Gravitational Production of Particles and its Difficulties

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based on:

- Aqeel Ahmed, BG, Anna Socha, Phys.Lett.B 831 (2022) 137201, e-Print: 2111.06065
- Aqeel Ahmed, BG, Anna Socha, e-Print: 2207.11218

AstroParticle Symposium 2022, Paris-Saclay University, Nov. 4^{th} 2022, Orsay

The α-attractor T-model

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$$

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{\sqrt{6\alpha}M_{\rm Pl}}\right)$$

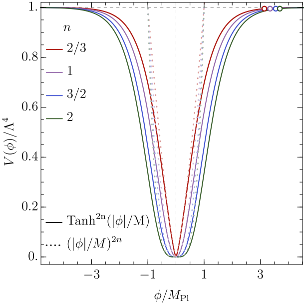
$$\simeq \begin{cases} \Lambda^4 & |\phi| \gg M_{\rm Pl} \\ \boxed{\Lambda^4 \left|\dfrac{\phi}{M_{\rm Pl}}\right|^{2n}} & |\phi| \ll M_{\rm Pl} \end{cases},$$

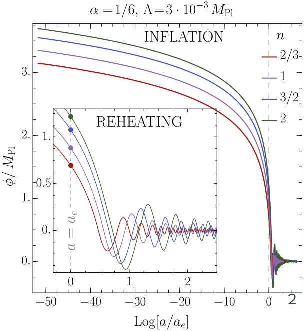
where $n > 0$, $\sqrt{6\alpha} \lesssim 10$, $\Lambda \lesssim 1.6 \times 10^{16}\ \mathrm{GeV}$.

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

where $H \equiv \dot{a}/a$ is the Hubble rate.





$$\phi(t) = \varphi(t) \cdot \mathcal{P}(t)$$

- $\mathcal{P}(t)$ is a quasi-periodic, fast-oscillating function,
- $\varphi(t)$ denotes a slowly-varying envelope:

$$\rho_\phi = V(\varphi) \simeq \Lambda^4 \left(\frac{\varphi}{M_{\rm Pl}}\right)^{2n}$$

$$\dot{\varphi}(t) = -\frac{3}{n+1} H\varphi(t) \quad \Rightarrow \quad \varphi(a) = \varphi_e \left(\frac{a_e}{a}\right)^{\frac{3}{n+1}}$$

$$\dot{\mathcal{P}} \simeq \pm \frac{m_\phi}{\sqrt{n(2n-1)}} \sqrt{1 - |\mathcal{P}|^{2n}} \quad \Rightarrow \quad \mathcal{P}(a) = \left[\mathcal{I}_z^{-1}\left(\frac{1}{2n}, \frac{1}{2}\right)\right]^{\frac{1}{2n}}$$

where $\mathcal{I}_z^{-1}(i,j)$ is an incomplete beta function $\mathcal{I}_z^{-1}(i,j)$. The period of the oscillations, $\mathcal{T}$,

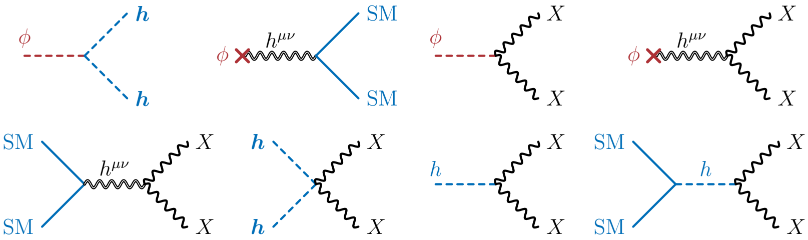
$$\mathcal{T} = \frac{\sqrt{4\pi}}{m_\phi} \sqrt{\frac{2n-1}{n}} \frac{\Gamma\left(\frac{1}{2n}\right)}{\Gamma\left(\frac{n+1}{2n}\right)} \quad \text{with} \quad m_\phi^2 \equiv \left.\frac{\partial^2 V(\phi)}{\partial \phi^2}\right|_{\phi=\varphi} = 2n(2n-1)\frac{\Lambda^4}{M_{\rm Pl}^2}\left(\frac{\rho_\phi}{\Lambda^4}\right)^{\frac{n-1}{n}}$$

$$\mathcal{P}(t) = \sum_{k=-\infty}^{\infty} \mathcal{P}_k \, e^{-ik\omega t}$$

Interactions

$$\mathcal{L}_{\rm DM} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2 X_\mu X^\mu,$$

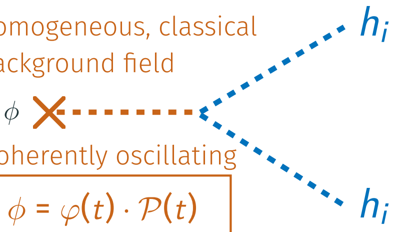
$$\mathcal{L}_{\rm int} = -\Bigg\{ \boxed{g_{h\phi} M_{\rm Pl}\,\phi |\mathsf{h}|^2} + \frac{h^{\mu\nu}}{M_{\rm Pl}}\left[T^{\phi}_{\mu\nu} + T^{\rm DM}_{\mu\nu} + T^{\rm SM}_{\mu\nu}\right] + \frac{\mathcal{C}_X^{\phi}\, m_X^2}{2M_{\rm Pl}}\phi X_\mu X^\mu + \frac{\mathcal{C}_X^{\mathsf{h}}\, m_X^2}{2M_{\rm Pl}^2}|\mathsf{h}|^2 X_\mu X^\mu \Bigg\},$$



The Higgs portal

$$\mathcal{L}_{int} = g_{h\phi} M_{\rm Pl} \phi |h|^2$$

homogeneous, classical background field



coherently oscillating

$$\phi = \varphi(t) \cdot \mathcal{P}(t)$$

rapidly-oscillating

slowly-varying envelope, $\rho_\phi \equiv V(\varphi)$

⇨ Reheating

i.e., energy transfer between the inflaton and the SM sector

$$\frac{1}{V}\frac{dE_g}{dt} \equiv \rho_\phi \Gamma_\phi = g_{h\phi}^2 M_{\rm Pl}^2 \frac{\varphi^2(t)}{8\pi} \sum_{i=0}^{3} \sum_{k=1}^{\infty} k\omega |\mathcal{P}_k|^2 \sqrt{1 - \left(\frac{2m_{h_i}}{k\omega}\right)^2}, \quad \mathcal{P}(t) = \sum_{k=-\infty}^{\infty} \mathcal{P}_k e^{-ik\omega t}$$

⇨ Higgs mass

induced by the oscillating inflaton background

$$m_{h_0}^2 = g_{h\phi} M_{\rm Pl} \varphi \begin{cases} |\mathcal{P}|, & \mathcal{P}(t) > 0 \\ 2|\mathcal{P}|, & \mathcal{P}(t) < 0 \end{cases} \qquad v_h = \begin{cases} 0, & \mathcal{P}(t) > 0 \\ \sqrt{|m_{h_0}^2|/(2\lambda_h)}, & \mathcal{P}(t) < 0 \end{cases}$$

The Higgs portal

$$m^2_{h_0} = g_{h\phi} M_{\rm Pl}\varphi \begin{cases} |\mathcal{P}|\,, & \mathcal{P}(t)>0 \\ 2|\mathcal{P}|\,, & \mathcal{P}(t)<0 \end{cases}$$

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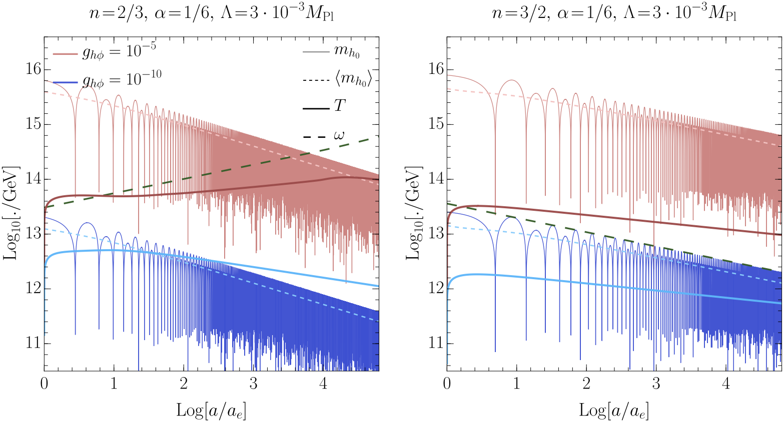


Kinematic suppression

The inflaton decay rate can be written as

effective mass

$$m_\phi^2(a) = V_{,\phi\phi}|_{\phi=\varphi}$$

$$\langle\Gamma_\phi\rangle = \frac{g_{h\phi}^2}{32\pi}\frac{M_{\rm Pl}^2}{m_\phi(a)}\gamma_h(a),$$

$$\gamma_h(a) \simeq \sum_{i=0}^{3}\sum_{k=1}^{\infty} k|\mathcal{P}_k|^2\left\langle\sqrt{1-\left(\frac{2m_{h_i}(a)}{k\omega(a)}\right)^2}\right\rangle$$

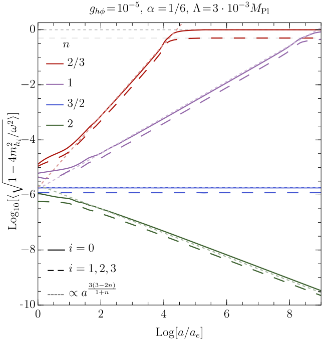
$$\omega \propto m_\phi$$

It turns out that

$$\gamma_h \propto a^{\frac{3(3-2n)}{1+n}}$$

which implies

$$\langle\Gamma_\phi\rangle \propto a^{\frac{6-3n}{1+n}} \equiv a^{-\beta}$$



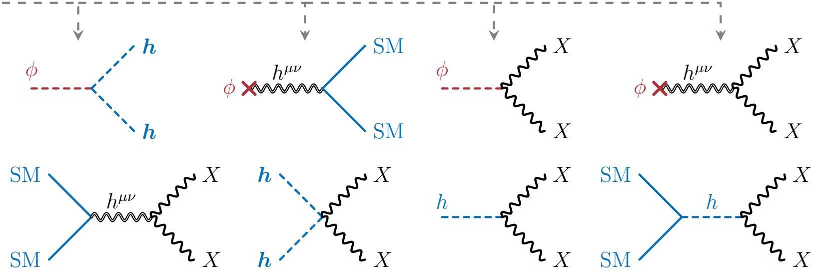
The time-averaged Boltzmann equations

$$\dot{\rho}_\phi + \frac{6n}{n+1} H\rho_\phi = -\boxed{\langle \Gamma_\phi \rangle}\rho_\phi$$

$$\dot{\rho}_{\mathrm{SM}} + 4H\rho_{\mathrm{SM}} = \langle \Gamma_{\phi\to \mathrm{SM\,SM}} \rangle \rho_\phi - 2\langle E_X \rangle \mathcal{S}_{\mathrm{SM}} - \langle E_{h_0} \rangle \mathcal{D}_{h_0}$$

$$\dot{n}_X + 3Hn_X = \mathcal{D}_\phi + \mathcal{S}_\phi + \mathcal{S}_{\mathrm{SM}} + \mathcal{D}_{h_0}$$

with the Hubble rate $H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left(\rho_\phi + \rho_{\mathrm{SM}} + \rho_X\right)$



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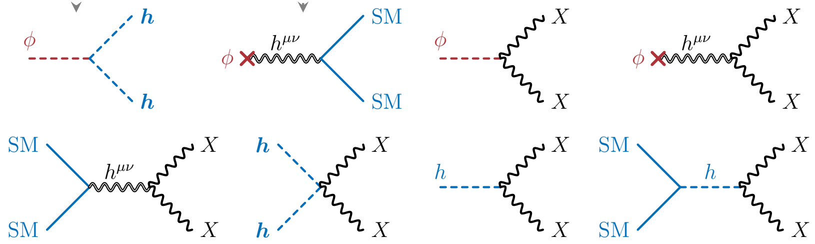

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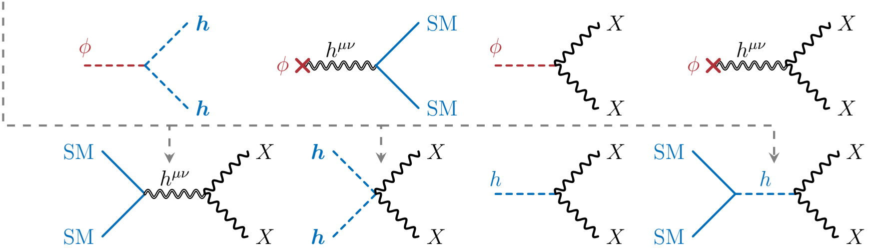

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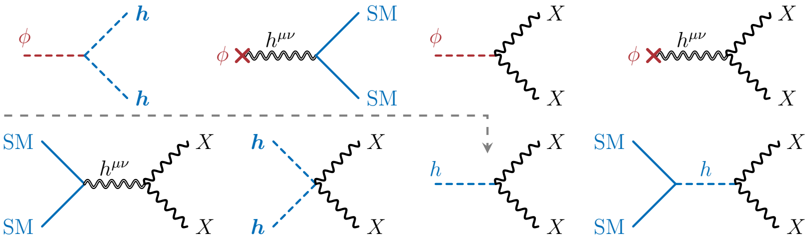

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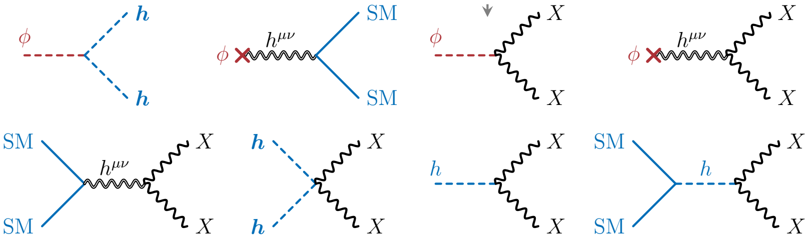

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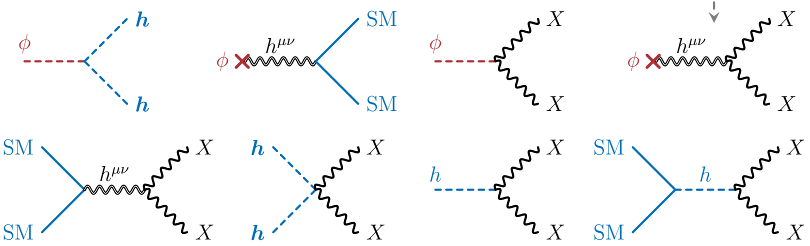
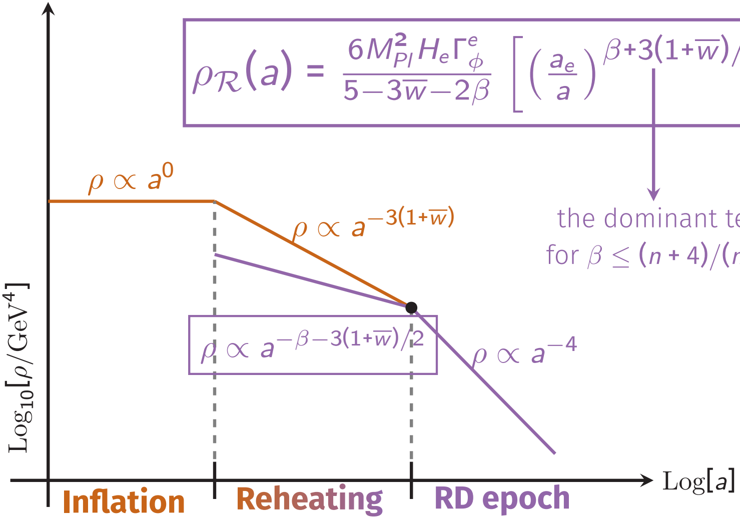


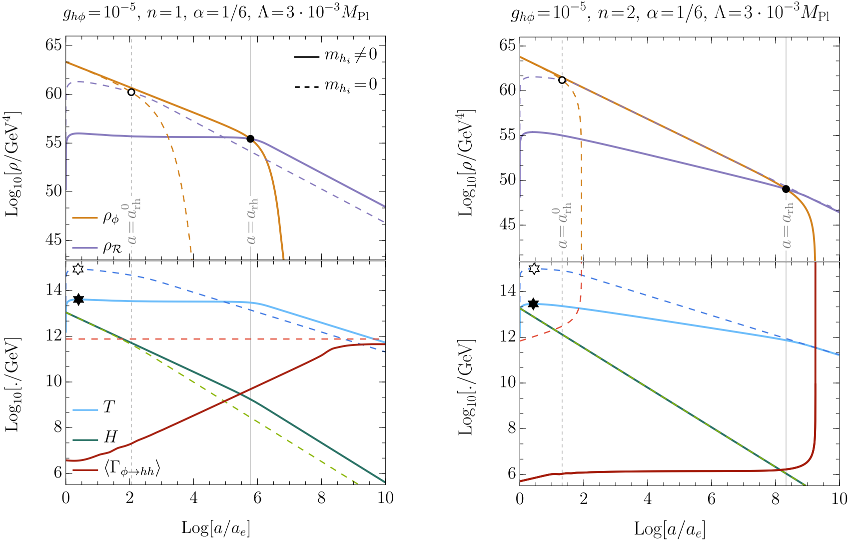
Non-instantaneous reheating

$$\rho_\phi(a) \overset{H \gg \Gamma_\phi}{\simeq} 3 M_{\rm Pl}^2 H_e^2 \left(\frac{a_e}{a}\right)^{3(1+\overline{w})}$$

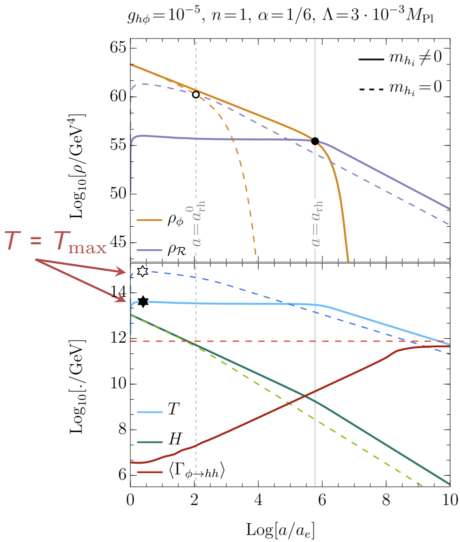
$$\rho_{\mathcal{R}}(a) = \frac{6 M_{Pl}^2 H_e \Gamma_\phi^e}{5 - 3\overline{w} - 2\beta} \left[\left(\frac{a_e}{a}\right)^{\beta + 3(1+\overline{w})/2} - \left(\frac{a_e}{a}\right)^4 \right]$$

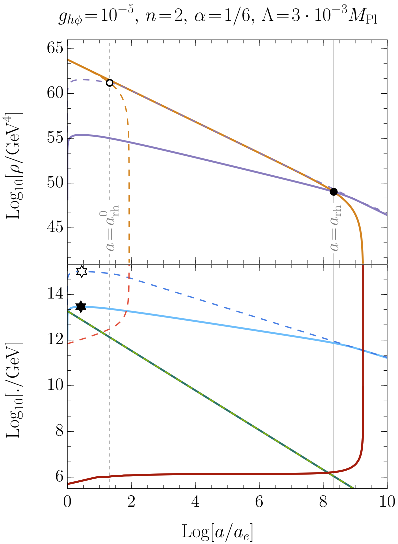
the dominant term for $\beta \leq (n+4)/(n+1)$





$g_{h\phi}=10^{-5}$, $n=1$, $\alpha=1/6$, $\Lambda=3\cdot10^{-3}M_{\rm Pl}$
$g_{h\phi}=10^{-5}$, $n=2$, $\alpha=1/6$, $\Lambda=3\cdot10^{-3}M_{\rm Pl}$
$m_{h_i}\neq0$
$m_{h_i}=0$
ρ_ϕ
$\rho_{\mathcal{R}}$
$a=a_{\rm rh}^0$
$a=a_{\rm rh}$
$\mathrm{Log}_{10}[\rho/\mathrm{GeV}^4]$
$\mathrm{Log}_{10}[\cdot/\mathrm{GeV}]$
T
H
$\langle\Gamma_{\phi\to hh}\rangle$
$\mathrm{Log}[a/a_e]$





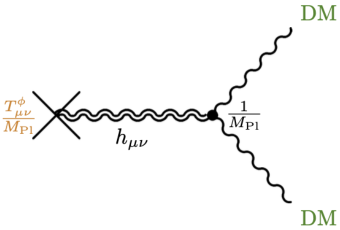
$m_{h_i} = 0$ | $\beta = 0,\ \rho_{\mathcal{R}} \propto a^{-3/2}$ | $\beta = -1,\ \rho_{\mathcal{R}} \propto a^{-1}$

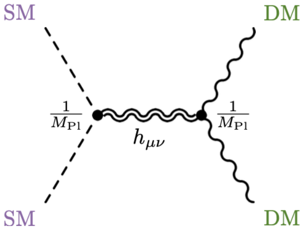
$m_{h_i} \neq 0$ | $\beta = -\frac{3}{2},\ \rho_{\mathcal{R}} \propto a^{0}$ | $\beta = 0,\ \rho_{\mathcal{R}} \propto a^{-2}$

Gravitational DM production

$$\mathcal{L}_{\rm DM} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2 X_\mu X^\mu + \mathcal{L}_{\rm int}$$

$$\mathcal{L}_{\rm int} = \frac{h_{\mu\nu}}{M_{\rm Pl}}\left(T_\phi^{\mu\nu} + T_X^{\mu\nu} + T_{\rm SM}^{\mu\nu}\right)$$





Y. Mambrini et al., arXiv:2102.06214

M.R. Haque et al., arXiv:2112.14668

S. Clery et al., arXiv:2112.15214

M. Garny et al., arXiv:1511.03278

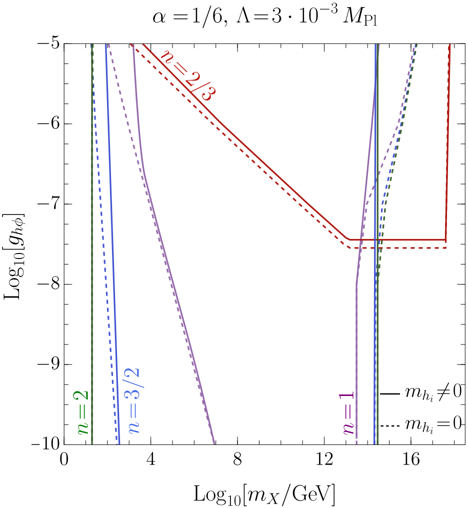
Y. Tang et al., arXiv:1708.05138

M. Garny et al., arXiv:1709.09688

Gravitational DM production

$$\Omega_X^{\rm grav} h^2 \simeq \frac{m_X}{\rho_c} \frac{N^{\rm grav}(a_{\rm rh})}{a_{\rm rh}^3} \frac{s_0}{s(a_{\rm rh})} h^2 .$$

$$\Omega_X^{\rm grav} h^2 = \Omega_X^{\rm (obs)} h^2 = 0.1198 \pm 0.0012$$



Gravitational DM production

Heavy DM particles are produced by the freeze-in from the SM sector

$$\Omega_X^{\rm grav} h^2 \simeq \frac{m_X}{\rho_c} \frac{N^{\rm grav}(a_{\rm rh})}{a_{\rm rh}^3} \frac{s_0}{s(a_{\rm rh})} h^2 .$$

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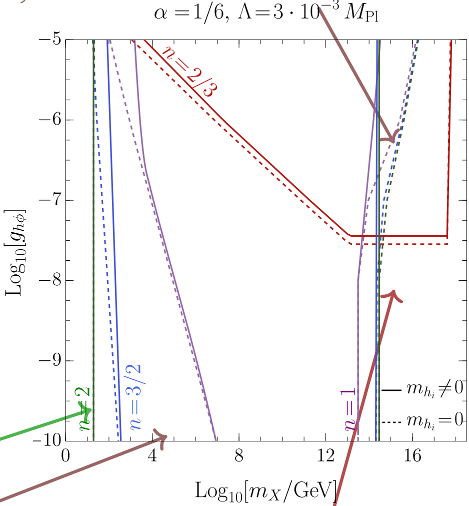


For the $n=2$ case, $\Omega_X^{\rm grav} h^2$ does not depend on $g_{h\phi}$

Light DM particles are dominantly

For the $n=2/3$ case, $\Omega_X^{\rm grav} h^2$ does not depend on m_X

Summary

- The α-attractor T-model potential for the inflaton field has been adopted:

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{\sqrt{6\alpha}M_{\rm Pl}}\right) \simeq \begin{cases} \Lambda^4 & |\phi| \gg M_{\rm Pl} \\ \boxed{\Lambda^4 \left|\frac{\phi}{M_{\rm Pl}}\right|^{2n}} & |\phi| \ll M_{\rm Pl} \end{cases},$$

- The reheating has been triggered by

$$\mathcal{L}_{int} = g_{h\phi} M_{\rm Pl} \phi |h|^2$$

- It has been shown that both duration of reheating and evolution of radiation energy density, $\rho_{\mathcal{R}}$, are sensitive to the shape of the inflaton potential (n).
- The role of kinematical suppression emerging from $\mathcal{L}_{int}$ has been investigated. It has been shown that the non-zero mass of the Higgs boson leads to the elongation of the reheating period, changes the $\rho_{\mathcal{R}}(a)$ and $T(a)$ evolution, and favors reduced $T_{\rm max}$.

Summary

- It has been shown that purely gravitational perturbative production of DM is possible.

- Purely gravitation perturbative reheating needs to be investigated.

Problems to consider

1. Classical inflaton background (condensate)

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

Assume:

$$V(\phi, h) = V_\phi(\phi) + V_h(h) + g_{h\phi} M_{\rm Pl}\phi|h|^2 + \lambda_{h\phi}\phi^2|h|^2$$

$$\Downarrow$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi, h) = 0$$

$$\ddot{h} + 3H\dot{h} + V_{,h}(\phi, h) = 0$$

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[\underbrace{\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{h}^2 + V(\phi, h)}_{\rho_\phi + \rho_h}\right]$$

Then

$$\phi(t) = \bar{\phi}(t) + \delta\phi(x), \qquad h(t) = \bar{h}(t) + \delta h(x)$$

with $\bar{\phi}(t)$, $\bar{h}(t)$ being classical homogeneous background while $\delta\phi(x)$, $\delta h(t)$ are quantum fluctuations ($\bar{\phi}(t)$, $\bar{h}(t) \to T_{\mu\nu}$)

2 Choice of the background metric: Minkowski versus FRWL?

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + \frac{h_{\mu\nu}}{M_{\rm Pl}}(x) \qquad \bar{g}_{\mu\nu}(x) = \begin{cases} \eta_{\mu\nu} \\ g_{\mu\nu}^{FLRW}(t) \end{cases}$$

- graviton propagator
- contractions
- scalar, fermion and vector propagators

3 Renormalization of $T_{\mu\nu}$

The equation of motion for longitudinally-polarized modes

$$\tilde{\mathcal{X}}_L'' + \omega_L^2(\tau)\tilde{\mathcal{X}}_L = 0$$

where the time-dependent frequency ω_L^2 reads

$$\omega_L^2(\tau) \equiv k^2 + m_X^2 a^2 - \frac{k^2}{k^2 + m_X^2 a^2}\frac{a''}{a} + 3\frac{k^2 m_X^2 a'^2}{(k^2 + m_X^2 a^2)^2}$$

For $max[am_X, aH_I] \ll k$

$$\frac{d\rho_L}{d\ln k} = \frac{k^3}{4\pi^2 a^4}\left\{\frac{k}{2} + \left(\frac{k^4}{(k^2 + a^2 m_X^2)^2} a^2 H_I^2 + k^2 + a^2 m_X^2\right)\frac{1}{2k}\right\}$$

$$= \frac{k^4}{4\pi^2 a^4} + \frac{k^2\left(H_I^2 + m_X^2\right)}{8\pi^2 a^2} - \frac{H_I^2 m_X^2}{4\pi^2} + \frac{3a^2 H_I^2 m_X^4}{8\pi^2 k^2} + \mathcal{O}\left(k^{-4}\right)$$

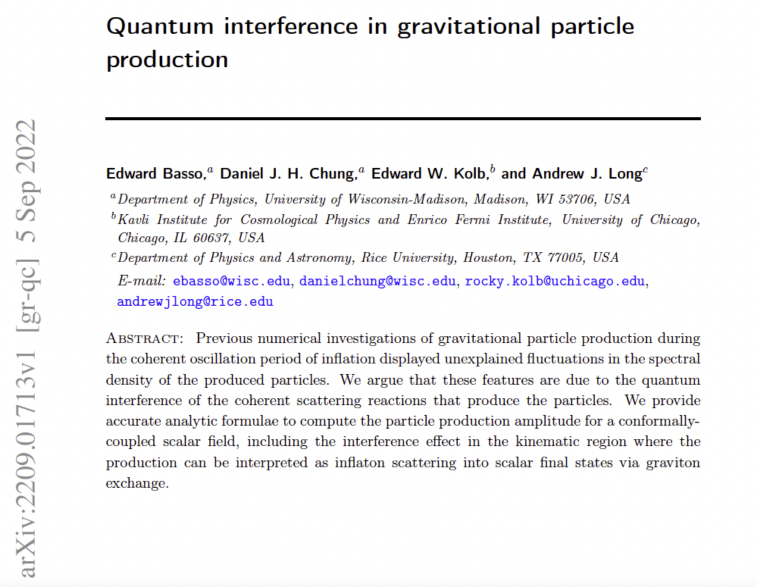
The first three terms are divergent in the limit $k \to \infty$.

Cristian Moreno-Pulido and J. Sola Peracaula, "Running vacuum in quantum field theory in curved space-time: renormalizing $\rho_{\rm vac}$ without $\sim m^4$ terms", Eur. Phys. J. C (2020) 80:692

- adiabatic regularization procedure (ARP) introducing a scale M
- subtraction prescription (equivalent to subtraction of the Minkowski result)

- The cut-off:

$$k < \Lambda \equiv a_e H_I$$



Quantum interference in gravitational particle production

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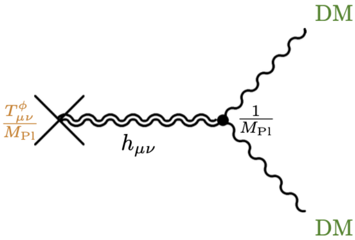
ABSTRACT: Previous numerical investigations of gravitational particle production during the coherent oscillation period of inflation displayed unexplained fluctuations in the spectral density of the produced particles. We argue that these features are due to the quantum interference of the coherent scattering reactions that produce the particles. We provide accurate analytic formulae to compute the particle production amplitude for a conformally-coupled scalar field, including the interference effect in the kinematic region where the production can be interpreted as inflaton scattering into scalar final states via graviton exchange.



$$S^{(2)}_{\phi\to xx} = \frac{\rho_\phi}{M_{\rm Pl}^2}\sum_k \mathcal{P}_k^{2n}\mathcal{M}_V(k)(2\pi)^4\delta(k\omega - p_1^0 - p_2^0)\delta^{(3)}(\vec{p}_1 + \vec{p}_2),$$

$$\Downarrow$$

- No interference between k and $k' \neq k$.
- However, periodicity of $\mathcal{P}(t)$ is approximate.
- The decomposition of $\phi(t) = \varphi(t)\cdot\mathcal{P}(t)$ is not unique.

5 Bogoliubov versus Boltzmann. $\Longrightarrow$ Kunio Kaneta

6 Thermal corrections to $V(\phi, h)$.

7 RGE corrections to parameters of $V(\phi, h)$.

Limits on $g_{h\phi}$

- Perturbativity ($h_i\phi \to h_i\phi$)

$$g_{h\phi} \lesssim \left(\frac{\Lambda^2}{\phi M_{\rm Pl}}\right),$$

- The inflationary dynamics is dominated by the cosmological constant term $\sim \Lambda^4$ therefore

$$g_{h\phi} \lesssim \sqrt{\lambda_h}\left(\frac{\Lambda^2}{\phi M_{\rm Pl}}\right),$$

- If $m_{h_0} > 3H_I/2$ the Higgs field fluctuations during inflation are strongly suppressed ensuring stability (J. R. Espinosa, et al. , [arXiv:1505.04825]), therefore

$$g_{h\phi} \gtrsim \frac{3}{4}\sqrt{6\alpha}\left(\frac{\Lambda^2}{\phi M_{\rm Pl}}\right)^2\left(\frac{\phi}{M}\right).$$

$$\boxed{6 \cdot 10^{-11} \lesssim g_{h\phi} \lesssim 3 \cdot 10^{-6}}$$