Adiabatic Matching Device

Physics Background

Eugene BULYAK

IJClab/NSC KIPT/Karazin U

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Positron Source Scheme

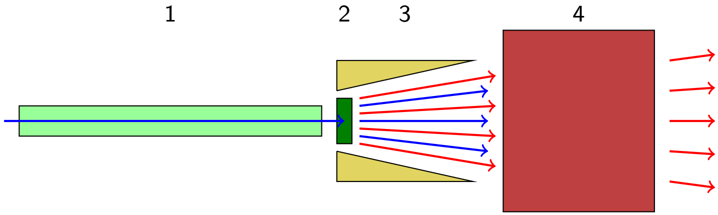
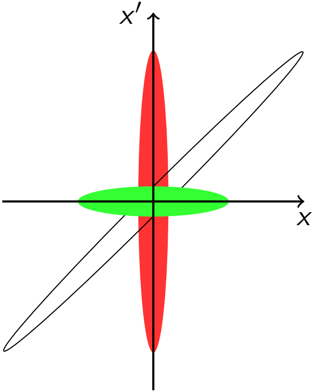


Figure: Scheme of positron source. 1 –electron beam, 2 (dark green) – converter, 3 (yellow) – AMD, 4 – next stage, collimating, removing electrons, accelerating, etc.

Matching Problem Setup



Positron beams from conversion target possesses specific properties:

- Large spread of the positrons energy, $0 \leq E^{+} \leq E^{-} - 2mc^{2}$
- Large angular spread (on accelerator's scale)
- Small initial radius, 1 mm or less

Adiabatic Matching Device

Magnetic horn – tapered solenoidal magnetic field

Magnetic field on axis z

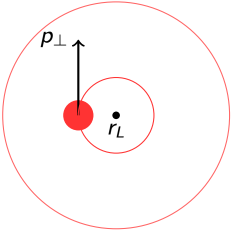
$$B_z(r=0) = \frac{B_0}{1+\alpha z}$$

$\alpha = 0$ – solenoid: $B_z(r,z) = B_0$; $B_r = 0$
$\alpha > 0$ – tapered (AMD)

$$B_r(r,z) = \frac{B_0 \alpha r}{2(1+\alpha z)^2}$$

Trajectories in Solenoidal Field

'Larmour rings' model: $p_{\|} = \text{const}$



Helical spring-like trajectory
Decomposed into longitudinal drift of Larmour rings
cyclotron frequency of 'heavy' positrons

$$\omega_c = \frac{eB}{\gamma m} \approx \frac{1.76 \times 10^{11} \times B[\mathrm{T}]}{\gamma}$$

corresponding radius of 'spring'

$$r_L = \frac{2\gamma c \sin(\theta_0)}{1.76 \times 10^{11} \times B[\mathrm{T}]}$$

spatial period

$$L \approx \frac{2\pi c \cos(\theta_0)}{\omega_c} = \frac{2\pi c \gamma \cos(\theta_0)}{1.76 \times 10^{11} \times B[\mathrm{T}]}$$
$$= 1.07 \times 10^{-2} \frac{\gamma \cos(\theta_0)}{B[\mathrm{T}]} [\mathrm{m}]$$

Adiabatic Approach in Mechanics

adiabatic invariants – Poincaré, Lee, Cartano

Periodic motion – a number of quantities remain constant (period, energy, amplitude, initial phase, amplitude squared, etc)
If system parameters subject to slow change (comp with the period) then some of that quantities changed much slower than others. They are the adiabatic invariants.
Action is one of the adiabatic integrals

$$J = \oint p \, \mathrm{d}q = \int_T E \, \mathrm{d}t$$

Action adiabatic invariant

$$J \approx \text{const} = \frac{2\pi E_\perp}{\omega_c}$$

Adiabatic Matching Device

Field decreased $D = 1 + \alpha Z$ times over AMD

$$r_Z = r_0(1 + \sqrt{D})$$
$$\theta_Z = \theta_0/\sqrt{D}$$

No explicit dependence on energy but $r_0 \propto 1/\gamma; \quad \omega_c \propto 1/\gamma$

Summary

- AMD has no resonant energy – all particles focused
- The higher field strength at input the better
- Emittance at the output $\approx$ at input
- Bunch length in AMD increased: less energetic positrons emitting at larger angles to the axis have passed longer paths

Discussion and Outlook

Does adiabatic reduction of the transverse energy contradict the energy conservation law in a static magnetic field?

NO: Reduction of $E_\perp$ compensated with increase in $E_{||}$ – non-radiating charged particle in magnetic field preserves the energy

Optimisation

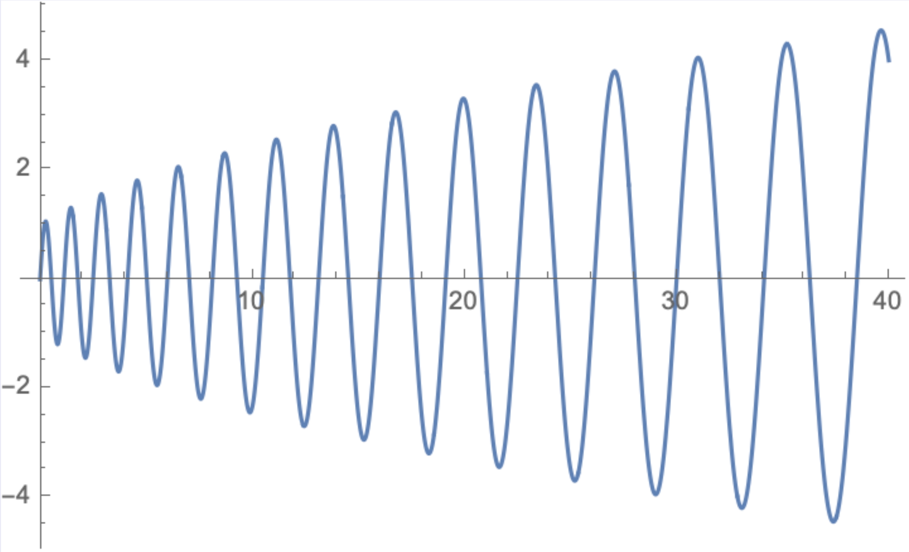
- Start with acceptance of further acceleration: $r_Z; \Delta\gamma$
- B_0^{max} available ?
- Optimal positrons density at the converter rear end on γ, θ plane (controlled with converter's thickness)

Bunch Lengthening

- Ultrarelativistic particles move at $v \approx c$
- Longer the path later to come

Path length (trajectory) within AMD required for evaluation of the bunch prolonging

Trajectory projection



Trajectory Addition Length

$$\Delta L = \frac{1}{4}\left[2L\sqrt{\frac{4\alpha L + \mathrm{r0}^2\left(\alpha^2 + 4\mathrm{w0}^2(\alpha L+1)\right)+4}{\alpha L+1}} + \frac{2\sqrt{\frac{4\alpha L+\mathrm{r0}^2\left(\alpha^2+4\mathrm{w0}^2(\alpha L+1)\right)+4}{\alpha L+1}}}{\alpha} + \frac{\alpha \mathrm{r0}^2 \sinh^{-1}\left(\frac{2\sqrt{(\alpha L+1)\left(\mathrm{r0}^2\mathrm{w0}^2+1\right)}}{\alpha \mathrm{r0}}\right)}{\sqrt{\mathrm{r0}^2\mathrm{w0}^2+1}} - \frac{2\sqrt{\mathrm{r0}^2\left(\alpha^2+4\mathrm{w0}^2\right)+4}}{\alpha} - \frac{\alpha \mathrm{r0}^2 \sinh^{-1}\left(\frac{2\sqrt{\mathrm{r0}^2\mathrm{w0}^2+1}}{\alpha \mathrm{r0}}\right)}{\sqrt{\mathrm{r0}^2\mathrm{w0}^2+1}}\right] - L$$

Dependence on r_0