Bureau International des Poids et Mesures

Redefinition of the kilogram based on a physical constant

Richard Davis
rdavis@bipm.org

with thanks to BIPM colleagues

www.bipm.org

- Organized under terms of the Metre Convention (1875).
- Supported by 53 Member States and 28 Associates.
- Carries out a scientific programme overseen by the International Committee for Weights and Measures (CIPM).
- Scientific Sections include Mass, Electricity, Time/Frequency/Gravimetry, Chemistry, Ionizing Radiation.
- Publishes the *SI Brochure*, produced by the Consultative Committee for Units (CCU) of the CIPM.
- Changes to the SI units are recommended by the CCU in cooperation with other relevant technical CCs, for ultimate approval by the General Conference on Weights and Measures (CGPM), which is at the diplomatic level and meets every 4 years [next in 2011].

Topics to be presented

- Reminder of the SI units and quantities.
- Why in particular the kilogram must be redefined.
- The constraints on a new definition.
- The relation between the SI and the fundamental constants of physics (as listed by CODATA); the role of the fine-structure constant.
- When could/should the kilogram be redefined?

In an ideal world, units would be defined by fundamental constants

“Yet, after all, the dimensions of our earth and its time of rotation, though, relative to our present means of comparison, very permanent, are not so by physical necessity. The earth might contract by cooling, or it might be enlarged by a layer of meteorites falling on it, or its rate of revolution might slowly slacken, and yet it would continue to be as much a planet as before.

But a molecule, say of hydrogen, if either its mass or its time of vibration were to be altered in the least, would no longer be a molecule of hydrogen.

If, then, we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules.”

James Clerk Maxwell, 1870

6/7 of the SI base units at present

second (s): has the effect of fixing value of ν_{hfs} , the hyperfine-transition frequency of the caesium atom. [atomic clock]

metre (m): has the effect of fixing the value of the speed of light, c, with additional reference to s. [laser interferometry]

kilogram (kg): defined by assigning 1 kg to the mass of the international prototype of the kilogram. [artefact definition!]

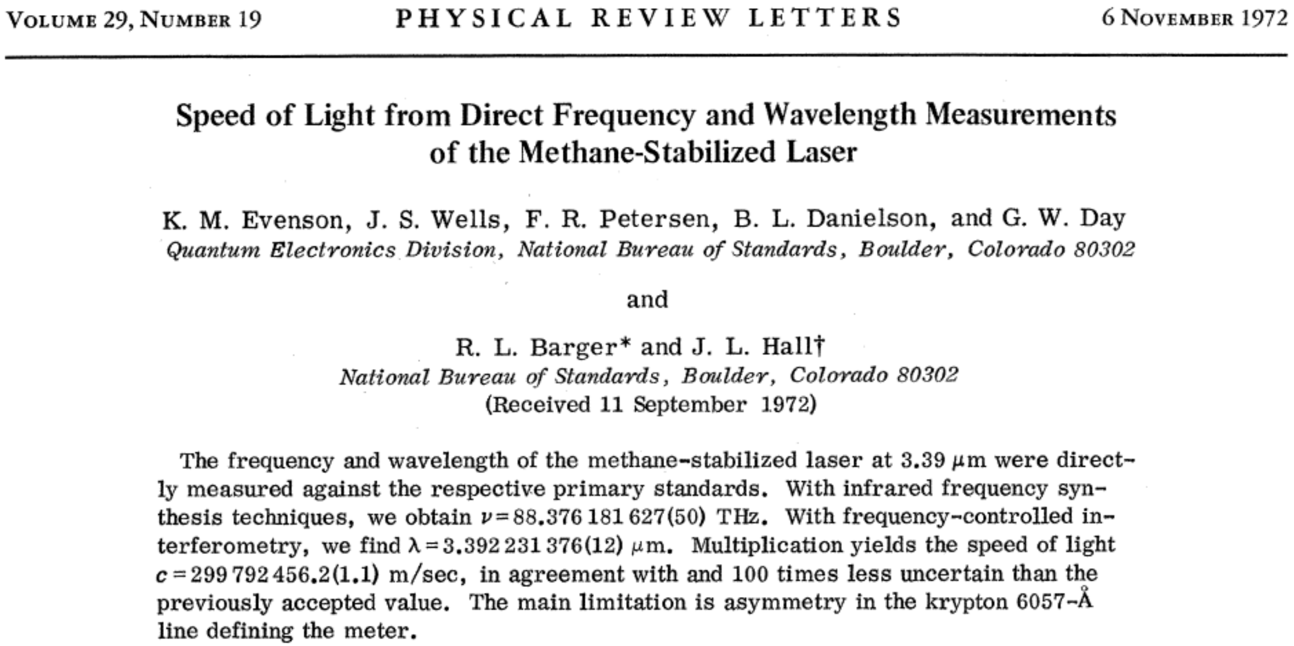
ampere (A): has the effect of fixing the value of μ_0 , the magnetic constant, with additional reference to kg, m, s

kelvin (K): has the effect of fixing the value of 273.16 K for T_{TPW}.

mole (mol): has the effect of fixing the value of the molar mass of ^{12}C to be $M(^{12}\mathrm{C}) = 0.012$ kg/mol. [note: $N_A = M(\mathrm{X})/m(\mathrm{X})$]

Before 1983, *c* was a measurable quantity

$$c = F_1\left(\nu_{\text{Cs-hfs}}, \lambda_{\text{Kr}}, \ldots\right)$$



VOLUME 29, NUMBER 19 PHYSICAL REVIEW LETTERS 6 NOVEMBER 1972

Speed of Light from Direct Frequency and Wavelength Measurements of the Methane-Stabilized Laser

K. M. Evenson, J. S. Wells, F. R. Petersen, B. L. Danielson, and G. W. Day
Quantum Electronics Division, National Bureau of Standards, Boulder, Colorado 80302

and

R. L. Barger* and J. L. Hall†
National Bureau of Standards, Boulder, Colorado 80302
(Received 11 September 1972)

The frequency and wavelength of the methane-stabilized laser at 3.39 μm were directly measured against the respective primary standards. With infrared frequency synthesis techniques, we obtain $\nu = 88.376\,181\,627(50)$ THz. With frequency-controlled interferometry, we find $\lambda = 3.392\,231\,376(12)$ μm. Multiplication yields the speed of light $c = 299\,792\,456.2(1.1)$ m/sec, in agreement with and 100 times less uncertain than the previously accepted value. The main limitation is asymmetry in the krypton 6057-Å line defining the meter.

"…in agreement with and 100 times less uncertain than the previously accepted value…limitation is asymmetry in the Kr…line defining the metre."

Since 1983, the SI value of *c* is *defined*

$$\lambda_{\mathrm{i}} = F_2\left(\nu_{\mathrm{Cs\text{-}hfs}}, c, \ldots\right)$$

Mise en pratique **(*MeP* ≈ practical realization) specifies a recommended set (λ_{i} , ν_{i})**
where $\lambda_{\mathrm{i}}\, \nu_{\mathrm{i}} = c$

Definition of the kilogram

3rd CGPM, 1901 :

“Le kilogramme est l’unité de masse;
il est égal à la masse du prototype international du kilogramme.”

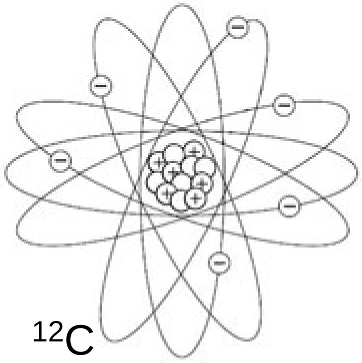
“The kilogram is the unit of mass;
it is equal to the mass of the international prototype of the kilogram.”

(international prototype manufactured in 1880s, put into service in 1889)

What the definition means

The mass in kilograms of any object X is given by :

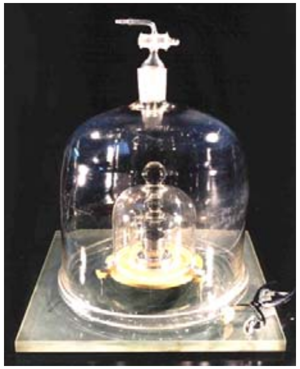
This ratio represents a <u>measurement</u> having an <u>experimental uncertainty</u>



÷ 10^{26}

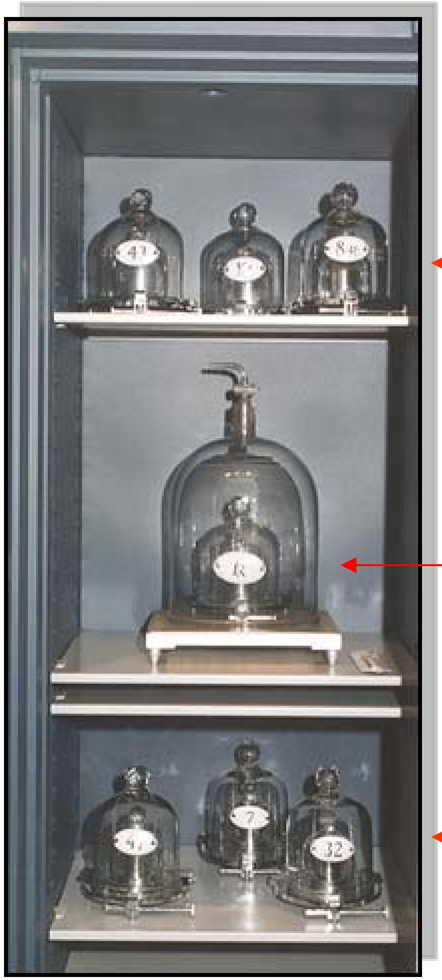
X 10^{1}

$$\{m_{\mathrm{X}}\}[\mathrm{kg}] = \left\{\frac{m_{\mathrm{X}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]$$



$$\{m_{\mathrm{X}}\}[\mathrm{kg}] = \left\{\frac{m_{\mathrm{X}}}{\cancel{m_{\mathrm{n}}}}\right\}\cdot\left\{\frac{\cancel{m_{\mathrm{n}}}}{\cancel{m_{\mathrm{n}-1}}}\right\}\cdot\ldots\cdot\left\{\frac{\cancel{m_{2}}}{\cancel{m_{1}}}\right\}\cdot\left\{\frac{\cancel{m_{1}}}{m_{\mathcal{K}}}\right\}\cdot[\mathrm{kg}]$$

Conservation of the international prototype



official copies (3 of 6)

international prototype

official copies (3 of 6)

Consequences of 1901 definition and 2007 physics

Even though m_X may represent something fundamental-- m_e, m_p, $m(^{12}\mathrm{C})$, $(\hbar c/G)^{1/2}$ etc., nevertheless

$$\{m_\mathrm{X}\}[\mathrm{kg}] = \left\{\frac{m_\mathrm{X}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]$$

This curious situation persists largely because:

Experimental uncertainties of $\{m_\mathrm{X}/m_{\mathcal{K}}\}$ are still much larger than the precision of the best commercial balances;

Inconclusive evidence that $\{m_\mathrm{X}/m_{\mathcal{K}}\}$ is changing, where m_X is "something" that can reasonably be considered more stable than the mass of $\mathcal{K}$;

When comparing molecular, atomic, and subatomic masses amongst themselves, it is traditional to use the dalton, Da, a non-SI unit which avoids correlations to $m_{\mathcal{K}}$.

1927

from ***La Création du BIPM et son œuvre***

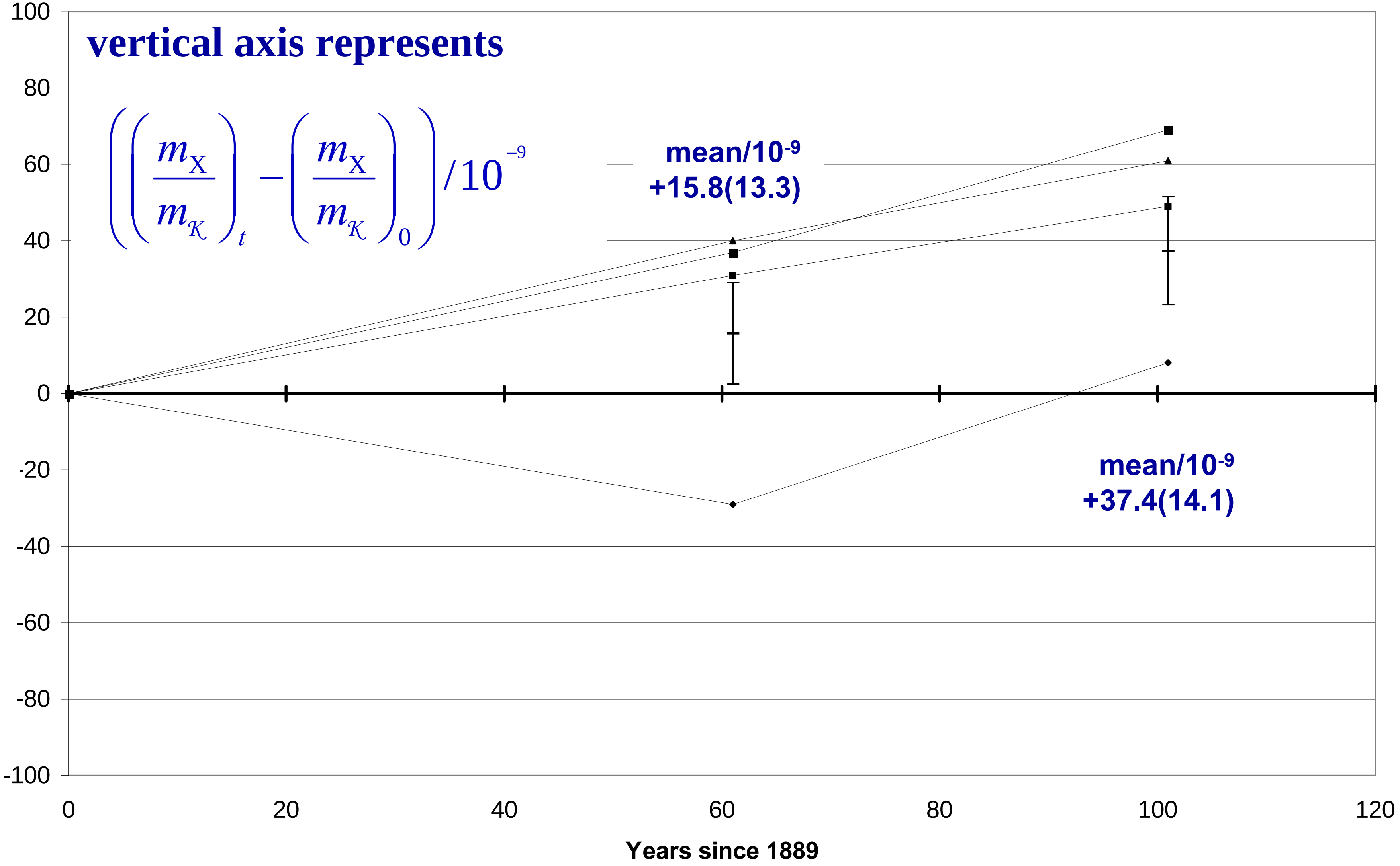
«Il semble donc que l'unité de masse soit guarantie au cent-millionième près pour plus de 10 000 ans, et cette durée est à peine commencée.

Sans doute, bien avant qu'elle soit écoulée, les travaux exécutés par les métrologistes des siècles futurs auront conduit à des solutions encore plus parfaites.»

Ch.-Ed. Guillaume

BIPM Director
Nobel laureate

Ensemble average of $\mathcal{K}$ and four oldest official copies



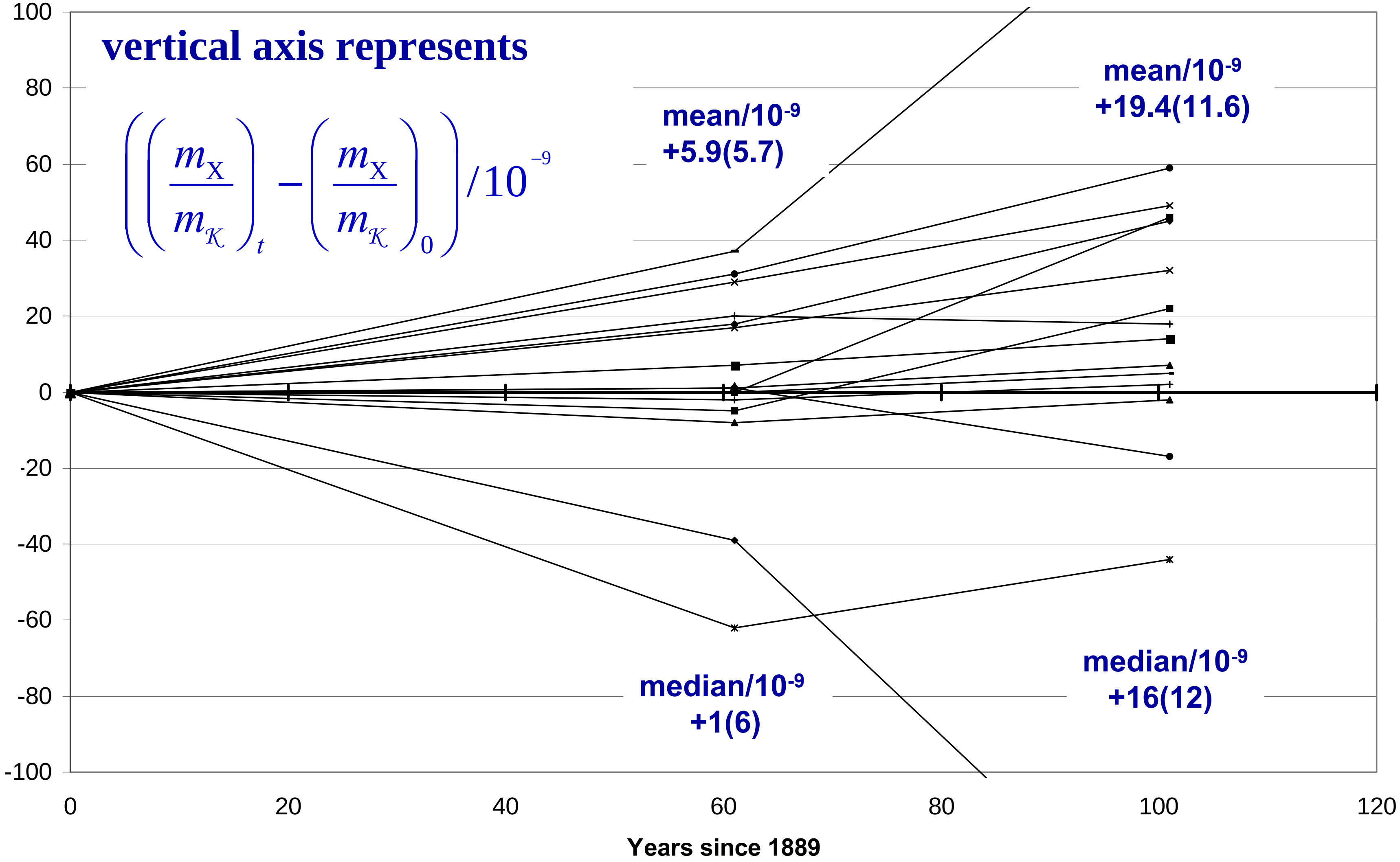
vertical axis represents

$$\left(\left(\frac{m_X}{m_{\mathcal{K}}}\right)_t-\left(\frac{m_X}{m_{\mathcal{K}}}\right)_0\right)/10^{-9}$$

mean/10^{-9}
+15.8(13.3)

mean/10^{-9}
+37.4(14.1)

Ensemble average of $\mathcal{K}$ and the oldest national prototypes



vertical axis represents

$$\left(\left(\frac{m_X}{m_{\mathcal{K}}}\right)_t - \left(\frac{m_X}{m_{\mathcal{K}}}\right)_0\right)/10^{-9}$$

mean/10^{-9}
+5.9(5.7)

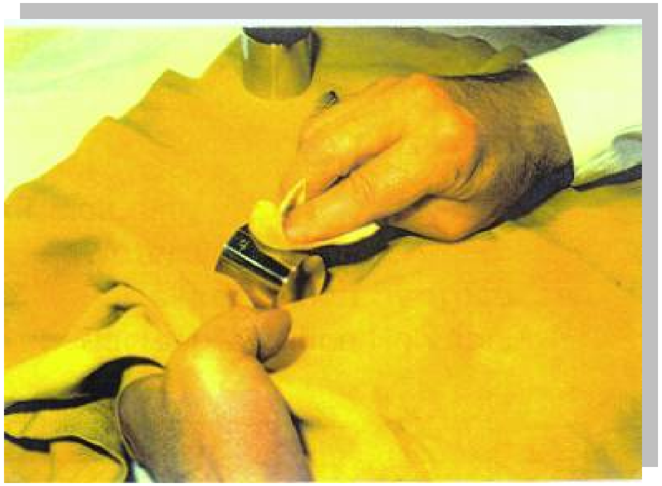
mean/10^{-9}
+19.4(11.6)

median/10^{-9}
+1(6)

median/10^{-9}
+16(12)

Summary: real problems with the artefact definition

- mass ratios of "identical" prototypes have clearly changed during a century;
- as experiments improve, we will find evidence of $\{m_{\mathrm{FC}}/m_{\mathcal{K}}\}$ change with time;
- mass artefacts suffer surface contamination over long periods of time;



in 1989, the International Committee for Weights and Measures (CIPM) decided to interpret the 1901 definition of the kilogram as referring to the mass of $\mathcal{K}$ just after cleaning and washing using “the BIPM method”

Mass metrologists are not alone

The definition of the kilogram has an impact on other areas of science and metrology:

- Electrical metrology
- Chemistry
- Physics (especially the fundamental constants)

Why electrical metrologists care about the kilogram definition-1

“The **ampere** is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 x 10^{-7} **newton** per metre of length.”

The definition ensures the consistency of SI units.
For instance, thanks to this definition, the SI unit “watt” is the same for

$I^{2}\cdot R$ **(electrical power) and** $m\cdot a\cdot v$ **(mechanical power)**

The ampere definition implicitly fixes the permeability of vacuum, μ_0, (sometimes called the magnetic constant) and the impedence of vacuum

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2} \quad (1\ \mathrm{N/A^2} = 1\ \mathrm{H/m})\ ; \quad \mu_0 c \equiv Z_0 = 376.7...\ \Omega$$

Josephson Effect

Voltage via the Josephson effect

According to theory,

$$\frac{U_n}{f} = n\left(\frac{h}{2e}\right)$$

The Josephson junction is a frequency-to-voltage converter, governed by a combination of fundamental constants.

Niemeyer and Grimm, PTB
Metrologia, 1988

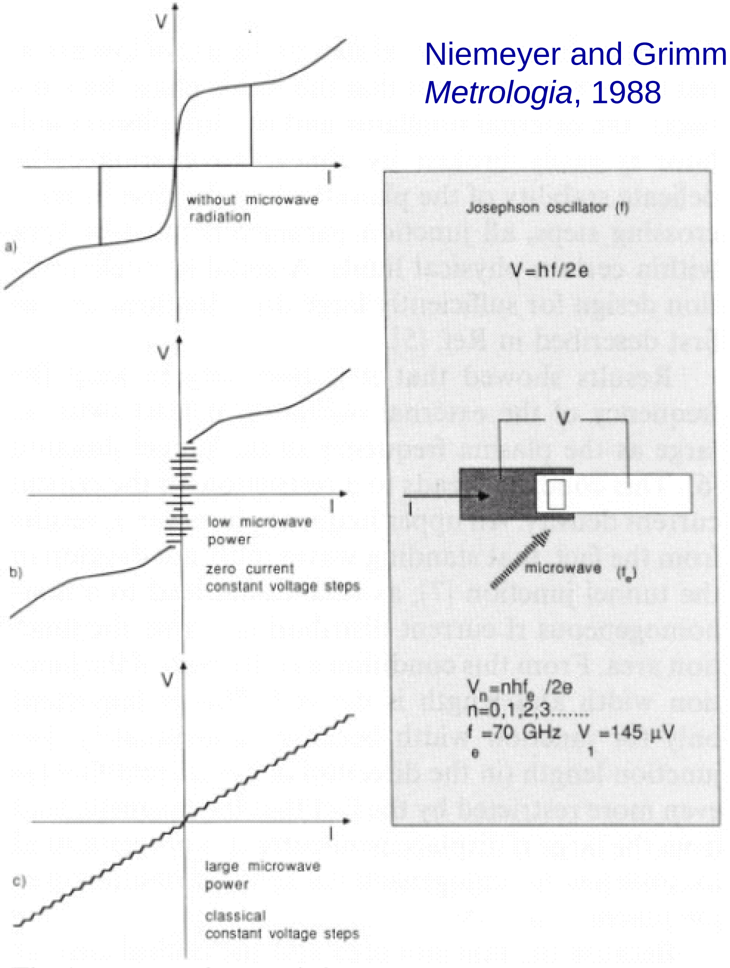


Fig. 1a–c. *I-V* characteristic of a highly hysteretic Josephson tunnel junction. **a** without microwave radiation; **b** with low-power microwave radiation; **c** with high-power microwave radiation

Quantized Hall Effect

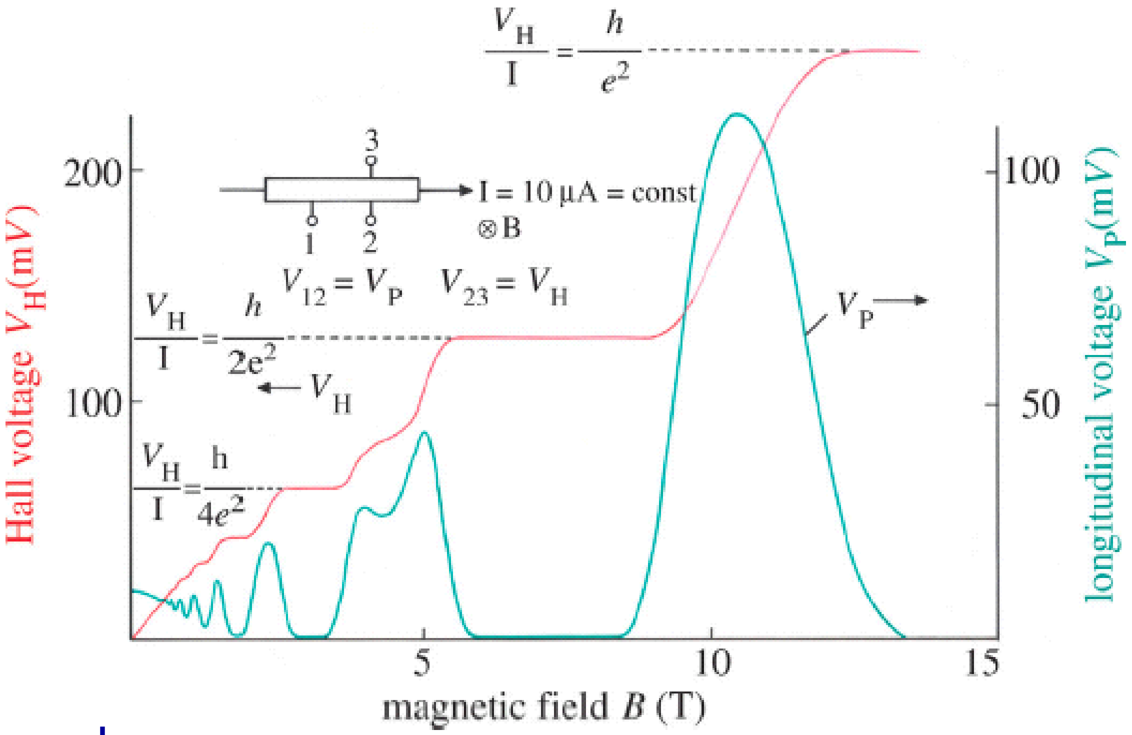
Resistance via the quantized-Hall effect

According to theory,

$$R_{\mathrm{H}}(i) = \frac{1}{i}\left(\frac{h}{e^2}\right)$$

$$\approx \frac{26\ \mathrm{k}\Omega}{i}$$

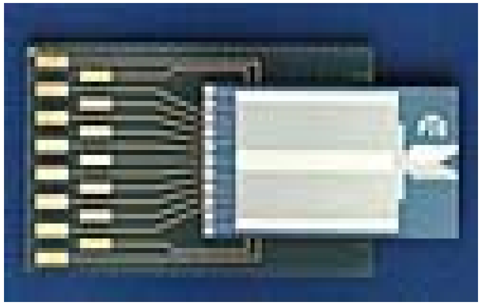
where *i* is the plateau number



From von Klitzing, *Phil. Trans. R. Soc. A* (2005)

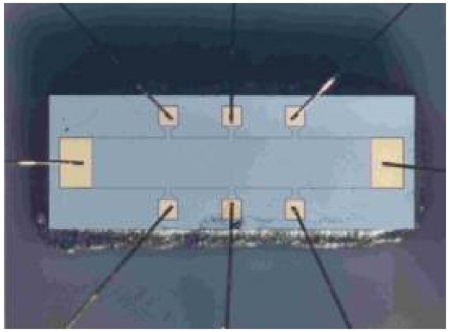
“1990 conventional representations”

Josephson effect



$$U_J(n) = \frac{n\,f}{K_J}, \qquad K_J = \frac{2e}{h}$$

quantized-Hall effect



$$R_H(i) = \frac{R_K}{i}, \qquad R_K = \frac{h}{e^2}$$

Conventional (non SI values)

$K_{J\text{-}90} \equiv 483\,597.9$ GHz/V

$R_{K\text{-}90} \equiv 25\,812.807\ \Omega$

Resumé

Present voltage and resistance metrology rely on quantum-mechanical phenomena and two 'fundamental' constants of physics.

Josephson constant, K_J ,
and
von Klitzing constant, R_K .

According to present knowledge, $K_J = 2e/h$ and $R_K = h/e^2$.

Therefore, <u>defining</u> the values of *h* and of *e*

- **defines the value of the Josephson constant,**
- **defines the value of the von Klitzing constant,**

thereby eliminating the need for conventional (non SI) values, $K_{J\text{-}90}$ and $R_{K\text{-}90}$ that are used today.

But…

c : speed of light in vacuum. This already has a fixed value in the SI.

μ_0 already has a fixed value in the SI due to the definition of the ampere

Therefore, it is impossible for *e* , *h* to be fixed as well:

$$\alpha = \frac{\mu_0 c e^2}{2h} = \frac{Z_0}{2R_\mathrm{K}}$$

Choices must be made.

Many interesting proposals, including those of the *Académie des Sciences*

A new SI?

http://www.bipm.org/en/committees/cc/ccu/

Presentation by Prof. Ian Mills (president of CC Units)

Present SI

second: ν_{hfs}(Cs)

metre: second and c

kilogram: international prototype

ampere: second, metre, kilogram and μ_0

kelvin: triple point of water (V-SMOW)

mole: number of atoms in 0.012 kg of ^{12}C

New SI ?

second: ν_{hfs}(Cs)

metre: second and c

kilogram: second, metre, h

ampere: second, e

kelvin: second, metre, kilogram, k_B

mole: a fixed number of entities

Constraint

$$\alpha = \frac{\mu_0 c e^2}{2h}$$

where α is measured and dimensionless (independent of unit systems)

Why chemists care about the kilogram definition

“The **mole** is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 **kilogram** of carbon 12; its symbol is ‘mol’.

“When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.”

This definition is closely linked to the Avogadro Constant

$$N_{\mathrm{A}} = \frac{0.012\left[\mathrm{kg}\cdot\mathrm{mol}^{-1}\right]}{m\left({}^{12}\mathrm{C}\right)[\mathrm{kg}]} = \frac{molar\ mass\ of\ x}{mass\ of\ x}$$

The dalton (example)

$$\{m_{^{12}\mathrm{C}}\}[\mathrm{kg}] = \left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}] \qquad = 12[\mathrm{u}]$$

$$\{m_\mathrm{e}\}[\mathrm{kg}] = \left\{\frac{m_\mathrm{e}}{m_{^{12}\mathrm{C}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}] \quad = 12\left\{\frac{m_\mathrm{e}}{m_{^{12}\mathrm{C}}}\right\}[\mathrm{u}]$$

From 2006 CODATA adjustment

quantity	relative uncertainty (ppm)
$m_{^{12}\mathrm{C}}$	0.05
m_e	0.05
$m_\mathrm{e} / m_{^{12}\mathrm{C}}$	0.000 42

(1 u = 1 Da)

$$[\mathrm{u}] = \frac{1}{12}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]$$

Summary of why physicists care about the kg definition

- An artefact manufactured in the 19th century has no interest for 21st century physics.

- Comparison to the international prototype is the dominant uncertainty in the SI values for many <u>fundamental constants</u> of physics; therefore: either large covariances must be taken into account or non-SI units like the dalton must be used.

Corollary: SI values of many fundamental constants change whenever there is a newly-measured link to $\mathcal{K}$,, and yet the link to $\mathcal{K}$ is not “fundamental”.

Large correlations due to links to a macroscopic kg

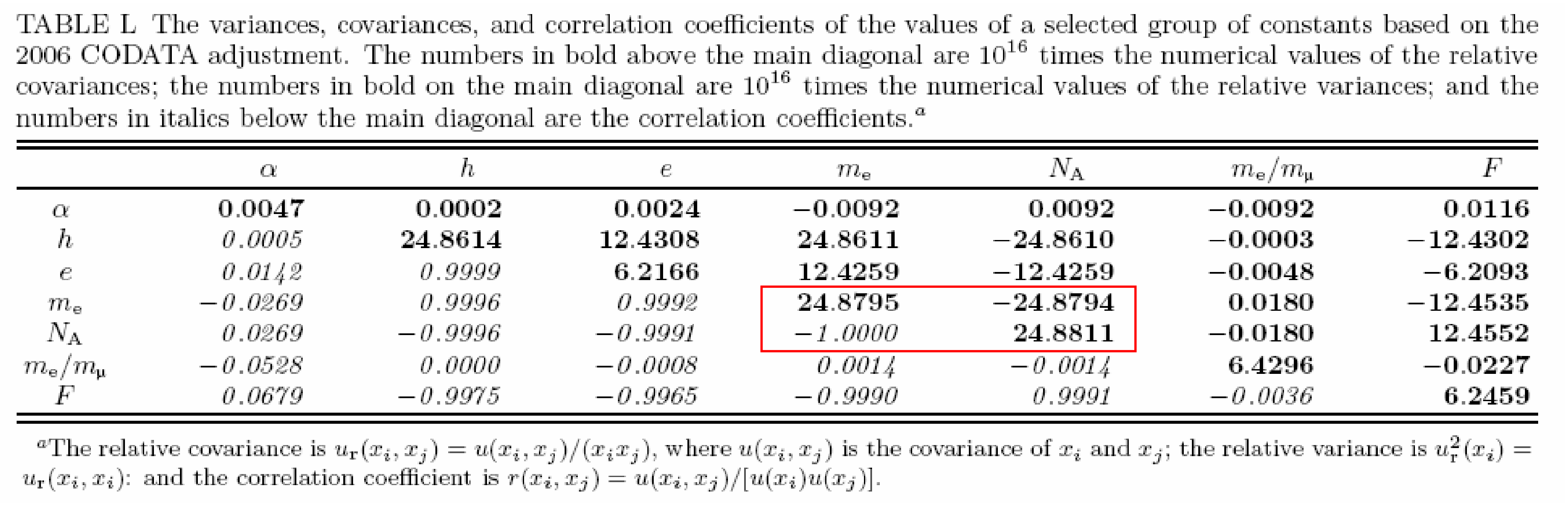


TABLE L. The variances, covariances, and correlation coefficients of the values of a selected group of constants based on the 2006 CODATA adjustment. The numbers in bold above the main diagonal are 10^{16} times the numerical values of the relative covariances; the numbers in bold on the main diagonal are 10^{16} times the numerical values of the relative variances; and the numbers in italics below the main diagonal are the correlation coefficients.[a]

	α	h	e	m_e	N_A	m_e/m_μ	F
α	**0.0047**	**0.0002**	**0.0024**	**−0.0092**	**0.0092**	**−0.0092**	**0.0116**
h	*0.0005*	**24.8614**	**12.4308**	**24.8611**	**−24.8610**	**−0.0003**	**−12.4302**
e	*0.0142*	*0.9999*	**6.2166**	**12.4259**	**−12.4259**	**−0.0048**	**−6.2093**
m_e	*−0.0269*	*0.9996*	*0.9992*	**24.8795**	**−24.8794**	**0.0180**	**−12.4535**
N_A	*0.0269*	*−0.9996*	*−0.9991*	*−1.0000*	**24.8811**	**−0.0180**	**12.4552**
m_e/m_μ	*−0.0528*	*0.0000*	*−0.0008*	*0.0014*	*−0.0014*	**6.4296**	**−0.0227**
F	*0.0679*	*−0.9975*	*−0.9965*	*−0.9990*	*0.9991*	*−0.0036*	**6.2459**

[a] The relative covariance is $u_r(x_i, x_j) = u(x_i, x_j)/(x_i x_j)$, where $u(x_i, x_j)$ is the covariance of x_i and x_j; the relative variance is $u_r^2(x_i) = u_r(x_i, x_i)$: and the correlation coefficient is $r(x_i, x_j) = u(x_i, x_j)/[u(x_i)u(x_j)]$.

From CODATA 2006 recommendation

http://physics.nist.gov/cuu/Constants/JPCRDMTN2008.pdf

Present and future definition of the kg

At present:

**The kilogram is the unit of mass;
it is equal to the mass of the
international prototype of the kilogram.**

In the future:

The kilogram, unit of mass, is such that the Planck constant is equal to exactly 6.626 06X XX $\times 10^{-34}$ joule second.

How can we realize the new definition?

Wanted: an experiment to measure F_2 to about 10^{-8} (10 µg/kg), where

$$\frac{h}{m(\mathcal{K})} = F_2\left(\nu_{\text{Cs-hfs}}, c, \ldots\right)$$

The experiments with the highest claimed accuracy are:

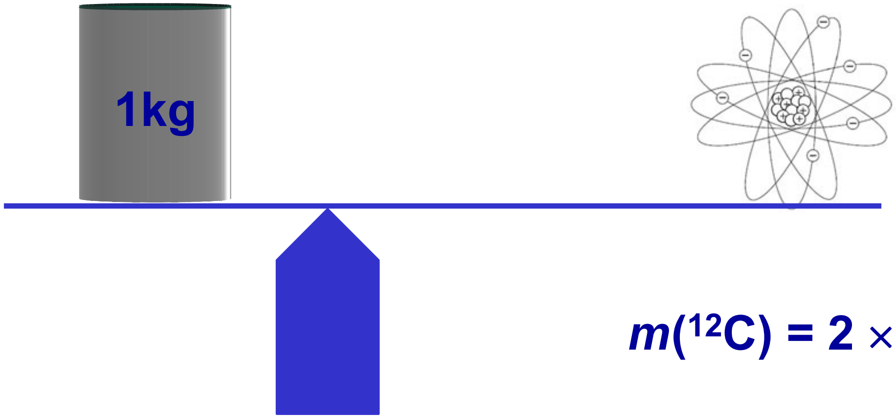
watt balance, to yield *h* and XRCD (IAC) to yield N_A

Q. why N_A?
A. $N_A h$ is already known with negligible uncertainty.

a measurement of N_A thus gives us *h* to comparable uncertainty

A large mismatch between 1 kg and the mass of a carbon-12 atom

$$\frac{1}{N_{\mathrm{A}}} = m\left({}^{12}\mathrm{C}\right)\frac{10^3}{12}[\mathrm{mol}]$$



$m(^{12}\mathrm{C}) = 2 \times 10^{-26}$ **kg**

recall: uncertainty of m_e / $m(^{12}\mathrm{C})$ is negligible in this context (0.42 ppb)

N_A by the X-ray Crystal Density (XRCD) method

n = number of ^{28}Si atoms in the sphere:

$$n=\left\{\frac{m_{\mathrm{sph}}}{m_{^{28}\mathrm{Si}}}\right\}=\left\{\frac{m_{\mathrm{sph}}}{m_{\mathcal{K}}}\right\}\left\{\frac{m_{\mathcal{K}}}{m_{^{12}\mathrm{C}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{^{28}\mathrm{Si}}}\right\}$$

$$n=8\frac{V}{a^3}$$

V = volume of sphere

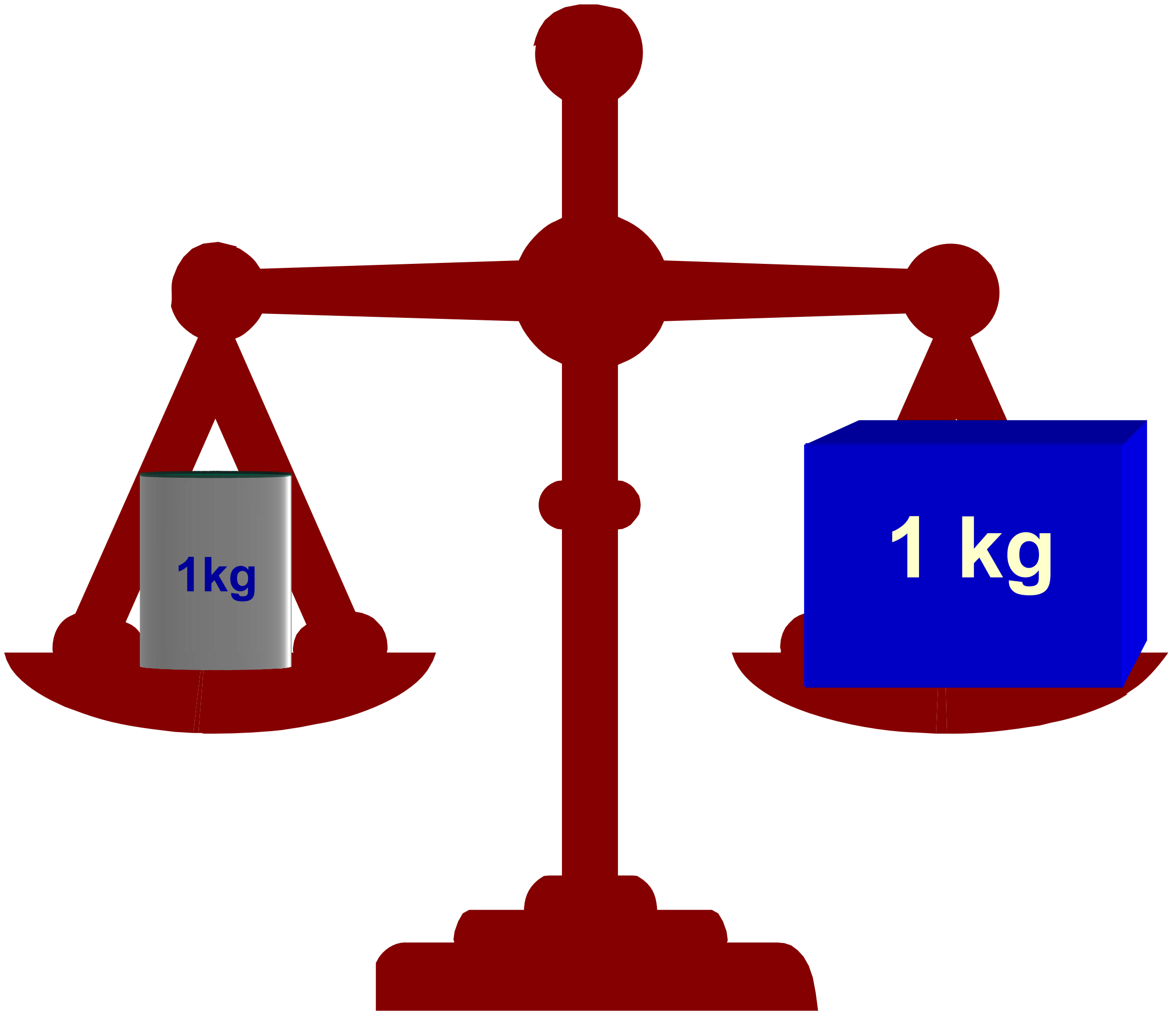
a^3 = volume of unit cell

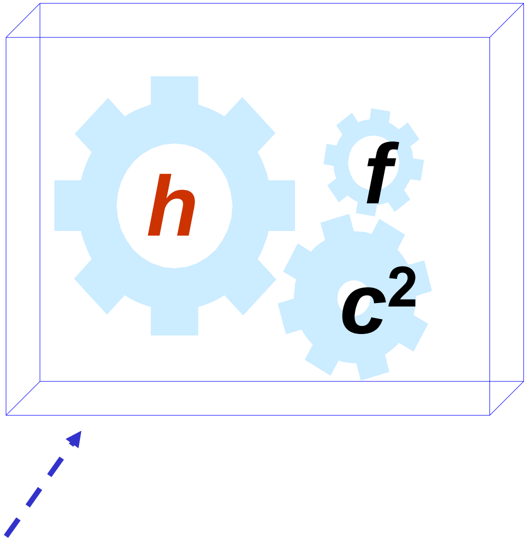
$$\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}=\left\{\frac{m_{\mathrm{sph}}}{m_{\mathcal{K}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{^{28}\mathrm{Si}}}\right\}\frac{a^3}{8V}$$

mass comparator ; mass spectrometer
X-ray interferometer; optical interferometer
purity: chemical, atomic, crystallographic

$$\frac{1}{N_{\mathrm{A}}}=\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}\frac{10^3}{12}[\mathrm{mol}]=\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]\cdot\frac{1}{12}\left[\frac{\mathrm{mol}}{\mathrm{kg}}\right]\cdot 10^3$$

How the Planck constant can be linked to the kilogram





$$m = h\frac{f}{c^2}$$

de Broglie-Compton equation;

watt balance equation for 1 kg is “similar”.

Watt Balance

Part 1

$$m_{\mathcal{K}}\,g = I \cdot f\left(\vec{B}, \vec{r}\right)$$

g is local grav. accel.; ***I*** is a current

Part 2

$$U = v \cdot f\left(\vec{B}, \vec{r}\right)$$

U is a voltage; ***v*** is a velocity

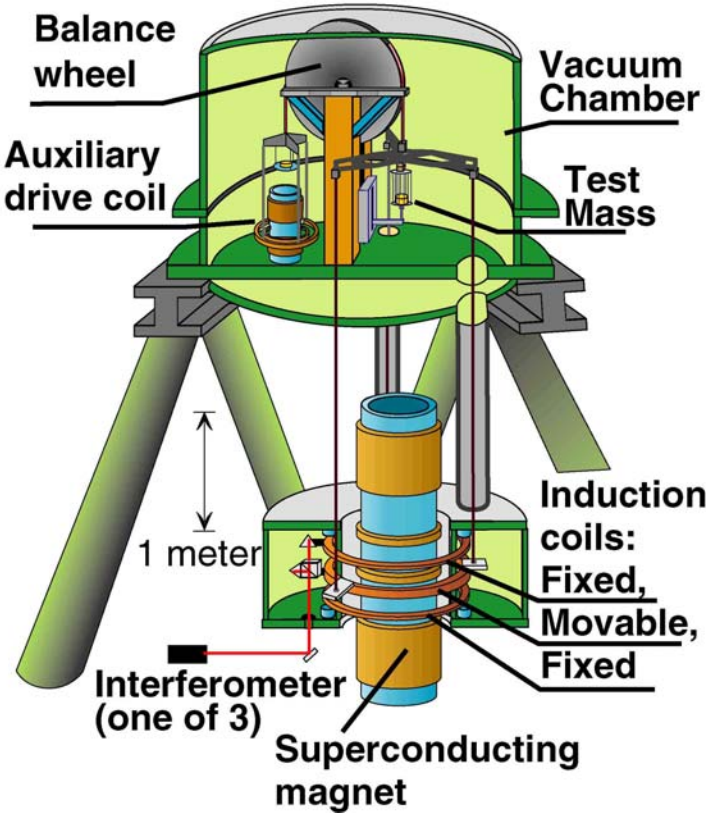
$$m_{\mathcal{K}}\,g v = I U$$

[Watt] = [Watt]

$$I = \frac{U'}{R}$$

$$\frac{m_{\mathcal{K}}}{h} = \left(\frac{U'_{90}\,U_{90}}{R_{90}\,g v} \cdot (\mathrm{cwnu}) \right)$$

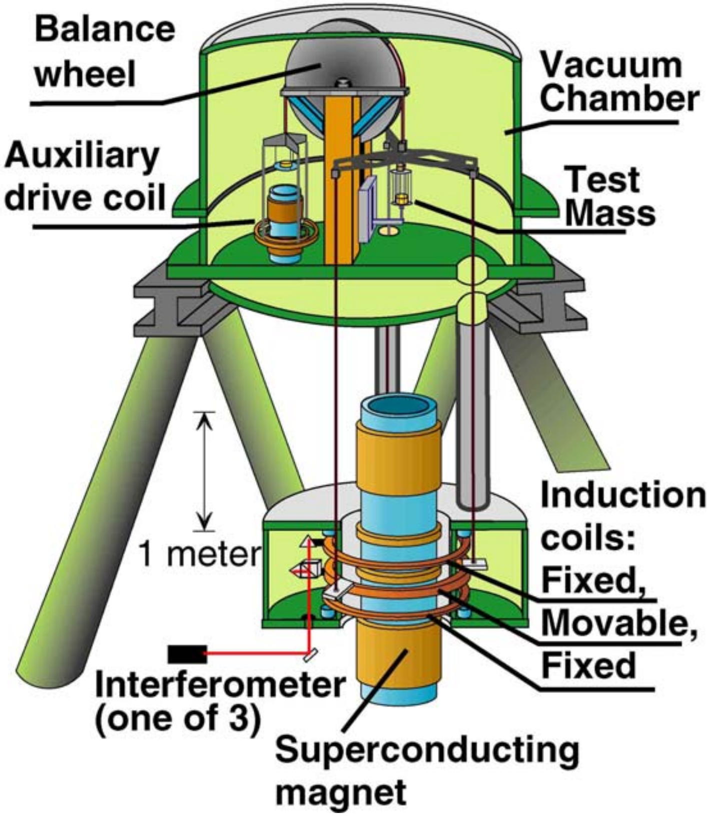
(cwnu) means "constants with no uncertainty"

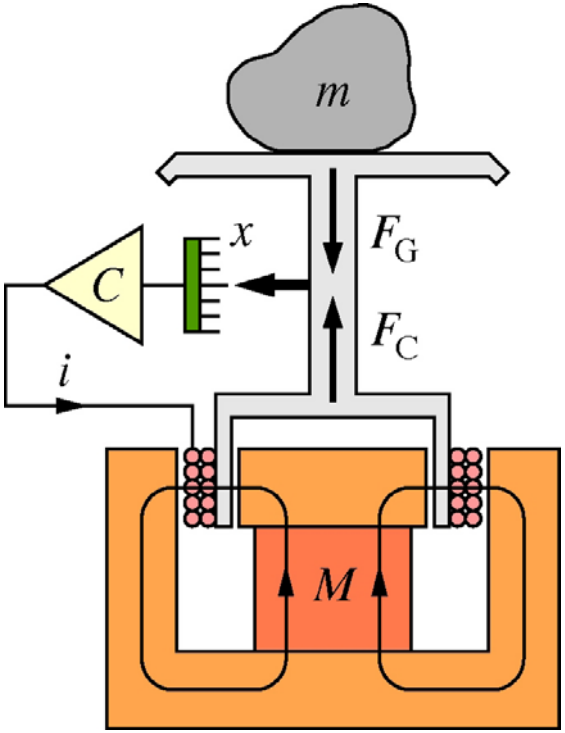


Schematic of NIST apparatus

(courtesy of NIST)

Another way to look at Part 1…



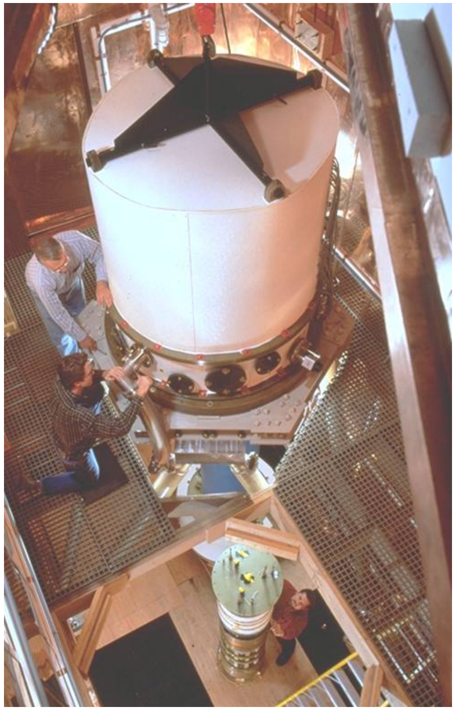


from Mettler-Toledo documentation describing servocontrol.

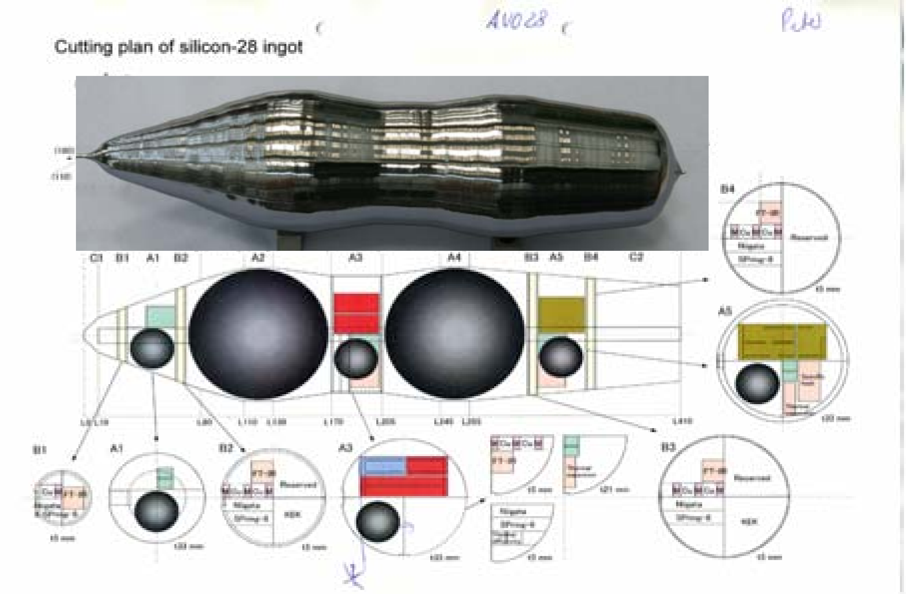
This fulfills our conceptual requirement

Wanted: an experiment to measure F_2 to about 10^{-8} (10 µg/kg), where

$$\frac{h}{m(\mathcal{K})} = F_2\left(\nu_{\text{Cs-hfs}}, c, \ldots\right)$$

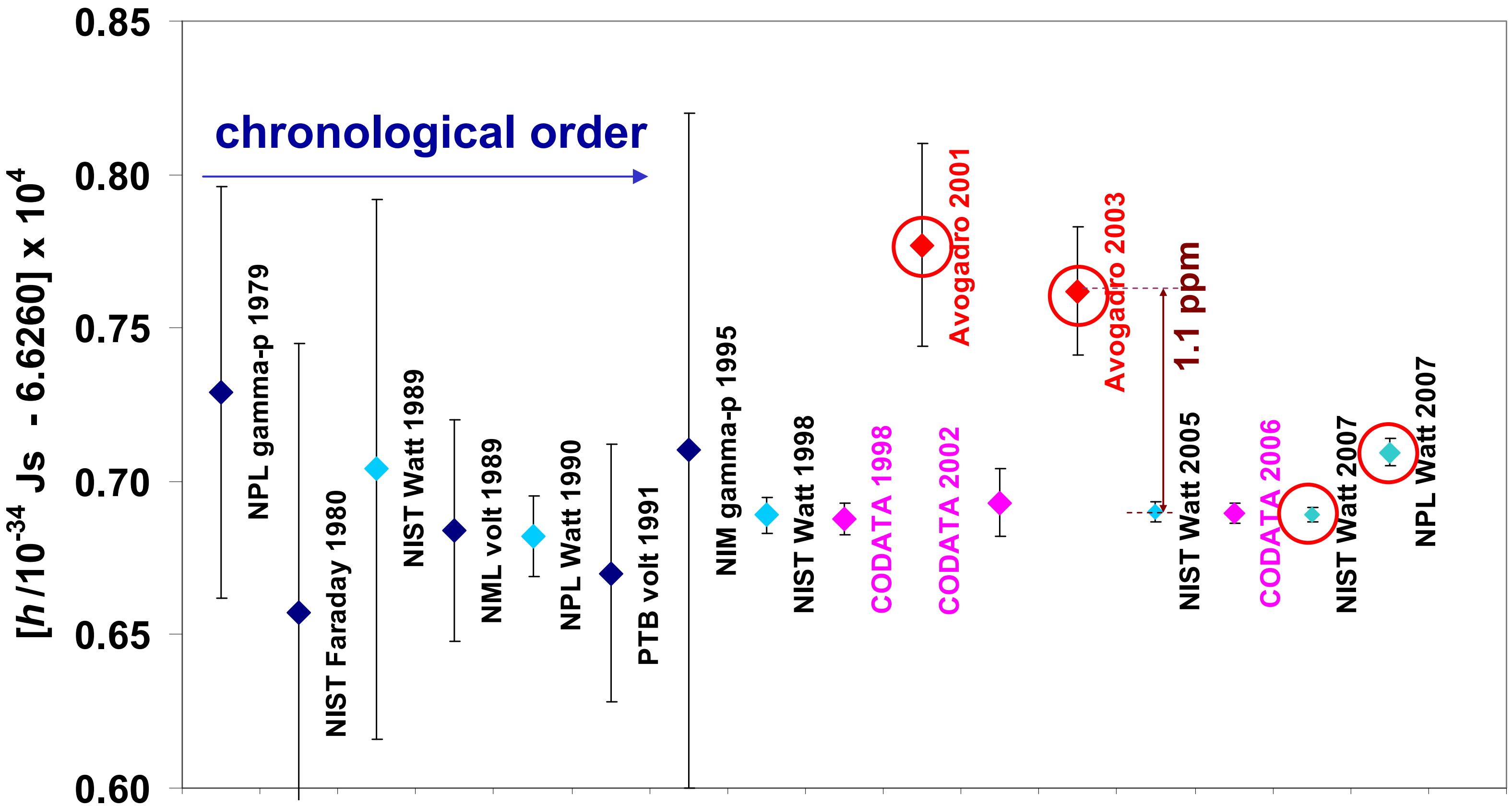


watt balance at NIST



cutting plan for 5 kg silicon-28 ingot

History of measurements of the Planck constant



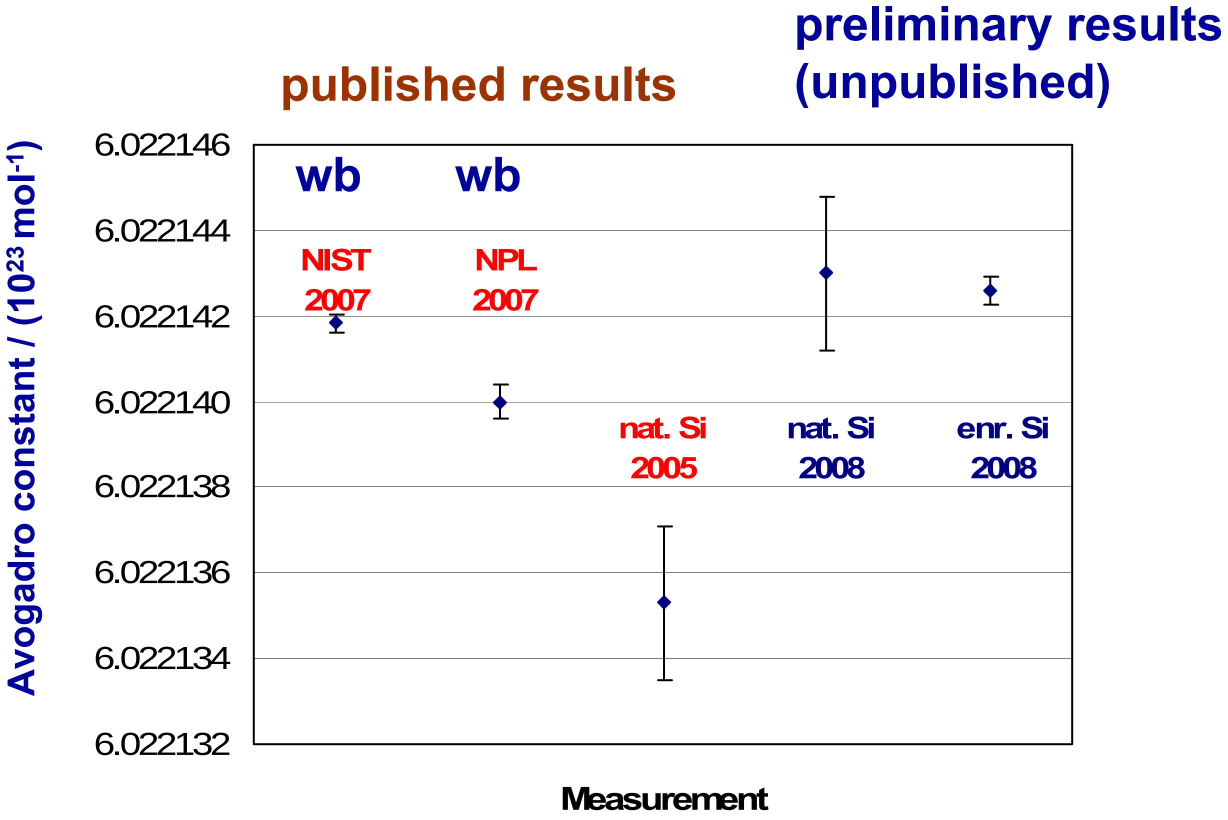
N_A and h are related by

$$N_A\, h = 0.012 \frac{m_e}{m\left({}^{12}\mathrm{C}\right)} \frac{c\alpha^2}{2R_\infty} \qquad u_r\left(N_A\, h\right) = 1.4\times10^{-9}$$

uncertainty of $N_A h$ dominated by contribution of α^2

What are the most recent results ?

Data provided by P. Becker, PTB



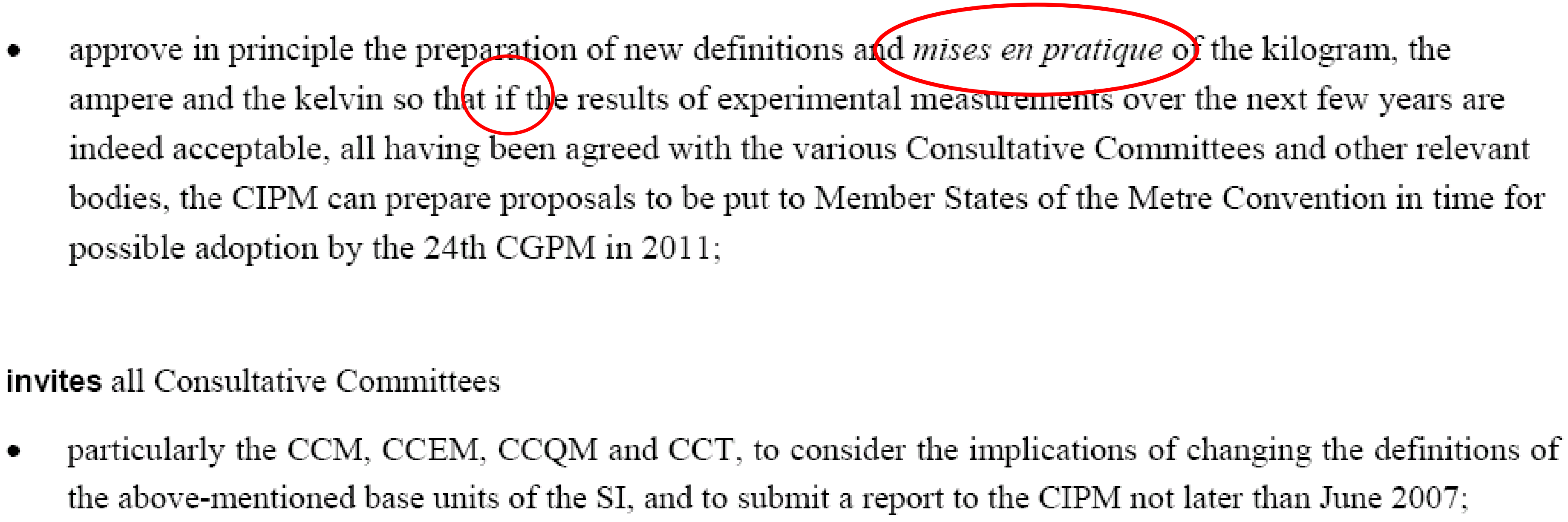
Note: Average observed drift of prototypes over 30 years would fit within the smallest of the uncertainty bars (NIST 2007).

Steps to redefine the kilogram

CCU, CCM, and other consultative committees send proposals/counter proposals to the CIPM.

The CIPM takes action, or it does not.

In 2005, the CIPM approved a Recommendation whose major points for mass metrology are:



- approve in principle the preparation of new definitions and *mises en pratique* of the kilogram, the ampere and the kelvin so that if the results of experimental measurements over the next few years are indeed acceptable, all having been agreed with the various Consultative Committees and other relevant bodies, the CIPM can prepare proposals to be put to Member States of the Metre Convention in time for possible adoption by the 24th CGPM in 2011;

invites all Consultative Committees

- particularly the CCM, CCEM, CCQM and CCT, to consider the implications of changing the definitions of the above-mentioned base units of the SI, and to submit a report to the CIPM not later than June 2007;

Some consequences for everybody

- There will be no discontinuity in the kilogram, therefore no immediate consequences to measurements.
- The relative uncertainties of all mass standards, including the international prototype, will have an additional (but identical) component.
- Because the relative uncertainty component is exactly the same for all mass standards and all masses derived from mass standards, this component does not increase the uncertainty of comparisons between mass standards.
- Therefore, the consequence for end users will be negligible.
- Nevertheless, we must forge the strongest possible experimental links to a new definition to ensure that values of macroscopic masses remain traceable to the SI to sufficient accuracy.
- It seems unlikely that the public will understand the new definition of the kilogram. (This is a challenging problem in communications.)

Resolution of the CGPM, November 2007 - 1

[CGPM: Conférence Générale des Poids et Mesures; meets every 4 yrs.]

…

The CGPM notes

- That any changes to the definitions of the SI units must be self-consistent.
- That it is desirable that definitions of the base units should be easily understood.

…

- **The importance of soliciting comments and contributions from the wider scientific and user communities.**

…

Resolution of the CGPM, November 2007 - 2

The CGPM Recommends that NMIs and the BIPM

- pursue the relevant experiments so that the CIPM can decide whether it may be possible to redefine the kg, A, K and mol using fixed values of the fundamental constants at the time of the 24th CGPM (2011).
- together with CIPM and appropriate Working Groups, work on practical ways of realizing the new definitions (MeP) and consider the most appropriate way of explaining the new definitions to users.
- initiate awareness campaigns to alert user communities to the possibility of redefinitions (technical and legislative implications, MeP)…

Requests the CIPM to report to the 24th CGPM in 2011 and to undertake whatever preparations are considered necessary so that, if the results of experiments are found to be satisfactory and the needs of users met, formal proposals for changes to the definitions of the kg, A, K, and mol can be put to the 24th CGPM.

Summary

- Artefacts are inherently unstable with respect to fundamental physical constants.
- The masses of “identical” 1 kg Pt-Ir artefacts are not stable with respect to each other.
- There is no experimental evidence (so far) that $m_{\mathcal{K}}$ is changing with respect to the fundamental physical constants.
- Inconsistent results between Watt Balance and XRCD values of h (or N_{A}) are nearly resolved.
- Two most recent Watt Balance results disagree by 3×10^{-7}.
- An immediate redefinition of the kilogram would benefit electrical metrology and some important areas of physics.
- Many groups are active in this research and progress is being made.

Conclusion

There is much to do in the next two years if we are to redefine the kilogram in a useful way for everybody.

Watt Balance details

$$m_{\mathcal{K}} = \frac{U'U}{Rgv}$$

In SI

$$m_{\mathcal{K}} = h\,\frac{f_1 f_2}{f_3 c^2}\left\{\frac{n_1 n_2}{4i\beta\gamma}\right\}$$

$$g = \beta c f_3$$

$$v = \gamma c$$

$$U = \frac{nf}{K_{\mathrm{J}}} = \frac{nf}{\left(\frac{2e}{h}\right)}$$

$$U_{90} = \frac{nf}{K_{\text{J-90}}}$$

$$m_{\mathcal{K}} = \frac{U'_{90}\,U_{90}}{R_{90}\,gv}\cdot\frac{K_{\mathrm{J}-90}{}^{2}}{\left(\frac{2e}{h}\right)^{2}}\,\frac{R_{\mathrm{K}-90}}{\frac{h}{e^{2}}} = h\,\frac{U'_{90}\,U_{90}}{R_{90}\,gv}\left\{\frac{K_{\mathrm{J}-90}{}^{2}R_{\mathrm{K}-90}}{4}\right\}$$