Probing extended Higgs sectors with the mass of the W boson

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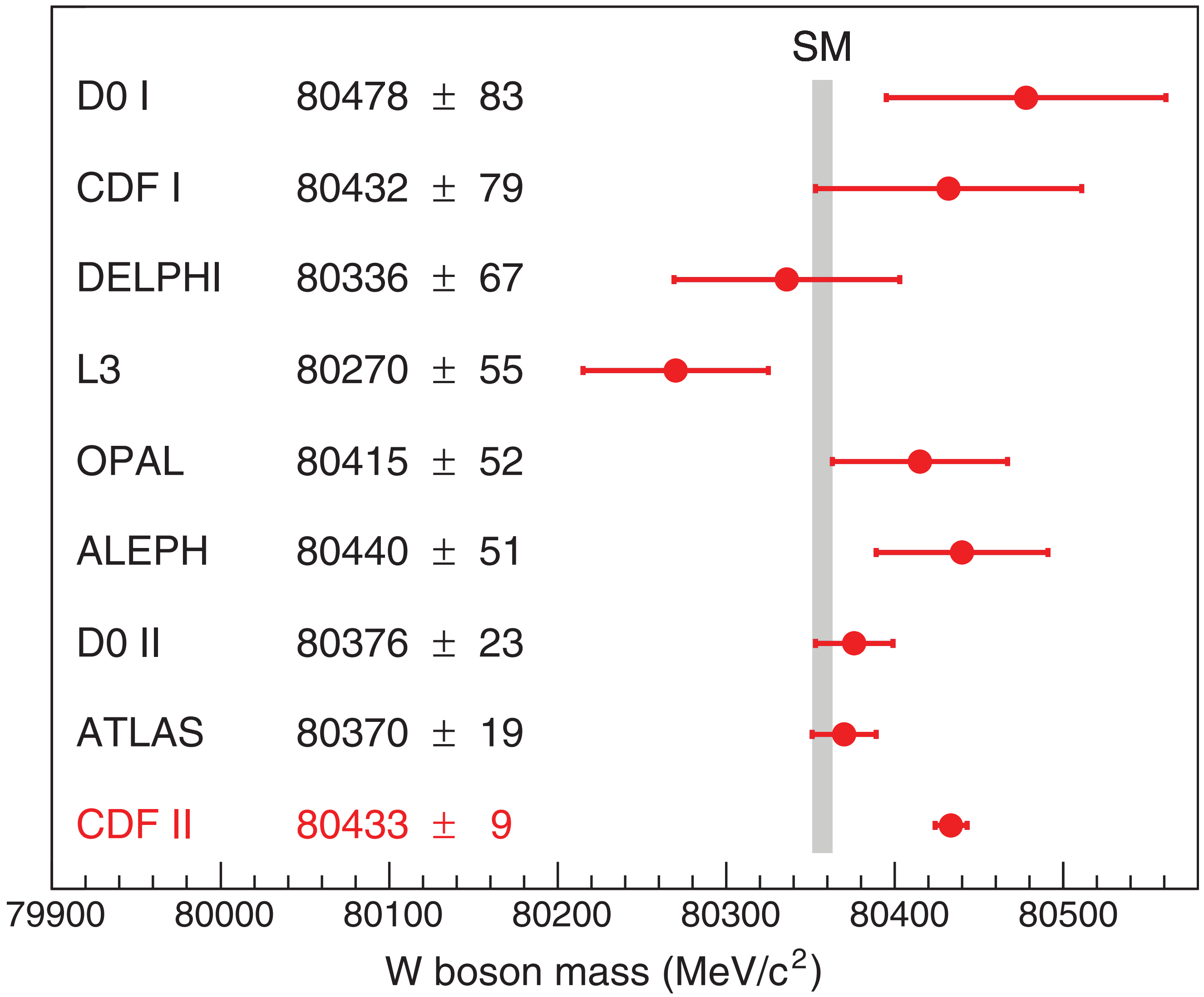
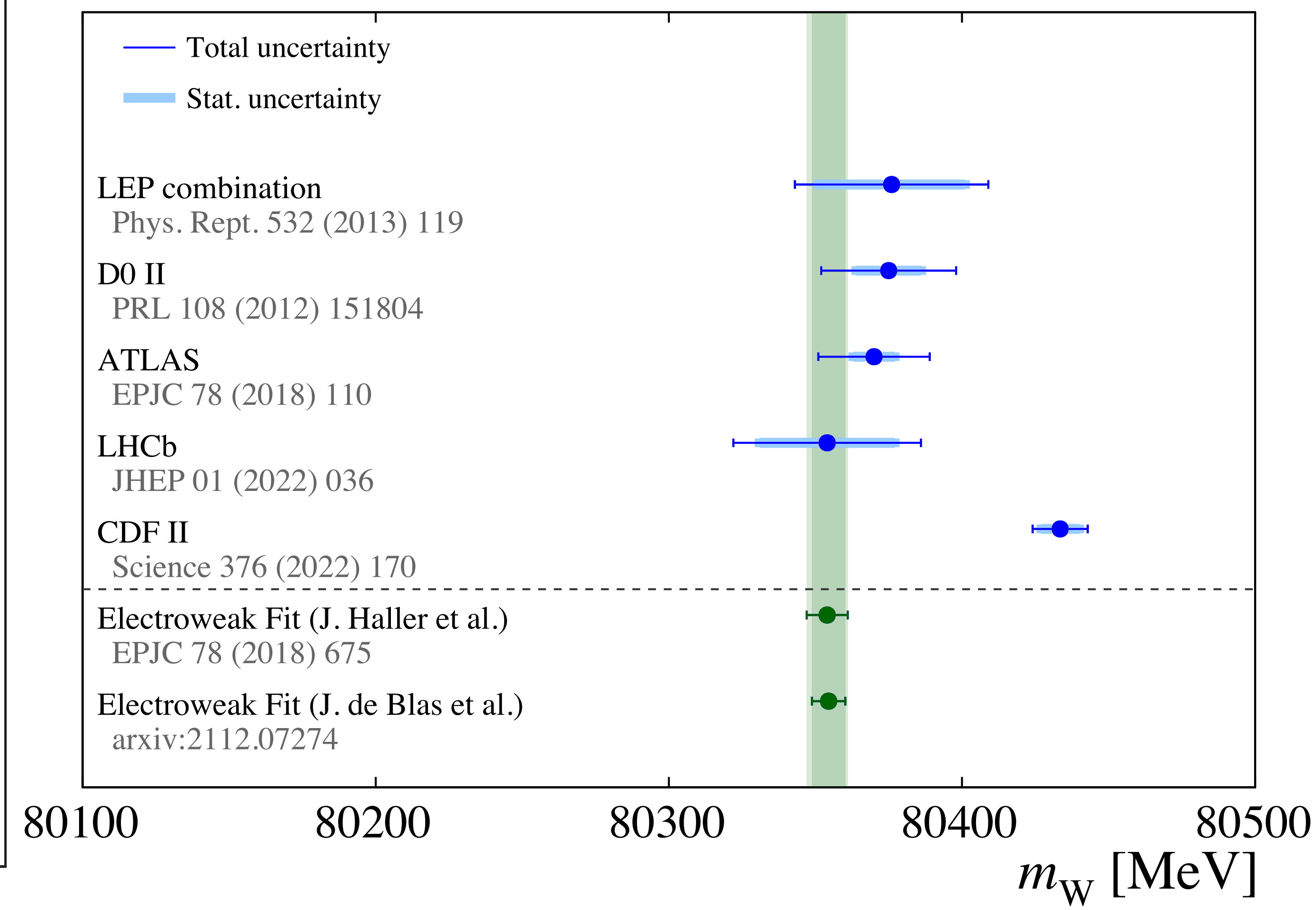
Introduction

Previous measurements of the W mass and recent CDF result:

[CDF Collaboration '22]





⇒ CDF result: large deviation from the SM; very small experimental error
Compatibility of the different M_W measurements? New world average?

W-mass measurement: past and present and future

- LEP: $e^+e^- \to W^+W^-$ in the continuum and at threshold (small amount of data); impact of fully hadronic final state suffered from uncertainties due to BE correlations, colour reconnections

- Tevatron, LHC: transverse mass / momentum distributions

PDG average (does not include new CDF result):
M_W^{exp} (PDG) = 80.377 +- 0.012 GeV (accuracy of 1.5×10^{-4})

New CDF measurement: *[CDF Collaboration '22]*
M_W^{exp} (CDF) 80.434 +- 0.009 GeV

Prospects for further experimental improvements of M_W from LHC (CMS, updates from ATLAS, LHCb), future e^+e^- collider
Tevatron: further D0 data?

Which parameter is actually measured?

- On the theory side M_W is a Lagrangian parameter. Its physical meaning, e.g. pole mass according to the real part of the complex pole, is determined by renormalisation order by order in perturbation theory

- On the experimental side masses of unstable particles are not directly physical observables (can only measure cross sections, branching ratios, kinematical distributions, …): masses are ``pseudo-observables" whose determination involves a deconvolution procedure (unfolding)
  Different parameterisations of the resonance: Breit-Wigner shape with running or constant width
  The experimental mass parameter is obtained from comparison data — Monte Carlo

⇒ The experimental mass parameters M_W, M_Z, m_t, … are not strictly model-independent

Theoretical uncertainties affecting the measurement of the mass of the W boson

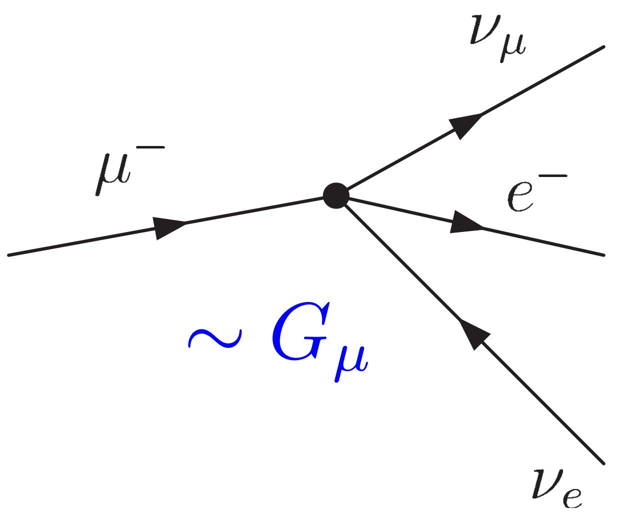
See talks by Alessandro, Tobias, Raoul and Mauro yesterday

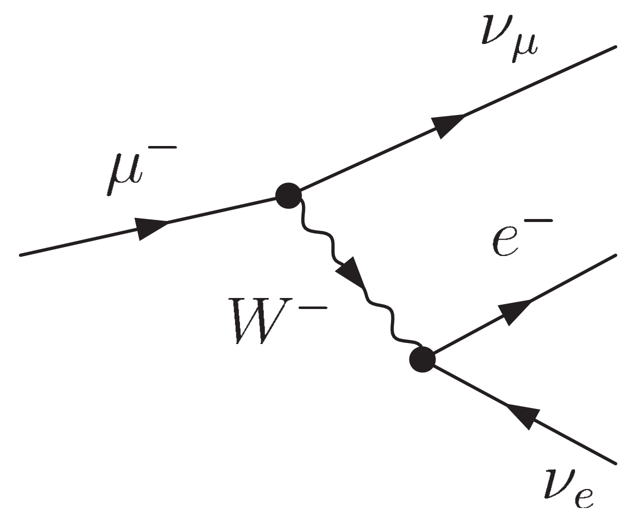
Not only the prediction for M_W in a particular model (SM and beyond, see below) but also its extraction from the experimental data is affected by theoretical uncertainties

Those theoretical uncertainties need to be taken into account as systematic uncertainties in the measurement of M_W together with pdf uncertainties, etc.

Theoretical prediction for the W-boson mass from muon decay: relation between M_W, M_Z, α, G_μ

Fermi model





SM

M_W: Comparison of prediction for muon decay with experiment (Fermi constant G_μ); QED corrections in Fermi model incl. in def. of G_μ

$$\Rightarrow \quad M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1 + \Delta r\right),$$

$\Updownarrow$ loop corrections

$$\Rightarrow M_W^2 = M_Z^2\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\alpha\pi}{\sqrt{2}G_\mu M_Z^2}\left(1 + \Delta r\right)}\right)$$

$\Rightarrow$ Theo. prediction for M_W in terms of M_Z, α, G_μ, $\Delta r(m_\mathrm{t}, m_{\tilde{\mathrm{t}}}, \ldots)$

Tree-level prediction: M_W^{tree} = 80.939 GeV, M_W^{exp} = 80.377 +- 0.012 GeV

⇒ Very high sensitivity to quantum effects in the SM and beyond

W-mass prediction in the SM

One-loop contribution:

$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2}\Delta\rho + \Delta r_{\mathrm{rem}}(M_{\mathrm{H}}, \ldots)$$

≈ 6% ≈ –3% < 1%

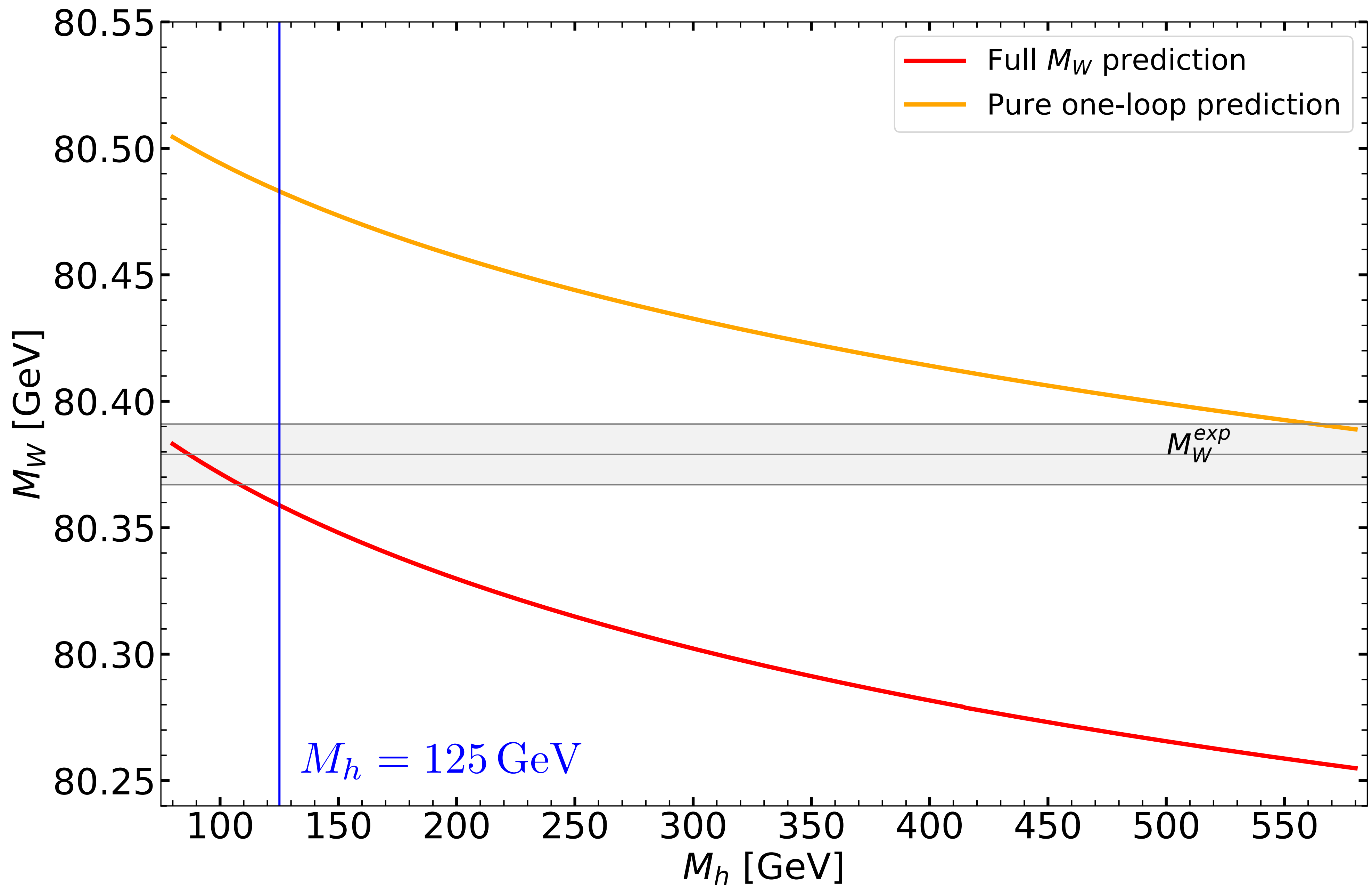
$$\Delta\rho = \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{WW}(0)}{M_W^2}$$

contribution from isospin splitting: $\sim (m_t^2 - m_b^2) \approx m_t^2$

custodial symmetry: $\rho = 1$ at lowest order

W-mass prediction within the SM: one-loop result vs. state-of-the-art prediction

[M. Berger, S. Heinemeyer, G. Moortgat-Pick, G. W. '22]



⇒ Pure one-loop result would imply preference for heavy Higgs, $M_h > 500$ GeV
Corrections beyond one-loop order are crucial for reliable prediction of M_W

M_W prediction in the Standard Model

Contributions beyond one-loop order:

$$\Delta r^{\mathrm{SM(h.o.)}} = \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(\alpha^2)}_{\mathrm{ferm}} + \Delta r^{(\alpha^2)}_{\mathrm{bos}} + \Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)} + \Delta r^{(G_\mu m_t^2 \alpha_s^3)}$$

Chetyrkin, Kuhn, Steinhauser, Djouadi, Verzegnassi, Awramik, Czakon, Freitas, Weiglein, Faisst, Seidensticker, Veretin, Boughezal, Kniehl, Sirlin, Halzen, Strong, ...

Impact of different contributions to Δr (x 10^4) for fixed M_W = 80.385 GeV and M_H^{SM} = 125.09 GeV:

[O. Stål, G. W., L. Zeune '15]

$\Delta r^{(\alpha)}$	$\Delta r^{(\alpha\alpha_s)}$	$\Delta r^{(\alpha\alpha_s^2)}$	$\Delta r^{(\alpha^2)}_{\mathrm{ferm}} + \Delta r^{(\alpha^2)}_{\mathrm{bos}}$	$\Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)}$	$\Delta r^{(G_\mu m_t^2 \alpha_s^3)}$
297.17	36.28	7.03	29.14	-1.60	1.23

Sources of theoretical uncertainties

- From experimental errors of the input parameters

$\delta m_t = 0.7$ GeV, $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$, $\delta M_Z = 2.1$ MeV

$\delta M_W^{\text{para},m_t} = 4$ MeV, $\delta M_W^{\text{para},\Delta\alpha_{\text{had}}} = 2$ MeV, $\delta M_W^{\text{para},M_Z} = 2.5$ MeV

- From unknown higher-order corrections ("intrinsic")

SM: Complete 2-loop result $+$ leading higher-order corrections known for M_{W} and $\sin^2\theta_{\text{eff}}$

$\Rightarrow$ Remaining uncertainties:

[*M. Awramik, M. Czakon, A. Freitas, G.W. '03, '04*]

[*M. Awramik, M. Czakon, A. Freitas '06*]

$$\Delta M_{\text{W}}^{\text{intr}} \approx 4 \text{ MeV}, \quad \Delta \sin^2\theta_{\text{eff}}^{\text{intr}} \approx 5 \times 10^{-5}$$

BSM prediction for M_W, example: MSSM, NMSSM

Δr in the MSSM and the NMSSM, treatment of higher-order contributions:
full one-loop + higher orders (SM) + higher orders (SUSY)

$$\Delta r^{\rm (N)MSSM} = \Delta r^{{\rm (N)MSSM}(\alpha)} + \Delta r^{\rm (N)MSSM(h.o.)}$$

$$\Delta r^{\rm (N)MSSM(h.o.)} = \Delta r^{\rm SM(h.o.)} + \Delta r^{\rm SUSY(h.o.)}$$

⇒ State-of-the art SM prediction recovered in decoupling limit, all available higher-order corrections of SUSY-type included

For light SUSY particles: additional theoretical uncertainty from higher-order SUSY-loop corrections

SUSY higher-order contributions

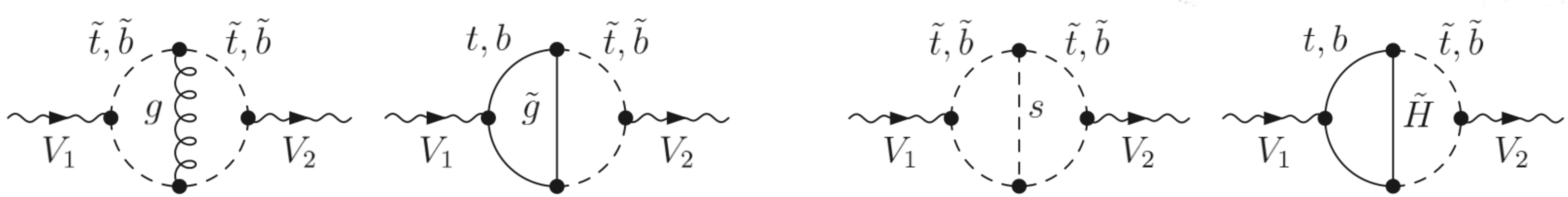
One-loop:

leading contributions from the scalar superpartners of the top and bottom quarks via Δϱ: additional source of isospin splitting

Two-loop:

leading reducible 2-loop corrections, gluon/gluino 2-loop corrections, higgsino 2-loop corrections

$$\Delta r^{\mathrm{SUSY(h.o.)}} = \Delta r_{\mathrm{red}}^{\mathrm{SUSY}(\alpha^2)} - \frac{c_W^2}{s_W^2}\Delta\rho^{\mathrm{SUSY},(\alpha\alpha_s)} - \frac{c_W^2}{s_W^2}\Delta\rho^{\mathrm{SUSY},(\alpha_t^2,\alpha_t\alpha_b,\alpha_b^2)}$$



Interpretation of the new CDF measurement?

- At present it seems neither justified to disregard the new CDF measurement nor any of the previous measurements
- New world average: how to deal with the tensions between the different measurements? Central value, systematic uncertainties?
- Ongoing effort:

Tevatron-LHC W-boson mass Combination Working Group
https://twiki.cern.ch/twiki/bin/view/LHCPhysics/MWCOMB

See Maarten's talk yesterday

New world average?

[M. Boonekamp '23]

Conclusions

- The W boson mass is arguably the most difficult measurement in HEP
  - Partial event reconstruction, incomplete kinematics
  - Calibrations
  - Physics modelling
  - Precision goal
- First measurement ~2017, with 2011 data. Being updated
- Next measurement will use low-pile-up data collected in 2017,2018.
- Combination
  - At present, it is difficult to quote a conclusive "world average". The most precise measurement is also discrepant.
  - Still important work : comparing LEP, Tevatron, LHC measurement results forces to look deep into the modelling aspects, to "translate" the measurements into eachother, allowing quantitative comparisons and better studies of model dependence

Interpretation of the new CDF measurement?

Note: already the present world-average lies above the SM prediction and therefore gives rise to a certain preference for a non-zero BSM contribution. The inclusion of the new CDF measurement is expected to further move up the central value.

From the theory side one may ask the question whether a prediction for a larger M_W value (somewhere between the current world average and the new CDF measurement) in BSM models would be compatible with other experimental and theoretical constraints

Some examples are shown in the following

BSM predictions for the W-boson mass

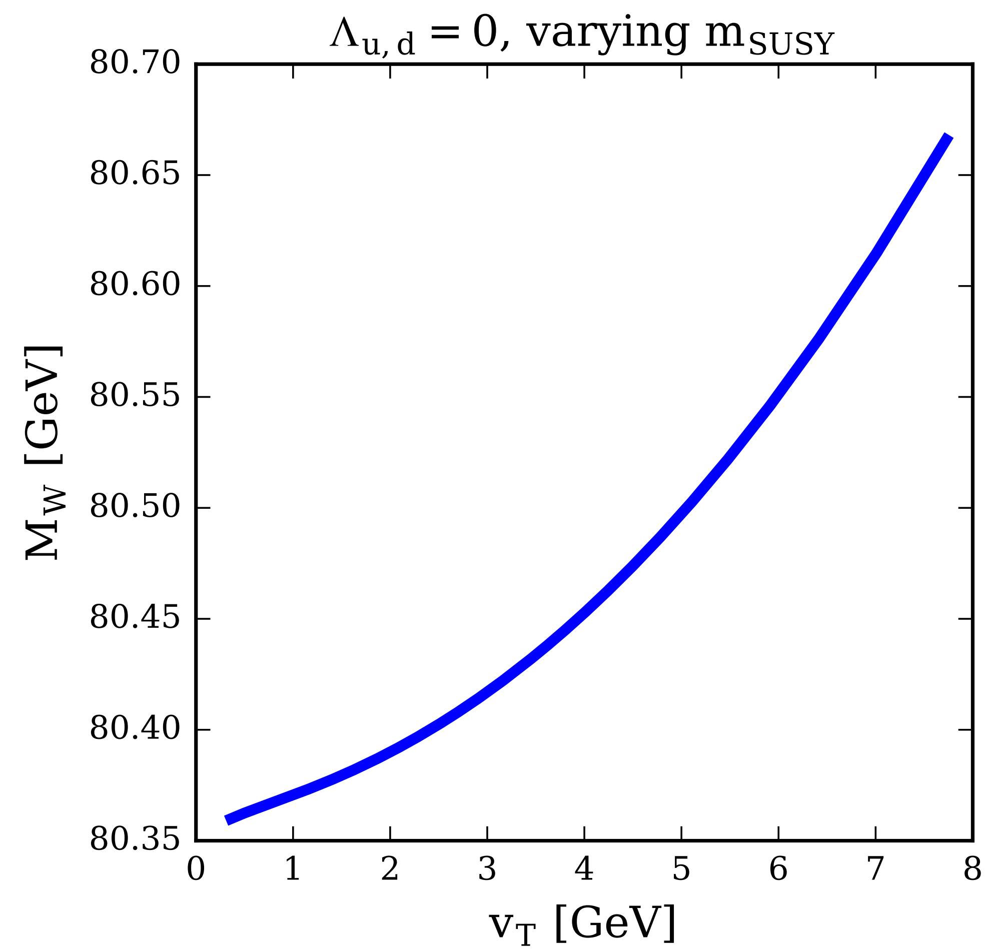
Extended Higgs sectors consisting of doublets and singlets:
custodial symmetry ⇒ $\varrho = 1$ at lowest order

Lowest-order charged Higgs exchange contribution: ~ $(m_\mu m_e)/M_W^2$

⇒BSM contributions enter at 1-loop level: $\Delta r(m_i^{SM}, m_j^{BSM}, \ldots)$

Extended Higgs sectors involving triplets:
tree-level contribution from triplet v.e.v. v_T:
$M_W^2 = 1/4\, g_2^2\, v^2 + g_2^2\, v_T^2$

Example: MRSSM

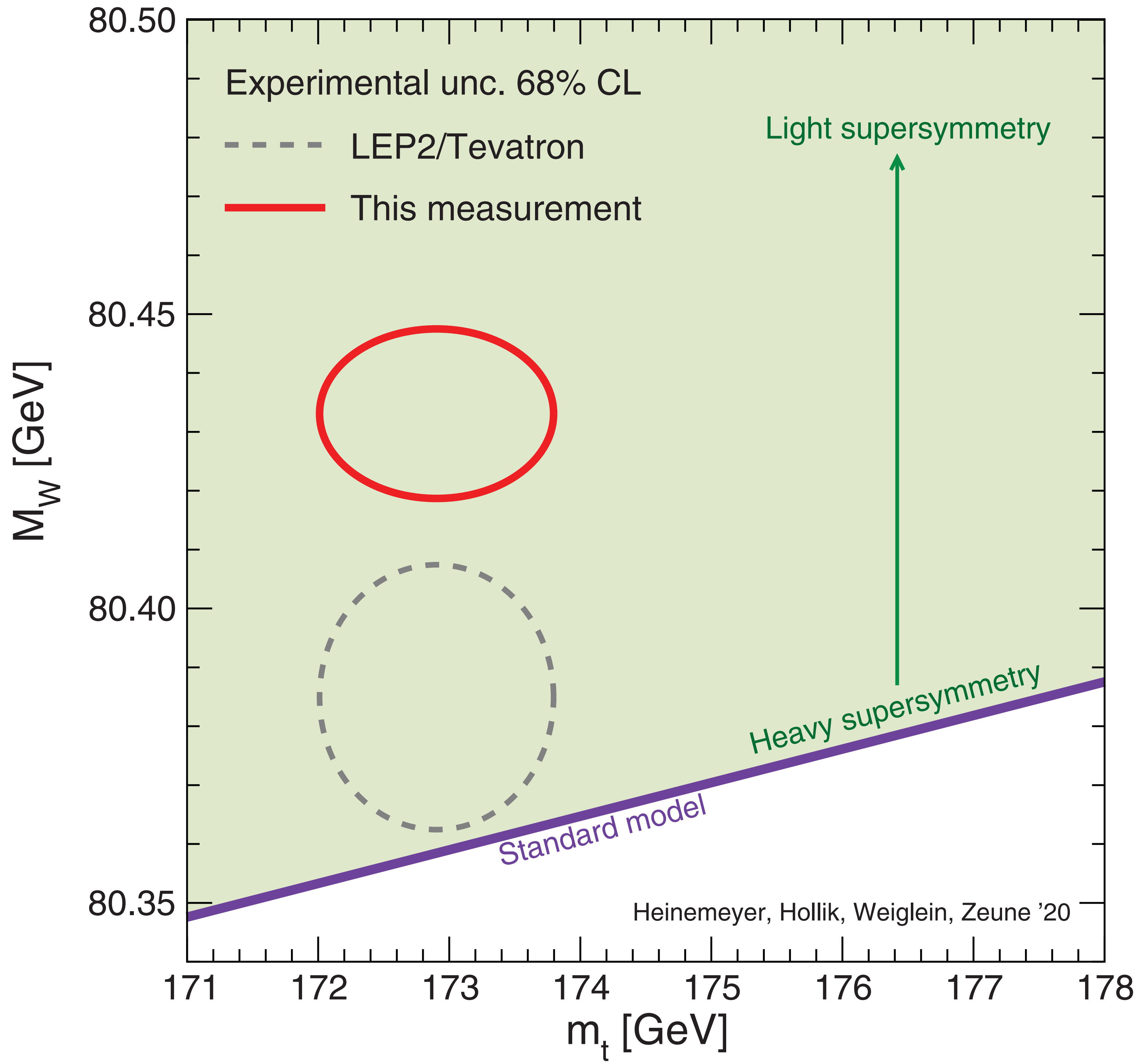


[P. Diessner, G. W. '19]

New CDF value for M_W: preference for BSM contribution

[CDF Collaboration '22] *[S. Heinemeyer, W. Hollik, G. W., L. Zeune '20]*

Example: MSSM

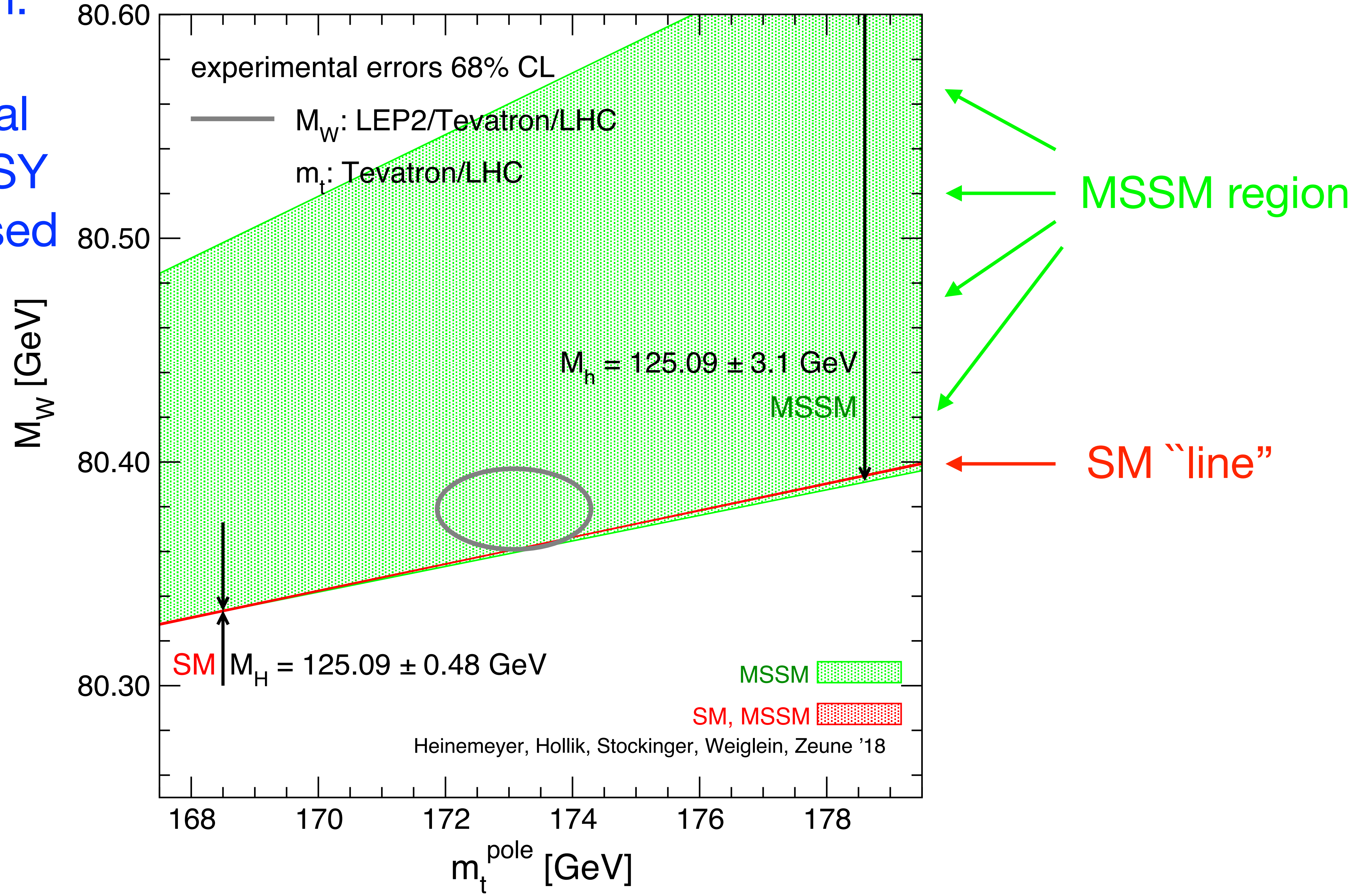


Prediction for M_W in the SM and the MSSM vs. experimental results for M_W and m_t

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]

Parameter scan:

No experimental bounds on SUSY particles imposed

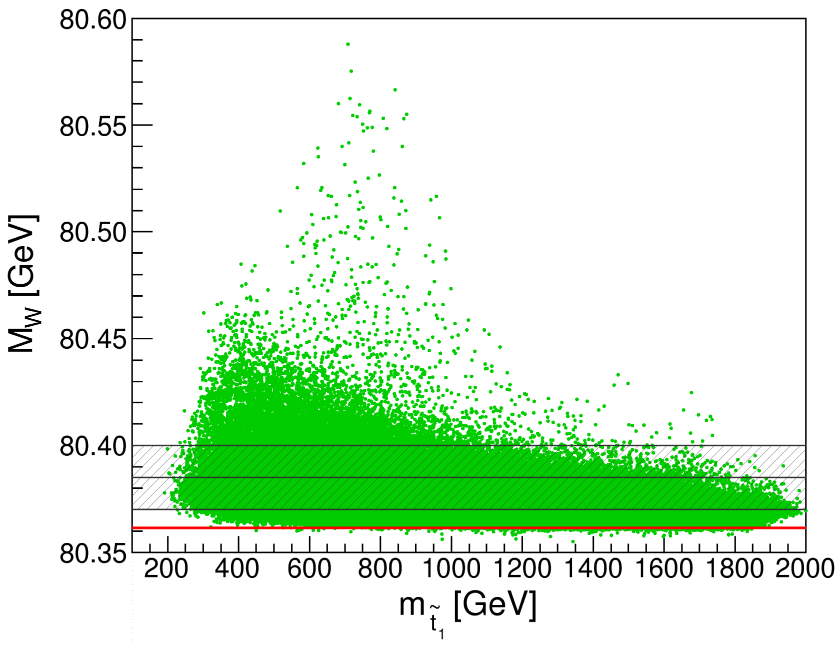


⇒ Large upward shift in M_W possible, large sensitivity to BSM effects

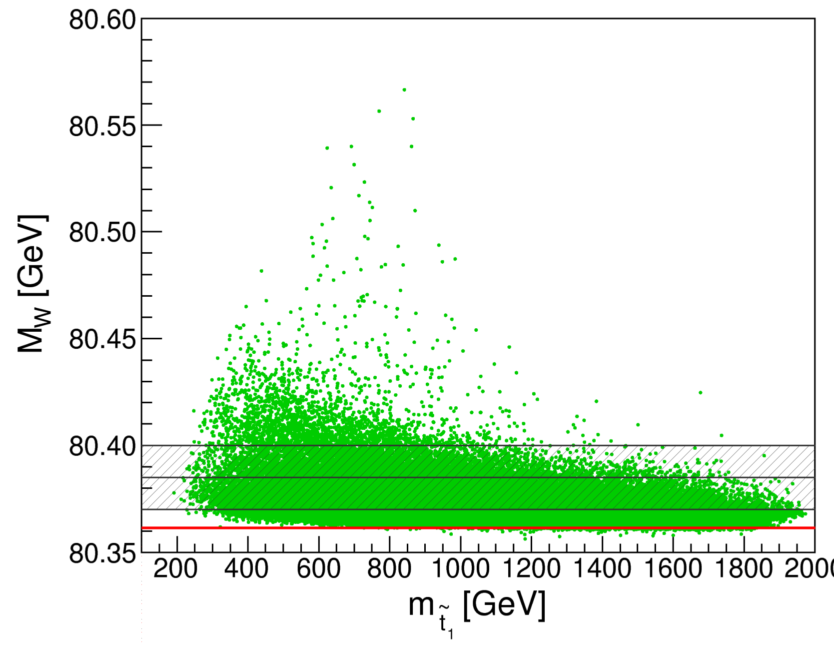
Prediction for M_W in the MSSM depending on the lighter stop mass (parameter scan)

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '13]

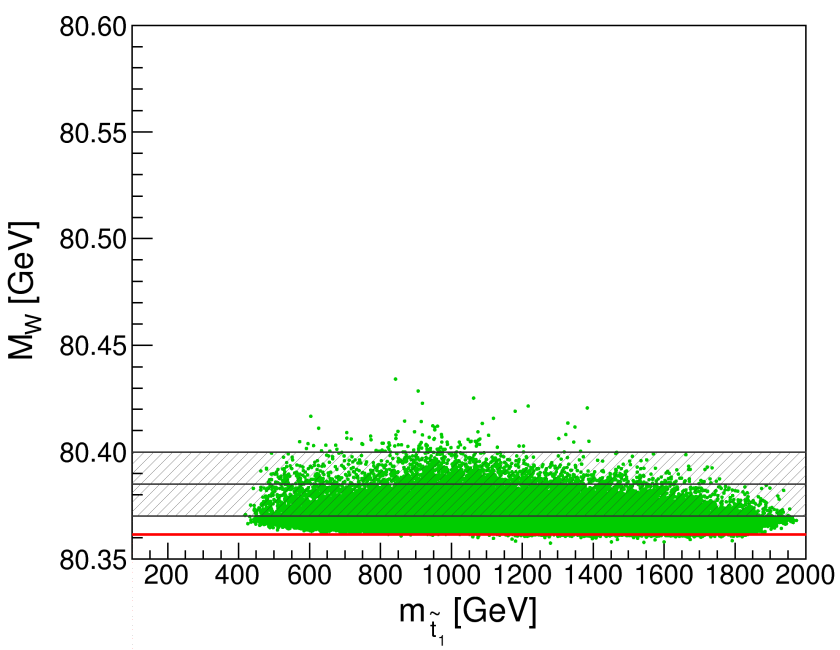
All points



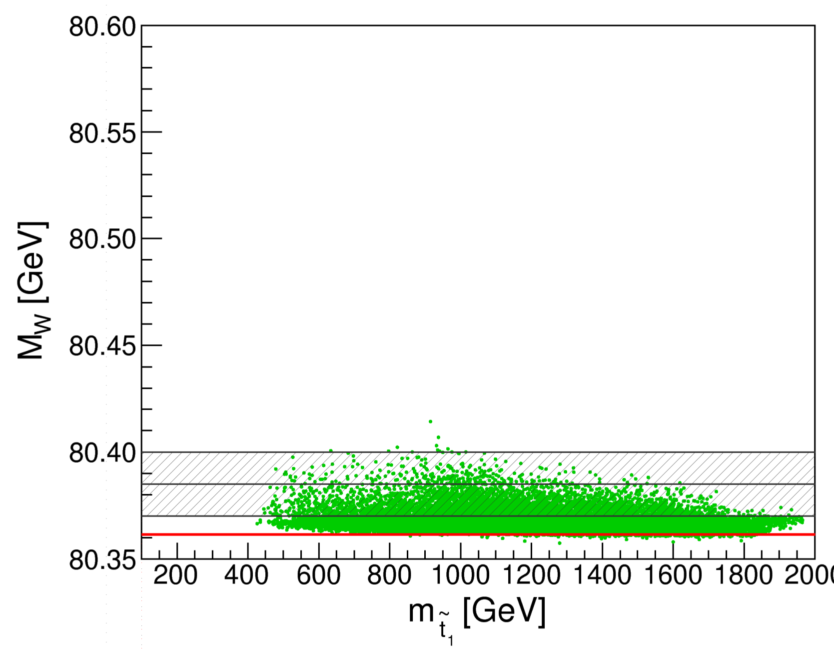
Heavy gluino, heavy first and second generation squarks



+ heavy sbottoms



+ heavy sleptons and charginos



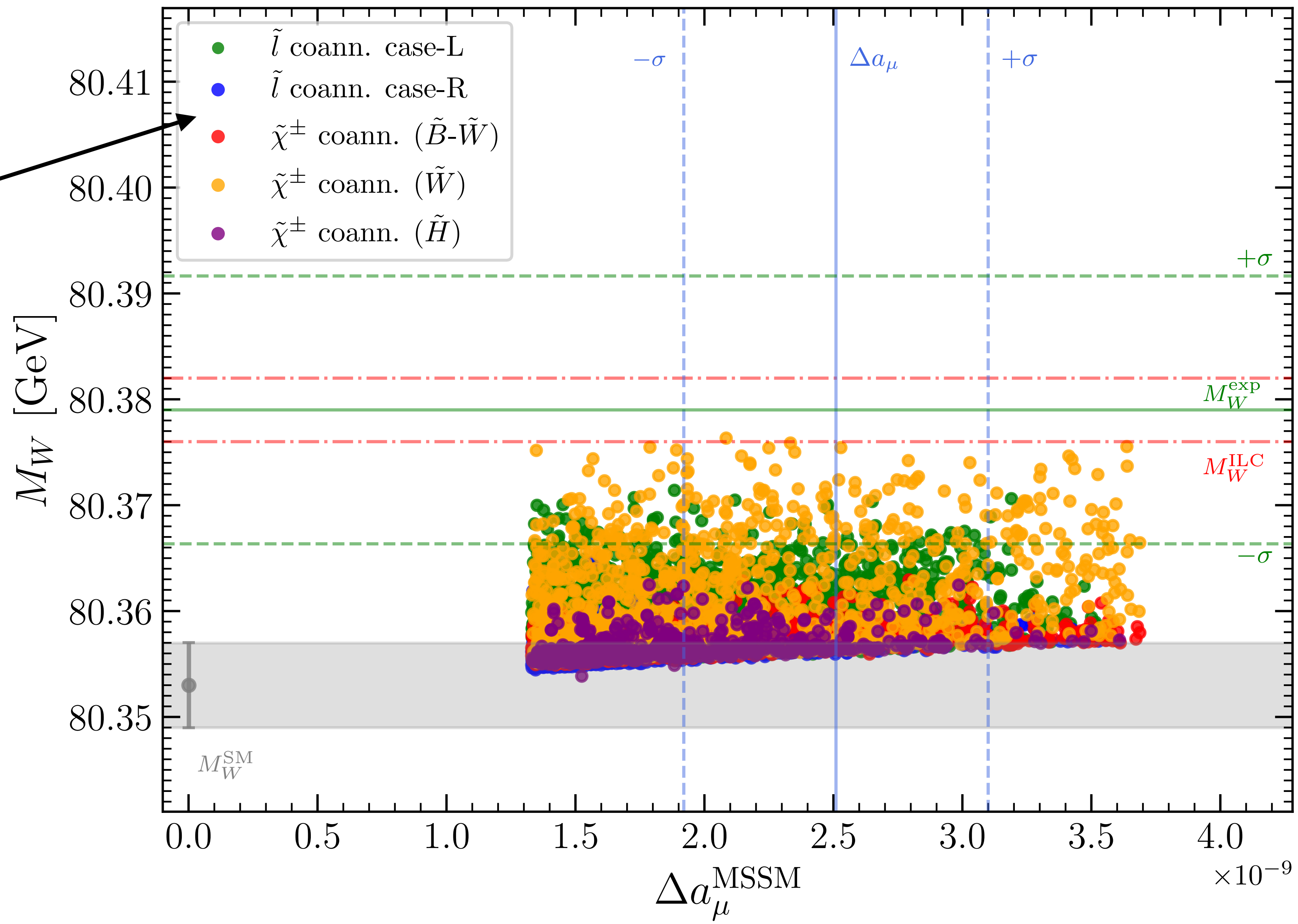
⇒Sizeable enhancements possible even for relatively heavy SUSY

Muon g–2 and M_W: a hint for light BSM particles?

[E. Bagnaschi, M. Chakraborti, S. Heinemeyer, I. Saha, G. W. '22]

Impact of light electroweak SUSY particles on $g_\mu - 2$, M_W (PDG average) and dark matter relic density (squarks assumed very heavy!)

Different mechanisms for obtaining the right amount of dark matter



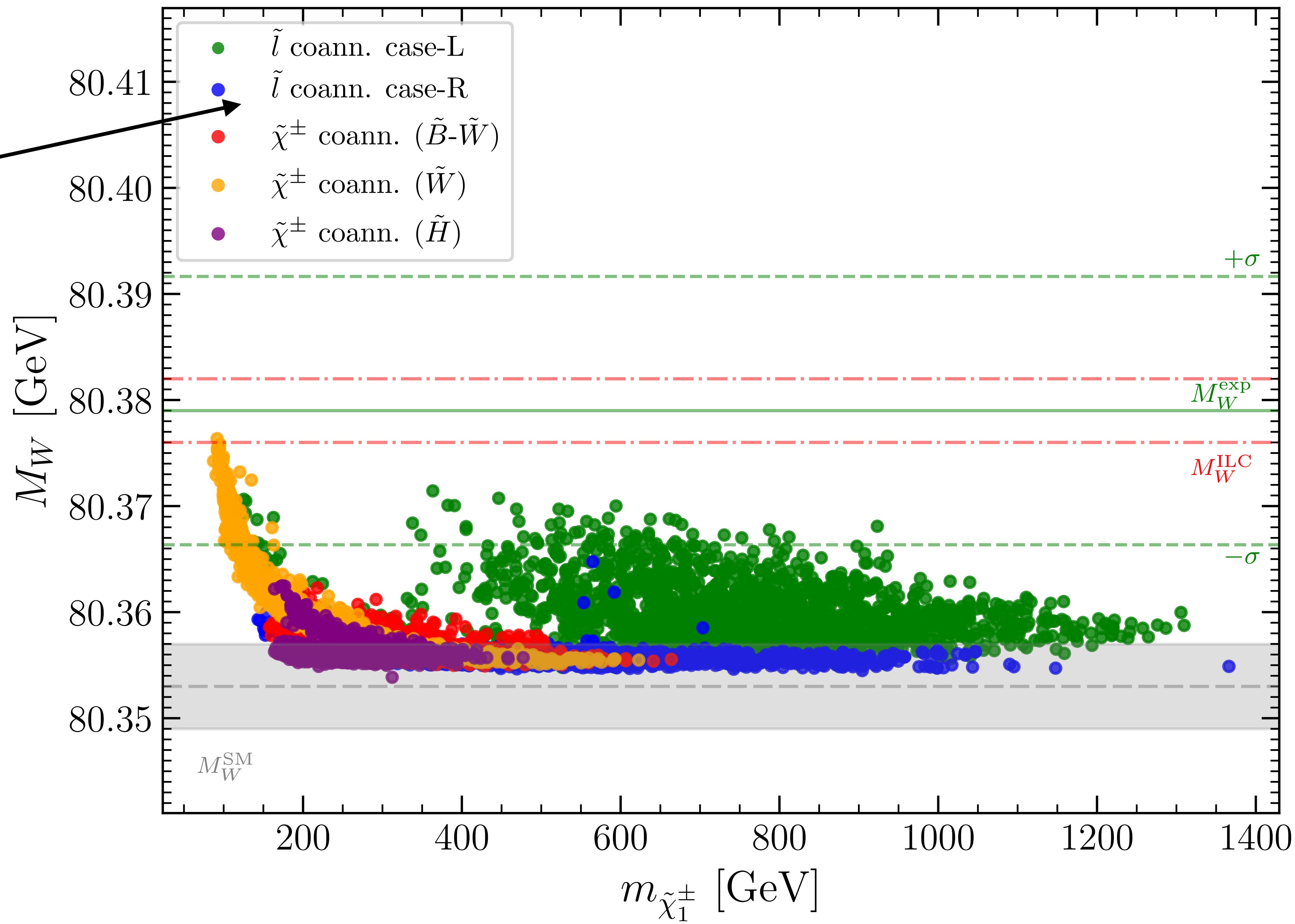
⇒ Improved precision on M_W can probe different dark matter mechanisms

Muon g–2 and M_W: a hint for light BSM particles?

[E. Bagnaschi, M. Chakraborti, S. Heinemeyer, I. Saha, G. W. '22]

Correlation with the mass of the lightest chargino:

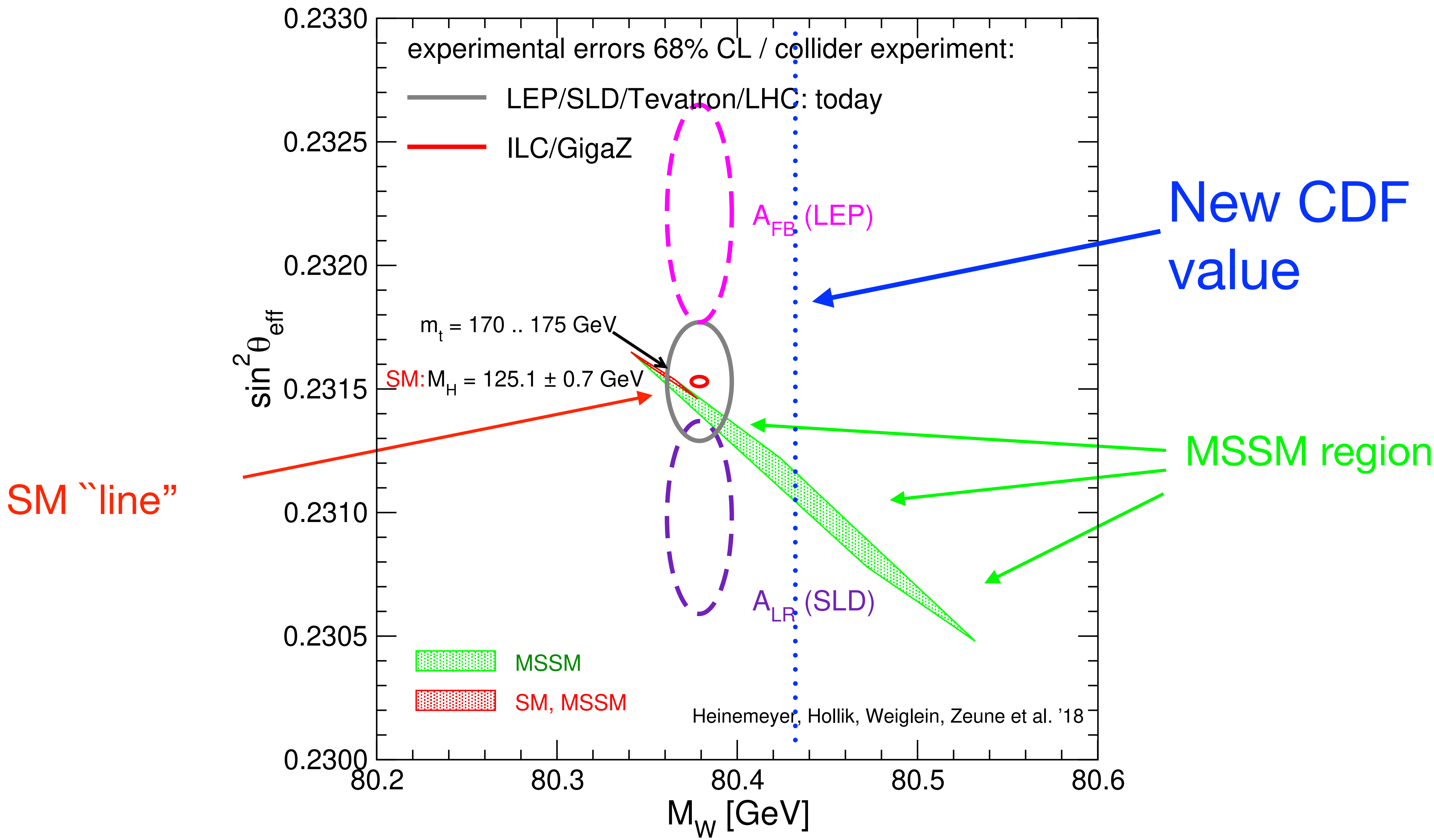
Different mechanisms for obtaining the right amount of dark matter



⇒ Possible hints for light charginos can be probed with future searches
Larger values for M_W possible if stops, sbottoms are close to the exp. bounds

Prediction for M_W and $\sin^2\theta_{eff}$ in the SM and MSSM vs. experimental accuracies (before new CDF result)

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]



⇒ M_W and $\sin^2\theta_{eff}$ have high sensitivity for model discrimination

Results for the 2HDM (alignment limit)

Leading BSM one-loop contribution: $\Delta M_W \simeq \frac{1}{2} M_W \frac{c_W^2}{c_W^2 - s_W^2} \Delta\rho$

$$\Delta\rho^{(1)}_{\text{non-SM}} = \frac{\alpha}{16\pi^2 s_W^2 M_W^2} \left\{ \frac{m_A^2 m_H^2}{m_A^2 - m_H^2} \ln \frac{m_A^2}{m_H^2} - \frac{m_A^2 m_{H^\pm}^2}{m_A^2 - m_{H^\pm}^2} \ln \frac{m_A^2}{m_{H^\pm}^2} - \frac{m_H^2 m_{H^\pm}^2}{m_H^2 - m_{H^\pm}^2} \ln \frac{m_H^2}{m_{H^\pm}^2} + m_{H^\pm}^2 \right\}$$

⇒ Large contribution possible for sizeable splitting between the BSM Higgs bosons

⇒ Prediction for the electroweak precision observables in the 2HDM (alignment limit) at 2-loop order *[H. Bahl, J. Braathen, G. W. '22]*

THDM_EWPOS *[S. Hessenberger, W. Hollik '16, '22]*

Plots on next slides:

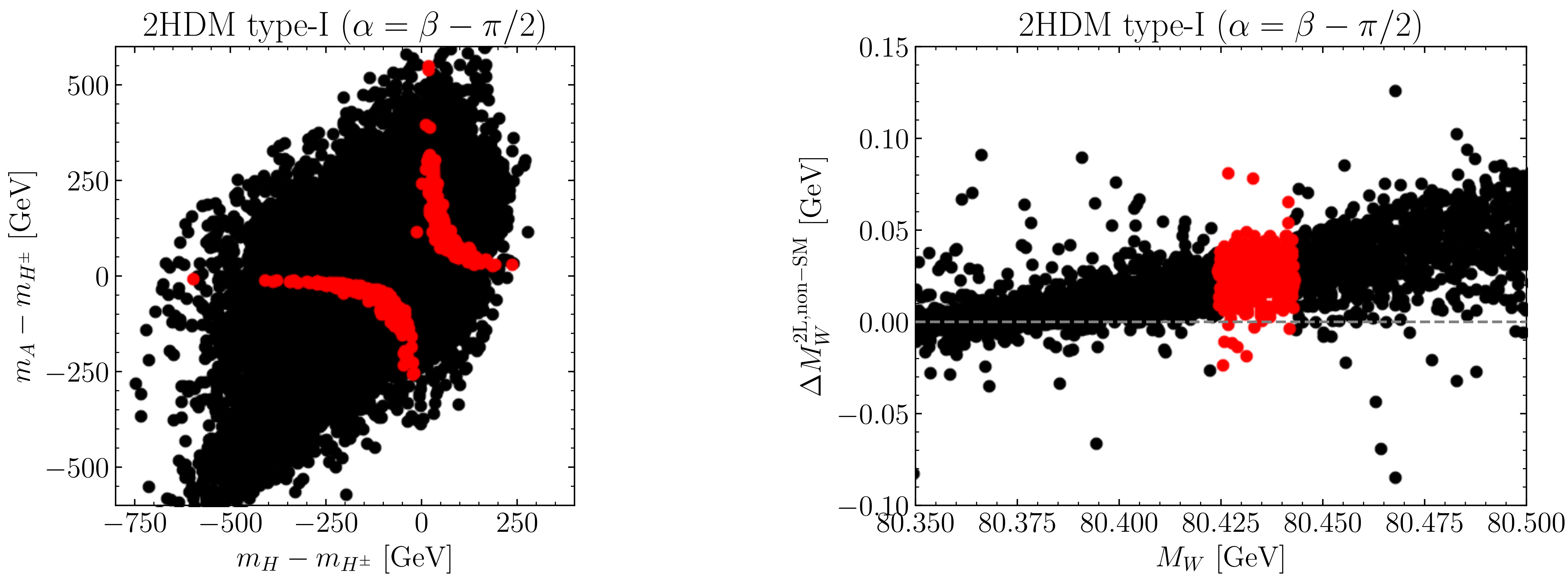
All displayed points are in agreement with other relevant experimental and theoretical constraints

Red: points in the 1-sigma range of the CDF measurement

Large corrections to M_W in the 2HDM

[H. Bahl, J. Braathen, G. W. '22]

Prediction for the electroweak precision observables at 2-loop order, 2HDM in the alignment limit; example: type I



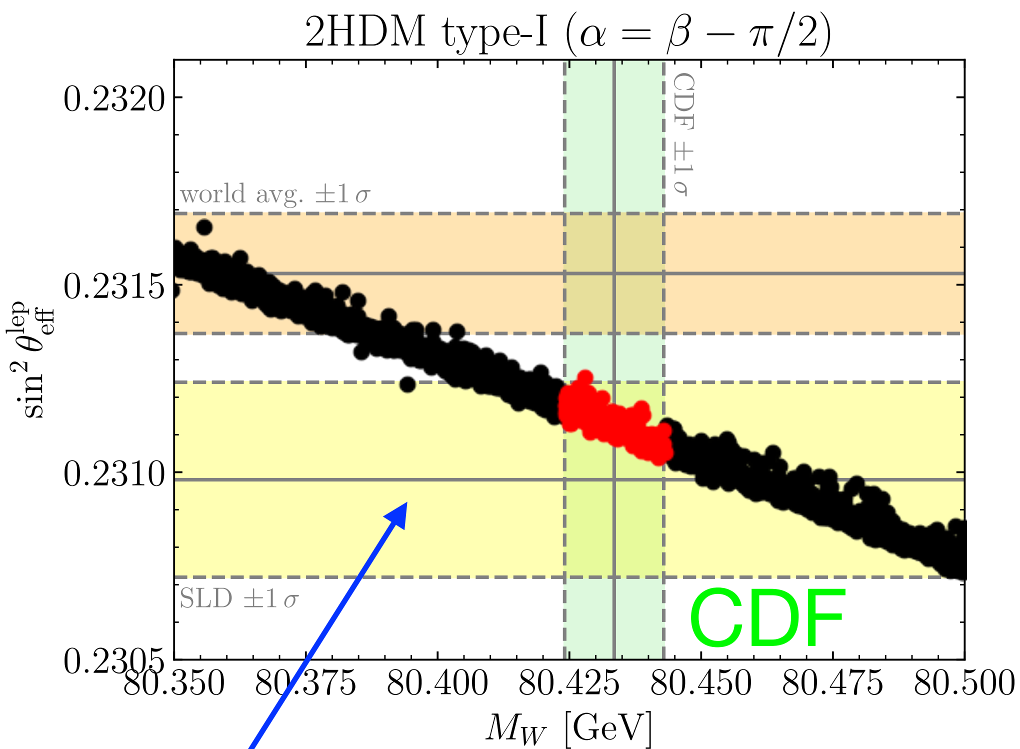
⇒ Large effects on M_W arise from mass splitting between heavy Higgses

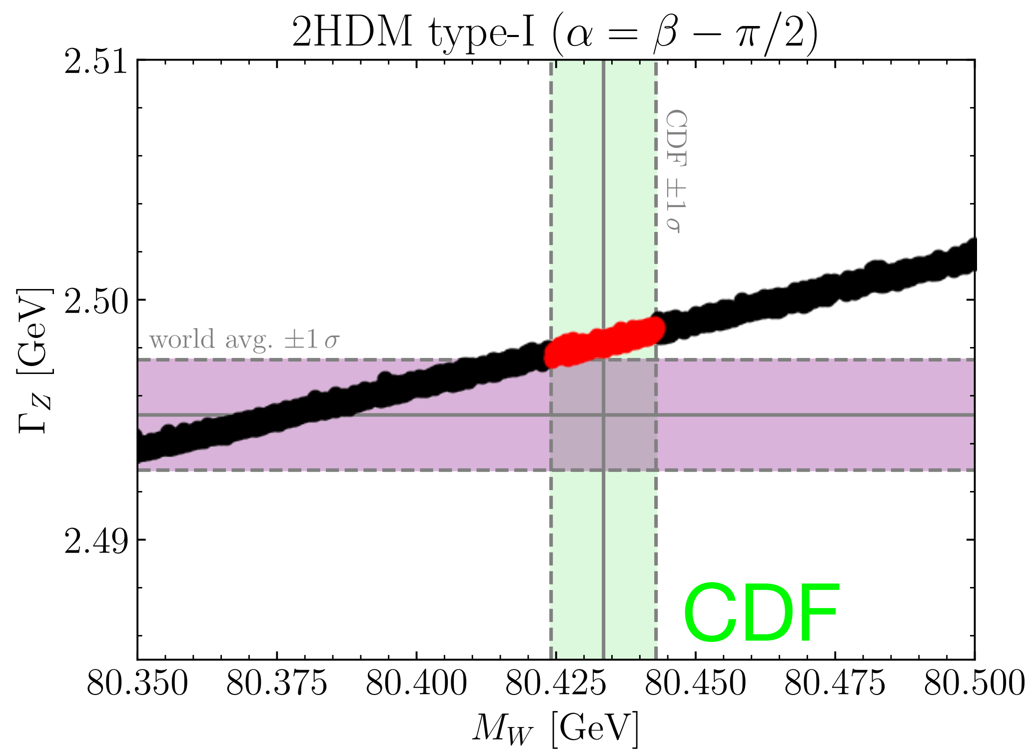
2-loop effects can be very important!

No significant impact on results for trilinear Higgs coupling

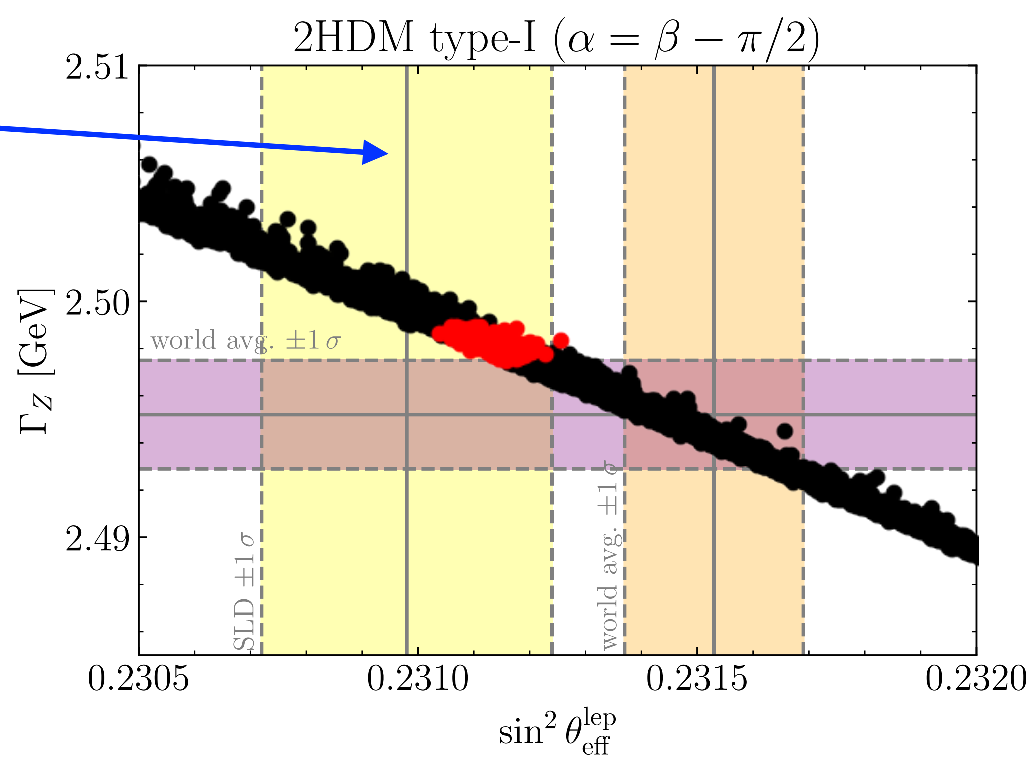
Large corrections to M_W in the 2HDM

[H. Bahl, J. Braathen, G. W. '22]





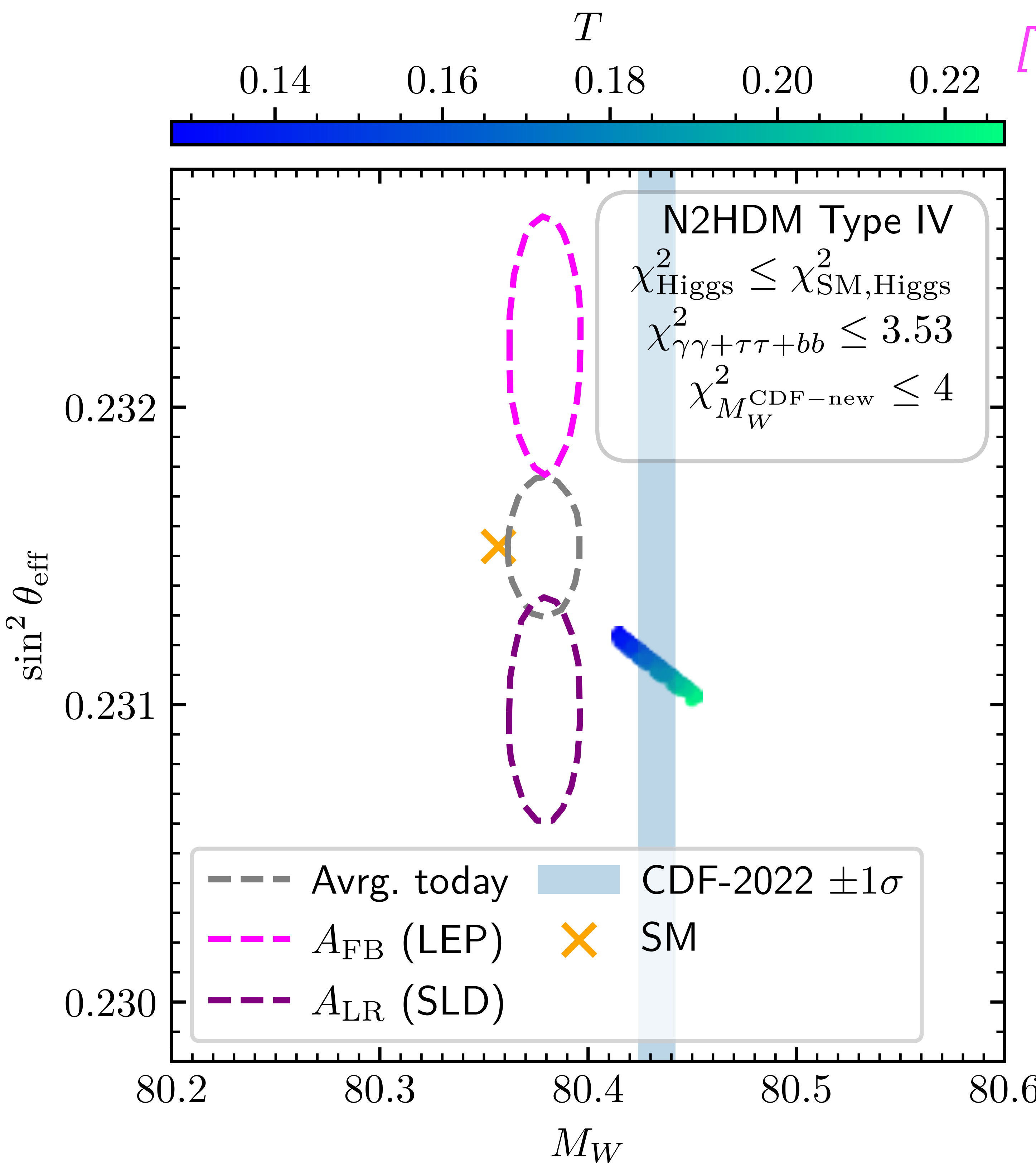
SLD value



$\Rightarrow$ M_W values as large as the CDF one can be accommodated in the 2HDM without violating other constraints
Better agreement with SLD value for $\sin^2\theta_{\rm eff}$

N2HDM: a 95 GeV Higgs and the CDF value of M_W

[T. Biekötter, S. Heinemeyer, G. W. '22]



⇒ The N2HDM of type IV can simultaneously accommodate the three excesses in the Higgs searches near 95 GeV and an M_W value that agrees with the new CDF measurement!

Conclusions

Experimental results for M_W: preference for non-zero BSM contribution

Extended Higgs sectors: prediction for the mass of the W boson in agreement with the recent CDF measurement is possible in the 2HDM, N2HDM, … without being in conflict with the relevant experimental and theoretical constraints

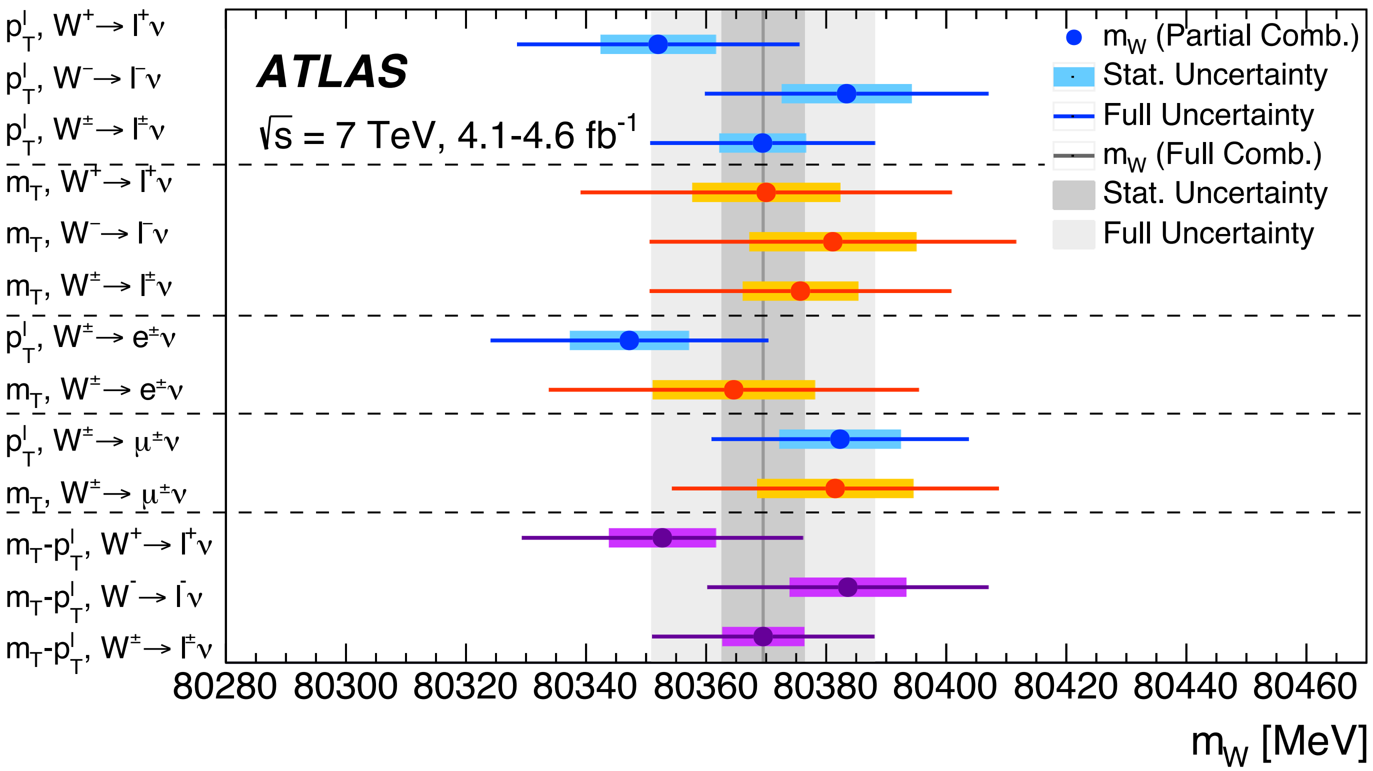
N2HDM: parameter region yielding large upward shift on M_W can also accommodate a light Higgs at 95 GeV that is compatible with the observed excesses

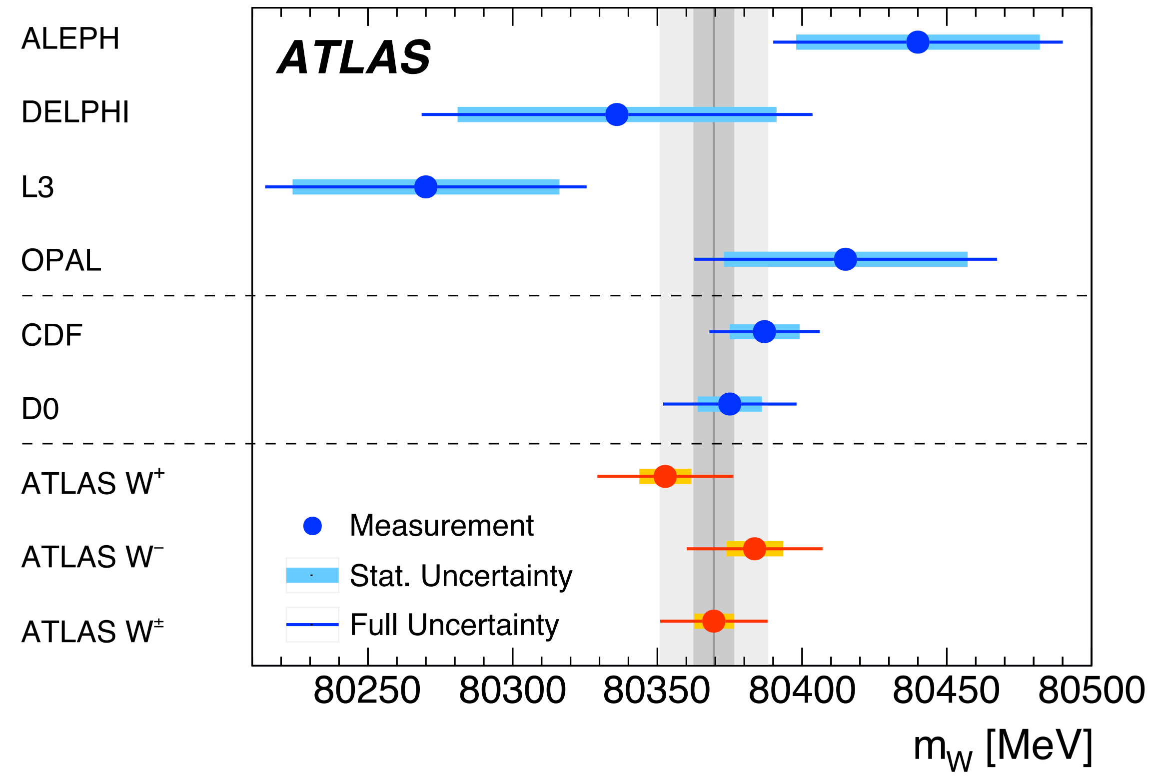
$\Rightarrow$ Further data will be crucial for probing the possible hints for new physics

Backup

W-mass measurement at the LHC

[ATLAS Collaboration '17]





$$
\begin{aligned}
m_W &= 80369.5 \pm 6.8\ \text{MeV(stat.)} \pm 10.6\ \text{MeV(exp. syst.)} \pm 13.6\ \text{MeV(mod. syst.)} \\
&= 80369.5 \pm 18.5\ \text{MeV},
\end{aligned}
$$

ATLAS error dominated by modelling systematics

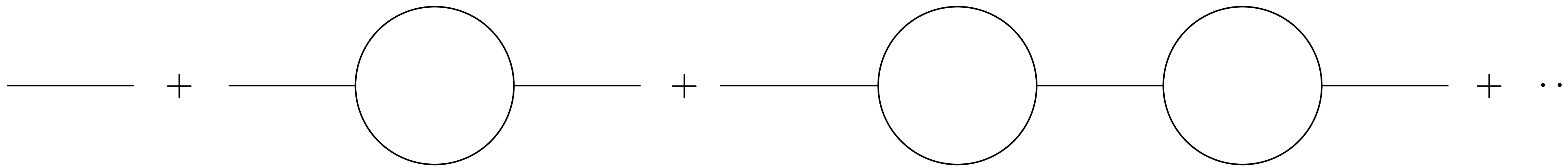
Accuracy of 2.3 x 10^{-4}, i.e. sub-per-mille level!

Very many subtle effects contribute at this level

Control of theory / systematic uncertainties is crucial!

What is meant by the mass of an unstable particle?

Mass of a physical particle: pole of the propagator



$\frac{i}{p^2-m^2}$ $\quad$ $\frac{i}{p^2-m^2}\; i\Sigma(p^2)\; \frac{i}{p^2-m^2}$ $\quad$ $\frac{i}{p^2-m^2}\; i\Sigma(p^2)\; \frac{i}{p^2-m^2}\; i\Sigma(p^2)\; \frac{i}{p^2-m^2}$

$$= \frac{i}{p^2 - m^2 + \Sigma(p^2)}$$

$\Rightarrow$ Pole of the propagator: $\mathcal{M}^2$

Renormalised self-energy

$$\mathcal{M}^2 - m^2 + \Sigma(\mathcal{M}^2) = 0$$

For a stable particle: $\Sigma(\mathcal{M}^2)$ is real

If $\Sigma(\mathcal{M}^2) \neq 0 \Rightarrow$ Pole shifted by higher-order contributions

Mass of an unstable (elementary) particle

For an unstable particle:

$\Sigma(\mathcal{M}^2)$ is complex $\Rightarrow$ Pole in the complex plane

$$\mathcal{M}^2 - m^2 + \Sigma(\mathcal{M}^2) = 0, \quad \mathcal{M}^2 = M^2 - iM\Gamma$$

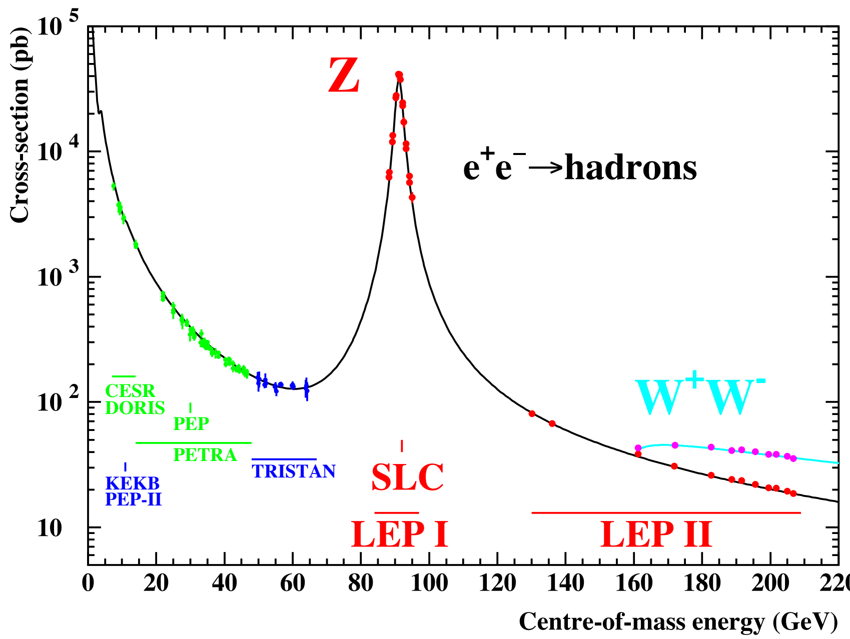
M: physical mass, Γ: decay width of the unstable particle

$\Rightarrow$ The mass of an unstable (elementary) particle is defined according to the real part of the complex pole

Example:

resonant production

of the Z boson and its decay



Deconvolution and residual model dependence

Example: model dependence of the Z-boson mass,
$M_Z^{exp} = 91.1876 \pm 0.0021$ GeV

In order to obtain this experimental value the Standard Model has been assumed. As a consequence, the experimental value quoted for the Z-boson mass actually depends (slightly) on the Higgs-boson mass of the Standard Model!

$\delta M_Z^{exp} \approx \pm 0.2$ MeV for 100 GeV $< M_H <$ 1 TeV, corresponds to about 10% of the experimental error

$\Rightarrow$ A careful assessment of similar effects will be needed for high-precision analyses at the LHC and future colliders!

M_W: which parameter is actually measured?

Experimentally, a ``Monte Carlo” mass parameter is measured. The problem of how to relate such a parameter to a theoretically well defined Lagrangian mass parameter is most apparent for the case of the top-quark mass (coloured object, renormalon ambiguities, …).

However, for an accuracy at the level of 10^{-4} this issue may also be of relevance for M_W

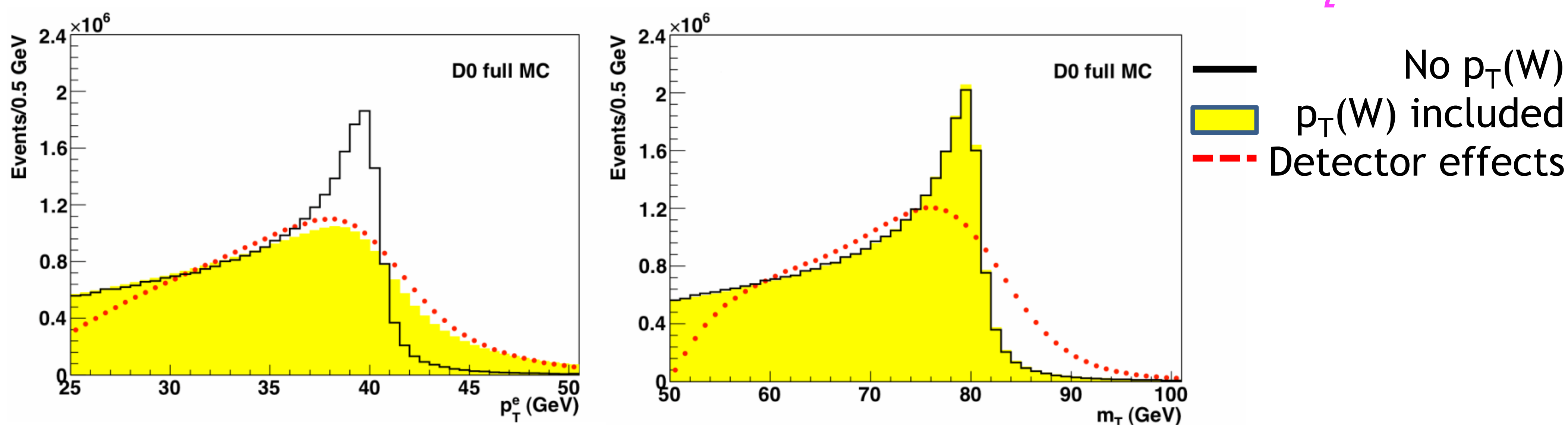
Example: the shift in M_W between the fixed-width and the running-width parameterisation of the resonance is formally an electroweak two-loop effect that amounts to about 27 MeV!

Are the Monte Carlo codes that are used for the experimental determination of M_W sufficiently accurate?

Measurement of the W mass at hadron colliders

problems are due to
- the smearing of the distributions due to difficult neutrino reconstruction
- strong sensitivity to the modelling of initial state QCD effects

[A. Vicini '22]



Templates for different M_W values:

The templates are perturbative predictions.
Their residual theoretical uncertainties will propagate as theoretical systematic errors on the determination of (G_μ, m_W, m_Z)

Given the very high precision goal $\delta m_W/m_W \sim 1\cdot 10^{-4}, \quad \delta \sin^2\theta_{eff}/\sin^2\theta_{eff} \sim 1\cdot 10^{-3}$
control on the shape of the distributions at the sub-percent level is needed, at a hadron collider…

- very large impact of initial-state QCD radiation on the ptlep distribution
- large radiative corrections due to QED final state radiation at the jacobian peak
- very large interplay of QCD and QED corrections redefining the precise shape of the jacobian peak

BSM predictions for the W-boson mass

S, T, U parameters: only BSM contributions taken into account that enter via gauge-boson self-energies (only one-loop contributions), external momentum neglected

$$M_W^2 = M_W^2\big|_{\mathrm{SM}} \left(1 + \frac{s_w^2}{c_w^2 - s_w^2} \Delta r'\right)$$

$$\Delta r' = \frac{\alpha}{s_w^2} \left(-\frac{1}{2} S + c_w^2 T + \frac{c_w^2 - s_w^2}{4 s_w^2} U\right)$$

SM prediction for the experimental values of M_{H}, m_{t}, …

Global fits to electroweak precision observables:
SM, SM + S, T, U parameters: *GFitter*, …
BSM models (SUSY, …): *MasterCode*, *Gambit*, …
EFT fits

The effective leptonic weak mixing angle: $\sin^2\theta_{\text{eff}}$

Effective leptonic weak mixing angle at the Z-boson resonance:

$$\sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\,\frac{g_V}{g_A}\right) = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa)$$

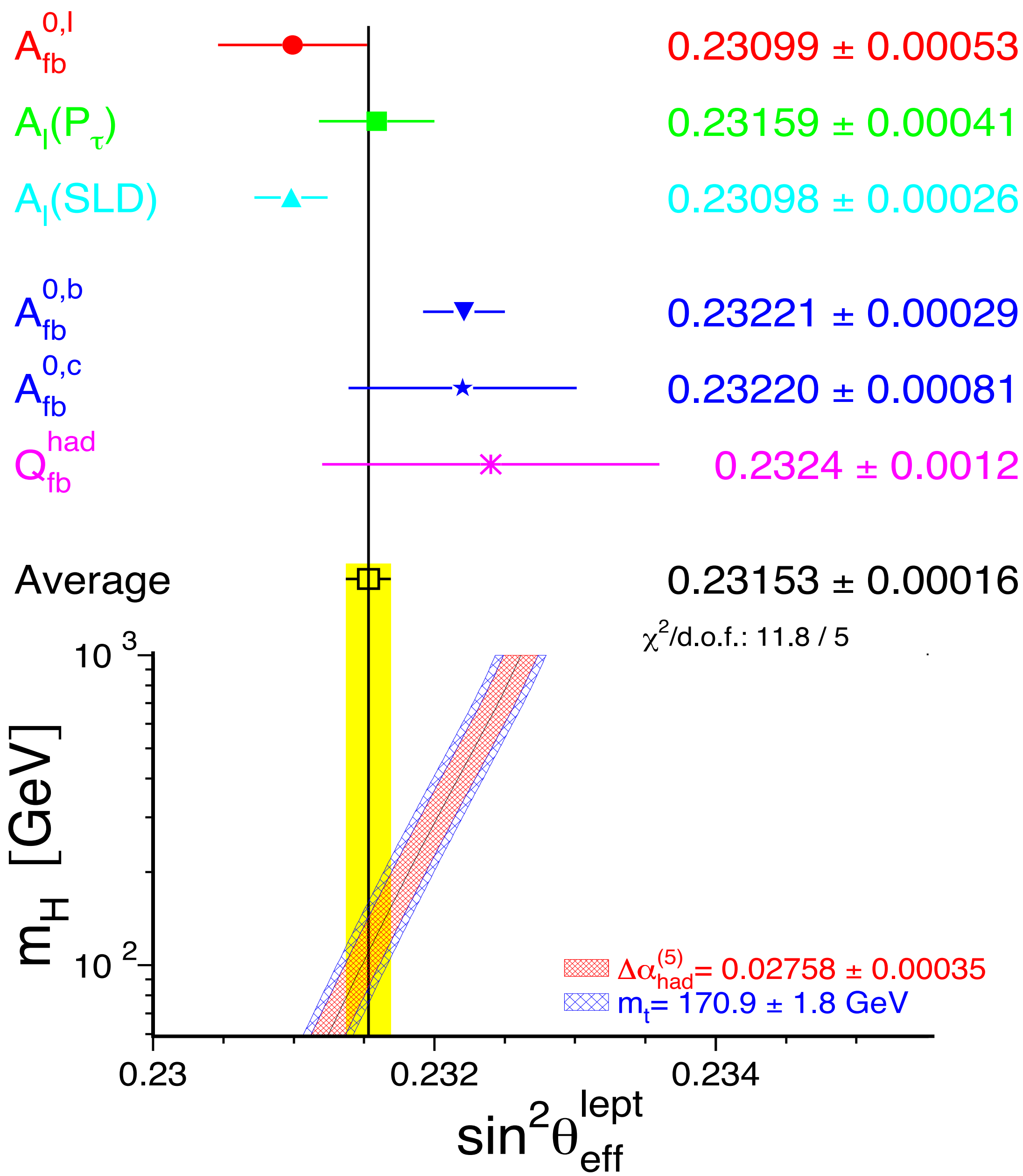
Current experimental value from LEP and SLD:

$\sin^2\theta_{\text{eff}} = 0.23153 \pm 0.00016 \Rightarrow$ Accuracy of 0.07%

However: the small experimental error of the world-average is driven by two measurements that are not well compatible with each other: A_{LR} (SLD) and A_{FB} (LEP)

$$\sin^2\theta_{\text{eff}}(A_{\text{LR}}) = 0.23098 \pm 0.00026, \quad \sin^2\theta_{\text{eff}}(A_{\text{FB}}) = 0.23221 \pm 0.00029$$

$\sin^2\theta_{eff}$: unclear experimental situation



[*LEPEWWG '07*]

$\sin^2\theta_{\rm eff}$ has a high sensitivity to $M_{\rm H}$ and effects of new physics

But:
large discrepancy between $A_{\rm LR}$ (SLD) and $A_{\rm FB}$ (LEP),

has big impact on constraints on new physics

Interpretation of constraints from $\sin^2\theta_{eff}$ is complicated by the fact that the two most precise individual measurements differ from each other by more than 3 σ

Interface between experiment and theory

The effective weak mixing angle at the Z-resonance, $\sin^2\theta_{eff}$, is also a ``pseudo observable'', whose extraction from the actually measured quantities requires a certain amount of ``unfolding''; as in the case of the W-boson mass not only its prediction within a certain model but also its extraction from the experimental data is affected by theoretical uncertainties

$\sin^2\theta_{eff}$ has a clear experimental prescription in terms of left-right and forward-backward asymmetries and a well-defined theoretical meaning; for higher precision more and more contributions have to be taken into account in the relation to the experimentally measured quantities in order to ensure that the measured quantity $\sin^2\theta_{eff}$ indeed matches the theoretically predicted one

Extraction of $\sin^2\theta_{\rm eff}$ at LEP

Form factors implemented in *ZFITTER*: *[M. Awramik, M. Czakon, A. Freitas '06]*

$$
\begin{aligned}
\mathcal{A}[e^+e^- \to f\bar{f}] = 4\pi i\,\alpha\,\frac{Q_{\rm e}Q_{\rm f}}{s}\,\gamma_\mu \otimes \gamma^\mu \\
+\, i\frac{\sqrt{2}G_\mu M_{\rm Z}^2}{1+i\Gamma_{\rm Z}/M_{\rm Z}}\, I_{\rm e}^{(3)}\, I_{\rm f}^{(3)}\, \frac{1}{s-\overline{M}_{\rm Z}^2+i\overline{M}_{\rm Z}\overline{\Gamma}_{\rm Z}} \\
\times\, \rho_{\rm ef}\Big[\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu(1+\gamma_5) \\
-\, 4|Q_{\rm e}|s_{\rm W}^2\,\kappa_{\rm e}\,\gamma_\mu\otimes\gamma^\mu(1+\gamma_5) \\
-\, 4|Q_{\rm f}|s_{\rm W}^2\,\kappa_{\rm f}\,\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu \\
+\, 16|Q_{\rm e}Q_{\rm f}|s_{\rm W}^4\,\kappa_{\rm ef}\,\gamma_\mu\otimes\gamma^\mu\Big]
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{\rm ef}(s) &= \kappa_{\rm e}(s)\kappa_{\rm f}(s) - \frac{M_{\rm Z}^2-s}{s}\,\frac{1}{(a_{\rm e}^{(0)}-v_{\rm e}^{(0)})(a_{\rm f}^{(0)}-v_{\rm f}^{(0)})} \\
&\quad\times\left[q_{\rm e}^{(1)}q_{\rm f}^{(0)} + q_{\rm f}^{(1)}q_{\rm e}^{(0)} - p_{\rm f}^{(1)}q_{\rm e}^{(0)}\frac{v_{\rm f}^{(0)}}{a_{\rm f}^{(0)}} - p_{\rm e}^{(1)}q_{\rm f}^{(0)}\frac{v_{\rm e}^{(0)}}{a_{\rm e}^{(0)}} - q_{\rm e}^{(0)}q_{\rm f}^{(0)}\frac{\Sigma_{\gamma\gamma}^{(1)}}{s} + \text{boxes}\right], \\
\kappa_{\rm e,f}(s) &= \kappa_{\rm Z}^{\rm e,f}(s) + \frac{M_{\rm Z}^2-s}{s}\left[\frac{q_{\rm e,f}^{(0)}}{a_{\rm e,f}^{(0)}-v_{\rm e,f}^{(0)}}\,\frac{p_{\rm f,e}^{(1)}}{a_{\rm f,e}^{(0)}} + \text{boxes}\right], \\
\kappa_{\rm Z}^{\rm f}(s) &= \kappa_{\rm Z}^{\rm f}(M_{\rm Z}^2) + (s-M_{\rm Z}^2)\,\frac{\hat{a}_{\rm f}^{(1)\prime}(M_{\rm Z}^2)\,v_{\rm f}^{(0)} - \hat{v}_{\rm f}^{(1)\prime}(M_{\rm Z}^2)\,a_{\rm f}^{(0)}}{a_{\rm f}^{(0)}(a_{\rm f}^{(0)}-v_{\rm f}^{(0)})}.
\end{aligned}
$$

Relation between $\sin^2\theta_{\rm eff}^{\rm f}$ determined from expansion around the complex pole and the one defined in *ZFITTER*:

$$
\sin^2\theta^f_{\rm eff,pole} = \overline{s}_{\rm W}^2\,{\rm Re}\left\{\overline{\kappa}_{\rm Z}^{\rm f}(M_{\rm Z}^2)\right\} = \sin^2\theta^f_{\rm eff,ZFITTER} - \frac{\Gamma_{\rm Z}}{M_{\rm Z}}\,\frac{q_{\rm f}^{(0)}}{a_{\rm e}^{(0)}(a_{\rm f}^{(0)}-v_{\rm f}^{(0)})}\,{\rm Im}\{p_{\rm e}^{(1)}\}
$$

$$
\overline{s}_{\rm W}^2 = \left(1-\frac{\overline{M}_{\rm W}^2}{\overline{M}_{\rm Z}^2}\right) = s_{\rm W}^2\left[1+\frac{c_{\rm W}^2}{s_{\rm W}^2}\left(\frac{\Gamma_{\rm W}^2}{M_{\rm W}^2}-\frac{\Gamma_{\rm Z}^2}{M_{\rm Z}^2}\right)\right]^{-1}.
$$

numerically small, but required at this order

M_W and the Z-pole observables

PDG list of Z-pole observables:

[PDG '22]

Quantity	Value	Standard Model	Pull
M_Z [GeV]	91.1876 ± 0.0021	91.1882 ± 0.0020	-0.3
Γ_Z [GeV]	2.4955 ± 0.0023	2.4941 ± 0.0009	0.6
$\sigma_{\rm had}$ [nb]	41.481 ± 0.033	41.482 ± 0.008	0.0
R_e	20.804 ± 0.050	20.736 ± 0.010	1.4
R_μ	20.784 ± 0.034	20.736 ± 0.010	1.4
R_τ	20.764 ± 0.045	20.781 ± 0.010	-0.4
R_b	0.21629 ± 0.00066	0.21582 ± 0.00002	0.7
R_c	0.1721 ± 0.0030	0.17221 ± 0.00003	0.0
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01617 ± 0.00007	-0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.6
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5
$A_{FB}^{(0,b)}$	0.0996 ± 0.0016	0.1029 ± 0.0002	-2.0
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0735 ± 0.0002	-0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1030 ± 0.0002	-0.4
$\bar{s}_\ell^2$	0.2324 ± 0.0012	0.23155 ± 0.00004	0.7
	0.23148 ± 0.00033		-0.2
	0.23129 ± 0.00033		-0.8
A_e	0.15138 ± 0.00216	0.1468 ± 0.0003	2.1
	0.1544 ± 0.0060		1.3
	0.1498 ± 0.0049		0.6
A_μ	0.142 ± 0.015		-0.3
A_τ	0.136 ± 0.015		-0.7
	0.1439 ± 0.0043		-0.7
A_b	0.923 ± 0.020	0.9347	-0.6
A_c	0.670 ± 0.027	0.6677 ± 0.0001	0.1
A_s	0.895 ± 0.091	0.9356	-0.4

Note: the fit for the electroweak precision observables is dominated by M_W and $\sin^2\theta_{\rm eff}$ (does not appear explicitly in the PDG list), the other Z-pole observables have only a relatively small impact

M_W and the Z-pole observables

[PDG ’22]

1. Physical Constants

Table 1.1: Revised 2021 by D. Robinson (LBNL). Reviewed by P. Mohr (NIST). Mainly from “CODATA Recommended Values of the Fundamental Physical Constants: 2018,” E. Tiesinga, D.B. Newell, P.J. Mohr, and B.N. Taylor, NIST SP961 (May 2019) [1]. The electron charge magnitude e, and the Planck, Boltzmann, and Avogadro constants h, k, and N_A, now join c as having defined values; the free-space permittivity and permeability constants ϵ_0 and μ_0 are no longer exact. These changes affect practically everything else in the Table. Figures in parentheses after the values are the 1-standard-deviation uncertainties in the last digits; the fractional uncertainties in parts per 10^9 (ppb) are in the last column. The full 2018 CODATA Committee on Data for Science and Technology set of constants are found at `https://physics.nist.gov/constants`. The last set of constants (beginning with the Fermi coupling constant) comes from the Particle Data Group. See also “The International System of Units (SI),” 9th ed. (2019) of the International Bureau of Weights and Measures (BIPM), `https://www.bipm.org/utils/common/pdf/si-brochure/SI-Brochure-9-EN.pdf`.

Quantity	Symbol, equation	Value	Uncertainty (ppb)
speed of light in vacuum	c	299 792 458 m s^{-1}	exact
Planck constant	h	6.626 070 15×10^{-34} J s (or J/Hz) §	exact
Planck constant, reduced	$\hbar \equiv h/2\pi$	1.054 571 817... × 10^{-34} J s	exact*
		= 6.582 119 569... × 10^{-22} MeV s	exact*
electron charge magnitude	e	1.602 176 634×10^{-19} C	exact
conversion constant	$\hbar c$	197.326 980 4... MeV fm	exact*
conversion constant	$(\hbar c)^2$	0.389 379 372 1... GeV^2 mbarn	exact*
electron mass	m_e	0.510 998 950 00(15) MeV/c^2 = 9.109 383 7015(28)×10^{-31} kg	0.30
proton mass	m_p	938.272 088 16(29) MeV/c^2 = 1.672 621 923 69(51)×10^{-27} kg	0.31
		= 1.007 276 466 621(53) u = 1836.152 673 43(11) m_e	0.053, 0.060
neutron mass	m_n	939.565 420 52(54) MeV/c^2 = 1.008 664 915 95(49) u	0.57, 0.48
deuteron mass	m_d	1875.612 942 57(57) MeV/c^2	0.30
unified atomic mass unit**	u = (mass ^{12}C atom)/12	931.494 102 42(28) MeV/c^2 = 1.660 539 066 60(50)×10^{-27} kg	0.30
permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	8.854 187 8128(13) ×10^{-12} F m^{-1}	0.15
permeability of free space	$\mu_0/(4\pi \times 10^{-7})$	1.000 000 000 55(15) N A^{-2}	0.15
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297 352 5693(11)×10^{-3} = 1/137.035 999 084(21)† ‡‡	0.15
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 3262(13)×10^{-15} m	0.45
(e^- Compton wavelength)/2π	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e\alpha^{-1}$	3.861 592 6796(12)×10^{-13} m	0.30
Bohr radius ($m_{\rm nucleus} = \infty$)	$a_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2 = r_e\alpha^{-2}$	0.529 177 210 903(80)×10^{-10} m	0.15
wavelength of 1 eV/c particle	$hc/(1\ {\rm eV})$	1.239 841 984... × 10^{-6} m	exact*
Rydberg energy	$hcR_\infty = m_e e^4/2(4\pi\epsilon_0)^2\hbar^2 = m_e c^2\alpha^2/2$	13.605 693 122 994(26) eV	1.9×10^{-3}
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	0.665 245 873 21(60) barn	0.91
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.788 381 8060(17)×10^{-11} MeV T^{-1}	0.30
nuclear magneton	$\mu_N = e\hbar/2m_p$	3.152 451 258 44(96)×10^{-14} MeV T^{-1}	0.31
electron cyclotron freq./field	$\omega^e_{\rm cycl}/B = e/m_e$	1.758 820 010 76(53)×10^{11} rad s^{-1} T^{-1}	0.30
proton cyclotron freq./field	$\omega^p_{\rm cycl}/B = e/m_p$	9.578 833 1560(29)×10^7 rad s^{-1} T^{-1}	0.31
gravitational constant‡	G_N	6.674 30(15)×10^{-11} m^3 kg^{-1} s^{-2}	2.2 × 10^4
		= 6.708 83(15)×10^{-39} $\hbar c$ (GeV/c^2)$^{-2}$	2.2 × 10^4
standard gravitational accel.	g_N	9.806 65 m s^{-2}	exact
Avogadro constant	N_A	6.022 140 76×10^{23} mol^{-1}	exact
Boltzmann constant	k	1.380 649×10^{-23} J K^{-1}	exact
		= 8.617 333 262... × 10^{-5} eV K^{-1}	exact*
molar volume, ideal gas at STP	$N_A k$ (273.15 K)/(101 325 Pa)	22.413 969 54... × 10^{-3} m^3 mol^{-1}	exact*
Wien displacement law constant	$b = \lambda_{\rm max}T$	2.897 771 955... × 10^{-3} m K	exact*
Stefan-Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	5.670 374 419... × 10^{-8} W m^{-2} K^{-4}	exact*
Fermi coupling constant‡‡	$G_F/(\hbar c)^3$	1.166 378 8(6)×10^{-5} GeV^{-2}	510
weak-mixing angle	$\sin^2\hat\theta(M_Z)$ ($\overline{\rm MS}$)	0.231 21(4)††	1.7 × 10^5
$W^\pm$ boson mass	m_W	80.377(12) GeV/c^2¶	1.5 × 10^5
Z^0 boson mass	m_Z	91.1876(21) GeV/c^2	2.3 × 10^4
strong coupling constant	$\alpha_s(m_Z)$	0.1179(9)	7.6 × 10^6

$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ldots$ $\qquad e = 2.718\ 281\ 828\ 459\ 045\ 235\ldots$ $\qquad \gamma = 0.577\ 215\ 664\ 901\ 532\ 860\ldots$

1 in ≡ 0.0254 m	1 G ≡ 10^{-4} T	1 eV = 1.602 176 634 × 10^{-19} J (exact)	kT at 300 K = [38.681 727 0718...]$^{-1}$$e$V (exact*)
1 Å ≡ 0.1 nm	1 dyne ≡ 10^{-5} N	(1 kg)c^2 = 5.609 588 603... × 10^{35}eV(exact*)	0 °C ≡ 273.15K
1 barn ≡ 10^{-28} m^2	1 erg ≡ 10^{-7} J	1 C = 2.997 924 58 × 10^9 esu	1 atmosphere ≡ 760 Torr ≡ 101 325Pa

§ CODATA recommends that the unit be J/Hz to stress that in $h = E/\nu$ the frequency ν is in cycles/sec (Hz), not radians/sec.
* These are calculated from exact values and are exact to the number of places given (*i.e.* no rounding).
** The molar mass of ^{12}C is 11.999 999 9958(36) g.
† At $Q^2 = 0$. At $Q^2 \approx m_W^2$ the value is ∼ 1/128.
‡ Absolute laboratory measurements of G_N have been made only on scales of about 1 cm to 1 m.
‡‡ See the discussion in Ch. 10, “Electroweak model and constraints on new physics.”
†† The corresponding $\sin^2\theta$ for the effective angle is 0.23153(4).
¶ See the “Mass and width of the W boson” review.

Beware: this is not an independent experimental observable!

Ongoing Tevatron — LHC combination

[J. Kretzschmar '22]

D0 (4.3+1.1 fb^{-1}) [*Phys. Rev.* ***D89*** *(2014) 012005*]

m_W = 80375 ± 11 (stat.) ± 20 (sys.) MeV

CDF (8.8 fb^{-1}) [*Science* ***376*** *(2022) 170*]

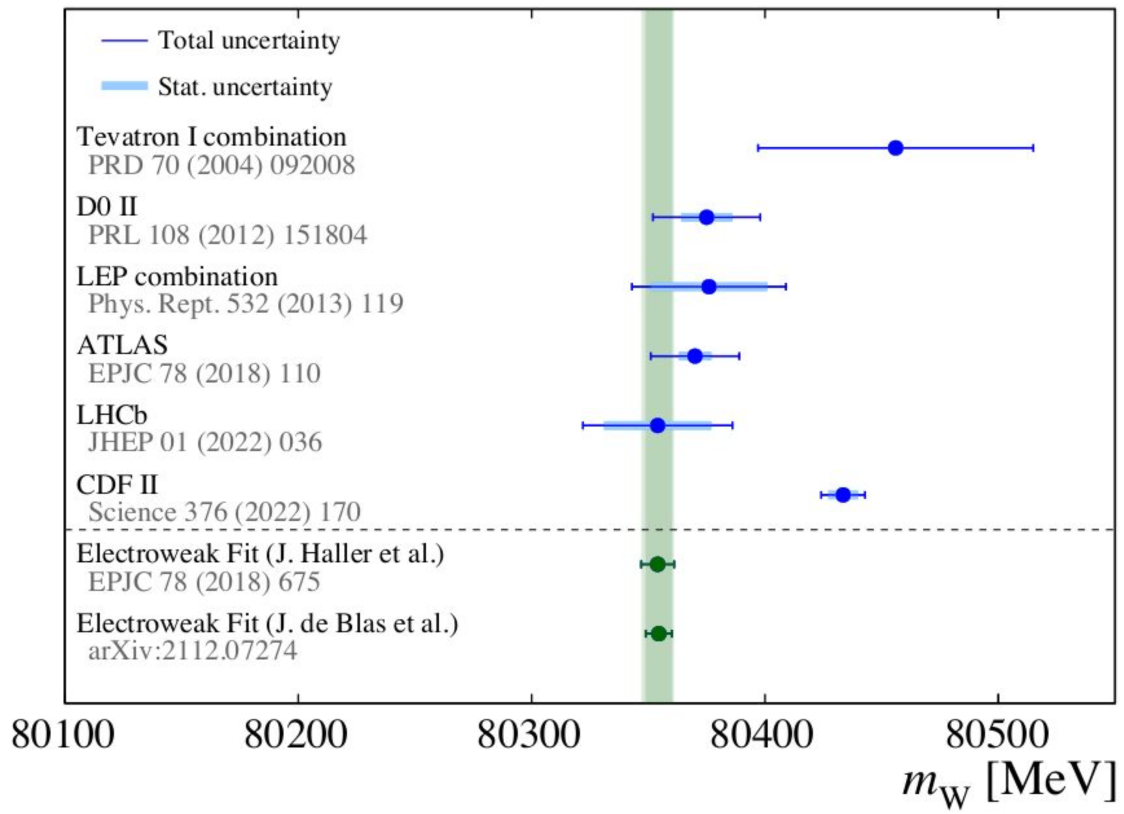
m_W = 80433.5 ± 6.4 (stat.) ± 6.9 (sys.) MeV

ATLAS (4.6 fb^{-1}) [*Eur. Phys. J.* ***C78*** *(2018) 110*]

m_W = 80370 ± 7 (stat.) ± 18 (sys.) MeV

LHCb (1.7 fb^{-1}) [*JHEP* ***01*** *(2022) 036*]

m_W = 80354 ± 23 (stat.) ± 22 (sys.) MeV



- Sensitive probe for BSM physics
- Dominated by hadron collider measurements: difficult measurement with significant theory input, non-trivial correlations e.g. from PDFs
- New CDF result significantly away from the SM and prior measurements

Ongoing Tevatron — LHC combination

[J. Kretzschmar '22]

- Created in 2020 with ATLAS, CMS, CDF, D0; LHCb added more recently
- Primary goals:
  - Official combinations of measurements with treatment of correlations of systematic uncertainties
  - Publication signed by corresponding collaborations
- Established a methodology to combine existing and future measurements; can also be used to enable physics modelling updates of past measurements (i.e. PDFs, pTW) or e.g. correlate measurements of m_W and $\sin^2\theta_W$
- Intermediate results presented at ICHEP2022 + public note released

CERN-LPCC-2022-06
FERMILAB-TM-2779-V
7th July 2022

Towards a combination of LHC and TeVatron *W*-boson mass measurements

The LHC–TeVatron *W*-boson mass combination working group[1]

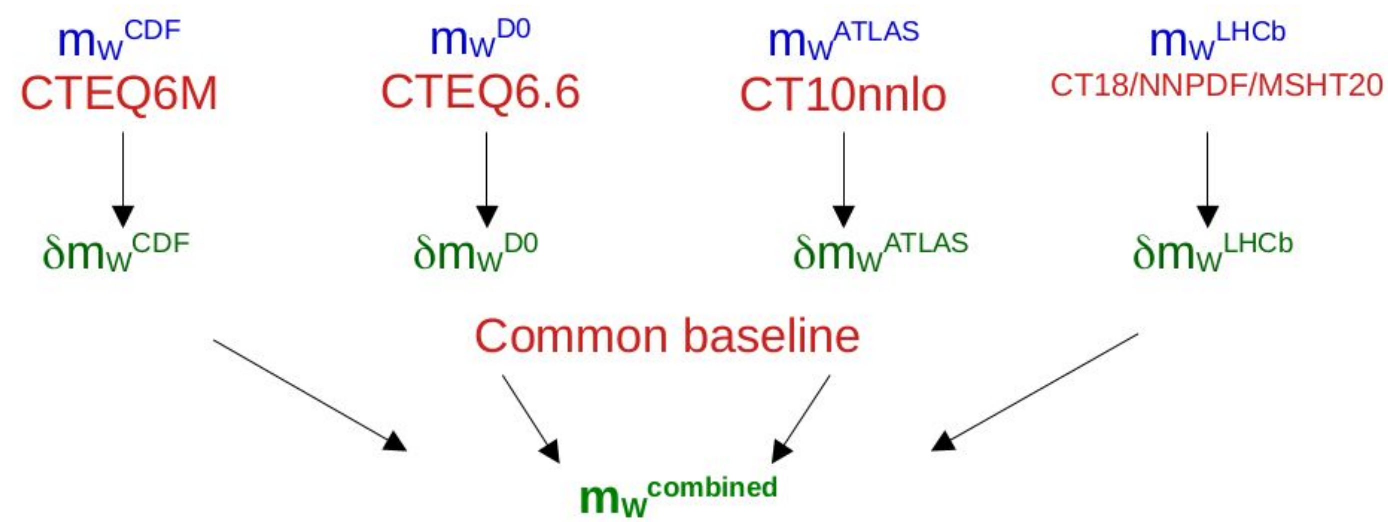
In this note methodological and modelling considerations towards a combination of the ATLAS, CDF and D0 measurements of the *W*-boson mass are discussed. As they were performed at different moments in time, each measurement employed different assumptions for the modelling of *W*-boson production and decay, as well as different fits of the parton distribution functions of the proton (PDFs). Methods are presented to accurately evaluate the effect of PDFs and other modelling variations on existing measurements, allowing to extrapolate them to any PDF set and to evaluate the corresponding uncertainties. Based on this approach, the measurements can be corrected to a common modelling reference and to the same PDFs, and subsequently combined accounting for PDF correlations in a quantitative way.

https://cds.cern.ch/record/2815187

Ongoing Tevatron — LHC combination

[J. Kretzschmar ’22]

- Measurements performed at different times, using different baseline PDFs and QCD tools, two-step procedure :
  - correct to common PDF & QCD accuracy
  - combination including correlations



- Full procedure, decomposed into generator and PDF effects

$$m_W^{updated} = m_W^{ref.} - \delta m_W^{QCD} - \delta m_W^{PDF}$$

published — Improved predictions, for reference PDF — PDF extrapolation

- Allows to improve existing experimental results for improved theory & PDFs

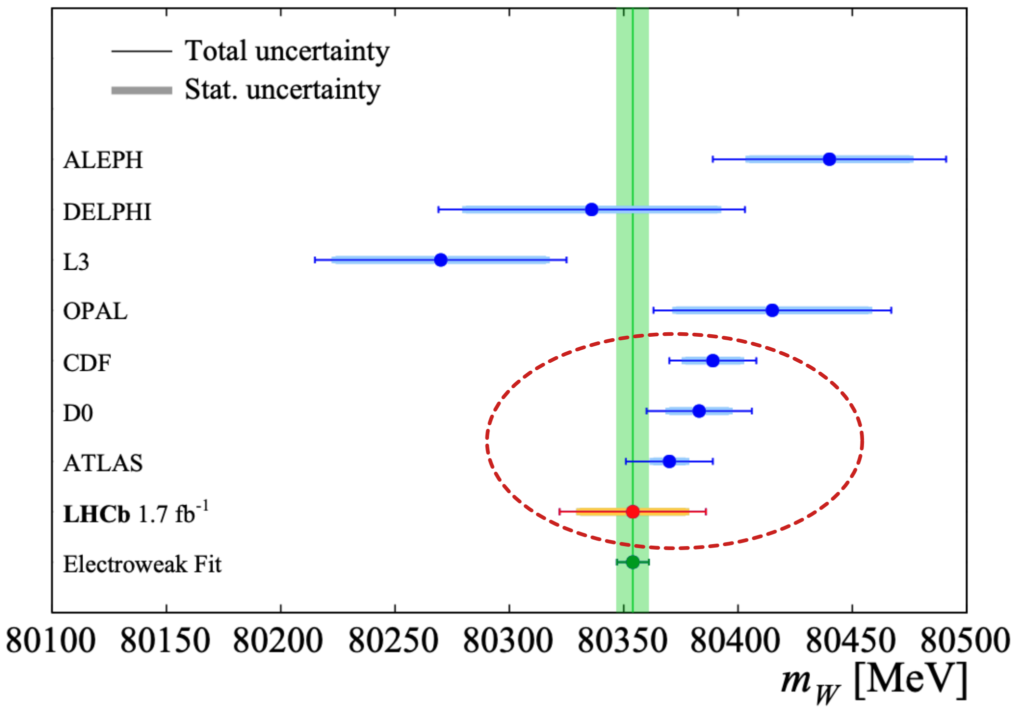
Ongoing Tevatron — LHC combination

[J. Kretzschmar '22]

- Significant progress towards combination (and understanding) of existing m_W measurements
  - Detailed study of generator features such as decay angle/polarization effects
  - PDF archeology
- Close to having a fully final set of numbers for QCD corrections, PDF shifts and correlations and combinations
- In contrast to initial remit, the LHCb measurement will be fully incorporated
- Expecting a complete documentation with combinations using different PDFs sets and different subsets of experiments:
  - TeVatron, LHC, All (including LEP); N-1
  - PDFs: ABMP16, CT18, NNPDF3.1, NNPDF4.0, MSHT20
- Also expecting remaining incompatibilities between measurements even after applying relevant theory updates – iterating on how to present & discuss the results

On-going M_W combination (with previous CDF value)

[LHC EW WG, M. Boonekamp '22]



Measurement extrapolations

- Full procedure, decomposed into generator and PDF effects :

$$m_W^{updated} = m_W^{ref.} + \delta m_W^{QCD} + \delta m_W^{PDF}$$

published — Improved predictions, for reference PDF — PDF extrapolation

- Published measurements :
  - CDF : Resbos1 (NLO) CTEQ6M (NLO)
  - D0 : Resbos1 (NNLO) CTEQ6.6 (NLO)
  - ATLAS : Powheg+Pythia; rapidy+spin corr. at NNLO CT10 (NNLO)
  - LHCb : Powheg+Pythia; spin corr. at NNLO <NNPDF3.1,CT18,MSHT20> (NLO)
- Extrapolations (δm_W) evaluated using generator-level reweightings and "emulation" of detector effects

- δm_W^{PDF} Main PDF targets : modern NNLO sets
  ~Finalized, including generator dependence of PDF extrapolations.
- δm_W^{QCD} Applies when generators or QCD improvements are beyond the quoted uncertainties.
  Long neglected, and subject of ongoing work : Powheg, MiNNLO, New Resbos

- Accuracy of Resbos1, compared to modern generators?
  - Resbos1 distributions obtained from the CDF publication sample, and D0 event generation grids (thanks for sharing!)
    - Resbos1 was a semi-private generator, and it is difficult to reproduce these distributions externally
  - Comparisons to Powheg, MiNNLO, and "Resbos 2"
    - "Resbos 2" is an upgrade of Resbos1, with (among others) improved NNLO QCD corrections, and improved treatment of spin correlations

On-going M_W combination (with previous CDF value)

[LHC EW WG, M. Boonekamp '22]

PDF extrapolations (including generator dependence)

- Example, for CDF (defines reference PDF):

Generator		Powheg	Powheg	MiNNLO	Resbos	Resbos
Sample type		Reweighted	Direct	Reweighted	Direct	Direct
QCD accuracy		NLO+NLL	NLO+NLL	NNLO+NLL	NLO+NLL	NNLO+NNLL
PDF set		Shift				
CTEQ6M	NLO	0	0	0	0	0
CTEQ66	NLO	-15.4 ± 0.8	-15.8 ± 0.8	-14.0 ± 1.3	-17.8 ± 1.0	-16.6 ± 1.0
CT10	NLO	-6.3 ± 0.8	-6.2 ± 0.8	-4.2 ± 1.3	–	–
CT10nnlo	NNLO	-16.2 ± 0.8	-16.6 ± 0.8	-16.8 ± 1.3	–	–
CT14	NNLO	-4,1 ± 0.8	-3.9 ± 0.8	-6.8 ± 1.3	-7.1 ± 1.0	-6.9 ± 1.0
CT18	NNLO	-6.2 ± 0.8	-6.6 ± 0.8	-8.5 ± 1.3	-9.4 ± 1.0	-7.2 ± 1.0
CJ15	NLO	7.7 ± 0.8	7.9 ± 0.8	10.1 ± 1.3	–	–
MMHT14	NNLO	-6.2 ± 0.8	-6.4 ± 0.8	-6.9 ± 1.3	-8.1 ± 1.0	-3.5 ± 1.0
MSHT20	NNLO	-5.0 ± 0.8	-4.9 ± 0.8	-4.9 ± 1.3	–	–
ABMP16	NNLO	5.2 ± 0.8	5.0 ± 0.8	-0.2 ± 1.3	–	–
NNPDF3.1	NNLO	-13.8 ± 0.8	-14.3 ± 1.4	-14.1 ± 1.3	-15.8 ± 1.0	-8.0 ± 1.0

→ Significant difference between CTEQ6M and CTEQ6.6 (not accounted for this far)

On-going M_W combination (with previous CDF value)

[LHC EW WG, M. Boonekamp '22]

New combinations

- Preliminary combinations for ATLAS+CDF+D0.
  - Central values may need corrections : hidden for now!
    - Model-dependence of PDF extrapolations?
    - Impact of generator mis-modellings?
  - Total (PDF) uncertainties : 11–13 MeV (3–7 MeV).
  - CT18, MSHT20, NNPDF4.0 now available too.

	CTEQ6M	CTEQ6.1	CTEQ6.6	CT10nnlo	MSTW2008
Central value	[redacted]				
PDF	9	9	9	9	5
Total	14	14	14	14	12
$\chi^2/ndof$	47/35	46/35	50/35	48/35	60/35

Table 1: Combination summary: Legacy PDFs

	CT10	CJ15	CT14nlo	MMHT2014nlo	NNPDF3.1nlo
Central value	[redacted]				
PDF	11	2	9	6	4
Total	16	11	14	13	11
$\chi^2/ndof$	46/35	53/35	48/35	58/35	49/35

Table 2: Combination summary: NLO PDFs

	CT14nnlo	MMHT2014nnlo	ABMP16nnlo	NNPDF3.1nnlo
Central value	[redacted]			
PDF	10	7	3	4
Total	15	13	11	11
$\chi^2/ndof$	45/35	45/35	55/35	50/35

Table 3: Combination summary: NNLO PDFs

$\Rightarrow$ On-going effort will hopefully soon result in a new world average for both the central value and the experimental uncertainty

Simple example of extended Higgs sector: 2HDM

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge ½
- CP-conserving 2HDM, with softly-broken Z_2 symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs

$$V^{(0)}_{\text{2HDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

- m_1,m_2 eliminated with tadpole equations, and $v_1^2 + v_2^2 = v^2 = (246\ \text{GeV})^2$
- 7 free parameters in scalar sector: m_3, λ_i (i=1,..,5), $\tan\beta \equiv v_2/v_1$
- Mass eigenstates: h, H: CP-even Higgses, A: CP-odd Higgs, $H^\pm$: charged Higgs, α: CP-even Higgs mixing angle
- λ_i (i=1,..,5) traded for mass eigenvalues m_h, m_H, m_A, $m_{H\pm}$ and angle α
- m_3 replaced by a Z_2 soft-breaking mass scale

$$M^2 = \frac{2m_3^2}{s_{2\beta}}$$

- **BSM-scalar masses** take form $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2\,, \quad \Phi \in \{H, A, H^\pm\}$

In alignment limit, α = β - π/2 : h couplings are SM-like at tree level