

# Do we understand $B \rightarrow D^{**}$ transitions ?

paper

*Phys.Rev.D* 105 (2022) 1, 013004

arXiv:2102.11608

Layout

Problems in  $B \rightarrow D^{**} l \nu$  decays for  $D^{**}$  = broad states

New model for  $B \rightarrow D^{**}$  transitions

Results for  $B \rightarrow D^{**} l \nu$  decays ,  $l = e/\mu/\tau$

Results for  $B \rightarrow D^{**} D_s^{(*)}$  decays

Conclusions and prospects



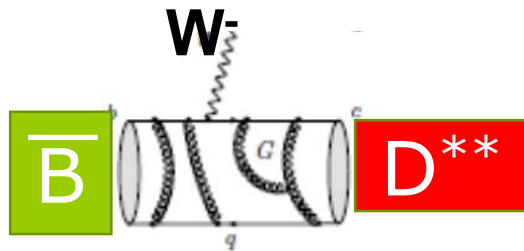
A. Le Yaouanc : [alain.le-yaouanc@th.u-psud.fr](mailto:alain.le-yaouanc@th.u-psud.fr)  
J.-P. Leroy : [jean-pierre.leroy@ijclab.in2p3.fr](mailto:jean-pierre.leroy@ijclab.in2p3.fr)  
P. Roudeau : [patrick.roudeau@ijclab.in2p3.fr](mailto:patrick.roudeau@ijclab.in2p3.fr)

Orsay, 2022, May 19

# Understanding broad $D^{**}$ production in $B \rightarrow D^{**}$ transitions ... (>20 years old pb.)

Consider tree topology only

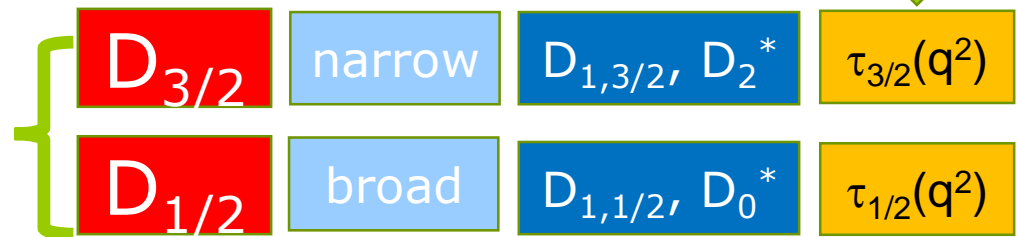
Some conventions



$$D_{1,3/2} = D_1(2420), D_2^* = D_2^*(2460)$$

$$D_{1,1/2} = D_1(2430), D_0^* = D_0(2300)$$

$$2 \text{ FF}(q^2) \\ (m_Q \rightarrow \infty)$$



+ 4 states grouped in 2 doublets labelled using  $J_q = L_q \oplus 1/2$  with  $L_q = 1$   
(D and  $D^*$  correspond to  $L_q = 0$ )

The problems:

Respectable theoretical evaluations give:  $\text{BR}(B \rightarrow D_{1/2}) \ll \text{BR}(B \rightarrow D_{3/2})$ .

Meanwhile, if PDG2021 quotes:  $\text{BR}(D_{1/2}) \ll \text{BR}(D_{3/2})$  in NL decays, and therefore agrees with these expectations, they give also  $\text{BR}(D_{1/2}) \sim \text{BR}(D_{3/2})$  for SL decays, which is in contradiction with factorization expectations.

# Some history (1)

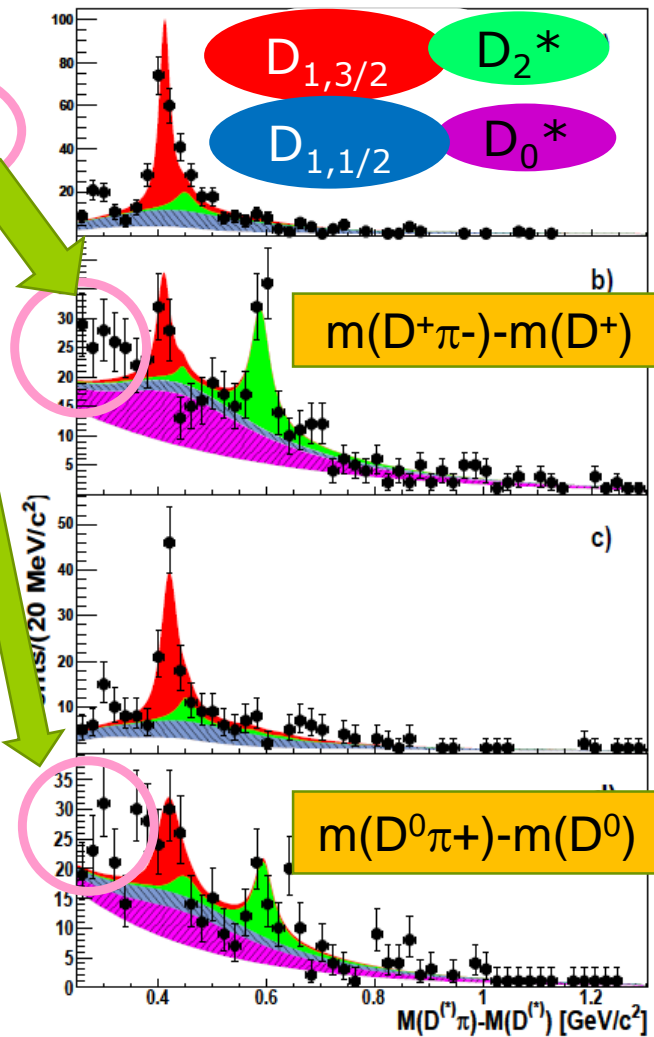
+  $B \rightarrow D_X \ell \nu$  expected to be dominated by D and  $D^*$  hadronic final states.

Yes but these channels correspond to  $(72 \pm 2)\%$  which is less than naively expected.

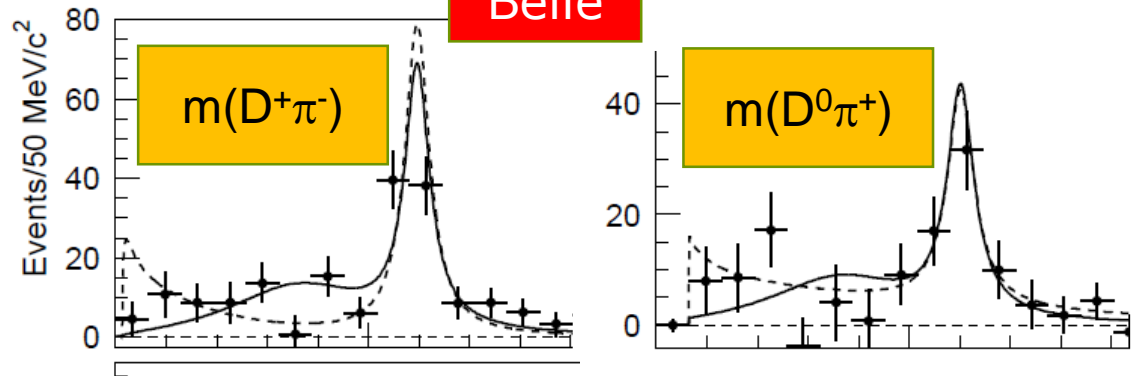
+  $D_{3/2}$  states are narrow (D-wave decays) and easier to measure than  $D_{1/2}$  states that are broad (S-wave decays)

+ no experimental proof that the measured broad  $D^{(*)}\pi$  mass distributions, in s.l. B decays, attributed to  $D_{1/2}$  decays, are S-waves (apart for a Belle  $D\pi$  measurement, with limited statistics).

**Babar**



**Belle**



# Some history (2)

<2000

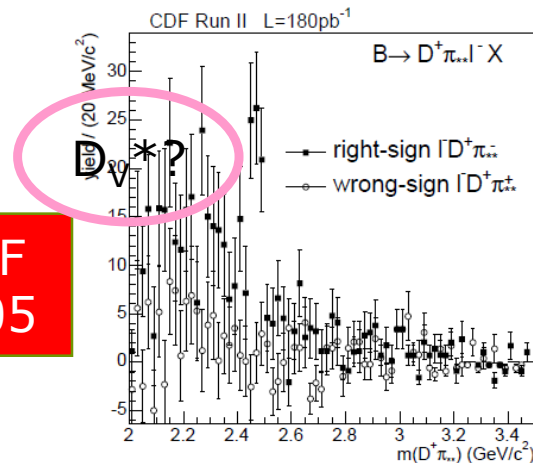
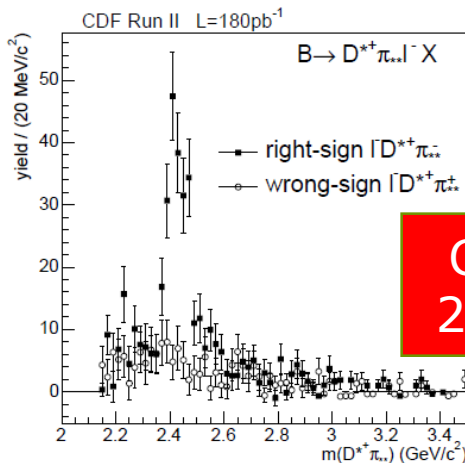
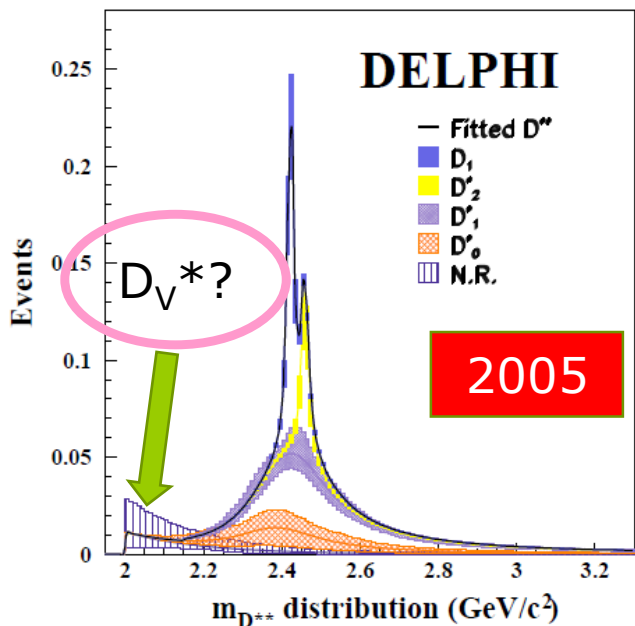
+ A. Abada, A. Le Yaouanc, V. Morénas, L. Oliver, O. Pène, J-C. Raynal (RQM, SR).  
 + I. Bigi, M. Shifman, N. Uraltsev (OPE)  
 + D. Becirevic, B. Blossier, J-P. Leroy...(LQCD)

$BR(D_{1/2}) \ll BR(D_{3/2})$  in SL decays

What are these guys?

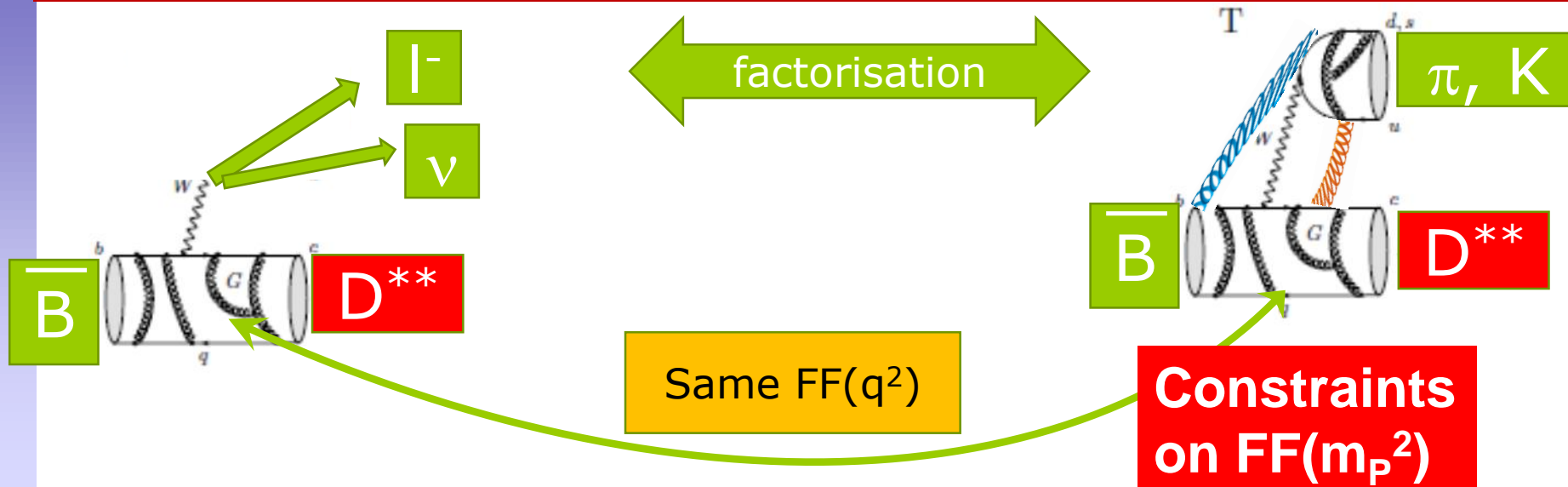
2021

Channel	BR x 10 <sup>3</sup>
$B\text{bar} \rightarrow D_2^*   \nu$	$3.1 \pm 0.3$
$B\text{bar} \rightarrow D_1   \nu$	$6.4 \pm 0.4$
$B\text{bar} \rightarrow D\pi(\text{broad})   \nu$	$4.2 \pm 0.6$
$B\text{bar} \rightarrow D^*\pi(\text{broad})   \nu$	$2.9 \pm 0.7$



- broad and narrow states of similar importance;
- two  $D\pi$  components;
- no spin analysis.

# Constraints from factorisation



Verified for  $D, D^*$  and  $P = \pi, K, \rho, \dots$  in  $B$  decay channels dominated by external- $W$  emission (tree):

$$R_P^{(*)} \equiv \frac{\text{BR}(\bar{B}_d^0 \rightarrow D^{(*)+} P^-)}{d\Gamma(\bar{B}_d^0 \rightarrow D^{(*)+} \ell^- \bar{\nu}_\ell)/dq^2|_{q^2=m_P^2}}$$

$$= 6\pi^2 \tau_{B_d} |V_P|^2 f_P^2 |a_1(D_q P)|^2 X_P,$$

$$a_1(D, D^*) \sim 0.9$$

We check that  $a_1(D_{3/2})$  is similar and assume that  $a_1(D_{1/2}) = a_1(D_{3/2})$

NL transitions to broad states are easier to measure (partial-wave analyses) than in SL decays.

# Another way to express this puzzle

## Factorization implies

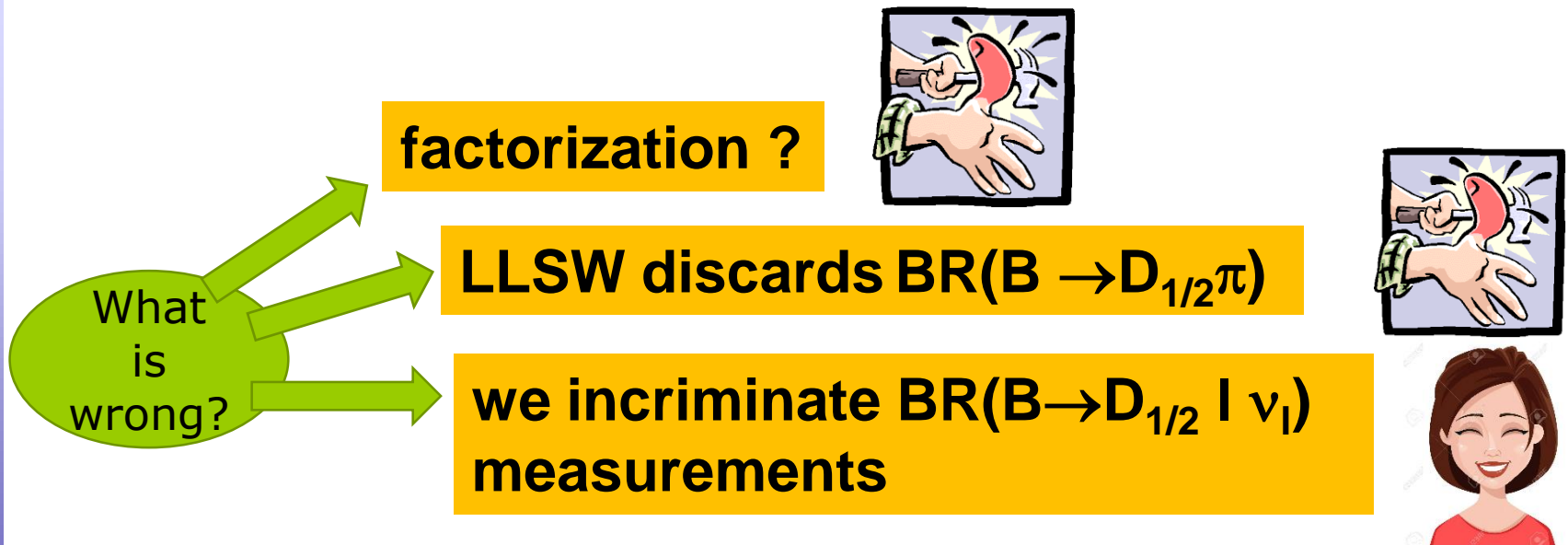
$$\text{BR}(B \rightarrow D_{3/2}\pi) / \text{BR}(B \rightarrow D_{3/2} | \nu_l) \sim \text{BR}(B \rightarrow D_{1/2}\pi) / \text{BR}(B \rightarrow D_{1/2} | \nu_l)$$

## While, according to PDG 2021 and HFAG

$$\text{BR}(B \rightarrow D_{3/2}\pi) / \text{BR}(B \rightarrow D_{3/2} | \nu_l) = 0.132 \pm 0.023$$

$$\text{BR}(B \rightarrow D_{1/2}\pi) / \text{BR}(B \rightarrow D_{1/2} | \nu_l) = 0.029 \pm 0.008$$

factor  $4.6 \pm 1.4$



# “Columbo” like presentation



## Summary

- + sl. and nl. decay channels are related through factorization, in a rather accurate way (**we have updated corresponding numbers**).
- +  $BR(D_{1/2}) \ll BR(D_{3/2})$  in B sl. decays (expected from theory) **agrees with measurements of nl.  $B \rightarrow D^{**} \pi$**
- + therefore, we consider that  $BR(B \rightarrow D_{1/2} | \nu_l) \sim BR(B \rightarrow D_{3/2} | \nu_l)$  is in contradiction with **theoretical** and **experimental** results.
- + the gap between expected broad  $D_{1/2}$  states contributions and measured  $D^{(*)}\pi$  mass distributions **can be explained by  $B \rightarrow D^*_\nu X$  decays**.
- + we have provided various BR evaluations for  $B \rightarrow D^{**} | \nu$  and  $B \rightarrow D^*_\nu | \nu$  with a light and the  $\tau$  lepton (**differences are expected for  $\tau$  lepton channels**).
- + following a proposal from G. Wormser we evaluate also  $BR(B \rightarrow D^{**} D_s^{(*)})$ . This can give **constraints, through factorization, in regions  $m(l \nu) \sim m(D_s)$** .
- +we check that, **surprisingly**, factorization still applies in  $B \rightarrow D^{(*)} D_s$  decays.

# Models for $B \rightarrow D^{**}$ transitions

Phenomenology based on HQET to obtain  $1/m_{b,c}$  expansion of FF as done by LLSW (Leibovich, Ligeti, Stewart and Wise) in PRD57(1998)308.

## Dynamical calculations (Relativistic Quark models, LQCD)

+  $\tau_{3/2}(w=1) > \tau_{1/2}(w=1)$  is mainly a **relativistic effect** (A. Le Yaouanc et al., PLB386 (1997) 364.)

$$(\tau_{3/2}(w=1) = \tau_{1/2}(w=1) \text{ in NRQM})$$

+ RQM expectations agree with LQCD at  $w=1$ . (B. Blossier et al., JHEP 0906 (2009) 022)

$$\tau_{3/2}(1) = 0.53 \pm 0.03; \quad \tau_{1/2}(1) = 0.30 \pm 0.03$$

+ **expectations limited at  $m_{b,c} \rightarrow \infty$**

+ RQM allow to provide the FF( $w$ ) dependence. It differs for the two FF.

+ these observations + **different kinematics**  
 $\rightarrow \text{BR}(D_{1/2}) \sim 1/10 \text{BR}(D_{3/2})$

## Current quantitative model (LLSW)

(Bernlochner et al., PRD95(2017) 014022, PRD97 (2018) 075011)

+ values for  $\tau_{3/2}^{\text{eff.}}(1)$ ,  $\tau_{1/2}^{\text{eff.}}(1)$  and slopes. are fitted on data ... but:

and three-body decays are applied. The smallness of  $\text{BR}(B^0 \rightarrow D_0^{*-} \pi^+)$  is puzzling [38, 39], and measurements using the full *BABAR* and Belle data sets would be worthwhile. It would also be interesting to measure

$\rightarrow$  they use SL information, only, for  $D_{1/2}$  states

$$\rightarrow \text{BR}(D_{1/2}) \sim \text{BR}(D_{3/2})$$

$$\text{BR}(B^0 \rightarrow D_0^{*-} \pi^+) \times 10^4$$

$$1.19 \pm 0.12 \text{ (expt.)} \leftrightarrow 10.0 \pm 2.5 \text{ (expect.)}$$

factorization



# Example of a sl. partial decay width (HQET)

$1/m_Q$  corrections (HQET) imply additional form factors and, in practice, **there are (much) more parameters than constraints.**

$$\frac{d^2\Gamma_{D_1}}{dw d\cos\theta} = 3\Gamma_0 r_1^3 \sqrt{w^2 - 1} \left\{ \sin^2\theta \left[ (w - r_1)f_{V_1} + (w^2 - 1)(f_{V_3} + r_1 f_{V_2}) \right]^2 \right. \\ \left. + (1 - 2r_1w + r_1^2) \left[ (1 + \cos^2\theta) [f_{V_1}^2 + (w^2 - 1)f_A^2] - 4\cos\theta \sqrt{w^2 - 1} f_{V_1} f_A \right] \right\}, \quad (2.2)$$

## Lorentz invariant FF.

$$\begin{aligned} \sqrt{6} f_A &= -(w + 1)\tau - \varepsilon_b \{ (w - 1)[(\bar{\Lambda}' + \bar{\Lambda})\tau - (2w + 1)\tau_1 + (w + 1)\eta_b \} \\ &\quad - \varepsilon_c [4(w\bar{\Lambda}' - \bar{\Lambda})\tau - 3(w - 1)(\tau_1 - \tau_2) + (w + 1)(\tau_1 - \tau_2 + 2\eta_1 - 3\eta_3)], \\ \sqrt{6} f_{V_1} &= (1 - w^2)\tau - \varepsilon_b (w^2 - 1)[(\bar{\Lambda}' + \bar{\Lambda})\tau - (2w + 1)\tau_1 + (w + 1)\eta_b] \\ &\quad - \varepsilon_c [4(w + 1)(w\bar{\Lambda}' - \bar{\Lambda})\tau - 3(w - 1)(\tau_1 - \tau_2) + (w + 1)(\tau_1 - \tau_2 + 2\eta_1 - 3\eta_3)], \\ \sqrt{6} f_{V_2} &= -3\tau - 3\varepsilon_b [(\bar{\Lambda}' + \bar{\Lambda})\tau - (2w + 1)\tau_1 + (w + 1)\eta_b] \\ &\quad - \varepsilon_c [(4w - 1)(w\bar{\Lambda}' - \bar{\Lambda})\tau - 3(w - 1)(\tau_1 - \tau_2) + (w + 1)(\tau_1 - \tau_2 + 2\eta_1 - 3\eta_3)], \\ \sqrt{6} f_{V_3} &= (w - 2)\tau + \varepsilon_b \{ (2 + w)[(\bar{\Lambda}' + \bar{\Lambda})\tau - (2w + 1)\tau_1 - \tau_2] - (2 - w)\eta_b \} \\ &\quad + \varepsilon_c [4(w\bar{\Lambda}' - \bar{\Lambda})\tau + (2 + 3w)\tau_2 \\ &\quad + (w - 2)\eta_{ke} - 2(w + 1)\eta_1 - 4(w - 1)\eta_2 - (3w - 2)\eta_3]. \end{aligned}$$

$\varepsilon_b = 1/(2m_b)$ ,  $\varepsilon_c = 1/(2m_c)$ ,  $\Lambda, \Lambda'$ : HQET parameters  
(constrained using hadron masses)  
 $\tau \sim \tau_{3/2}(w)$ ,  $\tau_1(w)$ ,  $\tau_2(w)$ ,  $\eta_1(w)$ ,  $\eta_2(w)$ ,  $\eta_3(w)$ , unknown

# Importance of $1/m_Q$ corrections (1) (HQET)

Those affecting the FF. normalisation at  $q^2_{\max}$

$$\Lambda, \Lambda_{3/2}, \Lambda_{1/2}, \varepsilon_Q = 1/(2 m_Q)$$

Are **evaluated with some accuracy** and give large effects:

**$D_{1,3/2}$**

$$+ \Delta f_{V1}(1) = 8/\sqrt{2} \varepsilon_c (\Lambda_{3/2} - \Lambda) \tau_{3/2}(1) \sim 0.7 \tau_{3/2}(1) \text{ it explains:}$$

$$R_{3/2} = \text{BR}(B \rightarrow D_2^* | \nu) / \text{BR}(B \rightarrow D_1 | \nu) = \mathbf{1.67} \text{ (th, } m_Q \rightarrow \infty) \leftrightarrow \mathbf{0.48} \pm 0.06 \text{ (expt.)}$$

**$D_0^*$**

$$+ \Delta g_+(1) = 3 (\varepsilon_c + \varepsilon_b) (\Lambda_{1/2} - \Lambda) \tau_{1/2}(1) \sim 0.45 \tau_{1/2}(1) \text{ can be sizable}$$

**$D_{1,1/2}$**

$$+ \Delta g_{V1}(1) = 2 (\varepsilon_c - 3\varepsilon_b) (\Lambda_{1/2} - \Lambda) \tau_{1/2}(1) \sim 0.0 \tau_{1/2}(1) \text{ no effect}$$

+ these effects are proportional to  $\tau_{3/2}(w)$  and  $\tau_{1/2}(w) \rightarrow$  will not change the hierarchy between  $D_{3/2}$  and  $D_{1/2}$  states

+  $D_0^*$  and  $D_{1,1/2}$  production rates can be different with  $D_0^* > D_{1,1/2}$

# Importance of $1/m_Q$ corrections (2) (HQET)

## Those proportional to sub-leading FF.

+  $\tau_1(w)$ ,  $\tau_2(w)$ ,  $\eta_1(w)$ ,  $\eta_2(w)$ ,  $\eta_3(w)$  for  $D_{3/2}$  states : unknown

+  $\chi_1(w)$ ,  $\chi_2(w)$ ,  $\zeta_1(w)$  for  $D_{1/2}$  states : unknown

Some « reasonable » hypotheses are used : it is assumed that these quantities have the same dependence versus  $w$  as  $\tau_{3/2}(w)$  and  $\tau_{1/2}(w)$ , respectively.

Ex. :  $\tau_1(w) = \hat{\tau}_1(1) \tau_{3/2}(w)$  try to adjust normalisation using data

## $1/m$ corrections to the normalization

One defines  $\tau_{3/2}^{eff.}(1)$  and  $\tau_{1/2}^{eff.}(1)$ , expected from th. in the  $m_Q \rightarrow \infty$  limit:

$$\tau_{3/2}^{eff.}(1) = \tau_{3/2}(1) + \frac{\eta_{ke}(1)}{2m_c} + \frac{\eta_b(1)}{2m_b} = \tau_{3/2}(1) \times |1 + \hat{\epsilon}_{3/2}|$$

$$\tau_{1/2}^{eff.}(1) = \tau_{1/2}(1) + \frac{\chi_{ke}(1)}{2m_c} + \frac{\chi_b(1)}{2m_b} = \tau_{1/2}(1) \times |1 + \hat{\epsilon}_{1/2}|$$

# Experimental inputs in the two models

- + HQET parameters are obtained using  $B$ ,  $B^*$ ,  $D$ ,  $D^*$  meson masses and  $\lambda_1$ .
- + Measurements of  $B \rightarrow D^{**} | \nu_l$  branching fractions are used.

**+  $BR(B \rightarrow D_{3/2} | \nu_l)$  are used in the two models**

**+  $BR(B \rightarrow D_{1/2} | \nu_l)$  from PDG, as they stand, in the LLSW model**

**+ on the contrary, we obtain  $BR(B \rightarrow D_{1/2} | \nu_l)$  by subtracting estimates for  $BR(B \rightarrow D_V^{(*)} | \nu_l)$  contributions, from measured  $BR(B \rightarrow D^{(*)} \pi | \nu_l)$ .**

- + Measurements of  $B^0 \rightarrow D^{*-} \pi^+ (K^+)$  branching fractions are used also to provide constraints at  $q^2 = m_\pi^2$  (using factorisation)

**+ on  $D_{3/2}$  only, in the LLSW model**

**+ on  $D_{3/2}$  and  $D_0^*$ , in our model**

# Expt. Results on $B \rightarrow D_{3/2} l \nu$ and $B \rightarrow D_{3/2} \pi / K$

decay channel	this analysis	LLSW	PDG(2020) or HFLAV ( <sup>3</sup> )
		analysis [4] (2016)	
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+} \pi^-) \times 10^4$	$5.85 \pm 0.42$	$5.9 \pm 1.3$	$5.85 \pm 0.43$
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+} K^-) \times 10^5$	$4.7 \pm 0.8$	unavailable	$5.0 \pm 0.9$
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+} \ell^- \bar{\nu}_\ell) \times 10^3$	$3.09 \pm 0.32$	$2.8 \pm 0.4$	$3.18 \pm 0.26$
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+ \pi^-) \times 10^4$	$7.12 \pm 1.13$	$7.5 \pm 1.6$	$6.6 \pm 2.0$
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+ \ell^- \bar{\nu}_\ell) \times 10^3$	$6.40 \pm 0.44$	$6.2 \pm 0.5$	$6.24 \pm 0.54$

$w$ bin	fraction (%)
[1.00, 1.08]	$6.0 \pm 2.3$
[1.08, 1.16]	$30.0 \pm 5.4$
[1.16, 1.24]	$37.5 \pm 6.2$
[1.24, 1.32]	$26.5 \pm 6.2$

$B \rightarrow D_2^* l \nu$  (Belle)

Similar input values for the two models

# Expt. Results on $B \rightarrow D_{1/2} l \nu$ and $B \rightarrow D_{1/2} \pi/K$

		LLSW		
	decay channel	this analysis	analysis [4] (2016)	PDG(2020) or HFLAV <sup>(3)</sup>
$D_0^*$	$\mathcal{B}(\overline{B}^0 \rightarrow D_0(2300)^+ \pi^-) \times 10^4$	$1.19 \pm 0.12$	discarded	$1.14 \pm 0.12$
	$\mathcal{B}(\overline{B}^0 \rightarrow D_0(2300)^+ \ell^- \bar{\nu}_\ell) \times 10^3$	$2.2 \pm 1.2$	$4.1 \pm 0.7$	$3.9 \pm 0.7$ <sup>(4)</sup>
$D_{1,1/2}$	$\mathcal{B}(\overline{B}^0 \rightarrow D_1(2430)^+ \pi^-) \times 10^4$	$0.21 \pm 0.27$ <sup>(1)</sup>	discarded	not quoted
	$\mathcal{B}(\overline{B}^0 \rightarrow D_1(2430)^+ \ell^- \bar{\nu}_\ell) \times 10^3$	$1.4 \pm 1.3$	$1.9 \pm 0.5$	$1.8 \pm 1.5$ <sup>(4)</sup>
	$\mathcal{B}(\overline{B}^0 \rightarrow D_0(2300)^+ K^-) / \mathcal{B}(\overline{B}^0 \rightarrow D_2^{*+} K^-)$ <sup>(2)</sup>	$0.84 \pm 0.36$	unavailable	

## LLSW model

F.U. Bernlochner, Z. Ligeti, [4]PRD95(2017)014022, [5]PRD97(2018)075011

not used

- + no constraint from factorization on  $D_{1/2}$  production in sl. Decays
- + small uncertainty on  $D_{1,1/2}$  not including the poor compatibility of data samples

## Our model

- + constraint on  $D_0^* l \nu$  from factorization
- + rates for  $D_{1/2} l \nu$  obtained using measurements of  $D^{(*)} \pi l \nu$  and subtracting measured contributions from  $D_{3/2} l \nu$  and those expected from  $D_V^{(*)}$

# Expected contributions from $B \rightarrow D_V^* l \nu$

+ coming from the D or the D\* pole

+ the  $g_{D^*D\pi}$  coupling is known and  $g_{D^*D^*\pi} = \sqrt{2 \frac{m_{D^*}}{m_D}} g_{D^*D\pi}$

+ uncertainty on the  $D^{(*)}\pi$  mass distributions (Blatt-Weisskopf factor)

## Our estimate (NL)

Expt.	BR( $B \rightarrow D_V^* \pi^-$ ) $\times 10^4$
Babar	$2.1 \pm 0.6$
Belle	$1.3 \pm 0.2$
LHCb	$1.2 \pm 0.3$

+  $BR(B \rightarrow D_V^* \pi) \times BR(D_V^* \rightarrow D\pi) = (1.6 \pm 0.6) \times 10^{-4}$

OK

## Our estimates (SL)

+  $BR(B \rightarrow D_V^* l \nu) \times BR(D_V^* \rightarrow D\pi) = (2.0 \pm 0.6) \times 10^{-3}$

+  $BR(B \rightarrow (D_V + D_V^*) l \nu) \times BR(D_V + D_V^* \rightarrow D^*\pi) = (1.4 \pm 0.6) \times 10^{-3}$

right  
order  
needed

# Constraints from theory (parameterizations)

## LLSW model

- + use the quantity  $(w-1)$  as expansion parameter ( $w_{\text{max.}} - 1 \sim 0.3 \sim 1/m_Q$ )
- + consider that the IW functions have a linear variation versus  $w$
- + use different scenarios (A, B, C) in which they fit the values (at  $w=1$ ) and slopes of IW functions. Different sub-leading IW or chromomagnetic contributions at  $w=1$  are fitted, depending on the scenario

## Our model

- + use full expressions without expansions in  $(w-1)$
- + use dipole parameterizations for IW functions
- + evaluate the fit sensitivity to the different variables to evaluate those which can be fitted and those that have to be guessed within some interval ( $\pm 0.5 \text{ GeV} \sim \pm \Lambda$ )



# Results for $B \rightarrow D_{3/2} l \nu$ decays

+ we check that using similar inputs and constraints we reproduce the classical results

analysis	$\tau_{3/2}^{eff.}$	$\sigma_{3/2}^2$	$\hat{\tau}_1 (GeV)$	$\hat{\tau}_2 (GeV)$	$\chi^2/NDF$
LLSW	$0.40 \pm 0.04$	$1.6 \pm 0.2$	$-0.5 \pm 0.3$	$2.9 \pm 1.4$	2.4/4
our code	$0.41 \pm 0.06$	$1.60 \pm 0.25$	$-0.66 \pm 0.42$	$5. \pm 2.$	1.8/3

+ **we have fitted the  $a_1(D_{3/2})$  factorization parameter**, used to relate sl and nl decays. **Its value is compatible with the one measured in D and D\* production.**

+ without any 1/m corrections  $P(\chi^2) < 10^{-13}$

+ we find that **it is possible to fit the values of 3 of 1/m corrections in addition to the normalisation and slope of  $\tau_{3/2}(w)$**

+ **we obtain similar expectations as in the LLSW model for decays with a tau**

$\mathcal{R}_{D_{3/2}} (\%)$	our analysis	LLSW	
		[4](2016)	[5](2017)
$\mathcal{R}_{D_2^*}$	$6.1 \pm 0.5 \pm 0.2$	$7 \pm 1$	$7 \pm 1$
$\mathcal{R}_{D_1}$	$9.9 \pm 0.7 \pm 0.1$	$10 \pm 1$	$10 \pm 2$

$$R_{D^{**}} = \frac{BR(B \rightarrow D^{**} \tau \nu)}{BR(B \rightarrow D^{**} l \nu)}$$

# Results for $B \rightarrow D_{1/2} l \nu$ decays

+ Measurements on  $B \rightarrow D_{1/2} l \nu$  decays are uncertain and scarce

## Value of $\tau_{1/2}(1)$

+  $a_1 = 1$

+ FF slope  $\sigma_{1/2}^2 = 0.8 \pm 0.6$

$$\tau_{1/2}^{eff.}(1) = 0.147 \pm 0.025,$$

$$\tau_{1/2}^{eff.}(1) = 0.20 \pm 0.06$$

LQCD

$$\hat{\epsilon}_{1/2} = \hat{\epsilon}_{3/2}$$

$$\tau_{1/2}(1) = 0.296 \pm 0.026.$$

$$\tau_{1/2}^{eff.}(1) = 0.24 \pm 0.02.$$

We measure that  $\tau_{1/2}(1) < \tau_{3/2}(1)$ ; mainly coming from  $\mathcal{B}(\bar{B}^0 \rightarrow D_0(2300)^+ \pi^-)$

## Fitting another parameter?

X param.	$\tau_{1/2}^{eff.}(1)$	$\sigma_{1/2}^2$	X	$\chi^2/NDF$
no X	$0.155 \pm 0.025$	$1.0 \pm 0.5$	no value	3.5/3
$\chi_1$	$0.21 \pm 0.06$	$0.80 \pm 0.50$	$-0.27 \pm 0.22$	2.6/2
$\chi_2$	$0.21 \pm 0.06$	$0.78 \pm 0.50$	$0.37 \pm 0.27$	2.5/2
$\zeta_1$	$0.22 \pm 0.06$	$0.76 \pm 0.50$	$0.75 \pm 0.44$	2.3/2

# Global fit of $D_{3/2}$ and $D_{1/2}$ production

## Fitted model parameters

$a_{1,eff}^{D^{**}\pi}$	$\tau_{3/2}(1)$	$\sigma_{3/2}^2$	$\tau_{1/2}^{eff.}(1)$	$\sigma_{1/2}^2$
$0.944 \pm 0.062$	$0.53 \pm 0.03$	$1.50 \pm 0.50$	$0.21 \pm 0.06$	$0.80 \pm 0.70$
$\hat{\epsilon}_{3/2}$	$\hat{\eta}_1 (GeV)$	$\hat{\eta}_3 (GeV)$	$\hat{\tau}_1 (GeV)$	$\hat{\chi}_1 (GeV)$
$-0.18 \pm 0.11$	$-0.32 \pm 0.13$	$-0.77 \pm 0.28$	$0.36 \pm 0.35$	$-0.24 \pm 0.26$

← constrained

## Systematic uncertainties

+ HQET parameters

+ dipole / linear w dependence of IW functions

+ variation of  $\pm 0.5$  GeV of the not-fitted parameters:  $\hat{\tau}_2(1)$ ,  $\hat{\eta}_2(1)$ ,  $\hat{\chi}_2(1)$ ,  $\hat{\zeta}_1(1)$   
combined linearly (dominant)

$$\hat{\eta}_1 (GeV) = -0.32 \pm 0.13 \pm 0.03$$

$$\hat{\eta}_3 (GeV) = -0.77 \pm 0.28 \pm 0.21$$

$$\hat{\tau}_1 (GeV) = 0.36 \pm 0.35 \pm 0.35.$$

$$\hat{\chi}_1 (GeV) = -0.24 \pm 0.26 \pm 0.60$$

# Expectations for $B \rightarrow D_{3/2} l \nu$ and $B \rightarrow D_{1/2} l \nu$ decays

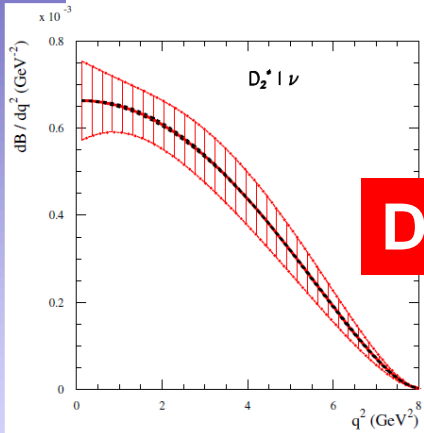
decay	$e/\mu$ ( $\times 10^3$ )	$\tau$ ( $\times 10^4$ )	R(D**) (%)
$D_2^* l \nu$	$3.15 \pm 0.30$	$1.90 \pm 0.29 \pm 0.05$	$6.03 \pm 0.52 \pm 0.15$
	$3.16 \pm 0.30$	$1.90 \pm 0.27 \pm 0.07$	$6.01 \pm 0.49 \pm 0.19$
$D_{1,3/2} l \nu$	$6.40 \pm 0.44$	$6.30 \pm 0.59 \pm 0.10$	$9.84 \pm 0.68 \pm 0.15$
	$6.40 \pm 0.44$	$6.19 \pm 0.56 \pm 0.15$	$9.67 \pm 0.62 \pm 0.24$
	$e/\mu$ ( $\times 10^4$ )	$\tau$ ( $\times 10^5$ )	R(D**) (%)
$D_0^* l \nu$	$5.1 \pm 1.2 \pm 1.2$	$5.0 \pm 1.3 \pm 1.7$	$9.9 \pm 1.5 \pm 1.0$
	$39.1 \pm 7.0 \pm 0.2$	$31.9 \pm 7.9 \pm 2.0$	$8.2 \pm 1.5 \pm 0.5$
$D_{1,1/2} l \nu$	$4.6 \pm 3.7 \pm 0.9$	$3.4 \pm 2.7 \pm 0.6$	$7.4 \pm 1.2 \pm 1.6$
	$17. \pm 15. \pm 2.$	$13. \pm 12. \pm 5.$	$7.6 \pm 1.0 \pm 2.0$
			$8 \pm 3$
			$5 \pm 2$

our analysis -

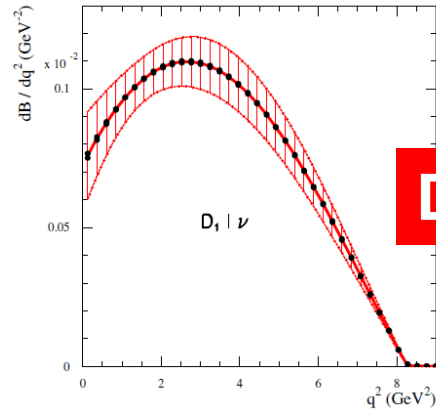
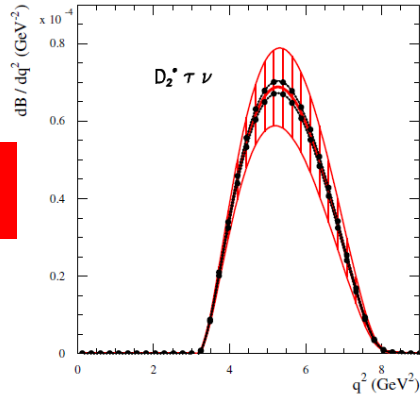
LLSW -

published arXiv:1711.03110

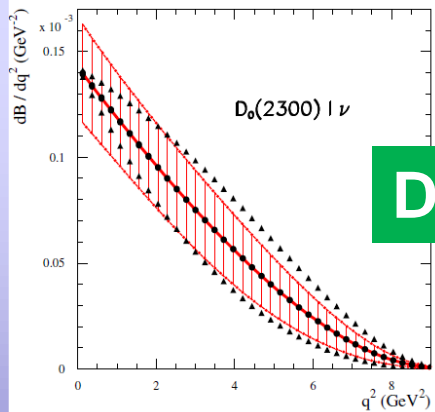
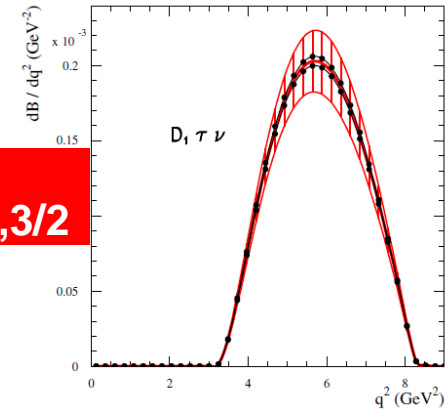
# $q^2$ distributions



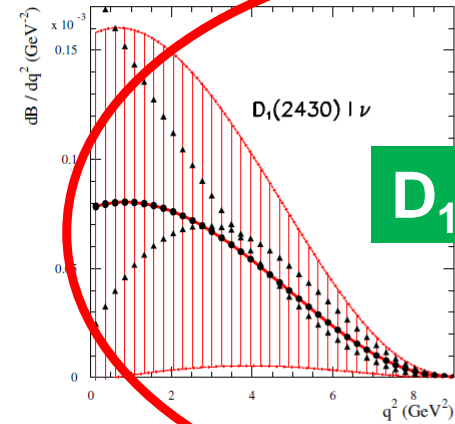
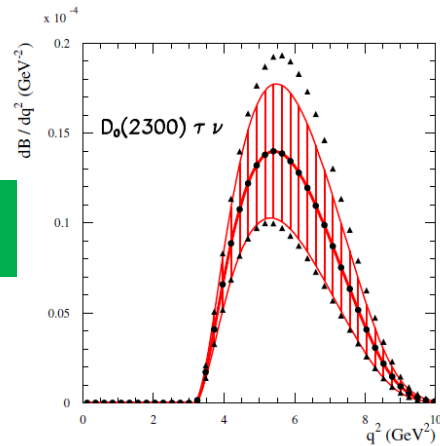
$D_2^*$



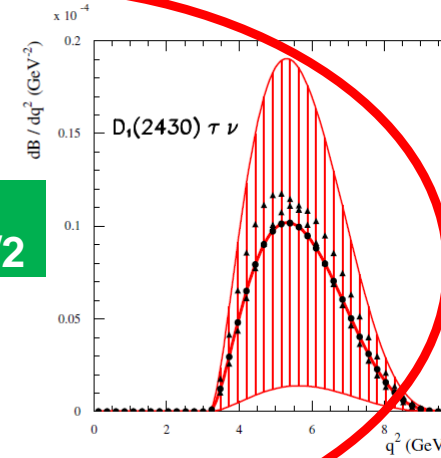
$D_{1,3/2}$



$D_0^*$



$D_{1,1/2}$



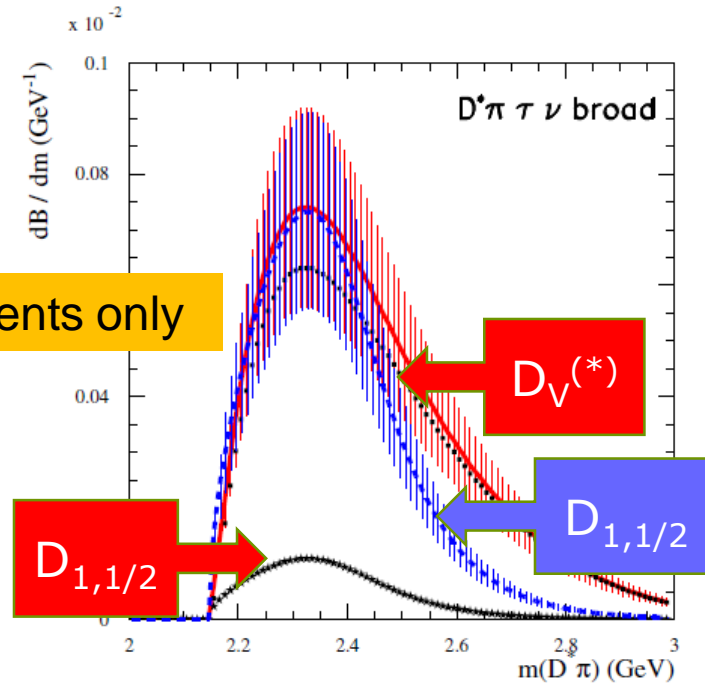
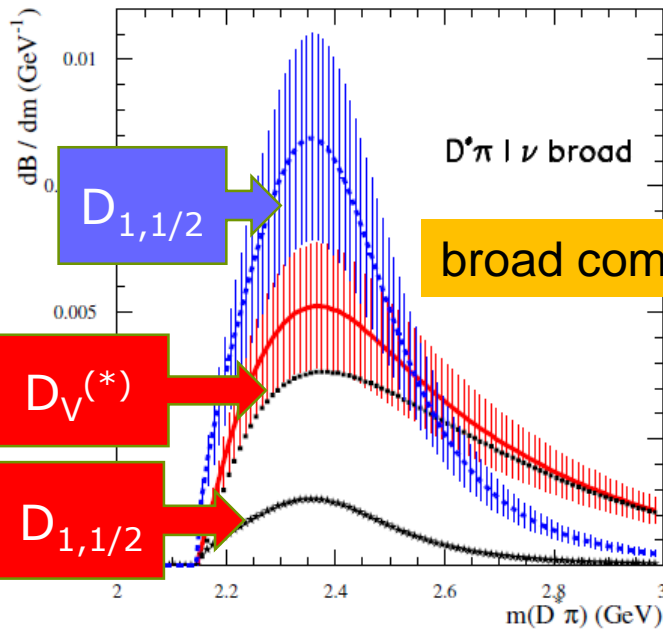
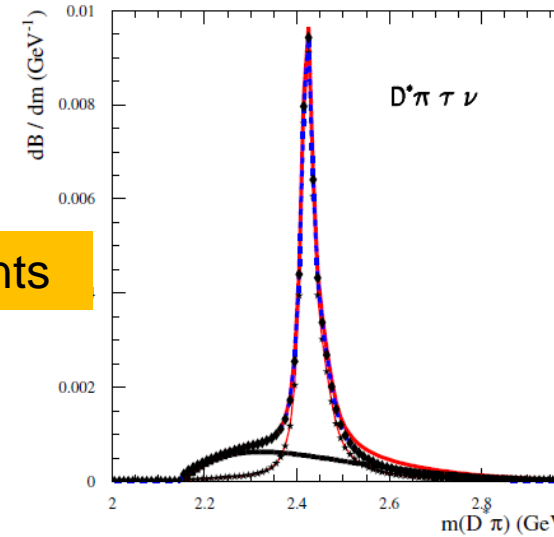
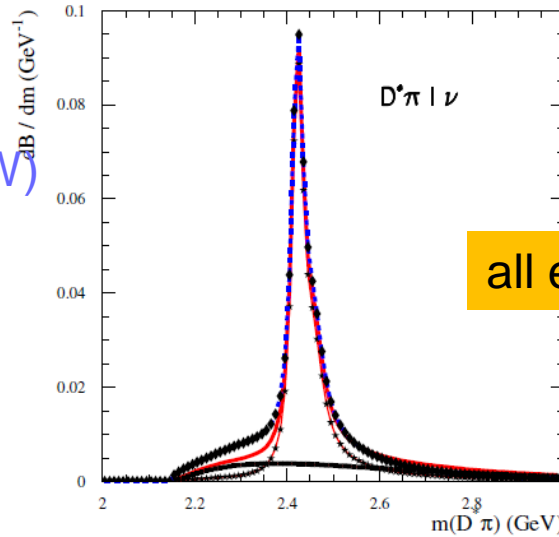
Terra incognita

Need for the measurement of  $BR(B^0 \rightarrow D_{1,1/2}^- \pi^+)$

# $B \rightarrow D^* \pi l \nu$ decays

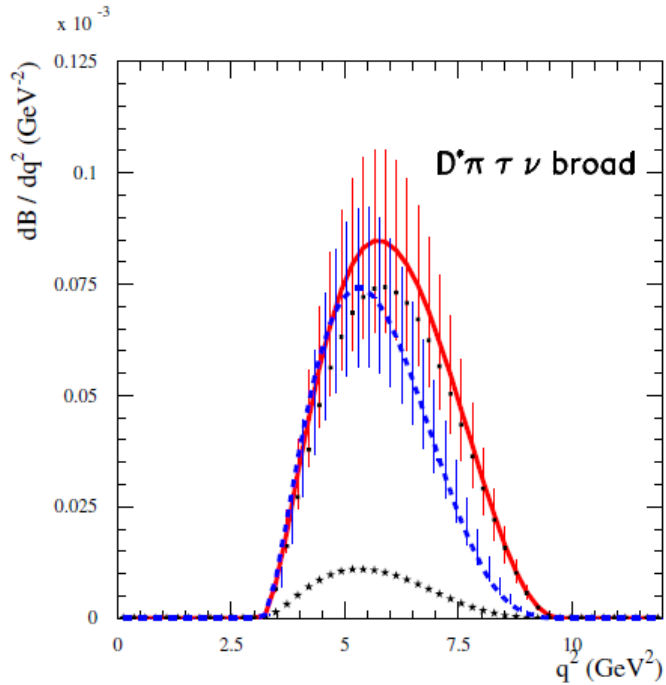
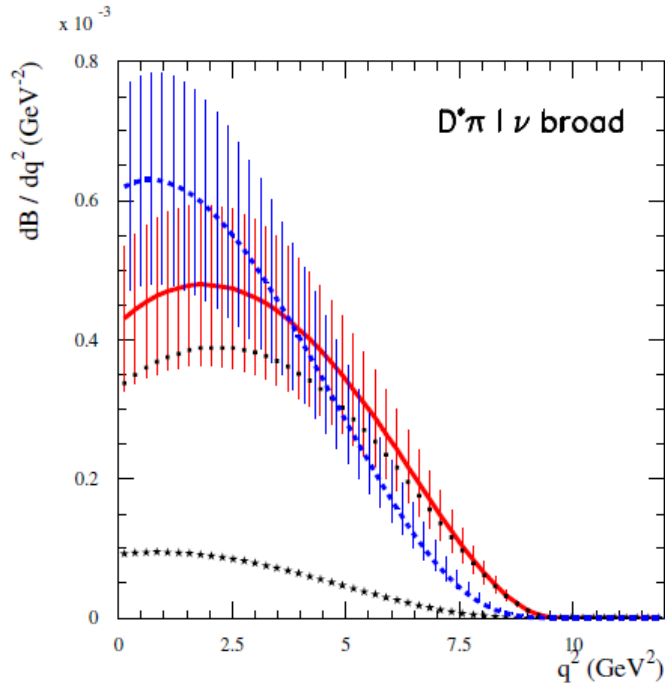
## Mass distributions

Compare dashed-blue (LLSW) and red (our model) lines (total distributions)



# $B \rightarrow D^* \pi l \nu$ decays

## $q^2$ distributions



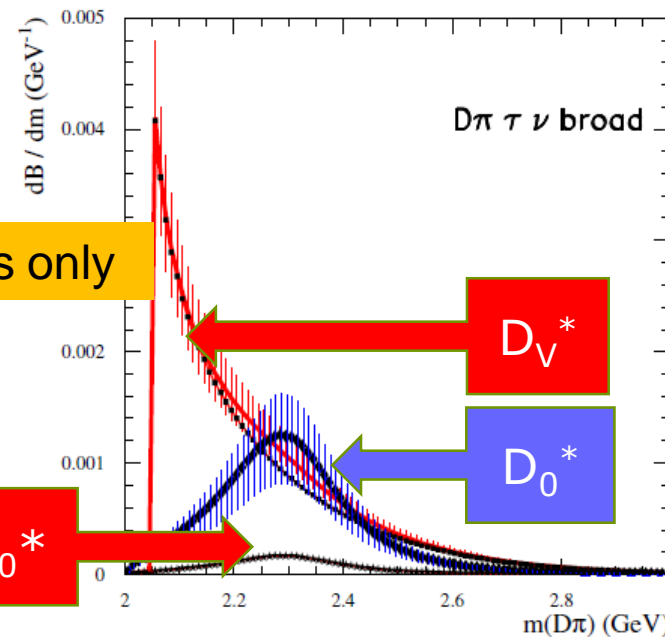
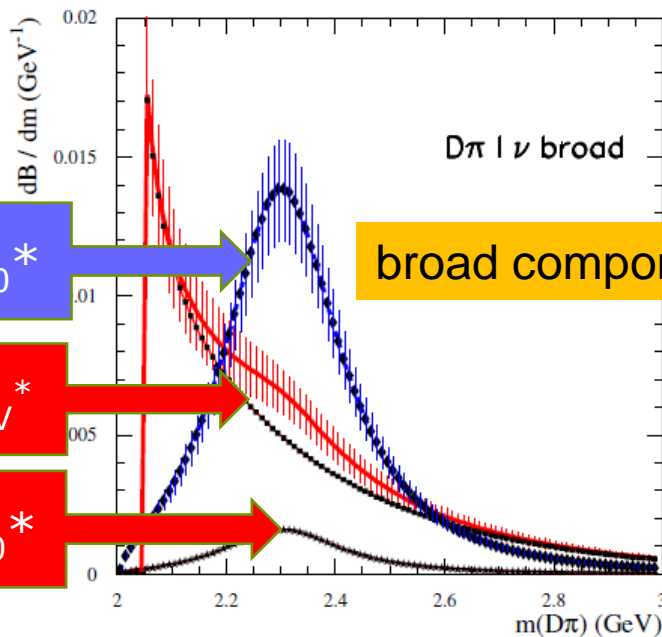
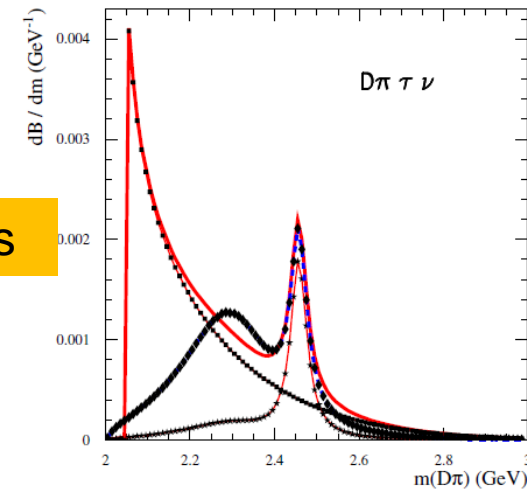
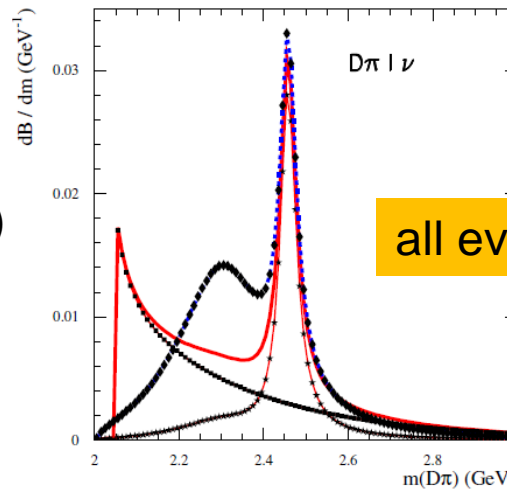
In our model : more events are expected in the  $\tau$  channel

# $B \rightarrow D\pi l \nu$ decays

## Mass distributions

Compare **dashed-blue** and **red** lines (total distributions)

Rather different expectations from the two models, in particular for tau events

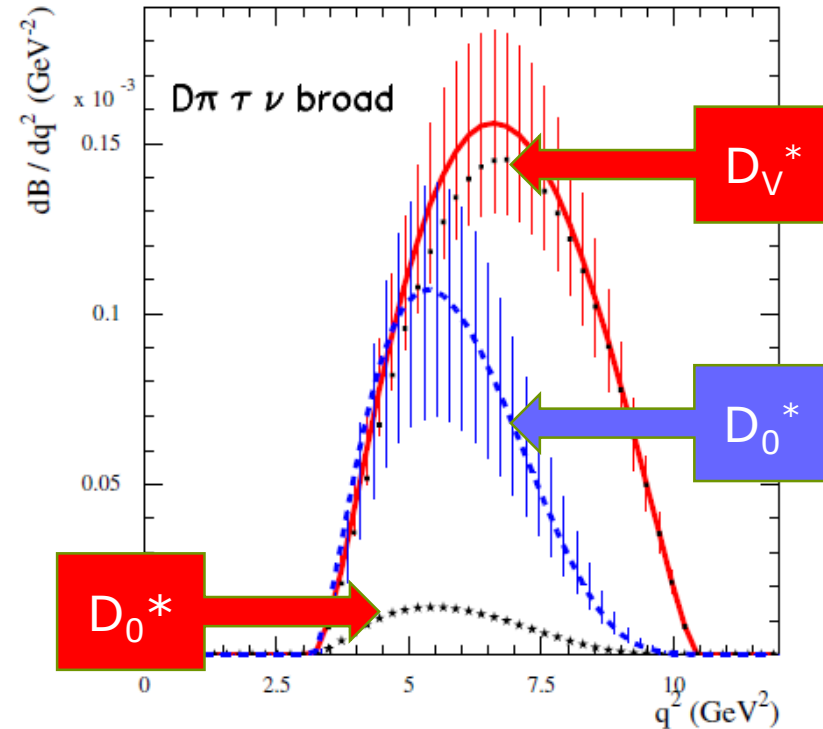
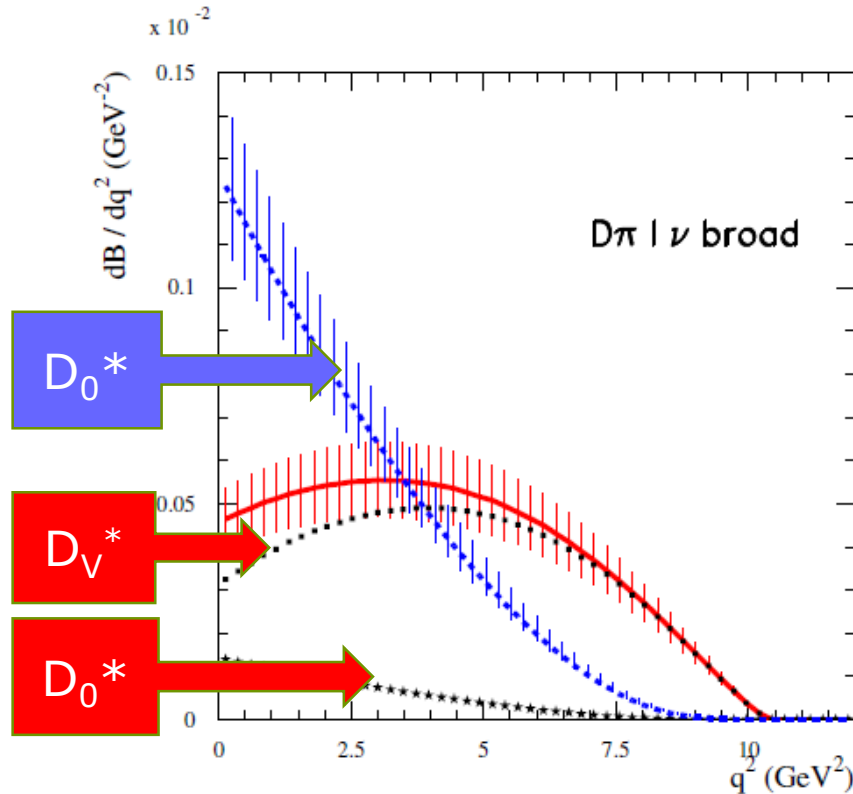




# $B \rightarrow D \pi l \nu$ decays

## $q^2$ distributions

broad components only



$q^2$  variation is different for  $D_0^*$  and  $D_V^*$  components

In our model : more events are expected in the  $\tau$  channel

# $B \rightarrow D^{**} D_s^{(*)}$ decays

## The idea (G. Wormser)

Have an experimental control of the background from  $B \rightarrow D_X l \nu$  decays in  $B \rightarrow D^{(*)} \tau \nu$  analyses, through factorization, using measurements of  $B \rightarrow D^{**} D_s^{(*)}$  decays.

$$q^2 \equiv m^2(D_s) \sim m^2(\tau).$$

+ Is factorization valid for such decays?

+ penguin contributions in case of  $D^{**}$ ?  
→ neglected at present ...

One can average  $B^0$  and  $B^+$  decays for such channels  
(no internal-W contribution)

# Factorization ( $a_1$ : data $\leftrightarrow$ computation, $m_Q \rightarrow \infty$ )

## Comparison with data

For  $D\pi$  and  $D^*\pi$  decays we obtain from present measurements:

$$a_1^{D\pi} = 0.861 \pm 0.024 \quad a_1^{D^*\pi} = 0.886 \pm 0.023$$

Using  $B \rightarrow D^{**} X$  decays we obtain:  $a_1^{D^{**}\pi} = [0.81 - 0.90] \pm 0.06$

$a_1$  is  $\sim 1 \rightarrow$  factorization  
« works »

Note that  $a_1(m_Q \rightarrow \infty) = 1.070 \pm 0.022$  !?

« Works » also in  $B \rightarrow D^{(*)} D_s$  decays, once penguins “removed”

$$\frac{|a_{1,eff}^{DD_s^-}|}{|a_1^{DK}|} = 0.873 \pm 0.053 \quad \text{expects : } 0.847 \pm ?$$

$$\frac{|a_{1,eff}^{D^*D_s}|}{|a_1^{D^*K}|} = 1.052 \pm 0.078 \quad \text{expects } 1.037 \pm ?$$

Importance of penguin amplitude in the D channel.  
What about  $D^{**}$ ?

# $B \rightarrow D^{**} D_s^{(*)}$ decays

channel	value $\pm$ fit	model
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+} D_s^-) \times 10^4$	$5.8 \pm 0.8$	0.6
$\mathcal{R}_{D_2^*}^{\tau, D_s}$	$0.33 \pm 0.06$	0.03
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+ D_s^-) \times 10^4$	$13.1 \pm 3.5$	0.9
$\mathcal{R}_{D_1}^{\tau, D_s}$	$0.48 \pm 0.12$	0.03
$\mathcal{B}(\bar{B}^0 \rightarrow D_0(2300)^+ D_s^-) \times 10^4$	$2.3 \pm 0.4$	0.2
$\mathcal{R}_{D_0(2300)}^{\tau, D_s}$	$0.22 \pm 0.04$	0.05
$\mathcal{B}(\bar{B}^0 \rightarrow D_1(2430)^+ D_s^-) \times 10^4$	$1.4 \pm 1.1$	0.9
$\mathcal{R}_{D_1(2430)}^{\tau, D_s}$	$0.25 \pm 0.05$	$^{+0.45}_{-0.10}$

channel	value $\pm$ fit	model
$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+} D_s^{*-}) \times 10^3$	$1.9 \pm 0.4$	0.00
$\mathcal{R}_{D_2^*}^{\tau, D_s^*}$	$0.100 \pm 0.014$	0.004
$\mathcal{R}_{D_2^*}^{D_s, D_s^*}$	$0.30 \pm 0.08$	0.03
$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+ D_s^{*-}) \times 10^3$	$4.7 \pm 0.9$	0.1
$\mathcal{R}_{D_1}^{\tau, D_s^*}$	$0.134 \pm 0.024$	0.0
$\mathcal{R}_{D_1}^{D_s, D_s^*}$	$0.28 \pm 0.10$	0.02
$\mathcal{B}(\bar{B}^0 \rightarrow D_0(2300)^+ D_s^{*-}) \times 10^4$	$2.4 \pm 0.7$	1.0
$\mathcal{R}_{D_0(2300)}^{\tau, D_s^*}$	$0.20 \pm 0.04$	0.02
$\mathcal{R}_{D_0(2300)}^{D_s, D_s^*}$	$0.96 \pm 0.26$	$^{+0.44}_{-0.24}$
$\mathcal{B}(\bar{B}^0 \rightarrow D_1(2430)^+ D_s^{*-}) \times 10^4$	$2.2 \pm 2.0$	$^{+0.6}_{-0.1}$
$\mathcal{R}_{D_1(2430)}^{\tau, D_s^*}$	$0.14 \pm 0.02$	0.02
$\mathcal{R}_{D_1(2430)}^{D_s, D_s^*}$	$0.52 \pm 0.02$	$^{+0.52}_{-0.36}$

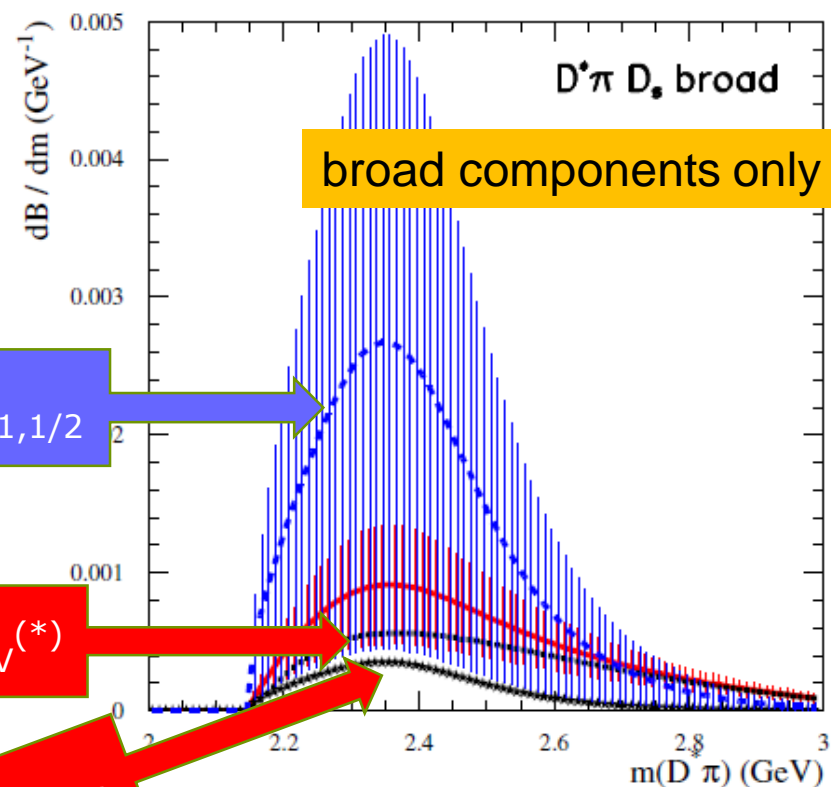
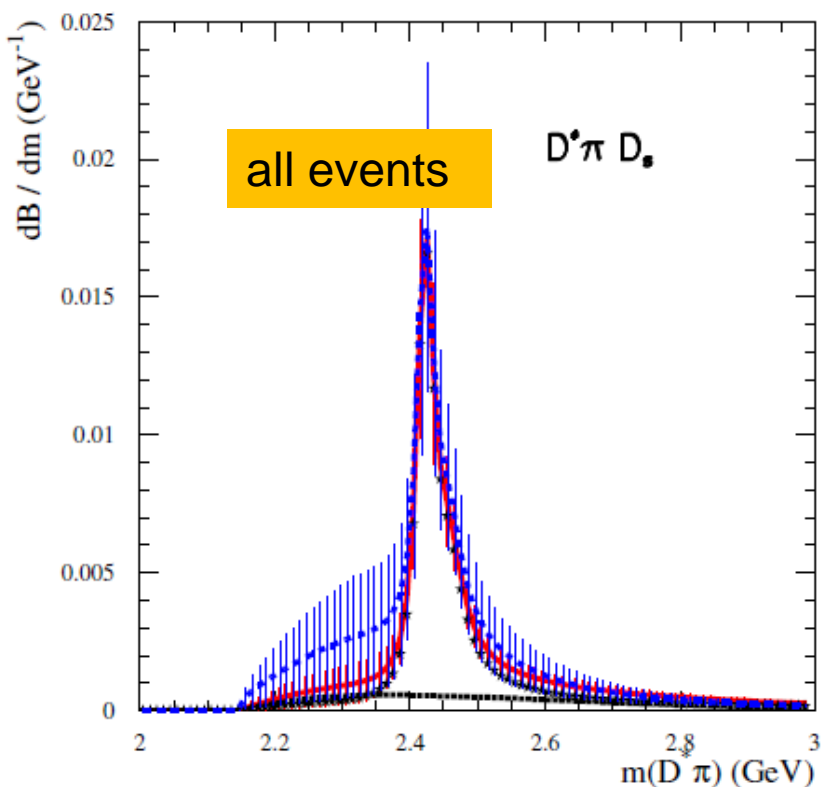
$$\mathcal{R}_{D_i^{**}}^{\tau, D_s} = \frac{\mathcal{B}(\bar{B}^0 \rightarrow D_i^{**,+} \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B}^0 \rightarrow D_i^{**,+} D_s^-)}$$

$$\mathcal{R}_{D_i^{**}}^{D_s, D_s^*} = \frac{\mathcal{B}(\bar{B}^0 \rightarrow D_i^{**,+} D_s^-)}{\mathcal{B}(\bar{B}^0 \rightarrow D_i^{**,+} D_s^{*-})}$$

Expectations for  $D_2^*$ ,  
 $D_{1,3/2}$  and  $D_0^*$  with 30%  
uncertainty and ~100%  
uncertainty for  $D_{1,1/2}$ .

... neglecting penguins!!!

# $B \rightarrow D^* \pi D_s$ decays



$$\mathcal{B}(\bar{B}^0 \rightarrow D_2^{*+} D_s^-) = (2.3 \pm 0.3 \pm 0.2) \times 10^{-4}$$

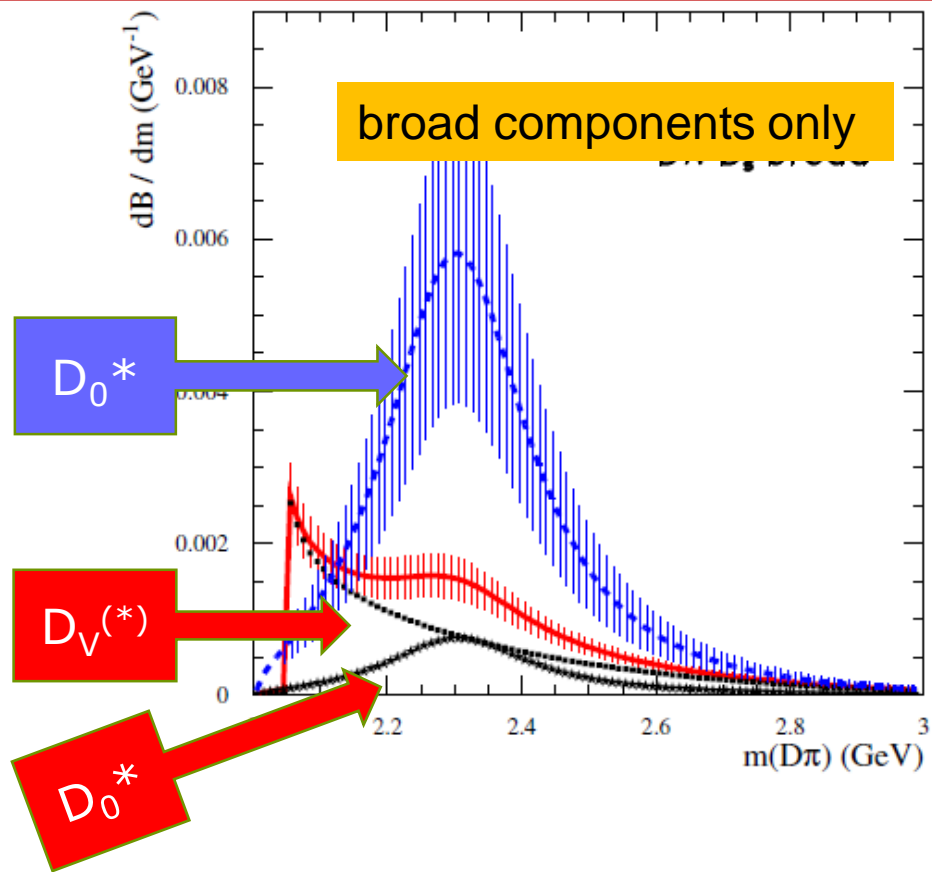
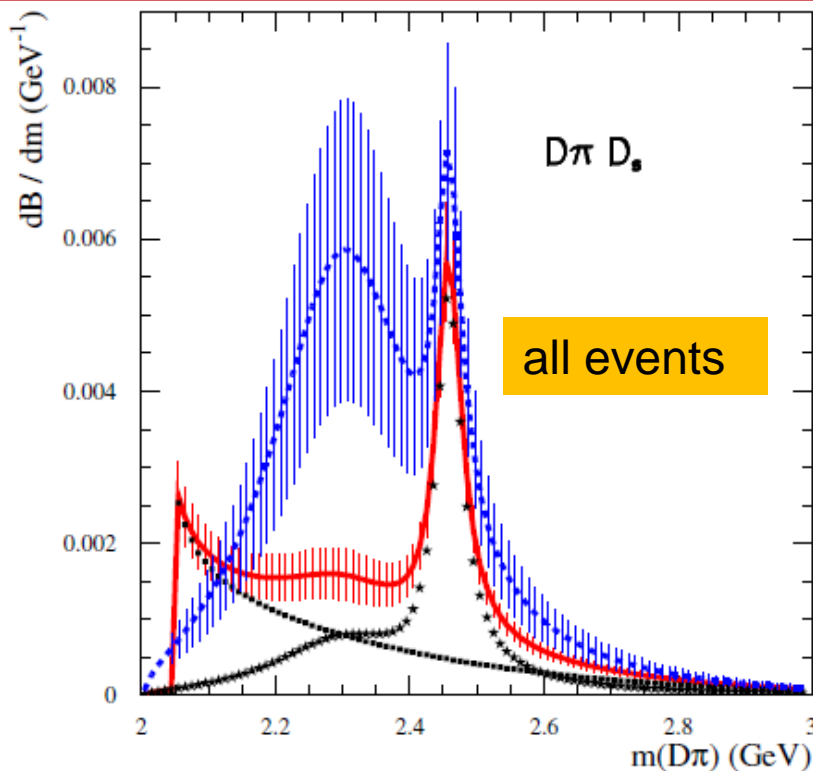
$$\mathcal{B}(\bar{B}^0 \rightarrow D_1^+ D_s^-) = (8.8 \pm 2.3 \pm 0.6) \times 10^{-4}$$

$$\mathcal{B}(\bar{B}^0 \rightarrow D_1(2430)^+ D_s^-) = (1.4 \pm 1.1 \pm 0.9) \times 10^{-4}$$

values include  
 $\mathcal{B}(D^{**} \rightarrow D^* \pi)$

**In our model : less events are expected**

# $B \rightarrow D\pi D_s$ decays



$$B(\bar{B}^0 \rightarrow D_2^{*+} D_s^-) = (3.48 \pm 0.46 \pm 0.33) \times 10^{-4}$$

$$B(\bar{B}^0 \rightarrow D_0(2300)^+ D_s^-) = (2.30 \pm 0.43 \pm 0.22) \times 10^{-4(30)}$$

values include  
 $B(D^{**} \rightarrow D \pi)$

rather different expected distributions from the two models

In our model : much less events are expected

# Conclusions

- We propose a model which includes all present experimental results and theoretical expectations. LLSW models are in large contradiction with existing measurements for  $D_{1/2}$  production:

channel	measured	our model	LLSW
$\mathcal{B}(\overline{B}^0 \rightarrow D_0(2300)^+ \pi^-) \times 10^4$	$1.19 \pm 0.12$	$1.21 \pm 0.12$	$10.0 \pm 2.5$
$\mathcal{B}(\overline{B}^0 \rightarrow D_1(2430)^+ \pi^-) \times 10^4$	$0.21 \pm 0.27$	$0.7 \pm 0.7$	$3.2 \pm 2.8$
$\mathcal{R}_{\mathcal{K}}(D_0(2300), D_2^*)$	$0.84 \pm 0.36$	$0.35 \pm 0.04$	$2.8 \pm 0.7$

- model parameters fitted on data, confirm low production for S-states
- $D^{**}$  production needs to be complemented by  $D_V + D_V^*$  components
- this analysis differs from « classical » ones and gives different expectations for decays with a  $\tau$  lepton.
- we provide also expectations for  $\mathbf{B} \rightarrow \mathbf{D}^{**} \mathbf{D}_s^{(*)}$  decays
- treatment of systematic uncertainties is improved by fitting more parameters (no scenario needed)

$D_0^*$

$D_{1,1/2}$

## From theory

- origin of the difference between  $a_1$  ( $m_Q \rightarrow \infty$ ) and  $a_1$  (expt.) ?
- evaluation of penguin contributions in  $\mathbf{B} \rightarrow \mathbf{D}^{**} \mathbf{D}_s^{(*)}$

## From experiment

- use our model in realistic event simulations to measure the variation induced in backg. evaluation for  $\mathbf{B} \rightarrow \mathbf{D}^{(*)} \tau \nu$  analyses
- use measurements of  $\mathbf{B}^- \rightarrow \mathbf{D}^{*+} \pi^- \mathbf{D}_s^{(*)-}$  to improve the present analysis and, in particular:
  - + to constrain  $\mathbf{D}_{1,1/2}$  production;
  - + to evaluate the contribution of radial excitations in B sl. decays



# To do list(?) in B decays

$\text{BR}(B_s \rightarrow D_{s0}^* 1 \nu)$  (Belle 2)

- this S-state is narrow and should be well identified at variance with the non-strange corresponding meson

$\text{BR}(B \rightarrow D^{(*)} \eta^{(\prime)} 1 \nu)$  (Belle 2 + theory)

- to reduce the fraction of « missing » channels in B sl decays

Evaluate if our model changes bckg. expectations in  $B \rightarrow D^{(*)} \tau \nu$  (LHCb, Belle 2, ...)?

$\text{BR}(B \rightarrow D_s^{(*)} K \pi 1 \nu, D^{(*)} K K 1 \nu), \text{BR}(\Lambda_b \rightarrow D_s X 1 \nu)$  (LHCb)

- to reduce uncertainties on fs/fd

$B^0 \rightarrow D^{*0} \pi^+ \pi^-$  Dalitz analysis (Belle 2) (to constrain  $D_{1,1/2}$  contribution)

Evaluate penguin amplitude in  $B \rightarrow D^{**} D_s^{(*)}$  decays (theory)

Understand difference between  $a_1(m_Q \rightarrow \infty)$  and expt.

# Backup

# Constraints from theory (parameters)

+ HQET parameters :  $m_b, m_c, \Lambda, \Lambda_{3/2}, \Lambda_{1/2}$

## Our analysis

	parameters list	evaluation	constraints from theory
Set 1	$m_{b,c}, \bar{\Lambda}, \bar{\Lambda}_{3/2}, \bar{\Lambda}_{1/2}$	using HQET	from spectroscopy and $\lambda_1$
Set 2 for $D_{3/2}$ mesons	$\tau_{3/2}(1)$	fitted	$0.53 \pm 0.03$
	$\hat{\epsilon}_{3/2}$	fitted	
	$\sigma_{3/2}^2$	fitted	$1.5 \pm 0.5$
	$\hat{\eta}_{1,3}, \hat{\tau}_1$	fitted	
	$\hat{\eta}_2, \hat{\tau}_2$	set to zero	$\pm 0.5 \text{ GeV}$
Set 2 for $D_{1/2}$ mesons	$\tau_{1/2}^{eff.}(1)$	fitted	$0.20 \pm 0.06$
	$\sigma_{1/2}^2$	fitted	$\sigma_{3/2}^2 - \sigma_{1/2}^2 = 0.7 \pm 0.5$
	$\hat{\chi}_1$	fitted	
	$\hat{\chi}_2, \hat{\zeta}_1$	set to zero	$\pm 0.5 \text{ GeV}$

# Wigner rotations

+  $B\bar{b} \rightarrow D^{**} X$  transitions are of interest by themselves ( $L=0 \rightarrow L=1$ )  
There are relativistic spin effects that are different for  $1/2$  and  $3/2$  states.

+ this is the reason why all RQM exhibit the fact that  $BR(D_{3/2}) \gg BR(D_{1/2})$

As shown in ref. Exact duality...Phys.Lett. **386** , 304, the relativistic boost of states generates a spin dependent operator for the :

$$(v'_z - v_z) i(\vec{\sigma} \times \frac{\vec{p}_T}{E_p + m_q})_z \quad (3)$$

for each spectator quark,  $Oz$  being the direction of hadron motion,  $v_z, v'_z$  the hadron initial and final velocities, and  $T$  denoting the transverse internal momentum. The two  $j = 3/2, j = 1/2$  wave functions contain both the spin wave functions  $S = 0, S = 1$ , but with reverted weights :  $1/3, 2/3$  or the reverse. Therefore, the matrix element of eqn. (3) is quite different in the two cases, since moreover the spatial matrix element of  $\frac{\vec{p}_T}{E_p + m_q}$  is large for a relativistic light quark motion. The transition matrix element vanishes at zero transfer for a  $L = 0 \rightarrow L = 1$  transition, but the slopes  $\tau_{j=3/2, j=1/2}(w = 1)$  are non zero and quite different for the two states. The expression for the quantitative difference is given in Phys. Rev D56, 5668, page 5673.

A. Le Yaouanc

# Other differences with the usual model

## Mass distributions

+ we use relativistic Breit-Wigner mass distributions for  $D^{**}$  resonances

→ How to account for mass effects in FF? We take:

$$FF(m, q^2) = FF(m_{D^*}, q^2)$$

$m \neq m_{D^{**}}$  are included in kinetic terms in helicity FF.

# Results for $B \rightarrow D_{1/2} l \nu$ decays

+ we check that using similar inputs and constraints we reproduce the classical results

analysis	$\tau_{1/2}^{eff.}$	$\sigma_{1/2}^2$	$\hat{\zeta}_1$	$\chi^2/NDF$
LLSW	$0.35 \pm 0.11$	$0.2 \pm 1.4$	$0.6 \pm 0.3$	9.1/4
our code ( $\Gamma(D_{1/2}) = 0$ )	$0.37 \pm 0.11$	$0.26 \pm 1.23$	$0.23 \pm 0.31$	7.0/3
our code ( $\Gamma(D_{1/2}) \neq 0$ )	$0.30 \pm 0.18$	$-1.6 \pm 3.2$	$0.45 \pm 0.28$	6.0/3

# Factorization (data $\leftrightarrow$ computation, $m_Q \rightarrow \infty$ )

## Comparison with data

$$|a_1^{DK}| = 0.884 \pm 0.033.$$

$$|a_1^{D^*K}| = 0.924 \pm 0.030$$

$$a_1^{(m_Q \rightarrow \infty)} = 1.070 \pm 0.022 \text{ (NNLO)}$$

$a_1$  is  $\sim 1 \rightarrow$  factorization  
« works » .... BUT



**>5  $\sigma$  difference**  
**( $1/m^n$   $n > 1$**   
**corrections?)**

« Works » also in  $B \rightarrow D^{(*)} D_s$  decays, once penguins “removed”

$$\frac{|a_{1,eff}^{DD_s^-}|}{|a_1^{DK}|} = 0.873 \pm 0.053 \quad \text{expects : } 0.847 \pm ?$$

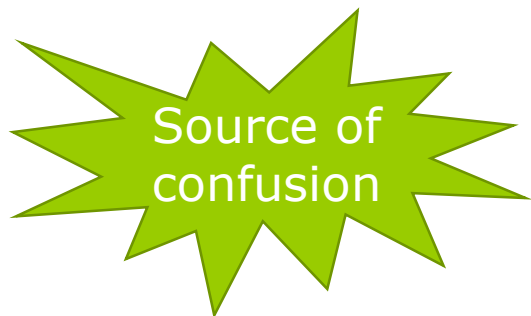
$$\frac{|a_{1,eff}^{D^*D_s}|}{|a_1^{D^*K}|} = 1.052 \pm 0.078 \quad \text{expects } 1.037 \pm ?$$

**Importance of**  
**penguin amplitude**  
**in the D channel.**  
**What about  $D^{**}$ ?**

# Hadrons produced in $B \rightarrow D x l \nu$ decays

## Rather well measured

- $B \rightarrow D^{(*)} l \nu$  and  $B \rightarrow$  narrow  $D^{**} l \nu$  decays.
- $B \rightarrow D^{(*)} \pi(\pi) l \nu$  decays without a clear identification of broad states.
- $B \rightarrow D_s^{(*)} K l \nu$  ( $BR = (6.1 \pm 1.0) \times 10^{-4}$ )



PDG values for « broad »  $D^{**}$  states:

$$BR(B \rightarrow D_0(2300) l \nu) = (0.39 \pm 0.07) \%$$

$$BR(B \rightarrow D_1(2430) l \nu) = (0.19 \pm 0.05) \%$$

In contradiction with theory and factorization by **about a factor 10**

## Not (well) measured

- broad  $D^{**}$ , radial excitations, non-resonant
- what about  $B \rightarrow D^{(*)} \eta(\prime) l \nu$ ,  $D^{(*)} \pi \pi l \nu$ ,  $D_s^{(*)} K \pi l \nu$ ,  $D^{(*)} K K l \nu$  decays ?



1- consider  $D^{(*)} \pi$  final states

2- factorization

3- fs/fd



# Comments on factorization (1)

## BBNS 2000, NLO + $\Lambda/m_b$

The main lesson from the previous discussion is that corrections to naive factorization in the class-I decays  $\bar{B}_d \rightarrow D^{(*)+} L^-$  are very small. The reason is that these effects are governed by a small Wilson coefficient and, moreover, are colour suppressed by a factor  $1/N_c^2$ . For these decays, the most important implications of the QCD factorization formula are to restore the renormalization-group invariance of the theoretical predictions, and to provide a theoretical justification for why naive factorization works so well. On the other hand, given the theoretical uncertainties arising, e.g., from unknown power-suppressed corrections, there is clearly no hope to confront the extremely small predictions for non-universal (process-dependent) “non-factorizable” corrections with experimental data. Rather, what we may do is ask whether data supports the prediction of a quasi-universal parameter  $|a_1| \simeq 1.05$  in these decays.

If this is indeed the case, it would support the usefulness of the heavy-quark limit in analyzing non-leptonic decay amplitudes. If, on the other hand, we were to find large non-universal effects, this would point towards the existence of sizeable power corrections to our predictions.

We will see that with present experimental errors the data are in good agreement with our prediction of a quasi universal  $a_1$  parameter. However, a reduction of the experimental uncertainties to the percent level would be very desirable for obtaining a more conclusive picture.

# Comments on factorization (2)

2016 : arXiv:1606.02888, NNLO +  $\Lambda/m_b$

a colour-suppressed tree topology. Therefore, a precise knowledge of the colour-allowed tree amplitude  $a_1$  allows to reliably estimate the size of power corrections to eq. (1) by comparison to experimental data, and at the same time provides a test of the QCDF framework. This requires that the perturbative expansion of the hard scattering kernel is

Given the fact that the results show rough agreement within errors for  $\bar{B}_d \rightarrow D^{(*)+}K^{(*)}$  decays, which receive only contributions from colour-allowed tree topologies, this may indicate a non-negligible impact from the  $W$ -exchange topologies appearing only in  $\bar{B}_d \rightarrow D^{(*)+}\pi^-$  and  $\bar{B}_d \rightarrow D^{(*)+}\rho^-$  decays. For  $\bar{B}_s$  decays, on the other hand, since