Big Bang Cosmology



Credit: NASA/WMAP Science Team

Recombination



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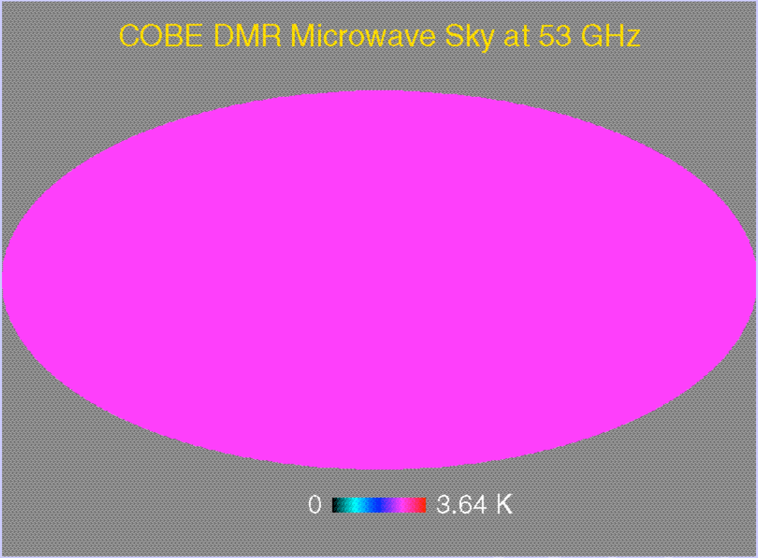
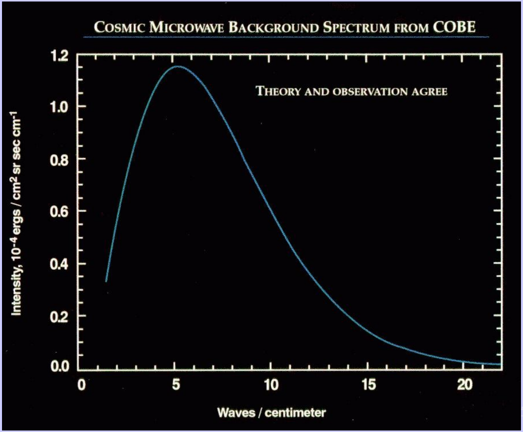


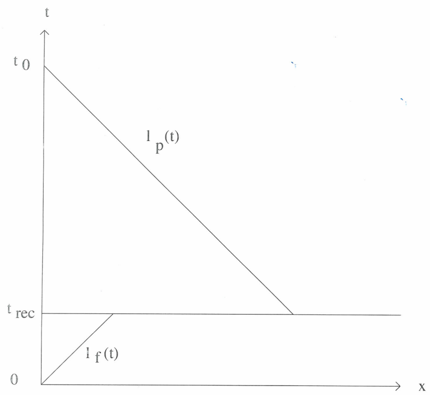
Spectrum of the CMB



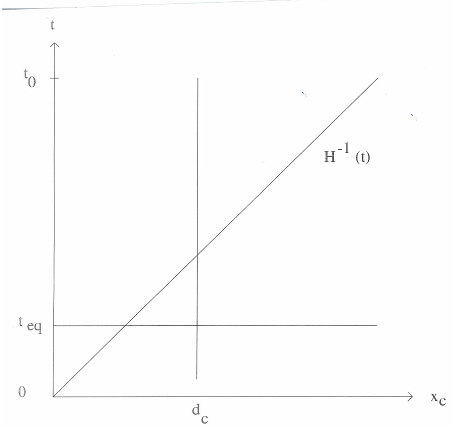
Conceptual Problems of the SBB Model

- Horizon Problem: No explanation for the isotropy of the CMB,
- Flatness Problem: No explanation of the spatial flatness of the universe.
- Formation of Structure Problem: No explanation of the observed inhomogeneities in the distribution of matter and anisotropies in the Cosmic Microwave Background.
- Singularity Problem: Breakdown of applicability of the model at the Big Bang singularity.

Horizon Problem of the SBB Model



Formation of Structure Problem SBB Model

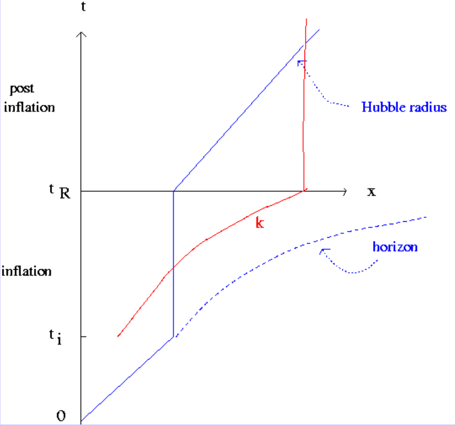


Scenario 2: Inflationary Universe Scenario

Time line of inflationary cosmology:

- t_i: inflation begins
- t_R: inflation ends, reheating

Space-Time Sketch of the Inflationary Scenario



Successes of Inflation

- inflation renders the universe large, homogeneous and spatially flat
- classical matter redshifts $\rightarrow$ matter vacuum remains
- quantum vacuum fluctuations: seeds for the observed structure [Chibisov & Mukhanov, 1981]
- sub-Hubble $\rightarrow$ locally causal

Obtaining Inflation

- Assumption: Space-time described by General Relativity.
- $\rightarrow$ require matter with $p < -\frac{1}{3}\rho$.
- Consider scalar field φ as matter: potential energy term has an equation of state $p = -\rho$.
- Require a slowly rollling scalar field:

$$\frac{V'}{V} \ll \frac{1}{m_{pl}} \, .$$

- Require rolling over large distances

$$\Delta\varphi \, > \, m_{pl} \, .$$

Formation of Structure in Inflationary Cosmology

V. Mukhanov and G. Chibisov,, *JETP Lett.* **33**, *532 (1981)*

- Quantum vacuum fluctuations are the seeds for the **observed structure**.
- $\rightarrow$ successful **predictions** for cosmological observations.

Problems of the Inflationary Scenario

- How does one obtain inflation?
- Inflation takes place at energy scales close to the Planck scale.
- At this scale quantum effects of gravity should be important.
- Setup of inflationary cosmology is unable to handle this problem.
- Singularity problem persists.

Question

- Superstring theory is a candidate **unified quantum theory of all four forces of nature.**
- Question: **What cosmological scenario for the very early universe emerges from superstring theory?**
- Are there testable predictions?
- Note: both SBB and Inflationary Cosmology are based on the hypothesis that point particles are the basic building blocks of nature.
- In superstring theory the basic building blocks are fundamental strings.

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Constraints on Effective Field Theories consistent with String Theory

The space of **effective scalar field theories** consistent with superstring theory is constrained by the swampland criteria (see e.g. E. Palti, 2019):

- $\Delta\varphi < \mathcal{O}(1) m_{pl}$
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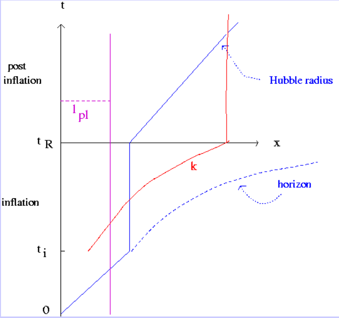
Trans-Planckian Problem



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- $\rightarrow$ new physics MUST enter into the calculation of the fluctuations.

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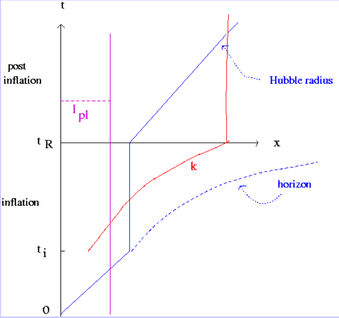

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Trans-Planckian Censorship Conjecture (TCC)

A. Bedroya and C. Vafa., arXiv:1909.11063

No trans-Planckian modes exit the Hubble horizon.

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

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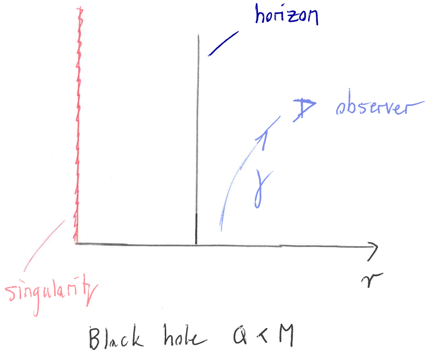
Justification

R.B. arXiv:1911.06056

Analogy with Penrose's Cosmic Censorship Hypothesis:



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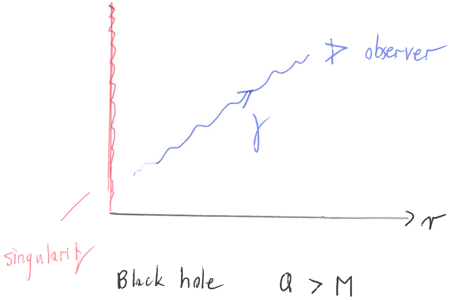

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- Effective field theory of General Relatiivty allows for solutions with timelike singularities: super-extremal black holes.
- $\rightarrow$ Cauchy problem not well defined for observer external to black holes.
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Cosmological Version of the Censorship Conjecture

Translation

- Position space $\rightarrow$ momentum space.
- Singularity $\rightarrow$ trans-Planckian modes.
- Black Hole horizon $\rightarrow$ Hubble horizon.

Observer measuring super-Hubble horizon modes must be shielded from trans-Planckian modes.

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Unitarity Problem

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- Recall: non-unitarity of **effective field theory** in an expanding universe (N. Weiss, Phys. Rev. D32, 3228 (1985); J. Cotler and A. Strominger, arXiv:2201.11658).
- $\mathcal{H}$ is the product Hilbert space of a harmonic oscillator Hilbert space for all **comoving** wave numbers k
- UV cutoff: time dependent k_{max} : $k_{max}(t)a(t)^{-1} = m_{pl}$
- Continuous mode creation $\rightarrow$ non-unitarity.
- Demand: **classical region be insensitive to non-unitarity**.
- $\rightarrow$ no trans-Planckian modes ever exit Hubble horizon.

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Effective Field Theory (EFT) and the CC Problem

- EFT: expand **fields** in comoving Fourier space.
- Quantize each Fourier mode like a harmonic oscillator $\rightarrow$ ground state energy.
- Add up ground state energies $\rightarrow$ CC problem.
- The usual quantum view of the CC problem is an artefact of an EFT analysis!

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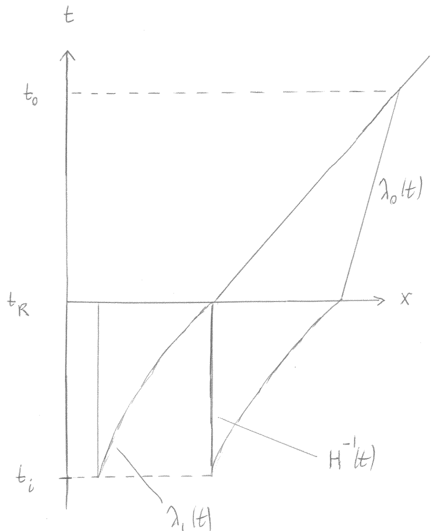
Application of the Second Law of Thermodynamics

S. Brahma, O. Alaryani and RB, arXiv:2005.09688

- Consider entanglement entropy density $s_E(t)$ between sub- and super-Hubble modes.
- Consider an **phase of inflationary expansion.**
- $s_E(t)$ increases in time since the phase space of super-Hubble modes grows.
- **Demand**: $s_E(t)$ remain smaller than the post-inflationary thermal entropy.
- $\rightarrow$ Duration of inflation is bounded from above, consistent with the TCC.

Application to EFT Description of Inflation

A. Bedroya, R.B., M. Loverde and C. Vafa., arXiv:1909.11106



Application to EFT Descriptions of Inflation

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String Cosmology

R. Brandenberger

TCC implies:

$$\frac{a(t_R)}{a(t_*)} l_{pl} < H(t_R)^{-1}$$

Demanding that inflation yields a causal mechanism for generating CMB anisotropies implies:

$$H_0^{-1} \frac{a(t_0)}{a(t_R)} \frac{a(t_R)}{a(t_*)} < H^{-1}(t_*)$$

Implications

Upper bound on the **energy scale of inflation**:

$$V^{1/4} < 3 \times 10^9 \mathrm{GeV}$$

$\rightarrow$ upper bound on the **primordial tensor to scalar ratio** r:

$$r < 10^{-30}$$

Note: Secondary tensors will be larger than the primary ones.

Implications for Dark Energy

- Dark Energy cannot be a bare cosmological constant.
- Quintessence models of Dark Energy are constrained (L. Heisenberg et al. arXiv:2003.13283]

Lessons

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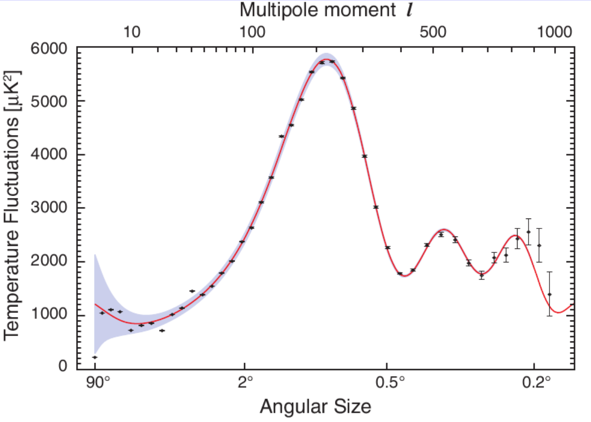
Challenge

- **Can superstring theory yield an improved early universe scenario?**
- **Can superstring theory resolve the Big Bang singularity?**
- **Can superstring theory yield a quantum theory of space, time and matter?**

Plan

Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Early Work

1970

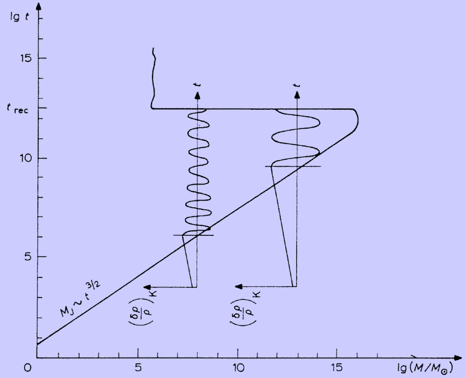


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

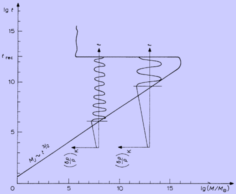
Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.

Early Work

1970 paper



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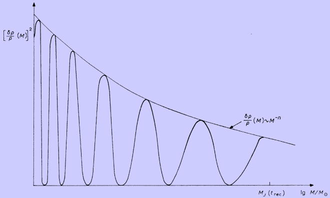

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

R. Sunyaev & Ya. Zeldovich, Astrophysics and Space Science 7 3-19 (1970)

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

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Key Challenge

How does one obtain such a spectrum?

- Inflationary Cosmology is the first scenario based on causal physics which yields such a spectrum.
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Hubble Radius vs. Horizon

- Horizon: Forward light cone of a point on the initial Cauchy surface.
- Horizon: region of causal contact.
- Hubble radius: $l_H(t) = H^{-1}(t)$ inverse expansion rate.
- Hubble radius: local concept, relevant for dynamics of cosmological fluctuations.
- In Standard Big Bang Cosmology: Hubble radius = horizon.
- In any theory which can provide a mechanism for the origin of structure: Hubble radius $\neq$ horizon.

Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the "horizon problem" of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Squeezing of fluctuations on super-Hubble scales in order to obtain the acoustic oscillations in the CMB angular power spectrum.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

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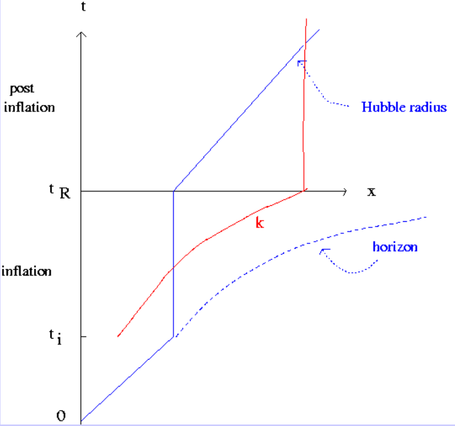
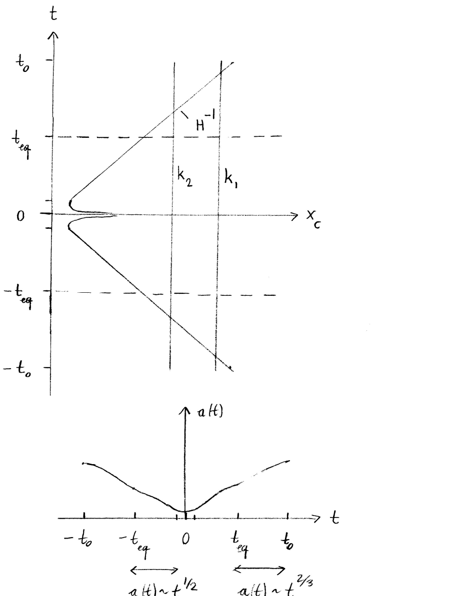
- Squeezing of fluctuations on super-Hubble scales in order to obtain the acoustic oscillations in the CMB angular power spectrum.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

Inflation as a Solution



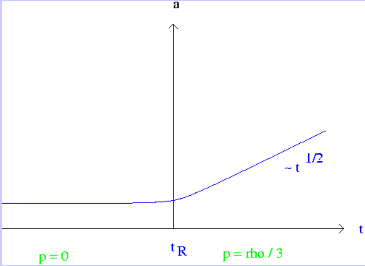
Matter Bounce as a Solution

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



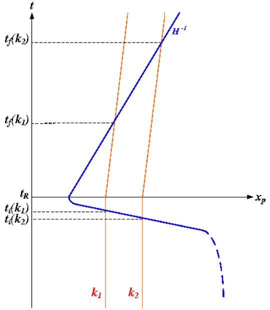
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Challenge for Superstring Theory

Question: **What kind of early universe cosmology emerges from superstring theory?**

Plan

Principles (String Gas Cosmology)

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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T-Duality

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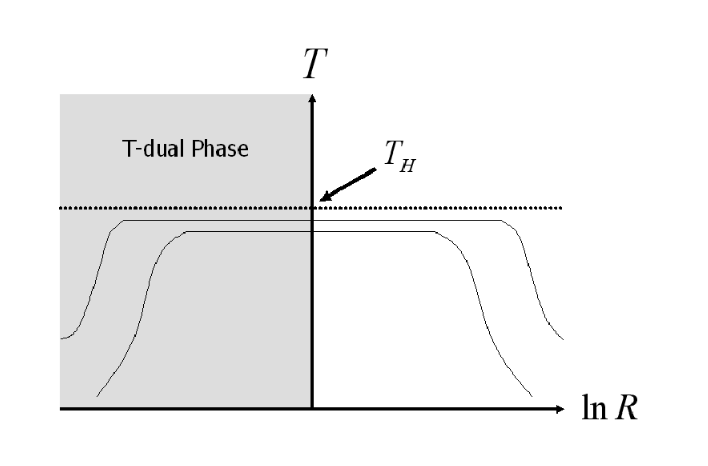
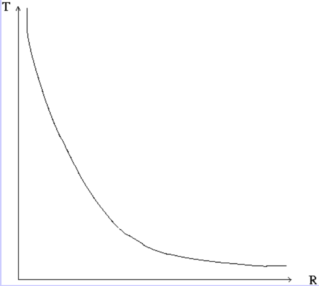
- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

Temperature-size relation in string gas cosmology



Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology



Position Operators

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Position operators (dual to momenta)

$$|x> = \sum_p \exp(ix \cdot p)|p>$$

Dual position operators (dual to windings)

$$|\tilde{x}> = \sum_w \exp(i\tilde{x} \cdot w)|w>$$

Note:

$$|x> = |x + 2\pi R> , \quad |\tilde{x}> = |\tilde{x} + 2\pi \frac{1}{R}>$$

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Heavy vs. Light Modes

- $R \gg 1$: momentum modes light.
- $R \ll 1$: winding modes light.
- $R \gg 1$: length measured in terms of $|x>$.
- $R \ll 1$: length measured in terms of $|\tilde{x}>$
- $R \sim 1$: both $|x>$ and $|\tilde{x}>$ important.

Conclusion: At string scale densities usual effective field theory (EFT) based on supergravity will break down.

Conclusion: If an effective field theory description is valid, it must be an EFT in 18 spatial dimensions.

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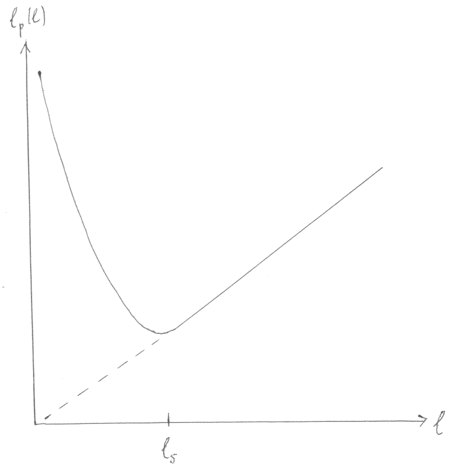
Physical length operator

$$
\begin{aligned}
l_p(R) &= R \quad R \gg 1 \\
l_p(R) &= \frac{1}{R} \quad R \ll 1
\end{aligned}
$$

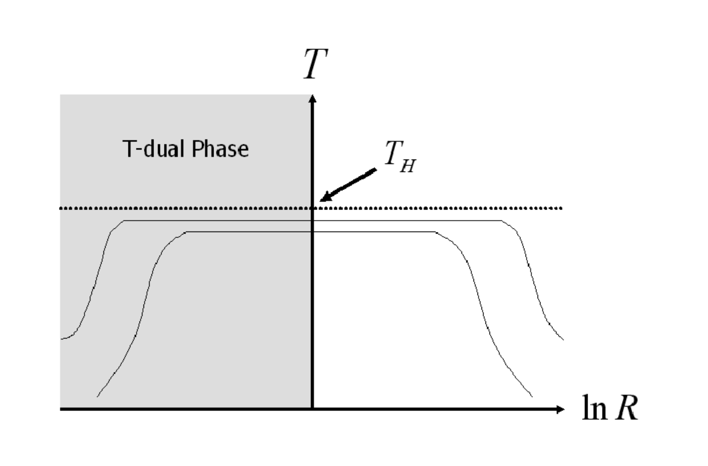
Physical length



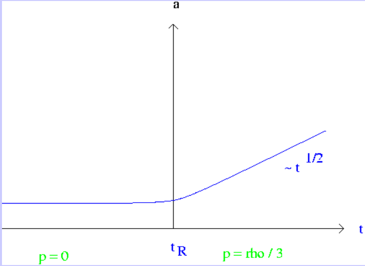
Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology

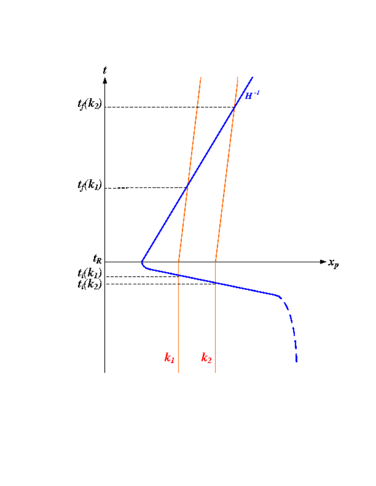


Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \,.$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} \,.$$

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$$C_V \approx 2 \frac{R^2 / \ell_s^3}{T (1 - T/T_H)} .$$

Power spectrum of cosmological fluctuations

$$
\begin{aligned}
P_\Phi(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\
&= 8G^2 k^2 < (\delta M)^2 >_R \\
&= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\
&= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H}
\end{aligned}
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Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

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P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\
&= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\
&\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H)
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Key ingredient for string thermodynamics

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< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)
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Key features:

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- slight blue tilt (unlike for inflation)

Requirements

- Static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Plan

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Starting point: BFSS matrix model at high temperatures.

- BFSS model is a quantum mechanical model of 10 $N \times N$ Hermitean matrices.
- Note: no space!
- Note: no singularities!
- Note: BFSS matrix model is a proposed non-perturbative definition of M-theory: 10 dimensional superstring theory emerges in the $N \rightarrow \infty$ limit.

BFSS Model (bosonic sector)

T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997)

$$L = \frac{1}{2g^2}\left[\mathrm{Tr}\left(\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2\right)\right]$$

- $X_i, i = 1, ...9$ are $N \times N$ Hermitean matrices.
- D_t: gauge covariant derivative (contains a matrix A_0)

't Hooft limit: $N \to \infty$ with $\lambda \equiv g^2 N = g_s l_s^{-3} N$ fixed.

Thermal Initial State

N. Kawahara, J. Nishimura and S. Takeuchi, JHEP **12**, 103 (2007)

- Consider a high temperature state.
- At high temperatures, the bosonic sector of the (Euclidean) BFSS model is well approximated by the bosonic sector of the (Euclidean) IKKT matrix model.
- $S_{BFSS} = S_{IKKT} + \mathcal{O}(1/T)$
- Matsubara expansion:

$$X_i(t) = \sum_n X_i^n e^{2\pi i T t}$$

$$A_i \equiv T^{-1/4} X_i^0$$

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IKKT Matrix Model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

Proposed as a non-perturbative definition of the IIB Superstring theory.

Action:

$$S_{IKKT} = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A^a, A^b][A_a, A_b] + \frac{i}{2}\bar{\psi}_\alpha(\mathcal{C}\Gamma^a)_{\alpha\beta}[A_a, \psi_\beta]\right),$$

Partition function:

$$Z = \int dA d\psi e^{iS}$$

Emergent Time from Matrix Theory

Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

- Eigenvalues of A_0 become emergent time.
- Work in the basis in which A_0 is diagonal.
- Numerical studies: $\frac{1}{N}\langle \mathrm{Tr} A_0^2 \rangle \sim \kappa N$
- $\rightarrow t_{max} \sim \sqrt{N}$
- $\rightarrow \Delta t \sim \frac{1}{\sqrt{N}}$
- $\rightarrow$ infinite continuous time.

Note: $\sum_{n=0}^{N} n^2 = \frac{1}{6} N(N+1)(2N+1)$

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Emergent Space from Matrix Theory

Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

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- Work in the basis in which A_0 is diagonal: A_i matrices elements decay when going away from the diagonal.
- $\sum_i \langle |A_i|^2_{ab} \rangle$ decays when $|a-b| > n_c$
- $\sum_i \langle |A_i|^2_{ab} \rangle \sim$ constant when $|a-b| < n_c$
- $n_c \sim \sqrt{N}$

Emergent Space from Matrix Theory

S. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540

- Eigenvalues of A_0 become emergent time, continuous in $N \rightarrow \infty$ limit.
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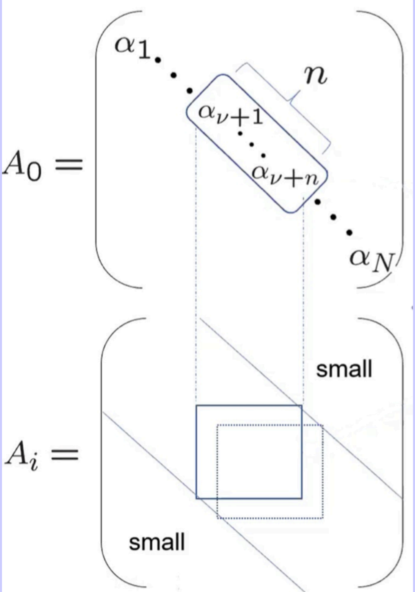
- Pick $n \times n$ blocks $\tilde{A}_i(t)$ about the diagonal $(n < n_c)$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α_1
n
$\alpha_{\nu+1}$
$\alpha_{\nu+n}$
α_N
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small
small

Spontaneous Symmetry Breaking in Matrix Theory

J. Nishimura, PoS CORFU **2019**, 178 (2020) [arXiv:2006.00768 [hep-lat]].

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
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- **Extent of space** in direction i

$$x_i(t)^2 \equiv \left\langle \frac{1}{n} \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- In a thermal state there is spontaneous symmetry breaking: $SO(9) \to SO(6) \times SO(3)$: three dimensions of space become larger, the others are confined. [J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000);]S.-W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **109**, 011601 (2012)]

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Emergent Metric from Matrix Theory

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal: pick n (comoving spatial coordinate) and consider the block matrix $\tilde{A}_i(t)$.
- Physical distance between $n = 0$ and n (emergent space):

$$l^2_{phys,i}(n,t) \equiv \left\langle \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- $l_{phys,i}(n) \sim n$ (for $n < n_c$)
- Emergent infinite and continuous space in $N \to \infty$ limit.
- Emergent metric (S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468).

$$g_{ii}(n)^{1/2} = \frac{d}{dn} l_{phys,i}(n)$$

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Emergent Metric from Matrix Theory

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

- Eigenvalues of A_0 become emergent time, continuous in $N \rightarrow \infty$ limit.
- Work in the basis in which A_0 is diagonal: pick n (comoving spatial coordinate) and consider the block matrix $\tilde{A}_i(t)$.
- Physical distance between $n = 0$ and n (emergent space):

$$l_{phys,i}^2(n,t) \equiv \left\langle \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- $l_{phys,i}(n) \sim n$ (for $n < n_c$)
- Emergent infinite and continuous space in $N \rightarrow \infty$ limit.
- Emergent metric (S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468).

$$g_{ii}(n)^{1/2} = \frac{d}{dn} l_{phys,i}(n)$$

No Flatness Problem in Matrix Theory Cosmology

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Emergent metric:

$$g_{ii}(n)^{1/2} = \frac{d}{dn} l_{phys,i}(n)$$

Result:

$$g_{ii}(n,t) = \mathcal{A}(t)\delta_{ii} \;\; i = 1,2,3$$

$SO(3)$ symmetry $\rightarrow$

$$g_{ij}(n,t) = \mathcal{A}(t)\delta_{ij} \;\; i = 1,2,3$$

$\rightarrow$ spatially flat.

Note: Local Lorentz invariance emerges in $N \rightarrow \infty$ limit.

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Late Time Dynamics

$$\mathcal{A}(t) \sim t^{1/2}$$

Note: no sign of a cosmological constant.

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

- We assume that the spontaneous symmetry breaking $SO(9) \to SO(3) \times SO(6)$ observed in the IKKT model also holds in the BFSS model.
- Using the Gaussian approximation method we have shown the existence of a symmetry breaking phase transition in the IKKT model (S. Brahma, RB and S. Laliberte, arXiv:2209.01255).
- **Thermal correlation functions** in the three large spatial dimensions calculated in the high temperature state of the BFSS model (following the formalism developed in String Gas Cosmology).
- $\to$ curvature fluctuations and gravitational waves.

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Matrix Theory Cosmology: Thermal Fluctuations

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- Start with the BFSS partition function .
- Note: $\frac{1}{T}$ correction terms in the BFSS action are crucial!
- Calculate matter correlation functions in the emergent phase.
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$.
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations.

Note: the matter correlation functions are given by partial derivatives of the **finite temperature partition function** with respect to T (density fluctuations) or R (pressure perturbations).

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Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Thermal fluctuations in the emergent phase $\rightarrow$

- Scale-invariant spectrum of curvature fluctuations
- With a Poisson contribution for UV scales.
- Scale-invariant spectrum of gravitational waves.

$\rightarrow$ BFSS matrix model yields emergent infinite space, emergent infinite time, emergent spatially flat metric and an emergent early universe phase with thermal fluctuations leading to scale-invariant curvature fluctuations and gravitational waves.

Note: Horizon problem automatically solved.

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Open Problems

- Include the effects of the fermionic sector.
- Understand **phase transition** to the expanding phase of Big Bang Cosmology.
- Understand the **emergence of GR** in the IR.
- Spectral indices?
- What about Dark Energy?
- Emergent low energy effective field theory for localized excitations.

Plan

Conclusions

- Inflation is not the only scenario of early universe cosmology consistent with current data.
- In light of the TCC and other conceptual problems effective field theory models of inflation are not viable.
- In light of the TCC and other conceptual problems Dark Energy cannot be a cosmological constant.
- We need to go beyond point particle EFT in order to describe the very early universe.

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Conclusions

- BFSS matrix model is a proposal for a non-perturbative definition of superstring theory. Consider a high temperature state of the BFSS model.
- $\rightarrow$ emergent time, space and metric. Emergent space is spatially flat and infinite.
- **Thermal fluctuations** of the BFSS model $\rightarrow$ scale-invariant spectra of cosmological perturbations and gravitational waves.
- **Horizon problem, flatness problem** and **formation of structure problem** of Standard Big Bang Cosmology resolved without requiring inflation.
- Transition from an emergent phase to the radiation phase of expansion. No cosmological constant.
- String theory testable with cosmological observations.