Neutrino Flavor Dynamics:
Quantum Evolution

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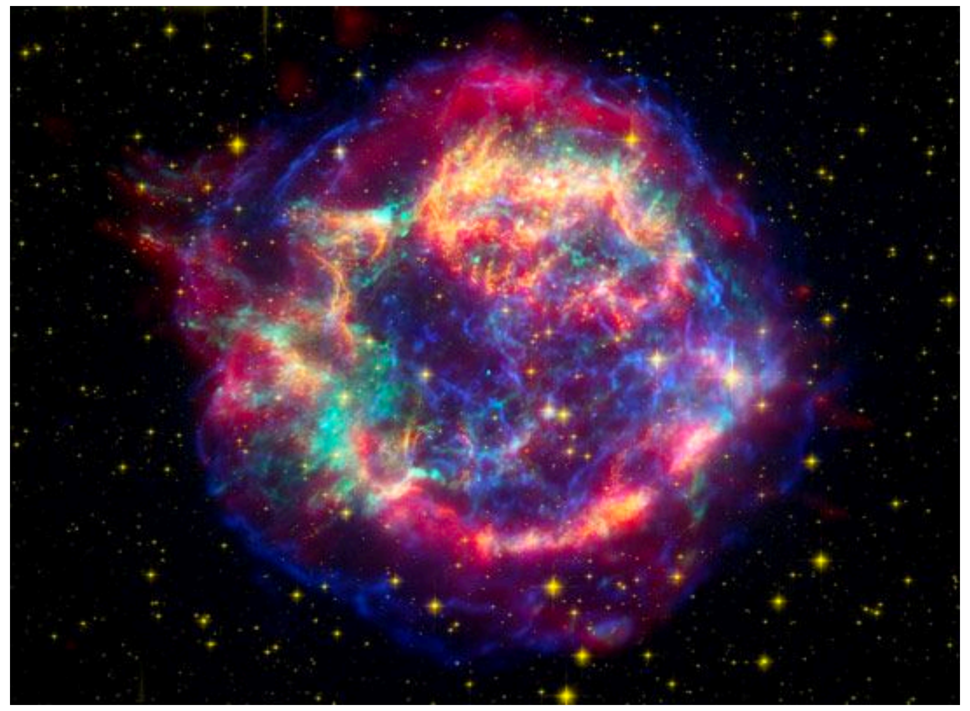
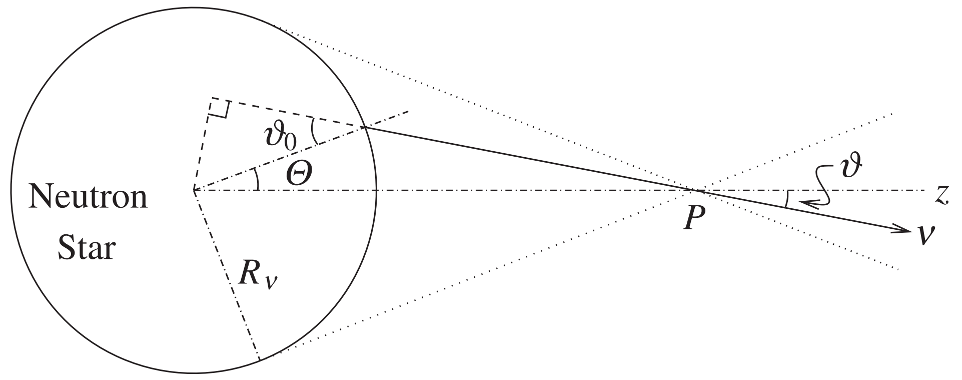
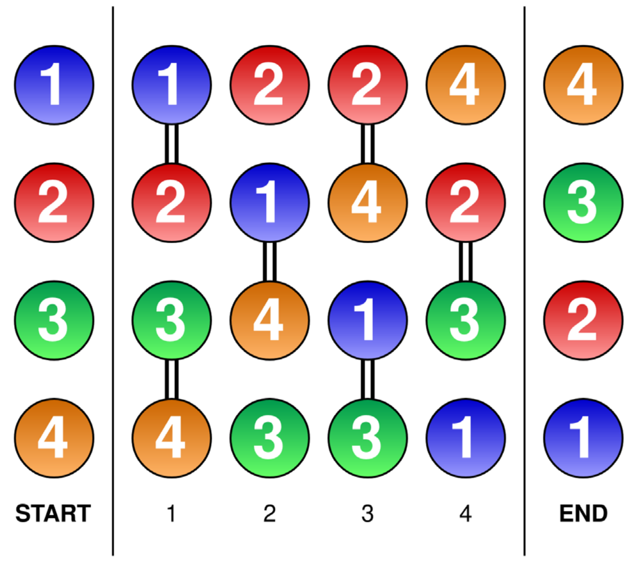
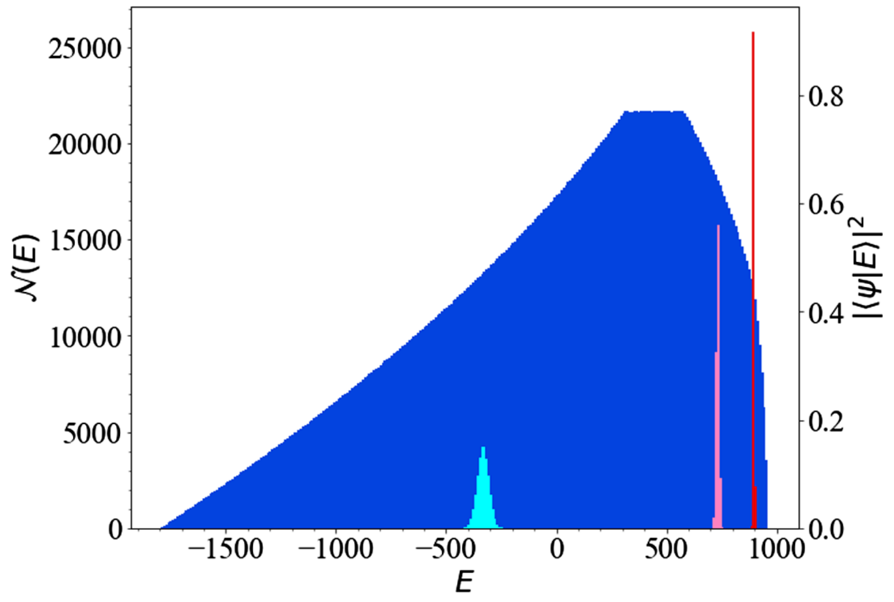
- Astrophysical Neutrinos (Supernovae, Neutron Star Mergers)
- Mean field (product state) treatments
- Quantum Computing Approaches
- Dynamics: Quench
- Dynamics: Time-Dependence
- Outlook



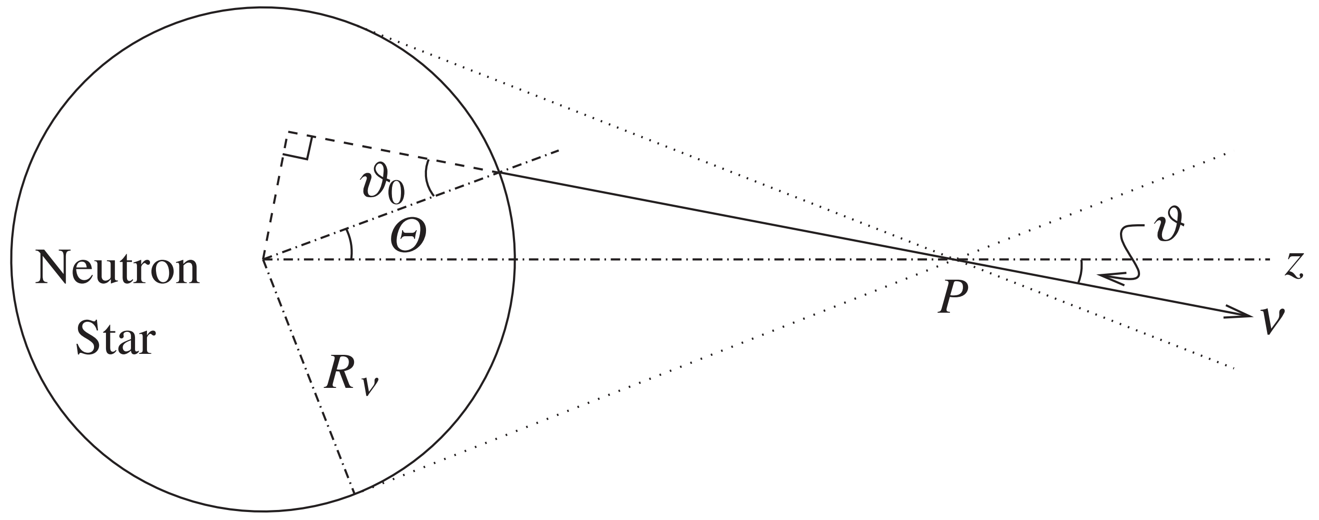






Flavor evolution of neutrinos and antineutrinos in supernovae and NS mergers (dense neutrino environments)

$$H = H_{1,\nu} + H_{1,m} + H_2$$

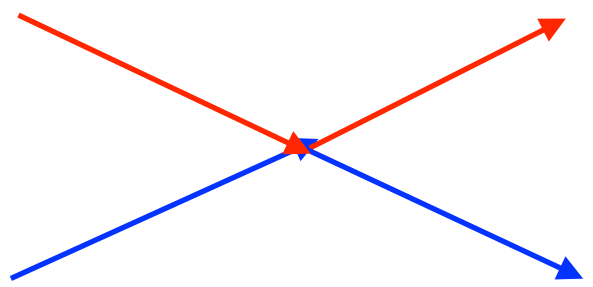


$$H_{1,\nu} = \begin{pmatrix} -\frac{\delta m^2}{2E_\nu}\cos(2\theta) & \frac{\delta m^2}{2E_\nu}\sin(2\theta) \\ \frac{\delta m^2}{2E_\nu}\sin(2\theta) & \frac{\delta m^2}{2E_\nu}\cos(2\theta) \end{pmatrix}$$

$$H_{1,m} = \begin{pmatrix} \sqrt{2}G_f n_e & 0 \\ 0 & \sqrt{2}G_F n_e \end{pmatrix}$$

Independent Neutrino Evolution
(Solar, Accelerators, Reactors,…)

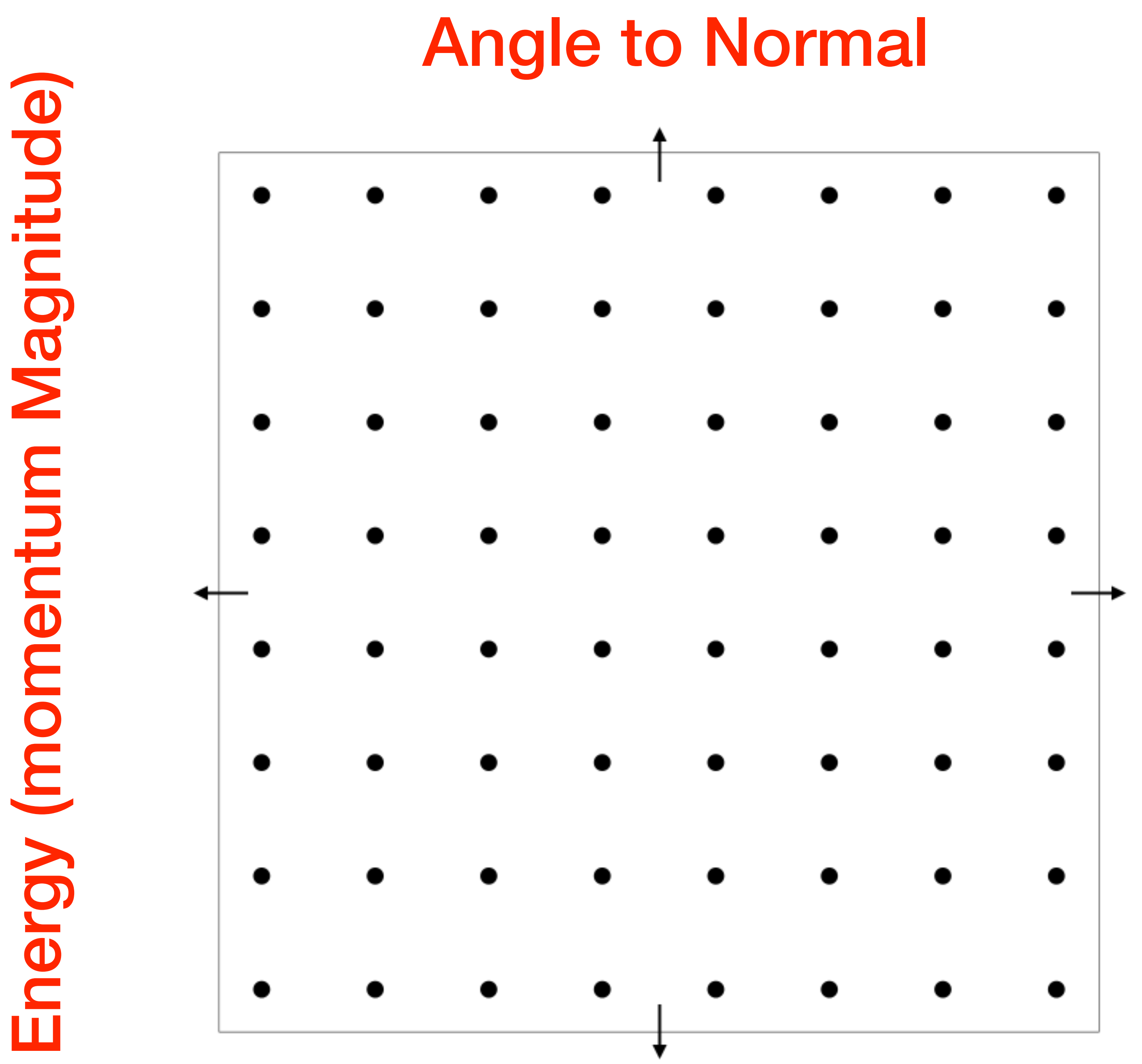
Flavor Exchange



$$H_2 = \frac{G_F}{2V}\sum_{i<j}\,[1 - \vec{v_i}\cdot\vec{v_j}]\ \sigma_i \cdot \sigma_j$$

Neutrino-Neutrino Forward Scattering
Flavor Exchange

Two Dimensional Lattice for Neutrinos
(Momentum space)



SU(2) [SU(3)] spin on each lattice site

Comparison of Neutrino Problem vs. Heisenberg-like spin problem

	Heisenberg (Typical)	Neutrinos
2-Body Spin Exchange	Nearest Neighbor	All-to-All
1-Body Magnetic Field	Uniform	Space-Time Magnitude & angle dependence
Initial State	Ground State, Finite T, Linear Response	Product State
Time Evolution	Quench,…	Geometry: 2-body decays
Measurement	Along H	Flavor Basis

Product Initial State and Time Evolution:

Initial State is a product of SU(2) spinors

$$\psi[t=0] = \prod_{i,j} \phi_{i,j}$$

- Assuming initial distribution is incoherent from coupling to dense hadronic matter in the interior
- Initial Flavor angular and energy distributions encoded in $\phi_{i,j}$ grid, couplings encode angular & energy dependent flux

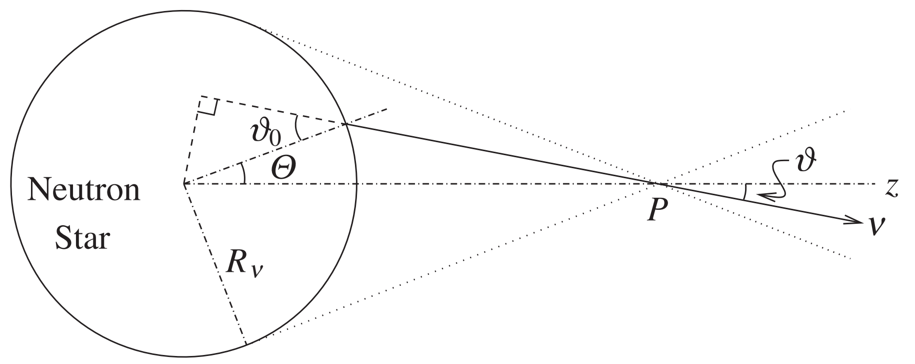
Mean-field (product state) evolution:

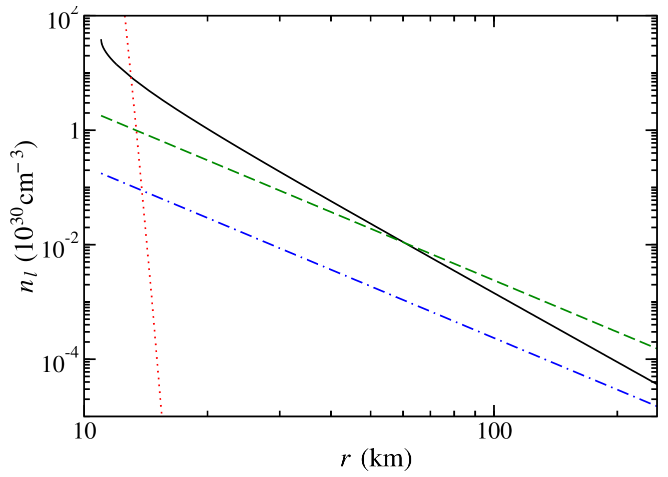
$$\Psi[t+\delta t] = \prod_{i,j} \phi_{i,j}(t+\delta t) \approx \ \exp[-iHt]\ \Psi[t]$$

- Approximate truncated Hilbert space to enable evolution
- 2N amplitudes instead of 2^N for full problem
- Typically of order 100 Energies x 100 angles = 10K amplitudes

Time dependent Hamiltonian: start at radius R0 and integrate to large radius

Time dependence from (1) geometry (relative angles decreasing with time)
Matter Density decreasing with distance (time)

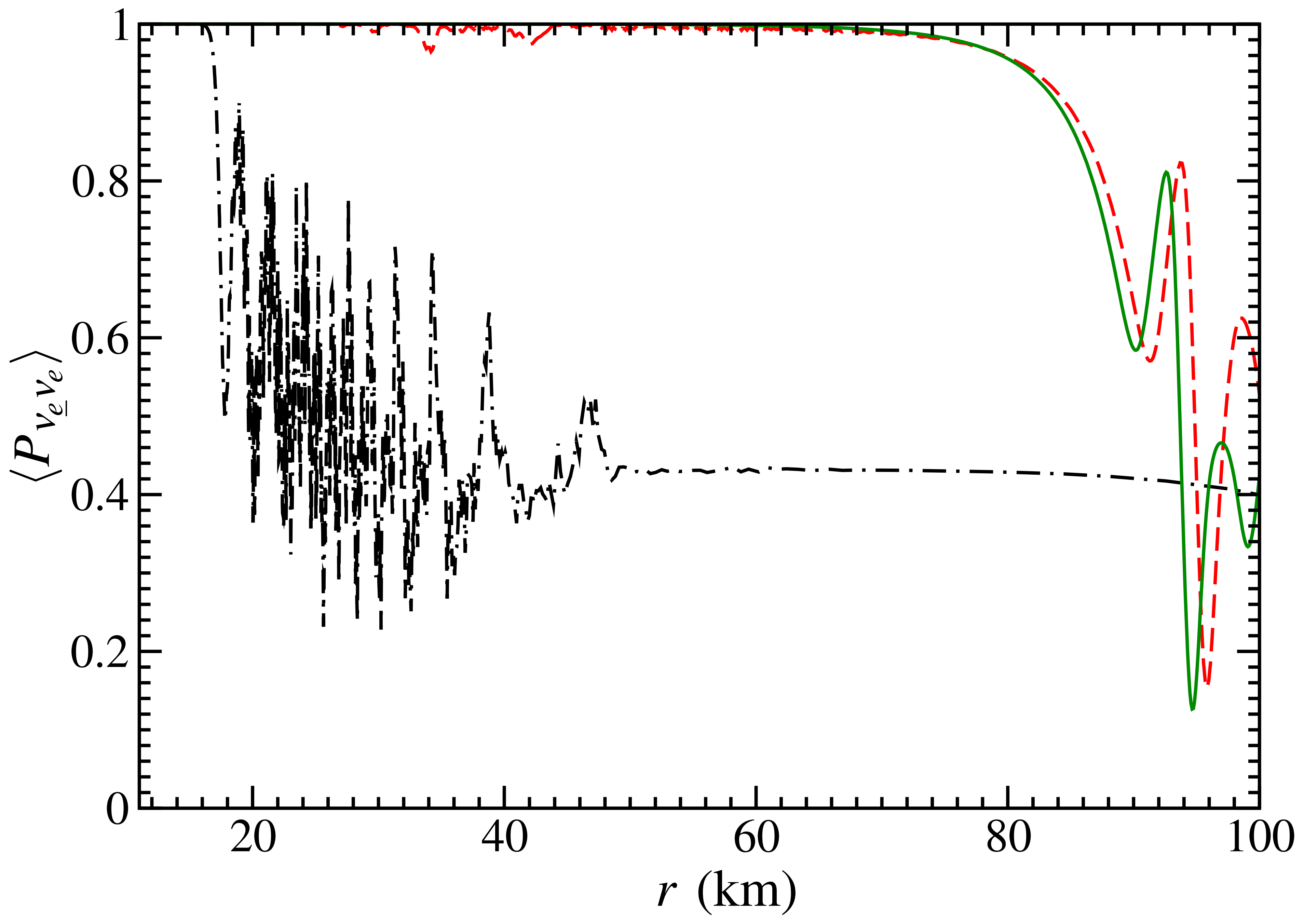




Time dependence from (1) geometry (relative angles decreasing with time)
Matter Density decreasing with distance (time)

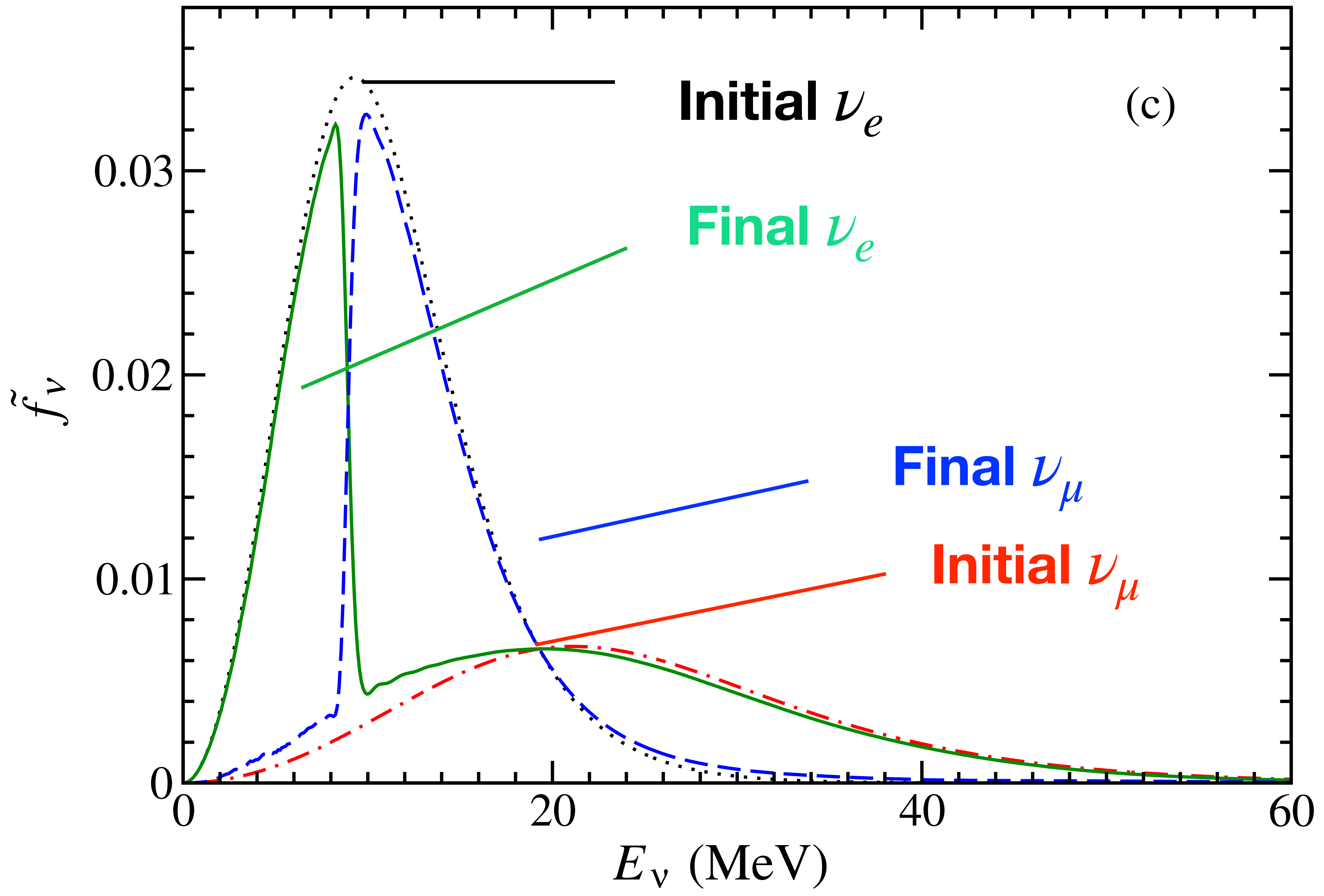
Even mean field can be computationally challenging

ν_e survival probability versus radius for different # angle bins, tolerances



Duan, Fuller, Carlson, Qian; PRD 2006

Spectral Swap



In many cases, find the neutrinos 'swap' spectra
at and above some energy

Above the transition muon neutrinos → electron neutrinos
and vice versa

Final state is what can be observed terrestrially (e.g. DUNE)

Depends upon many things:

- Initial energy spectra
- Initial angular distributions
- Normal/Inverted neutrino mass hierarchy

Intermediate time evolution influences

- Energy deposition in the shock
- Nucleosynthesis
- …

Rich possibilities, multiple swaps, etc.
CP-violations, matter effects, ….

Is product state evolution reliable? When?

Duan, Fuller, Carlson, Qian; PRD 2006

Beyond Mean Field

$$\Psi[t] = \sum_{\alpha=1}^{2^N} \phi_\alpha(t)$$

Binary representation of many-body state - each spin up (1) or down(0)

$$\Psi[t+\delta t] \ = \exp[-iHt] \ \Psi[t]$$

2^{16} = 65K amplitudes
2^{20} = 1M amplitudes

Hamiltonian is somewhat sparse. Each state connects to
N Spin flip states plus
N^2 spin exchange states

Fairly reasonable to do 16-24 spins
on a laptop/desktop

However, neutrino masses (with matter)
lead to short oscillation lengths

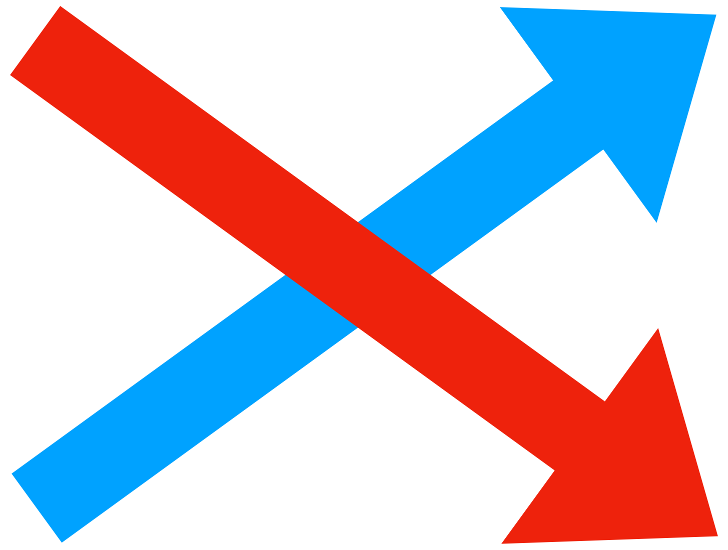
In flavor basis:

$H_{1,M} \propto \sigma_z$ is diagonal in flavor

$H_{1,v} \propto \cos(2\theta)\sigma_z + \sin(2\theta)\sigma_x$ Diagonal plus bit flip

$H_2 \propto \sigma_i \cdot \sigma_j$ Diagonal plus spin exchange

Simplify Problem by adding symmetries
Doing quench (time-dependent H) only



Two Beam Hamiltonian

$$\frac{H}{\mu} = \frac{\Omega}{2}\vec{B}\cdot(\vec{J}_{\mathrm{A}} - \vec{J}_{\mathrm{B}}) + \frac{2}{N}\vec{J}_{\mathrm{A}}\cdot\vec{J}_{\mathrm{B}},$$

$$\vec{J}_{\mathrm{A/B}} = \sum_{i\in \mathrm{A/B}} \vec{\sigma}_i/2.$$

No 2-body coupling within a beam
Total Hamiltonian has term
proportional to Beam A . Beam B spin

Initial State (product)

$$|\Psi\rangle = |\hat{n}_{\mathrm{A}}\rangle^{\otimes N_{\mathrm{A}}}|\hat{n}_{\mathrm{B}}\rangle^{\otimes N_{\mathrm{B}}}.$$

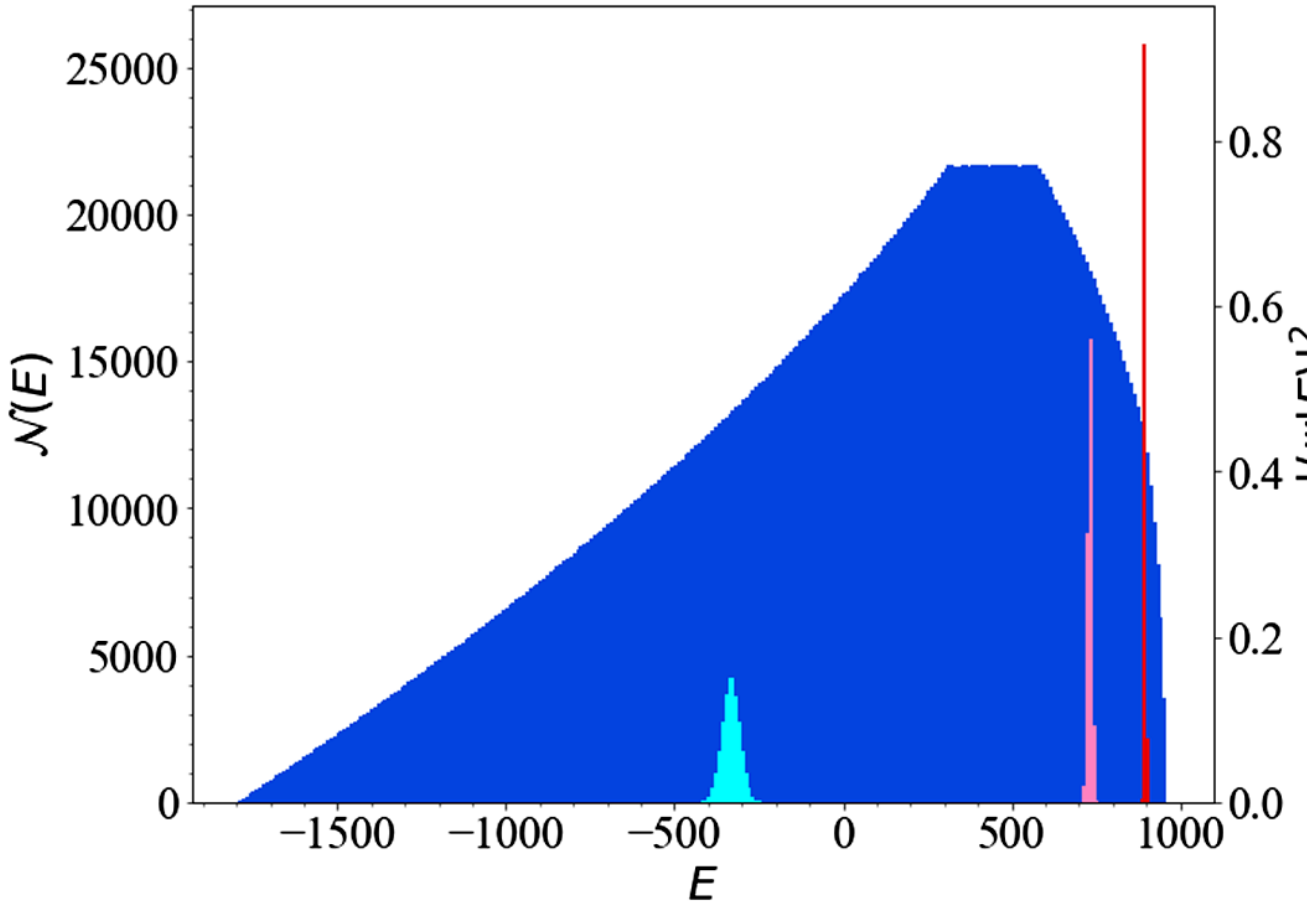
$$|\hat{n}_{\mathrm{A/B}}\rangle = \cos\left(\frac{\theta_{\mathrm{A/B}}}{2}\right)|\nu_1\rangle + \sin\left(\frac{\theta_{\mathrm{A/B}}}{2}\right)e^{i\phi_{\mathrm{A/B}}}|\nu_2\rangle,$$

Initial state and Hamiltonian
symmetric under exchange
Of any two spins within a beam

Propagate with time-independent Hamiltonian (quench):
Investigate Entanglement Entropy, Purity, …
When does the mean-field (product state approximation) work?
    # of spins, time, …

Possible dynamic phase transitions in low-energy regime of Hamiltonian

Density of States
And initial state distributions



3600 spins

Initial product states sharply peaked in energy

Dark Blue : Full many-body density of states

- Red: bipolar oscillation initial conditions
- Pink: collective precession of the spins
- Cyan: random polarizations of the individual spins

Initial state moments: consistent w/ gaussian for large N

$$\Delta H^2 = c_1 N + c_0.$$

$$c_0 = \frac{N_A N_B}{4N^2}(1 - (\hat{n}_A \cdot \hat{n}_B))^2,$$

$$\lim_{N\to\infty} \mathcal{M}_3 \equiv \frac{\langle (H - \langle H \rangle)^3 \rangle}{\Delta H^3} = 0,$$

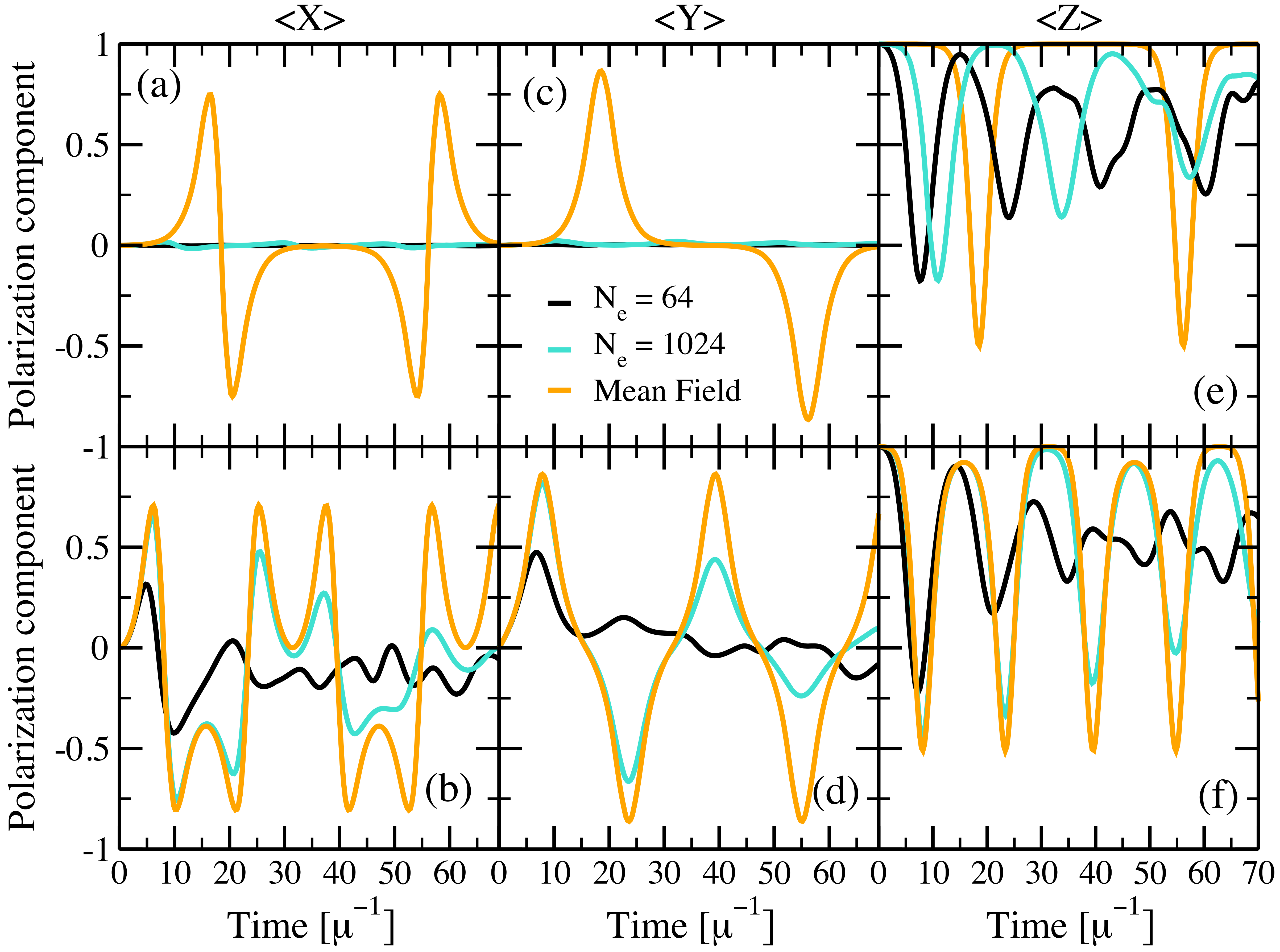
$$\lim_{N\to\infty} \mathcal{M}_4 \equiv \frac{\langle (H - \langle H \rangle)^4 \rangle}{\Delta H^4} = 3.$$

Martin, Roggero, Duan, Carlson and Cirigliano, PRD 2022

Comparison of mean-field and full time evolution
Bipolar Oscillations

$\theta = 0.001$

$\theta = 0.1$

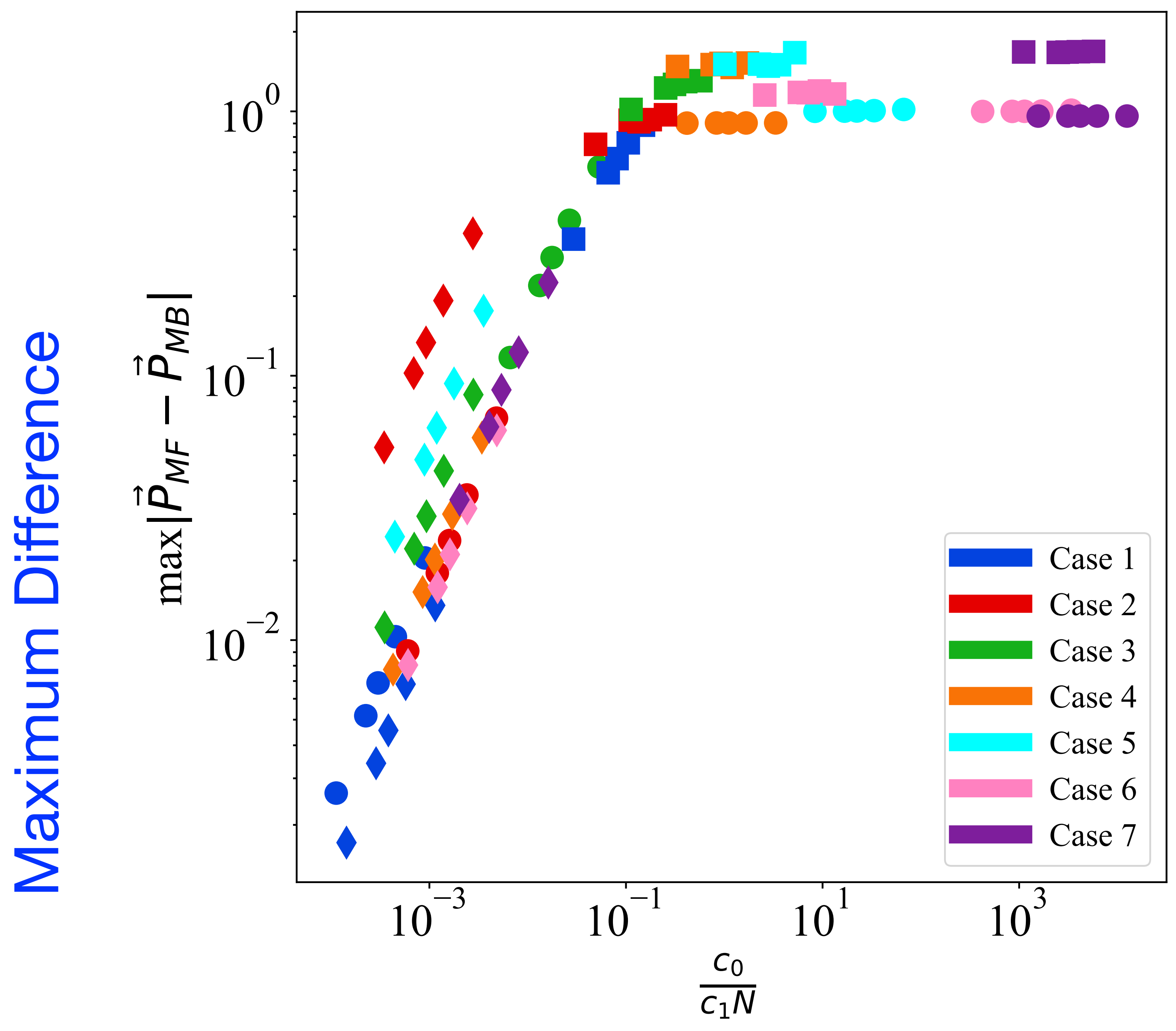


Agreement improves with larger N, higher density of states

Comparison of mean-field and full time evolution: Maximum Differences

Maximum Time $$t_f = 3\sqrt{\frac{N}{\Delta H^2}}$$

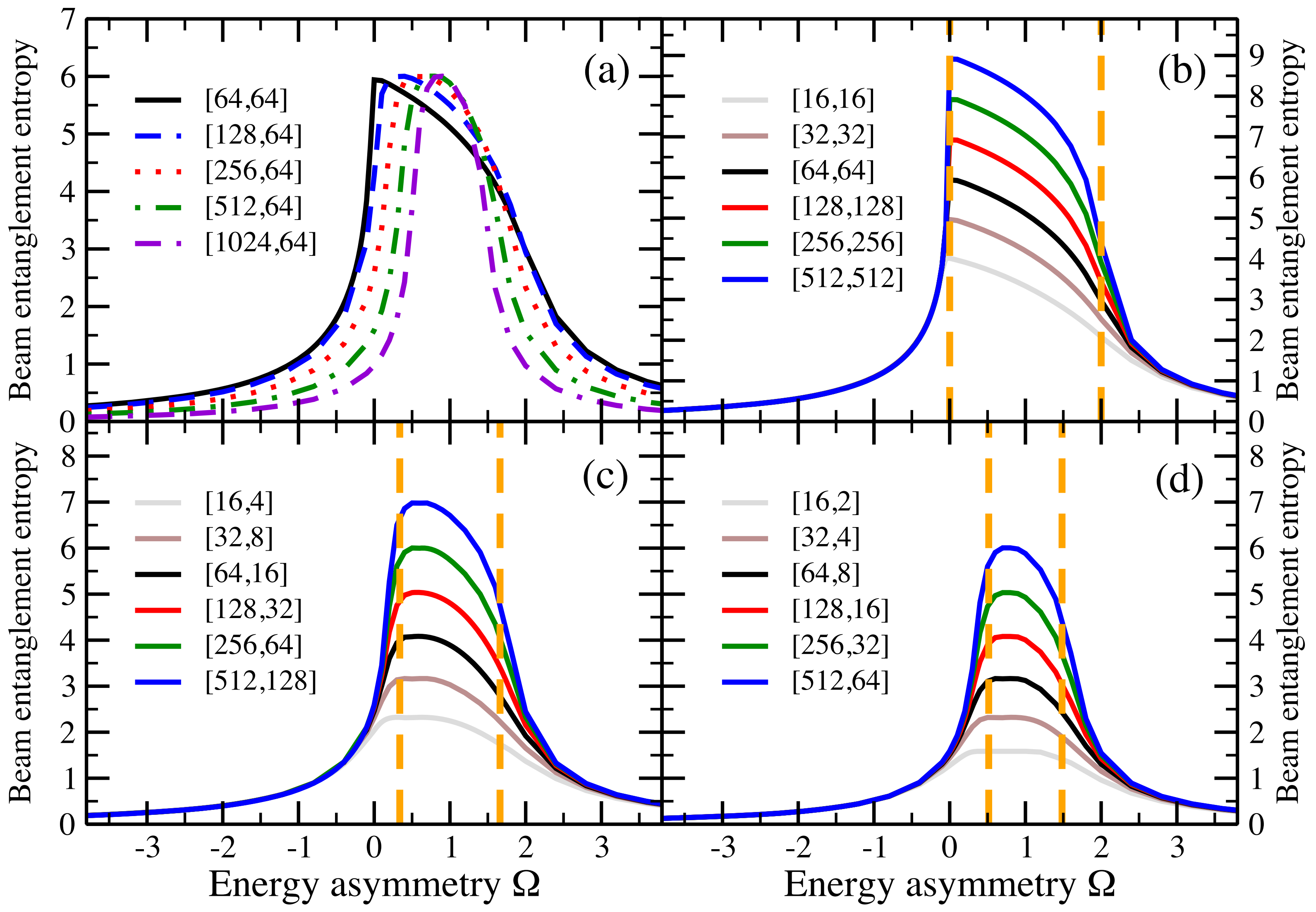
Beyond this time resolving energies
Within gaussian distribution



Increasing N

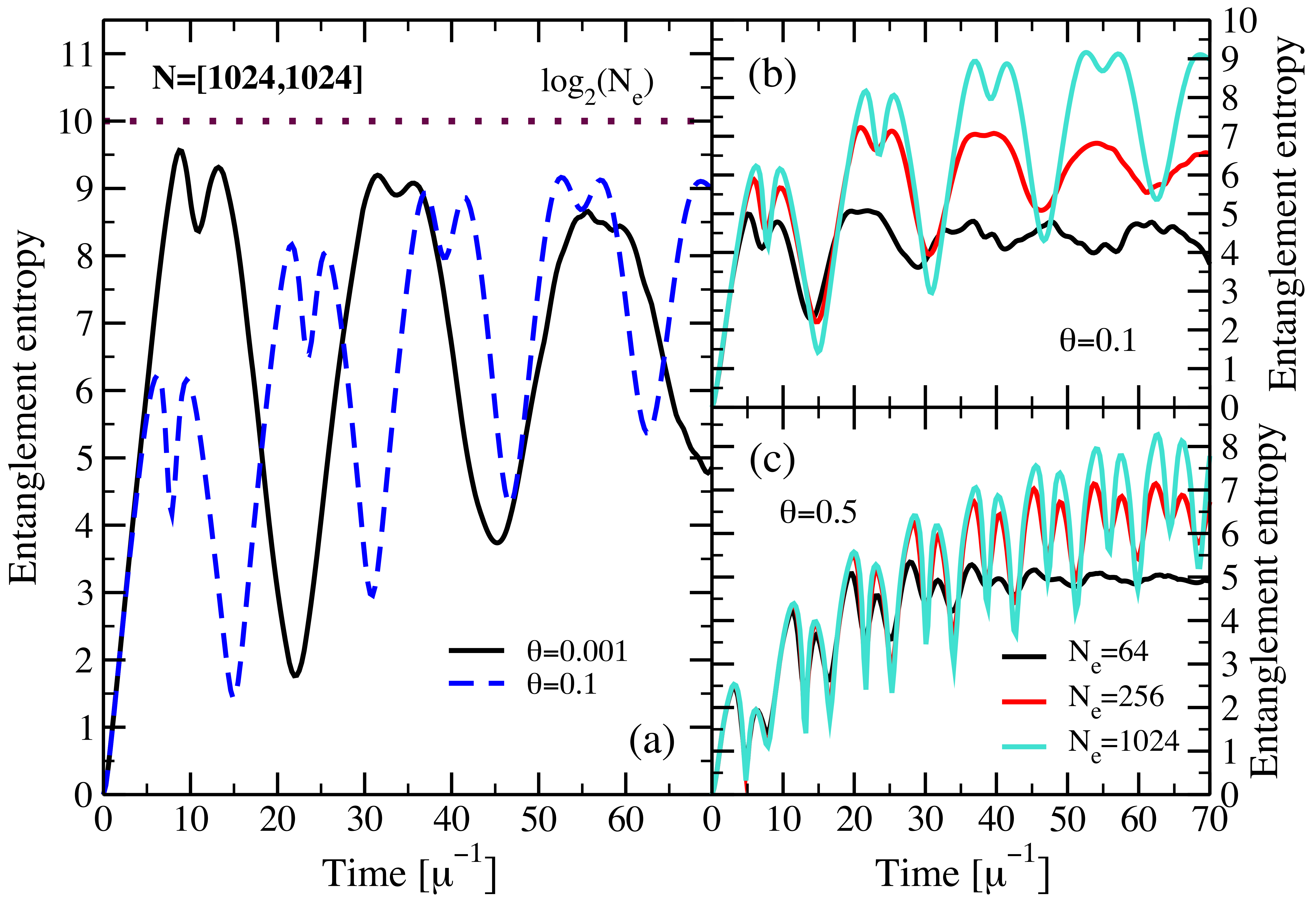
Mean Field works quite well for large N, modest times

Entanglement Entropy



Vertical Lines: threshold values for bipolar instability
As a function of the energy asymmetry

Time development of entanglement entropy vs. N
Symmetries limit entanglement entropy to $\log_2$ (N)



See also purity

What about time- (radius) dependent Hamiltonian?

Eliminate (most) symmetries

Limits studies to order 12-25 spins : 2^N states

Sample problem: Pehlivan, Balaneekin, Kajino, Yoshida PRD 2011

Patwardhan, Cervia, Balentekin (arXiv:2109.08995)

Hamiltonian

$$H = H_{\rm vac} + H_{\nu\nu}(t)$$

$$H_{\rm vac} = \sum_i \frac{\omega_i}{2} \vec{B} \cdot \vec{\sigma}_i, \qquad \omega_i = i\frac{16\omega_0}{N} \qquad (i \in [1, N]) \ .$$

$$H_{\nu\nu}(t) = \frac{\mu(t)}{2N} \sum_{i<j} (1 - \mathbf{v}_i \cdot \mathbf{v}_j)\, \vec{\sigma}_i \cdot \vec{\sigma}_j \ .$$

$$\mu(t) = 16\mu_0 \left(1 - \sqrt{1 - \left(\frac{R_\nu}{r_0 + t} \right)^2} \right)^2$$

Two-body term reduces with time

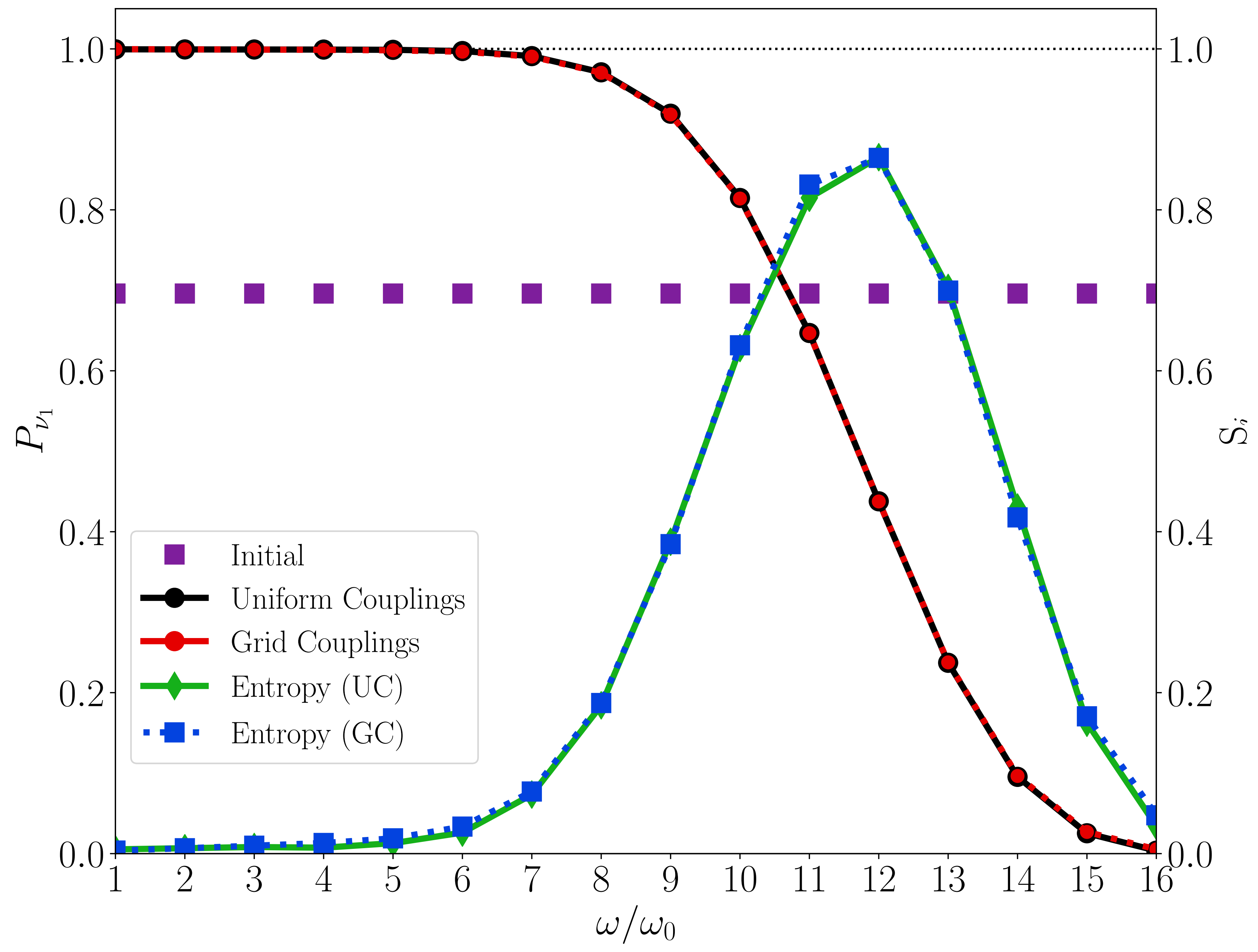
Note all magnetic fields aligned, can keep J_Z as a good quantum number.

J^2 commutes with $H_{\nu\nu}$

Initial State (1st case)

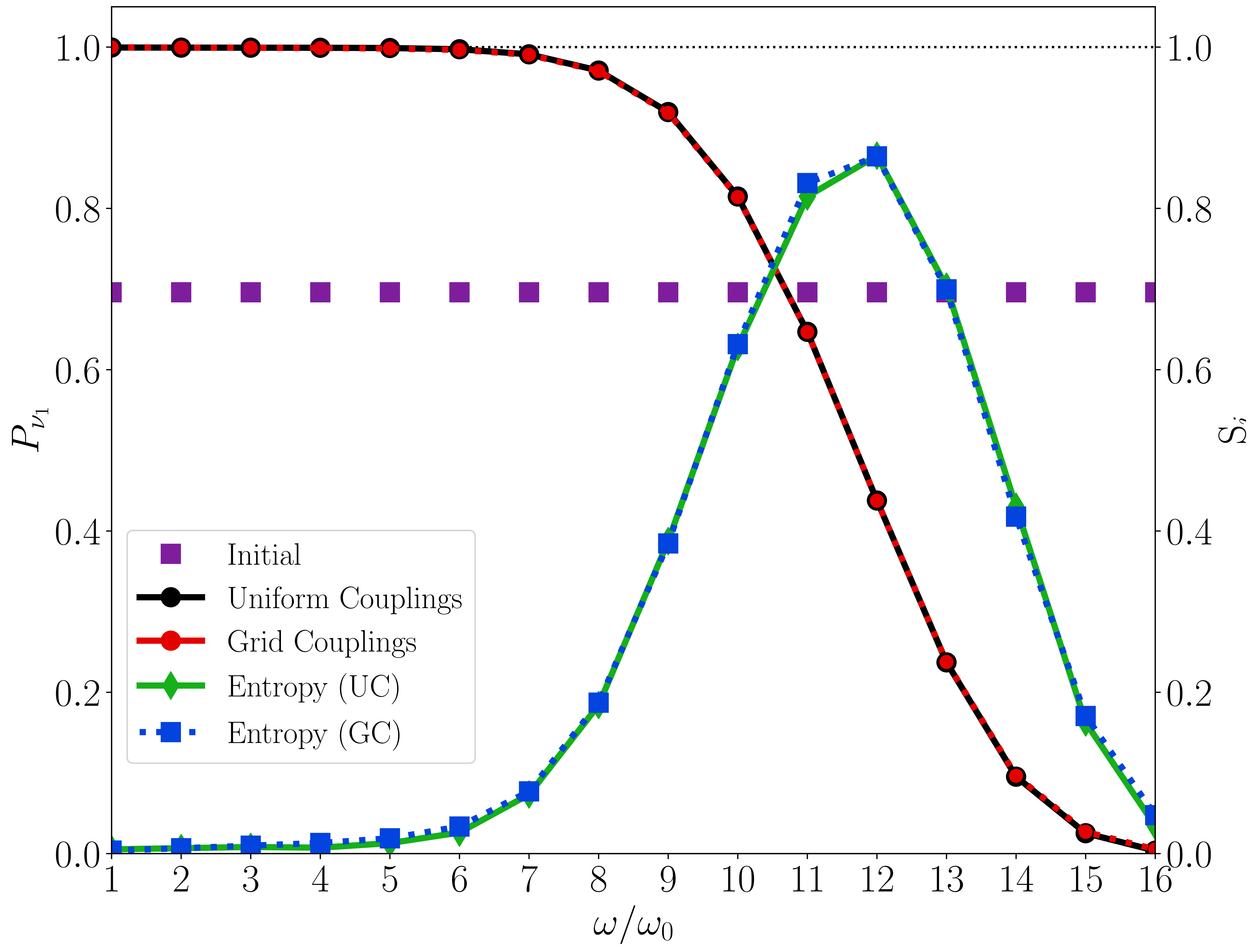
$$|\Psi(t=0;n)\rangle = \bigotimes_{i=1}^{n} |\nu_e\rangle_i \bigotimes_{j=n+1}^{N} |\nu_\tau\rangle_j \ .$$

Initial state all ν_e



Initial state all ν_e: all initial spins parallel

Spectral Split in full quantum treatment

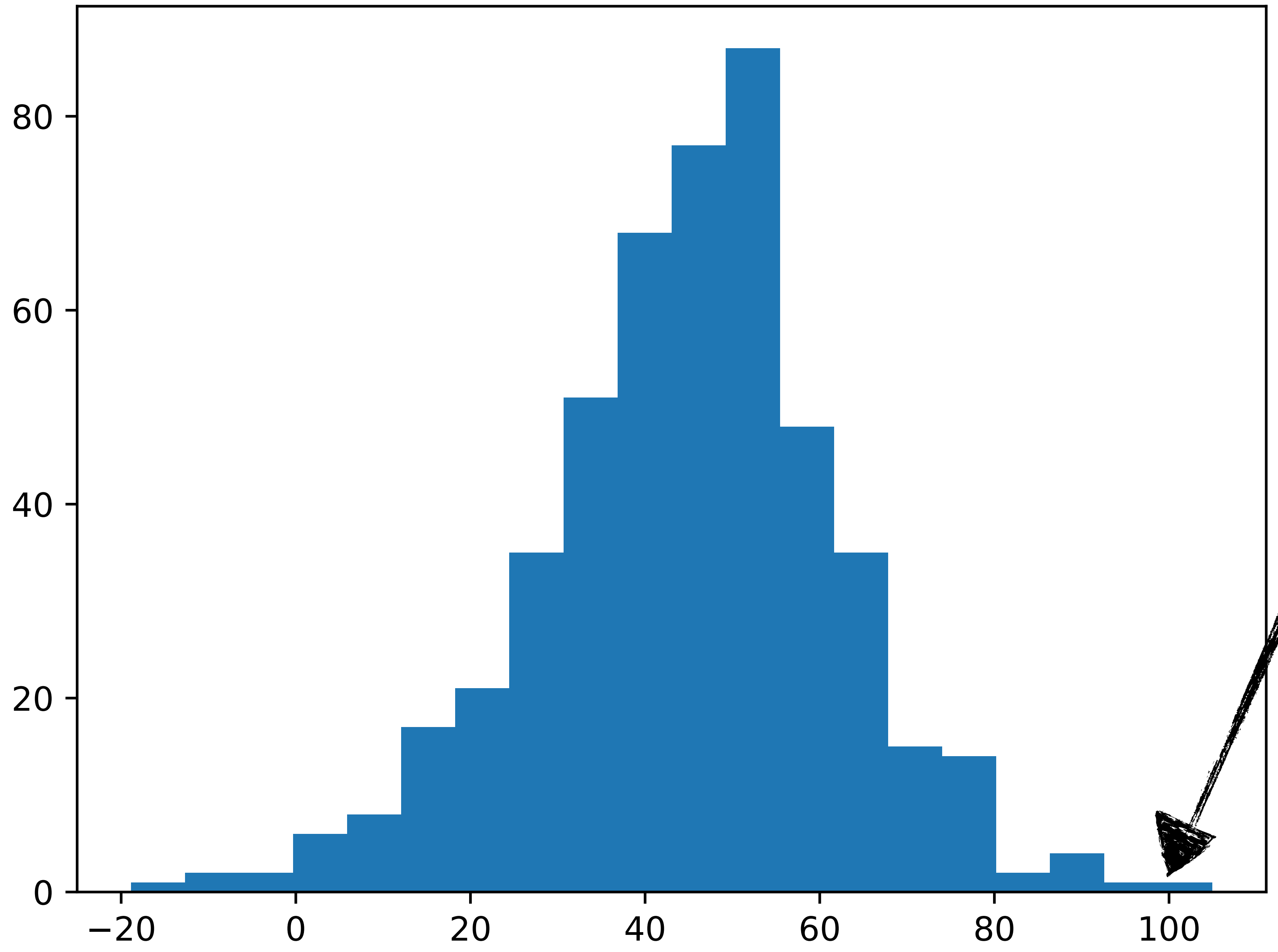


- Spectral Split observed!
- Agreement between mean-field and many-body
- Uniform coupling and grid couplings similar
- Entanglement entropy peaks near swap energy

Is this general or special behavior?

Properties of Initial State where spectral split observed

- All initial spins aligned: maximum of neutrino-neutrino interaction
- Initial state can be split into components of fixed M that do not mix
- Initial state is maximal energy state within each subspace



Density of states for N=12, # up = 8

Initial State component
w/ N=12, Nup=8

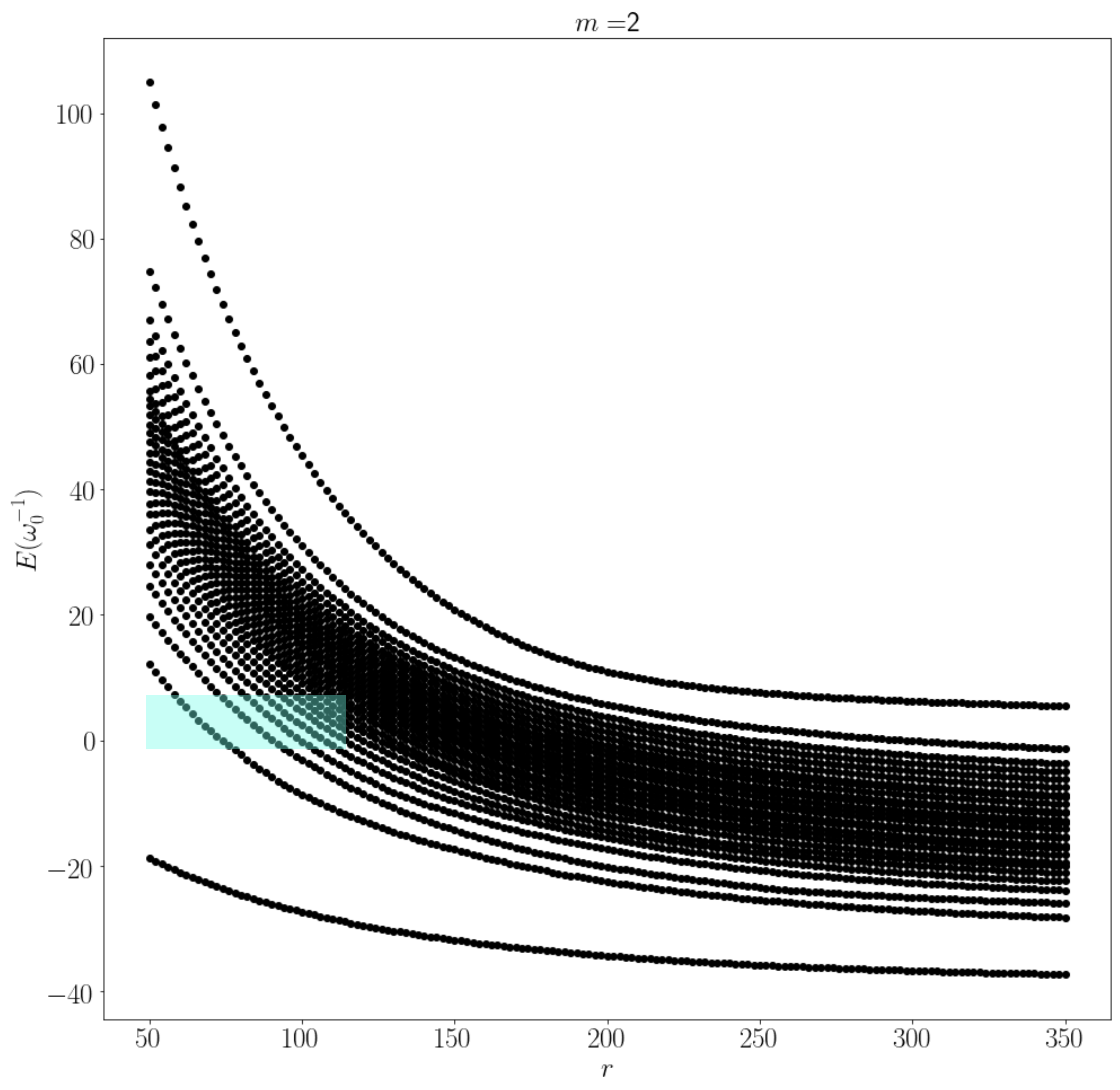
More typical initial product states

- More typically some finite fraction of both flavors
- (Neutronization burst ~90/10%, others 40/60%)
- Distributions spread in energy and angle
  with significant overlap

- All such initial states are far from
  the edges of the spectrum,
- Both the total spectrum and within individual M

This case is equivalent to adiabatic state preparation
within each subspace

Time-dependent spectrum

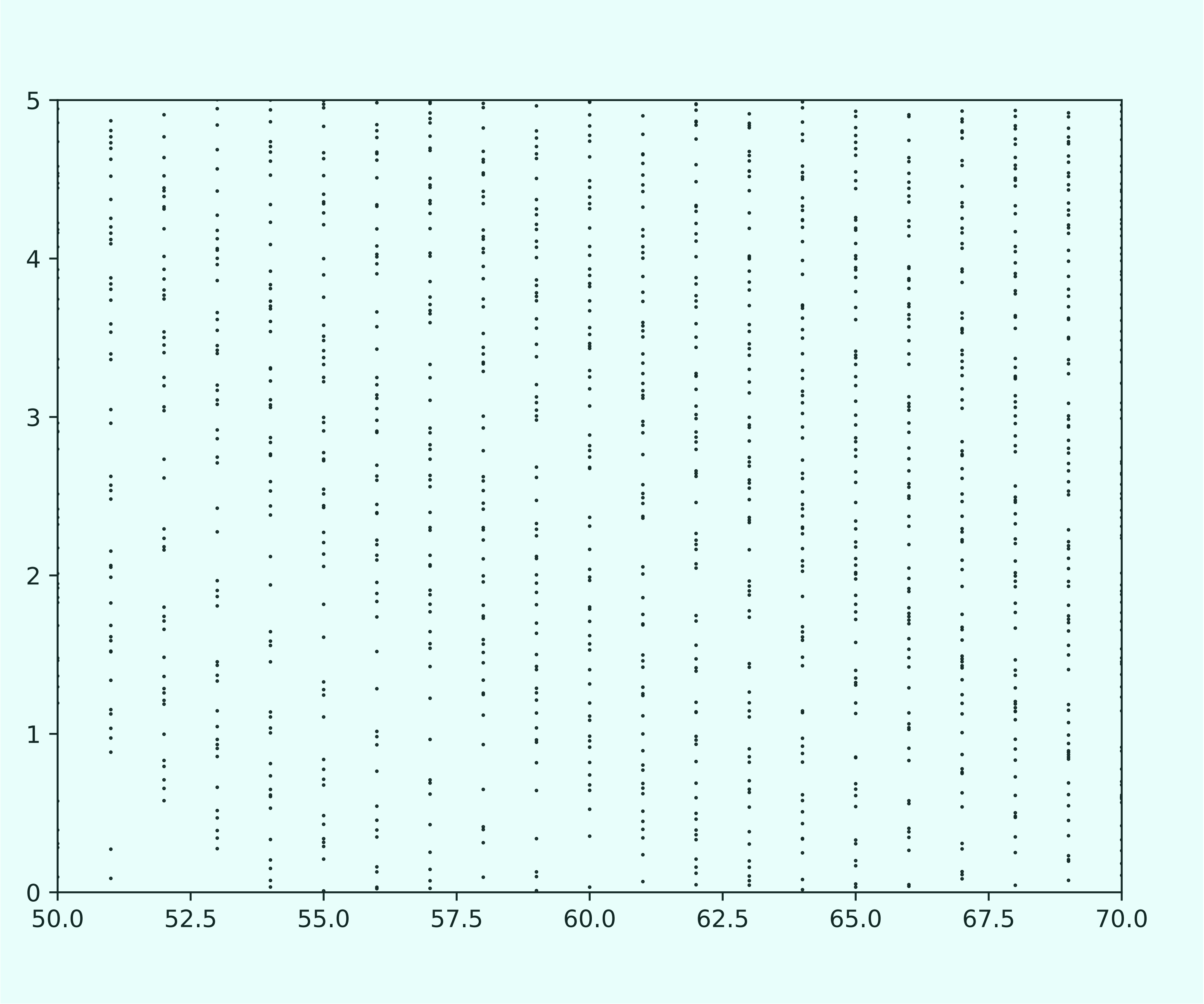


Energy levels vs time

Large Gap / Adiabatic evolution

Avoided level crossing
More general dynamics

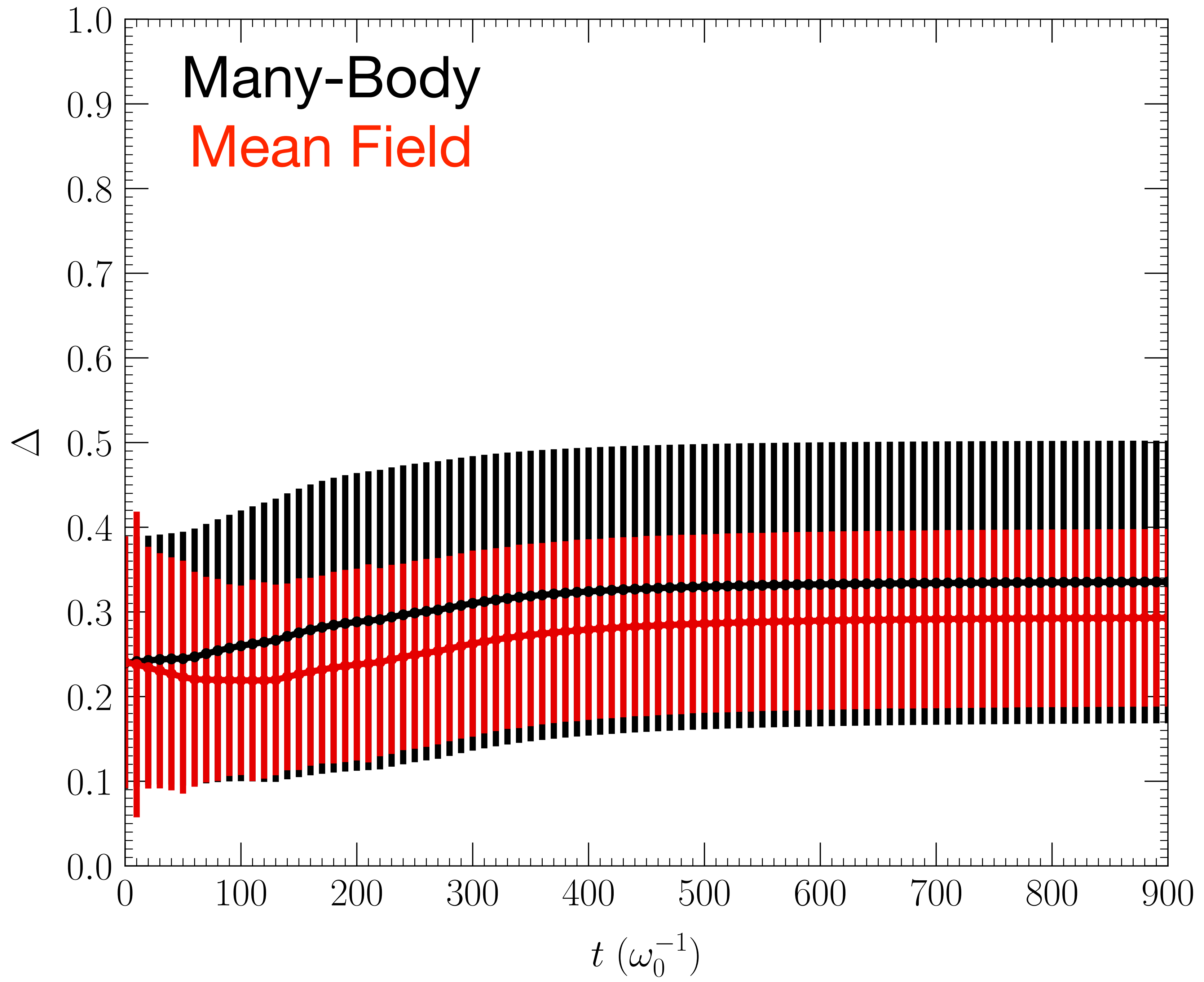
Large Gap / Adiabatic evolution
Spectral swap / Dynamical phase transition



Interior Spectrum

Energy and variance versus time

$$\Delta = \frac{\langle H \rangle - E_{min}}{E_{max} - E_{min}}$$



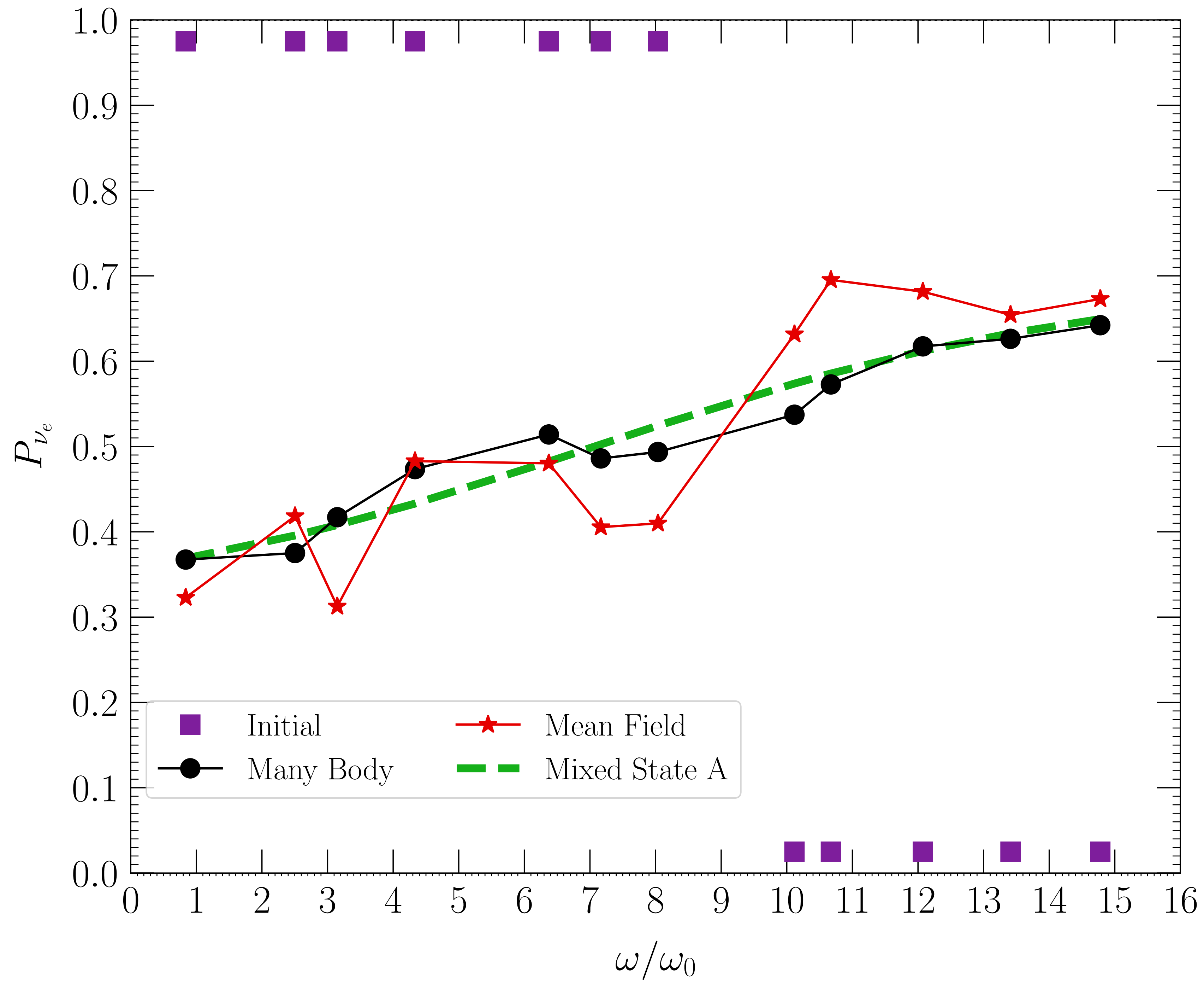
At end (large radius), Hamiltonian is H_1 only

Variance is $\dfrac{\langle H^2 \rangle - \langle H \rangle^2}{N} = (1/N) \sum_i w_i^2 [1 - \sigma_z(i)^2]$

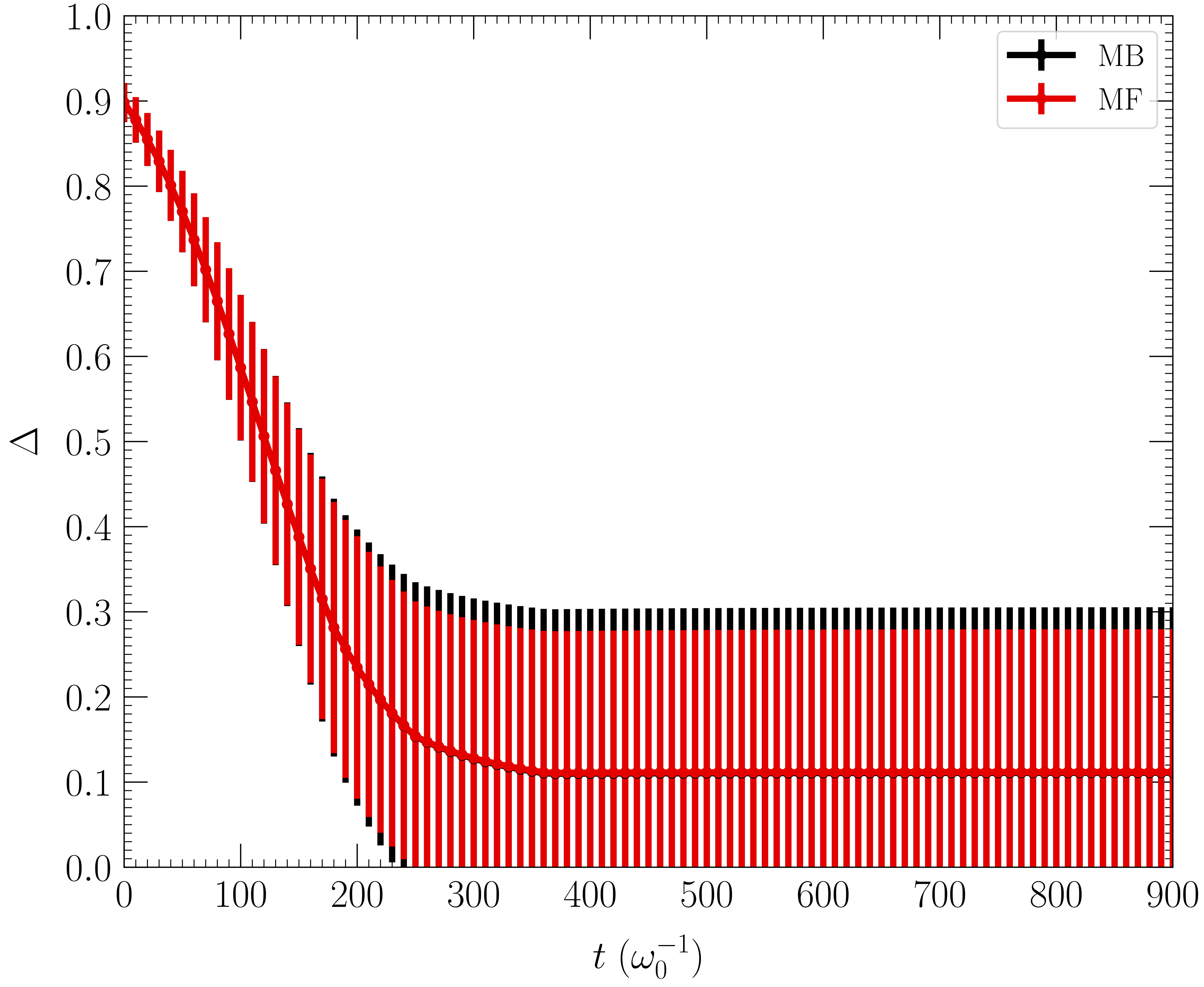
Knowledge of N moments = knowledge of all $\langle \sigma_z(i) \rangle$

'Perfect' spectral split: variance computed with final H same with initial and final state

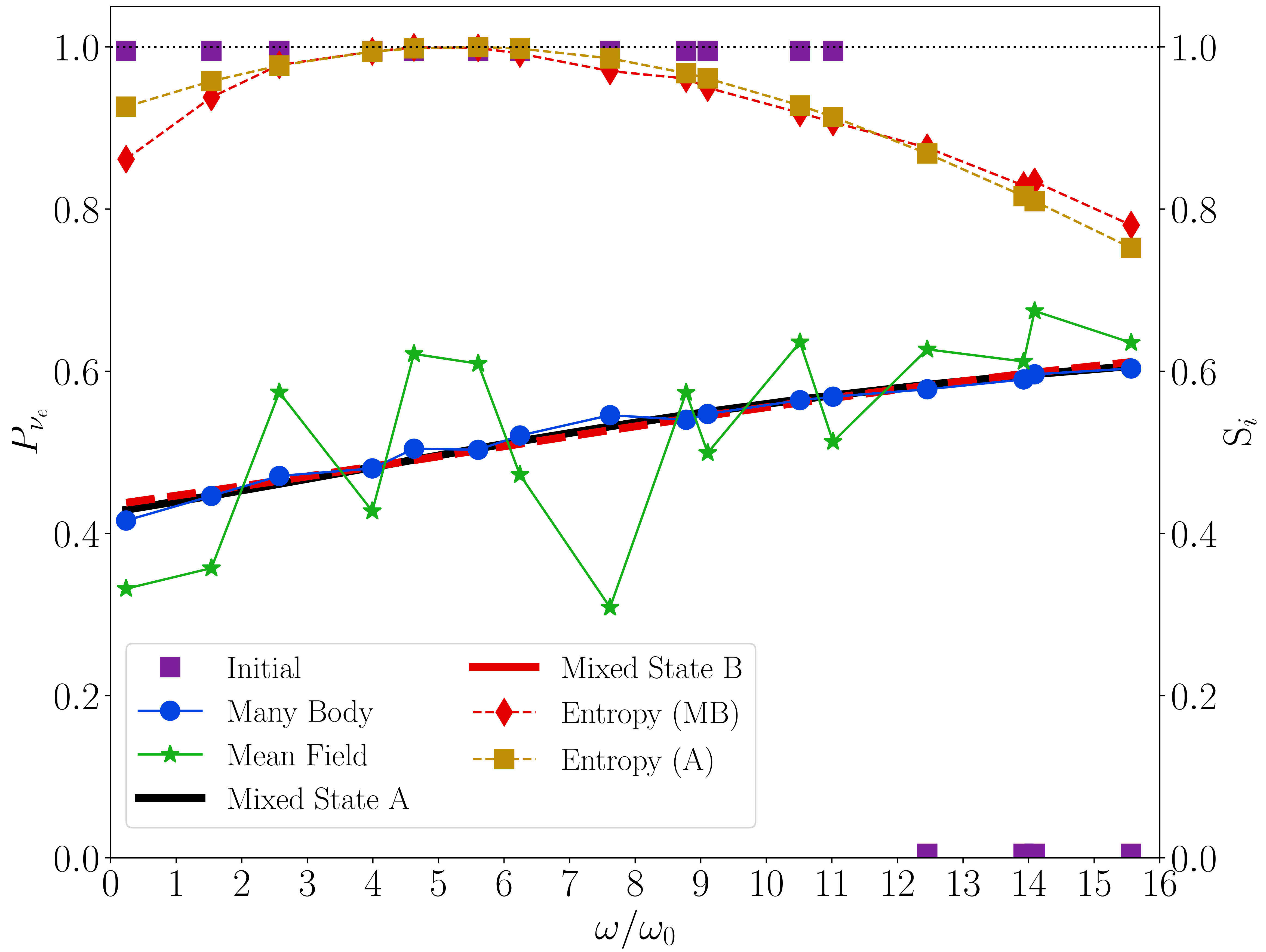
Single-spin expectation values (Initial and Final)



What does evolution look like in spectral swap case?
Mean Field works:



A `typical' case



Initial state all low-E ν_e; three highest states ν_μ

Mean Field and Many-Body differ significantly

- One-body expectation values can be fit with $\exp[-\tilde{\beta}E_i]$
- Overall fit shown
- Can be fit within subspace with different $\tilde{\beta}_M$

Knowledge of 1 moment apparently enough to reconstruct single spin expectation values

On occasion mean-field off in <H> by ~20%

Coherent Neutrinos Conclusion/Outlook:

- Mean-field solutions produced enticing
  neutrino flavor physics in dense neutrino environments

- Spectral splits survive in full many-body treatment
  when adiabatic evolution applies

- In general full quantum treatment modifies the solution
  from standard mean-field evolution

- Single-spin observables appear to be describable in
  a statistical treatment

- Hopes for improved mean-field like treatment (<< 2^N states)

- Intriguing analogues to traditional (local) quantum spin Hamiltonians