Mixed QCD-electroweak corrections and W-mass determination

Raoul Röntsch

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Delto, Jaquier, Melnikov, RR [hep-ph/1909.08428]

Buccioni, Caola, Delto, Jaquier, Melnikov, RR, [hep-ph/2005.10221]

Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, RR [hep-ph/2009.10386, **hep-ph/2103.02671**]

Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, RR, Signorile-Signorile [hep-ph/2203.11237]

W-mass Workshop

Orsay, 23 February 2023

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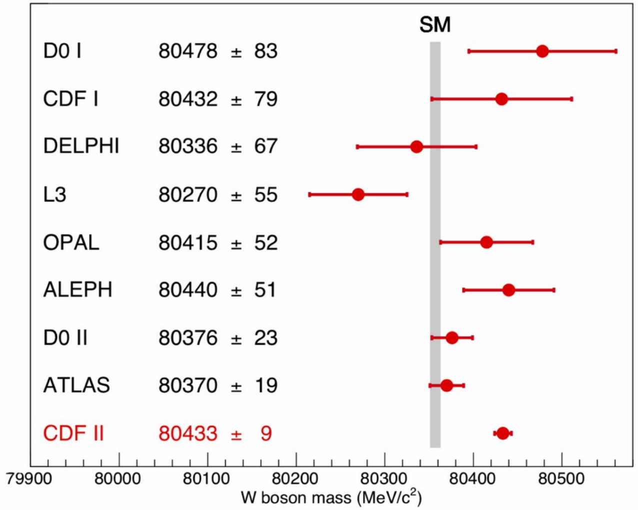
W-mass measurements at the Tevatron and LHC

- We are trying to measure a parameter of the SM with precision ~ 0.1 per mille at a *hadron collider.*

  → Per mille effects are "big" !

- Extraordinary precision demanded from experimental measurements and theoretical predictions:
  - ✔ Fixed-order calculations – expansions in α_s and α.
  - ✔ Parton showers
  - ✔ Resummations
  - ✔ Non-perturbative effects
  - ✔ Numerical effects?
  - ✔ ...



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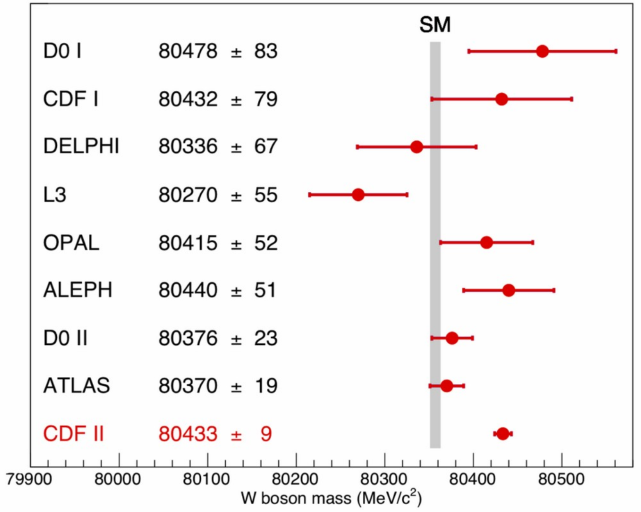


Precision Predictions for Drell-Yan Production

$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0,0)} + \alpha_s \hat{\sigma}_{ij}^{(1,0)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2,0)} + \alpha_s^3 \hat{\sigma}_{ij}^{(3,0)} + \ldots + \alpha\, \hat{\sigma}_{ij}^{(0,1)} + \alpha_s \alpha\, \hat{\sigma}_{ij}^{(1,1)} + \ldots$$

Mixed QCD-EW corrections

[Bonciani, Buccioni, Mondini, Vicini (‘17); De Florian, Der, Fabre (‘18); Delto, Jaquier, Melnikov, R.R. (‘19); Bonciani, Buccioni, Rana, Triscari, Vicini (‘19); Buccioni *et al.* (‘20); Cieri, De Florian, Der, Mazzitelli (‘20); Bonciani, Buccioni, Rana, Vicini (‘20); Behring *et al.* (‘20); Buonocore, Grazzini, Kallweit, Savoini, Tramontano (‘21); Bonciani *et al.* (‘21); Armadillo *et al.* (‘22); Buccioni *et al.* (‘22)]

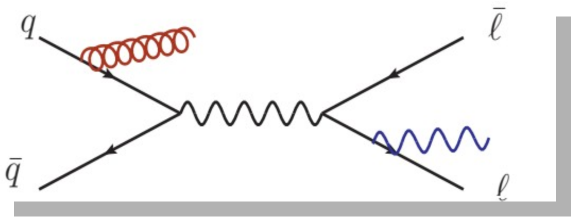
- For W-mass determination, offshell effects are secondary → focus on onshell vector boson production $pp \to V \to \ell\bar{\ell}$

QCD-EW corrections to onshell vector boson production

- Consider onshell vector boson production and decay $pp \to V \to \ell\bar{\ell}$
- In resonance region, QCD-EW corrections to production and decay processes can be treated separately.

[Dittmaier, Huss, Schwinn ('14, '15)]

QCD (production) x EW (decay)



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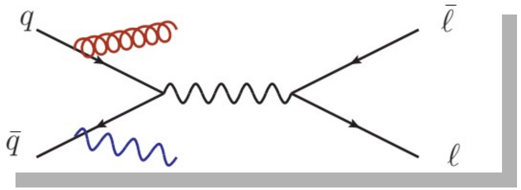

Non-factorizable resonant contributions (not shown) suppressed by Γ_V/m_V

[Dittmaier, Huss, Schwinn ('14, '15)]

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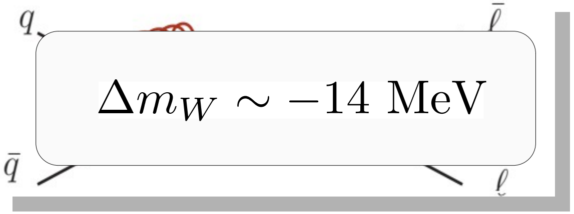

$\Delta m_W \sim -14\ \mathrm{MeV}$

[Dittmaier, Huss, Schwinn ('14, '15)]

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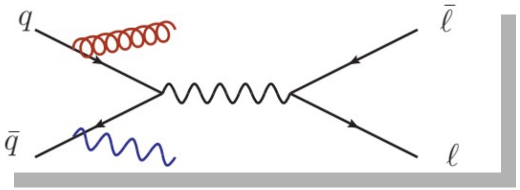

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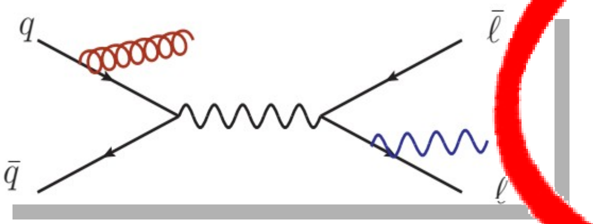

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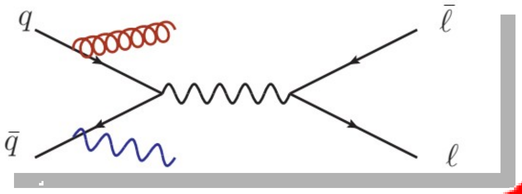

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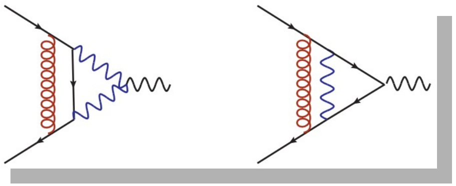
QCD-EW corrections to onshell vector boson production

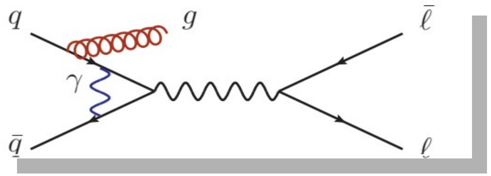
- Two major challenges in computing higher order corrections:
  1. Loop amplitudes
  2. Handling infrared singularities

- Loop amplitudes:
  - Two-loop form factors



    - Known for Z production. [Kotikov, Kühn, Veretin ('08)]
    - Computed for first time for W production. [Behring *et al.* ('20)]

  - One-loop real-virtual amplitudes



    - Standard one-loop programs, e.g. `OpenLoops`

[Cascioli, Maierhöfer, Pozzorini ('12); Buccioni, Pozzorini, Zoller ('18); Buccioni *et al.* ('19)]

QCD-EW corrections to onshell vector boson production

- Two major challenges in computing higher order corrections:
  1. Loop amplitudes
  2. Handling infrared singularities

- Infrared singularities with different origins appear simultaneously:
  - *Virtual* photons;
  - *Virtual* partons;
  - *Unresolved real* photons;
  - *Unresolved real* partons.

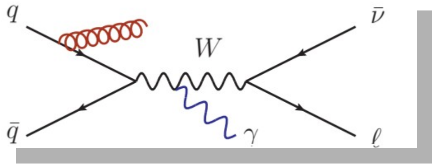
- Insight from NNLO QCD: treatment of IR singularities with non-trivial structure.

IR subtraction for mixed QCD-EW corrections

- Use nested soft-collinear subtraction scheme.

  [Caola, Melnikov, R.R. (‘17); Caola, Melnikov, R.R. (‘19); Asteriadis, Caola, Melnikov, R.R. (‘19); Delto, Frellesvig, Caola, Melnikov (‘18); Delto, Melnikov (‘19)]

- Straightforward modification to handle e.g. soft limit of photon radiated in W-production.



- Subtraction simpler in some cases, e.g. soft limits of photon and gluon decouple.
- Used to compute mixed QCD-EW corrections to production of onshell Z and W bosons.

  [Delto, Jaquier, Melnikov, RR (‘19); Buccioni, Caola, Delto, Jaquier, Melnikov, RR, (‘20); Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, RR (‘20)]

- Further extended method for dilepton production $pp \to Z/\gamma^* \to \ell^+\ell^-$

  [Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, RR, Signorile-Signorile (‘22)]

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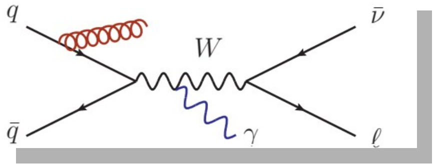

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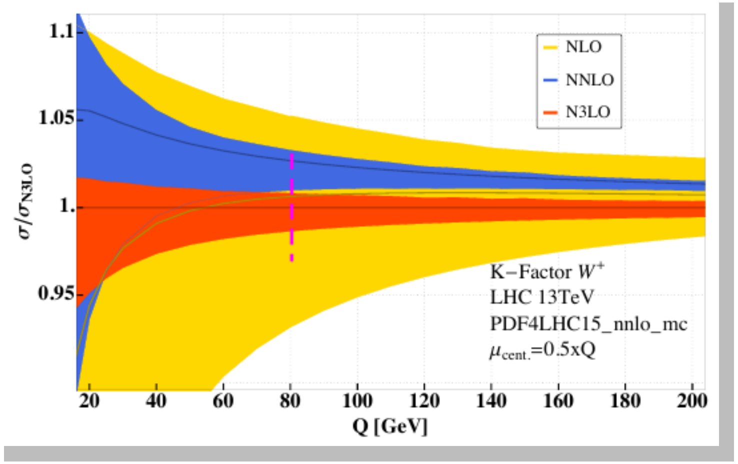
- **How could these corrections impact the measurement of the W-mass?**

W-mass measurements

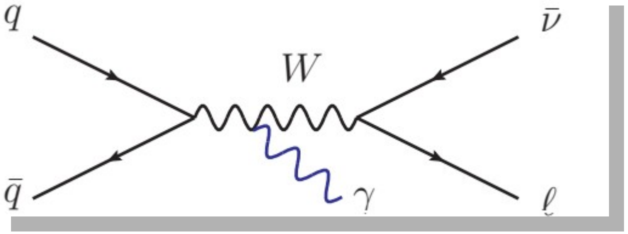
- Aiming at 0.1 per mille precision, but theoretical precision limited to ~ 1%.
- Sources of theoretical uncertainty:
  - higher order corrections
  - subleading logs
  - pdfs
  - non-perturbative effects
  - quark masses
  - …
- Use excellent experimental control of $pp \to Z \to \ell\bar{\ell}$ to tune generators and verify results.



[Duhr, Dulat, Mistlberger '20]

W-mass measurements

- Implicit assumption: higher-order corrections to W and Z production strongly correlated.
- Reasonable for QCD corrections:
  - Minor differences: pdfs, masses, helicity structures, …
- EW corrections: qualitatively different – *W* charged, can radiate:



- Mixed QCD-EW corrections potentially decorrelated.
- Possible impact on W-mass measurements at desired precision.

Impact on W mass determination

- ***Estimate*** effect of QCD-EW corrections on W mass measurement, due to decorrelations between *Z* and *W* production.
- Correlation between average transverse momentum of leptons and mass of boson:

$$\frac{m_W}{m_Z} = \frac{\langle p_{T,l}^W \rangle}{\langle p_{T,l}^Z \rangle} \Rightarrow m_W^{\text{meas.}} = m_Z \frac{\langle p_{T,l}^{W,\text{meas.}} \rangle}{\langle p_{T,l}^{Z,\text{meas.}} \rangle} C_{\text{th.}}$$

- Theoretical correction: assume input masses, compute W-mass, and compare with input W-mass.

$$\Rightarrow C_{\text{th.}} = \frac{m_W^{\text{in}}}{m_Z^{\text{in}}} \frac{\langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

→ estimate impact of decorrelations in *W* and *Z* spectra from higher order corrections:

$$\frac{\delta m_W^{\text{meas.}}}{m_W^{\text{meas.}}} = \frac{\delta C_{\text{th.}}}{C_{\text{th.}}} = \frac{\delta \langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{Z,\text{th.}} \rangle} - \frac{\delta \langle p_{T,l}^{W,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

[Behring *et al.* ('21)].

Impact on W mass determination

Shifts in W-mass, inclusive:

- NLO EW: $\Delta m_W = 1 \text{ MeV}$
- QCD-EW: $\Delta m_W = -7 \text{ MeV}$

➔ Impact of QCD-EW corrections larger than NLO EW:

- NLO EW corrections suppressed in G_μ scheme.
- NLO EW corrections more correlated between W and Z production.

$$\frac{\delta m_W^{\text{meas.}}}{m_W^{\text{meas.}}} = \frac{\delta C_{\text{th.}}}{C_{\text{th.}}} = \frac{\delta\langle p_{T,l}^{Z,\text{th.}}\rangle}{\langle p_{T,l}^{Z,\text{th.}}\rangle} - \frac{\delta\langle p_{T,l}^{W,\text{th.}}\rangle}{\langle p_{T,l}^{W,\text{th.}}\rangle}$$

NLO EW: $\Delta m_W = -31 \text{ MeV} + 32 \text{ MeV}$

QCD-EW: $\Delta m_W = +54 \text{ MeV} - 61 \text{ MeV}$

$\sqrt{s} = 13 \text{ TeV}$

G_μ scheme

$m_Z = 91.1876 \text{ GeV}$

$m_W = 80.398 \text{ GeV}$

$m_t = 173.2 \text{ GeV}$

$m_H = 125 \text{ GeV}$

$G_F = 1.16339 \cdot 10^{-5} \text{ GeV}^{-2}$

NNPDF31_luxQED

$\mu_R = \mu_F = m_V/2$

Impact on W mass determination

Shifts in W-mass: fiducial setup

- Inclusive setup: $\Delta m_W = -7 \text{ MeV}$
- “ATLAS” cuts: $\Delta m_W = -17 \text{ MeV}$
- “Tuned” cuts: $\Delta m_W = -1 \text{ MeV}$

  - Cuts can have dramatic impact: shifts vary by factor of ~20.
    - “ATLAS” cuts have stronger cuts on leptons from (lighter) *W* than from *Z* → decorrelation.
  - QCD-EW shifts potentially relevant for target precision of 8 MeV.

$p^{Z}_{T,\ell} > 25 \text{ GeV}; |\eta^{Z}_{\ell}| < 2.4$

“ATLAS” cuts: $\mathrm{p}^{W}_{T,\ell} > 30 \text{ GeV}; \mathrm{p}^{\mathrm{W}}_{\mathrm{T,miss}} > 30 \text{ GeV}; |\eta^{\mathrm{W}}_{\ell}| < 2.4.$

“Tuned” cuts: $\mathrm{p}^{W}_{T,\ell} > 25.44 \text{ GeV}; \mathrm{p}^{\mathrm{W}}_{\mathrm{T,miss}} > 25.44 \text{ GeV}; |\eta^{\mathrm{W}}_{\ell}| < 2.4,$ such that $C_{\mathrm{th.}} = 1$ at LO.

Interpretation

- These results are **estimates** of impact of QCD-EW corrections on W-mass measurements at the LHC.
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W-mass measurements.
- Further investigations are essential:

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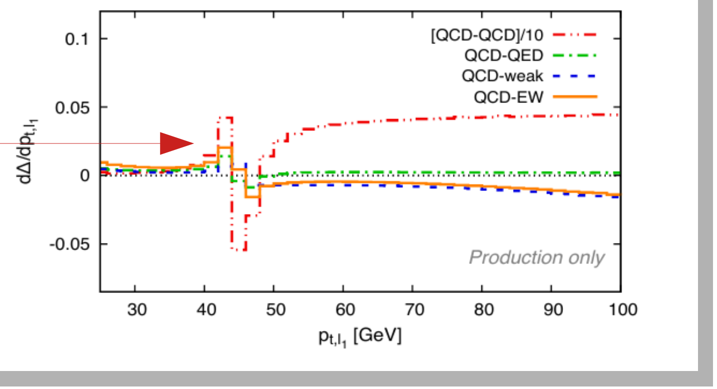
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W-mass measurements.
- Further investigations are essential:
  - What is the impact when using the full transverse momentum spectrum?
    - Cannot be evaluated using purely fixed-order results due to Sudakov shoulder.
    - Need to include multiple photon radiation.
    - Match photon shower to QCDxEW?
  - Can we gain further insight from using higher moments of lepton distribution?



[Buccioni, Caola, Delto, Jaquier, Melnikov, R.R. (2020)]

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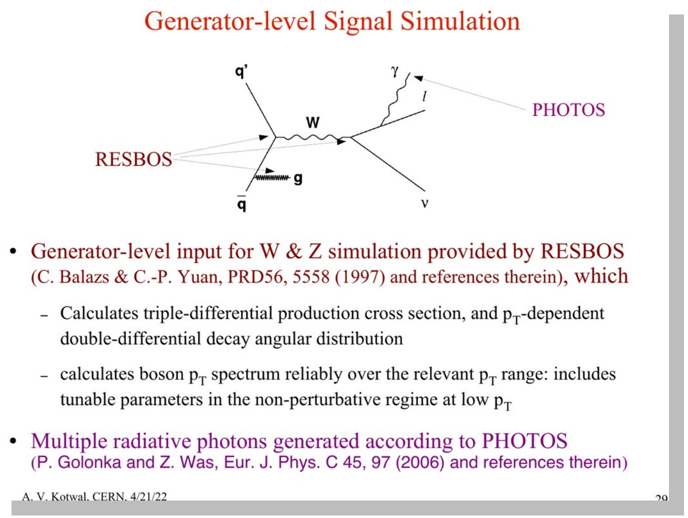
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W-mass measurements.
- Further investigations are essential:
  - How well are these captured with standard experimental simulation tools?
    - (Some) Effects of simultaneous QCD and EW radiation included in experimental analyses through e.g. `RESBOS+PHOTOS`.
    - Include multiple emissions but miss virtual contributions.
    - Missing effects accounted for in uncertainties?



[A. V. Kotwal, CERN EP Seminar 21/4/2022]

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- Further investigations are essential:
  - What is the impact on other observables?
  - ...

Dialogue between theorists and experimentalists is crucial!

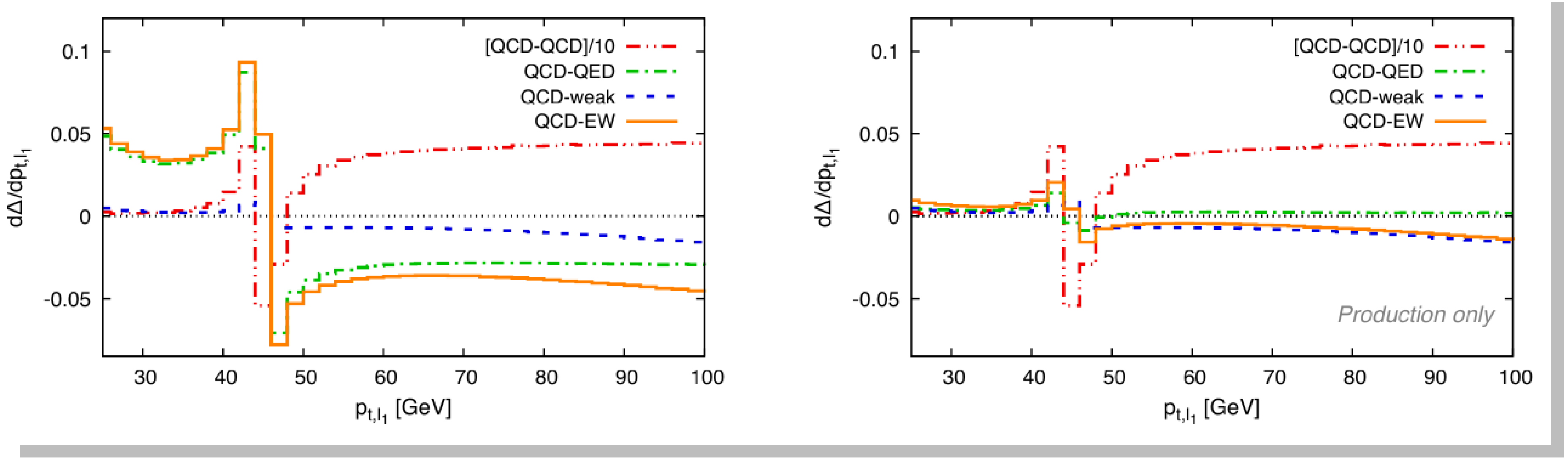
Conclusions

- Calculation of mixed QCD-EW corrections to onshell vector boson production using nested soft-collinear subtraction scheme.
- Analogous calculation exists for dilepton production.
- Estimated impact on measurement of W-mass at LHC ~ 10 MeV.
  - Looked at decorrelation in ratios of average lepton transverse momentum.
  - Strongly cut-dependent.
  - Potentially relevant for target uncertainty of 0.1 per mille.
  - More refined analysis is required, and discussions are encouraged!

THANK YOU FOR YOUR ATTENTION

BACKUP SLIDES

QCD-EW corrections to Z boson production



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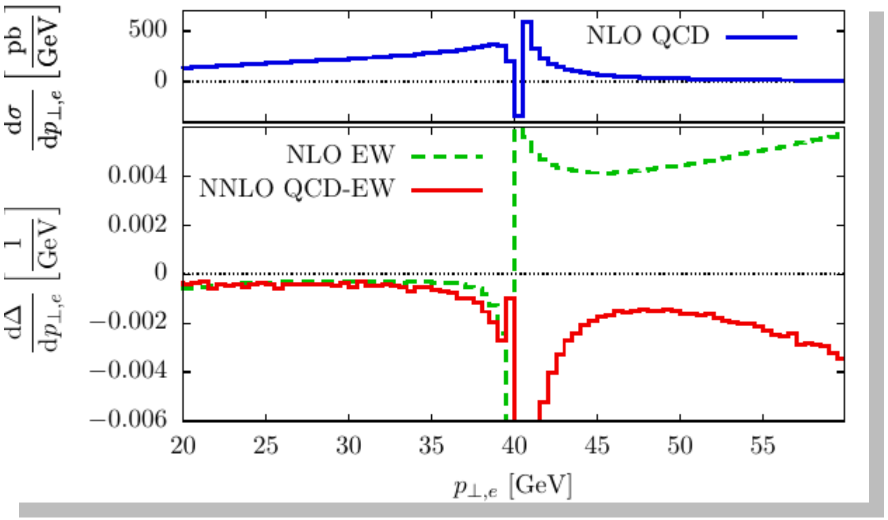
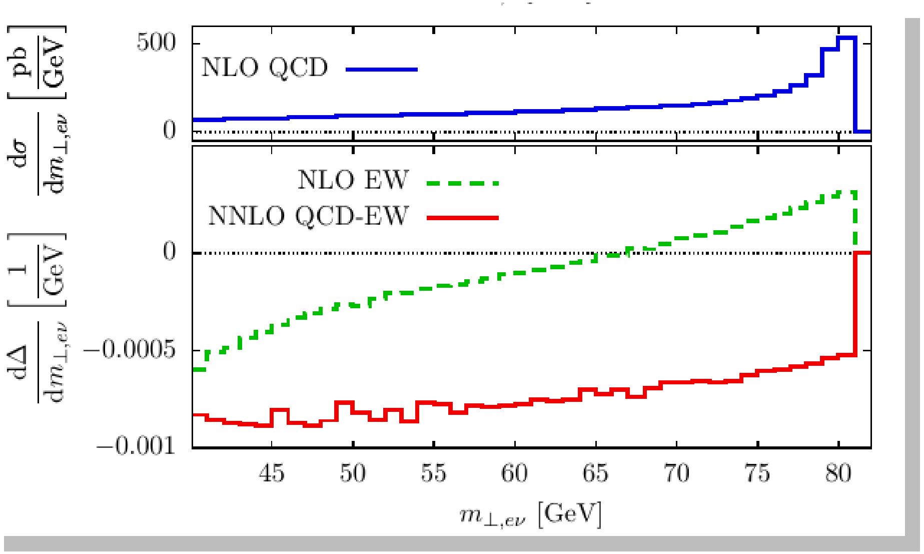
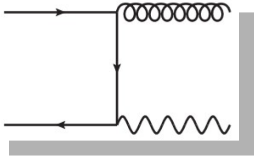




Infrared singularities in QCD

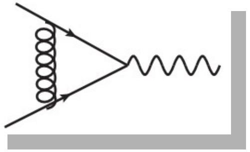
Higher order corrections contain IR singularities from soft and/or collinear radiation.

- Real corrections
  - Integrate over phase space of radiated parton:



$$\longrightarrow \int |\mathcal{M}|^2 F_J d\phi_g \text{ diverges}$$

- Virtual corrections
  - Explicit IR singularities from loop integration



$$\longrightarrow \mathcal{M}_{1-\text{loop}} = \frac{c_{-2}}{\epsilon^2} + \frac{c_{-1}}{\epsilon} + c_0$$

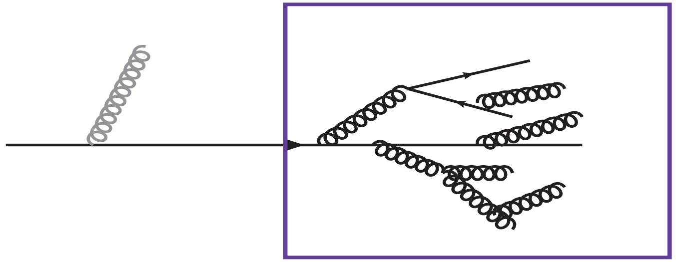
- Singularities unphysical, guaranteed to cancel in sum (KLN theorem).
- Cancellation only manifest after integrating over full phase space of emitted parton:

  → lose kinematic information.

Nested soft-collinear subtractions

[Caola, Melnikov, R.R. (‘17)]
[Caola, Melnikov, R.R. (‘19); Asteriadis, Caola, Melnikov, R.R. (‘19)]
[Delto, Frellesvig, Caola, Melnikov (‘18); Delto, Melnikov (‘19)]

- Extension of FKS subtraction to NNLO.
- Exploits color coherence of onshell, gauge-invariant amplitudes
  - Used in resummation & parton showers; not manifest in subtractions.



- Soft gluon cannot resolve details of collinear splittings; only sensitive to total color charge.

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Nested soft-collinear subtractions

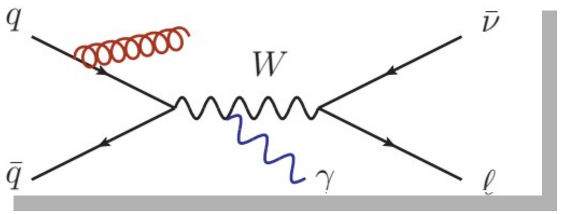
- No overlap between soft and collinear limits – treated independently:
  - Energies and angles decouple.
  - Regularize soft singularities first, then collinear singularities – iterative subtraction of divergences.
  - Overlapping soft singularities separated by energy ordering.
  - Overlapping collinear singularities separated using partitioning and sectoring of phase space. [Czakon ('10, '11)]
    - Natural splitting by rapidity.

- Straightforward adaptation for mixed QCD-EW singularities.

QCD-EW corrections to onshell vector boson production

Consider onshell vector boson production $pp \to V \to \ell\bar{\ell}$

- Z production: subtractions proceed as "abelianized NNLO QCD".
- W production: qualitatively new feature – photon radiated off W.
- Collinear limits regulated by W-mass, but soft limit is singular.
- Changes form of eikonal function in soft limit:



Soft gluon → $$\mathrm{Eik}_g(p_1,p_2;p_g) = \frac{2C_F(p_1\cdot p_2)}{(p_1\cdot p_g)(p_2\cdot p_g)}$$

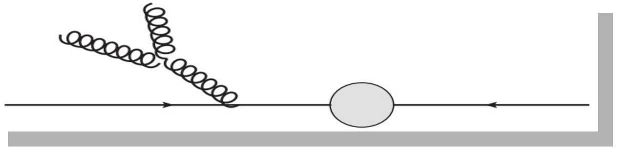
Soft photon → $$\mathrm{Eik}_\gamma(p_1,p_2,p_W;p_\gamma) = Q_u Q_d \frac{2(p_1\cdot p_2)}{(p_1\cdot p_\gamma)(p_2\cdot p_\gamma)} - Q_W^2 \frac{p_W^2}{(p_W\cdot p_\gamma)^2} + Q_W\left(Q_u \frac{2(p_W\cdot p_1)}{(p_W\cdot p_\gamma)(p_1\cdot p_\gamma)} - Q_d \frac{2(p_W\cdot p_2)}{(p_W\cdot p_\gamma)(p_2\cdot p_\gamma)}\right)$$

➡ More complicated function, but method is conceptually unchanged!

QCD-EW corrections to onshell vector boson production

Can make subtraction scheme simpler:

- NNLO QCD: soft limits of gluons overlap → introduced energy ordering.
- Mixed QCD-EW: soft limits of gluons and photons are independent → no energy ordering needed.
- Soft subtraction: iterated NLO-like soft limits.

- Genuine NNLO-like singularities in collinear limits → require phase-space partitioning and sectoring.
- Fewer collinear limits, e.g.  disappears.

→ Fewer sectors required.

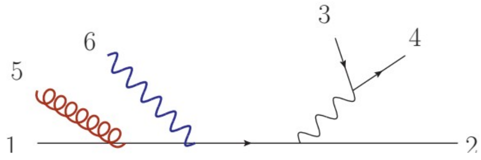
Phase space partitioning for dilepton production

$$q(p_1)\bar{q}(p_2) \to e^-(p_3)e^+(p_4)g(p_5)\gamma(p_6)$$

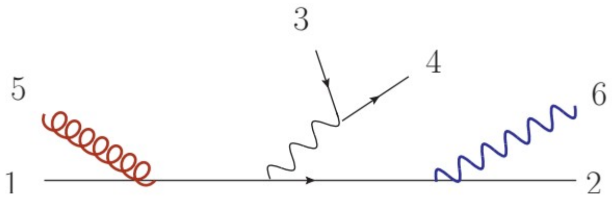
- Partitioning:

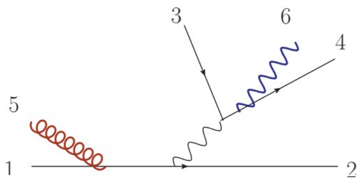
$$1 = w^{15,16} + w^{25,26} + w^{15,26} + w^{16,25} + w^{15,36} + w^{15,46} + w^{25,36} + w^{15,46}$$

- Triple collinear sectors



- Double collinear sectors





- Additional partitions have only double collinear limits
  - ~ NLO x NLO – simple!

Phenomenological results: Fiducial cross section

LO — Corrections $\sim \mathcal{O}(\alpha_s^i \alpha^j)$

σ[fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	−49.907	44.60	−16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		−32.66	1.03
$q\gamma$			−0.305		−0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	−47.046	13.88	−15.71

- LHC 13.6 TeV
- `NNPDF31_nnlo_as_0118_luxqed`
- G_μ input scheme for EW parameters.
- Massless leptons, clustered with photons if $\Delta R_{\ell\gamma} < 0.1$ ("lepton jets")
- $\mu_R = \mu_F = \mu = m_{\ell\ell}/2$
- $m_{\ell\ell} > 200$ GeV

  $p_{T,\ell^\pm} > 30$ GeV

  $|y_{\ell^\pm}| < 2.5$

$$\sqrt{p_{T,\ell^+} p_{T,\ell^-}} > 35 \text{ GeV}$$

Phenomenological Results: Fiducial cross section

- Fiducial cross section to NNLO QCD + NLO EW:

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%}\ \mathrm{fb}$$

- Theoretical uncertainty:
  - Vary scale μ by factor of 2 in either direction.
  - Change input scheme for EW parameters to $\alpha(m_Z)$-scheme.
  - Take envelope of these results.
- Mixed QCD-EW corrections (~ -1%) comparable to theoretical uncertainty.
- Including mixed QCD-EW corrections decreases theoretical uncertainty (mainly through decreasing dependence on EW input scheme)

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0\%}\ \mathrm{fb}$$

(*) Uncertainties from pdfs not included

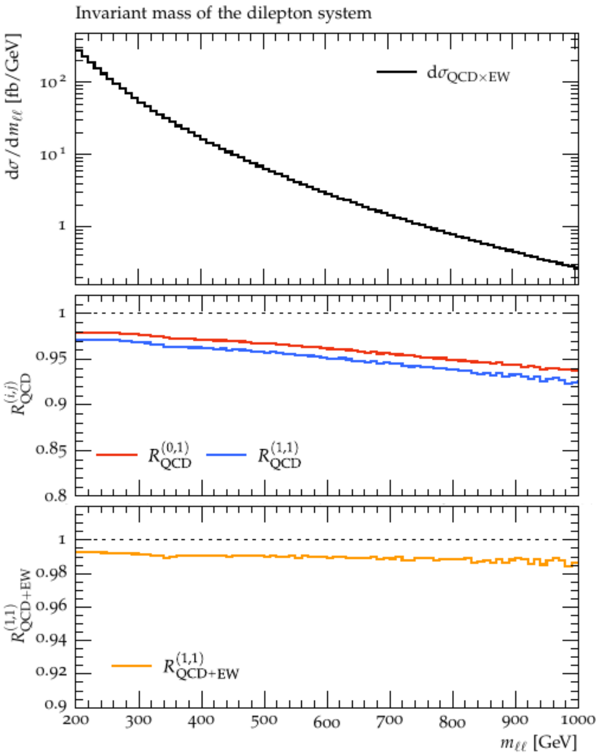
Phenomenological Results: Cross sections in mass windows

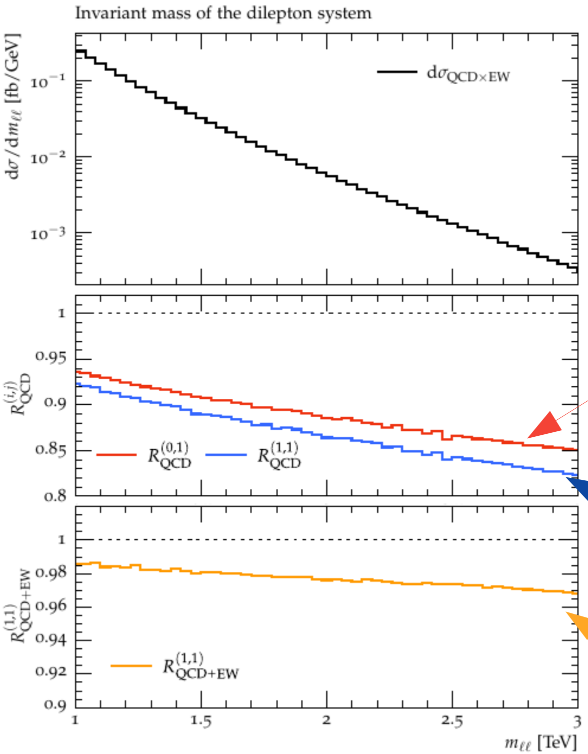
- At high energies, EW corrections dominated by universal Sudakov logarithms.
- Look at fiducial cross section in 4 mass windows:

$$\Phi^{(1)}: \quad 200 \text{ GeV} < m_{\ell\ell} < 300 \text{ GeV},$$
$$\Phi^{(2)}: \quad 300 \text{ GeV} < m_{\ell\ell} < 500 \text{ GeV},$$
$$\Phi^{(3)}: \quad 500 \text{ GeV} < m_{\ell\ell} < 1.5 \text{ TeV},$$
$$\Phi^{(4)}: \quad 1.5 \text{ TeV} < m_{\ell\ell} < \infty.$$

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma^{(1,1)}_{\text{fact.}}$	$\sigma_{\text{QCD}\times\text{EW}}$
$\Phi^{(1)}$	1169.8	254.3	-30.98	10.18	-10.74	-6.734	$1392.6^{+0.75\%}_{-0\%}$
$\Phi^{(2)}$	368.29	71.91	-11.891	2.85	-4.05	-2.321	$427.1^{+0.41\%}_{-0.02\%}$
$\Phi^{(3)}$	82.08	14.31	-4.094	0.691	-1.01	-0.7137	$91.98^{+0.22\%}_{-0.14\%}$
$\Phi^{(4)} \times 10$	9.107	1.577	-1.124	0.146	-0.206	-0.1946	$9.500^{+0\%}_{-0.97\%}$

Phenomenological results: Distributions





$$R^{(0,1)}_{\mathrm{QCD}} = \frac{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)}}{\sigma^{(0,0)} + \delta\sigma^{(1,0)}}$$

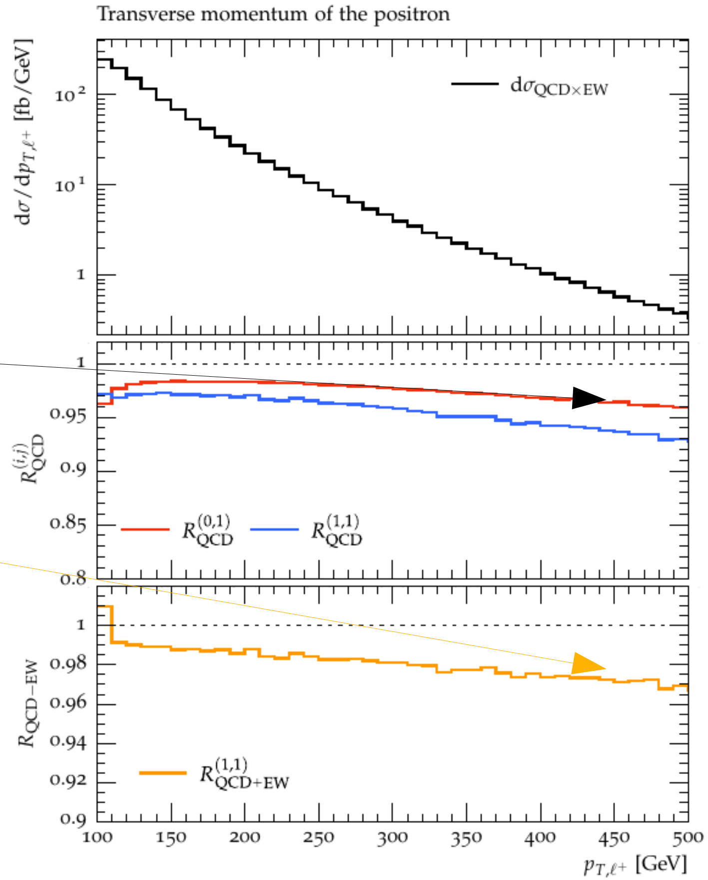
$$R^{(1,1)}_{\mathrm{QCD}} = \frac{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(1,1)}}{\sigma^{(0,0)} + \delta\sigma^{(1,0)}}$$

$$R^{(1,1)}_{\mathrm{QCD+EW}} = R^{(1,1)}_{\mathrm{QCD}} / R^{(0,1)}_{\mathrm{QCD}} = \frac{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(1,1)}}{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)}}$$

Phenomenological results: Distributions

Similar pattern for p_{T,ℓ^+} :

- NLO EW and QCD-EW corrections become more important at high transverse mass.
- QCD-EW corrections have slightly stronger dependence on transverse momentum compared to NLO EW corrections.
- Reach ~ -3% at $p_{T,\ell^+} \simeq 500\ \mathrm{GeV}$



Phenomenological results: Distributions

- NLO EW corrections to angular and rapidity distributions show minor shape changes.
- Mixed QCD-EW corrections very flat.