

Nuclear shell-model Hamiltonians in a quantum computer

Antonio Márquez Romero - a.marquez.romero@ub.edu 24th November, 2022

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• Implementation in quantum devices comes with other challenges.

• Variational quantum eigensolvers (VQE) are a promising tool for the description of nuclear structure with quantum circuits.

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- In its implementation, different strategies are to be pursued for the following challenges¹:
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 - Fermionic operator encoding
 - Iterative optimization and convergence
 - Measurement
 - Error mitigation
- Our algorithm of choice is the recent Adaptive Derivative-Assemled Problem-Tailored Variational Quantum Eigensolver (ADAPT-VQE). Adequate for the NISQ era.

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Simulation

Adaptive variational quantum eigensolver (ADAPT-VQE)¹

Starting ingredients: a reference state $|\Psi_{\rm ref}\rangle$ and a pool of operators $\hat{A}_m:a_i^+a_j,\,a_i^+a_j^+a_la_k\ldots$

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$$\begin{split} |\Psi_{0}\rangle &= |\Psi_{\mathrm{ref}}\rangle \\ |\Psi_{1}\rangle &= e^{i\theta_{1}\hat{A}_{1}}|\Psi_{0}\rangle \\ |\Psi_{2}\rangle &= e^{i\theta_{2}\hat{A}_{2}}|\Psi_{1}\rangle = e^{i\theta_{2}\hat{A}_{2}}e^{i\theta_{1}\hat{A}_{1}}|\Psi_{0}\rangle \\ &\vdots \\ |\Psi_{n}\rangle &= e^{i\theta_{n}\hat{A}_{n}}\cdots e^{i\theta_{1}\hat{A}_{1}}|\Psi_{0}\rangle \end{split}$$

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Operators \hat{A}_m selected according to the largest gradient $\partial F^{(k)}$

$$\frac{\partial E^{(\alpha)}}{\partial \theta_m}\Big|_{\theta_m=0} = i \langle \Psi_k | [H, \hat{A}_m] | \Psi_k \rangle$$

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Parameters θ_m obtained minimizing the energy surface at every iteration.

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Good scaling!

Previous work with ADAPT-VQE in the Lipkin and shell-model²:

Lipkin model of N particles with interaction strength y

USDB, KB3G phenomenological interactions



 $^{2}\mbox{A.}$ M. Romero, J. Engel, Ho Lun Tang, and Sophia E. Economou. Phys. Rev. C 105, 064317

Quantum implementation

Implementation in quantum devices: fermionic mapping



Implementation in quantum devices: fermionic mapping



Jordan-Wigner mapping is used for convenience:

$$a_i^{\dagger} = \left(\prod_{k=0}^{i-1} Z_k\right) \sigma_i^-, \quad a_i = \left(\prod_{k=0}^{i-1} Z_k\right) \sigma_i^+,$$

but leads to unnecessarily large matrices and number of qubits

Choice of ansatz

We simply chose the lowest-energy Slater determinant. Straightforward to implement

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Example: Two particles in a four-state space

 $|\psi\rangle = |02\rangle$



Variational part: pool operators

Implementation of ADAPT pool operators is straightforward. We chose all the possible two-body excitations

$$T_{rs}^{pq} = i(a_p^{\dagger}a_q^{\dagger}a_ra_s - a_r^{\dagger}a_s^{\dagger}a_pa_q)$$

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 $e^{i\theta T_{pq}^{rs}} = e^{i\theta P_{pq}^{rs} X_p Y_q Y_r Y_s} e^{i\theta P_{pq}^{rs} Y_p X_q Y_r Y_s} e^{-i\theta P_{pq}^{rs} Y_p Y_q X_r Y_s} e^{-i\theta P_{pq}^{rs} Y_p Y_q Y_r X_s}$ $\times e^{-i\theta P_{pq}^{rs} Y_p X_q X_r X_s} e^{-i\theta P_{pq}^{rs} X_p Y_q X_r X_s} e^{i\theta P_{pq}^{rs} X_p X_q Y_r X_s} e^{i\theta P_{pq}^{rs} X_p X_q X_r Y_s}$ $P_{pq}^{rs} = \prod_{m=p+1, m \notin [r,s]}^{r-1} Z_m \prod_{n=r+1, n \notin [p,q]}^{s-1} Z_n$



Figure 2: Quantum circuit to implement $e^{-i\theta X_0 Z_1 Y_4 Z_5}$

Measurement of the energy

$$H = \sum_{i} \varepsilon_{i} a_{i}^{\dagger} a_{i} + \frac{1}{4} \sum_{ijkl} \bar{v}_{ijkl} a_{i}^{\dagger} a_{j}^{\dagger} a_{l} a_{k}$$

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We disentangle them in terms of the form

$$\begin{split} n_i &= a_i^{\dagger} a_i = \frac{1}{2} (1 - Z_i) \\ h_{ijij} &= -2a_i^{\dagger} a_i a_j^{\dagger} a_j = -\frac{1}{2} (1 - Z_i) (1 - Z_j), \\ h_{ijki} &= a_i^{\dagger} a_i (a_j^{\dagger} a_k + a_k^{\dagger} a_j) \\ &= \frac{1}{2} (1 - Z_i) \left(\prod_{n=j+1}^{k-1} Z_n \right) (|01\rangle \langle 10| + |10\rangle \langle 01|)_{jk}, \\ h_{ijkl} &= P_{ij}^{kl} (|0011\rangle \langle 1100| + |1100\rangle \langle 0011|)_{ijkl}, \end{split}$$

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But this is not the problem!

Dimension of two-body excitation operators increases quartically with space dimensionality and can not be measured simultaneously. We need to find the minimum partition group of commuting operators

Measurement: minimum clique problem

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NP-hard problem! We used a greedy algorithm for approximate solutions

Results

- Number of CNOTS grows linearly with the number of iterations!
- In lower dimensionality spaces, results converge quickly
- Different convergence behavior for different nuclei



A. Perez-Obiol, A. M. Romero et al, Nuclear shell-model simulation in a quantum device, TBP

shell	N_q	$N_{\rm SD}$	nucleus	$N_{\rm iters}$	ϵ_r bound
р	6	5	⁶ He	2	10^{-5}
	12	10	⁶ Li	9	10^{-7}
		51	⁸ Be	48	10^{-5}
		51	^{10}Be	48	10^{-5}
		42	^{13}C	17	10^{-5}
	12	14	¹⁸ 0	5	10^{-6}
		74	¹⁹ O	32	10^{-6}
		81	²⁰ O	70	10^{-6}
sd		142	²² O	119	10^{-6}
	24	640	²⁰ Ne	167	2×10^{-2}
		4206	^{22}Ne	236	2×10^{-2}
		7562	24 Ne	345	2×10^{-2}
pf	20	30	^{42}Ca	9	10^{-8}
		565	^{44}Ca	132	10^{-2}
		3952	^{46}Ca	124	10^{-2}
		12022	48 Ca	101	10^{-2}
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A. Perez-Obiol, A. M. Romero et al, Nuclear shell-model simulation in a quantum 11

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A. Perez-Obiol, A. M. Romero et al, Nuclear shell-model simulation in a quantum 12 device. TRP

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A. Perez-Obiol, A. M. Romero et al, Nuclear shell-model simulation in a quantum 14 device. TRP

Model circuit



Figure 3: Quantum circuit to prepare the exact ground state for the *sd*-shell with two neutrons, ¹⁸O. X gates prepare the reference state and FSWAPS are used to change basis so that exponentials of pool operators operate on adjacent qubits. Multiqubit gates in the boxes are defined as $U_{ij}^{kl}(\theta) \equiv e^{i\theta T_{ij}^{kl}}$ and $\theta_0 = -0.157263$, $\theta_1 = -0.437238$, $\theta_2 = 0.604663$, $\theta_3 = 0.214431$, $\theta_4 = -0.785469$.

Number of circuits for measurement

shell	N_q	N_{h}	N_{hh}	Total
	6	2	10 (9)	13 (12)
ρ	12	4	109 (44)	114 (49)
sd	12	8	203 (86)	212 (95)
	24	16	1389 (518)	1406 (535)
pf	20	20	1507 (570)	1528 (591)
	40	40	10572 (3459)	10613 (3500)

Table 1: Number of different circuits needed to measure the expectation value of the Hamiltonian. N_q indicates the number of qubits used, corresponding to the number of orbitals in the shell. $N_{\rm h}$ and $N_{\rm hh}$ are the number of single and double hopping terms in the Hamiltonian, defining the number of different circuits needed to measure these parts. The values in parenthesis correspond to the minimum number of groups containing $h_{ijkl}^{(h,h)}$ terms found such that all operators in the group commute with each other and can be measured with the same circuit. In the last column, we list the total number of circuits corresponding to $N_{\rm h} + N_{\rm hh} + 1$, accounting also for the single circuit needed to measure $\langle h_i \rangle$.

Conclusions and ongoing work

• Promising results were obtained using ADAPT-VQE applied with phenomenological shell-model interactions in the classical and quantum simulation of the algorithm.

 $^{^{3}\}mbox{A}.$ Perez-Obiol, A. M. Romero et al, Nuclear shell-model simulation in a quantum device, TBP

⁴Selection of six sites to host the first European quantum computers

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- We developed a baseline code with the quantum implementation of the nuclear shell-model to explore and study these upcoming challenges³.

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- Barcelona Supercomputing Center will host one of the first European quantum computers⁴.

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In collaboration with

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University of Barcelona: Arnau Ríos, Javier Menéndez and Bruno Juliá Barcelona Supercomputing Center: **Axel Pérez Obiol** and Artur García



Merci beaucoup!

Extra slides

Fermionic mapping: example and potential alternative

The general Hamiltonian:

$$H = \sum_{i} \varepsilon_{i} a_{i}^{\dagger} a_{i} + \frac{1}{4} \sum_{ijkl} \bar{v}_{ijkl} a_{i}^{\dagger} a_{j}^{\dagger} a_{l} a_{k} , \qquad (1)$$

under a JW mapping, the corresponding matrix representation will have $2^{\dim}\times 2^{\dim}$ elements.

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Example:

Take the case of $^6{\rm Li}$ in the $M{\rm -scheme:}$ ground-state is formed with 10 M=0 states, but with JW the Hamiltonian matrix is $2^{12}\times2^{12}!$

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Compact encoding of the Hamiltonian⁵:

 $H = 0.598IIII - 0.088IIIX + \dots - 0.037ZZZX - 0.059ZZZZ.$ (2)

Only 4 qubits are needed! But what do projections 0 and 1 represent?

⁵RMN Pesce, PD Stevenson: H2ZIXY, arXiv:2111.00627

Why ADAPT and not Unitary Coupled Clusters approach?

$$|\Psi(\boldsymbol{\theta})\rangle = e^{\hat{T}(\boldsymbol{\theta})}|0\rangle \longrightarrow E_{\text{UCC}} = \min_{\boldsymbol{\theta}} \frac{\langle \Psi(\boldsymbol{\theta})|H|\Psi(\boldsymbol{\theta})\rangle}{\langle \Psi(\boldsymbol{\theta})|\Psi(\boldsymbol{\theta})\rangle},\tag{3}$$

 $^{^{6}}$ l. Stetcu et al. Phys. Rev. C 105, 064308 7 O. Kiss et al, Phys. Rev. C 106, 034325

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• ADAPT requires no Trotter approximation

$$e^{A+B} = \lim_{n \to \infty} \left(e^{A/n} e^{B/n} \right)^n, \tag{4}$$

Good results are obtained with only one Trotter step⁶. Although symmetries can be broken in the wavefunction and be more crucial in strongly-correlated systems

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- ADAPT does not depend on the cluster operator order
- However, it is possible that ADAPT leads to deeper circuits⁷

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⁷O. Kiss et al, Phys. Rev. C 106, 034325