

Monte Carlo event generators and electroweak corrections for charged-current Drell-Yan

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Theory predictions for DY

From the theory point of view, DY is one of the hadron collider processes known with the highest accuracy:

- FO QCD corrs up to N³LO differential (X. Chen et al. 2205.11426)
- Resummation of ISR QCD logs up to N³LO+N³LL (T. Neumann et al. 2207.07056, X. Chen et al. 2203.01565)
- Full mixed QCD-EW corrections at ($\alpha\alpha_S$) (R. Bonciani et al. 2106.11953, F. Buccioni et al. 2203.11237)

(*) just citing some very recent publications: NOT complete biblio

Event generators for MC: accuracy

Event generators reach lower accuracy compared to fixed-order (or FO+resummation)

- up to NNLO-QCD+QCD-PS: UN²LOPS (1405.3607), MiNNLO (1908.06987), GENEVA (1311.0286) methods
- up to NLO-QCD+NLO-EW matched to QCD and EW PS (POWHEG framework 1202.0465, 1302.4606, 1906.11569)

here QCD (and QED) effectively perform LL^(*) resummation of QCD (QED) logs

(*) ongoing studies to assess *actual* accuracy of different PS implementations available on the market

Tuned comparison of tools for CC DY in 1606.02330

Table 3 Tuned comparison of total cross sections (in pb) for $pp \rightarrow W^+ \rightarrow l^+ \nu_l + X$ at the 8 TeV LHC, with ATLAS/CMS cuts and *bare* leptons. (×) indicates that although POWHEG_BW provides NLO EW results also for *bare* electrons, due to the smallness of the electron mass

it would require very high-statistics to obtain per-mille level precision. Thus, we recommend to use the *bare* setup in POWHEG_BW only for muons

Code	LO	NLO QCD	NLO EW μ	NLO EW e
HORACE	2897.38(8)	×	2988.2(1)	2915.3(1)
WZGRAD	2897.33(2)	×	2987.94(5)	2915.39(6)
RADY	2897.35(2)	2899.2(4)	2988.01(4)	2915.38(3)
SANC	2897.30(2)	2899.9(3)	2987.77(3)	2915.00(3)
DYNNLO	2897.32(5)	2899(1)	×	×
FEWZ	2897.2(1)	2899.4(3)	×	×
POWHEG-w	2897.34(4)	2899.41(9)	×	×
POWHEG_BMNNP	2897.36(5)	2899.0(1)	2988.4(2)	2915.7(1)
POWHEG_BW	2897.4(1)	2899.2(3)	2987.7(4)	(×)

event generators including NLO EW: HORACE (NO QCD) and the two implementations in POWHEG

NLO EW corrections at FO available in a bunch of different tools (mainly thanks to automation), for instance

MCFM, Madgraph_aMG5, Sherpa+RECOLA

Resonance-improved treatment of FSR QED radiation implemented in W_ew-BMNNP

Independent implementation of CC DY at NLO QCD+NLO EW plus matching to QCD and QED PS in 1612.04292

very first steps towards inclusion of NLO EW (and QED PS matching) in MC generators at NNLO QCD+QCD PS

crucial to describe FSR effects

ingredient in matched calculations

- PHOTOS
- PYTHIA

more recently

- HERWIG
- SHERPA

QED PS for DY available in HORACE

Comparisons for NC DY (CERN EWWG)

focus on pure weak corrections, FO, context: $\sin\theta_W^{\text{eff}}$ measurement
codes involved

- (G_μ, M_W, M_Z)

MCSANC, POWHEG_EW, RADY, WZGRAD2

- (α_0, M_W, M_Z)

MCSANC, POWHEG_EW, RADY, WZGRAD2

- $(G_\mu, \sin^2\vartheta_{\text{eff}}^\ell, M_Z), (\alpha_0, \sin^2\vartheta_{\text{eff}}^\ell, M_Z)$

POWHEG_EW, RADY

- (α_0, G_μ, M_Z)

DIZET (used in TAUSPINNER+DIZET and KKMC_HH)

* POWHEG_EW=Z_ew-BMNNPV

Event gen's at NLO QCD+ EW matched to QCD and QED PS

	Z_ew-BMNPNV	W_ew-BMNPNP	VV_dec_ew	vbs-ssww-nloew
Process	$pp \rightarrow l^+ l^-$	$pp \rightarrow l\nu$	$pp \rightarrow 4l/2l2\nu/3l\nu$	$pp \rightarrow l^+ \nu l^- \nu jj$
FS leptons (*)	massive ($l = e, \mu$)	massive ($l = e, \mu$)	massless ($l = e, \mu, \tau$)	massless ($l = e, \mu, \tau$)
Identical l			in progress (§)	in progress (§)
Model	SM	SM	SM (**)	SM (**)
POWHEG-BOX-	V2	V2	RES	RES
Resonance-aware PS matching (RES)	Yes	Yes	Yes	Yes
Dedicated PS interface	Yes (Py8, Photos)	Yes (Py8, Photos)	Yes (Py8) (¶)	Yes (Py8) (¶)
Matrix elements	internal	internal	Reco1a2	Reco1a2
PHPS restrictions	None (‡)	None	None (‡)	VBS
Approx. in Mat.els	None	None	None	None (‡)
NLO QCD	Yes	Yes	Yes	No (‡)
NLO EW	Yes	Yes	Yes	Yes (‡)
Unstable Z/W	CMS/fact/pole (fix Γ)	CMS/CLA (fix Γ)	CMS (fix Γ)	CMS (fix Γ)
Renorm schemes	$G_\mu M_W M_Z$ (††) $\alpha_0 M_W M_Z$ $\alpha(M_Z) M_W M_Z$ $\sin\theta^{\text{eff}} M_Z G_\mu$ $\sin\theta^{\text{eff}} M_Z \alpha_0$	$G_\mu M_W M_Z$ $\alpha_0 M_W M_Z$	$G_\mu M_W M_Z$ $\alpha_0 M_W M_Z$ $\alpha(M_Z) M_W M_Z$	$G_\mu M_W M_Z$
γ -induced (††)	NLO (not on svn)	NLO (not on svn)	No	No

(*) massless: valid only for dressed lepton analyses.

(§) process-specific code is there, but fixes in the common POWHEG-BOX-RES code needed.

(**) generalization to BSM feasible if the corresponding Reco1a2 model file exists.

(¶) Photos interface can be developed upon request.

(‡) $M(l^+ l^-) > M(\text{cut})$ to avoid on-shell γ propagators at LO.

(†) considering only LO $\mathcal{O}(\alpha^6)$ (EW production) and NLO $\mathcal{O}(\alpha^7)$.

NLO+PS matching with EW corrections

- NLO EW corrections: $d\sigma = d\sigma_0 [1 + \delta_\alpha]$

- QED-PS: **all order γ radiation** in **leading log approx.**

$$d\sigma = d\sigma_0 \left[1 + \sum_{n=1}^{\infty} \delta'_{\alpha^n} \right]$$

- NLO EW+QED-PS: $d\sigma = d\sigma_0 \left[1 + \delta_\alpha + \sum_{n=2}^{\infty} \delta'_{\alpha^n} \right]$

matching replaces first PS radiation with NLO real radiation

- HORACE NLO EW+QED-PS: $d\sigma = d\sigma_0 \left[1 + \delta_\alpha + \sum_{n=2}^{\infty} \delta'_{\alpha^n} \right]$

- POWHEG NLO (QCD+EW)+(QCD+QED)-PS:

$$d\sigma = d\sigma_0 \left[1 + \delta_{\alpha_s} + \delta_\alpha + \sum_{m=1, n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n} \right]$$

POWHEG-BOX-V2 VS POWHEG-BOX-RES

POWHEG-BOX-V2

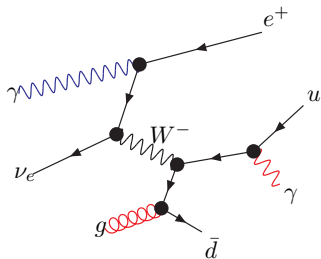
- try to generate one radiation from each α_r ($p_T^{\alpha_r}$)
- find the hardest radiation (p_T^{max})
- p_T^{max} is the starting scale of the PS

POWHEG-BOX-RES

- try to generate one radiation from each α_r ($p_T^{\alpha_r}$)
- for each resonance r , find the hardest radiation emitted by the resonance ($p_{T,r}^{max}$)
- $p_{T,r}^{max}$ is the starting scale of the PS radiation from r

- POWHEG-BOX-RES (like) events contain up to one radiation from each resonance
- PS radiation from each resonance must be vetoed independently
- dedicated interface to PS unavoidable (no LHE accord for multiple scales, scalup works for one radiation only)

POWHEG-BOX-RES (like) treatment of resonances



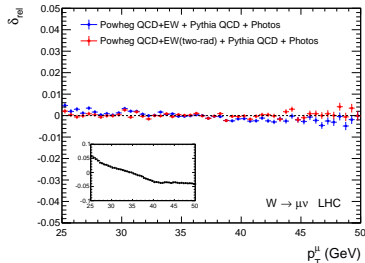
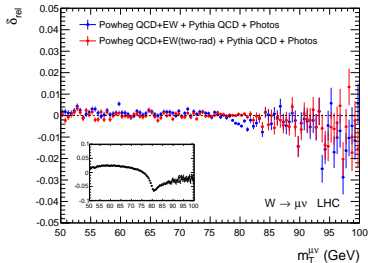
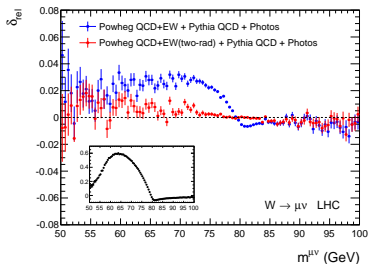
- 3 radiation regions:
QCD ISR, QED ISR, QED FSR
- 2 resonances: IS, W

The events contain up to 2 radiations:

- 1 one ISR QED or QCD radiation setting the scale of the IS shower
- 2 one FSR QED radiation setting the scale of the FS shower

POWHEG-BOX-RES (like) treatment of resonances (2)

non negligible effect for observables sensitive to QED FSR corrections but rather insensitive to QCD corrections



Theoretical uncertainties in M_W measurement: strategy

Impact of different EW effects (an theory uncertainties from weak and mixed QCD-EW corrs) on W -mass measurement in: arXiv:1612.02841 (W_ew-BMNNP)

1 pseudodata

- Monte Carlo samples with a given theoretical accuracy
- play the role of experimental data

2 templates

- MC samples at NLO QCD+QCD-PS (or LO) generated for different values of M_W
- will be fitted to the pseudodata

3 $\Delta M_W = M_W(\text{pseudodata}) - M_W(\text{fit output})$

Theory uncertainties in M_W measurement: event generators

- HORACE (Carloni Calame et al. hep-ph/0303102, hep-ph/050626)
 - MC event generator for DY
 - can generate events at NLO EW+QED-PS, and NLO EW+QED-PS+unresolved l^+l^- radiation

- POWHEG-BOX-V2/W_ew-BMNNP (Barze et al. arXiv:1202.0465)
 - MC event generator for charged DY
 - can generate events at NLO QCD+QCD-PS and NLO (QCD+EW)+(QCD+QED)-PS
 - relies on external shower MC programs (i.e. PYTHIA, PYTHIA+PHOTOS)

Theory unc. in M_W measurement: shower MC tools)

- **PYTHIA** (Sjostrand et al. hep-ph/0603175; arXiv:0710.3820)
 - general purpose shower MC generator
 - can generate multiple QCD and QED radiation
 - used for ISR multiple QCD (and QED) radiation AND non-perturbative QCD effects
 - in some runs used for QED FSR (see later)
- **PHOTOS** (Barberio et al. CPC 66 (1991), CPC 79 (1994), Golonka et al. hep-ph/0506026)
 - general purpose shower MC generator
 - can generate multiple QED radiation off fermions (from W decay)
 - in some runs used for QED FSR (see later)
- **HORACE** (has its own implementation of QED PS algorithm)

Mixed QCD-EW corrections (1)

$pp \rightarrow \mu^+ \nu_\mu$, fit to $M_T(\mu^+ \nu_\mu)$

	Templates	Pseudodata	M_W shifts (MeV)
1	LO	POWHEG(QCD) NLO	56.0 ± 1.0
2	LO	POWHEG(QCD)+PYTHIA(QCD)	74.4 ± 2.0
3	LO	HORACE(EW) NLO	-94.0 ± 1.0
4	LO	HORACE (EW,QEDPS)	-88.0 ± 1.0
5	LO	POWHEG(QCD,EW) NLO	-14.0 ± 1.0
6	LO	POWHEG(QCD,EW) two-rad+PYTHIA(QCD)+PHOTOS	-5.6 ± 1.0

	samples	M_W shift (MeV)
$\sum_{m=1, n=1}^{\infty} \delta'_{\alpha_s^m} \alpha^n + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n}$	[6]-[5]	8.4 ± 1.4 MeV
$\sum_{m=2}^{\infty} \delta'_{\alpha_s^m}$	[2]-[1]	18.4 ± 2.2 MeV
$\sum_{n=2}^{\infty} \delta'_{\alpha^n}$	[4]-[3]	6.0 ± 1.4 MeV

$$\sum_{m=1, n=1}^{\infty} \delta'_{\alpha_s^m} \alpha^n = ([6]-[5]) - ([2]-[1]) - ([4]-[3]) = -16.0 \pm 3.0 \text{ MeV}$$

in agreement with the results of Dittmaier et al. 1511.08016 for the full $\mathcal{O}(\alpha\alpha_s)$ corrections in pole approx. (-14 MeV)

Mixed QCD-EW corrections (2)

- mixed QCD-EW corrections from POWHEG $\sum_{m=1, n=1}^{\infty} \delta'_{\alpha_s^m} \alpha^n$
 - factorized approx
 - spurious H.O. effects ($PS \times \Delta \times \overline{B}$)
- Full $\mathcal{O}(\alpha\alpha_s)$ (arXiv:2102.12539, arXiv:2201.01754): it would be nice to study their impact on M_W extraction
 - per se
 - to assess the uncertainties coming from the factorized approach in MC generator at NLO QCD+NLO EW with PS matching (e.g. W_ew-BMNNP in POWHEG)

Theory uncertainties from QED PS (1)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD _{PS}			$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu(\text{dres})$	
Pseudodata accuracy			QED FSR		M_T	p_T^ℓ
			M_T	p_T^ℓ	M_T	p_T^ℓ
1	NLO-QCD+(QCD+QED) _{PS}	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED) _{PS}	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED) _{PS} two-rad	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED) _{PS} two-rad	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2

- difference between QED-PS in PHOTOS and PYTHIA at $\mathcal{O}(\alpha)$

- PHOTOS $\propto \frac{1}{1-\beta\cos\theta_{l\gamma}}$

- PYTHIA-QED $\propto \frac{1}{p_T}$

- 32 MeV (p_T)/ 7 MeV (M_T) effect for bare μ

Theory uncertainties from QED PS (2)

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- first QED radiation generated by POWHEG
- difference between QED-PS in PHOTOS and PYTHIA at $\mathcal{O}(\alpha^2)$

Theory uncertainties from QED PS (3)

Theory uncertainties from QED PS estimated from the difference in the shifts from PYTHIA and PHOTOS

- might be an overestimate (photon spectrum suggests that PHOTOS is more accurate)
- comparison should be extended to other QED PS: HERWIG, SHERPA, ...
- how would the shifts change if a QED PS beyond LL accuracy was used?

non-log QED, weak and mixed EW-QCD contributions (1)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$			M_W shifts (MeV)			
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- impact of non-log QED, weak and mixed EW-QCD contributions
- different effects for PHOTOS or PYTHIA (different non-log QED terms)
- more stable results for M_T (less sensitive to mixed EW-QCD corrections)

non-log QED, weak and mixed EW-QCD contributions (2)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$			M_W shifts (MeV)			
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uncertainties from
non-log QED,
weak,
mixed QCD-EW corr.

		ΔM_W (MeV) bare muons	
QED FSR model		M_T	p_T^ℓ
LHC	PYTHIA	$+6.2 \pm 0.8$	$+29 \pm 4$
	PHOTOS	-0.6 ± 0.8	-2 ± 4

non-log QED, weak and mixed EW-QCD contributions (3)

- QED, WEAK, and mixed effects inevitably have an interplay with IS QCD effects (e.g. $PS \times \overline{B}$)
- in our simulation we only used PYTHIA8 for ISR QED and QCD shower and non-perturbative effects with a default PYTHIA tuning
- how do the shifts change if we use another shower MC, say HERWIG?
- how do the estimates change when changing the PYTHIA tune? (having in mind the ATLAS procedure of tuning PYTHIA to reproduce the $Z p_T$ data)
- how do the shifts change if we use another description of IS effects, say for instance RESBOS like in TEVATRON analyses?

- Main progress in FO calculations for DY including EW effects: exact $\alpha\alpha_S$ corrections
- On the MC side, most accurate generators including EW effects still NLO QCD+NLO EW plus matching to QCD and QED PS
 - Some refinements of the existing codes
 - very first steps towards inclusion of NLO EW (and QED PS matching) in MC generators at NNLO QCD+QCD PS
- impact of weak, non-log QED, and mixed QCD-EW corrs (approximated, factorised assumption) in 1612.02841, but the estimate could be generalized in many ways

Backup Slides

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$			M_W shifts (MeV)			
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■ dressed e : recombine γ with e if $\Delta R(\gamma e) < 0.1$

■ bare μ : corrections enhanced by logs $\alpha \log\left(\frac{m_\mu^2}{Q^2}\right)$

Theory uncertainties from QED PS (1)

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Theory uncertainties from QED PS (2)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD _{PS}			$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu(\text{dres})$	
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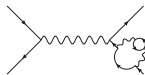
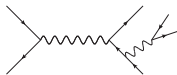
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Higher order effects: pair radiation (1)



same order as 2 γ radiation

Unresolved pair radiation can be included in the Sudakov through the running ¹ of α

$$\alpha \implies \alpha(s) = \begin{cases} \alpha / \left(1 - \frac{\alpha}{3\pi} \ln \frac{s}{m_e^2} \right) & \text{electrons only} \\ \alpha / \left(1 - \frac{\alpha}{3\pi} \ln \frac{s}{m_e^2} - \theta(s - m_\mu^2) \frac{\alpha}{3\pi} \ln \frac{s}{m_\mu^2} \right) & \text{electrons + muons} \end{cases}$$

$pp \rightarrow W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo-data accuracy		M_W shifts (MeV)			
		$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu$	
		M_T	p_T^ℓ	M_T	p_T^ℓ
1	HORACE FSR-LL	-89 ± 1	-97 ± 1	-179 ± 1	-195 ± 1
2	HORACE FSR-LL + Pairs	-94 ± 1	-102 ± 1	-182 ± 2	-199 ± 1

$\Delta M_W(\mu^+ \nu) \sim 5 \pm 1$ MeV (from μ) and $\sim 3 \pm 2$ MeV (from e)

¹alternative implementation: N. Davidson et al arXiv:1011.0937

Higher order effects: pair radiation (2)

$pp \rightarrow W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo-data accuracy		M_W shifts (MeV)			
		$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu$	
		M_T	p_T^ℓ	M_T	p_T^ℓ
1	HORACE FSR-LL	-89 ± 1	-97 ± 1	-179 ± 1	-195 ± 1
2	HORACE FSR-LL + Pairs	-94 ± 1	-102 ± 1	-182 ± 2	-199 ± 1

- pair corrections estimated using HORACE: no interplay with QCD effects possible
- similar strategy can be implemented in POWHEG (actually is already there...)
 - ⇒ one could repeat the study within POWHEG-BOX-V2/W_ew-BMNNP to see what are the changes in the shifts due to the presence of QCD pert and non-pert effects

NNLO uncertainty: input parameter schemes (1)

scheme choice = choice of the 3 independent EW params

- all choices formally equivalent at a given order in P.T.
- numerical differences in predictions from missing H.O. terms

⇒ difference in predictions from different schemes at a given order can be taken as an estimate of the theory uncertainty from missing H.O.

However....

- not conclusive: basically impossible to consider all possible choices of IPS
- might be over-estimate: we might consider some schemes as “more precise” than others
 - parametric uncertainties
 - perturbative convergence
 - ...

NNLO uncertainty: input parameter schemes (2)

- $\alpha(0)$, M_W and M_Z
- G_μ , M_W and M_Z to be preferred in the CC DY
- we can define

$$\begin{aligned}\alpha_\mu^{tree} &\equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2 \vartheta \\ \alpha_\mu^{1l} &\equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2 \vartheta (1 - \Delta r)\end{aligned}$$

The expressions for the cross section differ at $\mathcal{O}(\alpha^2)$

$$\begin{aligned}\alpha_0 &: \quad \sigma = \alpha_0^2 \sigma_0 + \alpha_0^3 (\sigma_{SV} + \sigma_H), \\ G_\mu I &: \quad \sigma = (\alpha_\mu^{tree})^2 \sigma_0 + (\alpha_\mu^{tree})^2 \alpha_0 (\sigma_{SV} + \sigma_H) - 2\Delta r (\alpha_\mu^{tree})^2 \sigma_0, \\ G_\mu II &: \quad \sigma = (\alpha_\mu^{1l})^2 \sigma_0 + (\alpha_\mu^{1l})^2 \alpha_0 (\sigma_{SV} + \sigma_H)\end{aligned}$$

NNLO uncertainty: input parameter schemes (3)

- potentially effects on M_W because of the different sharing among different photon multiplicities

$p\bar{p} \rightarrow W^+, \sqrt{s} = 1.96 \text{ TeV}$ Templates accuracy: LO		Input scheme	M_W shifts (MeV)	
Pseudodata accuracy			$W^+ \rightarrow \mu^+ \nu$	M_T
			p_T^ℓ	
1	HORACE NLO-EW	α_0	-101±1	-117±2
2		$G_\mu - I$	-112±1	-130±1
3		$G_\mu - II$	-101±1	-117±1
4	HORACE NLO-EW+QED-PS	α_0	-70±1	-81±1
5		$G_\mu - I$	-72±2	-83±1
6		$G_\mu - II$	-72±1	-82±2

- differences present at NLO, after matching with higher orders, become much smaller

$$\Delta M_W \sim 2 \text{ MeV} \pm 1 - 2 \text{ MeV}$$

NNLO uncertainty: input parameter schemes (4)

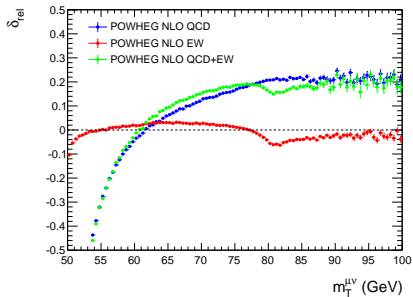
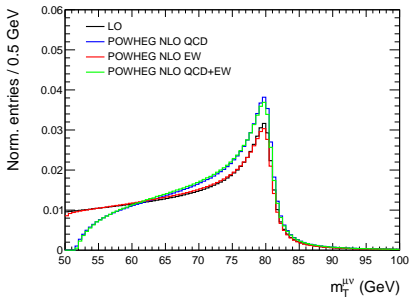
- uncertainties from IPS choice evaluated with HORACE: no interplay with QCD
- how do the shifts from different IPS change in the presence of QCD effects?

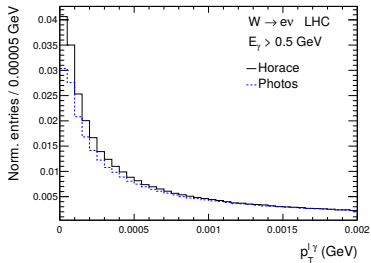
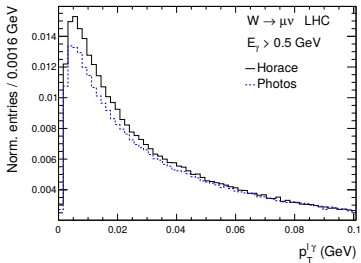
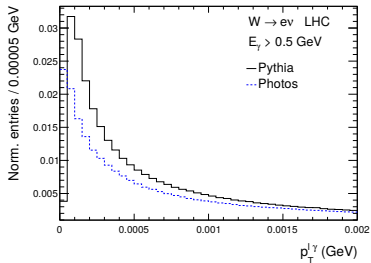
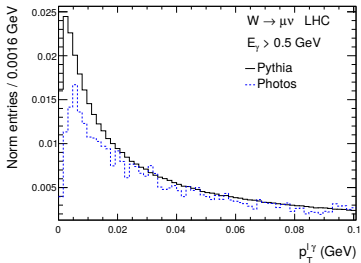
order	G_μ	$\alpha(0)$	$\delta(G_\mu - \alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

the LO + NLO-EW result would suffer of only 0.55% spread;

the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence ($\rightarrow 0.88\%$)

- one could estimate the H.O. corrections including the universal fermionic corrections connected to $\Delta\alpha$ and $\Delta\rho$





HORACE

$$d\sigma^\infty = F_{SV} \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=0}^n F_{H,i} \right) |\mathcal{M}_{n,LL}|^2 d\Phi_n$$

POWHEG

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) d\Phi_n \left\{ \Delta^{f_b}(\Phi_n, p_T^{\min}) \right. \\ \left. + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{\left[d\Phi_{rad} \theta(k_T - p_T^{\min}) \Delta^{f_b}(\Phi_n, k_T) R(\Phi_{n+1}) \right]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

taken from 1701.07240

W -boson charge Kinematic distribution	W^+		W^-		Combined	
	p_T^ℓ	m_T	p_T^ℓ	m_T	p_T^ℓ	m_T
δm_W [MeV]						
Fixed-order PDF uncertainty	13.1	14.9	12.0	14.2	8.0	8.7
AZ tune	3.0	3.4	3.0	3.4	3.0	3.4
Charm-quark mass	1.2	1.5	1.2	1.5	1.2	1.5
Parton shower μ_F with heavy-flavour decorrelation	5.0	6.9	5.0	6.9	5.0	6.9
Parton shower PDF uncertainty	3.6	4.0	2.6	2.4	1.0	1.6
Angular coefficients	5.8	5.3	5.8	5.3	5.8	5.3
Total	15.9	18.1	14.8	17.2	11.6	12.9

Table 3: Systematic uncertainties in the m_W measurement due to QCD modelling, for the different kinematic distributions and W -boson charges. Except for the case of PDFs, the same uncertainties apply to W^+ and W^- . The fixed-order PDF uncertainty given for the separate W^+ and W^- final states corresponds to the quadrature sum of the CT10nnlo uncertainty variations; the charge-combined uncertainty also contains a 3.8 MeV contribution from comparing CT10nnlo to CT14 and MMHT2014.