Electrons for the LHC
The LHeC/FCCeh and PERLE Workshop at Orsay

Exclusive processes with a leading neutron at the LHeC

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Based on PLB 752 (2016) 76, PRD 93 (2016) 054025, PRD 94 (2016) 014009, PRD 97 (2018) 074002 *and* PRD 103 (2021) 034021.

In collaboration with D. Spiering, F. S. Navarra, B. D. Moreira and F. Carvalho.

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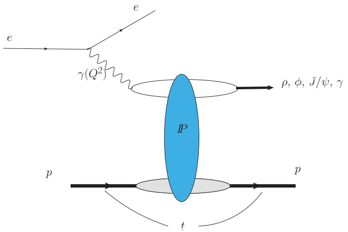
Definitions and Motivations

- Exclusive processes in *ep* collisions are characterized by the production of a given state $E = \rho, \phi, J/\Psi$, Υ and γ, an intact proton and a rapidity gap between these two systems .
- They are the most promising processes to probe the QCD dynamics at high energies and the spatial distribution of gluons inside the proton.



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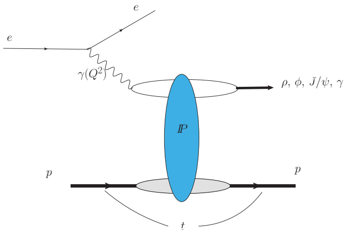

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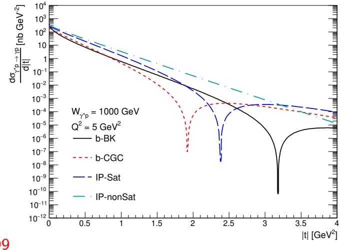

Bendova, Cepila, VPG and Sena Eur.Phys.J.C 82 (2022) 2, 99

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- In the last decades, the HERA *ep* collider has released very precise data about the leading neutron production, but the description of these data is still a theoretical challenge.
- The study of leading particle processes is one of goals of the future electron - hadron colliders (EIC, LHeC and EicC).

Goals

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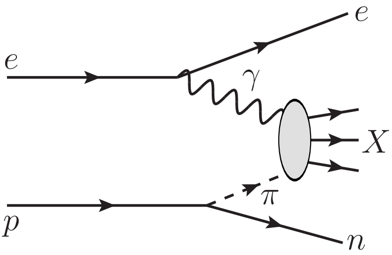
- Estimate the impact of non-linear effects.
- Predict the magnitude of cross sections for inclusive and exclusive processes with LN production in future *ep* colliders.

Inclusive and exclusive processes with a leading neutron

inclusive

$$e + p \rightarrow e + n + X$$



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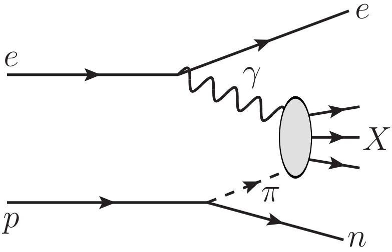

exclusive

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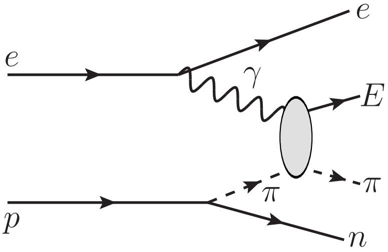

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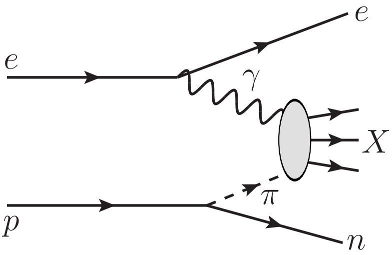

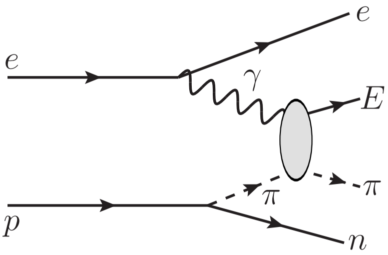

$$\frac{d^2\sigma(\hat{W}^2, Q^2, x_L, t)}{dx_L dt} = f_{\pi/p}(x_L, t)\sigma_{\gamma^*\pi}(\hat{W}^2, Q^2)$$

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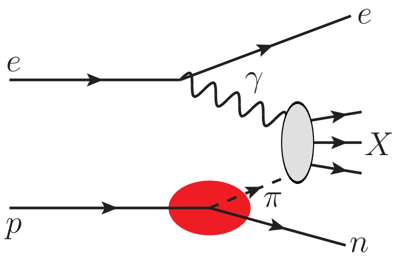

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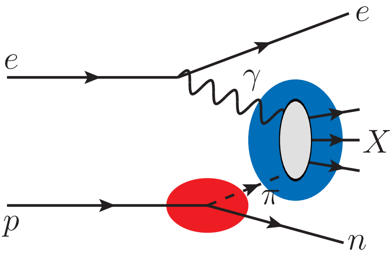

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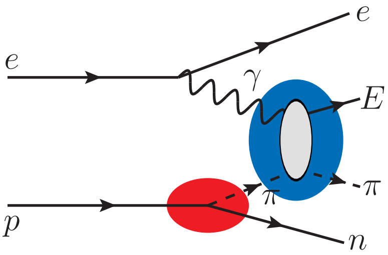

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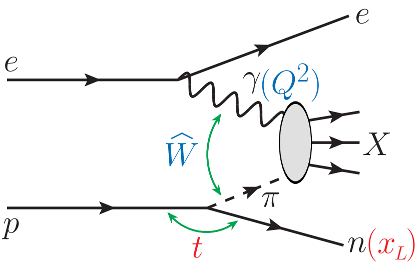

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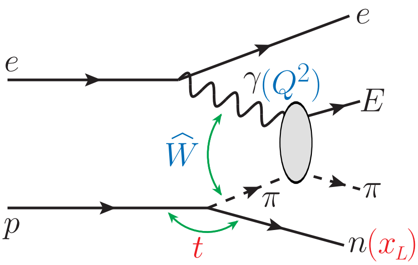

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Flux of pions / Pion splitting function

$$f_{\pi/p}(y) = \frac{1}{4\pi}\frac{g_{p\pi B}^2}{4\pi}\int_{-\infty}^{t_{min}} dt \frac{\mathcal{B}(t, m_p, m_B)}{(t - m_\pi^2)^2} y^{1-2t}[F(t)]^2$$

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Form factor: $F(t) = \exp\left[b(t - m_\pi)\right]$

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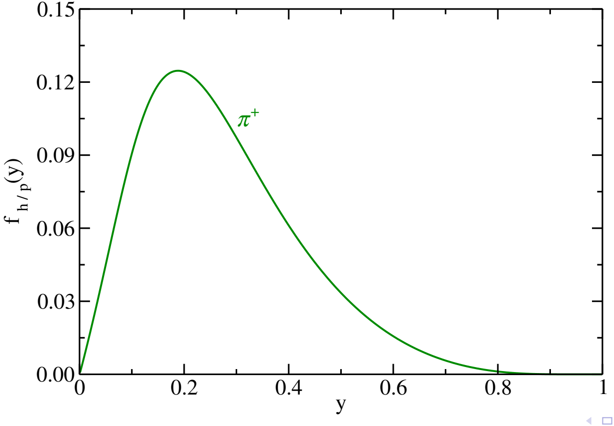

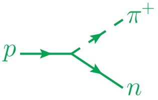

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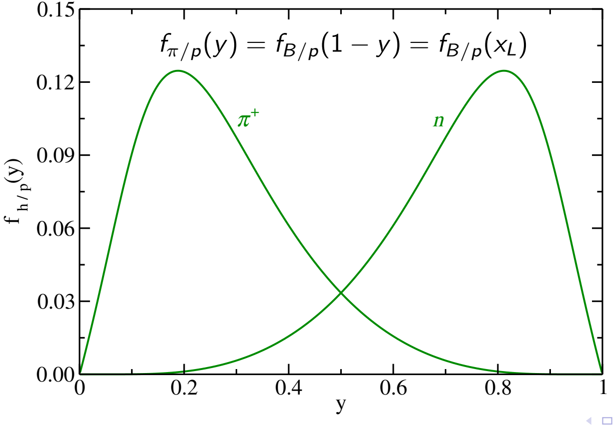

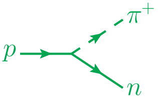

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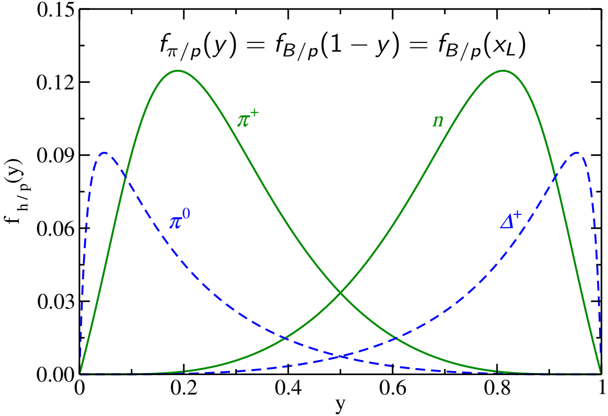

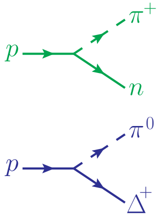

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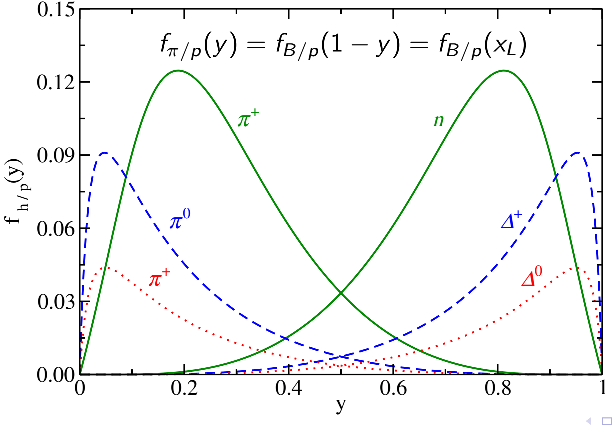
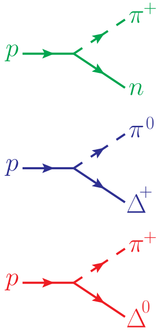
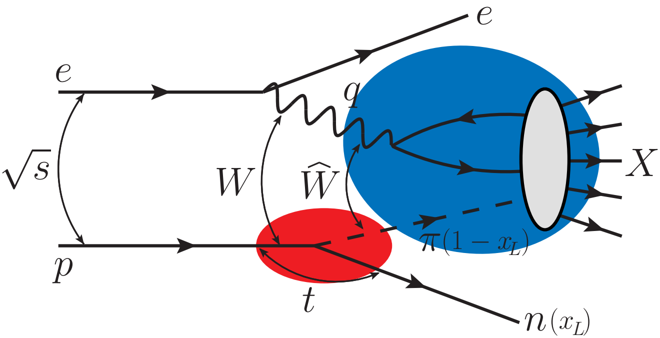




Leading neutron production in inclusive processes using the dipole formalism [1]

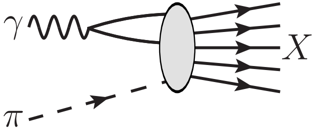


$$\frac{d^2\sigma(W,Q^2,x_L,t)}{dx_L dt} = f_{\pi/p}(x_L,t)\,\sigma^{inc}_{\gamma^*\pi}(\hat{W}^2,Q^2)$$

[1] VPG et al. PLB 752 (2016) 76

Photon-pion inclusive cross section

$$\sigma^{inc}_{\gamma^*\pi}(\hat{x}, Q^2) = \int dz \int d^2\vec{r} \sum_{L,T} \left|\psi_{L,T}(z, \vec{r}, Q^2)\right|^2 \sigma_{d\pi}(\hat{x}, \vec{r})$$



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Photon wave function:

$$\left|\psi_L(\vec{r}, Q^2)\right|^2 = \frac{3\alpha_{em}}{2\pi^2} \sum_f e_f^2 4Q^2 z^2 (1-z)^2 K_0^2(\epsilon r)$$

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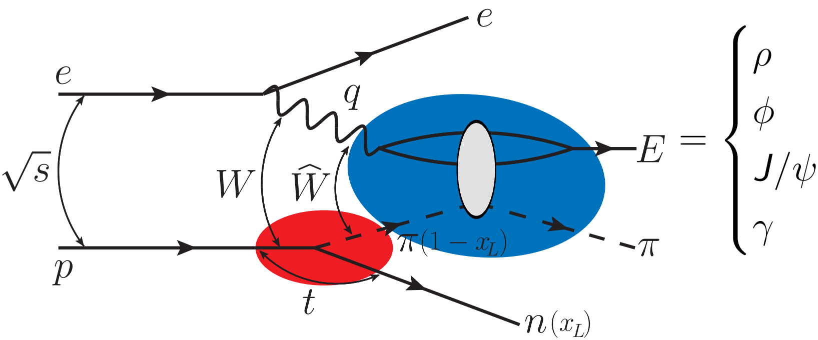

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[2] VPG et al. PRD 93 (2016) 054025

Photon-pion exclusive cross section

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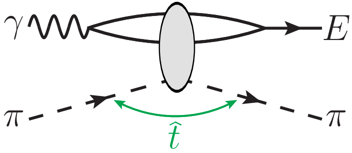

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$$\mathcal{A}_{L,T}^{\gamma^*\pi \to E\pi} = i \int dz\, d^2\vec{r}\, d^2\vec{b}\, e^{-i[\vec{b}-(1-z)\vec{r}]\cdot\vec{\Delta}} \left[\Psi_E^* \Psi(z, \vec{r}, Q^2)\right]_{L,T} 2\,\mathcal{N}^\pi(\hat{x}, \vec{r}, \vec{b})$$

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Overlap functions for Vector Mesons ($V = \rho,\ \phi,\ J/\psi$): $\quad \phi_{L,T} \equiv \phi_{L,T}(z, r)$

$$(\Psi_V^*\Psi)_L = \frac{\hat{e}_f e}{4\pi} \frac{N_c}{\pi} 2Qz(1-z)K_0(\epsilon r) \left[M_V \phi_L + \delta \frac{m_f^2 - \nabla_r^2}{M_V z(1-z)} \phi_L \right]$$

$$(\Psi_V^*\Psi)_T = \frac{\hat{e}_f e}{4\pi} \frac{N_c}{\pi z(1-z)} \left\{ m_f^2 K_0(\epsilon r)\phi_T - \left[z^2 + (1-z)^2\right] \epsilon K_1(\epsilon r) \partial_r \phi_T \right\}$$

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Overlap functions for DVCS (real photon):

$$\left(\Psi_\gamma^* \Psi\right)_T^f = \frac{N_c \alpha_{em} e_f^2}{2\pi^2} \left\{ \left[z^2 + (1-z)^2\right] \epsilon_1 K_1(\epsilon_1 r) \epsilon_2 K_1(\epsilon_2 r) + m_f^2 K_0(\epsilon_1 r) K_0(\epsilon_2 r) \right\}$$

$$\epsilon_i^2 = z(1-z)Q_i^2 + m_f^2$$

Photon-pion exclusive cross section

$$\sigma^{exc}_{\gamma^*\pi}(\hat{x},Q^2) = \sum_{i=L,T}\int_{-\infty}^{0}\frac{d\sigma_i}{d\hat{t}}d\hat{t} = \sum_{i=L,T}\int_{-\infty}^{0}\left|\mathcal{A}_i^{\gamma^*\pi\to E\pi}(\hat{x},Q^2,\Delta)\right|^2 d\hat{t}$$

Scattering amplitude:

$\hat{t} = -\Delta^2$ $\qquad$ $\hat{x} = \frac{Q^2+m_f^2}{\hat{W}^2+Q^2} = \frac{Q^2+m_f^2}{(1-x_L)W^2+Q^2}$

$$\mathcal{A}_{L,T}^{\gamma^*\pi\to E\pi} = i\int dz\, d^2\vec{r}\, d^2\vec{b}\, e^{-i[\vec{b}-(1-z)\vec{r}]\cdot\vec{\Delta}}\left[\Psi_E^*\Psi(z,\vec{r},Q^2)\right]_{L,T} 2\,\mathcal{N}^\pi(\hat{x},\vec{r},\vec{b})$$

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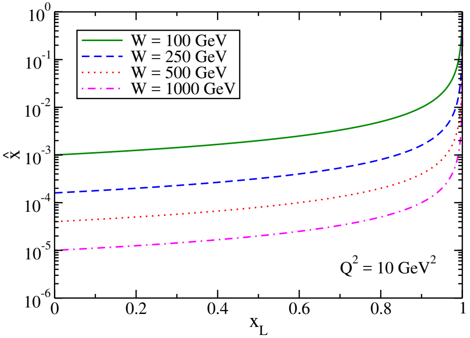
$$\epsilon_i^2 = z(1-z)Q_i^2 + m_f^2$$

Typical values of $\hat{x}$ probed in future *ep* colliders



$$\hat{x} = \frac{Q^2 + m_f^2}{\hat{W}^2 + Q^2} = \frac{Q^2 + m_f^2}{(1 - x_L)W^2 + Q^2}$$

Leading processes at the LHeC are sensitive to the small - x physics.

Dipole-pion scattering amplitude

Main assumption $\Longrightarrow$ $\mathcal{N}^{\pi}(\hat{x}, \vec{r}, \vec{b}) = R_q \cdot \mathcal{N}^{p}(\hat{x}, \vec{r}, \vec{b})$

$$R_q = \text{cte}$$

$\mathcal{N}^{p}(\hat{x}, \vec{r}, \vec{b}) \Rightarrow$ constrained by HERA data for inclusive and exclusive processes

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- GBW $\Leftrightarrow$ Golec-Biernat−Wüsthoff
- IIMS $\Leftrightarrow$ Iancu−Itakura−Munier−Soyez
- bCGC $\Leftrightarrow$ Kowalski−Motyka−Watt
- rcBK $\Leftrightarrow$ running coupling Balitsky−Kovchegov equation
- DGLAP $\Leftrightarrow$ Dokshitzer−Gribov−Lipatov−Altarelli−Parisi equation (CTEQ)

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Dipole-proton scattering amplitude

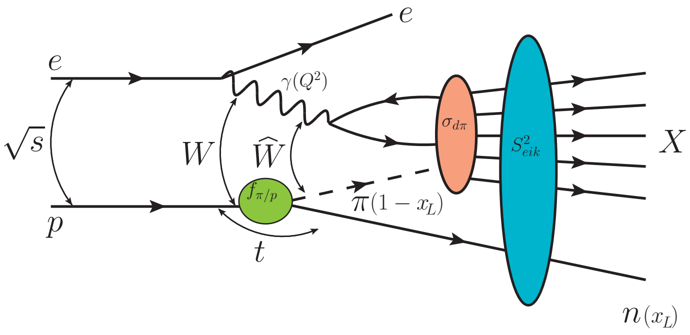
bCGC

$$\mathcal{N}^{p}(\hat{x}, \vec{r}, \vec{b}) = \begin{cases} \mathcal{N}_0 \left(\frac{r\, Q_s(b)}{2}\right)^{2\left(\gamma_s + \frac{\ln(2/r\, Q_s(b))}{\kappa\, \lambda\, Y}\right)} & rQ_s(b) \leq 2 \\ 1 - e^{-A\, \ln^2(B\, r\, Q_s(b))} & rQ_s(b) > 2 \end{cases}$$

Saturation scale

$$Q_s(b) \equiv Q_s(\hat{x}, b) = \left(\frac{x_0}{\hat{x}}\right)^{\frac{\lambda}{2}} \left[\exp\left(-\frac{b^2}{2B_{CGC}}\right)\right]^{\frac{1}{2\gamma_s}}$$

Absorptive effects



Absorptive corrections

$$\sigma(\gamma^*\pi \to X) = \mathcal{K}_{inc} \cdot \int dz \int d^2\vec{r} \sum_{L,T} \left|\psi_{L,T}(z,\vec{r},Q^2)\right|^2 \sigma_{d\pi}(\hat{x},\vec{r})$$

$$\sigma(\gamma^*\pi \to E\pi) = \mathcal{K}_{exc} \cdot \frac{1}{16\pi} \sum_{L,T} \int_{-\infty}^{0} \left|\mathcal{A}_{L,T}^{\gamma^*\pi\to E\pi}(\hat{x},\Delta)\right|^2 d\hat{t}$$

Open questions $\Rightarrow$
$$\begin{cases} \mathcal{K}_{inc} \stackrel{?}{=} \mathcal{K}_{exc} \Leftrightarrow \exists \mathcal{K}\,? \\ \mathcal{K}_{inc} = \mathcal{K}_{inc}(\hat{W}, x_L, Q^2) \\ \mathcal{K}_{exc} = \mathcal{K}_{exc}(\hat{W}, x_L, Q^2) \end{cases}$$

Our initial assumption:

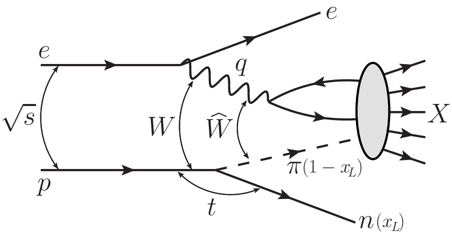
$\mathcal{K}_{inc} \neq \mathcal{K}_{exc}$

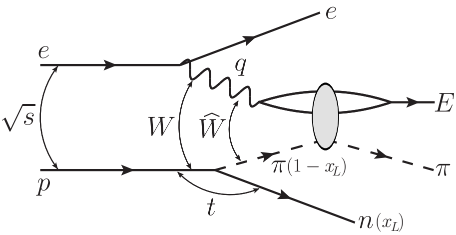
$\mathcal{K}_i = \text{cte}$

$(\mathcal{K}_i \leq 1.0)$

For a recent calculation of $\mathcal{K}_{inc}$ see VPG et al. PRD 103 (2021) 034021

Results for electron-proton collisions





Results for **inclusive** processes at HERA: $\mathcal{K}_{inc}$ and R_q dependence

$\mathcal{K}_{inc} = 1$
$R_q = 2/3$

$\mathcal{K}_{inc} = 1$
$R_q = 1/3$

or

$\mathcal{K}_{inc} = 1/2$
$R_q = 2/3$

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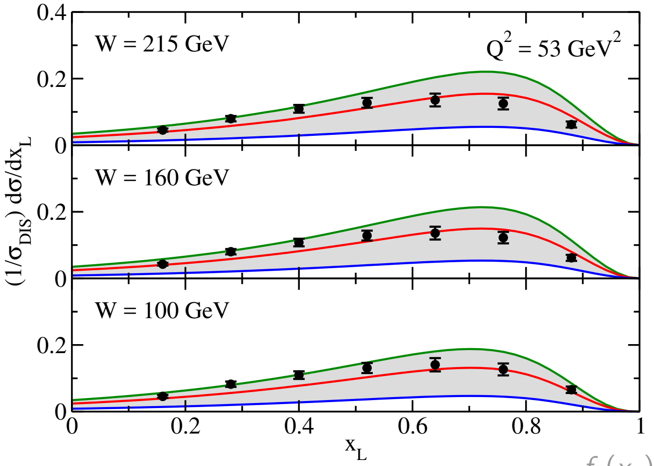
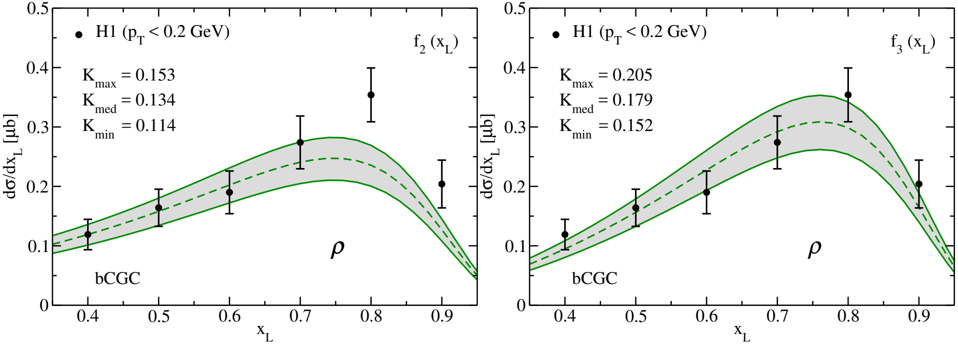


$f_2(x_L)$

Results for **exclusive** processes at HERA: upper and lower bounds

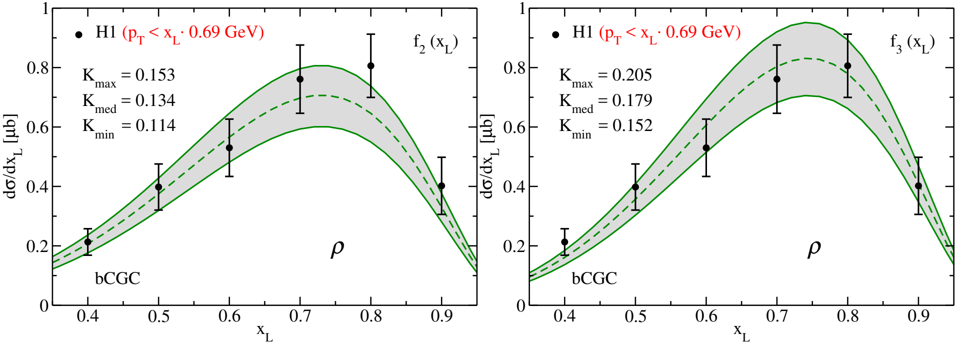
Bounds associated with the experimental uncertainty present in the H1 data.



$W = 60$ GeV and $Q^2 = 0.04$ GeV^2.

Results for **exclusive** processes at HERA: testing $\mathcal{K}$-factor

A cross-check: the same range of $\mathcal{K}$ to describe different H1 data.



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Predictions for future *ep* colliders

Dependence on the photon virtuality for the LHeC energy:

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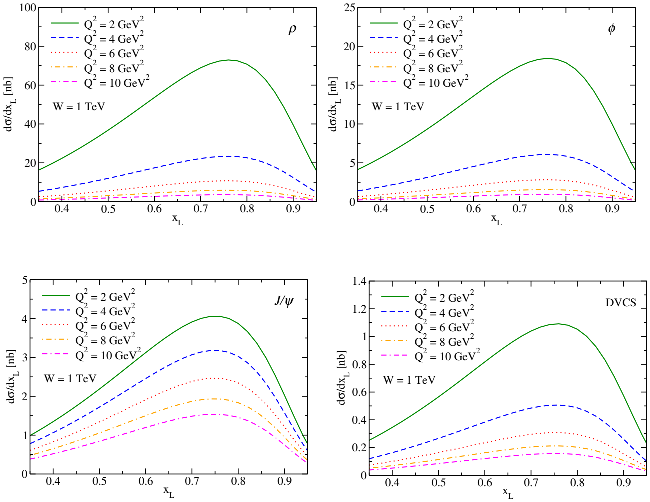

Predictions for future *ep* colliders

Dependence on the energy for exclusive processes:

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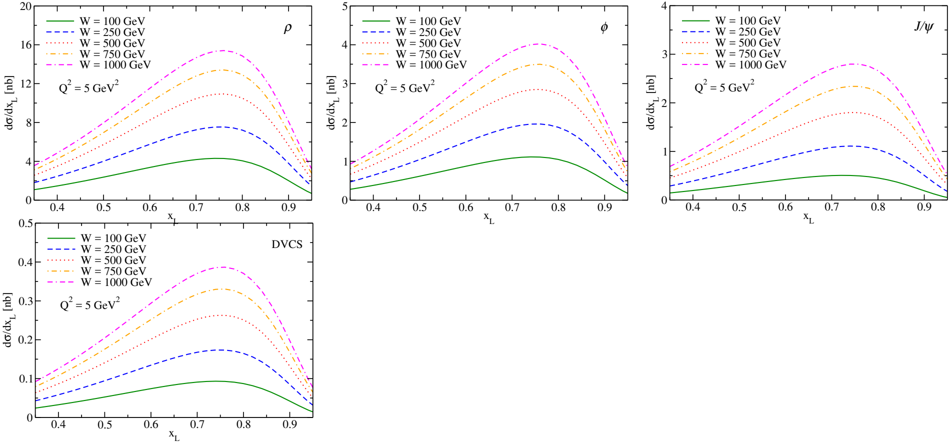

[bCGC and $f_3(y)$]

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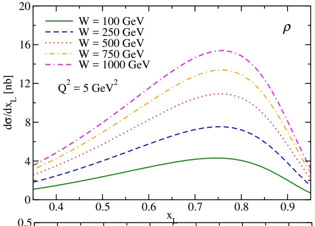

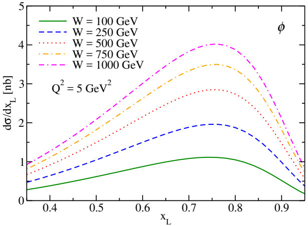

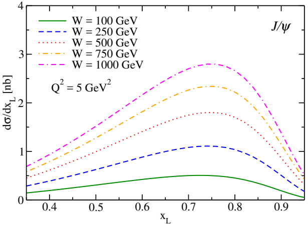

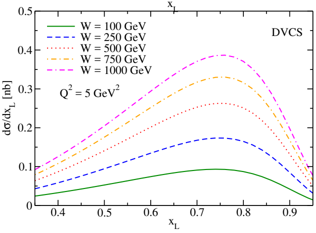

LHeC: $W = 1000$ GeV and $Q^2 = 5$ GeV^2

$$\sigma(\gamma p \to \rho\pi n) = 6.55 \pm 0.95 \text{ nb}$$

$$\sigma(\gamma p \to \phi\pi n) = 1.71 \pm 0.25 \text{ nb}$$

$$\sigma(\gamma p \to J/\psi\pi n) = 1.20 \pm 0.17 \text{ nb}$$

$$\sigma(\gamma p \to \gamma\pi n) = 0.16 \pm 0.002 \text{ nb}$$

[bCGC and $f_3(y)$]

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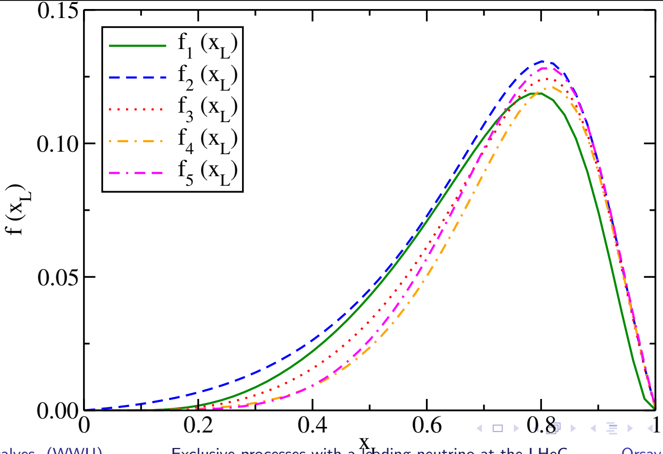
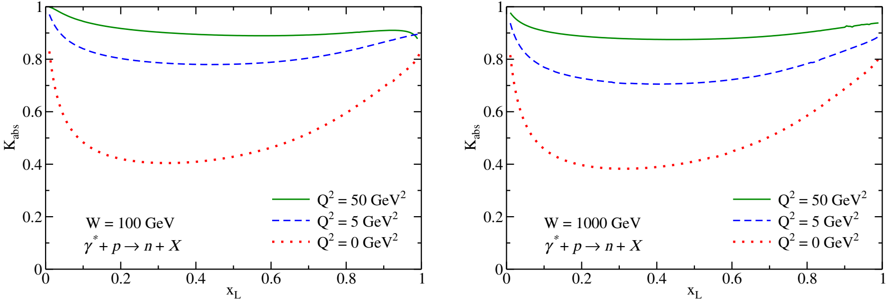
Thank you for your attention!

Flux of pions / Pion splitting function

$$f_{\pi/p}(x_L, t) = \frac{1}{4\pi} \frac{2g_{p\pi p}^2}{4\pi} \frac{-t}{(t - m_\pi^2)^2} (1 - x_L)^{1-2\alpha(t)} [F(x_L, t)]^2$$

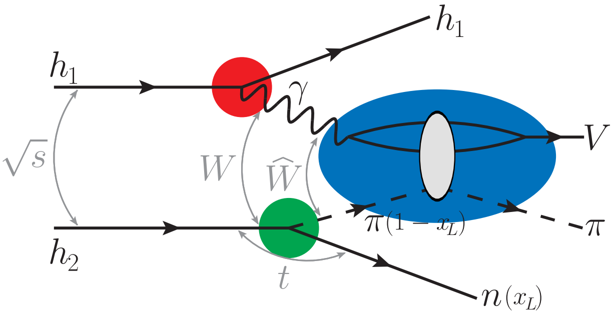


Absorptive factor



$$S_{eik}^2(r, b_{rel}) = \left\{1 - \Lambda_{\rm eff}^2 \frac{\sigma_{dn}(x_n, r)}{2\pi} \exp\left[-\frac{\Lambda_{\rm eff}^2 b_{rel}^2}{2}\right]\right\},$$

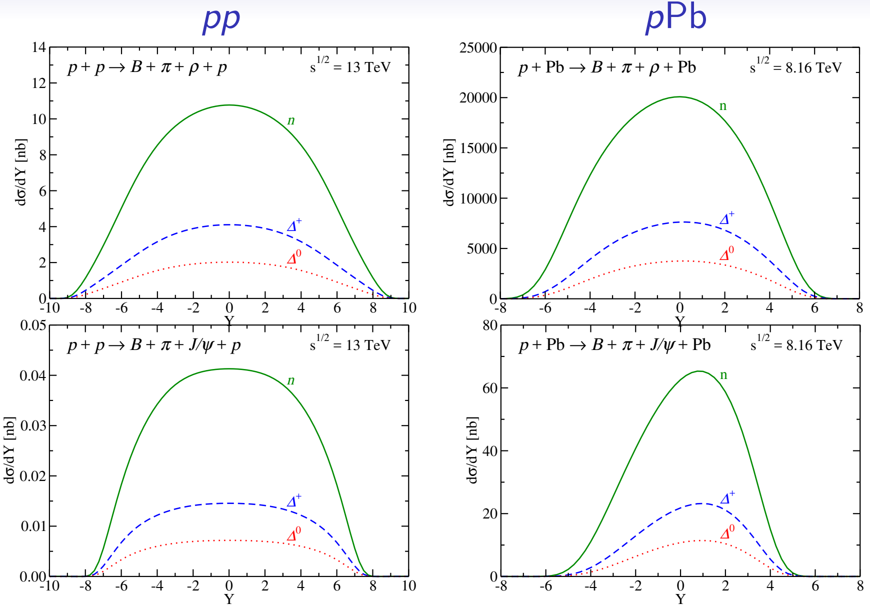
VM γ-production with proton dissociation in *hh* collisions in the color dipole picture



$$W^2 = m_V e^Y \sqrt{s}$$
$$\hat{W}^2 = (1 - x_L) W^2$$

$$\frac{d\sigma(h_1 + h_2 \to h_1 + V + \pi + n)}{dY dx_L dt} = N_{\gamma/h_1}(Y) \otimes f_{\pi/p}(x_L, t) \otimes \sigma_{\gamma\pi}(\hat{W})$$

VPG et al. PRD 94 (2016) 014009 and Phys.Rev.D 97 (2018) 7, 074002


pp
pPb
p + p → B + π + ρ + p
s^1/2 = 13 TeV
p + Pb → B + π + ρ + Pb
s^1/2 = 8.16 TeV
p + p → B + π + J/ψ + p
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p + Pb → B + π + J/ψ + Pb
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dσ/dY [nb]
Y
n
Δ+
Δ0

Total cross section

VM	ρ		J/ψ	
$\sqrt{s}$/TeV (pp)	8.0	13.0	8.0	13.0
$\sigma(n\pi^+)$/nb	97.846	121.348	0.324	0.451
$\sigma(\Delta^0\pi^+)$/nb	17.992	22.450	0.056	0.079
$\sigma(\Delta^+\pi^0)$/nb	36.533	45.593	0.114	0.160

VM	ρ		J/ψ	
$\sqrt{s}$/TeV (pPb)	5.02	8.16	5.02	8.16
$\sigma(n\pi^+)/\mu$b	124.026	163.124	0.244	0.376
$\sigma(\Delta^0\pi^+)/\mu$b	22.090	29.349	0.040	0.063
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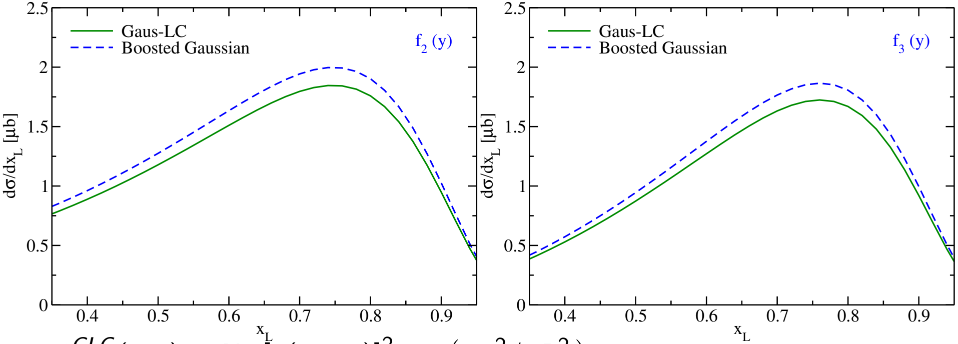
The case with a leading neutron is smaller by $\sim 10^2$ of magnitude.
The case with a leading delta is smaller by $\sim 10^3$ of magnitude.

Results for **exclusive** processes: wave function dependence



$$\phi_T^{GLC}(r,z) = N_T\left[z(1-z)\right]^2 \exp\left(-r^2/2R_T^2\right)$$

$$\phi_L^{GLC}(r,z) = N_L z(1-z)\exp\left(-r^2/2R_L^2\right)$$

$$\phi_{T,L}^{BoG}(r,z) = \mathcal{C}_{T,L}\, z(1-z)\exp\left[-\frac{m_f^2R^2}{8z(1-z)} - \frac{2z(1-z)r^2}{R^2} + \frac{m_f^2R^2}{2}\right]$$